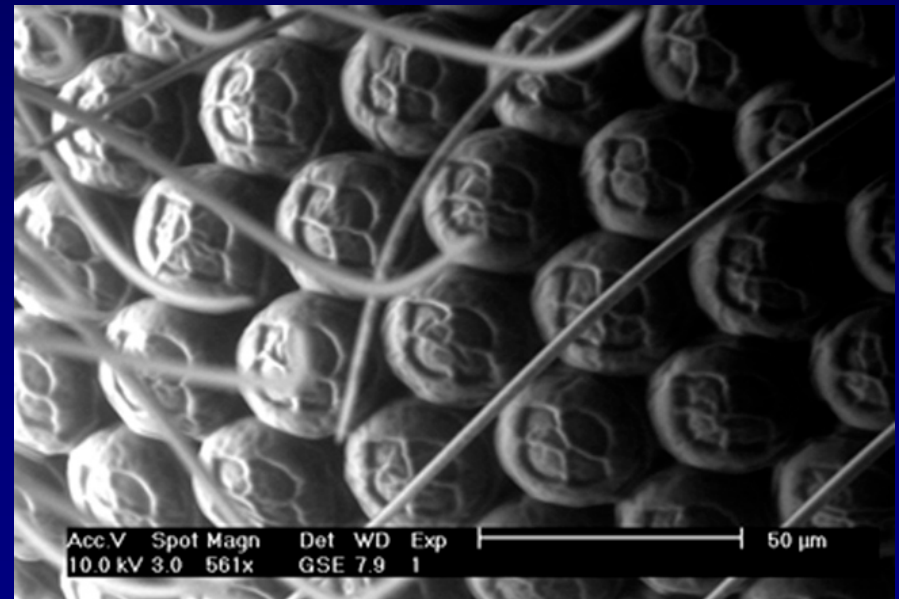
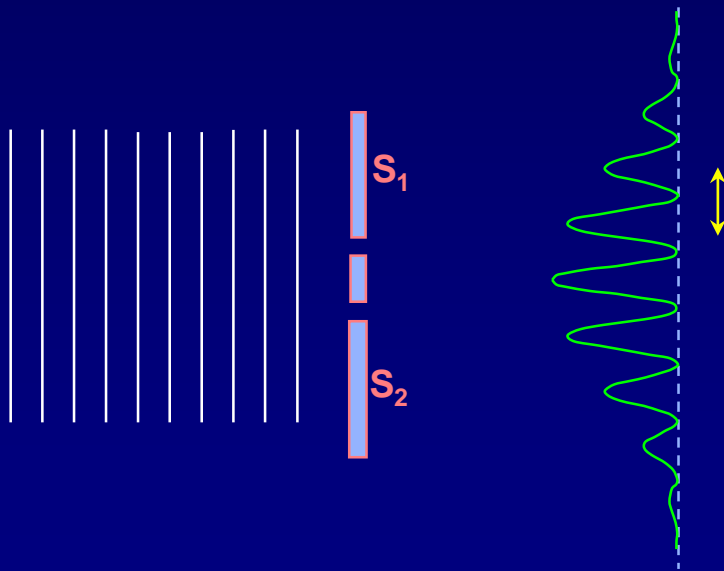


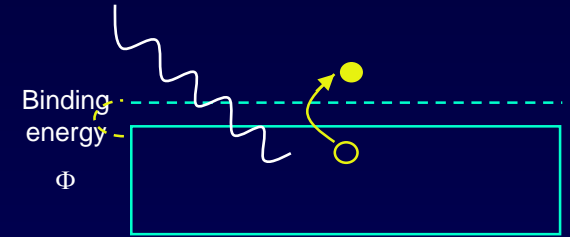
# Lecture 9:

## Introduction to QM: Review and Examples



# Photoelectric Effect

$$KE_{\max} = e \cdot V_{\text{stop}} = hf - \Phi$$

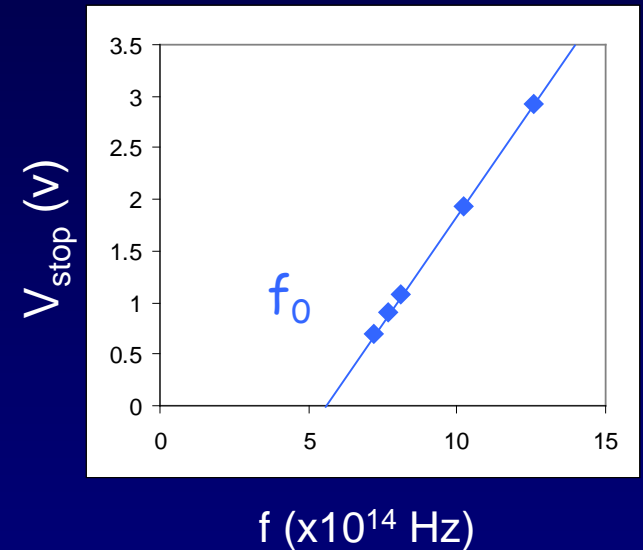


The work function:

- $\Phi$  is the *minimum* energy needed to strip an electron from the metal.
- $\Phi$  is defined as **positive**.
- Not all electrons will leave with the maximum kinetic energy (due to losses).

Conclusions:

- Light arrives in “packets” of energy (photons).
- $E_{\text{photon}} = hf$
- Increasing the intensity increases # photons, not the photon energy. Each photon ejects (at most) one electron from the metal.



Recall: For EM waves, frequency and wavelength are related by  $f = c/\lambda$ .

Therefore:  $E_{\text{photon}} = hc/\lambda = 1240 \text{ eV}\cdot\text{nm}/\lambda$

# Photoelectric Effect Example

1. When light of wavelength  $\lambda = 400 \text{ nm}$  shines on lithium, the stopping voltage of the electrons is  $V_{\text{stop}} = 0.21 \text{ V}$ . What is the work function of lithium?

# Photoelectric Effect: Solution

1. When light of wavelength  $\lambda = 400 \text{ nm}$  shines on lithium, the stopping voltage of the electrons is  $V_{\text{stop}} = 0.21 \text{ V}$ . What is the work function of lithium?

$$\begin{aligned}\Phi &= hf - eV_{\text{stop}} \\ &= 3.1 \text{ eV} - 0.21 \text{ eV} \\ &= 2.89 \text{ eV}\end{aligned}$$

Instead of  $hf$ , use  $hc/\lambda$ :  $1240/400 = 3.1 \text{ eV}$

For  $V_{\text{stop}} = 0.21 \text{ V}$ ,  $eV_{\text{stop}} = 0.21 \text{ eV}$

## UNITS

1 eV = energy an electron gains moving across a one volt potential difference:

$$1 \text{ eV} = (1.6022 \times 10^{-19} \text{ Coulomb})(1 \text{ volt}) = 1.6022 \times 10^{-19} \text{ Joules}.$$

Therefore,

SI units:  $h = 6.626 \times 10^{-34} \text{ J-s}$  and  $hc = 1.986 \times 10^{-25} \text{ J-m}$

eV units:  $h = 4.14 \times 10^{-15} \text{ eV-s}$ , and  $hc = 1240 \text{ eV-nm}$ .

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$$

$E_{\text{photon}}$  is in electron volts  
 $\lambda$  is in nanometers

Example: A red photon with  $\lambda = 620 \text{ nm}$  has  $E = 2 \text{ eV}$ .

# Photoelectric Effect: Solution

1. When light of wavelength  $\lambda = 400 \text{ nm}$  shines on lithium, the stopping voltage of the electrons is  $V_{\text{stop}} = 0.21 \text{ V}$ . What is the work function of lithium?

