VIJAY PANDHARIPANDE on NUCLEAR MATTER and NUCLEAR FORCES

Robert Wiringa, Physics Division, Argonne National Laboratory

Work supported by U.S. Department of Energy, Office of Nuclear Physics

Vijay's Work

Vijay authored or co-authored 155 papers in refereed scientific journals from 1963-2006.

Those papers have garnered more than 8,000 citations to date (> 50/paper).

He also wrote or contributed to $\sim$ 40 conference proceedings , as well as giving a huge number of invited talks at conferences, colloquia, and seminars around the world.

Perhaps most importantly, he supervised 15 Ph.D. theses (with a few more yet to be completed).

VIJAY'S PUBLICATIONS

His publications can be divided (arbitrarily) into five periods:

1. The Young Experimentalist (1963-1968)
   7 experimental papers from the Tata Institute with two to four authors/paper
2. The Young Theorist (1967-1973)
   16 theoretical papers from Tata and from visits to Niels Bohr Institute and Cornell
   11 are single-author papers, 4 two-author
3. The Young Teacher (1973-1981)
   33 papers beginning his career at Illinois
   5 are single-author (last one in 1978); 22 are two-author with a postdoc or student
4. Chief Colleague (1982-1991)
   44 papers including many with both students and young visiting colleagues
   14 are two-author, 26 three-author, 4 four-or-more-author
5. Leader of the Pack (1992-2006)
   58 papers with increasingly larger collaborations
   16 are two-author, 20 three-author, 12 four-author, 10 five-or-more-author

Of the 132 papers from Urbana, 67 (half) were with students; student's name first on 62 of them.

VIJAY'S THESIS STUDENTS

1. Robert Bruce Wiringa (1978) *A variational theory of nuclear matter*
2. Kevin Edward Schmidt (1979) *Variational theory of quantum fluids*
3. Isaac Lagaris (1981) *Nuclear matter with realistic Hamiltonians*
4. Joseph Allen Carlson (1983) *Few body problems in nuclear and particle physics*
5. Efstratios Manousakis (1985) *On the microscopic theory of liquid* ^4He
6. Rocco Schiavilla (1987) *Monte Carlo studies of momentum distributions and longitudinal response functions of A=3 and 4 nuclei*
7. Thomas Jeffrey Schlagel (1990) *Classical models of heavy-ion collisions*
8. Aleksandar Belić (1992) *Deep inelastic scattering by quantum liquids*
9. Brian Scott Pudliner (1996) *blue's function Monte Carlo calculation of few nucleon systems*
10. Roger J. Loucks (1996) *Electro-pion production from p, d, and* ^3He
11. Jun L. Forest (1998) *Relativistic Hamiltonians and short range structure of nuclei*
12. Arya Akmal (1998) *Variational studies of nucleon matter with realistic potentials*
13. Mark Wayne Paris (2000) *Quantum Monte Carlo calculations of three and six-quark states*
14. Shannon Tracy Cowell (2004) *Quenching of weak interactions in nucleon matter*
15. Soon Yong Chang (2006) *Study of the properties of dilute Fermi gases in the strongly interacting regime*

Early Work

Vijay began his scientific life as an experimental research assistant at the Tata Institute at age 21. He co-authored seven experimental papers on nuclei in the A=115-125 region from 1963-1968:

Energy levels of Te^{125}

Girish Chandra and V. R. Pandharipande, Nucl. Phys. **46**, 119 (1963)

⋮

Level structure of ^{117}In *from the decay of* ^{117}Cd

V. R. Pandharipande, K. G. Prasad, R. P. Sharma and B. V. Thosar, Nucl. Phys. **A109**, 81 (1968)

Thosar was his supervisor at Tata and also uncle to Rajeshwari Sinha. He played an important role in making sure Raj and Vijay met; that's what we call full-service advising!!

Vijay had an itch to do theoretical work, however, and wrote his first theoretical paper in 1967:

An effective, residual interaction for shell-model calculations

V. R. Pandharipande, Nucl. Phys. **A100**, 449 (1967)

Here he used a modified δ-function form to calculate spectra in nuclei from ^{18}O to ^{210}Bi. Already he was looking for a universal nuclear Hamiltonian.

Later given out as homework assignment in Urbana in 1973.

EARLY RECOGNITION

Several more single-author theoretical papers followed, including

On the calculation of core-polarization effects

V. R. Pandharipande, Phys. Lett. **27B**, 199 (1968)

On the calculation of nuclear compressibility

V. R. Pandharipande, Phys. Lett. **31B**, 635 (1970)

which brought him to the attention of Hans Bethe and others.

