Do we properly understand the basis for independent particle motion in atomic nuclei?

I context for the early discussion 1949-58

II text book answer:
Pauli principle endowes Fermi distribution with rigidity

but this seems to be too much

III what are the alternatives?
only two alternatives and choice depends on "quantality"

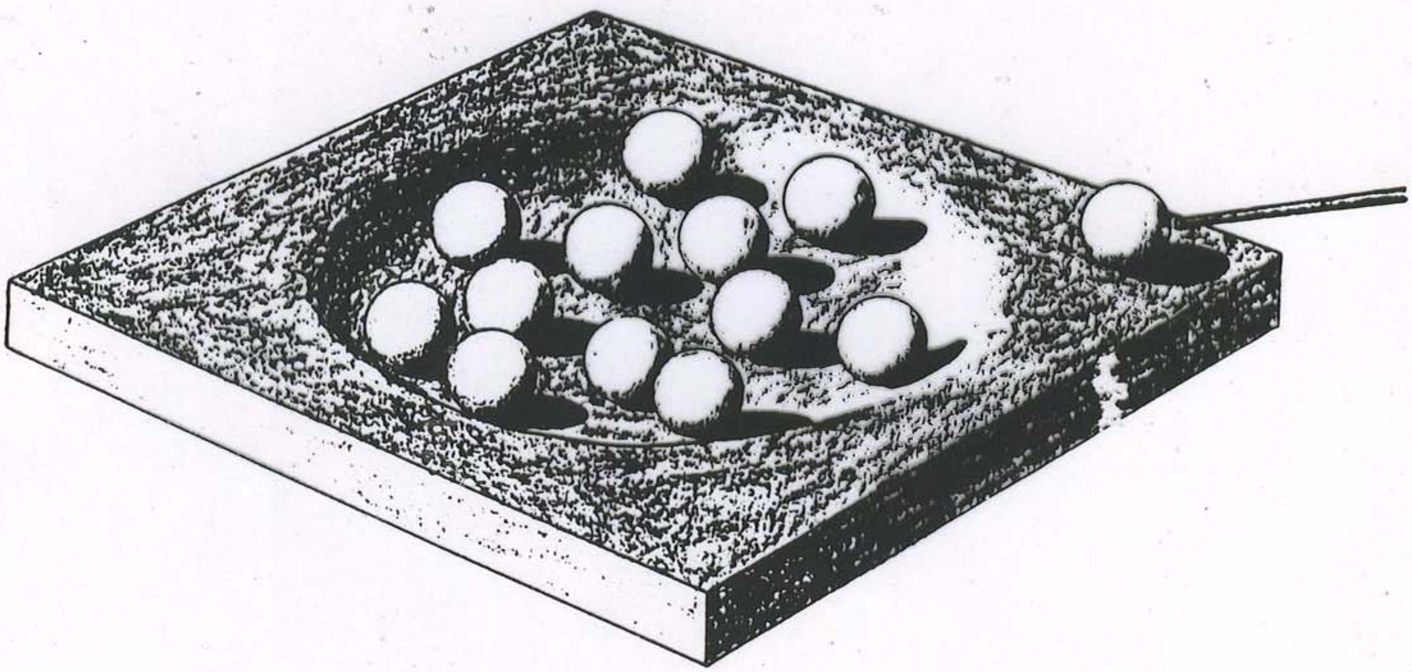
$$\Lambda = \frac{\hbar^2}{m a^2} \frac{1}{V_0}$$

but this is (almost) blind to statistics

IV a sharp look (by Vjay) at the role of statistics by study of artificial nuclei

