

Einstein's paper is a "bold, not to say reckless, hypothesis...which flies in the face of thoroughly established facts of interference."

--R. Millikan

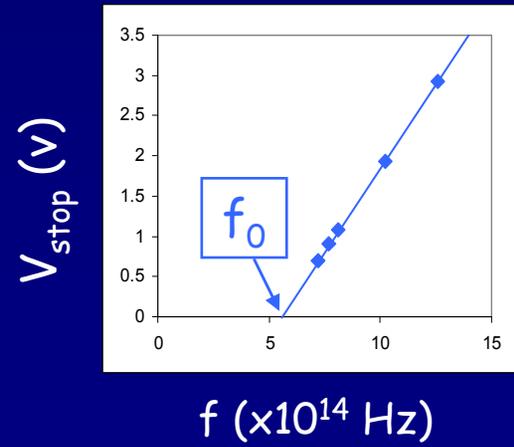
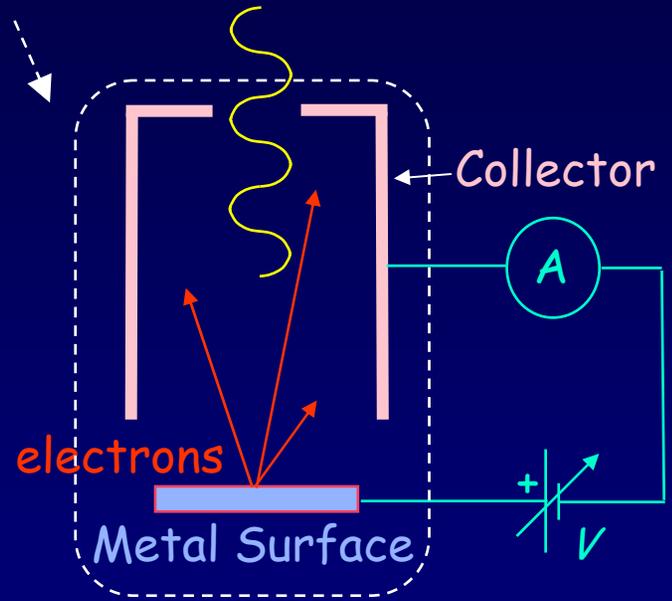
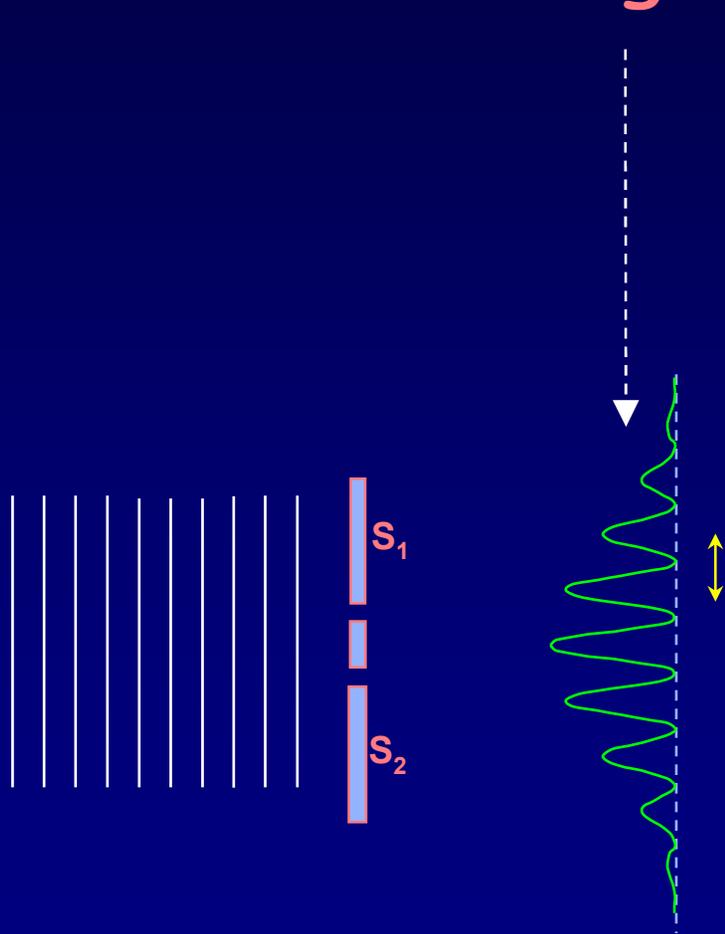
(discover of electric charge)

"Its unambiguous experimental verification in spite of its unreasonableness since it seems to violate everything that we knew about the interference of light."

