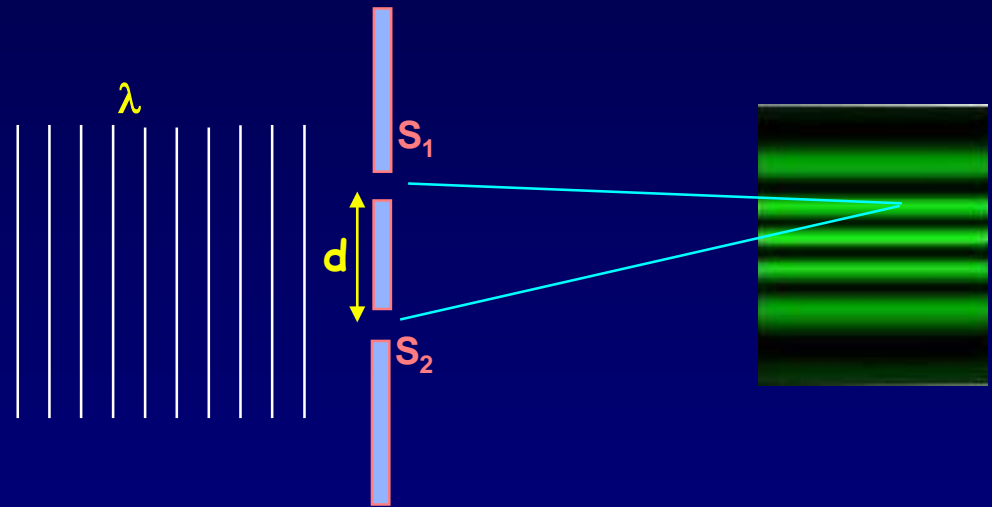
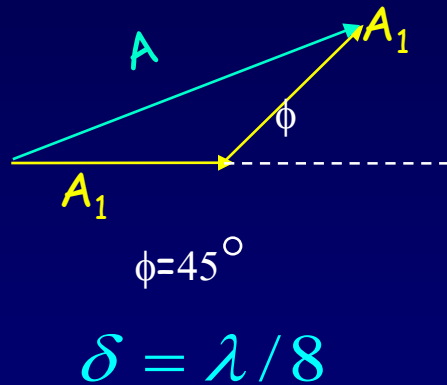


# Lecture 3:

## Review, Examples and Phasors



# Review: The Harmonic Waveform

$$y(x, t) = A \cos\left(\frac{2\pi}{\lambda}(x - vt)\right) \equiv A \cos(kx - 2\pi ft) \equiv A \cos(kx - \omega t)$$

$y$  is the displacement from equilibrium.

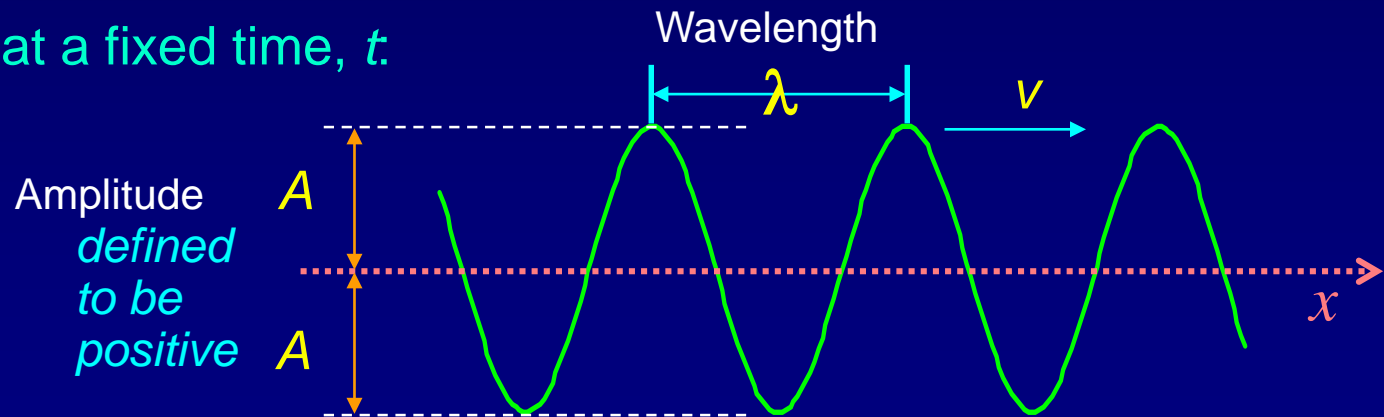
$v \equiv$  speed                       $A \equiv$  amplitude (defined to be positive)

$\lambda \equiv$  wavelength               $k \equiv \frac{2\pi}{\lambda} \equiv$  wavenumber

$f \equiv$  frequency                 $\omega \equiv 2\pi f \equiv$  angular frequency

A function of  
two variables:  
 $x$  and  $t$ .

A snapshot of  $y(x)$  at a fixed time,  $t$ .



This is review from Physics 211/212.

For more detail see Lectures 26 and 27 on the 211 website.