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For  $V_{\text{stop}} = 0.21 \text{ V}$ ,  $eV_{\text{stop}} = 0.21 \text{ eV}$

## UNITS

OR stick with SI units completely

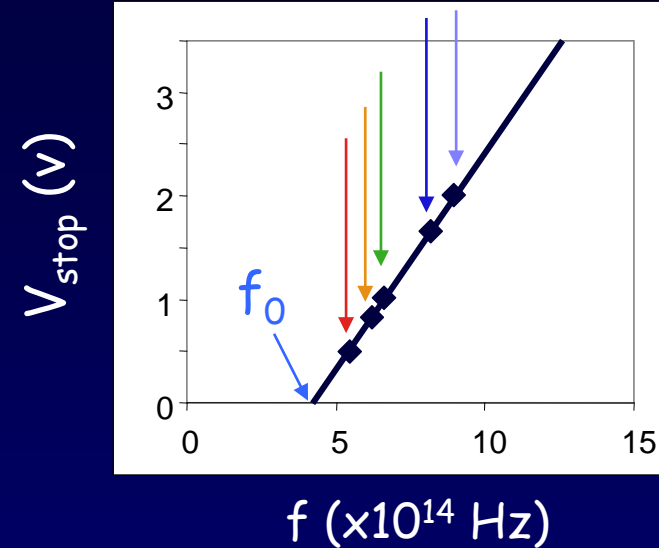
Calculate  $\Phi = hf - eV_{\text{stop}}$  in Joules

Joules/quantum of charge = eV, where quantum of charge =  $e = 1.602 \times 10^{-19}$

# Act 1

1. If the workfunction of the material increased, how would the graph change?

- a. Increased slope
- b. Increased  $f_0$
- c. Both a and b



2. We now shine on light with frequency  $2f_0$ . What effect does doubling the intensity (i.e., the power) of the incident light have on the current of emitted electrons?

- a. doubles
- b. quadruples
- c. stays the same decreases
- d. decreases

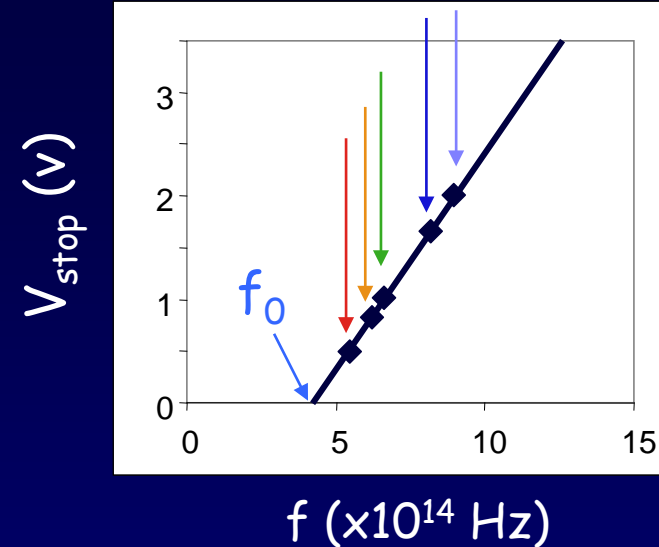
# Act 1

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The y intercept moves down, so the slope is unchanged and  $f_0$  increases  $\rightarrow$  the photons need more energy to be able to free the electrons from the increased binding.

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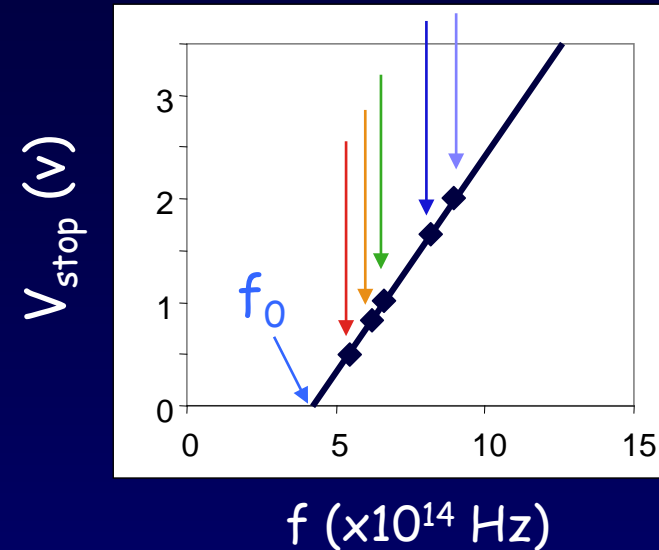
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- c. stays the same decreases
- d. decreases

Because the frequency is higher than  $f_0$ , each incident photon has a chance to emit an electron. Doubling the number of photons doubles the number of photoelectrons.

# Momentum of a Photon

What is the momentum of a photon?

Combine the two equations:

$$E_{\text{photon}} = hf = hc/\lambda \quad \text{– quantum mechanics}$$

$$p = E/c \quad \text{– Maxwell's equations, or special relativity}$$

This leads to the relation between momentum and wavelength:

$$p_{\text{photon}} = hf/c = h/\lambda$$

These are the key relations of quantum mechanics  
(they hold for *all* particles):

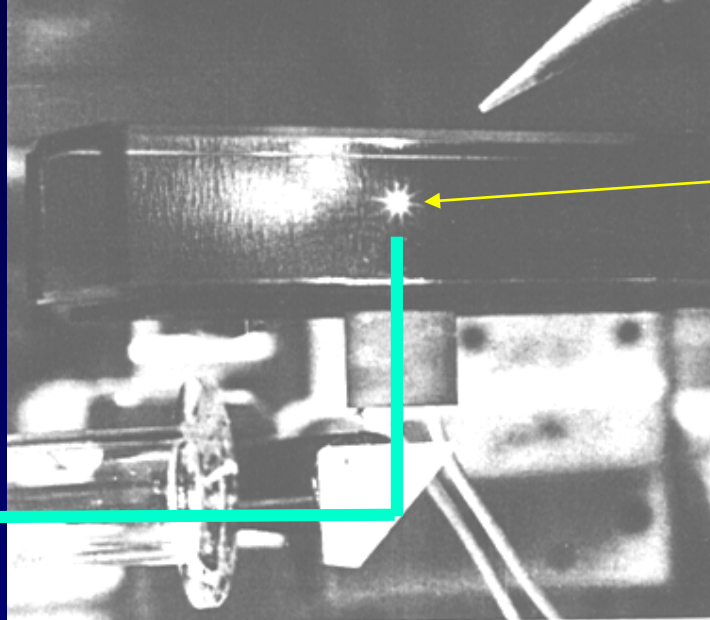
$$E = hf$$
$$p = h/\lambda$$

They relate an object's particle properties  
(energy and momentum)  
to its wave properties  
(frequency and wavelength).

Remember:  
 $E = hc/\lambda$   
 $p = hf/c$   
are only valid  
for photons

# Exercise: Optical "Levitation"

What laser power is required to suspend a glass bead weighing 0.01 g?



Glass bead 'floating' on a laser beam!

(AT&T Bell Labs)

Assume the bead absorbs all the incident light.

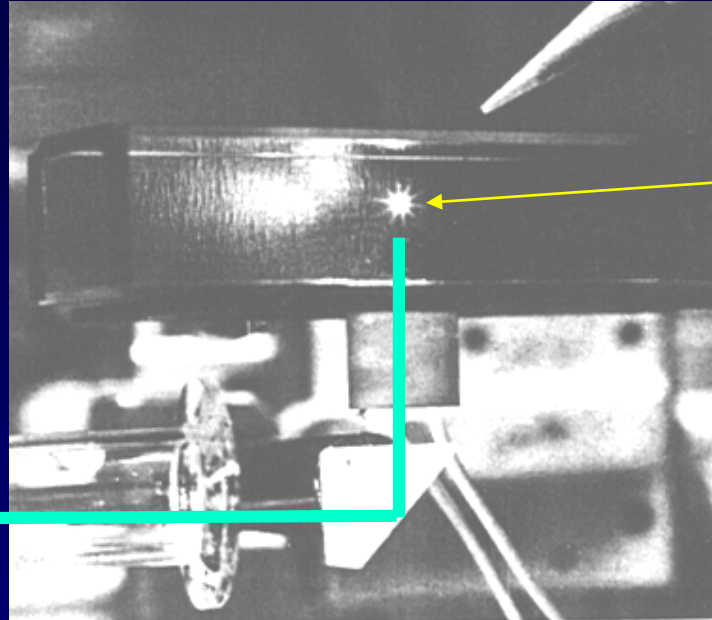
Laser

$$P = \Delta E / \Delta t = ?$$

Answer: 30 kW

# Exercise: Optical "Levitation"

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(AT&T Bell Labs)

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Laser

$$P = \Delta E / \Delta t = ?$$

$$F = mg = \frac{\Delta p}{\Delta t} = (\text{momentum for each absorbed photon}) \times (\# \text{ photons/sec})$$

$$\therefore \# \text{ photons/s} = \frac{F}{p} = \frac{mg}{h / \lambda}$$

$$\text{Power} = (\text{energy/photon}) \times (\# \text{ photons/sec}) = \frac{hc}{\lambda} \frac{mg}{h / \lambda} = mg c$$

$$= (10^{-5} \text{ kg}) (9.8 \text{ m/s}^2) (3 \times 10^8 \text{ m/s}) = 30 \text{ kW}$$

# Measurement

How does what we measure determine whether we observe wave or particle properties?