V wider perspectives from artificial nuclei

A16



Magic Numbers for Atomic Nuclei

N or Z = 2, 8, 20, (28), 50, 82, 126

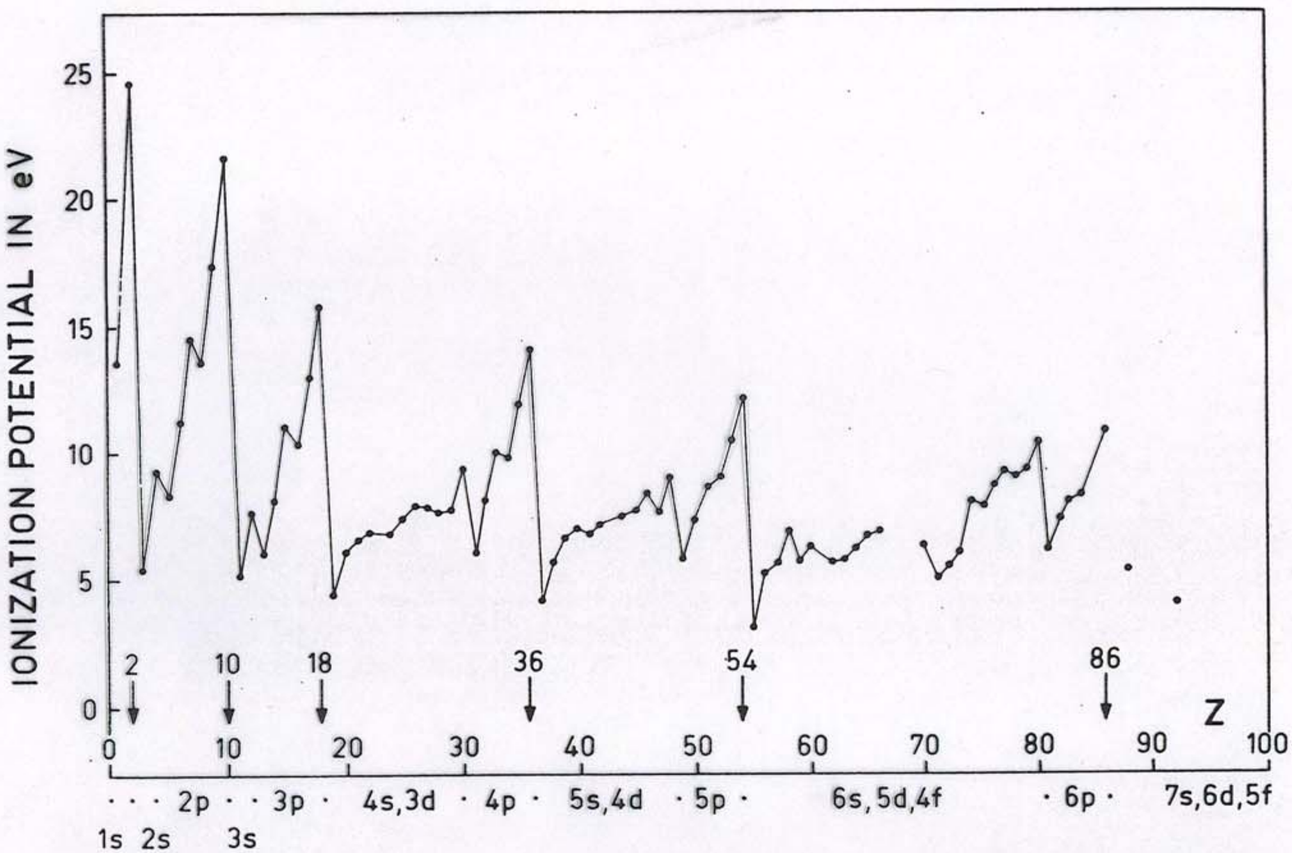


Figure 2-13 The values of the atomic ionization potentials are taken from the compilation by Moore (1949). The dots under the abscissa indicate closed shells.