# Act 1

The speed of sound in air is a bit over **300 m/s**, and the speed of light in air is about **300,000,000 m/s**.

Suppose we make a sound wave and a light wave that both have a wavelength of **3 meters**.

1. What is the ratio of the frequency of the light wave to that of the sound wave?

(a) About **1,000,000** (b) About **0.000001** (c) About **1000**

2. What happens to the **frequency** if the light passes under water?

(a) Increases (b) Decreases (c) Stays the same

3. What happens to the **wavelength** if the light passes under water?

(a) Increases (b) Decreases (c) Stays the same

# Act 1 - Solution

The speed of sound in air is a bit over **300 m/s**, and the speed of light in air is about **300,000,000 m/s**.

Suppose we make a sound wave and a light wave that both have a wavelength of **3 meters**.

1. What is the ratio of the frequency of the light wave to that of the sound wave?

- (a) About **1,000,000** (b) About **0.000001** (c) About **1000**

$$f = \frac{v}{\lambda} \quad \text{and} \quad \frac{v_{light}}{v_{sound}} \cong 1,000,000 \quad \Rightarrow \quad \frac{f_{light}}{f_{sound}} \cong 1,000,000$$

# Act 1 - Solution

The speed of sound in air is a bit over **300 m/s**, and the speed of light in air is about **300,000,000 m/s**.

Suppose we make a sound wave and a light wave that both have a wavelength of **3 meters**.

2. What happens to the **frequency** if the light passes under water?

- (a) Increases      (b) Decreases      (c) Stays the same

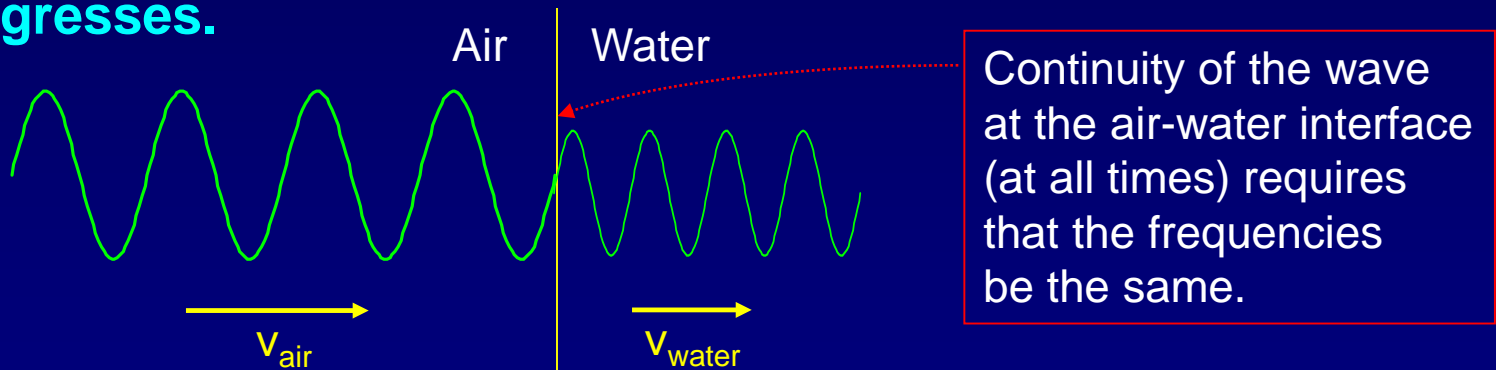
3. What happens to the **wavelength** if the light passes under water?

- (a) Increases      (b) Decreases      (c) Stays the same

# Act 1 - Discussion

Why does the **wavelength** change but not the **frequency**?

The frequency does not change because the time dependence in the air must match the time dependence at the air/water boundary. Otherwise, the wave will not remain continuous at the boundary as time progresses.



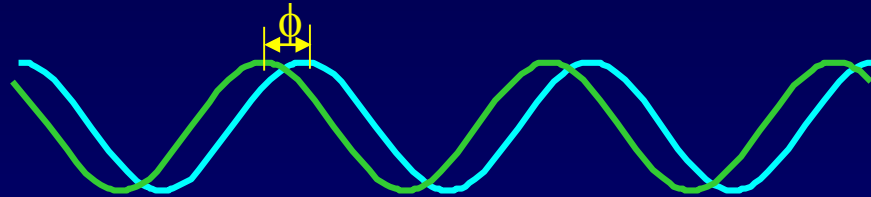
**Question:** Do we 'see' frequency or wavelength?