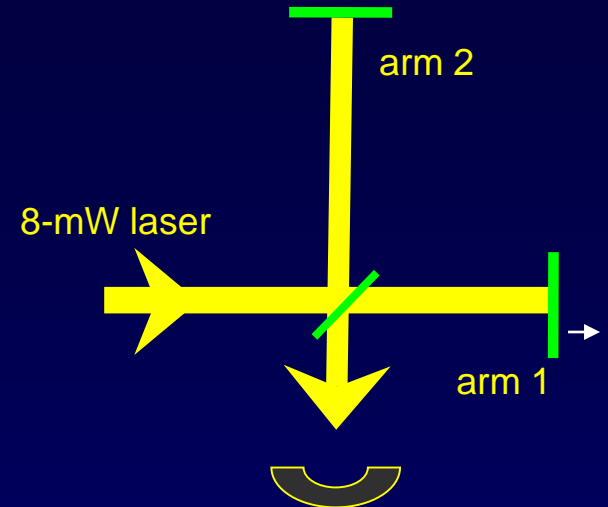
Waves have wavelength,  $\lambda$ , and frequency,  $f$ . So, if we measure momentum (wavelength) or energy (frequency), we have observed the wave properties of our object.

Particles have position (and trajectories). If we measure position (e.g., which slit it went through) we have observed a particle property. That's why the “which slit” measurement destroys the interference pattern.

Note that particle and wave properties are incompatible. One can't simultaneously measure both wavelength and position. This is the basis of Heisenberg's “uncertainty principle”. (more later)

# Act 2

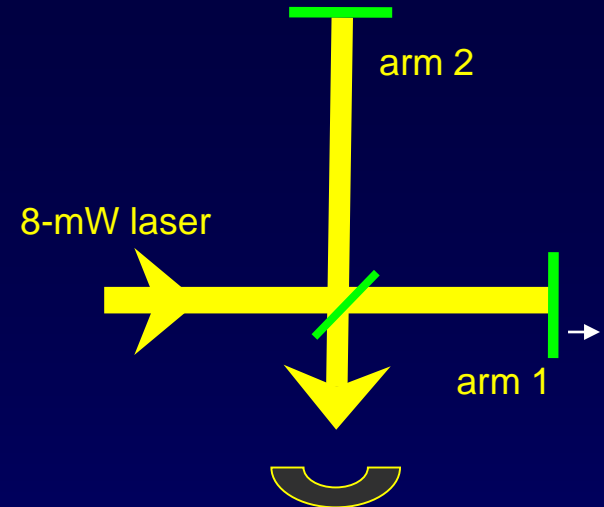
1. Consider a 500-nm laser sent into a Michelson interferometer, one of whose mirrors is moving rapidly (thereby shifting the color of the light in that arm to 502 nm). What will we observe at the output detector ?
- a. a single interference pattern, i.e., fringes with spacing 501 nm.
  - b. a double interference pattern, i.e., fringes with spacing 500 nm, plus fringes at 502 nm.
  - c. no light whatsoever
  - d. light (half the intensity of the laser), but no interference pattern



# Act 2

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Interference only occurs when the contributing processes are *indistinguishable*. In the current case, you could tell which path a given photon took, by measuring it's color.

Therefore, there's no interference.

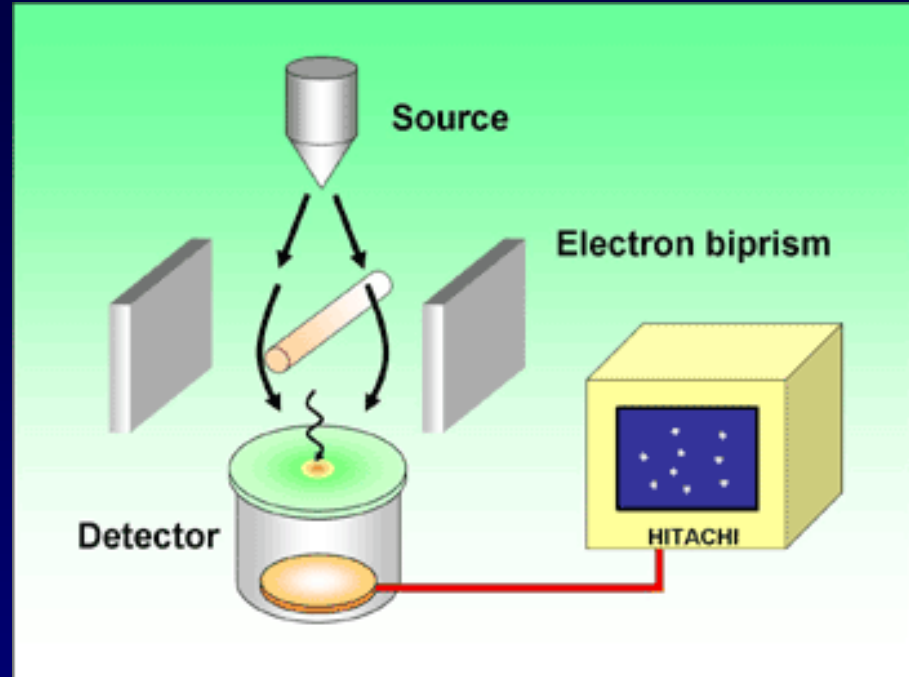
# "Double-slit" Experiment for Electrons

Electrons are accelerated to 50 keV  
 $\rightarrow \lambda = 0.0055 \text{ nm}$

Central wire is positively charged  $\rightarrow$  bends electron paths so they overlap.

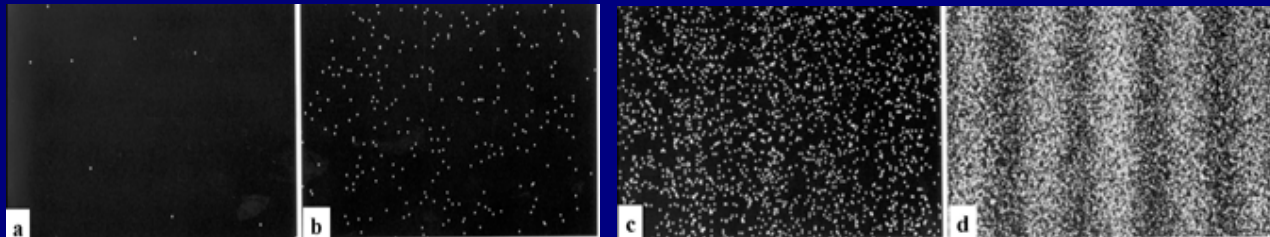
A position-sensitive detector records where they appear.

$\ll 1$  electron in system at any time



Video by A. TONOMURA (Hitachi) --pioneered electron holography.

<http://www.hqrd.hitachi.co.jp/rd/moviee/doubleslite.wmv>



Exposure time: 1 s

10 s

5 min

20 min

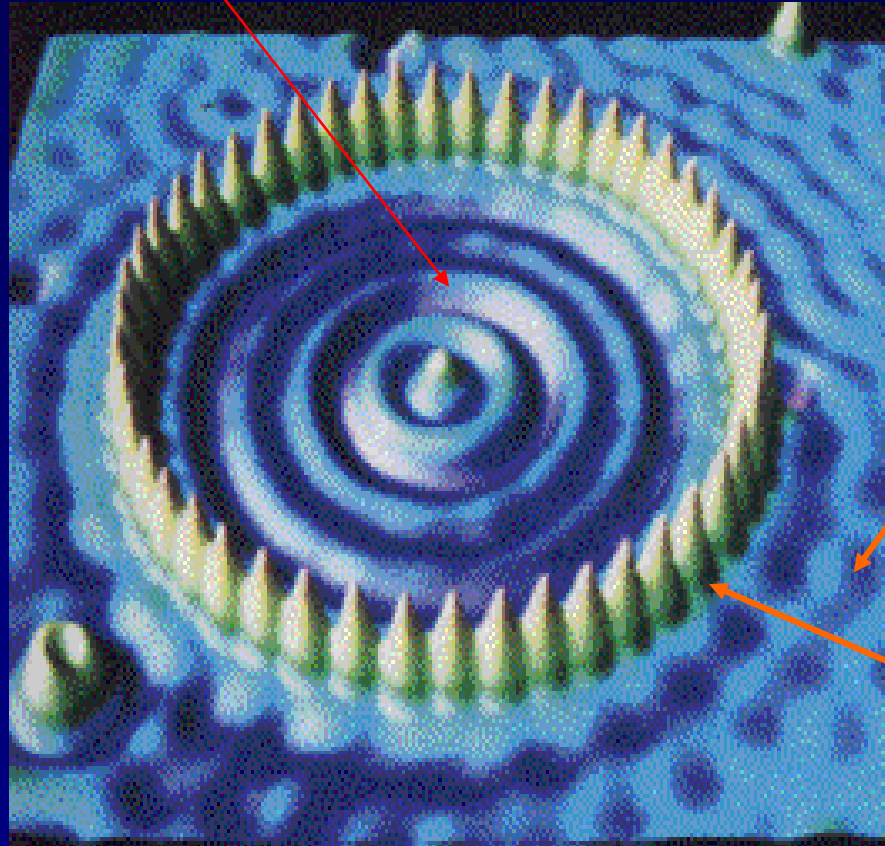
See also this Java simulation: <http://www.quantum-physics.polytechnique.fr/index.html>