Bethe arranged for him to come to the Niels Bohr Institute for one year in 1970-71.

Others present at that time included Gordon Baym, Gerry Brown, Franco Iachello, Mikkel Johnson, John Negele, Chris Pethick, David Pines, and Phil Siemens:
apparently a very exciting and productive year.

Neutron Stars

Pulsars discovered and identified as neutron stars in 1967.

Vijay interested in dense matter equation of state from bare NN forces.

Lowest-order Brueckner theory (LOBT) limited to densities $\rho \leq 0.6$ fm^{-3}.

Starts developing lowest-order constrained variation (LOCV) method to reach higher ρ.

Construct Jastrow wave function with correlation f and model wave function Φ

$$\Psi(1...N) = \prod_{i<j} f_{ij} \Phi(1...N)$$

and evaluate variational energy:

$$E_V = (\Psi, H\Psi)/(\Psi, \Psi)$$

Good f obtained by minimizing two-body cluster contribution to E_V

$\Rightarrow$ solve "Schrödinger" equation (in partial-wave expansion) for $u_l = f_l j_l$

$$-\frac{\hbar^2}{m}[\frac{d^2 u_l}{dr^2} - \frac{l(l+1)}{r^2} u_l] + v_l u_l = [\frac{\hbar^2}{m} k^2 + \lambda_l(k)] u_l$$

with λ_l determined by boundary condition on f — at some distance d the wave function "heals" such that the volume integral of correlation is unity — on average only nearest neighbors are correlated and distant neighbors contribute only to the average field λ.

These ideas developed in series of papers submitted from NBI and Tata after his return:

Dense neutron matter with realistic interactions (paper #0 to read as a student!)

V. R. Pandharipande, Nucl. Phys. **A174**, 641 (1971)

Neutron matter computations in Brueckner and variational theories

P. J. Siemens and V. R. Pandharipande, Nucl. Phys. **A173**, 561 (1971)

Hyperonic matter (mixtures of N, Λ, Σ, Δ, and leptons)

V. R. Pandharipande, Nucl. Phys. **A178**, 123 (1971)

Variational calculation of nuclear matter

V. R. Pandharipande, Nucl. Phys. **A181**, 33 (1972)

NN potentials used: Reid soft-core, Hamada-Johnston, Bressel-Kerman-Rouben.
HJ (1962) contained one-pion-exchange (OPE) + state-independent infinite hard-core at 0.49 fm; intermediate-range Yukawas with 28 parameters to fit data in different (S, T) states.

$$V = V_C + V_T S_{12} + V_{LS}(\mathbf{L} \cdot \mathbf{S}) + V_{LL} L_{12}$$

BKR (1968) used same structure, but replaced hard core with finite square-wells at 0.7 fm.
Reid SC (1968) made V_C, V_T, V_{LS} from sum of Yukawas, different in every LSJ partial wave ($J \leq 2$); smoother behavior (∞ at r=0 but finite integral) but inconsistent operator structure.

Vijay returned to Tata in 1971, but Bethe and Baym recruited him to come to the U.S. in 1972, first to Cornell as a research associate, and then to Urbana as an assistant professor.
At Cornell he and Bethe released the constraint in LOCV and summed higher-order terms in E_V with the hypernetted chain (HNC) equation of van Leeuwen, Groeneveld, and de Boer (1959).

Variational method for dense systems

V. R. Pandharipande and H. A. Bethe, Phys. Rev. C **7**, 1312 (1973)

Treated atomic helium interacting by Lennard-Jones potential; Bose ^{4}He by HNC, HNC/4

$$ln(\frac{g_{mn}}{f_{mn}^2 e^{E_{mn}}}) = \rho \int [g_{m1} - 1 - ln(\frac{g_{m1}}{f_{m1}^2 e^{E_{m1}}})](g_{n1} - 1) d^3 r_1$$

Fermi ^{3}He added exchange diagrams in cluster expansion; PB kinetic energy introduced.
Conclusion: HNCV method worked well for helium and simple neutron gas model.
(Helium liquids much denser than nuclear matter; LOBT does not work well because $\kappa > 0.5$)

I arrived in Urbana in 1972; in spring 1973 signed up for course: "Introduction to Nuclear Physics with Astrophysical Applications" being given by a young neutron-star hotshot. Vijay turned out to be a very gifted teacher, and the course was informative, challenging, and fun.