--R. Millikan

(after testing Einstein's photoelectric effect predictions)

# Lecture 7: Introduction to Quantum Mechanics: Light as Particles



# This week and next week are critical for the course:

Week 3, Lectures 7-9:

Light as Particles  
Particles as waves  
Probability  
Uncertainty Principle

Week 4, Lectures 10-12:

Schrödinger Equation  
Particles in infinite wells, finite wells  
Simple Harmonic Oscillator

**Midterm Exam** Monday, week 5

It will cover lectures 1-12 (except Simple Harmonic Oscillators)

Practice exams: Old exams are linked from the course web page.

Review: Sunday before Midterm

Office hours: Sunday and Monday

# Thumbnail Summary of Waves

Wave relationships:

Wavelength, frequency, speed, amplitude, intensity

$$v = f\lambda, I = A^2, \text{ etc.}$$

2-slit interference:

Phase difference depends on source phases and path lengths.

$$A_{\text{tot}} = 2A_1 \cos(\phi/2), \text{ etc.}$$

N-slit interference:

Diffraction gratings, Rayleigh's criterion.

1-slit diffraction:

Circular apertures, Rayleigh's criterion, limits on optics.

Interferometers.

We'll use many of these results when we study quantum mechanics.

# Crystal diffraction

How do we know the atomic scale structure of matter around us?

A crystal is a very large number of atoms or molecules arranged in a periodic fashion. It acts like a grating with an extremely large number ( $\sim$ Avagadro's number) of units that diffract waves coherently.

Every crystal has its own "signature" of the various spacings between atoms.

By measuring the diffraction, we can determine the atomic scale structure.

Typical distances between atoms are of order 0.1-0.3 nm ( $1-3 \times 10^{-10}$  m).

What characteristic wavelengths are needed to study crystals?

We want:

- $\lambda < \text{spacing}$  (so that we can get  $\delta > \lambda$ ).
- $\lambda$  not too small (so that  $\theta$  isn't too small).

That is:  $\lambda \sim 10^{-10}$  m.  $\Rightarrow$  **X-rays!**

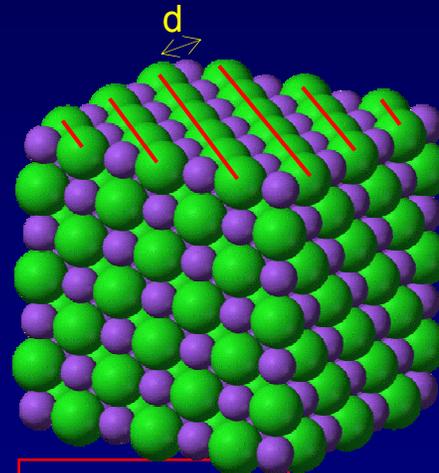
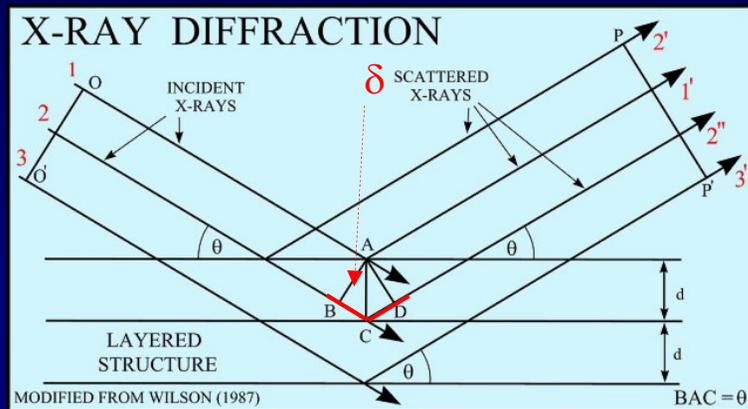
# FYI: Crystal diffraction

The structure of the crystal can be found using almost the same law we have for optical gratings!