# Observation of an electron wave "in a box"

Image taken with a scanning tunneling microscope (more later)

(Note: the color is not real! - it is a representation of the electrical current observed in the experiment)

Real standing waves of electron density in a "quantum corral"



IBM  
Almaden

Cu

Single  
atoms (Fe)

# "Double-slit" Experiment for Atoms!

This is the best example of how measuring which-way information destroys interference

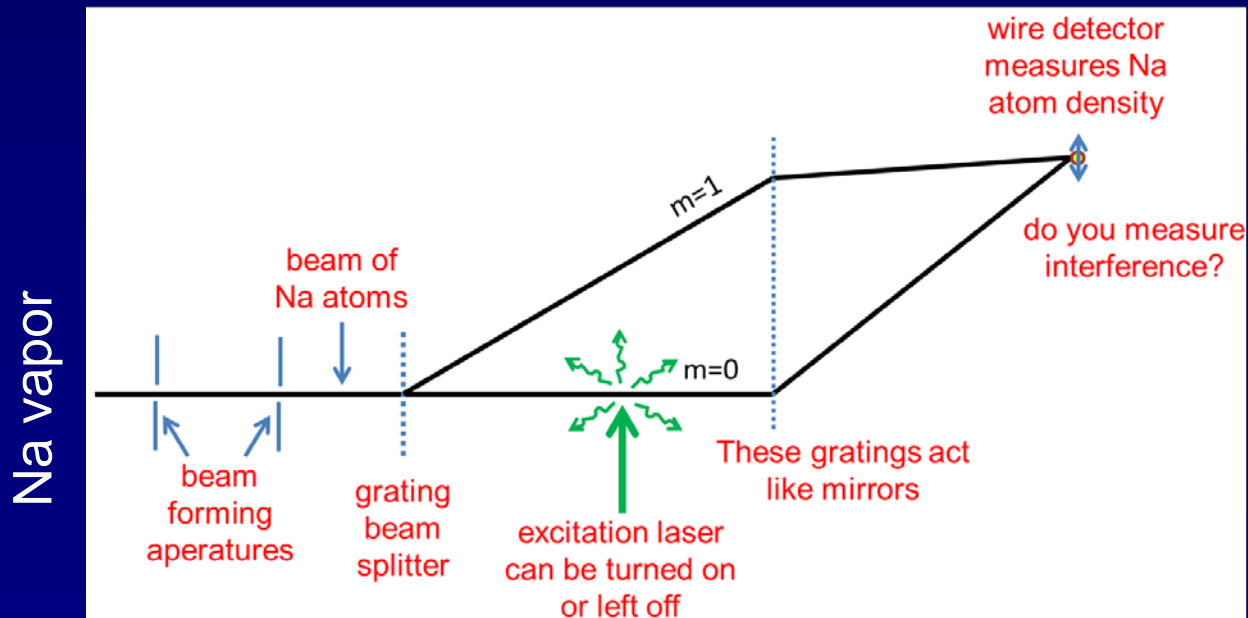
## Photon Scattering from Atoms in an Atom Interferometer: Coherence Lost and Regained

Michael S. Chapman,<sup>1</sup> Troy D. Hammond,<sup>1</sup> Alan Leneff,<sup>1</sup> Jörg Schmiedmayer,<sup>1,2</sup> Richard A. Rubenstein,<sup>1</sup>  
Edward Smith,<sup>1</sup> and David E. Pritchard<sup>1</sup>

<sup>1</sup>*Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology,  
Cambridge, Massachusetts 02139*

<sup>2</sup>*Institut für Experimentalphysik, Universität Innsbruck, A-6020 Innsbruck Austria*  
(Received 8 May 1995)

We have scattered single photons from interfering de Broglie waves in an atom interferometer and observed contrast loss and revivals as the separation of the interfering paths at the point of scattering is increased. Additionally, we have demonstrated that the lost coherence can be recovered by observing only atoms that are correlated with photons emitted into a limited angular range.



# "Double-slit" Experiment for Atoms!

Relatively cold beam of Na atoms

deBroglie Wavelength  $\sim \lambda \sim 10^{-10} \text{ m} < d = 2 \times 10^{-7} \text{ m}$

Grating formed by etching  $\text{Si}_3\text{N}_4$  membrane using Focused Ion Beam. Highly precise!

Grating acts as a beam-splitter. Pay attention to the 0-order and +1 order beams

$\ll 1$  atom in system at any time

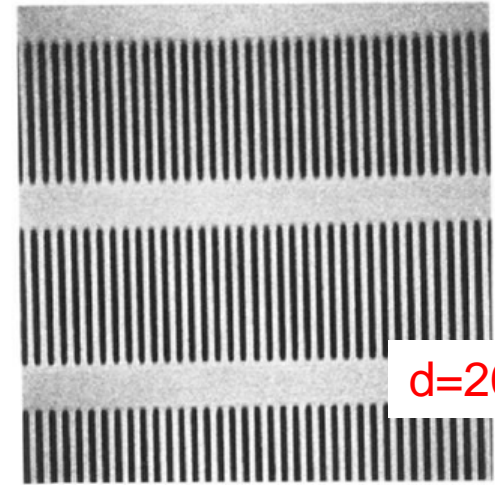
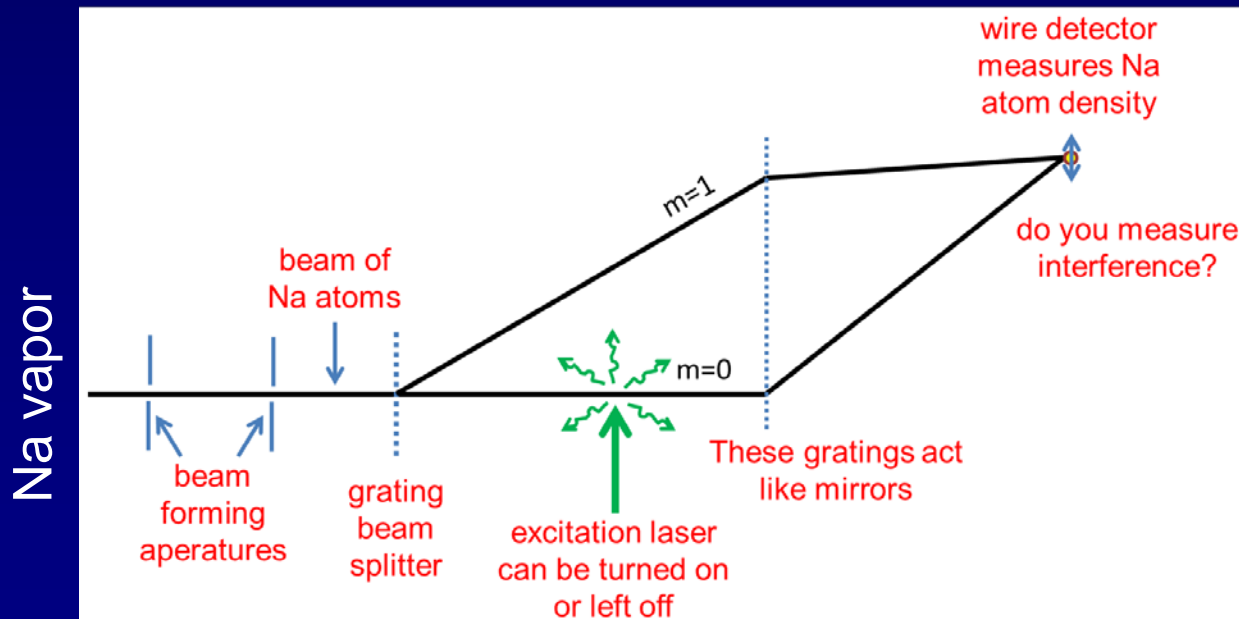


FIG. 2. A scanning electron micrograph of a completed 200-nm-period grating.