VIJAY CROSSING THE PUDDLE

Workshops and Conferences

Workshop on Dense Matter organized by Vijay in Urbana in 1973.

Important question: Can neutron matter solidify?

Pines, *et al.* thought pulsar "glitches" showed stars had solid cores.

Nosanow, Canuto, others produced supporting calculations; Vijay said "NO!"

Solidification of neutron matter

V. R. Pandharipande, Nucl. Phys. **A217**, 1 (1973)

Bethe suggested "homework" problem to sort things out:

Boltzmann neutrons interacting with repulsive core of Reid $V(^1S_0)$.

After people did their homework, they agreed neutron matter did not solidify.

XVI Solvay Conference on Physics on "Astrophysics and Gravitation", Brussels, Sept. 1973

Dozen lecturers including Bahcalls, Cameron, Giaconni, Pines, Wheeler.

Vijay lectured on "Physics of High Density and Nuclear Matter."

After discussing solidification question, he listed several subjects needing more work:

1) minimal relativity, 2) $NN - N\Delta$ coupled-channel problem, 3) three-body forces, and

4) pion condensation; all things he would proceed to investigate.

$NN - N\Delta$ COUPLED-CHANNELS

Neutron matter might solidify if neutrons were spin-aligned in layers, due to tensor correlation, forming a π^0 condensate. NN T=1 tensor too weak, but taking Δs into account might do it... Starting from ideas of Sugawara and von Hippel (1968) and Green and Haapakoski (1974), Vijay and Roger Smith investigated the addition of transition potentials:

$$V_{NN\to N\Delta}(r) = \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi}\left(\frac{m_\pi c^2}{3}\right)[Y_\pi(r)\sigma_1 \cdot \mathbf{S}_2 + T_\pi(r) S_{12}^{II}](\tau_1 \cdot \mathbf{T}_2)$$

$$V_{NN\to \Delta\Delta}(r) = \frac{f_{\pi N\Delta}^2}{4\pi}\left(\frac{m_\pi c^2}{3}\right)[Y_\pi(r)\mathbf{S}_1 \cdot \mathbf{S}_2 + T_\pi(r) S_{12}^{III}](\mathbf{T}_1 \cdot \mathbf{T}_2)$$

A model neutron solid with π^0 condensate

V. R. Pandharipande and R. A. Smith, Nucl. Phys. **A237**, 507 (1975)

Nucleon-nucleon potentials including $\pi N\Delta$ coupling effects

R. A. Smith and V. R. Pandharipande, Nucl. Phys. **A256**, 327 (1976)

They demonstrated that a generalized OPE potential would represent the time-ordered diagrams for a nonrelativistic interaction Lagrangian: box diagrams approximated the 2nd Born terms to $\sim$10%, while cross-box diagrams largely canceled out. The simple short-range behavior they added did not fit NN data very well, but this work paved the way for subsequent potentials.

The Crisis in Nuclear Matter

I started working with Vijay in 1974, applying HNCV to nuclear matter and letting the pair correlation f_{ij} vary without constraint. Results came out significantly below LOCV and LOBT. Vijay decided we should solve some more homework potentials and got Ben Day to help:

Do lowest-order approximations adequately describe nuclear matter? (submitted same day as SP paper)

V. R. Pandharipande, R. B. Wiringa, and B. D. Day, Phys. Lett. **57B**, 205 (1975)

This helped set off a "crisis" in nuclear matter (which John Clark had already warned about.) Vijay and I continued to develop our methods in a series of papers:

A variational theory of nuclear matter

V. R. Pandharipande and R. B. Wiringa, Nucl. Phys. **A266**, 269 (1976)

A variational theory of nuclear matter (II)

R. B. Wiringa and V. R. Pandharipande, Nucl. Phys. **A299**, 1 (1978)

A variational theory of nuclear matter (III)

R. B. Wiringa and V. R. Pandharipande, Nucl. Phys. **A317**, 1 (1979)

If the titles look unimaginative, remember Bohm and Pines took papers I-IV for the electron gas!

NUCLEAR MATTER CONTINUED

In 1975 Bethe visited and pointed us to new work by young Italians Fantoni and Rosati: using FOUR coupled equations the antisymmetry from exchange diagrams in hypernetted chains could be handled exactly — Fermi HNC (FHNC)!
New student Kevin Schmidt assigned to read their paper and report back.