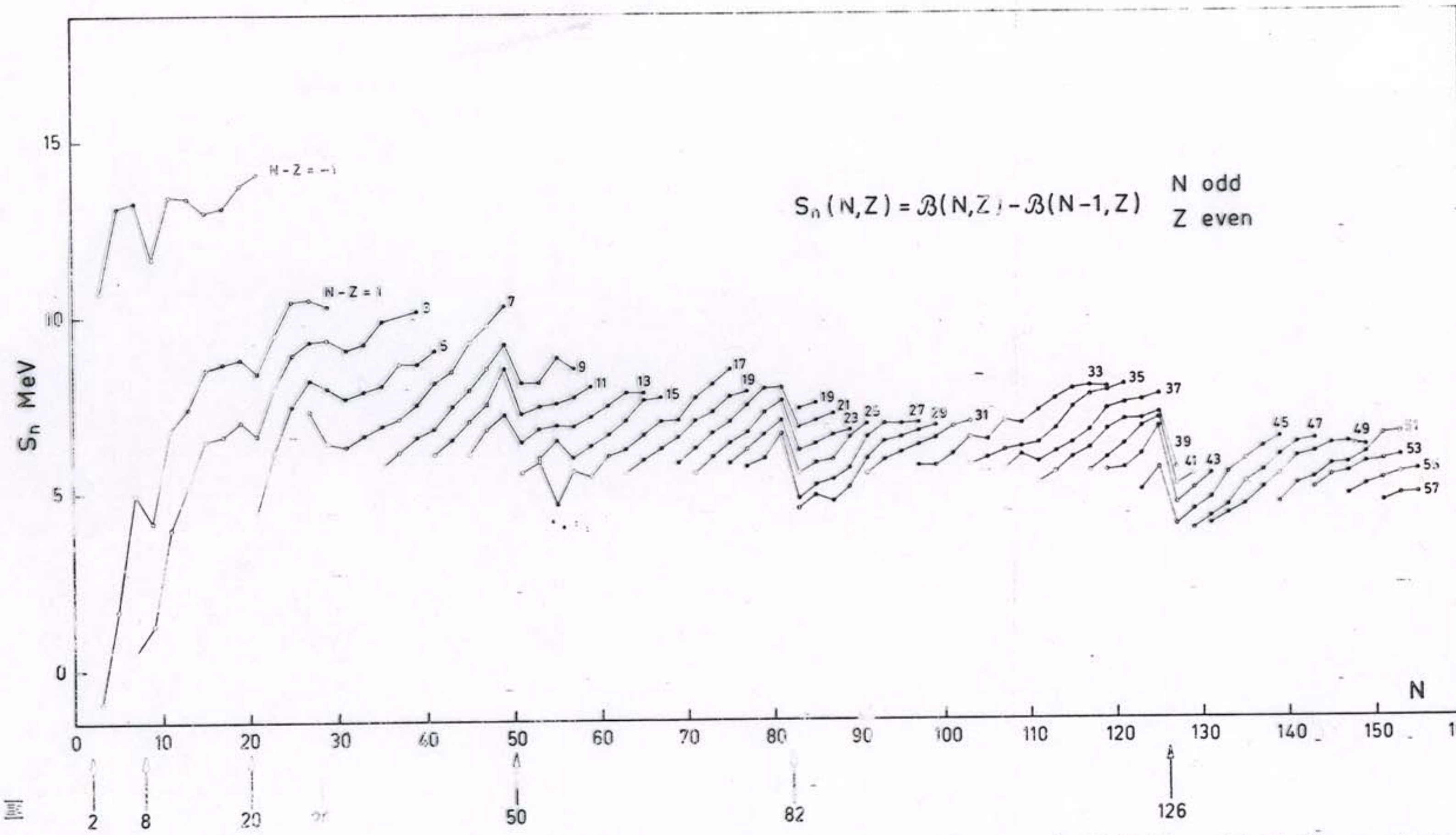
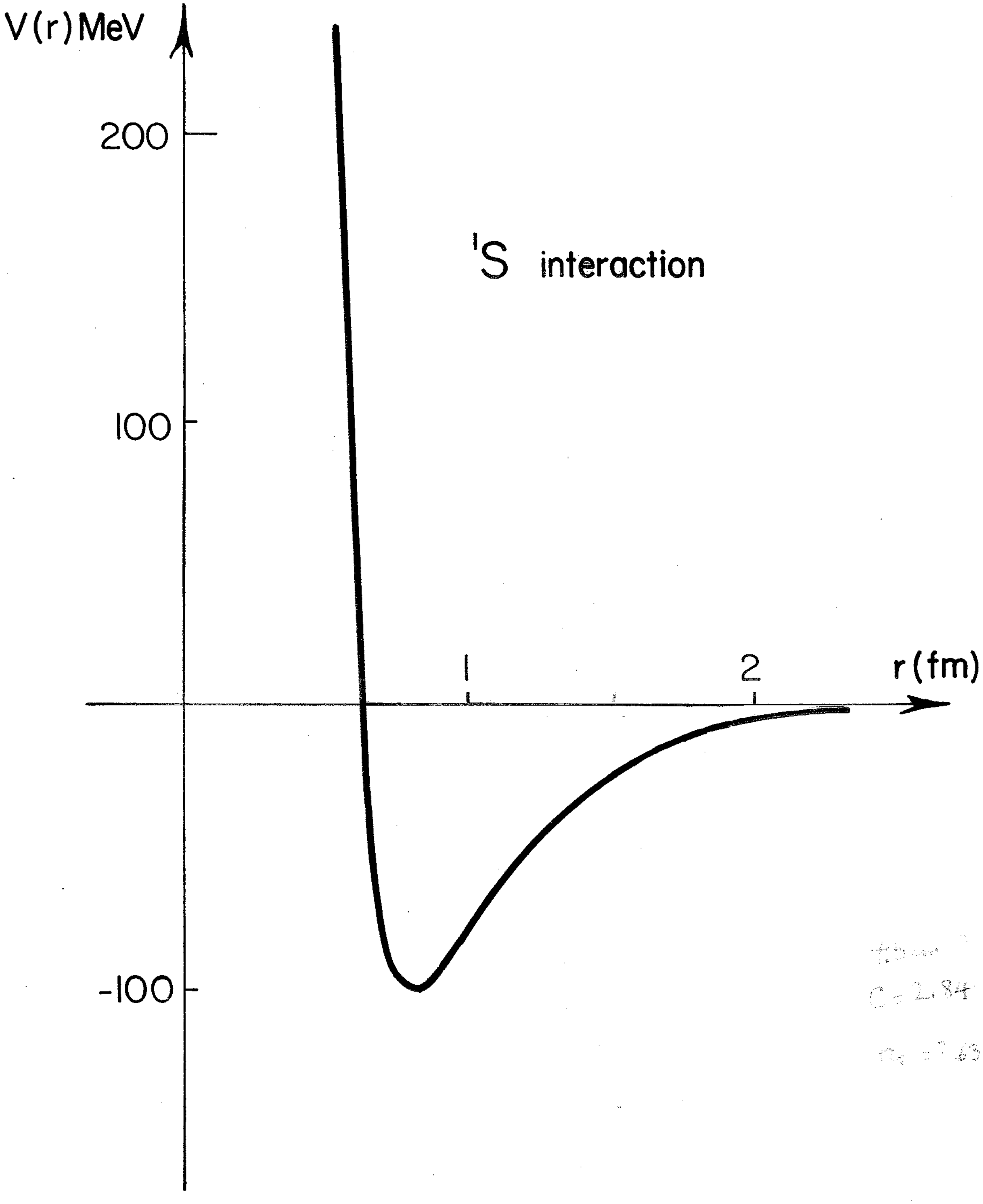


Figure 2-15 The neutron separation energies, S_n are taken from the compilation by J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, *Nuclear Phys.* **67**, 1 (1965)



Nuclear forces are indeed strong
(and very complicated)