# "Double-slit" Experiment for Atoms!

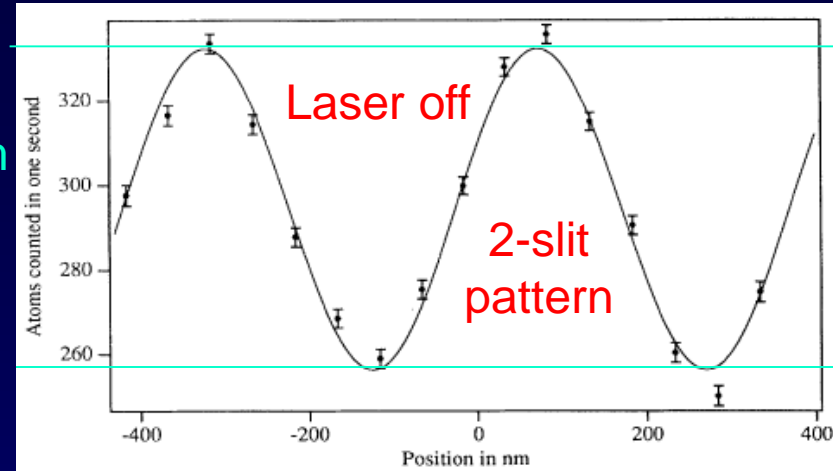
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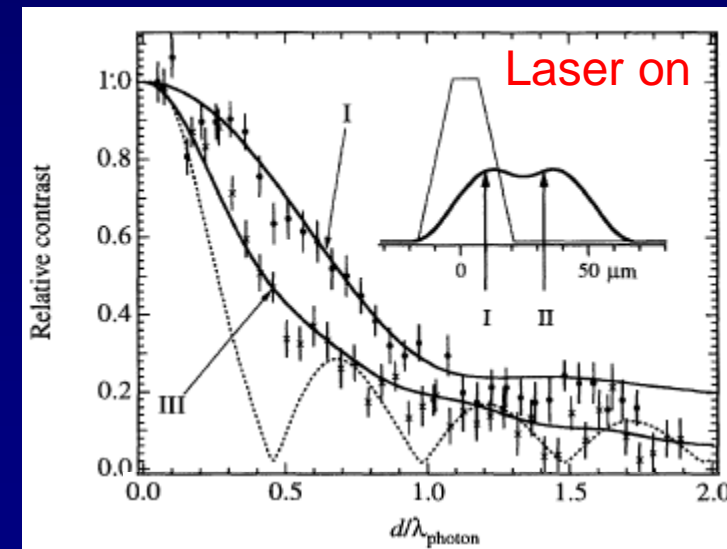
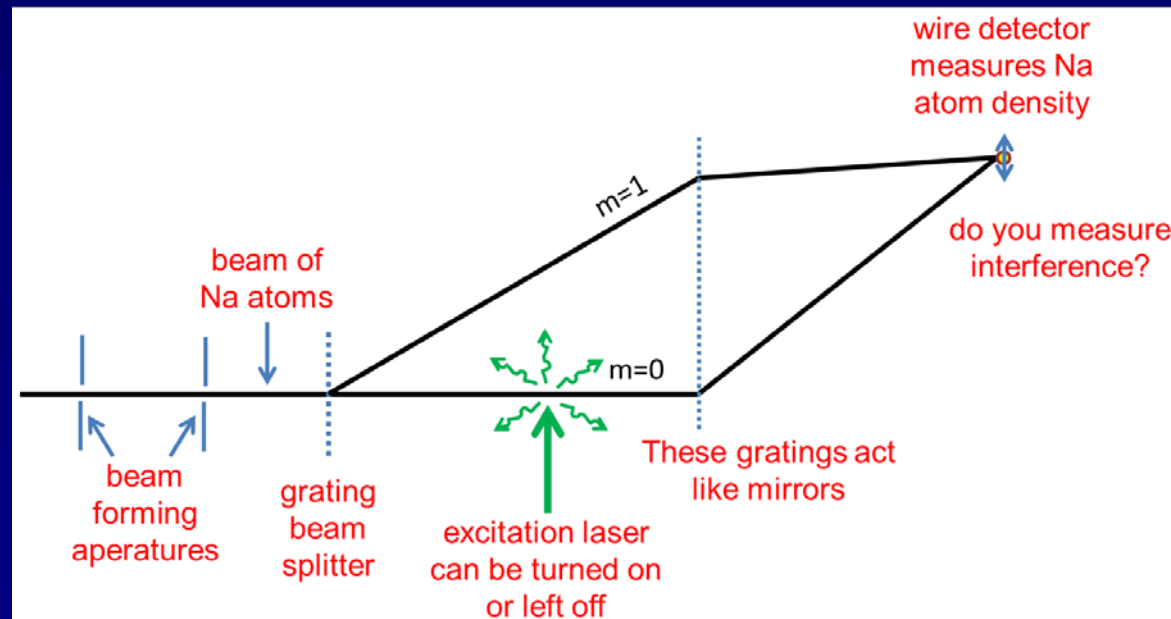
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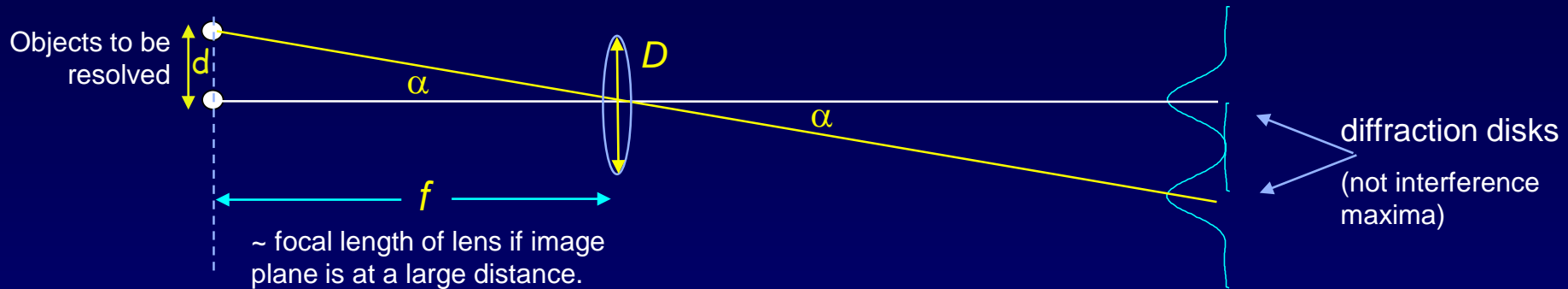


Contrast is reduced to zero if the path taken is detected!



# Application of Matter Waves: Electron Microscopy

The ability to resolve tiny objects improves as the wavelength decreases.  
Consider the microscope:



Rayleigh's  
criterion:

$$\alpha_c = 1.22 \frac{\lambda}{D}$$

The minimum  $d$  for which we  
can still resolve two objects is  
 $\alpha_c$  times the focal length:

$$d_{\min} \approx f \alpha_c = 1.22 \lambda \frac{f}{D}$$

the "f-number"

The objective lens of a good optical microscope has  $f/D \cong 2$ ,  
so with  $\lambda \sim 500$  nm the microscope has a resolution of  $d_{\min} \sim 1$   $\mu$ m.