**Bragg Law** for constructive interference:  $\delta = 2d \sin\theta = m\lambda$

$d$  = lattice spacing,  $\lambda$  = x-ray wavelength

$\theta$  = x-ray angle (with respect to plane of crystal)



Example of planes in a NaCl crystal



Diffraction pattern from single crystal of Nd<sub>2</sub>CuO<sub>4</sub>

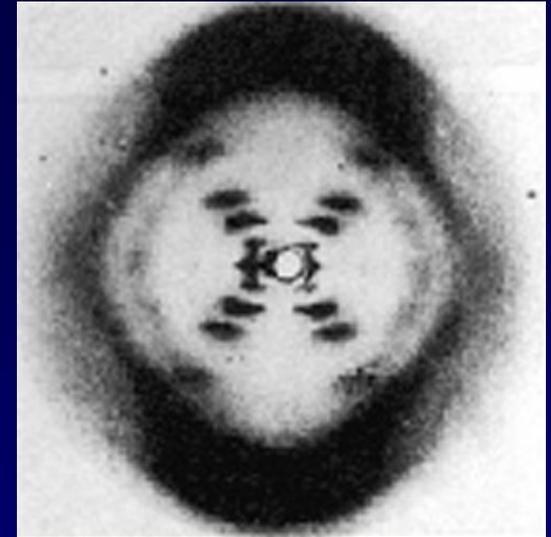
Each crystal has many values of  $d$  - the distances between different planes. For a known wavelength  $\lambda$  the observed angles  $\theta$  can be used to determine the crystal structure.

# FYI: Application: Structure of DNA

How do we know the structures of DNA, proteins and other biological molecules? X-ray diffraction!

The molecules are crystallized to create a crystal in which the molecules are arranged in a periodic lattice. By using the sharp Bragg diffraction from many molecules, the structure of each molecule is determined - the positions of thousands of atoms.

The original diffraction image of DNA taken by Rosalind Franklin in 1953

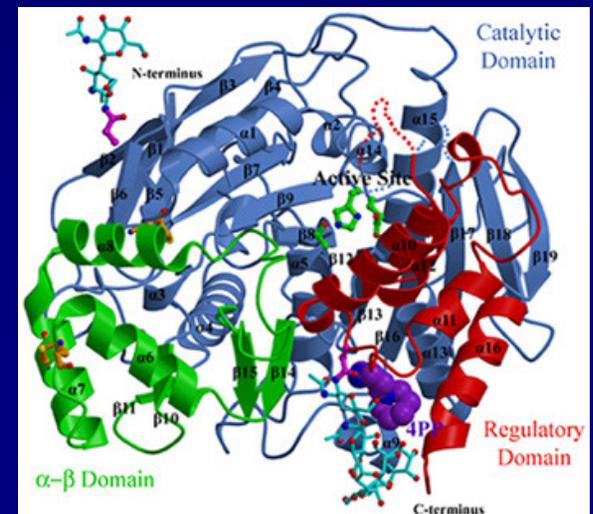


See figure and more details on this and other X-ray scattering in the text, sec. 36.6

X-rays remain the primary methods for establishing the atomic scale structures of complex molecules.

Example: Rabbit liver carboxylesterase (one molecule showing atomic groups and attached large scale structures with atoms not shown).

Source: St. Jude Children Research Hospital  
[http://www.stjude.org/faculty/0,2512,407\\_2030\\_4278,00.html](http://www.stjude.org/faculty/0,2512,407_2030_4278,00.html)



# Today

Photoelectric Effect:

Light as particles

Photons - Quanta of electromagnetic waves

The fundamental QM relations:

Energy:  $E = hf$  or  $E = \hbar\omega$  ( $\hbar = h/2\pi$  and  $\omega = 2\pi f$ )

Momentum:  $p = h/\lambda$

These equations relate the wave and particle properties of all quantum mechanical entities.

Reading in the text: 38.1-2,9 and 39.1-5

Note: All reading assignments are listed on the syllabus page.

# Wave-particle Duality for Light and Matter

In Physics 212 and the first 4 Lectures of Physics 214, we considered “light” to be a wave.