# Review: Adding Sine Waves

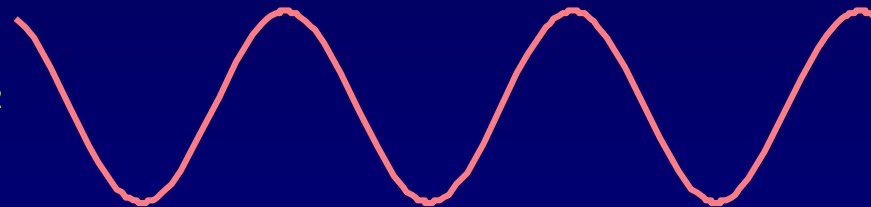
Suppose we have two sinusoidal waves with the same  $A_1$ ,  $\omega$ , and  $k$ . Suppose one starts at phase  $\phi$  after the other:

$$y_1 = A_1 \cos(kx - \omega t) \quad \text{and} \quad y_2 = A_1 \cos(kx - \omega t + \phi)$$

Spatial dependence  
of 2 waves at  $t = 0$ :



Resultant wave:  $y = y_1 + y_2$



Use this trig identity:

$$A_1 (\cos \alpha + \cos \beta) = 2A_1 \cos\left(\frac{\beta - \alpha}{2}\right) \cos\left(\frac{\beta + \alpha}{2}\right)$$

Note: the amplitude factor

$$2A_1 \cos(\phi / 2)$$

↓  
 $y_1 + y_2$

↓      ↓  
 $(\phi / 2) \quad (kx - \omega t + \phi / 2)$

Can be used for non-collinear  
equal amplitude waves as well

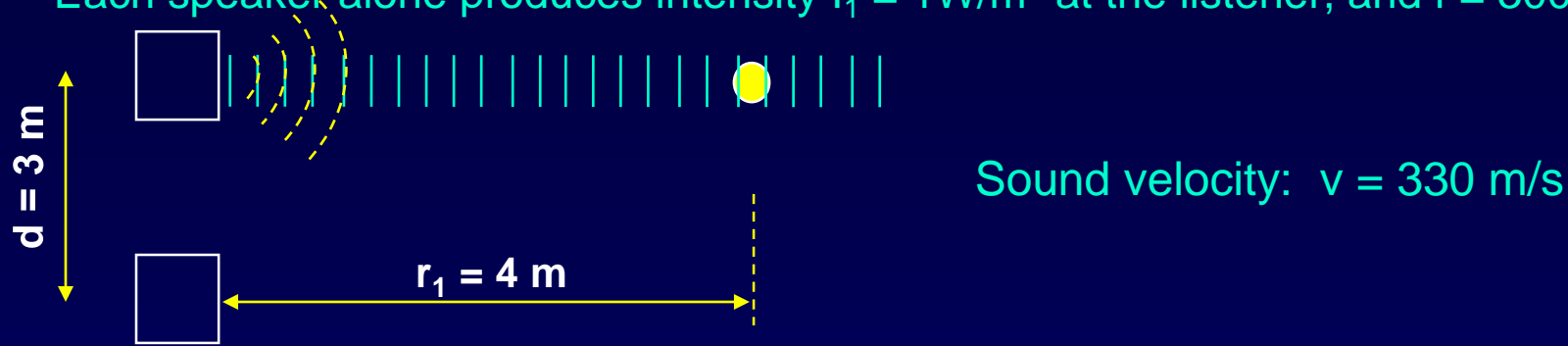
$$y = 2A_1 \cos(\phi / 2) \cos(kx - \omega t + \phi / 2)$$

Amplitude

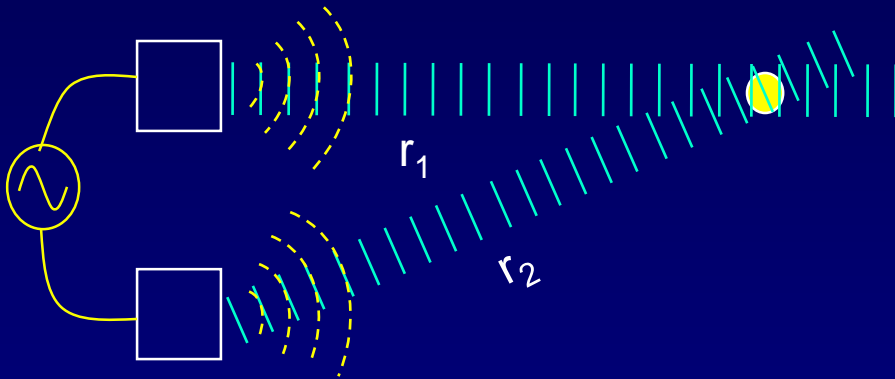
Oscillation

# Example: Path-Length Dependent Phase

Each speaker alone produces intensity  $I_1 = 1 \text{ W/m}^2$  at the listener, and  $f = 300 \text{ Hz}$ .



Drive speakers in phase. Compute the intensity  $I$  at the listener in this case:

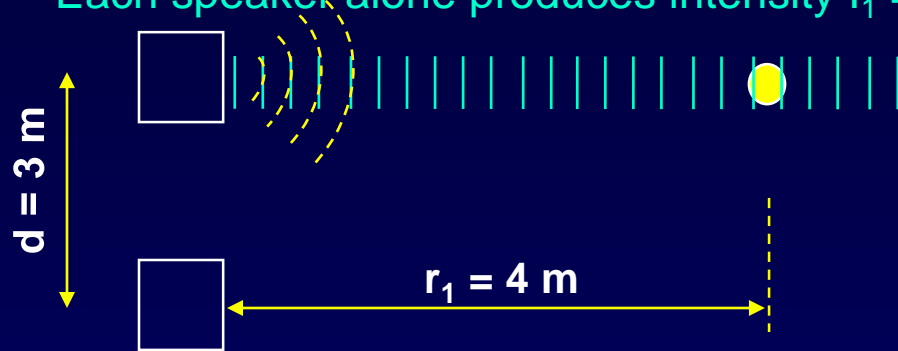


Procedure:

- 1) Compute path-length difference:  $\delta =$
- 2) Compute wavelength:  $\lambda =$
- 3) Compute phase difference:  $\phi =$
- 4) Write formula for resultant amplitude:  $A =$
- 5) Compute the resultant intensity:  $I = A^2 =$

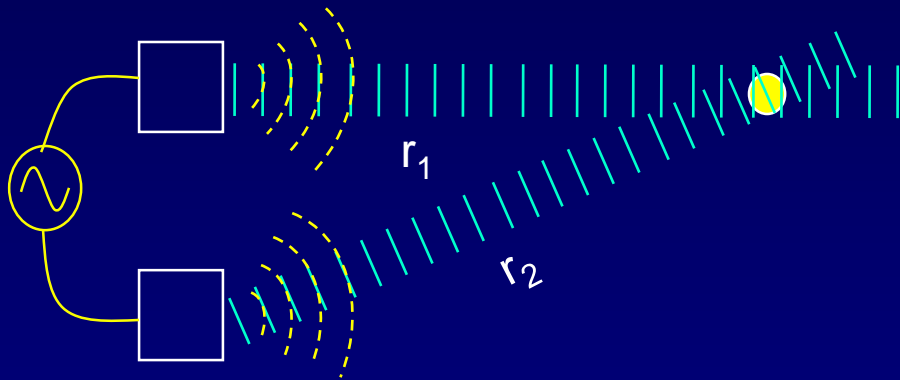
# Solution

Each speaker alone produces intensity  $I_1 = 1 \text{ W/m}^2$  at the listener, and  $f = 300 \text{ Hz}$ .



Sound velocity:  $v = 330 \text{ m/s}$

Drive speakers in phase. Compute the intensity  $I$  at the listener in this case:



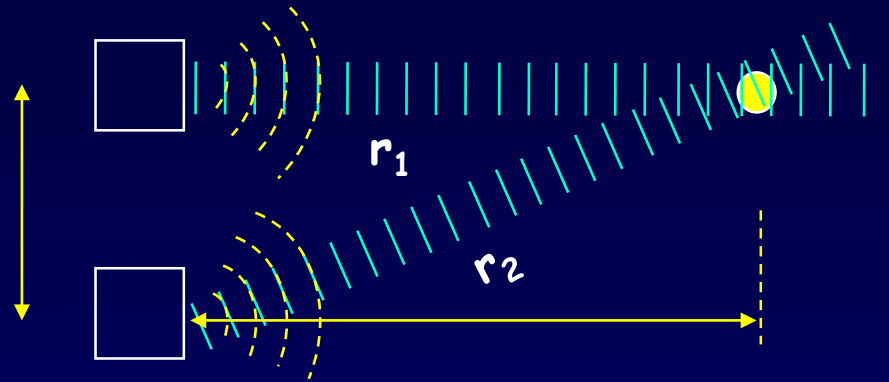
Procedure:

- 1) Compute path-length difference:  $\delta = 5 \text{ m} - 4 \text{ m} = 1 \text{ m}$
- 2) Compute wavelength:  $\lambda = v/f = 330 \text{ m/s} / 300 \text{ Hz} = 1.1 \text{ m}$
- 3) Compute phase difference:  $\phi = 2\pi(1 \text{ m} / 1.1 \text{ m}) = 5.71 \text{ rad} = 327^\circ$
- 4) Write formula for resultant amplitude:  $A = 2A_1 \cos(\phi/2) = 2 \cdot 1 \cdot \cos(2.86) = -1.92$
- 5) Compute the resultant intensity:  $I = A^2 = 3.69 \text{ W/m}^2$

Nice demo on web: [www.falstad.com/interference](http://www.falstad.com/interference)

The - sign is not significant.  
We care about  $|A|$ .

# Act 2: Speaker interference

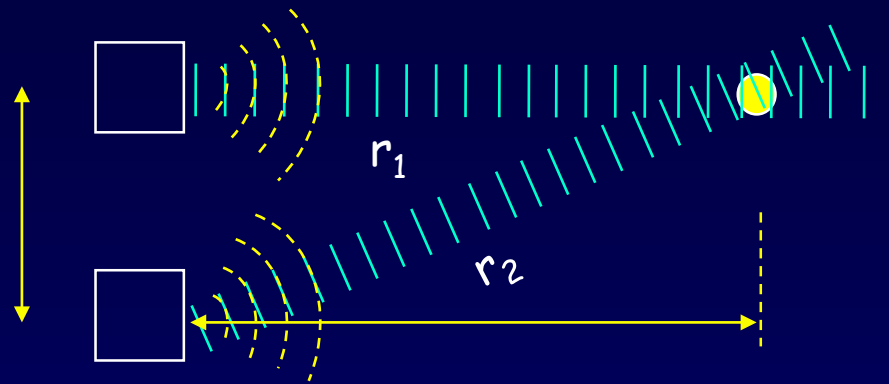


What happens to the intensity at the listener if we decrease the frequency  $f$  by a small amount?