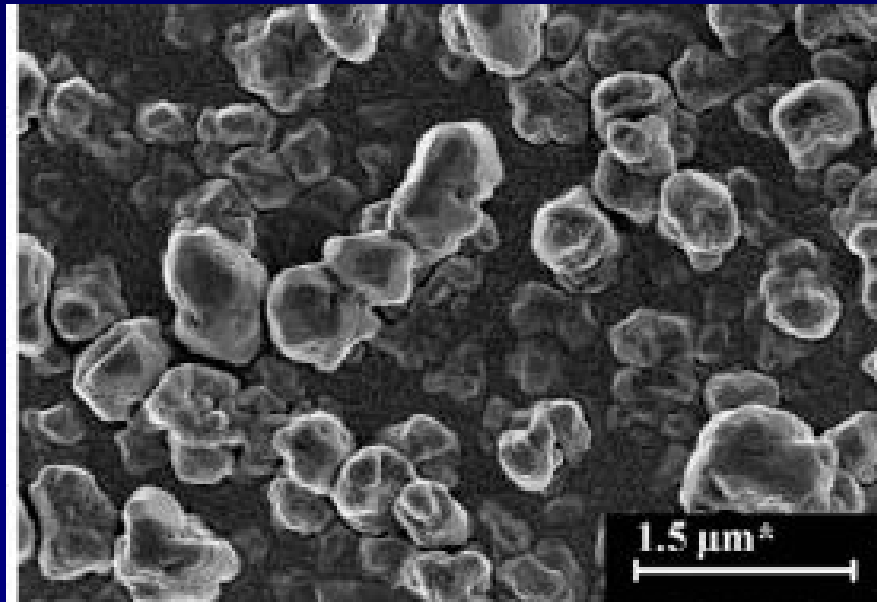
We can do much better with matter waves because electrons with  
energies of a few keV have wavelengths much less than 1 nm.

The instrument is known as an "electron microscope".

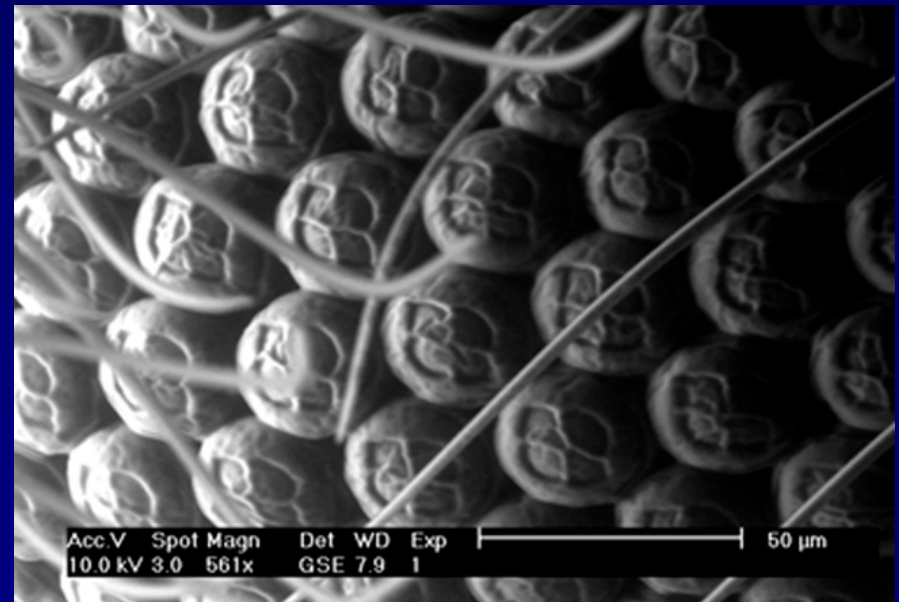
# Application of Matter Waves: Electron Microscopy

Scientists and engineers - such as those here at the **Materials Research Lab** and the **Beckman Institute** - use “electron microscopy” to study **nanometer-scale structures** in materials and biological systems

Cu-In alloy



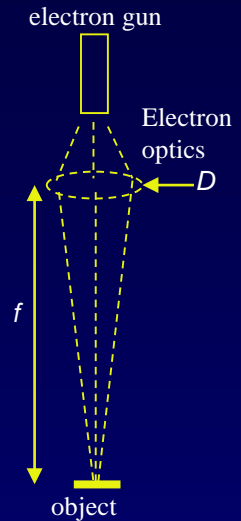
Compound eye of a fly



# Example: Imaging a Virus

You wish to observe a virus with a diameter of 20 nm, much too small to observe with an optical microscope. Calculate the voltage required to produce an electron wavelength suitable for studying this virus with a resolution of  $d_{\min} = 2 \text{ nm}$ . The “f-number” for an electron microscope is quite large:  $f/D \approx 100$ .

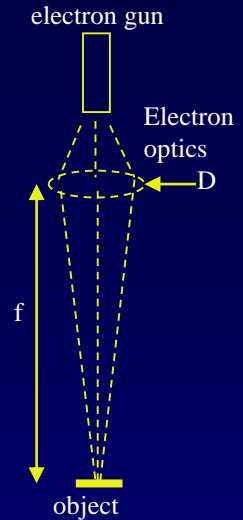
Hint: First find  $\lambda$  required to achieve  $d_{\min}$ .  
Then find  $E$  of an electron from  $\lambda$ .



# Solution

You wish to observe a virus with a diameter of 20 nm, much too small to observe with an optical microscope. Calculate the voltage required to produce an electron wavelength suitable for studying this virus with a resolution of  $d_{\min} = 2 \text{ nm}$ . The “f-number” for an electron microscope is quite large:  $f/D \approx 100$ .

Hint: First find  $\lambda$  required to achieve  $d_{\min}$ .  
Then find  $E$  of an electron from  $\lambda$ .



$$d_{\min} \approx 1.22 \frac{\lambda}{D} f \quad \leftarrow \text{Rayleigh plus geometry}$$

$$\lambda \approx d_{\min} \left( \frac{D}{1.22f} \right) = 2 \text{ nm} \left( \frac{D}{1.22f} \right) = 0.0164 \text{ nm}$$

Note:

$1.22\lambda/D$  is the diffraction angle,  $\theta$   
 $f$  is the lever arm,  
So,  $\theta f$  is the spot size.



$$E = \frac{h^2}{2m\lambda^2} = \frac{1.505 \text{ eV} \cdot \text{nm}^2}{(0.0164 \text{ nm})^2} = 5.6 \text{ keV}$$

To accelerate an electron to an energy of 5.6 keV requires 5.6 kilovolts .  
(The convenience of electron-volt units)

# Interference of larger particles

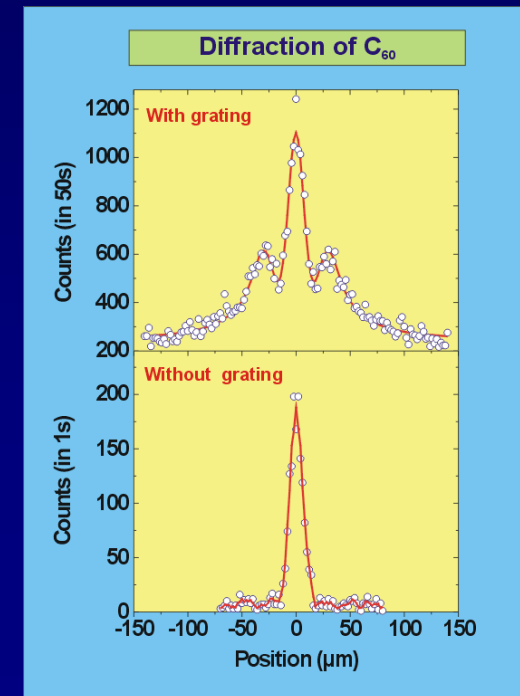
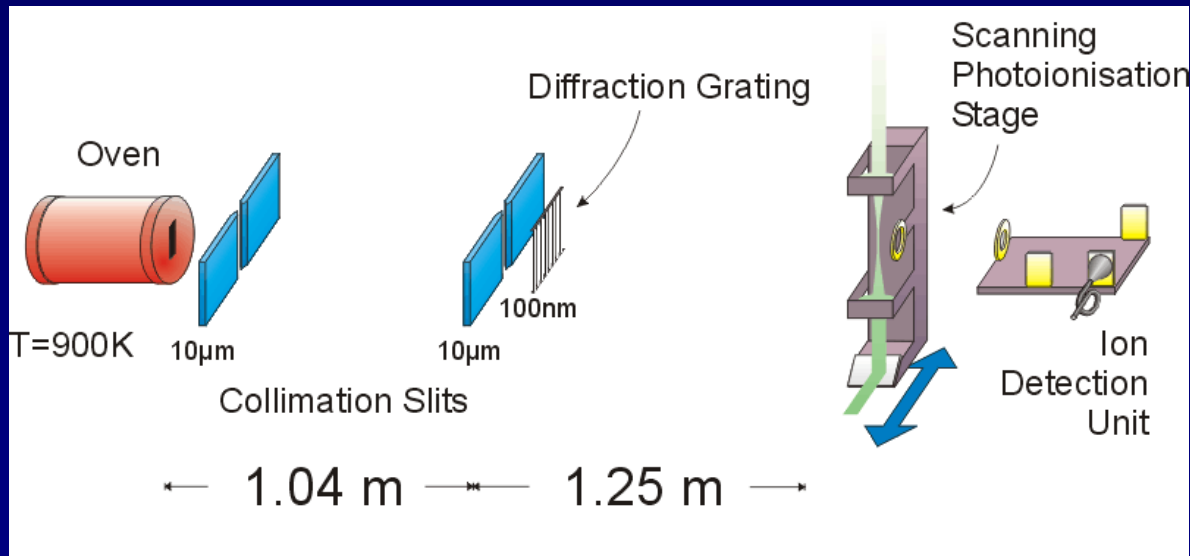
- Matter-wave interference has now been demonstrated with electrons, neutrons, atoms, small molecules, BIG molecules, & biological molecules
- Recent Example: Interference of  $C_{60}$ , a.k.a. "fullerenes", "buckyballs"