This was established by experiment in the 19<sup>th</sup> century (cf. Poisson spot)  
Electromagnetic waves exhibit interference and diffraction.

Surprise:

In the early 20<sup>th</sup> century, it was discovered that  
**light has particle-like properties**  
(e.g., localized lumps of energy) in some situations!

Furthermore, **matter exhibits wave-like properties**  
(e.g., electrons, protons, etc.) under certain circumstances.

It may seem surprising that an entity might exhibit both  
“wave-like” and “particle-like” properties!

Let's look at some of the evidence.

# Photoelectric Effect (1)

Not phase difference!

Electrons in a metal are bound by an energy  $\Phi$ , called the **work function**.

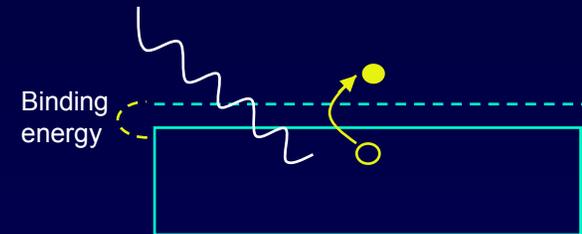
If you shine light on a clean metal surface, electrons can emerge  $\rightarrow$  the light gives the electrons enough energy ( $> \Phi$ ) to escape.

Measure the flow of electrons with an ammeter.

How will the current depend on intensity and frequency?

We might expect:

- Increasing the intensity should increase the current. Or maybe the energy of the electrons.
- Increasing the frequency shouldn't matter much. Perhaps a decrease in current due to rapid oscillations.
- With low intensity, there should be a time delay before current starts to flow, to build up enough energy.



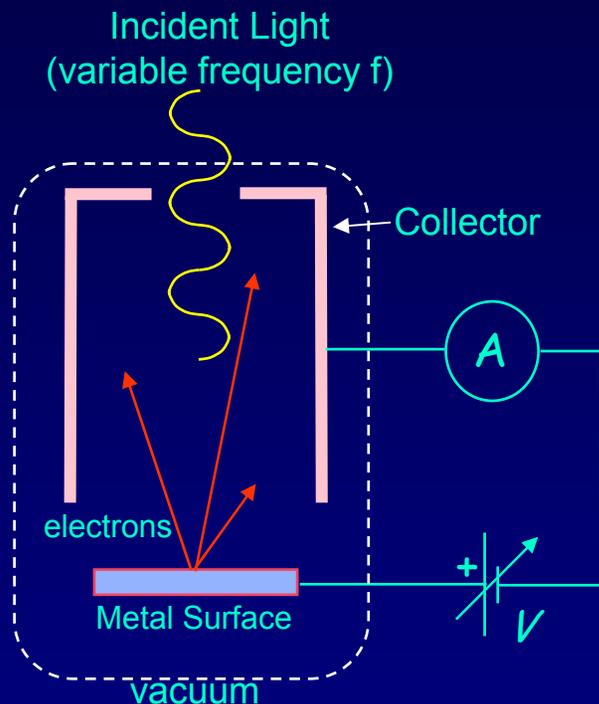
Workfunction  $\Phi$ :

- minimum energy needed to liberate an electron from the metal.
- defined to be positive.
- usually given in eV units

This follows from the idea that light is a continuous wave that consists of an oscillating E and B field. The intensity is proportional to  $E^2$ .

# Photoelectric Effect (2)

Experiment 1: Measure the maximum energy ( $KE_{\max}$ ) of ejected electrons



Bias the collector with a negative voltage to repel ejected electrons.

Increase bias voltage until flow of ejected electrons decreases to zero.

Current = 0 at  $V = V_{\text{stop}}$ . (the definition of  $V_{\text{stop}}$ )