I. the first decade 1948–1958

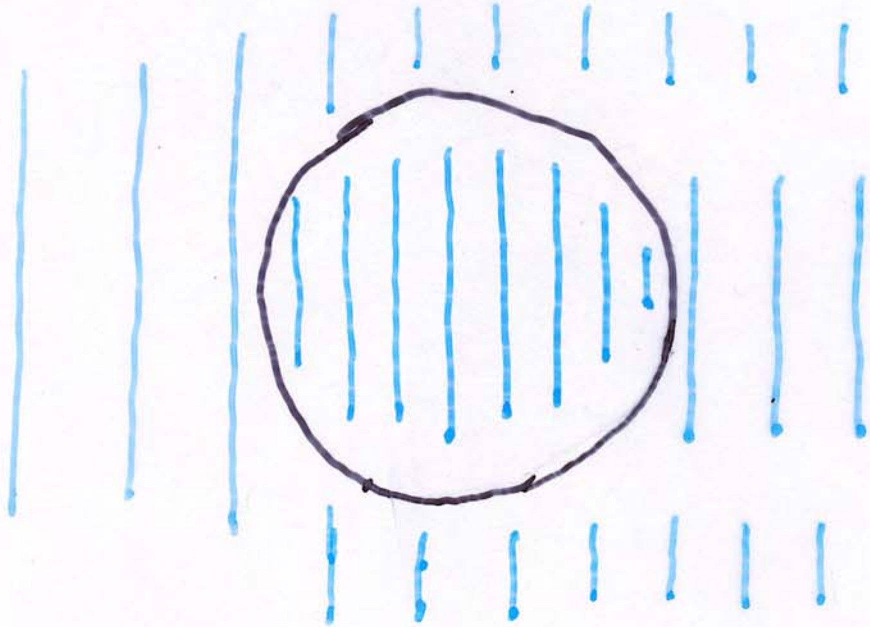
0. neutron reactions suggest mean free path, λ, in nucleus is $\lambda \ll R$
$\Rightarrow$ "compound nucleus" (1935)
this is completely dominating picture of nuclear structure for more than a decade

1. "magic numbers" (1948) suggest shell structure (1949)
but this requires $\lambda > R$ (independent particle motion)
developing nuclear spectroscopy (1949–59) confirms that IPM provides the correct degrees of freedom for low energy nuclear spectra

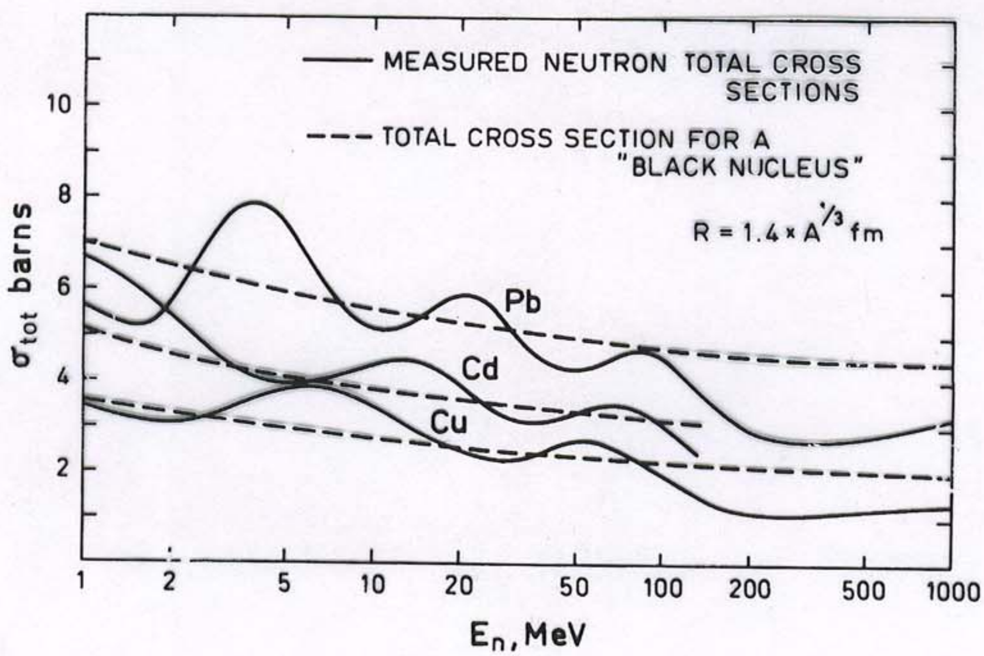
2. growing knowledge of nucleon–nucleon force seem to support $\lambda \ll R$ $R = 3-6$ fm
see f.x. Blatt and Weisskopf (1952) estimate
$$\lambda = (\rho\sigma)^{-1}$$
$$= 0.4 \text{ fm} \quad E^* \sim 10 \text{ MeV}$$

3. the mean free path is experimentally measured in nucleon–nucleus scattering (1953–63) and
$\boxed{\lambda \sim 30 \text{ fm}}$ for $E^* \sim 10$ MeV

4. the text book explaination for long mean



the transmitted wave is
phase shifted with respect
to the incident wave which
gives rise to scattering



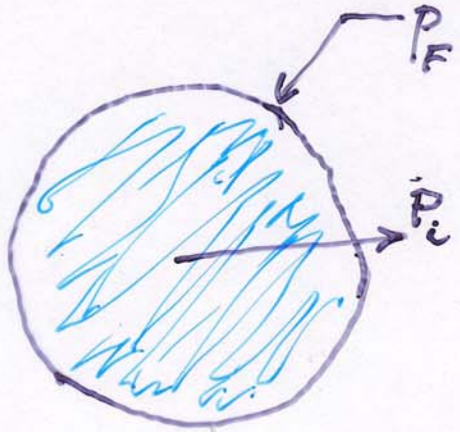
for low incident energies

$$W \approx 1\,\text{MeV} \Rightarrow \tau_c = \frac{\hbar}{2W} \approx 3\times10^{-22}\ \text{sec}$$