$$\text{Mass} = (60 \text{ C})(12 \text{ g/mole})/N_A = 1.2 \times 10^{-24} \text{ kg}$$

$$\frac{\langle p^2 \rangle}{2m} = K.E. \approx \frac{3}{2} kT \Rightarrow \langle p \rangle = \sqrt{3kTm} = 2.1 \times 10^{-22} \text{ kg m/s}$$

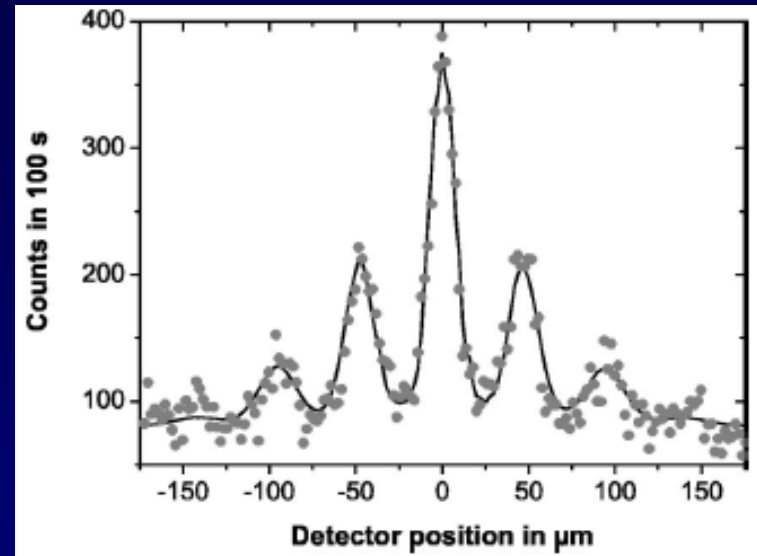
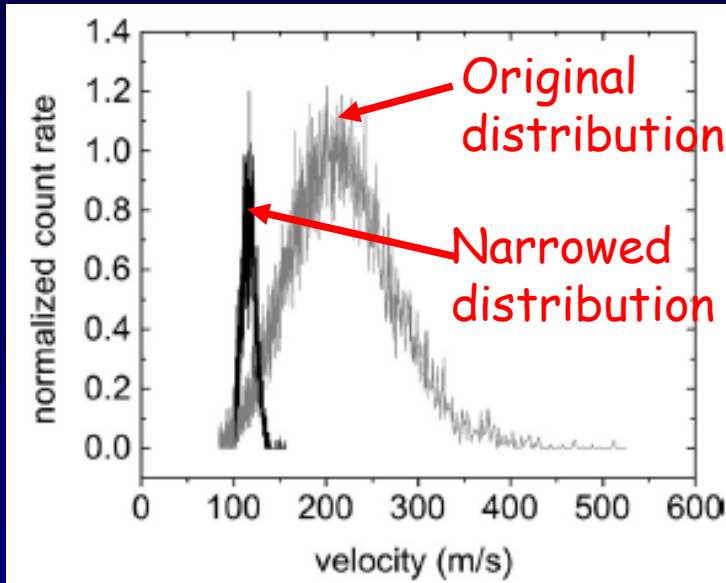
$$\lambda = h/p = 2.9 \text{ pm} \quad (\text{c.f. } C_{60} \text{ is } \sim 1 \text{ nm across!})$$



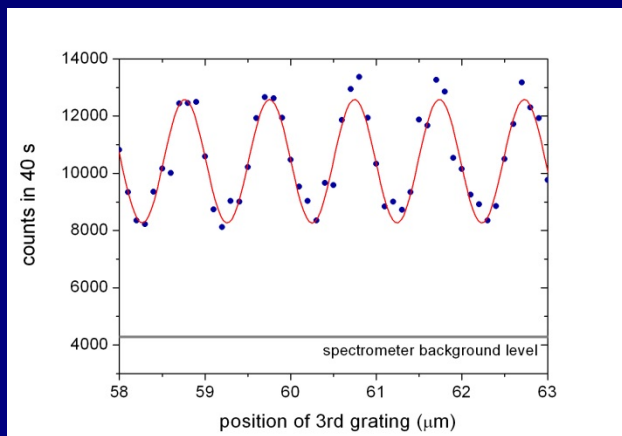
[A. Zeilinger (U. Vienna), 1999]

# FYI: More on Interference of larger particles

- Using a velocity selector, they could make the atoms more monochromatic → improved interference:



- In 2003 interference was observed with porphyrin, a bio. molecule:



Now they're trying to do something like this with a virus!

# Where do we go from here?

Two approaches pave the way:

## Uncertainty principle

- In quantum mechanics one can only calculate a probability distribution for the result of a measurement.
- The Heisenberg uncertainty principle provides a way to use simple arguments and a simple inequality to draw important conclusions about quantum systems.

## Schrödinger equation (next week)

- This differential equation describes the evolution of the quantum wave function,  $\Psi$ .  $\Psi$  itself has no uncertainty.
- $|\Psi|^2$  will then tell us the probabilities of obtaining various measurement results. That's where the uncertainty enters.

# Heisenberg Uncertainty Principle

All QM objects (we think that includes everything) have wave-like properties. One mathematical property of waves is:

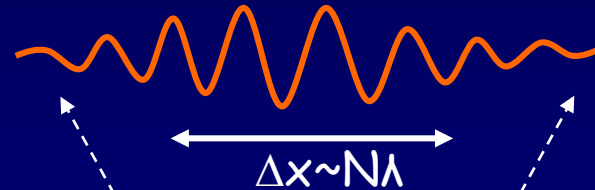
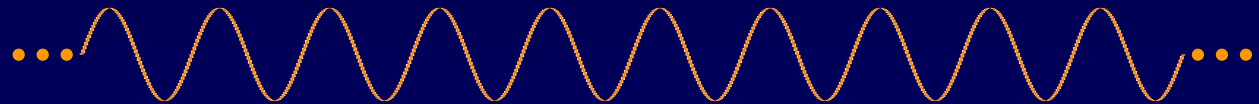
$$\Delta k \cdot \Delta x \geq 1$$

(See the supplementary slide for some discussion)

$$k = 2\pi/\lambda$$

Examples:

- Infinite sine wave:  
A definite wavelength must extend forever.
- Finite wave packet:  
A wave packet requires a spread\* of wavelengths.



We need a spread of wavelengths in order to get destructive interference.

$$\Delta k \sim k/2\pi N$$

$$k \sim 2\pi/\lambda$$

$$\Delta k \cdot \Delta x \geq 1$$

Using  $p = h/\lambda = \hbar k$ , we have:

$$\Delta k \cdot \Delta x \geq 1$$

$$\hbar \times (\Delta k \cdot \Delta x \geq 1) \Rightarrow (\hbar \Delta k) \cdot \Delta x \geq \hbar \Rightarrow \boxed{\Delta p \cdot \Delta x \geq \hbar}$$

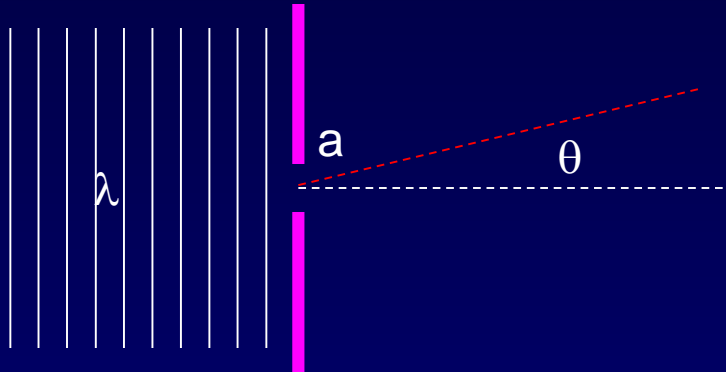
This relation is known as the **Heisenberg Uncertainty Principle**.