$V_{\text{stop}}$  tells us the maximum kinetic energy:

$$KE_{\max} = eV_{\text{stop}}$$

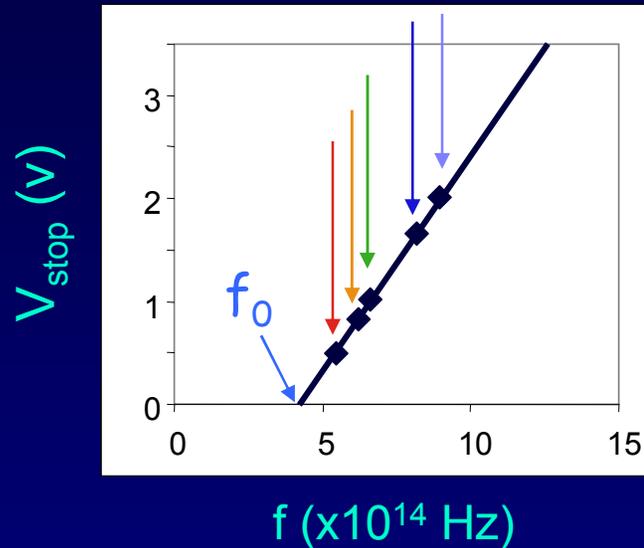
The result:

The stopping voltage is independent of light intensity.

Therefore, increasing the intensity does not increase the electron's KE!

# Photoelectric Effect (3)

Experiment 2: Measure  $V_{\text{stop}}$  vs  $f$



$$KE_{\text{max}} = e \cdot V_{\text{stop}} = h(f - f_0) = hf - \Phi$$

The slope:  $h$ , is Planck's constant.

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$$

The -y intercept:  $\Phi$ , is the work function.

Note that  $\Phi = hf_0$ .  $\Phi$  is positive.

The results:

The stopping voltage  $V_{\text{stop}}$  (the maximum kinetic energy of electrons) increases linearly with frequency.

Below a certain frequency  $f_0$ , no electrons are emitted, even for intense light! This makes no sense classically. Increasing the electric field should have an effect.

# Photoelectric Effect (4)

## Summary of Results:

- Electron energy depends on frequency, not intensity.
- Electrons are not ejected for frequencies below  $f_0$ .
- Electrons have a probability to be emitted immediately.

## Conclusions:

- Light arrives in “packets” of energy (photons).
- $E_{\text{photon}} = hf$  ← We will see that this is valid for all objects. It is the fundamental QM connection between an object’s wave and particle properties.
- Increasing the power increases # photons, not the photon energy. Each photon ejects (at most) one electron from the metal.

Recall: For EM waves, frequency and wavelength are related by:  $f = c/\lambda$ .

Therefore:  $E_{\text{photon}} = hc/\lambda$

Beware: This is only valid for EM waves,  
as evidenced by the fact that the speed is  $c$ .

# Convenient Units for Quantum Mechanics

Because most of the applications we will consider involve atoms, it is useful to use units appropriate to those objects.

We will express wavelength in **nanometers** (nm).

We will express energy in **electron volts** (eV).

1 eV = energy an electron gains moving across a one volt potential difference:  
1 eV =  $(1.6022 \times 10^{-19} \text{ Coulomb})(1 \text{ volt}) = 1.6022 \times 10^{-19} \text{ Joules}$ .

Therefore, SI units:  $h = 6.626 \times 10^{-34} \text{ J-s}$  and  $hc = 1.986 \times 10^{-25} \text{ J-m}$   
eV units:  $h = 4.14 \times 10^{-15} \text{ eV-s}$ , and  $hc = 1240 \text{ eV-nm}$ .

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$$

$E_{\text{photon}}$  in electron volts  
 $\lambda$  in nanometers

Example: A red photon with  $\lambda = 620 \text{ nm}$  has  $E = 2 \text{ eV}$ .

# Act 1

1. It is observed that shining light with wavelength 310 nm on a material will eject electrons, while 312 nm will not. What is the workfunction of the material?

Hint: What is  $V_{\text{stop}}$  at the maximum wavelength (minimum frequency)?

- a) +2 V      b) -2 V      c) +2 eV      d) -4 eV      e) +4 eV

2. If the same light is shined onto a material with  $\phi = 2 \text{ eV}$ , what stopping voltage will prevent all electrons from making it to the collector?