FHNC incorporated into paper I above, then a diagrammatic cluster expansion and single-operator chain (SOC) equations for operator correlations $F_{ij} = \sum_p f_p(r_{ij}) O^p_{ij}$ in II.
By paper III we felt we had reliable upper bounds for static v_6 potentials:

$$V = \sum_p v_p(r_{ij}) O^p_{ij} \qquad O^p_{ij} = 1, \tau_i \cdot \tau_j, \sigma_i \cdot \sigma_j, (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j), S_{12}, S_{12}(\tau_i \cdot \tau_j)$$

We projected v_6 models from Reid and BJ-II and started to add $\mathbf{L} \cdot \mathbf{S}$ terms for v_8 models.
Nuclear matter did not saturate at the empirical energy and density with these interactions.

Intermediate results were presented at

This was followed by the first "Recent Progress in Many-Body Theories" meeting, organized by Fantoni and Rosati at the ICTP, Trieste, Oct. 1978.

They invited me not knowing I was only a student; Vijay and I flew to Trieste one week after my defense; sat in smoking section :(and edited and extended the thesis into review article:

Variations on a theme of nuclear matter (Einstein approved!)

V. R. Pandharipande and R. B. Wiringa, Rev. Mod. Phys. **51**, 821 (1979)

I went to Los Alamos as a postdoc with Mikkel Johnson (arranged by Baird Brandow). Worked on extending FHNC/SOC to include explicit Δs via transition potentials and thus build in many-body forces. However, available models provided far too much saturation in nuclear matter.

Kevin finished thesis on helium liquids; also found neat trick for matter at finite temperature:

Variational theory of nuclear matter at finite temperatures

K. E. Schmidt and V. R. Pandharipande, Phys. Lett. **87B**, 11 (1979)

A New Hamiltonian ...

Isaac Lagaris joined group in 1978 and worked on adding spin-orbit terms to calculation:

Variational calculations of v_8 models of nuclear matter

I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A334**, 217 (1980)

Vijay and Isaac tried to use new parametrized Paris potential (1980) which had 14 operator terms, including four p^2-dependent and two quadratic spin-orbit terms. However, the strong momentum dependence was tricky to handle and they did not trust their results enough to publish. Instead, they made their own Urbana v_{14} potential:

$$V = v_\pi + v_I + v_S = v_\pi + \sum_p [I^p T_\pi^2(r) + S^p W(r)] O^p$$

$$O_{ij}^{p=1,14} = [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2 \sigma_i \cdot \sigma_j, (\mathbf{L} \cdot \mathbf{S})^2] \otimes [1, \tau_i \cdot \tau_j]$$

The $T_\pi^2(r)$ provided two-pion-exchange (TPE) range, while Woods-Saxon core was large ($\sim$ 2.5 GeV) but finite. Phase shifts fit by eye (!!) for $J \leq 4$ with 14 I^p and 14 S^p parameters.

Phenomenological two-nucleon interaction operator

I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A359**, 331 (1981)

… AND NEW MATTER RESULTS

Variational calculations of realistic models of nuclear matter

I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A359**, 349 (1981)

With Urbana v_{14}, nuclear matter had about the right binding, $E_0 = -16$ MeV,
but the saturation density was nearly double the empirical value of ρ_0 = 0.16 fm^{-3}.
Possible solution: add some form of three-nucleon interaction (TNI):

$$v_{14} + \mathrm{TNR} = v_\pi + v_I \exp(-\gamma_1 \rho) + v_S \qquad \mathrm{TNA} = \gamma_2 \rho^2 exp(-\gamma_3 \rho)[3 - 2(\frac{N-Z}{A})^2]$$

and adjust γ_i to get empirical saturation energy and density!

Barry Friedman and Vijay used Urbana v_{14} + TNI and Kevin's finite temperature idea to construct dense matter equation of state (EOS) for T = 0–20 MeV:

Hot and cold, nuclear and neutron matter (Best-seller with $>$ 500 citations!)

B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981)

TNI made EOS stiffer, producing larger maximum neutron star mass, fatter stars
with lower central density for a given mass.

Friedman-Pandharipande EOS on top for many years!