It limits the accuracy with which we can know the position and momentum of objects.

\* We will not use the statistically correct definition of “spread”, which, in this context, we also call “uncertainty”.

# Diffraction and the Uncertainty Principle

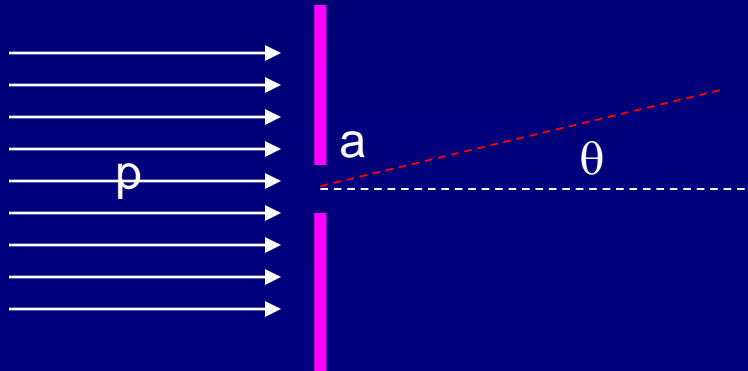
Remember single-slit diffraction:



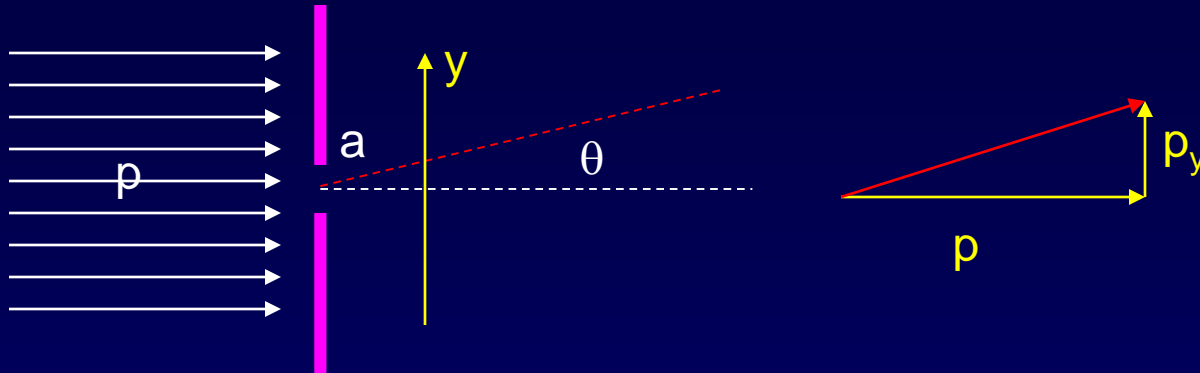
Wavelength:  $\lambda$   
Slit width:  $a$   
Diffraction angle:  $\theta = \lambda/a$   
angle to first zero

Let's analyze this problem using the uncertainty principle.

Suppose a beam of electrons of momentum  $p$  approaches a slit of width  $a$ . How big is the angular spread of motion after it passes through the slit?



# Solution



Consider the momentum uncertainty in the  $y$ -direction.

- Before the slit, the  $y$ -position is not known, so the uncertainty of  $p_y$  can be zero. We know that  $p_y = 0$ .
- Just after the slit, the  $y$ -position has an uncertainty of about  $a/2$ . Therefore  $p_y$  must have an uncertainty  $\Delta p_y \geq 2\hbar/a$ . This corresponds to a change of direction by an angle,  $\theta = \Delta p_y / p = 2\hbar/ap$ . Using  $p = h/\lambda$ , we have  $\theta = \lambda/(\pi a)$ .

This is almost the diffraction answer:  $\theta = \lambda/a$ . The extra factor of  $\pi$  is due to our somewhat sloppy treatment of the uncertainty.

The important point is that the uncertainty principle results because matter behaves as a wave.

# Example

The position of an electron in the lowest-energy state of a hydrogen atom; is known to an accuracy of about  $\Delta x = 0.05 \text{ nm}$  (the radius of the atom).

What is the minimum range of momentum measurements? Velocity?

# Solution

The position of an electron in the lowest-energy state of a hydrogen atom; is known to an accuracy of about  $\Delta x = 0.05 \text{ nm}$  (the radius of the atom).

What is the minimum range of momentum measurements? Velocity?

$$\Delta x \Delta p \geq \hbar$$

Heisenberg's Uncertainty Principle

$$\Delta p \geq \hbar / \Delta x$$

$$\hbar = h / 2\pi$$

$$= 2.1 \times 10^{-24} \text{ J} \cdot \text{s/m}$$

$$= 2.1 \times 10^{-24} \text{ kg} \cdot \text{m/s}$$

$$\Delta v = \Delta p / m_e$$

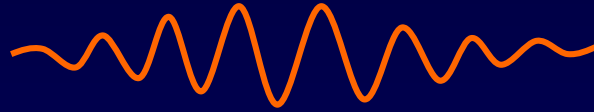
$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$= 2.3 \times 10^6 \text{ m/s}$$

# Why "Uncertainty"?

"Uncertainty" refers to our inability to make definite predictions.

Consider this wave packet:



- Where is the object?
- What is its momentum?

The answer is, We don't know. We can't predict the result of either measurement with an accuracy better than the  $\Delta x$  and  $\Delta p$  given to us by the uncertainty principle.

Each time you look, *you find a local blip* that is in a different place (in fact, it is your looking that *causes* the wavefunction to "collapse"!).

If you look many times, you will find a *probability distribution* that is spread out.

But you're *uncertain about where that local blip will be in any one of the times you look* -- it could be anywhere in the spread.

An important point: *You never observe the wave function itself.*

The wave merely gives the probabilities of obtaining the various measurement results. A measurement of position or momentum will always result in a definite result. You can infer the properties of the wave function by repeating the measurements (to measure the probabilities), but that's not the same as a direct observation.

# Uncertainty Principle -Implications

The uncertainty principle explains why electrons in atoms don't simply fall into the nucleus: If the electron were confined too close to the nucleus (small  $\Delta x$ ), it would have a large  $\Delta p$ , and therefore a very large average kinetic energy ( $\approx (\Delta p)^2/2m$ ).

The uncertainty principle does not say “everything is uncertain”. Rather, it tells us what the limits of uncertainty are when we make measurements of quantum systems.

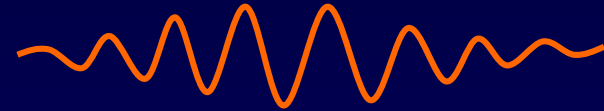
Some classical features, such as paths, do not exist precisely, because having a definite path requires both a definite position and momentum. One consequence, then, is that electron orbits do not exist. The atom is not a miniature solar system.

Other features can exist precisely. For example, in some circumstances we can measure an electron's energy as accurately as technology allows.

Serious philosophical issues remain open to vigorous debate, e.g., whether all outcomes or only one outcome actually occur.

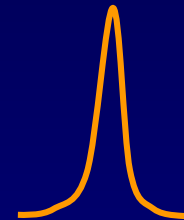
# Supplement: Wave Uncertainty

Mathematically, one can produce a localized function by superposing sine waves with a “spread” of wave numbers,  $\Delta k$ :  $\Delta k \cdot \Delta x \geq 1$ .



This is a result of Fourier analysis, which most of you will learn in Math.

It means that making a short wave packet requires a broad spread in wavelengths. Conversely, a single-wavelength wave would extend forever.



So far, this is just math. The physics comes in when we make the wavelength-momentum connection:  $p = h/\lambda = \hbar k$ .

Example:

How many of you have experienced a close lightning strike (within a couple hundred feet)? If you were paying attention, you may have noticed that the sound, which is a very short pulse, is very weird. That weirdness is a result of the very broad range of frequencies that is needed to construct a very short pulse. One doesn't normally experience such a broad frequency range.