- a) +2 V      b) + 4 V      c) +2 eV      d) -4 eV      e) +4 eV

# Act 1 - Solution

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e) +4 eV

The workfunction must have units of energy, and is defined to be *positive*, so c) and e) are the only possible candidates.  $V_{\text{stop}} = 0$ , so  $E_{\text{photon}} = \Phi$ .  
Use  $E = hc / \lambda \rightarrow \lambda = hc / E = 1240 \text{ eV}\cdot\text{nm} / 310 \text{ nm} = 4 \text{ eV}$

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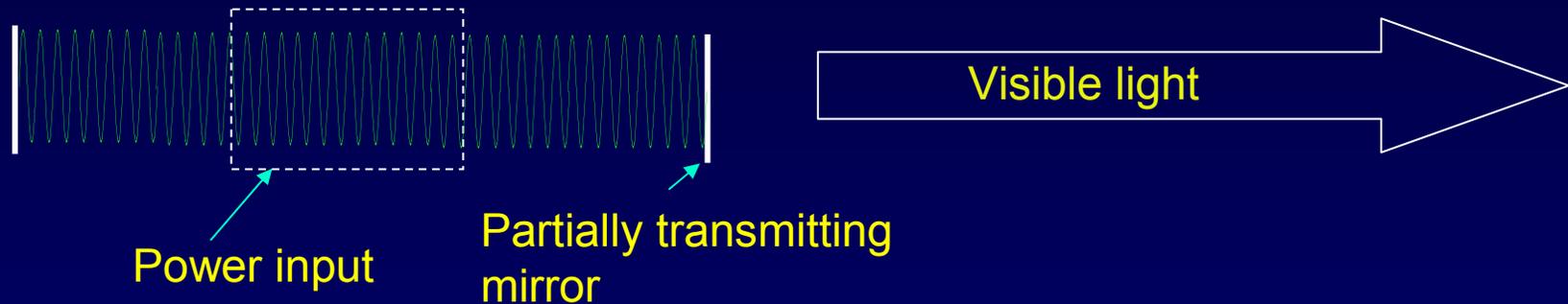
d) -4 eV

e) +4 eV

The answer must have units of voltage. Since 2 eV of every incident photon's energy (4 eV) goes to freeing each electron, they can leave with  $\leq 2 \text{ eV}$  in kinetic energy. A potential of 2 V is needed to slow such electrons to 0.

# Discrete vs Continuous

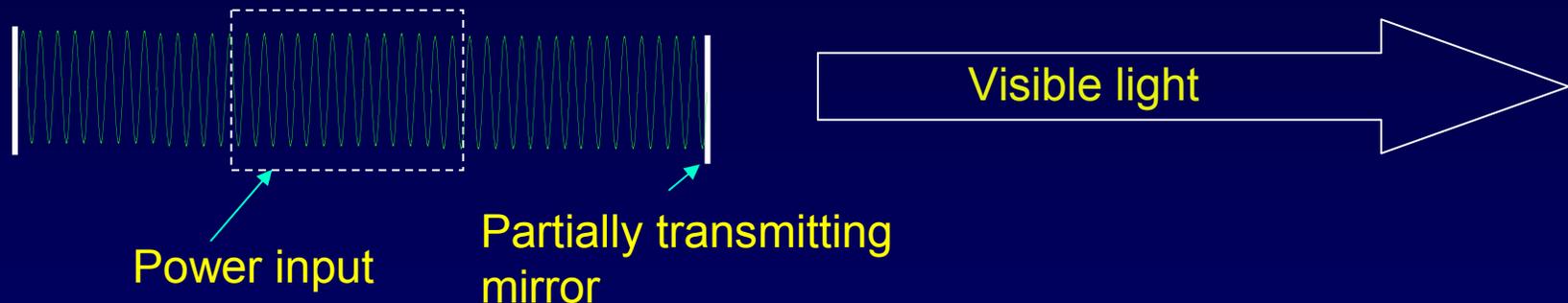
Can we reconcile the notion that light comes in 'packets' with our view of an electromagnetic wave, e.g., from a laser?



How many photons per second are emitted from a 1-mW laser ( $\lambda=635$  nm)?

# Solution

Can we reconcile the notion that light comes in 'packets' with our view of an electromagnetic wave, e.g., from a laser?



How many photons per second are emitted from a 1-mW laser ( $\lambda=635$  nm)?

