Looking to the Future:
Nuclear Intersections with New Physics

Vijay's interest in astrophysics and weak interactions and his appreciation for the role of nuclear structure in these fields ⇒

- the CNO cycle and its importance in stellar evolution
- the r-process
- neutrino physics: mass scales, unknown couplings, CP-violation
- associated nuclear structure issues that would have concerned Vijay

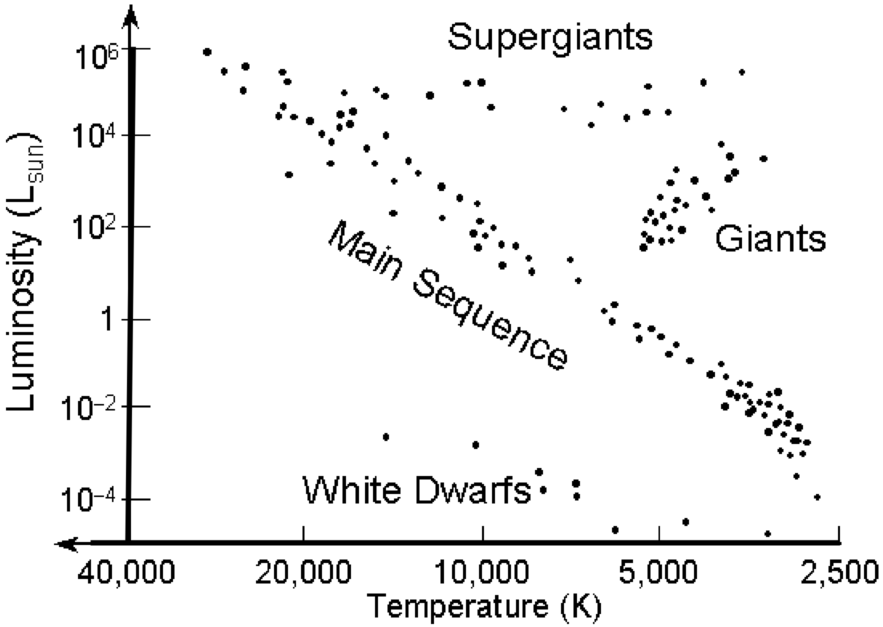
I. H-burning stars: the HR diagram

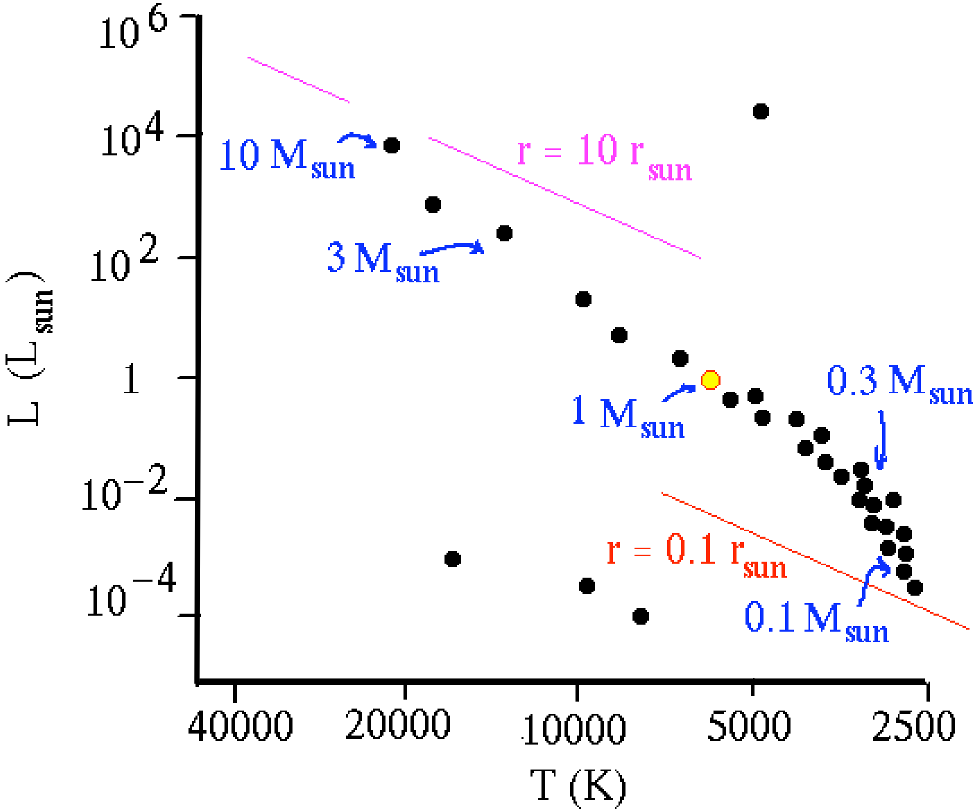
- T_s, L, R simplest stellar properties
- Stefan-Boltzmann black-body:

$$L = 4\pi R^2 \sigma T_s^4$$

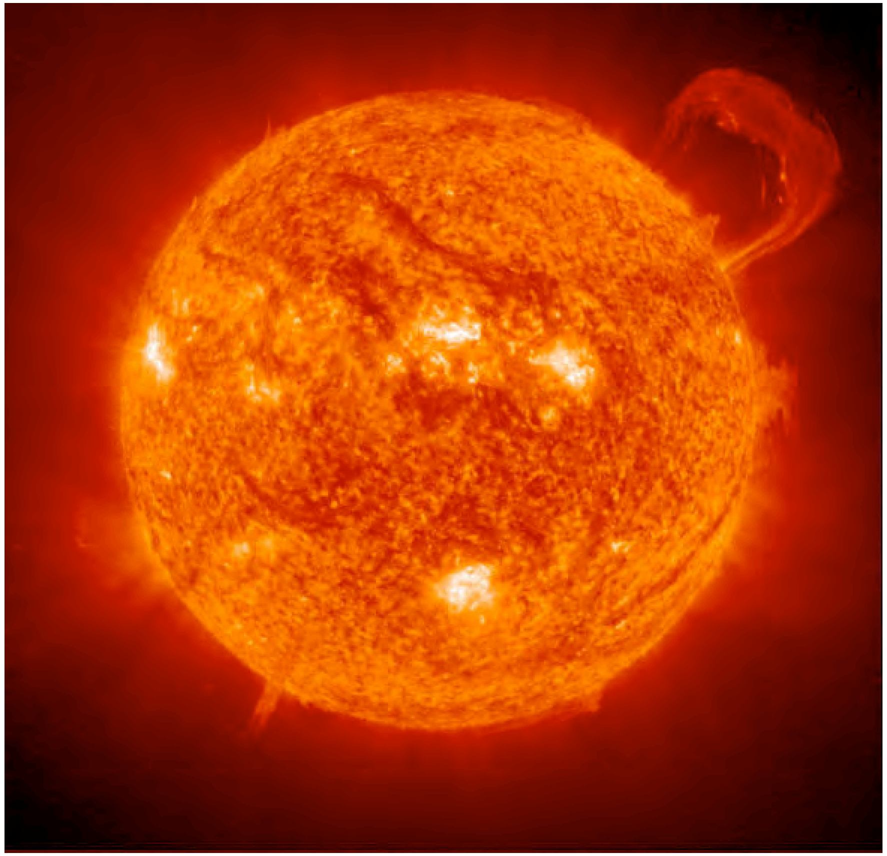
$$\left[\frac{L}{L_\odot}\right] = \left[\frac{R}{R_\odot}\right]^2 \left[\frac{T}{T_\odot}\right]^4$$

- ⇒ one-parameter HR trajectory
- "main sequence" of H-burning stars to which sun belongs
- sun a stellar-evolution test case: M, L, R, T_s, age, surface composition





Low-mass stars: Standard Solar Model



- sun burns in hydrostatic equilibrium: gravity balanced by gas pressure gradient
  (need EOS)

- energy transported: radiation, convection
  (convective envelope, radiative core)

- solar energy generated by H fusion

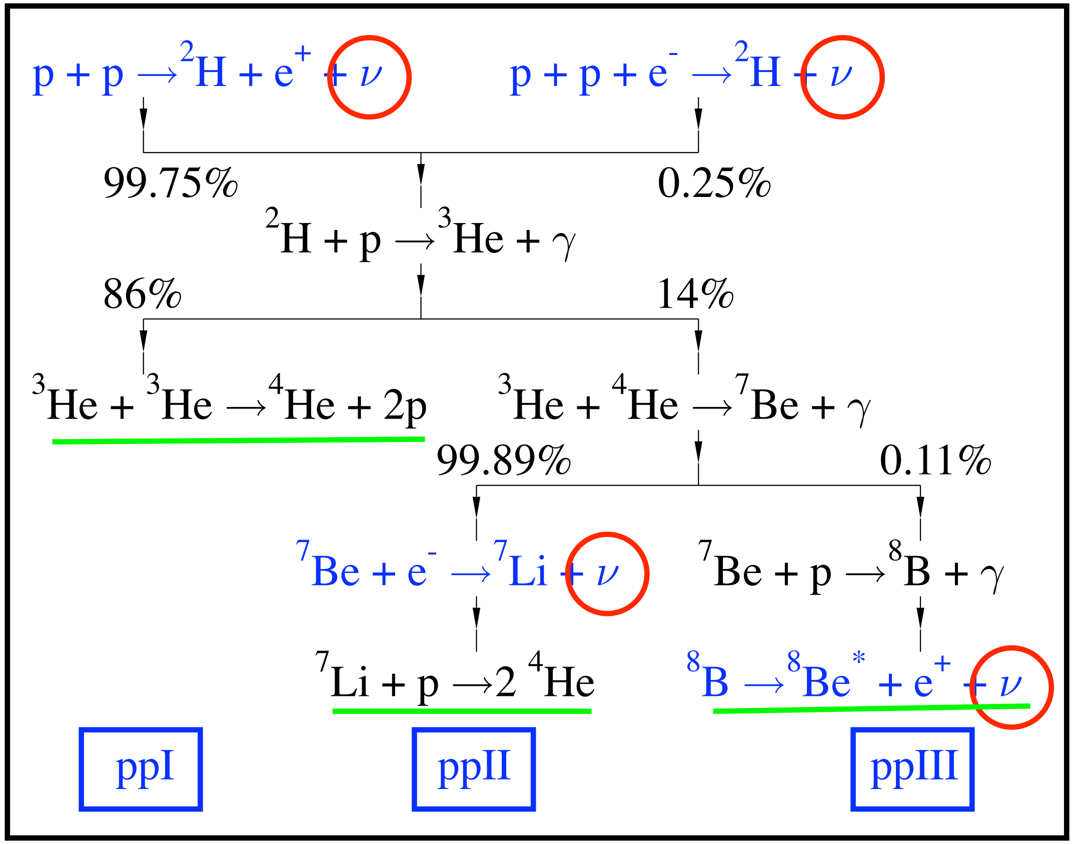
$$4\mathrm{p} \quad \rightarrow \quad {}^{4}\mathrm{He} + 2\mathrm{e}^{+} + 2\nu_{\mathrm{e}}$$

$$T_c \quad \sim \quad 1.5 \cdot 10^{7}\mathrm{K} \leftrightarrow \mathrm{E}_{\mathrm{cm}} \sim 2\mathrm{keV}$$

pp chain, CNO cycle

- boundary conditions: solar age, *L*, *M*, *R*, today's surface composition
  H : He : Z needed, where Z denotes metals (A ≥ 5)
  assume t=0 sun homogenized ⇒ Z from today's surface abundances
  H+He+Z = 1, He/H adjusted to produce today's *L*

Two quantitative SSM tests



$p + p \rightarrow {}^{2}H + e^{+} + \nu$ $p + p + e^{-} \rightarrow {}^{2}H + \nu$

99.75% 0.25%

$^{2}H + p \rightarrow {}^{3}He + \gamma$

86% 14%

$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$ $^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