Isaac finished thesis with a study of asymmetric matter, demonstrating an effective way of interpolating between symmetric nuclear matter (SNM) and pure neutron matter (PNM) for arbitrary asymmetry $\beta = (\rho_n - \rho_p)/\rho$:

$$E(\rho, \beta) = T_F(\rho, \beta) + V_0(\rho) + \beta^2 V_2(\rho)$$

$$V_0(\rho) = E_{\rm SNM} - T_F(\rho, 0) \;\; ; \;\; V_2(\rho) = E_{\rm PNM} - T_F(\rho, 1) - V_0(\rho)$$

Variational calculations of asymmetric nuclear matter

I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A369**, 470 (1981)

Many visitors and postdocs came to Urbana in early 1980s, including Stefano Fantoni, Bengt Friman, and Qamar Usmani, to work on pion production from hot matter, the single-particle potential and nucleon optical potential, and various problems in liquid helium, which I will have to pass over. Barry Friedman decided he did not like quantum mechanics and switched to another advisor to do classical physics (The Billiard Problem)!

Light Nuclei

New project initiated in 1979 by Vijay with Jorge Lomnitz-Adler: adapt generalized Jastrow correlations to finite nuclei — might scale to larger nuclei better than Faddeev methods.
Roger Smith and Joe Carlson developed a Metropolis Monte Carlo (VMC) algorithm.
Vijay was intrigued by the "hole" in the central density of ^{3}He observed in electron scattering experiments; he and Joe formulated several V_{ijk} models - Urbana I, II, and III - to force a hole.
(It later became clear that the "hole" was an effect of two-body current operators.)

A simple and realistic triton wave function

J. Lomnitz-Adler and V. R. Pandharipande, Nucl. Phys. **A342**, 404 (1980)

Monte Carlo calculations of triton and ^{4}He *nuclei with the Reid potential*

J. Lomnitz-Adler, V. R. Pandharipande, and R. A. Smith, Nucl. Phys. **A361**, 399 (1981)

A study of three-nucleon interaction in three- and four-body nuclei

J. Carlson and V. R. Pandharipande, Nucl. Phys. **A371**, 301 (1981)

I moved to Argonne in 1981 as a postdoc. When FP paper appeared, Fritz Coester complained to me about the density-dependent TNI. He wanted a direct evaluation of real V_{ijk}.
I realized we had the tools — a $g_3(r_i, r_j, r_k)$ *in FHNC/SOC approximation — to do that in matter. I tested Urbana model III and found it blew matter apart!*

THREE-BODY FORCES

Discussions with Sid Coon about Tucson-Melbourne TPE TNI inspired us to combine a simplified (Fujita-Miyaza) TPE and short-range TNR to make Urbana models IV, V, and VI:

$$V_{ijk}^{2\pi} = A_{2\pi}\left(\sum_{cyc}\{X_{ij}, X_{ik}\}\{\tau_i\cdot\tau_j, \tau_i\cdot\tau_k\} + \frac{1}{4}[X_{ij}, X_{ik}][\tau_i\cdot\tau_j, \tau_i\cdot\tau_k]\right)$$

$$X_{ij} = Y_\pi(r_{ij})\sigma_i\cdot\sigma_j + T_\pi(r_{ij})S_{ij}$$

$$V_{ijk}^{R} = U_0\sum_{cyc} T_\pi^2(r_{ij})T_\pi^2(r_{ik})$$

$A_{2\pi}, U_0$ adjusted to give ^{3}H binding in VMC and nuclear matter ρ_0 in FHNC/SOC calcs:

Three-nucleon interaction in 3-, 4-, and ∞-body systems

J. Carlson, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A401**, 59 (1983)

Joe also developed a modified R-matrix method for NA scattering at low energies:

Variational calculations of resonant states in ^{4}He

J. Carlson, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A424**, 47 (1984)

ARGONNE NN POTENTIALS

Existing NN potentials with explicit Δs had deficiencies, e.g., SP was too simple to give good fit to data. We needed good phase-equivalent potentials with and without Δs to really test their effect in matter. So Roger Smith, Tom Ainsworth, and I constructed Argonne v_{14} and v_{28}:

Nucleon-nucleon potentials with and without $\Delta(1232)$ degrees of freedom

R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984)