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{635 \text{ nm}} \approx 2 \text{ eV}$$

$$\text{Power output: } P = (\# \text{ photons/sec}) \times E_{\text{photon}}$$

$$\# \text{ photons/sec} = \frac{P}{E_{\text{photon}}} = \frac{10^{-3} \text{ J}}{\text{s}} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \times \frac{1 \text{ photon}}{2 \text{ eV}} = 3 \times 10^{15} \text{ s}^{-1}$$

This is an incredibly huge number. Your eye cannot resolve photons arriving every femtosecond (though the rods can detect single photons!).

## Act 2: Counting photons

Which emits more photons, a 1-mW cell phone ( $f = 830 \text{ MHz} \rightarrow \lambda = 0.36 \text{ m}$ ) or a 1-mW laser ( $\lambda = 635 \text{ nm}$ )?

- a) Laser emits more
- b) They emit the same number
- c) Cell phone emits more

## Act 2: Solution

Which emits more photons, a 1-mW cell phone ( $f = 830 \text{ MHz} \rightarrow \lambda = 0.36 \text{ m}$ ) or a 1-mW laser ( $\lambda = 635 \text{ nm}$ )?

- a) Laser emits more
- b) They emit the same number
- c) Cell phone emits more

Because the cell frequency is much less than the optical frequency, each cell-phone photon has much less energy. Therefore, you need many more of them to get the same total energy.

$$\text{Rate} \propto \lambda \quad \therefore \frac{\text{Rate}_{\text{cell}}}{\text{Rate}_{\text{laser}}} = \frac{\lambda_{\text{cell}}}{\lambda_{\text{laser}}} = \frac{0.36 \text{ m}}{635 \times 10^{-9} \text{ m}} = 5.7 \times 10^5$$

Cell phones actually emit  $\sim 1\text{W} \rightarrow \sim 10^{24}$  photons/sec

# Formation of Optical Images

The point:

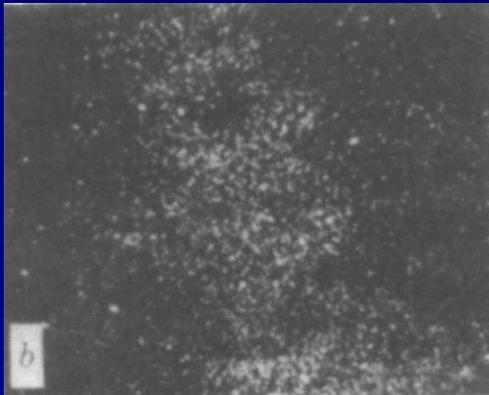
Processes that seem to be continuous may, in fact, consist of many microscopic “bits”.  
(Just like water flow.)

For large light intensities, image formation by an optical system can be described by classical optics.



For very low light intensities, one can see the statistical and random nature of image formation. Use a sensitive camera that can detect single photons.

A. Rose, J. Opt. Sci. Am. 43, 715 (1953)

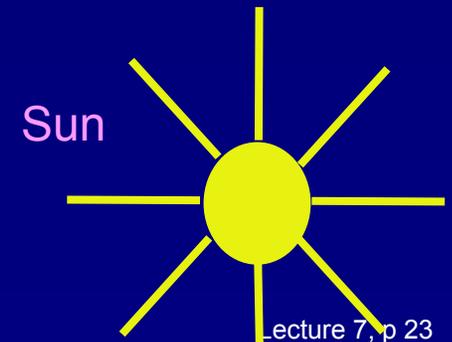
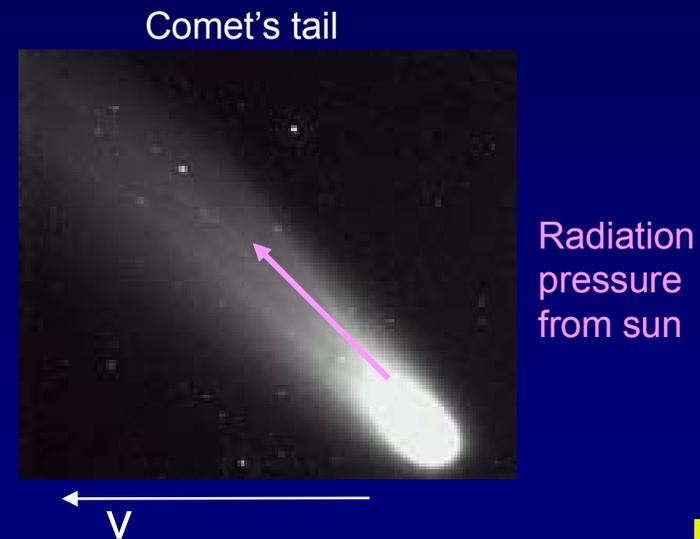
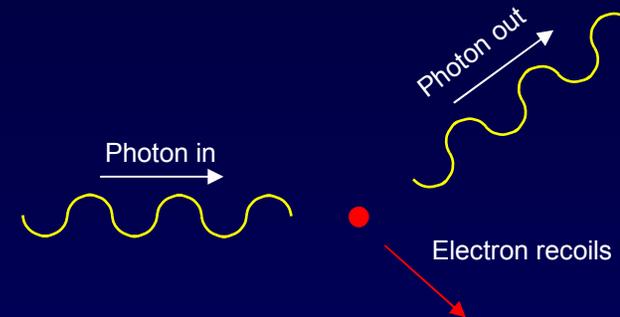