99.89% 0.11%

$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu$ $^{7}Be + p \rightarrow {}^{8}B + \gamma$

$^{7}Li + p \rightarrow 2\,{}^{4}He$ $^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu$

ppI ppII ppIII

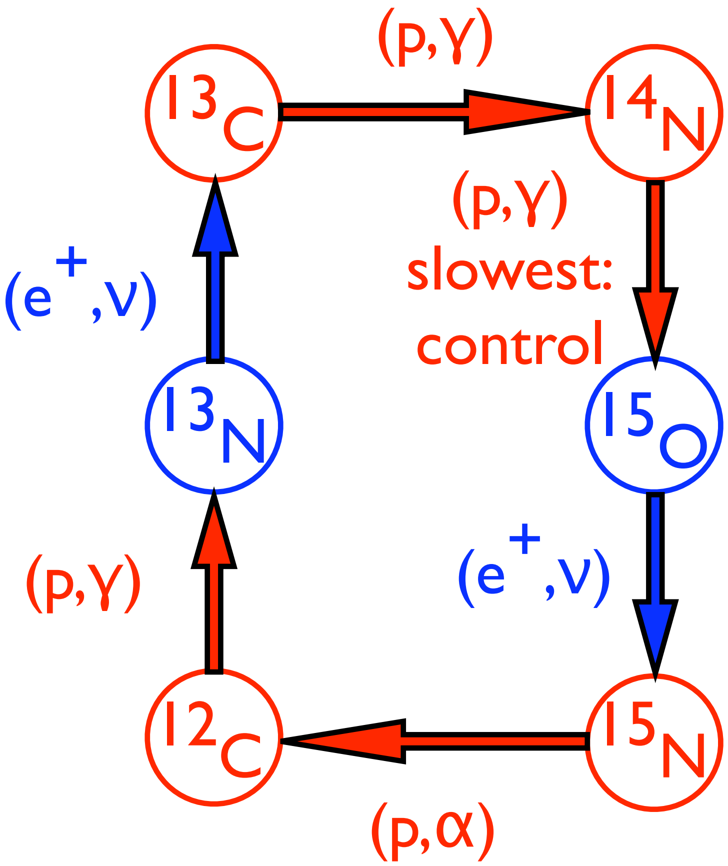
neutrinos:

- three competing cycles comprise pp chain, very different dependences on T_c
- each cycle tagged by a distinct neutrino: ν spectroscopy can measure core temperature
- SNO, SuperK results (after accounting for new physics) effectively verifies the SSM prediction of T_c to 1%

helioseismology:

- delicate probe of c: low-*l* acoustic modes dominant in convective envelope, frequency distribution sensitive to convective/radiative boundary ⇒ major accomplishment of the SSM;
  high-*l* g-modes (bouyancy) probe interior (difficult to see at surface)

CNO-cycle developments



^{13}C (p,γ) ^{14}N

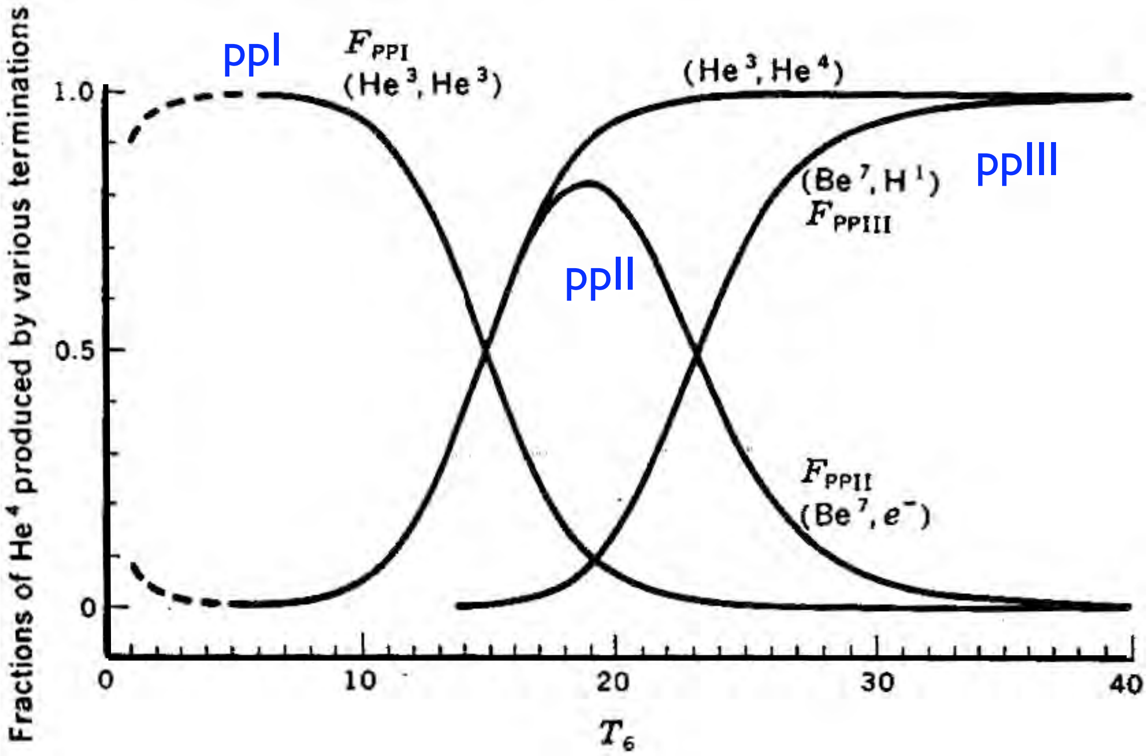
(p,γ) slowest: control

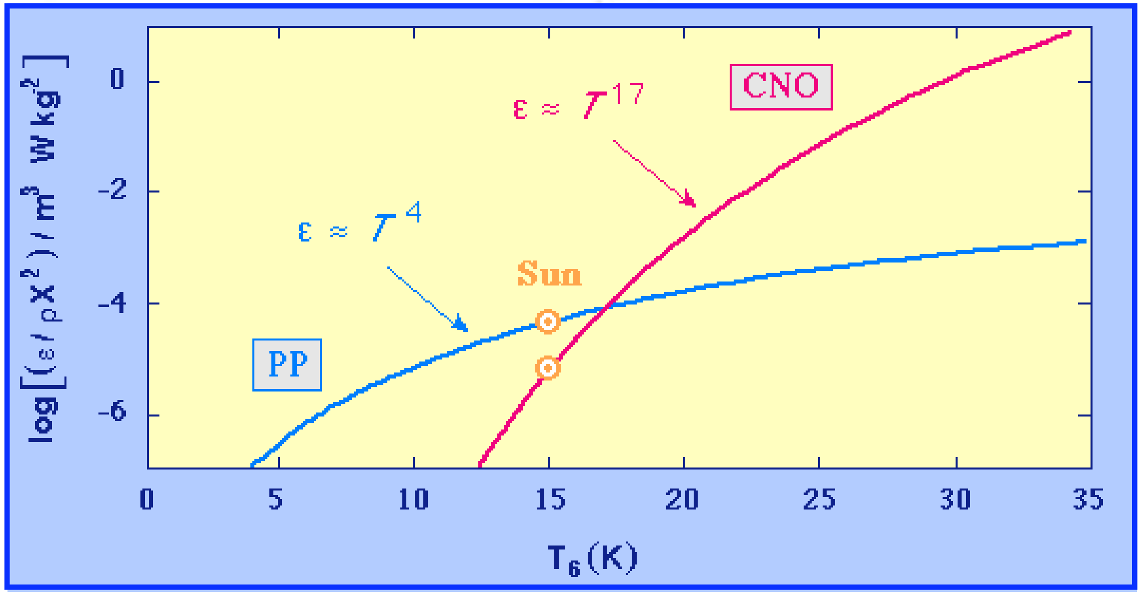
^{15}O (e$^+$,ν) ^{15}N

(p,α) ^{12}C

(p,γ) ^{13}N

(e$^+$,ν) ^{13}C

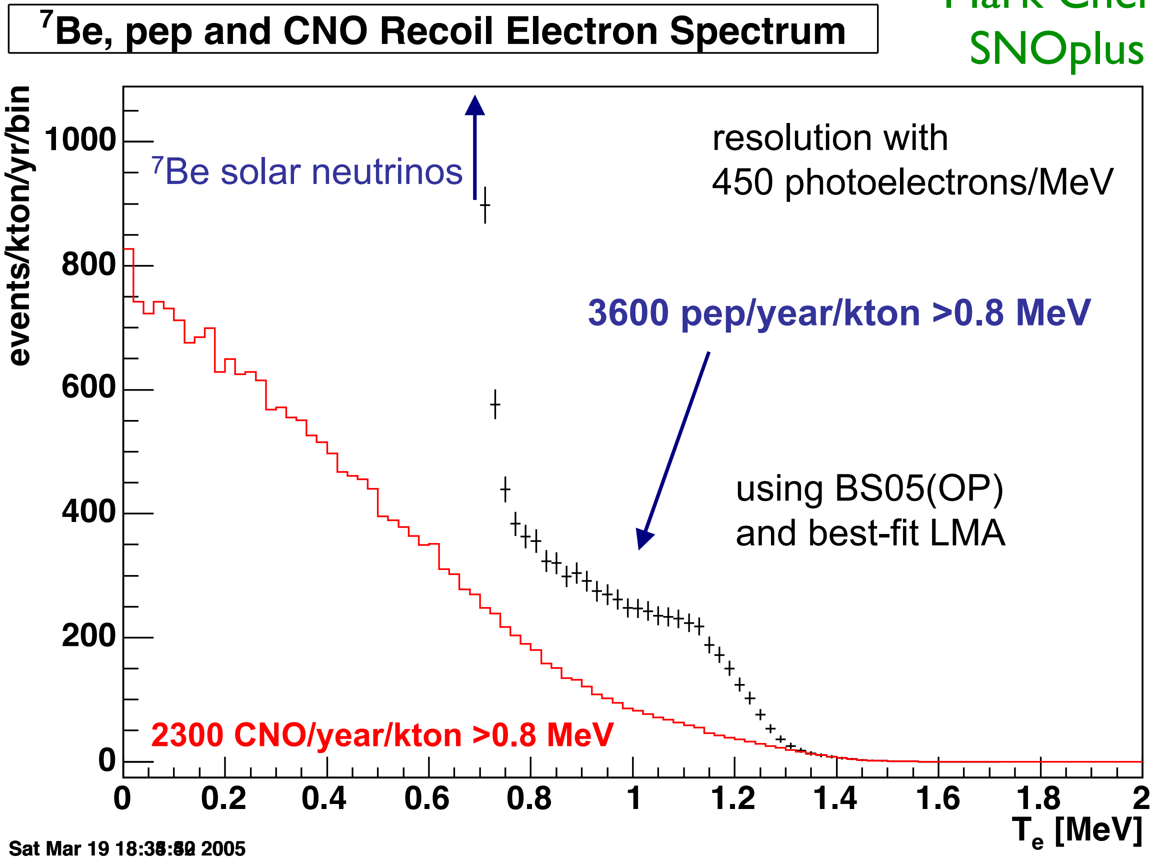




- Bethe first recognized that massive stars required a more efficient burning mechanism
- A minor contributor (~1%) to SSM energy generator, but nevertheless produces measurable νs

Mark Chen:
SNOplus

CNO role in SSM, elsewhere:



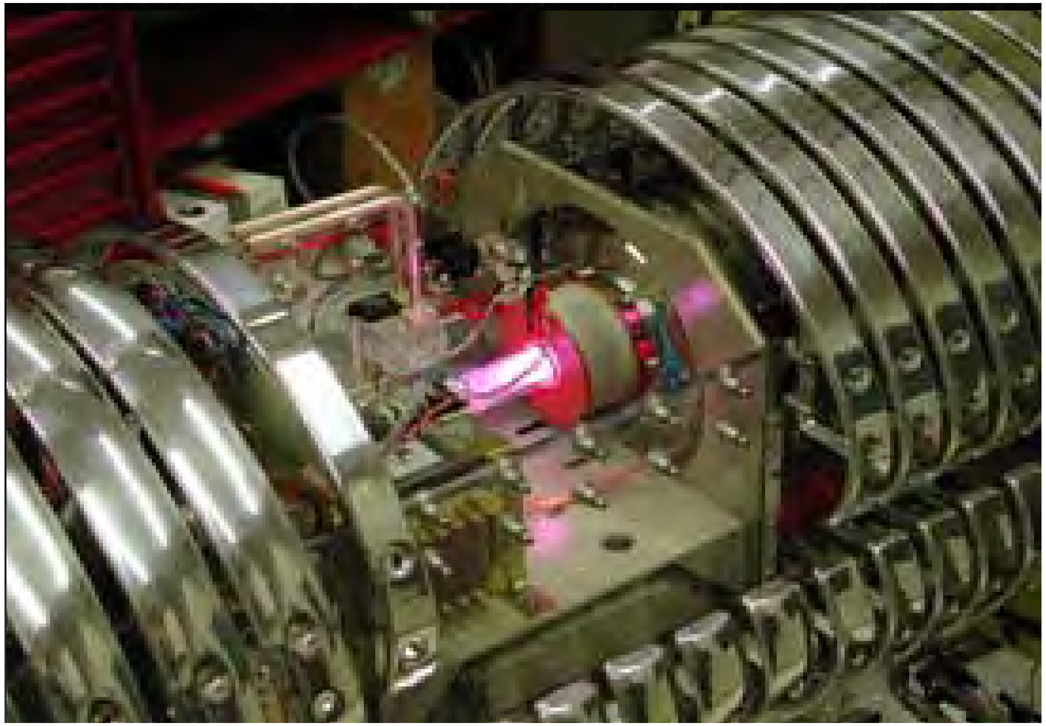
- directly tests key SSM assumption, equating core and current surface Zs
- out-of-equilibrium CNO burning thought to have powered early core convection for $\sim 10^8$ yr
- connected with current efforts to understand the formation and evolution of the first generation of massive, very-metal-poor stars:
  - H-burning must proceed through inefficient pp chain
  - T_cs in excess of 10^8 achieved
  - triple-α reaction turns on, synthesizing C
  - this allows CNO cycle to take over, restoring efficient H-burning

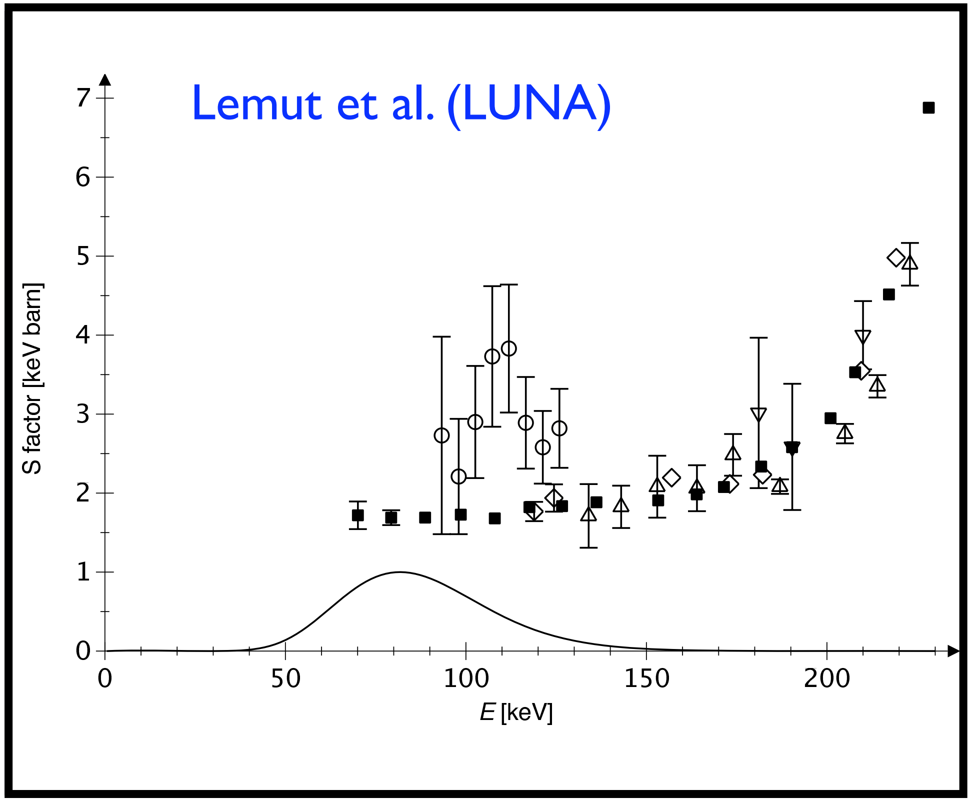
LUNA measurement of $^{14}N(p,\gamma)$

- controlling CNO rate measured in Gamow peak, 50% smaller than previously believed

3D atmospheric analysis of solar absorption lines

- surface Z revised downward by 30% ⇒ consequently SSM zero-age metallicity reduced





- Consequences for age determinations: solar-like stars within globular clusters important to host galaxy age determinations
    - new S(0) delays onset of CNO cycle and the evolution along main sequence and onto RG/AGB branches, with variable star "clocks"
    - effect is estimated to be an increase in the ages of the oldest such stars of 0.8 Gy

- Consequences for the SSM
    - predicted CNO ν flux reduced by more than a factor of two
    - reduced Z enhances radiative transport, reducing convective zone thickness, leading to significant SSM helioseismology discrepancies
    - SSM calculation already take into account heavy-element settling, though the modeling of such effects is uncertain
    - new abundances bring the sun into better accord with galactic stellar composition trends, He/H ~0.075 + 44.6 O/H (Turck-Chieze et al.)

⇒ a lot of important nuclear astrophysics could be put on firmer ground if SNOplus directly measures core metallicity via the CNO ν flux

II. More metals: the r-process

- half the heavy elements were created in an explosive, n-rich environment

- $(n,\gamma) \leftrightarrow (\gamma,n)$ equilibrium, not $\beta-$decay, determines the "valley of stability": the stable (for a second or so) species are exotic, n-rich nuclei

- n-capture occurs when $n \rightarrow p$ opens up a whole mass flow proportional to β-decay rates, so mass piles up at $A \sim 130, 195$ -- slow rates + shell gaps

Progress on sites/scenarios:

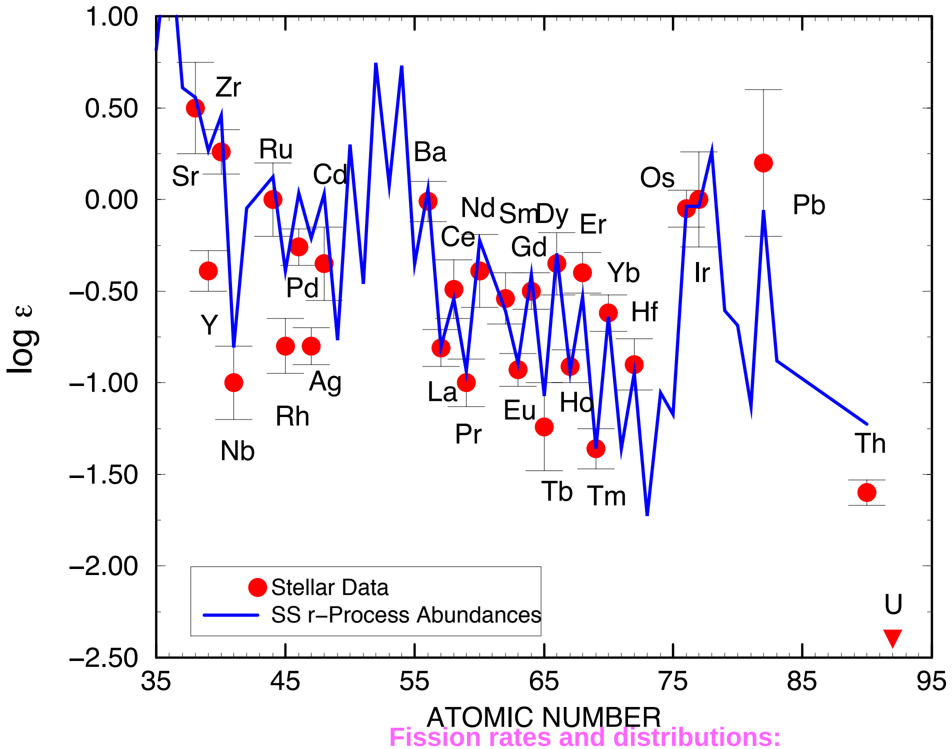
- site questions clearer: Of the two favored sites (decompressing n-rich matter from NS mergers; the expanding, cooling, ν-driven winds coming off a Type II supernova) recent data favor the latter -- frequency consistent with the enrichments seen in old, metal-poor stars

- "hot bubble" r-process: n-rich n/p gas undergoes an α-rich freezeout, $\alpha+\alpha$ reactions to produce heavy seed nuclei, and n-capture on the seeds

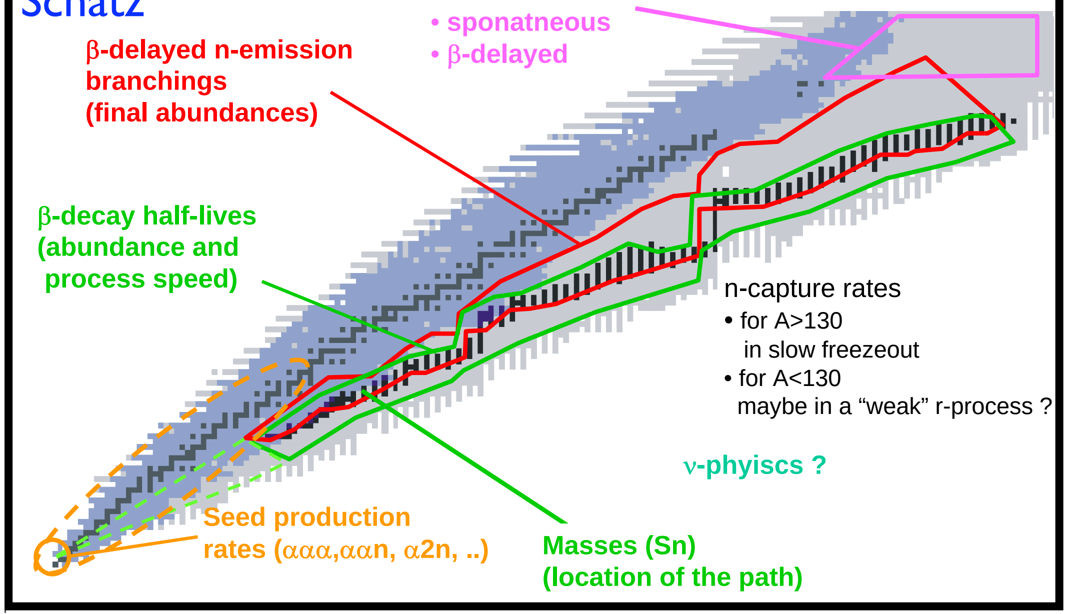
New r-process constraints

- metal-poor halo stars: r-process distribution for Z>56 (A>130) matches solar abundance
- detailed modeling of supernova winds as a site: frequency/yields and mixing consistent with observation -- but only if the gas entropy is made unrealistically high
- these same models do not explode
- halo star yields for Z<56 variable, and there are chronometers that suggest different frequencies for the astrophysical events responsible for the high- and low-mass nuclei

Cowan et al.