- a. decrease      b. stay the same      c. increase

Hint: How does intensity vary with  $\phi$  when  $\phi = 327^\circ$ ?

# Solution



What happens to the intensity at the listener if we decrease the frequency  $f$  by a small amount?

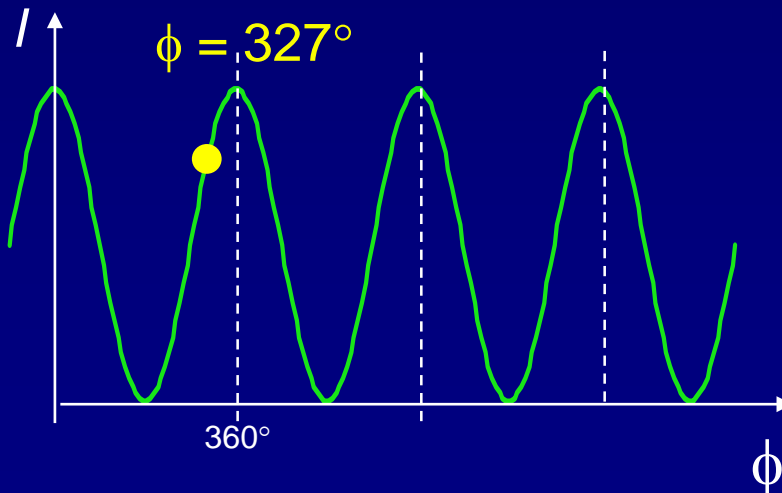
a. decrease

b. stay the same

c. increase

Hint: How does intensity vary with  $\phi$  when  $\phi = 327^\circ$ ?

Draw the graph of  $I(\phi)$ :



$f$  decreases:

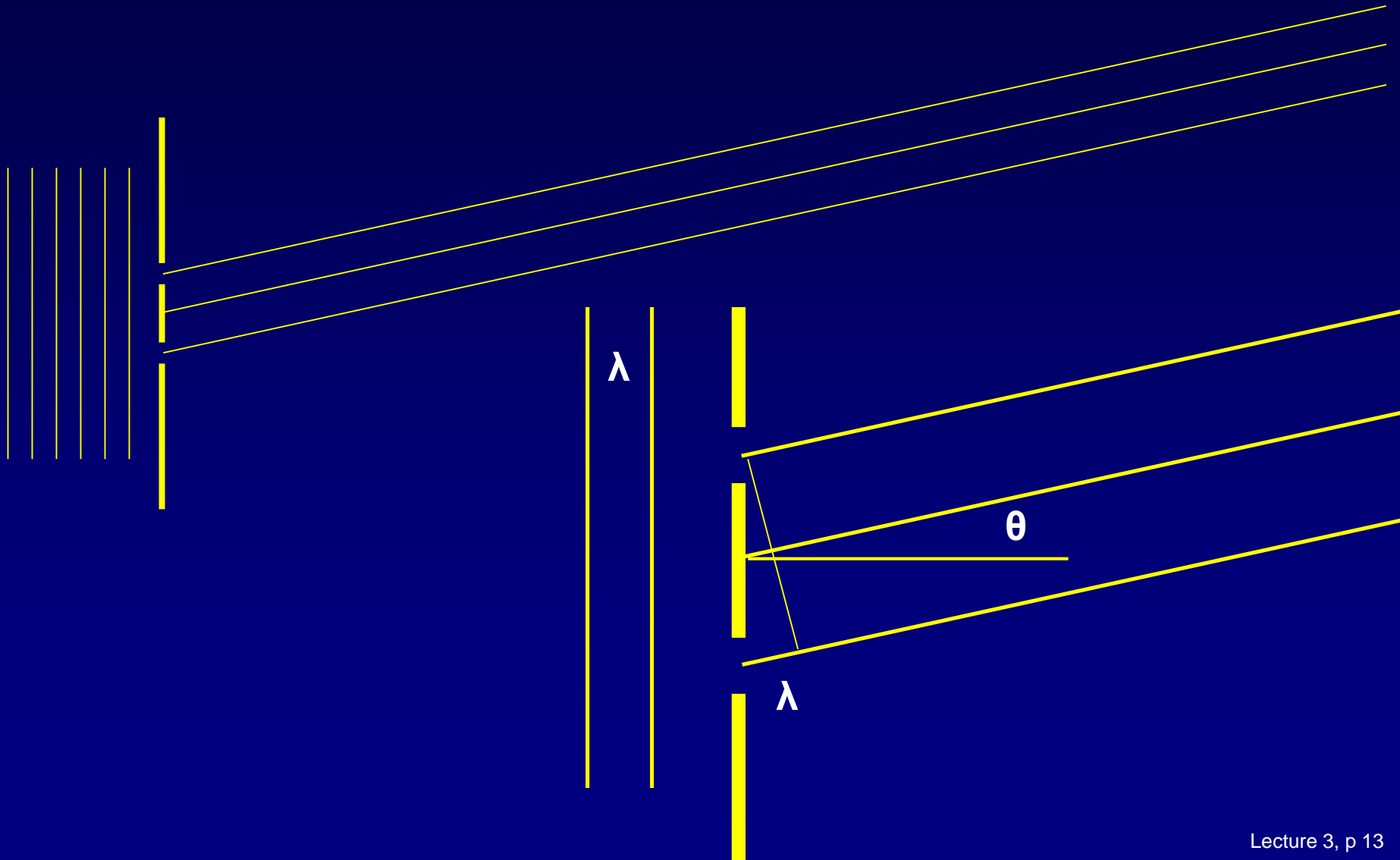
→  $\lambda$  increases

→  $\delta/\lambda$  decreases

→  $\phi$  decreases

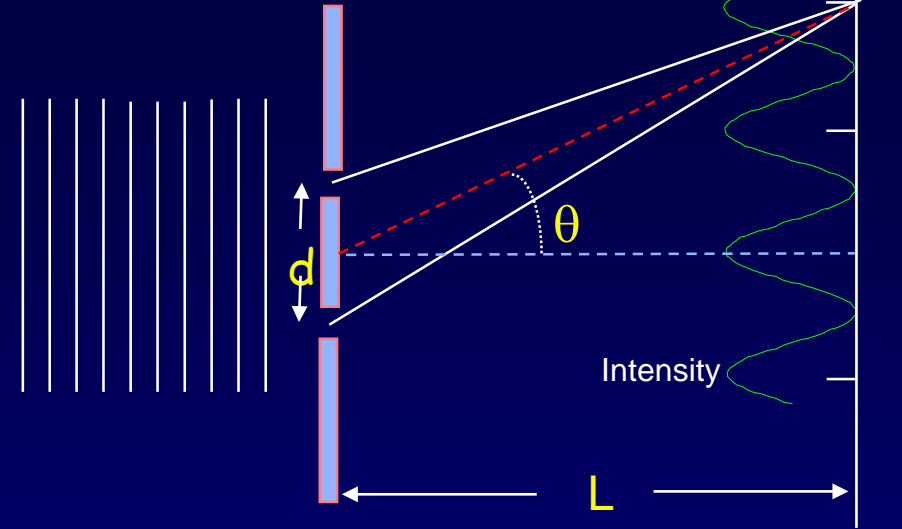
→  $I$  decreases

# The basic idea of two slit diffraction/interference and by extension, multi-slit diffraction/interference



# Example: 2-slit interference

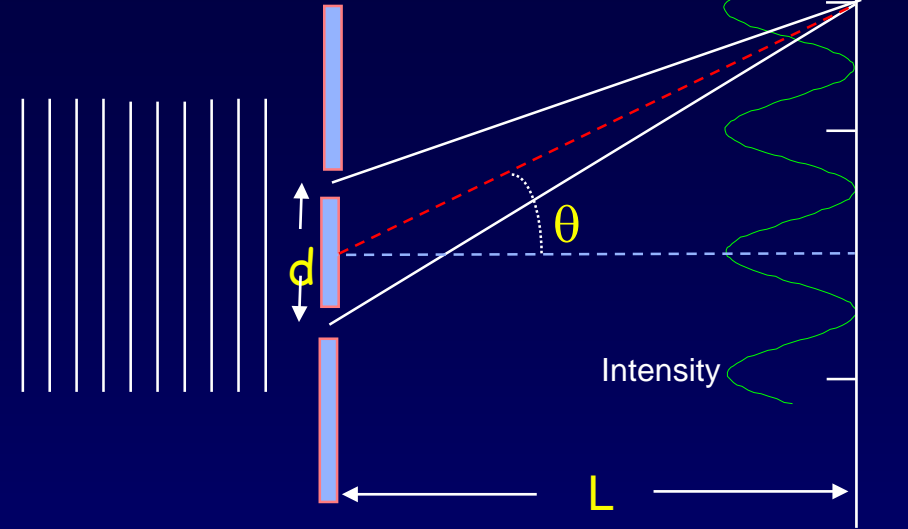
A laser of wavelength  $532\text{ nm}$  is incident on two slits separated by  $0.125\text{ mm}$ .



1. What is the angle of the second principle maximum?
2. What is the spacing  $\Delta y$  between adjacent fringe maxima (*i.e.*,  $\Delta m = 1$ ) on a screen  $2\text{m}$  away?

# Solution

A laser of wavelength 532 nm is incident on two slits separated by 0.125 mm.



1. What is the angle of the second principle maximum?

First: Can we use the small angle approximation?