Exposure time →

# Momentum of a Photon (1)

Between 1919 and 1923, A.H. Compton showed that x-ray photons collide elastically with electrons in the same way that two particles would elastically collide! “Compton Scattering”

Photons carry momentum!  
Perhaps this shouldn't surprise us:  
Maxwell's equations also predict that light waves have  $p = E/c$ .



Why not  $E_{\text{photon}} = p^2/2m$  ? (Physics 211) Because photons have no mass.  
 $E_{\text{photon}} = pc$  comes from special relativity, which more generally says  $E^2 = m^2c^4 + p^2c^2$ . For the photon,  $m = 0$ .

# Momentum of a Photon (2)

What is the momentum of a photon?

Combine the two equations:

$$E_{\text{photon}} = hf = hc/\lambda \quad \text{– quantum mechanics}$$

$$p = E/c \quad \text{– Maxwell's equations, or special relativity}$$

This leads to the relation between momentum and wavelength:

$$p_{\text{photon}} = hf/c = h/\lambda$$

These are the key relations of quantum mechanics:

$$E = hf$$
$$p = h/\lambda$$

They relate an object's particle properties  
(energy and momentum)  
to its wave properties  
(frequency and wavelength).

Remember:  
 $E = hc/\lambda$   
 $p = hf/c$   
are only valid  
for photons

So far, we discussed the relations only for light.

**But they hold for all matter!** We'll discuss this next lecture.

# Summary

Photoelectric Effect → light as particles

Measurement shows directly that light (all electromagnetic waves) are quantized with quanta denoted by “Photons”

Energy and momentum of a photon

Key relations of quantum mechanics:

$E=hf$  or  $E = \hbar\omega$  and  $p=h/\lambda$  where  $\hbar = h/2\pi$

Derived here for light – but valid for all matter – more later!

# Next Lecture

Interference, revisited

Only indistinguishable processes can interfere

Wave nature of particles

Proposed by DeBroglie in 1923, to explain atomic structure.  
Demonstrated by diffraction from crystals – just like X-rays!

Matter-wave Interference

Double-slit interference pattern, just like photons  
Electron microscopy

# FYI: The origins of quantum mechanics

- 1900 Planck “solves” the blackbody problem by postulating that the oscillators that emit light have quantized energy levels.
  - “Until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me...the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be.”
- 1905 Einstein proposes that light energy is quantized with quanta called “photons” - waves behave like particles
  - Photoelectric effect for which he got the Nobel Prize
- 1913 Bohr proposes that electron orbits are quantized
  - Idea that electrons act like waves - “explained” H atom, but wrong in crucial ways
- 1923 de Broglie proposes that particles behave like waves
  - The step that paved the way for understanding all of nature
- 1925 Pauli introduces “exclusion principle” – only 2 electrons/orbital
  - The step that leads to understanding of electrons in atoms, molecules, solids
- 1926 Schrödinger introduces the wave-formulation of QM
  - The fundamental equation that predicts the nature of matter
- 1927 Heisenberg uncertainty principle
  - The principle that shows the fundamental uncertainty in any one measurement
- 1928 Dirac combines quantum mechanics and special relativity
  - The step that made QM “the most successful theory in the history of physics” – description of atoms, nuclei, elementary particles, prediction of antimatter, . . .