Schatz



Fission rates and distributions:
- n-induced
- sponatneous
- β-delayed

β-delayed n-emission branchings (final abundances)

β-decay half-lives (abundance and process speed)

n-capture rates
- for A>130 in slow freezeout
- for A<130 maybe in a “weak” r-process ?

ν-phyiscs ?

Seed production rates (ααα,ααn, α2n, ..)

Masses (Sn) (location of the path)

- Similar to BBN, where nucleosynthesis placed an important constraint on η

- Differs from BBN in the degree of nuclear physics uncertainty: masses, weak rates, possibility of fission cycling, neutron emission accompanying β-decay back to the valley of stability -- one of the arguments for RIA (which Vijay supported)

- Complex connections with ν physics: CC reactions tend to drive n-rich nucleon gases to α-gases, destroying the needed excess neutrons; poorly understood aspects of neutrino oscillations also affect the gas

- SNII model deficiencies $\Rightarrow$ ρ,S,T wind trajectories uncertain

- The empirical evidence fits a scenario where two distinct classes of stars are exploding, producing different quantities of ejecta, different entropies; metal-poor stars are seen with very similar Z>56 r-process enrichment, but Fe differing greatly -- low-mass, O-Ne-Mg stars??

III. What we have learned about neutrino oscillations (vacuum)

$$\boxed{\begin{array}{c}\text{flavor}\\ \text{states}\end{array}} \quad \begin{array}{ccccc} |\nu_e > & & \leftrightarrow & |\nu_L > & m_L \\ |\nu_\mu > & & & |\nu_H > & m_H \end{array} \quad \boxed{\begin{array}{c}\text{mass}\\ \text{states}\end{array}}$$

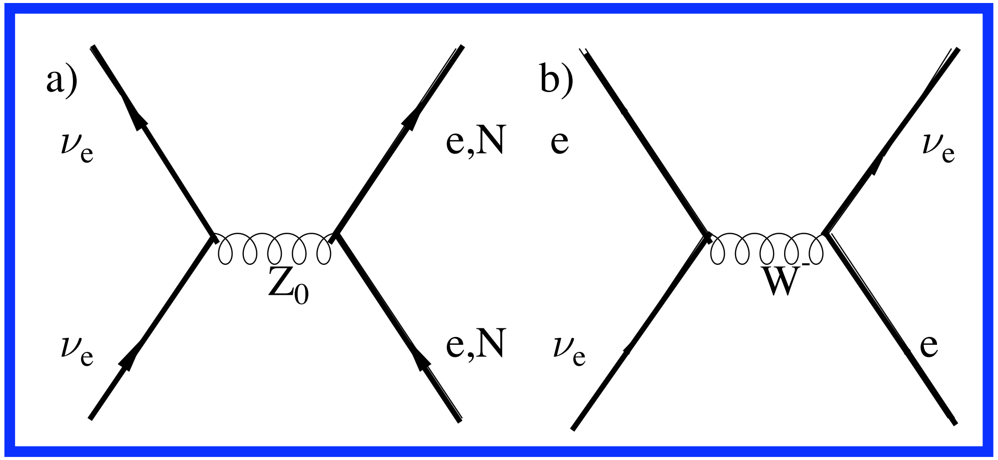
Noncoincident bases ⇒ oscillations down stream:

$$\begin{aligned} |v_e > &= \cos\theta|\nu_L > + \sin\theta|\nu_H > \\ |v_\mu > &= -\sin\theta|\nu_L > + \cos\theta|\nu_H > \end{aligned}$$

$$\begin{aligned} |\nu_e^k > &= |\nu^k(x=0, t=0) > \qquad E^2 = k^2 + m_i^2 \\ |\nu^k(x \sim ct, t) > &= e^{ikx}\left[e^{-iE_Lt}\cos\theta|\nu_L > + e^{-iE_Ht}\sin\theta|\nu_H >\right] \\ | < \nu_\mu|\nu^k(t) > |^2 &= \sin^2 2\theta \sin^2\left(\frac{\delta m^2}{4E}t\right), \qquad \delta m^2 = m_H^2 - m_L^2 \end{aligned}$$

ν_μ appearance downstream ⇔ vacuum oscillations

Inclusion of matter alters oscillations



$$|\nu^k(0)>= a_e(0)|\nu_e> + a_\mu(0)|\nu_\mu>$$

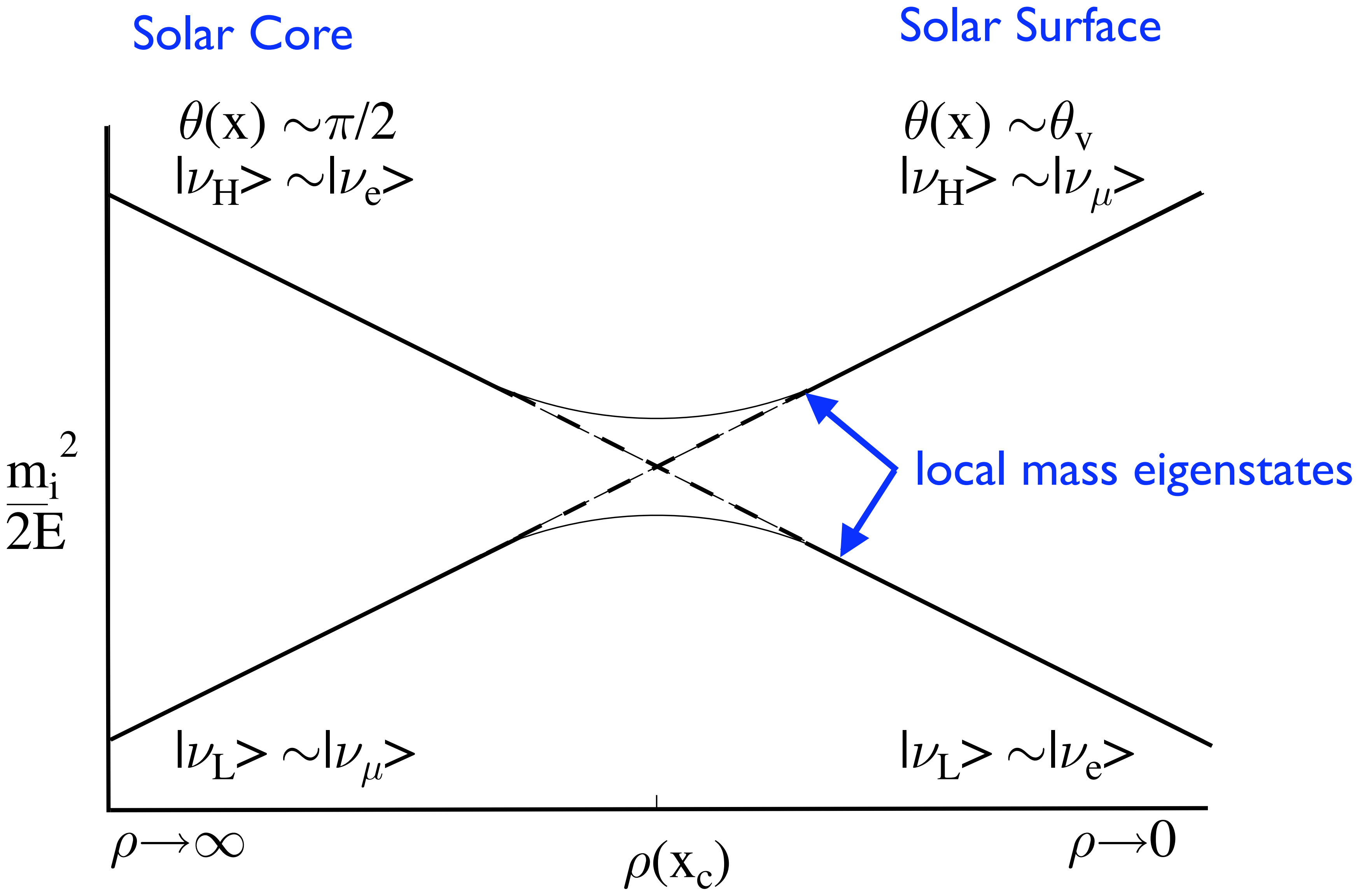
$$i\frac{d}{dx}\left[\begin{array}{c} a_e(x) \\ a_\mu(x) \end{array}\right] = \frac{1}{4E} \times$$

$$\left[\begin{array}{cc} -\delta m^2\cos 2\theta + \boxed{2E\sqrt{2}G_F\rho_e(x)} & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -\boxed{2E\sqrt{2}G_F\rho_e(x)} + \delta m^2 \cos 2\theta \end{array}\right]\left[\begin{array}{c} a_e(x) \\ a_\mu(x) \end{array}\right]$$

- Degeneracy (level crossing) for $\rho_e(x) = \rho_c = \delta m^2 \cos 2\theta / 2E\sqrt{2}G_F$

- A ν_e that is primarily made up of the light mass eigenstate becomes heavy at high densities

- Passage through the avoided level crossing leads to enhanced flavor transformation


Solar Core
Solar Surface
$\theta(x) \sim \pi/2$
$|\nu_H\rangle \sim |\nu_e\rangle$
$\theta(x) \sim \theta_v$
$|\nu_H\rangle \sim |\nu_\mu\rangle$
$\frac{m_i^2}{2E}$
local mass eigenstates
$|\nu_L\rangle \sim |\nu_\mu\rangle$
$|\nu_L\rangle \sim |\nu_e\rangle$
$\rho \rightarrow \infty$
$\rho(x_c)$
$\rho \rightarrow 0$

- the local oscillation length L(ρ) becomes extended at crossing density ρ_c

- if $L(\rho_c)$ is small compared to solar density scale height ⇒ adiabatic ⇒

$$P_{\nu_e}^{adiab} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i$$

path independent

- "adiabaticity" γ_c defined as the density scale height/$L(\rho_c)$ ratio:

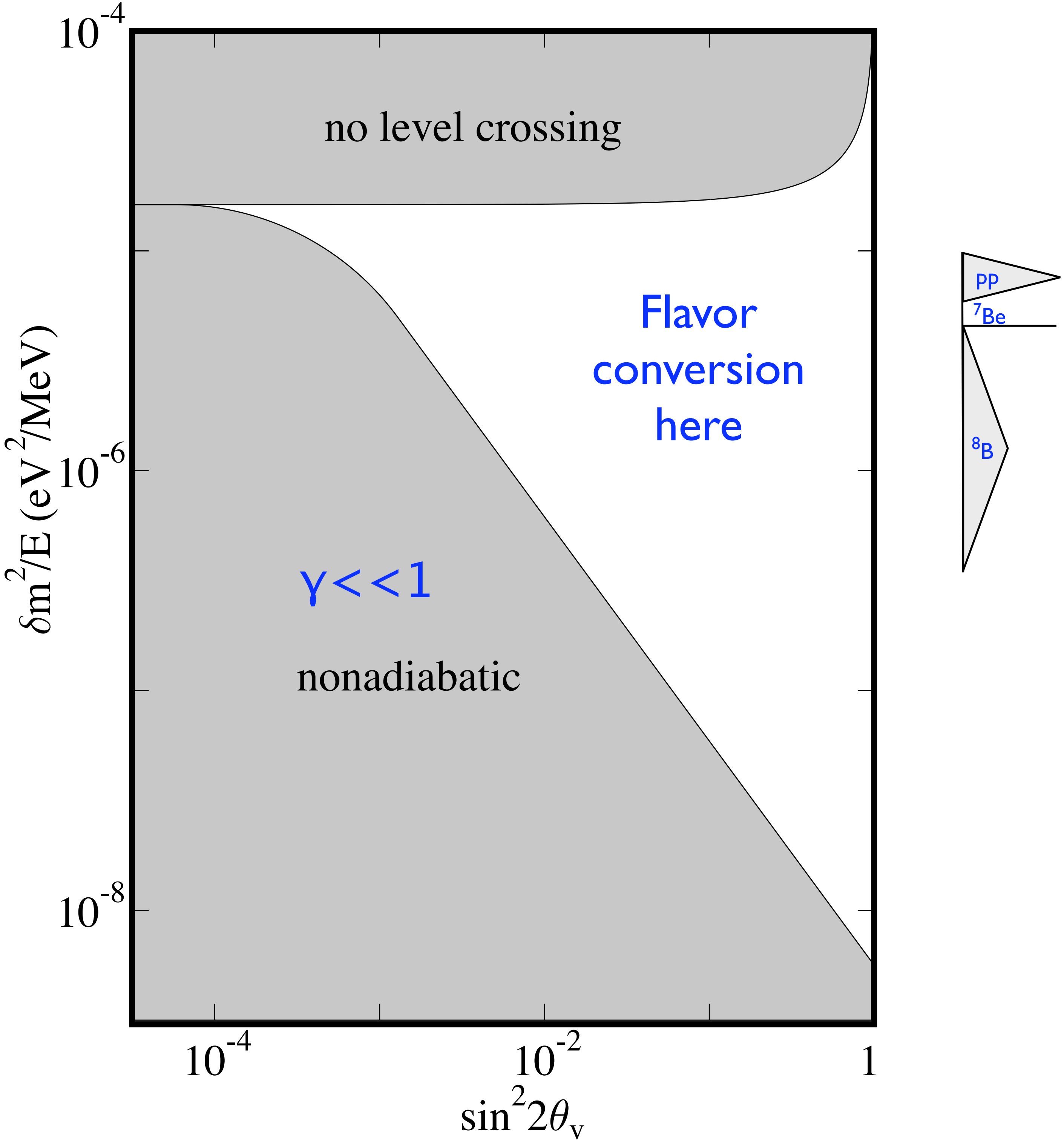
nonadiabatic if γ_c <<1 ("hops" at crossing - no flavor conversion)

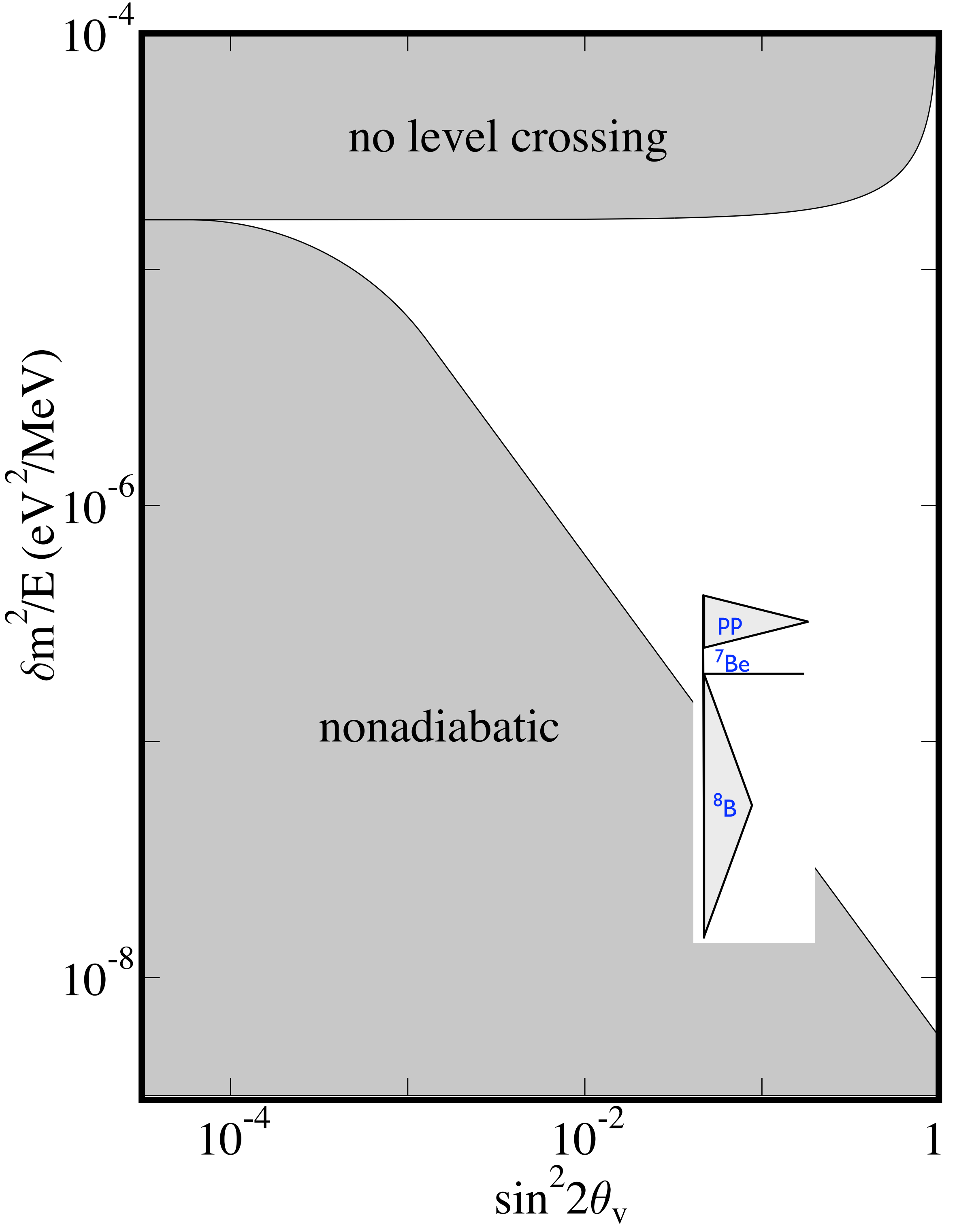
$$P_{\nu_e}^{LZ} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop})$$

Landau-Zener

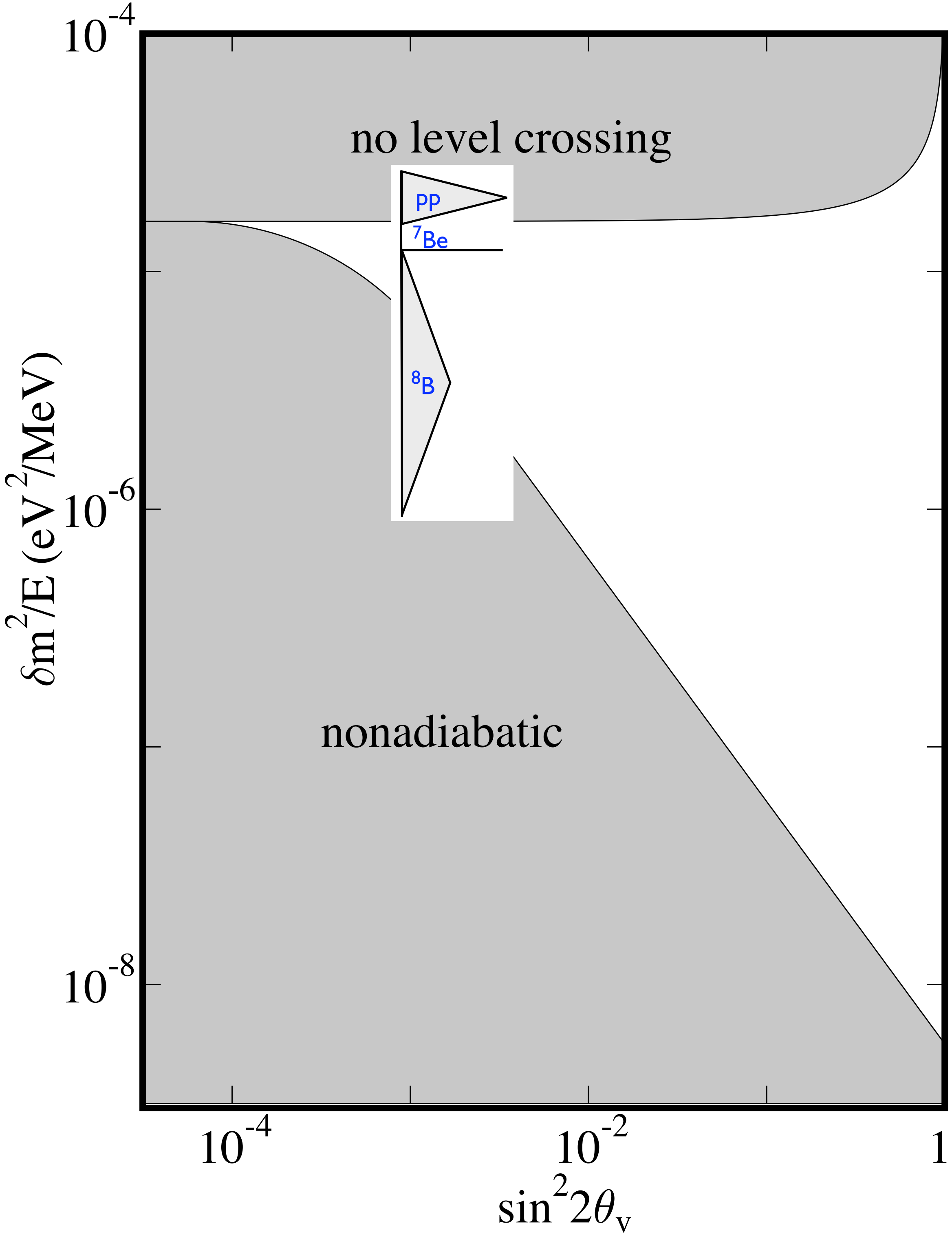
$$P_{hop} = e^{-\pi\gamma_c/2} \qquad \gamma_c = \frac{\sin^2 2\theta}{\cos 2\theta}\,\frac{\delta m^2}{2E}\,\frac{1}{|\frac{1}{\rho_c}\frac{d\rho}{dx}|}$$

two conditions for flavor conversion	a level crossing must occur ($\theta_i \sim \pi/2$) the crossing must be adiabatic