$$\lambda_c = \tau_c v_0 \approx 30\ \text{fm} \gg R$$

$$v_0 = \sqrt{\frac{2V_0}{M_n}} \approx 1\times10^{10}\ \text{cm/sec}$$

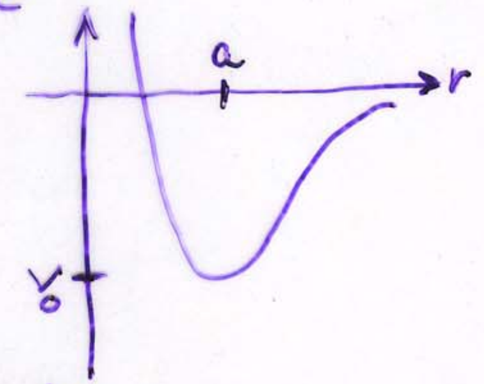
Fermi gas forbids most collisions for a particle with $\epsilon \simeq \epsilon_F$



Pauli principle:
phase space for scattering $\sim (p - p_F)^2$

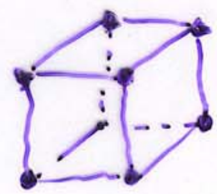
only two basically different

$\boxed{\text{Grand Designs}}$



1. forces dominate, as always in classical mechanics

<u>localized</u>, optimal positions
molecule / crystal



2. quantal kinetic energy associated with localization can dominate

$$\frac{\hbar^2}{Ma^2}$$

and yield ⇒ <u>delocalized</u> quantum liquid

quantality parameter

$$\Lambda = \frac{\hbar^2}{Ma^2}\frac{1}{V_0}$$ de Boer (1938)

comparison of "quantality" for atomic and nuclear matter

$$\boxed{\Lambda = \frac{\hbar^2}{M a^2 V_0}}$$

constituents	M	(eV) V_0	(cm) a	Λ	$T=0$ matter
^3He	3	9(-4)	2.9(-8)	0.21	liquid
^4He	4	9(-4)	2.9(-8)	0.16	liquid
H_2	2	3(-3)	3.3(-8)	0.07	solid
Ne	20	3(-3)	3.1(-8)	0.007	solid
nuclei	1	1(+8)	9(-14)	0.4	liquid

Urbana, IL.
Oct. 21, 1998

Dear Ben:

This is with reference to the role of Bose/Fermi statistics in determining the extent of independent particle motion in quantum liquid drops.

We have not done new calculations, it is not obvious that they are needed. I have looked over the results obtained by Lewart, self and Pieper (Phys. Rev. B37, 4950, 1988) for single-particle orbitals in Bose liquid ^4He and Fermi liquid ^3He drops with 70 atoms, using variational Monte Carlo method.

In the Bose drop we find 25.3 of the 70 atoms in the condensate suggesting that $\sim$ 36% of the atoms are moving independently. In the Fermi drop we find 70 single particle orbitals, which may be labeled $^1s, ^1p, ^1d, ^2s, ^1f, ^2p, ^1g, ^2d, ^3s$ in the standard fashion without spin-orbit, with an avarage occupation probability of 0.71. Higher orbitals, such as 1h have occupation numbers of $\sim$ 0.06 so that the discontinuity $Z \sim 0.65$. We can use Z to measure the extent of single particle motion in the Fermi drops. At first sight it then appears that the extent of independent particle motion in Fermi drops is twice that in Bose.

However it is very likely that much of this difference could be due to that in the densities of the drops. It is believed that the difference between the masses of ^3He and ^4He atoms is much less important than that in the density. The central densities of the Bose and Fermi drops are ~ 0.36 and 0.23 atoms$/\sigma^3$.

Crude estimates of the density dependence of the condensate fraction in ^4He and Z in ^3He liquid are given in that paper. These are:

$$n_c(\rho) = (1-0.68\,\rho/\rho_B)^2 \quad \text{Bose} \quad \rho_B = 0.365$$

$$Z(\rho) = (1-0.45\,\rho/\rho_F)^2 \quad \text{Fermi} \quad \rho_F = 0.277$$

$$= (1-0.59\,\rho/\rho_B)^2 \quad \text{Fermi using Bose } \rho_B.$$

The similarity of 0.59 and 0.68 already indicates that at same density $n_c(\rho) \sim Z(\rho)$ and thus a small effect of statistics on independent particle motion.

Using the above estimates I obtain, from $Z = 0.65$, for the Fermi drop an effective density of 0.12, while $n_c = 0.36$ gives an effective density of 0.21 for the Bose drop. The above estimate gives Bose $n_c \sim 0.6$ at $\rho = 0.12$ the effective density of the Fermi drop.

Thus, as you observed, much of the difference could be due to density. Statistics itself may have little effect on the extent of independent particle motion.

Bye, Vijay Pandharipande.