Argonne v_{14} used exactly the same operator structure and radial shapes as Urbana v_{14}.
28 parameters I^p, S^p fit to SAID WI81 np phase shifts by computer:

```
x=(138.03/197.33)*r   ;  cutoff=1-exp(-2.*r**2)
ypi=cutoff*exp(-x)/x  ;  tpi=cutoff*(1+3/x+3/x**2)*ypi
tpi2=tpi*tpi          ;  ws=1/(1+exp((rr-.5)/.2))
v(1)=          -4.801125*tpi2+2061.5625*ws
v(2)=            .798925*tpi2 -477.3125*ws
v(3)=           1.189325*tpi2 -502.3125*ws
v(4)=3.72681*ypi +.182875*tpi2  +97.0625*ws
v(5)=             -.1575*tpi2 +108.75*ws
v(6)=3.72681*tpi   -.7525*tpi2 +297.25*ws
v(7)=              .5625*tpi2 -719.75*ws
v(8)=              .0475*tpi2 -159.25*ws
v(9)=            .070625*tpi2   +8.625*ws
v(10)=          -.148125*tpi2   +5.625*ws
v(11)=          -.040625*tpi2  +17.375*ws
v(12)=          -.001875*tpi2  -33.625*ws
v(13)=            -.5425*tpi2 +391.*ws
v(14)=             .0025*tpi2 +145.*ws
```

Argonne v_{28} used same NN form plus all possible couplings to one- and two-Δ states with $f_{\pi N\Delta} = 2f_{\pi NN} = 10f_{\pi\Delta\Delta}$ plus diagonal $V^c_{N\Delta\to N\Delta}(r) = V^c_{\Delta\Delta\to\Delta\Delta}(r) = V^c_{NN\to NN}(r)$.
The 28 I^p, S^p parameters were then refit to make AV28 phase-equivalent to AV14.

AV28 proved difficult to use and gave disappointing results:
In FHNC/SOC calculations of nuclear matter it saturated at $E = -5$ MeV, $\rho = 0.7\rho_0$.
Picklesimer, Rice, and Brandenburg found $E(^3\mathrm{H}) = 7.14$ MeV. (Took 5 papers!)
However, it was a useful learning exercise, the Δs were used later in electroweak transitions, and AV14 became a popular NN potential.

In 1988, Adelchi Fabrocini, Vitaly Fiks, and I updated the FP EOS using UV14 and AV14, with and without UVII V_{ijk}, using latest FHNC/SOC methods, for β-stable matter.
Obtained fair nuclear matter saturation with V_{ijk}, neutron stars with $M_{\max} \sim 2.1 M_\odot$, evidence for π^0-condensate in neutron matter for $\rho > 2\rho_0$.

Equation of state for dense nucleon matter (My intended thesis - 10 years late!)

R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988)

Fast Forward

A great deal of work by Vijay, Joe, Stratos Manousakis, Steve Pieper, Dan Lewart and others on a range of nucleon, nuclear, and quantum-liquid calculations in late 1980s - early 1990s. Some notable finite nuclei papers to be mentioned:

Momentum distributions in $A = 3$ and 4 nuclei

R. Schiavilla, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A449**, 219 (1986)

Initiated a major effort on electron scattering, form factors, exchange current contributions, etc., in light nuclei.

Alpha particle structure

J. Carlson, Phys. Rev. C **38**, 1879 (1988)

First Green's function Monte Carlo (GFMC) and first accurate solution of four-nucleon system with realistic potentials.

Variational calculation of the ground state of ^{16}O

Steven C. Pieper, R. B. Wiringa, and V. R. Pandharipande, Phys. Rev. C **46**, 1741 (1992)

A semi-successful attempt to solve a larger nucleus which gave some important physical insight.

NEW NN POTENTIALS

In 1993 Nijmegen group published new partial-wave analysis fitting 4300 pp and np data for $E_{\rm lab} < 350$ MeV with $\chi^2 = 1$. Also produced three new partial-wave potentials, NijmI, NijmII, and Reid93, that gave equally good fits to data — required $v^N_{pp}(^1S_0) \neq v^N_{np}(^1S_0)$

Variational and GFMC methods geared to operator format $\Rightarrow$ need new Argonne potential!

Appeal to Nijmegen for assistance brought Vincent Stoks into collaboration.
Result was Argonne v_{18} potential with full electromagnetic potential, charge-dependent OPE, more general short-range parametrization, four new charge-independence-breaking operators:

$$V = v_{\rm EM} + v_\pi(NN) + \sum_{p=1,18} \{I^p T^2_\pi(r) + [P^p + Q^p r + R^p r^2] W(r)\} O^p_{ij}$$

$$O^{p=15,18}_{ij} = T_{ij}, (\sigma_i \cdot \sigma_j) T_{ij}, S_{ij} T_{ij}, (\tau_{zi} + \tau_{zj})$$

Accurate nucleon-nucleon potential with charge-independence breaking

R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995)

AV18 had 40 adjustable parameters to obtain $\chi^2 = 1.1$ and consistent two-body exchange currents constructed by Rocco to compute deuteron observables, including A, B, T_{20}.
Urbana V_{ijk} updated to model IX (UIX) to fit ^{3}H and ^{4}He binding when used with AV18.