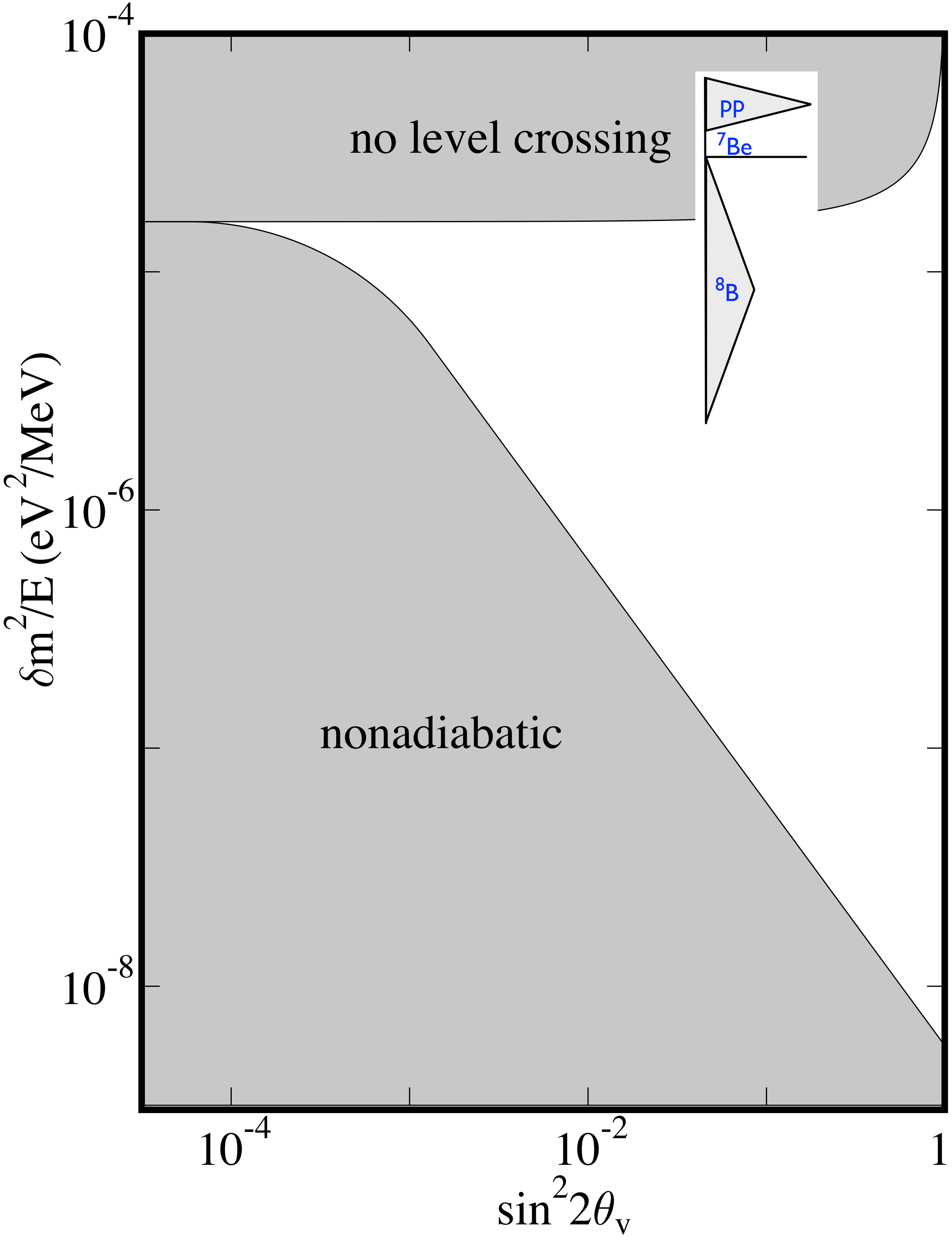

10^{-4}
10^{-6}
10^{-8}
$\delta m^2/E$ (eV^2/MeV)
no level crossing
Flavor conversion here
γ<<1
nonadiabatic
10^{-4}
10^{-2}
1
$\sin^2 2\theta_v$
pp
^{7}Be
^{8}B



Low
solution



Small angle solution



Large angle solution

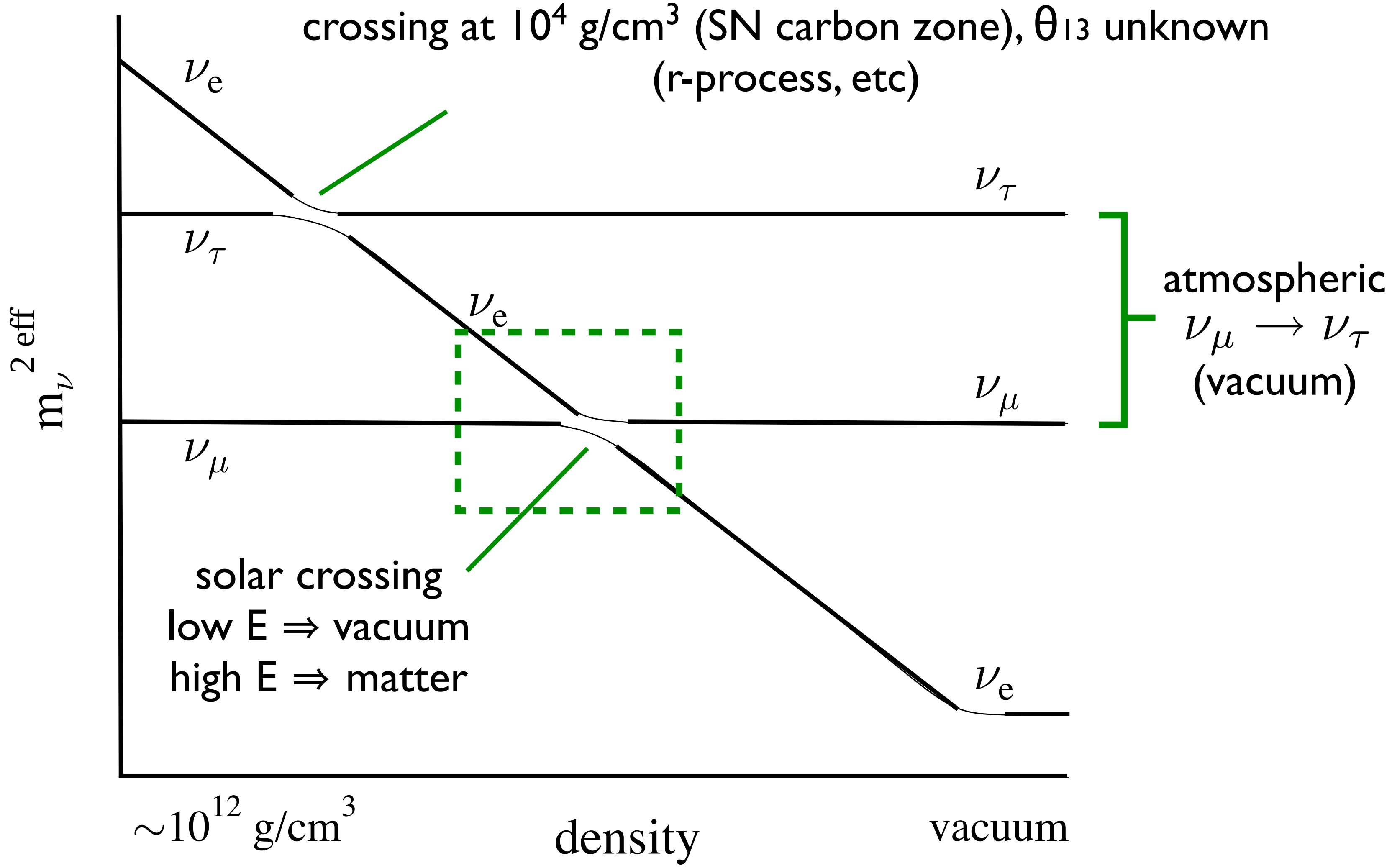
this is the solution matching SNO and SuperK results

+

Ga/Cl/KII

$\tan^2\theta_v \sim 0.40$

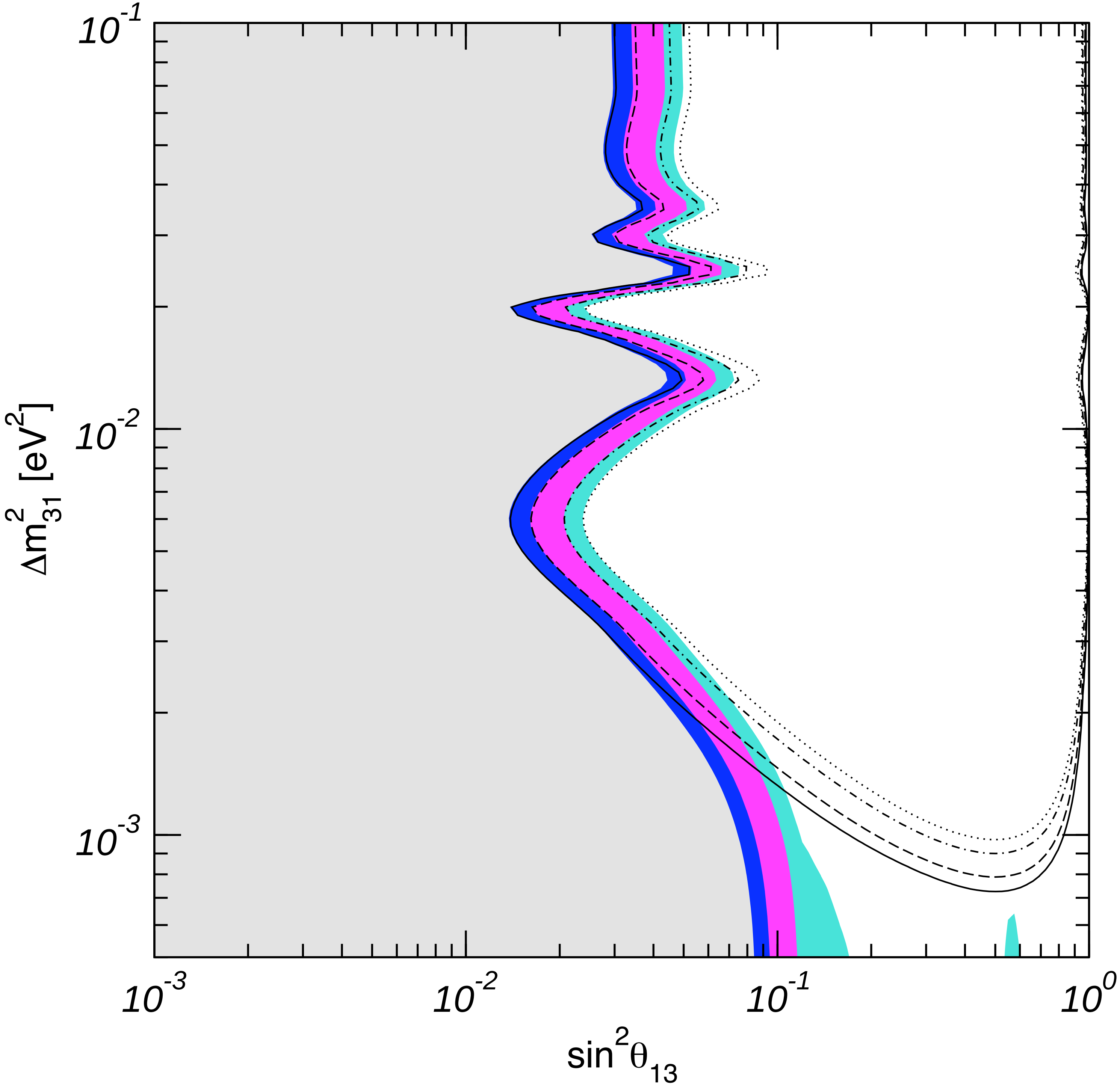
Generalization to three flavors:



crossing at 10^4 g/cm^3 (SN carbon zone), θ_{13} unknown
(r-process, etc)

atmospheric
$\nu_\mu \longrightarrow \nu_\tau$
(vacuum)

solar crossing
low E ⇒ vacuum
high E ⇒ matter



Maltoni et al.

SNO and SuperK, KamLAND, K2K, MINOS, ...

What do we know about mass differences?