Single-particle orbitals in liquid helium drops
D.S. Lewart, V.R. Pandharipande, and S.C. Pieper
Physical Review 37, 4950 (1988)

outline of the argument

1. solve the N-body problem for $(^3He)_N$ and $(^4He)_N$
$N = 20, 40, \underline{70}, \dots$

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\vec{\nabla}_i + \sum_{i<j}^{N} V(|\vec{r}_i - \vec{r}_j|)$$

$$H\Psi_0 = E_0\Psi_0 \qquad \text{ground state}$$

2. study the one-body orbitals determined by Ψ_0

mean field orbitals $\phi_{nl}(r)$ $\rho(r)$
natural orbitals $\psi_{nl}(r)$ $\rho(\vec{r}, \vec{r}')$
quasiparticle orbitals $\chi^{(\pm)}_{nl}(r)$ $\langle \Psi^{(N\pm1)}_{0l} | \Psi^{(N)}_0 \rangle$

determine occupation probabilities in one body orbita

3. correct occupation probabilities for different quantality in ^3He and ^4He

local density approximation one body orbitals

compare the corrected occupations to exhibit role of statistics in independent particle picture for He drops

2. single particle orbitals derived from Ψ_0

(i) mean field orbitals $\phi_{n\ell}(r)$

reproduce the one particle density, $\rho(r)$ with an assumed one particle potential $\mathcal{U}(r)$

$$\rho(r) = \langle \Psi_0 | \sum_i a_i^+(\vec{r})\, a_i(\vec{r}) | \Psi_0 \rangle$$

$$= \begin{cases} N|\phi_{1s}(r)|^2 & \text{Bose system} \\ \sum_{n\ell \ni FS} 2(2\ell+1)|\phi_{n\ell}(r)|^2 & \text{Fermi system} \end{cases}$$

FS = Fermi Sphere

where $\left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + \mathcal{U}(r)\right] \phi_{n\ell} = \epsilon_{n\ell}\, \phi_{n\ell}(r)$

(ii) natural orbitals $\psi_{n\ell}(r)$

reproduce the eigen values and eigen functions of the density matrix $\rho(\vec{r},\vec{r}')$

$$\rho(\vec{r},\vec{r}') = \langle \Psi_0 | a^+(\vec{r}')\, a(\vec{r}) | \Psi_0 \rangle$$

$$= \sum_i n_i\, \psi_i^*(r)\, \psi_i(r')$$

$$= \sum_\ell \frac{2\ell+1}{4\pi} P_\ell(\hat{r}\cdot\hat{r}')\, \rho_\ell(r,r')$$

$$\rho_\ell(r,r') = \sum_n n_{n\ell}\, \psi_{n\ell}^*(r)\, \psi_{n\ell}(r')$$

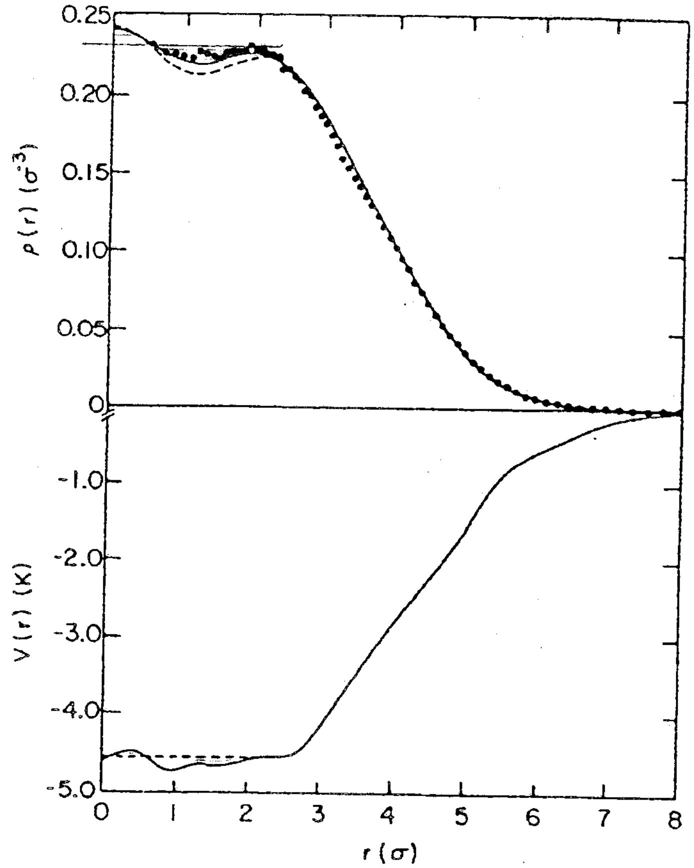


FIG. 2. The density distribution $\rho(r)$ (curves) obtained by filling the lowest 70 states in the single-particle potential $V(r)$, compared with the $\rho(r)$ obtained in Ref. 1 for the $N=70$ liquid ^{3}He drop by a Monte Carlo calculation with Ψ_v (data points). The lower panel shows $V(r)$. The solid curves are for the $V(r)$ used in this work and the dashed curves are for a flat-bottom well.

