

## Solution

Physics 214

Problem 2

Week 7

### Is the Electron Ever Inside the Proton?

The proton has a radius of  $r_p = 0.7 \times 10^{-15}$  m ( $\equiv 0.7$  femtometers\*, or 0.7 fm). The electron in the ground state of the hydrogen atom is described by the wave function,

$$\psi_{1S}(r) = \frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o} \quad a_o = 0.053 \text{ nm is the Bohr radius.}$$

a) What is the probability of finding the electron inside the proton?

Hint: In three dimensions  $\psi^2$  has units of probability per meter<sup>3</sup>. Also,  $r_p \ll a_o$ .

Because  $r_p \ll a_o$ ,  $\psi$  is nearly constant:  $\Psi^2(0) = 1/\pi a_o^3$ . So, the probability is:

$$P(\text{inside}) = \frac{1}{\pi a_o^3} \frac{4\pi r_p^3}{3} = \frac{4}{3} \left( \frac{r_p}{a_o} \right)^3 = 3.07 \times 10^{-15}.$$

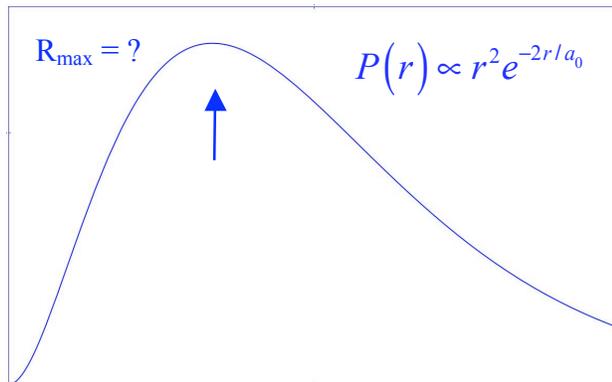
The electron is occasionally inside the proton, but the probability is tiny.

COMMENT: For heavy atoms (e.g., gold) the probability is much larger, because the orbit is smaller, and the nucleus is bigger.

b) Near what distance from the nucleus,  $r$ , are you most likely to find the electron? Hint: The probability of finding the particle within a thin shell of radius  $r$  and thickness  $dr$  is

$|\psi(r)|^2 d\text{Vol} = |\psi(r)|^2 (4\pi r^2 dr) \equiv P(r) dr$ . Sketch  $P(r)$ . How do you find the maximum of this?

This probability will be maximum at the peak. To find this, we simply set  $dP(r)/dr = 0$ :



$$\frac{dP(r)}{dr} \propto \frac{d}{dr} (r^2 e^{-2r/a_o}) = (2r) e^{-2r/a_o} + r^2 \left( \frac{-2}{a_o} e^{-2r/a_o} \right) = r e^{-2r/a_o} \left( 1 - \frac{r}{a_o} \right) = 0.$$

$$\therefore r = a_o$$

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\* The femtometer [fm] is also called the “fermi”, after Enrico Fermi, the eponym for Fermilab.