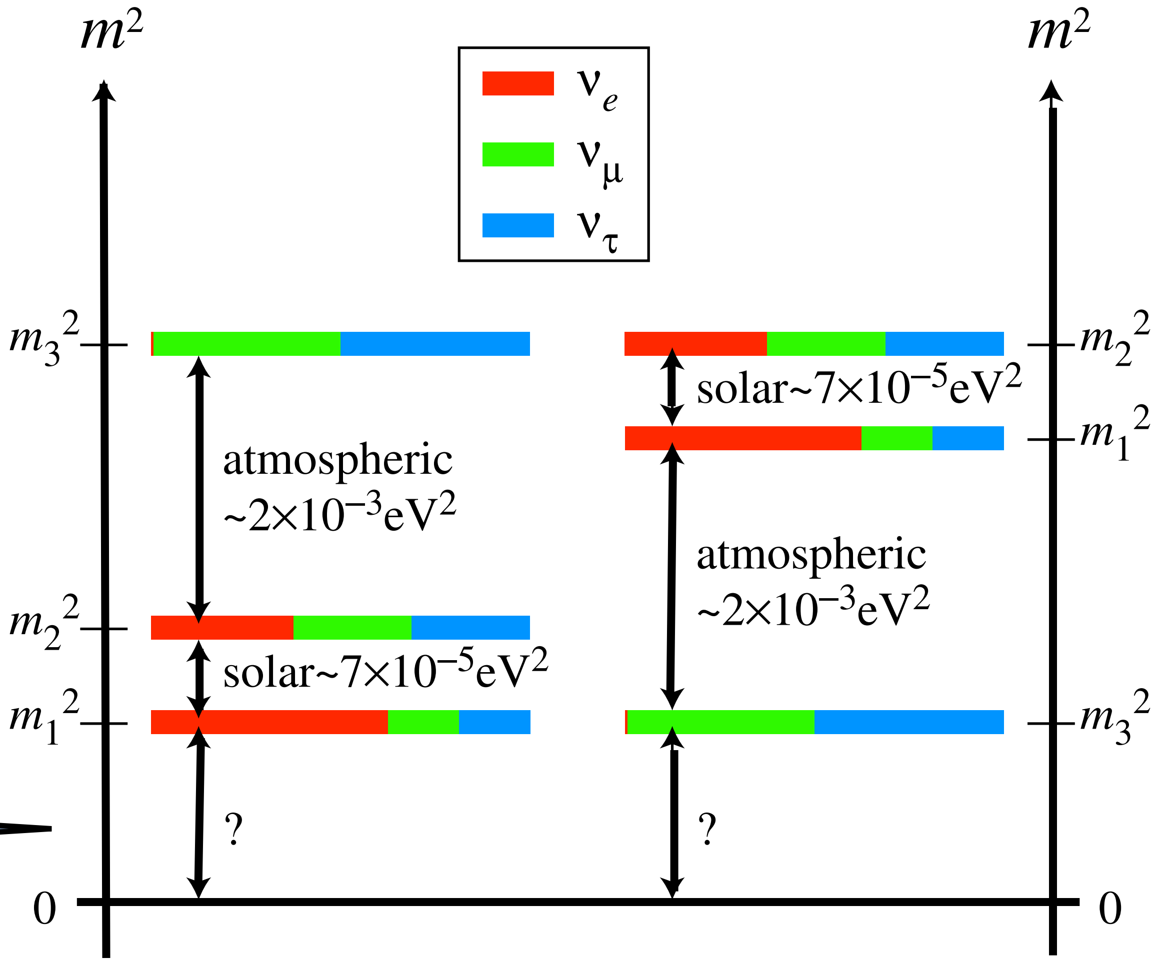
$m_{12}^2 \sim (8 \pm 1)\times 10^{-5}$ eV^2

$|m_{23}^2| \sim (2.2 \pm 0.8)\times 10^{-3}$ eV^2

And mass? WMAP + LSSS

$\sum m_i < 1$ eV

Degenerate or hierarchical schemes allowed within this constraint



Mohapatra et al., APS study

Neutrino mixing status: θ_{12}, θ_{23}

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

	atmospheric	ν_e disappearance	solar
results:	$\theta_{23} \sim 45^\circ$	$\sin\theta_{13} \leq 0.17$	$\theta_{12} \sim 30^\circ$

Neutrino mass may be the first signature of physics at the GUT scale

$$\bar{\psi}_R m_D \psi_L + h.c. \qquad \bar{\psi}^c_L m_L \psi_L + \bar{\psi}^c_R m_R \psi_R$$ (violates L)

neither allowed in the minimal standard model

$$\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \Rightarrow m_\nu^{light} = m_D \left(\frac{m_D}{m_R}\right)$$

SM's only dim-5
operator

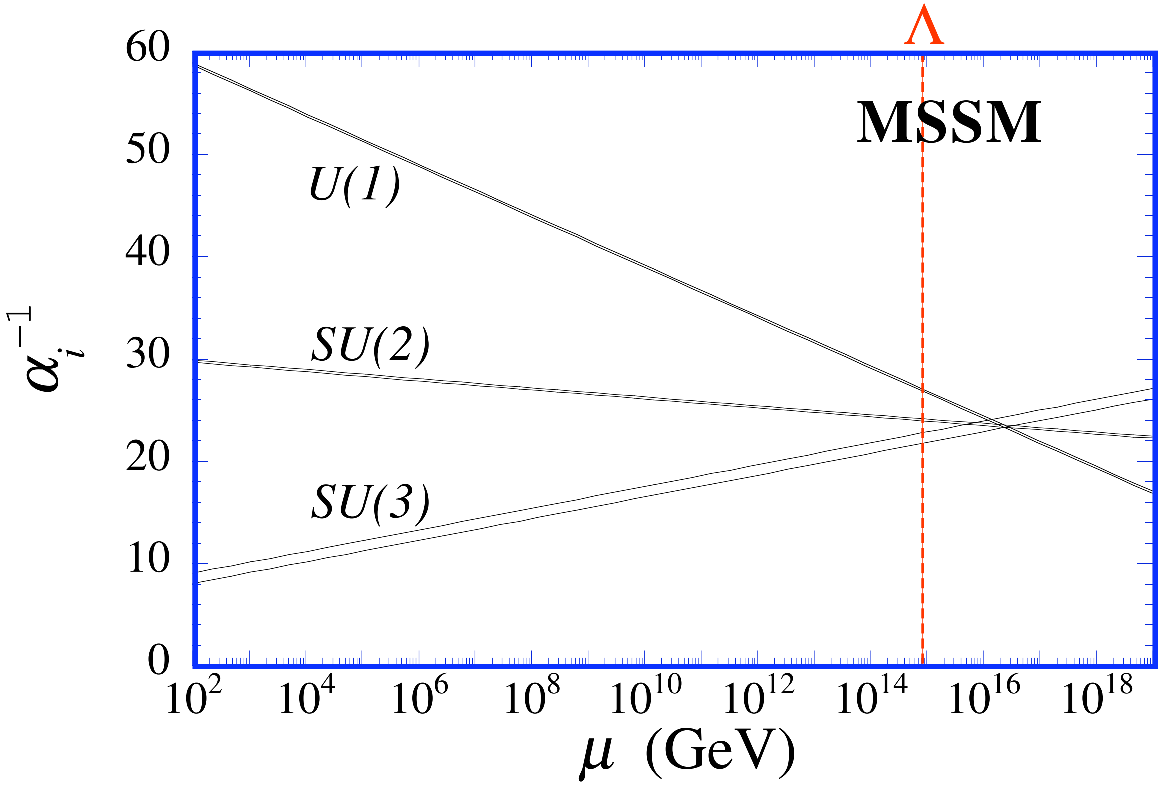
a natural explanation of the suppression (m_D/m_R) of light ν masses relative to the Dirac scale (m_D) of other SM fermions

using $m_3 \sim 0.05$ eV

$m_D \sim m_{top} \sim 180$ GeV

$\Rightarrow m_R \sim 0.3 \times 10^{15}$ GeV

Mohaptra et al.
APS study



Future program of HE/nuclear ν physics has been mapped out (APS study)

- constrain the absolute scale of neutrino mass: near-term ββ exps. and cosmological tests should reach 50 meV; future efforts to 10 meV

- measuring the unknown mixing angle θ_{13} in reactor or LB off-axis exps.

- demonstrating that Majorana masses exist in ββ decay

- distinguishing between the inverted and normal hierarchies in LB or next-generation atmospheric ν studies of subdominant oscillations

- seeing the Dirac CP phase in LB exps: $\nu_\mu \leftrightarrow \nu_\tau$ vs. $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau$

- once the masses and mixing angles are known, do the nuclear physics to high precision to constrain the Majorana phases in ββ decay

- includes future solar, supernova experiments to do astro-ν physics

IV. Vijay ⇔ quantitative nuclear structure tools: the problems he might see

Solar neutrinos:

- Vijay did a lot of work on pp-chain cross sections, particularly the driving p+p reaction and the weak branch ^{3}He+p important to the high energy tail of the solar neutrino spectrum
- He was also a member of the 1998 INT group that tried to quantify H-burning uncertainties, including ^{14}N(p,γ)

$$S(0) = 3.5^{+0.4}_{-1.6} \text{ keV b} \leftrightarrow S_{LUNA}(0) = 1.61 \pm 0.08 \text{ keV b}$$

- This is enough to allow us to exploit CNO νs to probe solar core Z
- But at the hotter temperatures important to popIII massive stars and RG evolution, significant nuclear uncertainties arise -- new cycles involving ^{16}O(p,γ), ^{17}O(p,α), etc.

Building the numerical tools to attack p-shell radiative capture reactions important to astrophysics ↔ talks by Joe, Bob

The r-process: halo nuclei ↔ Ben’s shell model talk ↔ effective theory

compact state:
$-E_1$: potential
important

extended state:
$-E_2 \sim 0$ MeV:
potential plays
virtually no role

Vijay spent much of his research career thinking about exact solutions of the nuclear many-body problem; but early in his career he also thought about effective interactions and the possibility of simple parameterizations

Suppose we viewed/reformulated the shell model as an exact effective theory. Suppose we lived in a world where the deuteron had two bound states, one deep and one barely bound.

How, say in a small Hilbert space of a few H.O. shells, would this come about? An extended state and a compact state (whose energy must reflect hard-core scattering from omitted high-q states) both reproduced?

A nice result derived recently:

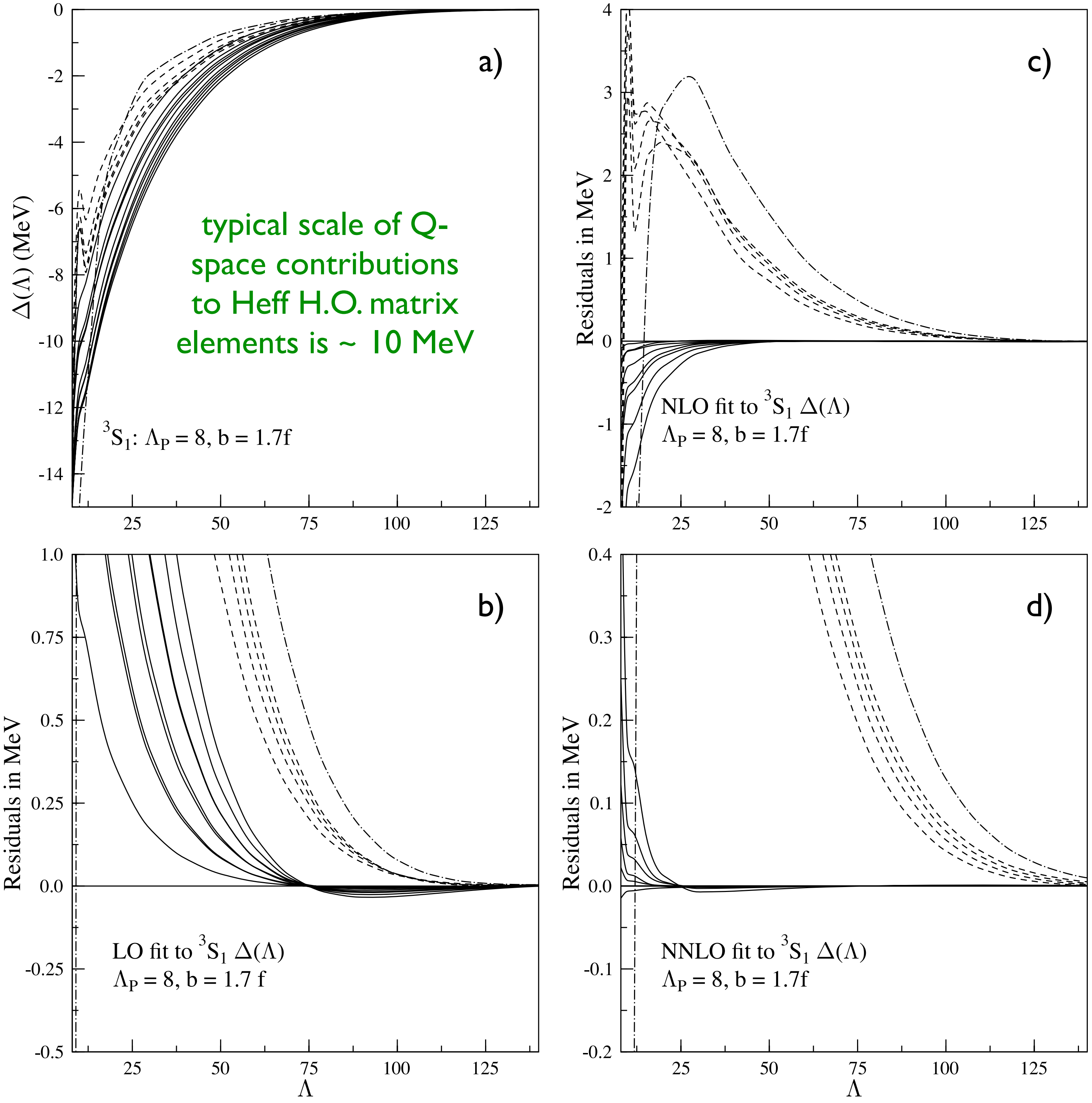
$$H^{eff}_{(lS)J}(r,E) = \sum_i g^i_{(lS)J}(E/\hbar\omega) e^{-r^2/2} V_i e^{-r^2/2}$$