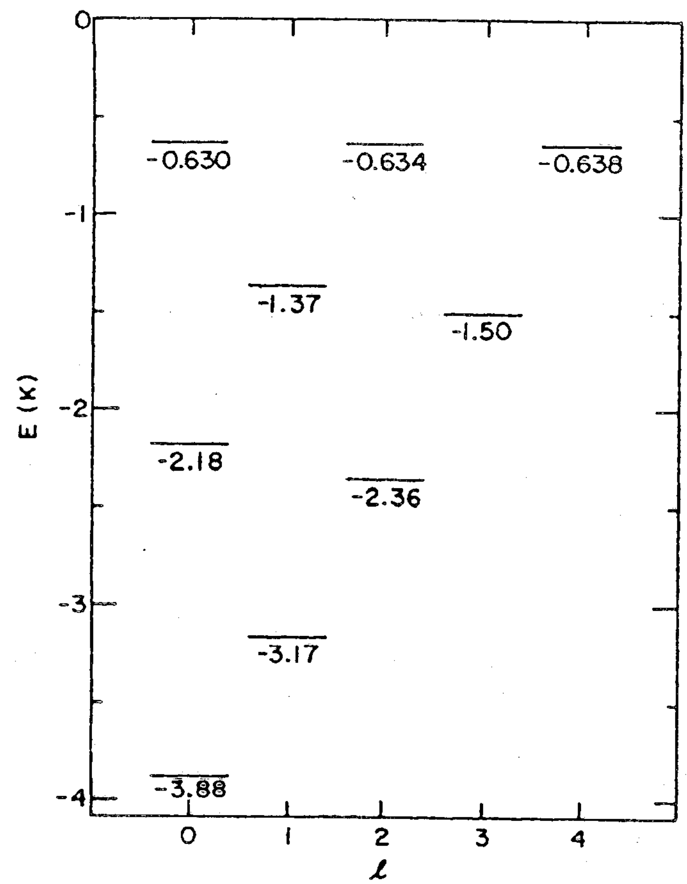


FIG. 1. The energies of single-particle states in the single-particle potential $V(r)$ shown in Fig. 2.

TABLE II. Occupation numbers of natural orbitals of the $N = 70$ Bose-liquid ^{4}He drop.

n,l	$n_{n,l}$	n,l	$n_{n,l}$	n,l	$n_{n,l}$
1s	25.33	1h	0.24	1k	0.104
1p	0.49	2f	0.22	2i	0.086
1d	0.44	3p	0.22	3g	0.078
2s	0.44	1i	0.19	4d	0.077
1f	0.37	2g	0.17	5s	0.100
2p	0.35	3d	0.16	1l	0.063
1g	0.30	4s	0.19	2j	0.060
2d	0.28	1j	0.14	3h	0.046
3s	0.30	2h	0.12	4f	0.049
		3f	0.11	5p	0.046
		4p	0.11		

$$n_c \equiv \frac{n_{1s}}{N} = 0.36$$

bulk ^{4}He

$$n_c = \frac{\rho_c}{\rho_0} \approx 0.1$$

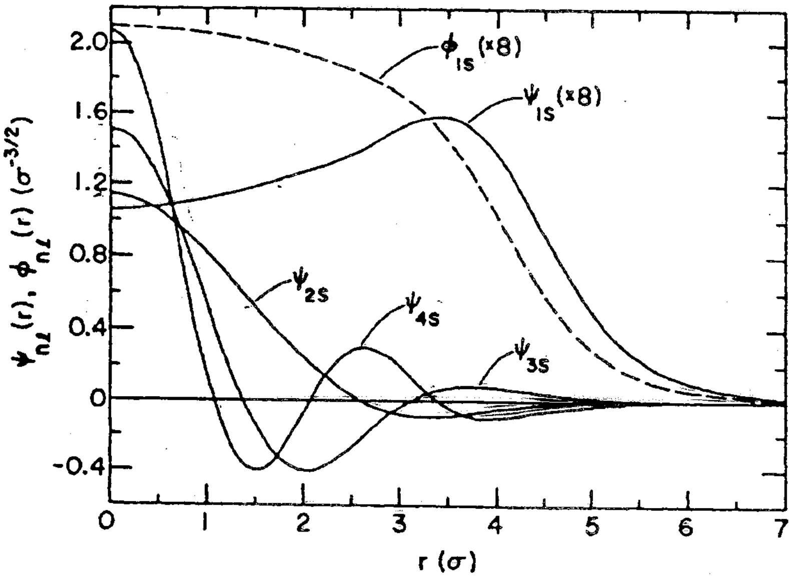


FIG. 4. The s-wave natural orbitals (1s to 4s) of the 70-particle Bose-liquid ^{4}He drop (solid lines). The dashed curve shows the 1s mean-field orbital. The ψ_{1s} and ϕ_{1s} have been multiplied by 8.

occupation numbers for natural orbitals of $(^3He)_{70}$

nl	n_{nl}
1s	0.54
1p	0.58
1d	0.60
2s	0.63
1f	0.69
2p	0.77
1g	0.75
2d	0.84
3s	0.85

nl	n_{nl}
1h	0.059
2f	0.074
3p	0.081
1i	0.048
2g	0.062
3d	0.071
4s	0.074
1j	0.034
2h	0.033
3f	0.039
4p	0.045
1k	0.024
2i	0.022
⋮	