FORWARD TO THE p-SHELL!

Joe had developed GFMC for ^{4}He, Steve was showing us how to do parallel computing on new IBM SP1, and I had improved VMC and timed ^{6}Li. Vijay saw that we could combine all this for the first accurate p-shell calculations with realistic interactions and set Brian Pudliner to work:

Quantum Monte Carlo calculations of $A \leq 6$ nuclei

B. S. Pudliner, V. R. Pandharipande, J. Carlson, and R. B. Wiringa, Phys. Rev. Lett. **74**, 4396 (1995)

Quantum Monte Carlo calculations of nuclei with $A \leq 7$

B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. C **56**, 1720 (1997)

Vijay was awarded the 1999 Tom W. Bonner Prize in Nuclear Physics
"For fundamental contributions in determining the structure of light nuclei by solving the Schroedinger problem with more than three nucleons using realistic nucleon-nucleon interactions supplemented by three-body forces."

Steve took the lead in pushing the calculations to A=8,9,10 with important algorithmic developments (constrained path) from Joe and an important assist from Kalman Varga. Calculations of ^{12}C now being done with new trial function from Vijay.

Team Leader and Team

DOES RELATIVITY MATTER?

Vijay, Jun Forest, and Jim Friar studied relativistic kinetic energy and Lorentz invariance in:

Relativistic nuclear Hamiltonians

J. L. Forest, V. R. Pandharipande, and J. L. Friar, Phys. Rev. C **52**, 568 (1995)

Variational Monte Carlo calculations of ^{3}H *and* ^{4}He *with a relativistic Hamiltonian*

J. L. Forest, V. R. Pandharipande, J. Carlson, and R. Schiavilla, Phys. Rev. C **52**, 576 (1995)

At two-body level relativistic Hamiltonian fit to NN data behaves same as nonrelativistic one fit to the same data. At three-body level "boost" correction δv appears, which light nuclei calculations showed scaling like fraction of TNR component of Urbana V_{ijk}.

Boost and TNR scale differently with density, so explicit treatment needed for matter.
New EOS studies made with AV18+δv+UIX* by Arya Akmal, Vijay, and Geoff Ravenhall:

Spin-isospin structure and pion condensation in nucleon matter

A. Akmal and V. R. Pandharipande, Phys. Rev. C **56**, 2261 (1997)

Equation of state of nucleon matter and neutron-star structure

A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998)

APR is the current best microscopic EOS available; produces neutron star $M_{\rm max} \sim 2.2 M_\odot$.

New Three-Body Forces

In the p-shell, AV18+UIX was found to underbind ^{8}Be by 2 MeV, ^{8}He by 4 MeV, with too small spin-orbit splitting between various excited states. Vijay led search for additional TNI terms and Steve did much testing. Eventually they found three-pion rings with intermediate Δs could solve the problem, by differentiating between $T = \frac{1}{2}$ and $T = \frac{3}{2}$ triples.

Realistic models of pion-exchange three-nucleon interactions

Steven C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001)

Adding two-pion s-wave scattering from chiral theory, new Illinois TNI has form:

$$V_{ijk} = A_{2\pi}^{PW} O_{ijk}^{2\pi,PW} + A_{2\pi}^{SW} O_{ijk}^{2\pi,SW} + A_{3\pi}^{\Delta R} O_{ijk}^{3\pi,\Delta R} + A_R O_{ijk}^R$$

Five different models IL1-IL5 constructed by fitting four strengths A_x and one cutoff factor to 17 ground or narrow excited states in $A \leq 8$ nuclei.

AV18+IL2 reproduces 38 narrow states in $A \leq 10$ nuclei with rms deviation of 0.75 MeV.

Very recent A=5 studies by Ken Nollett show AV18+IL2 describes $n\alpha$ scattering remarkably well, including splitting between $p_{3/2}$ and $p_{1/2}$ states.

FINAL HAMILTONIAN STUDIES

Vijay returned to question: what is best representation for velocity dependence in NN force? We investigated using meson-theory preferred p^2 instead of our standard L^2 operators to make AV18pq potential phase-equivalent to AV18:

$$v(r,\mathbf{p}^2) = v^c(r) + \mathbf{p}^2 v^{p2}(r) + v^{p2}(r)\mathbf{p}^2 \qquad \mathbf{p}^2 = -\left[\frac{1}{r}\frac{d^2}{dr^2}r - \frac{\mathbf{L}^2}{r^2}\right]$$

Quadratic momentum dependence in the nucleon-nucleon interaction

R. B. Wiringa, A. Arriaga, and V. R. Pandharipande, Phys. Rev. C **68**, 054006 (2003)

Generated p^2 terms weaker than L^2 and slightly larger wave functions at short distance, but no significant change in binding in VMC calculations of A=3,4 nuclei.