where the g^i are known analytically (generated by missing LR physics) and the Vi can be expanded systematically (hard-core SR scattering)

$$\begin{gathered} a^{ss}_{LO}(\Lambda_P,b)\delta(\mathbf{r}) + a^{ss}_{NLO}(\Lambda_P,b)(\overleftarrow{\nabla}^2\delta(\mathbf{r}) + \delta(\mathbf{r})\overrightarrow{\nabla}^2) + \\ a^{ss,22}_{NNLO}(\Lambda_P,b)\overleftarrow{\nabla}^2\delta(\mathbf{r})\overrightarrow{\nabla}^2 + a^{ss,40}_{NNLO}(\Lambda_P,b)(\overleftarrow{\nabla}^4\delta(\mathbf{r}) + \delta(\mathbf{r})\overrightarrow{\nabla}^4) + \\ a^{ss,42}_{N^3LO}(\Lambda_P,b)(\overleftarrow{\nabla}^4\delta(\mathbf{r})\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2\delta(\mathbf{r})\overrightarrow{\nabla}^4) + a^{ss,60}_{N^3LO}(\Lambda_P,b)(\overleftarrow{\nabla}^6\delta(\mathbf{r}) + \delta(\mathbf{r})\overrightarrow{\nabla}^6) \end{gathered}$$

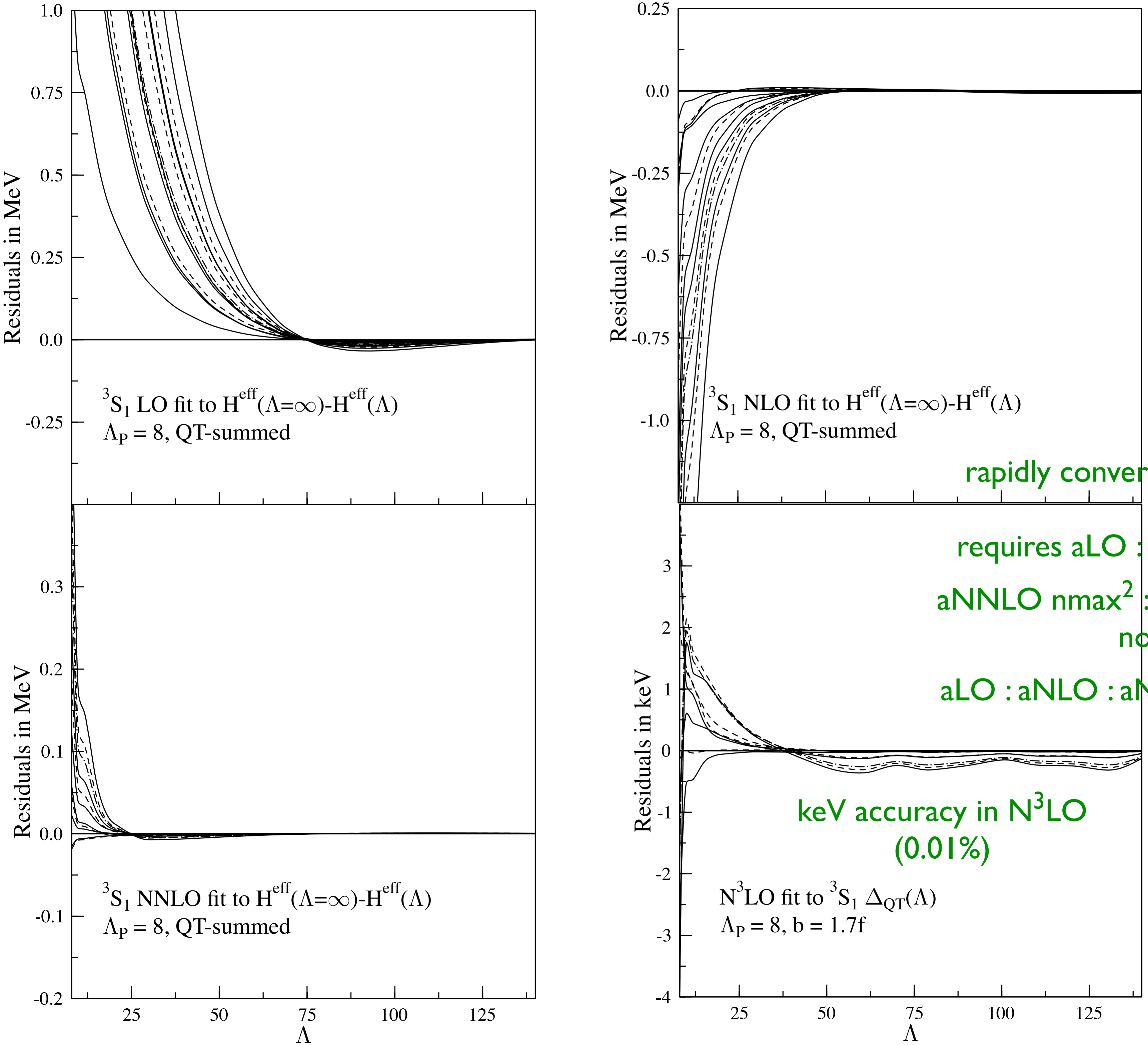
The g^i are sharply energy dependent and → 0 as E → 0; the a_i are virtually E-independent, determined only by the parameters describing the SM space ($\Lambda_P, \hbar\omega$) and represent high-q scattering; rapidly converging

e.g., 8hw SM-space, av18 potential, $^3S_1$: from 14 (LO) to 9 (N^3LO) free DoFs



typical scale of Q-space contributions to Heff H.O. matrix elements is ~ 10 MeV

most general E-independent nonlocal SR V ⇒ failure: no general systematic improvement as a function of the integration scale Λ



rapidly converging for Heff:

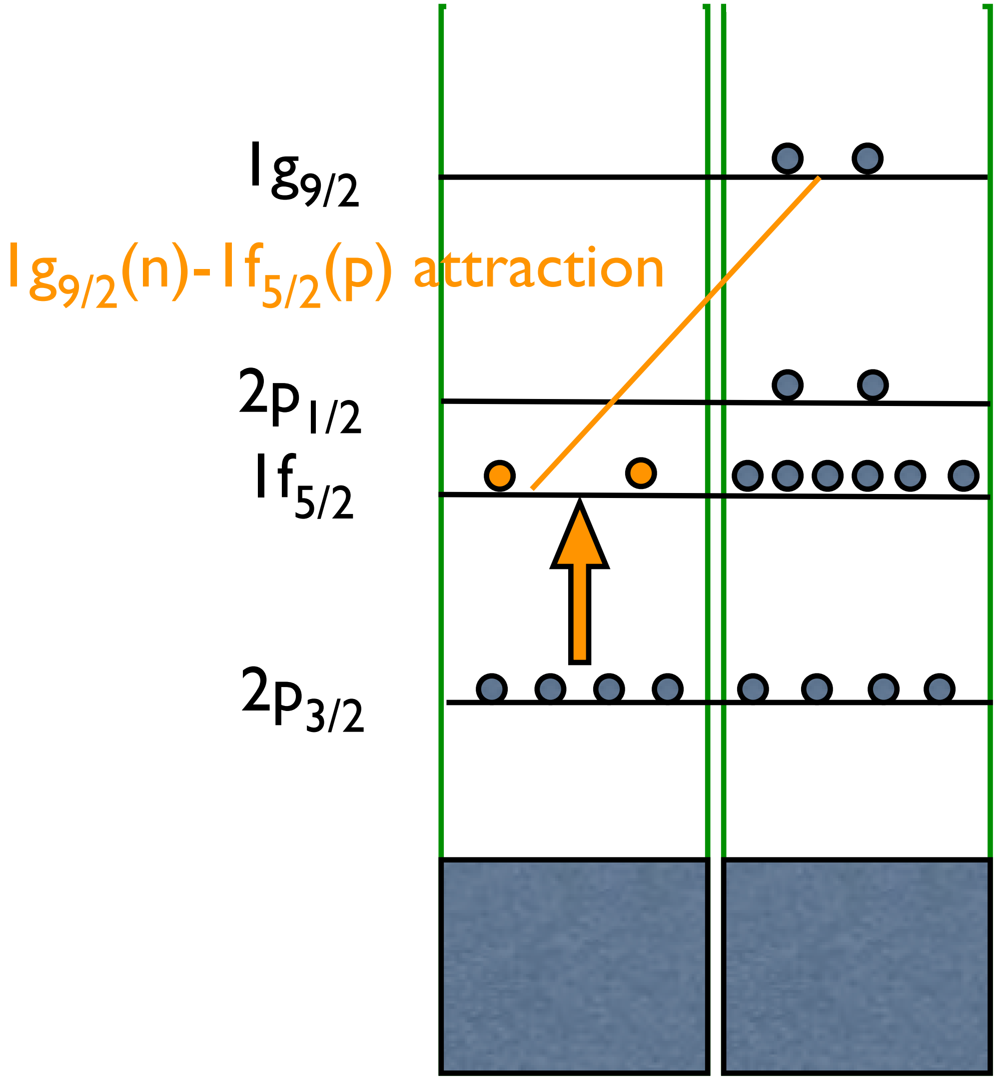
requires aLO : aNLO nmax : aNNLO nmax^2 : aN^3LO nmax^3

not

aLO : aNLO : aNNLO : aN^3LO

keV accuracy in N^3LO (0.01%)

ββ-decay: to say anything quantitative about ν mass, Majorana phases, must deal with subtle structure physics, unknown effective operators



^{74}Ge

(Majorana and Gerda collaborations will study ^{76}Ge in next-generation experiments)

Long-baseline oscillation physics: superbeams or a ν factory to determine hierarchy, measure θ_{13}, see leptonic CP violation

Precision measurements depend critically on the accuracy of the event generators used to analyze CC, NC(!) ν-target interactions (e.g., C, Fe)

Superbeam (and NuMi beam) energies typically range from 0.5-few GeV: transition range from quasi-elastic to resonance regions -- difficult

Current event generators (NUANCE, NUGENT, etc) based on relativistic Fermi gas models, resonance models developed in NP 30 years ago: do not incorporate what we have learned at JLab

Nuclear theorists have neglected to define an "interface" for these experimentalists: formulations that would allow one to go from threshold (SM, QRPA) → quasi-elastic→ resonance → deep inelastic regions -- there are ways to build on current JLab scaling analyses to do so

(August INT program)

It has been a sad year for nuclear physics and nuclear astrophysics

Vijay and his friends John and Hans