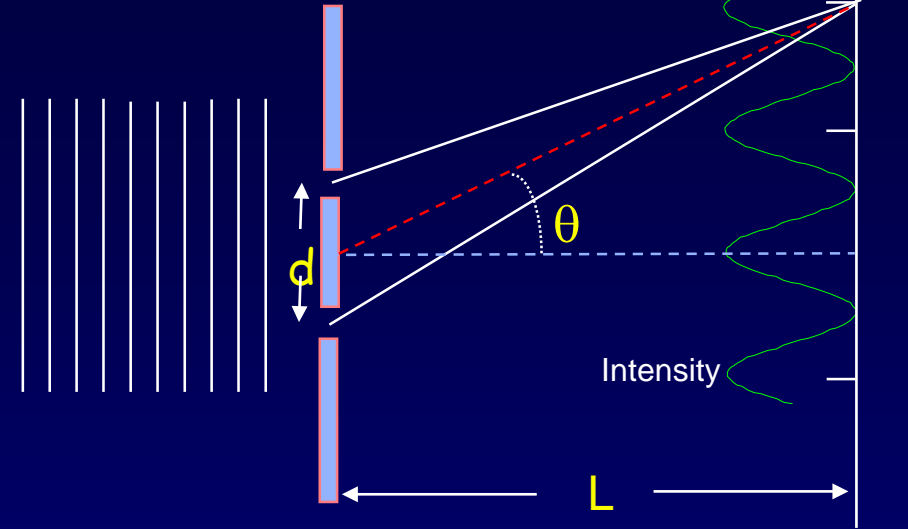
$d = 125 \mu\text{m}$ ;  $\lambda = 0.532 \mu\text{m}$   $d \sin \theta = m\lambda \rightarrow d \gg \lambda \rightarrow \sin \theta$  is small  $\rightarrow \theta$  is small.

$d \sin \theta_m = m\lambda \sim d\theta_m \rightarrow \theta_m \approx m(\lambda/d) = 2 (0.532/125) = 0.0085 \text{ rad} = 0.49^\circ$  (small!)

2. What is the spacing  $\Delta y$  between adjacent fringe maxima (*i.e.*,  $\Delta m = 1$ ) on a screen 2m away?

# Solution

A laser of wavelength 532 nm is incident on two slits separated by 0.125 mm.



1. What is the angle of the second principle maximum?

First: Can we use the small angle approximation?

$$d = 125 \mu\text{m}; \lambda = 0.532 \mu\text{m} \rightarrow d \gg \lambda \rightarrow \theta \text{ is small.}$$

$$d \sin \theta_m = m\lambda \sim d\theta_m \rightarrow \theta_m \approx m(\lambda/d) = 2 (0.532/125) = 0.0085 \text{ rad} = 0.49^\circ \text{ (small!)}$$

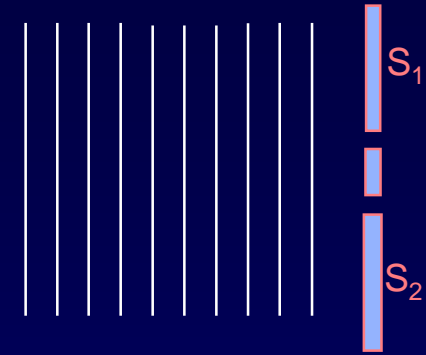
2. What is the spacing  $\Delta y$  between adjacent fringe maxima (*i.e.*,  $\Delta m = 1$ ) on a screen 2m away?

$$\Delta y \approx L(\theta_2 - \theta_1) \approx L(2 - 1)(\lambda/d) = L\lambda/d = (2 \text{ m})(0.532 \mu\text{m})/125 \text{ mm} = 0.0085 \text{ m} \sim 1 \text{ cm}$$

Could have also used 1 – 0 (or 6 – 5).

# Act 3: 2-slit interference

We now increase the wavelength by 20 and decrease the slit spacing by 10, i.e., direct a  $10.6\text{-}\mu\text{m}$  laser onto two slits separated by  $12.5\text{ }\mu\text{m}$ .



How *many* interference peaks may be observed?  
(Hint: Does the small angle approximation hold?)

a. 0

b. 1

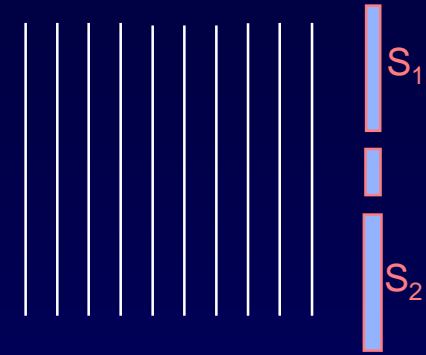
c. 3

d. 4

e.  $\infty$

# Solution

We now increase the wavelength by 20 and decrease the slit spacing by 10, i.e., direct a 10.6- $\mu\text{m}$  laser onto two slits separated by 12.5  $\mu\text{m}$ .



How *many* interference peaks may be observed?  
(Hint: Does the small angle approximation hold?)

a. 0

b. 1

c. 3

d. 4

e.  $\infty$

First: Can we use the small angle approximation?