Vijay was writing textbook on nuclear forces and light nuclei during much of his last 18 months. In discussing recent chiral effective theories he asked why they predicted stronger TPE TNI than needed in Urbana/Illinois models? Investigation with Dan Phillips and Bira van Kolck found chiral models overestimate strength unless Δ treated carefully.

Delta effects in pion-nucleon scattering and the strength of the two-pion-exchange three-nucleon interaction

V. R. Pandharipande, D. R. Phillips, U. van Kolck, Phys. Rev. C **71**, 064002 (2005)

This was his last publication on the nuclear Hamiltonian.

Final Matter Studies

Vijay continued to work to refine the variational chain-summation (FHNC/SOC) type of matter calculation; with Jaime Morales and Geoff Ravenhall, he borrowed techniques from the VMC finite nuclei calculations to make improvements:

Improved variational calculations of nucleon matter

J. Morales, Jr., V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **66**, 054308 (2002)

and with Joe Carlson, they checked GFMC simulations of neutron matter in a periodic box:

Quantum Monte Carlo calculations of neutron matter

J. Carlson, J. Morales, Jr., V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **68**, 025802 (2002)

He also started work with Shannon Cowell on studies of weak interactions in matter, particularly the effect of correlations as implemented through correlated basis perturbation theory:

Quenching of weak interactions in nucleon matter

S. Cowell and V. R. Pandharipande, Phys. Rev. C **67**, 035504 (2003)

⋮

Weak interactions in hot nucleon matter

S. Cowell and V. R. Pandharipande, Phys. Rev. C **73**, 025801 (2006)

These were the last of 37 publications on nuclear, neutron, nucleon, dense or hyperonic matter.

WHERE DO WE GO FROM HERE?

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

Nonrelativistic kinetic energy adequate for finite nuclei; boost correction needed in dense matter.
v_{ij} should fit elastic NN data with $\chi^2 \sim 1$.
V_{ijk} needs both long-range pion-exchange and short-range repulsion with isospin-dependence.

Next goal: more consistency between v_{ij}, V_{ijk}, and current operators.
Meson-exchange theory provided such a framework in past,
Chiral effective theories are reformulating this framework.
Need to remember π are gradient-coupled to N and Δ excitation is important!

Variational chain-summation calculations of matter are near their limit, but still produce best available EOS for astrophysics; finite-temperature EOS needs to be updated.
Long-term future of matter calculations probably lies with quantum Monte Carlo methods like AFDMC being developed by Kevin, Stefano, et al.

A Eureka Moment ...

AND THE ANSWER IS

Calculation of the probability P_n of finding n particles of a Fermi gas in volume $V \ll \Omega$

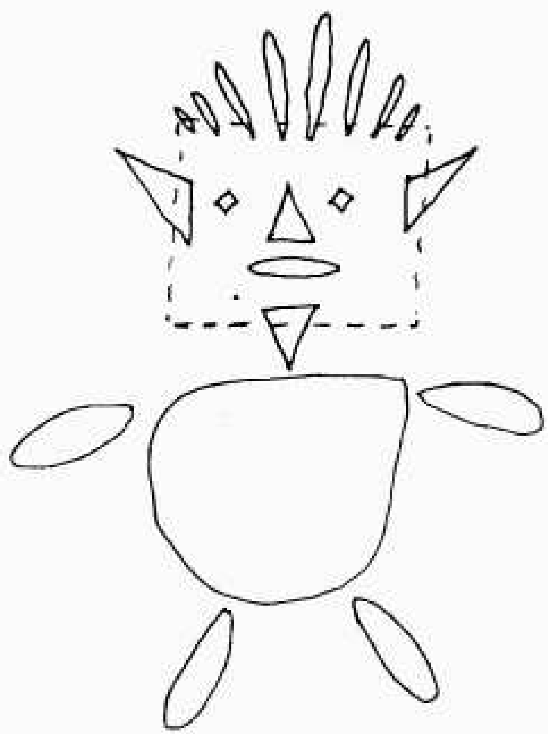

Fig 1.