$$N_{FS} = \sum_{nl \ni FS} 2(2l+1)\, n_{nl} = 49.5$$

$$\frac{N_{FS}}{N} = 0.71$$

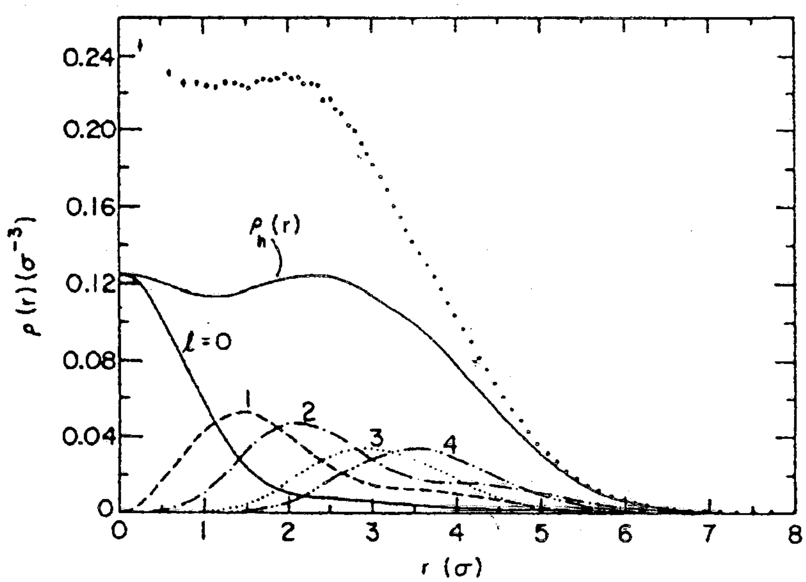


FIG. 11. The partial contribution of hole-state natural orbitals to the density $\rho_h(r)$ compared with the total $\rho(r)$ (error bars) of the $N=70$ ^{3}He drop. The curves labeled 0 to 4 show contributions of hole states with $l=0$ to 4.

3. a local density approximation accounts very well for the radial dependence of the condensate

for Bosons

$$\psi_{1s}(r) \approx A\left[1 - c\rho(r)\right]\sqrt{\frac{\rho(r)}{N}}$$

c can be determined from bulk ^4He

$$c = (1 - \sqrt{n_c})\,\rho_0^{-1}$$

similarly for Fermions

$$\psi_{nl}(r) = A_{nl}\left[1 - c'\rho(r)\right]\phi_{nl}(r)$$

yields

$$Z(r) = \left[1 - c'\rho(r)\right]^2$$

$$c' = (1 - \sqrt{Z})\,\rho_0^{-1}$$

Z and ρ_0 from bulk ^3He

and makes possible correction for the difference in quantality between $^{(3)}He$ and $^{(4)}He$

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if ρ_0 were the same for Fermions and Bosons with equal quantality, the local density approximation implies, for same quantality

$$\boxed{n_c = Z}$$

6. the difference between Z and n_c is quantitatively accounted for by the difference in bulk density of ^4He and ^3He which results from their different mass

$$\rho_0^{(3)} = 0.01635\ \text{Å}^{-3}$$

$$\rho_0^{(4)} = 0.02186\ \text{Å}^{-3}$$

density dependence of condensate is well described by local density approximation

$$\psi_{1s}(r) \simeq A\left[1 - c^{(4)}\,\frac{\rho^{(4)}(r)}{\rho_0^{(4)}}\right]\phi_{1s}(r)$$

similar local density description of ρ dependence of Z

the resulting values obey

$$\frac{c^{(4)}}{\rho_0^{(4)}} \simeq \frac{c^{(3)}}{\rho_0^{(3)}}$$

thus if the Bose and Fermi systems had been calculated for equal masses we would obtain

$$\boxed{n_c \approx Z}$$

including more recent data from

Moroni, Senatore, and Fantoni Phys Rev $\underline{B55}$ 1040 (1997)

Glyde, Azuah, and Sterling Phys Rev $\underline{B62}$ 14337 (2000)

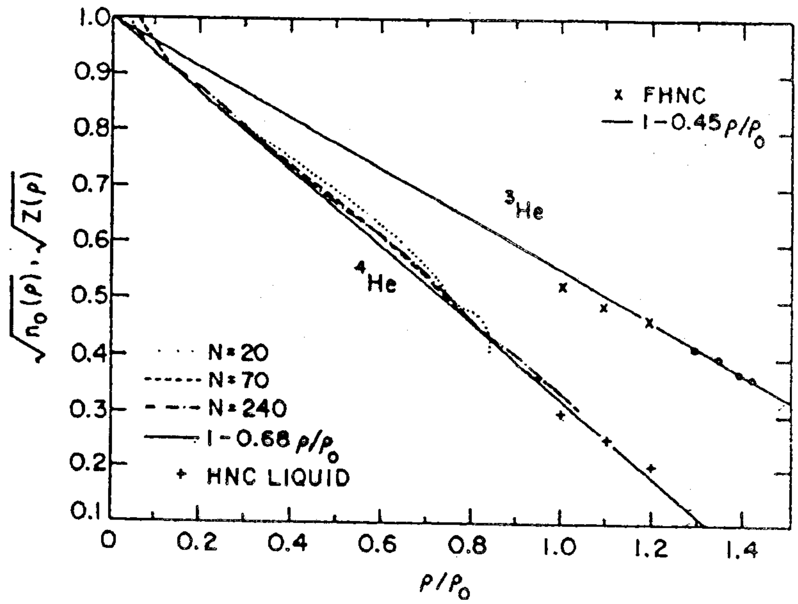

FIG. 6. Condensate amplitudes $\sqrt{n_0}$ as a function of density for liquid ^{4}He (lower curves and symbols) and the $\sqrt{z(\rho)}$ for liquid ^{3}He (upper line and symbols). The solid lines are the approximations $n_0(p)=(1-0.68\rho/\rho_0)^2$ (^{4}He) and $Z(\rho)=(1-0.45\rho/\rho_0)^2$ (^{3}He). The plus signs are from Ref. 4, the $\times$'s are from Ref. 5, and the circles are obtained by assuming that the experimental effective mass (Ref. 8) is given by $0.8/Z$ (Ref. 5). The ratio $\chi_{1s}(r)/\sqrt{\rho(r)}$, as described in the text, is shown for the 20-atom (dotted), 70-atom (dashed), and 240-atom (dot-dash) ^{4}He drops.