$d = 12.5 \mu\text{m}$ ;  $\lambda = 10.6 \mu\text{m}$   $\rightarrow d \sim \lambda \rightarrow \theta$  is *not* small.

$$d \sin\theta_m = m\lambda$$

$$\text{Because } \sin\theta_m \leq 1, m < d/\lambda = 12.5/10.6 = 1.17$$

$$\therefore m_{\text{max}} = 1 \quad (\theta_1 = 58^\circ)$$

Note: This ALWAYS has a solution for  $m = 0 \rightarrow$  there's *always* a central peak

Note: The pattern is symmetric, so there's a peak corresponding to  $m = -1$  too.



"Beam me up Scotty -  
It ate my phasor!"

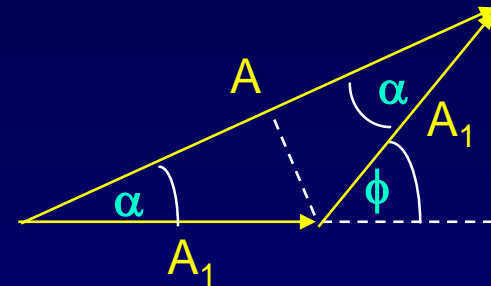
# Phasors

Let's find the resultant amplitude of two waves using phasors.

Pick a time ( $t=0$ ). Take a snapshot of waves.

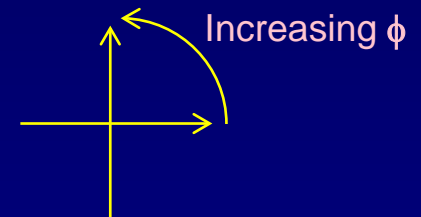
Represent them by 2D vectors,  $(x,y) \rightarrow (A, \phi)$   
polar coordinates

**Two-slit case.** Suppose the amplitudes are the same. Represent each wave by a vector with magnitude ( $A_1$ ) and direction ( $\phi$ ). One wave has  $\phi = 0$ .



Isosceles triangle:  $\alpha = \phi/2$ . So,  $A = 2A_1 \cos\left(\frac{\phi}{2}\right)$

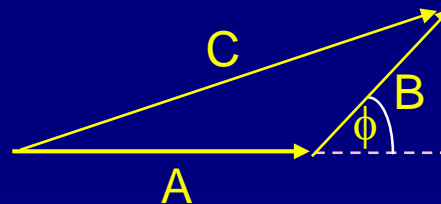
This is identical to our previous result !



WHY PHASORS ARE SO USEFUL:

More generally, if the phasors have different amplitudes  $A$  and  $B$ :

$$C^2 = A^2 + B^2 + 2AB \cos \phi$$



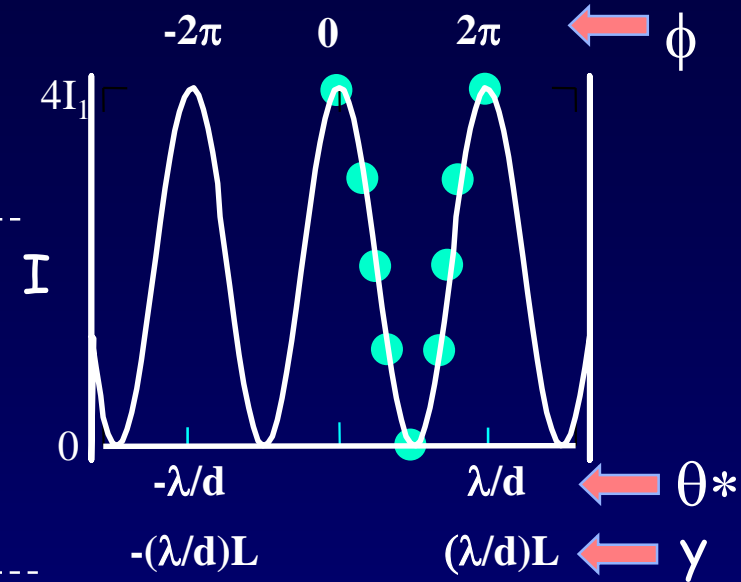
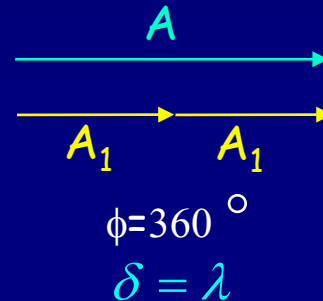
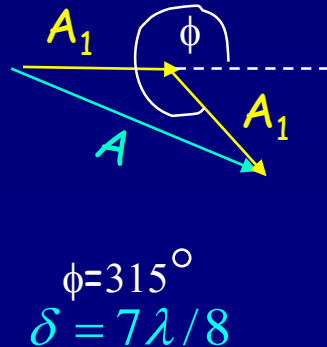
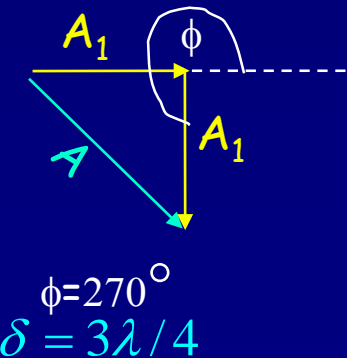
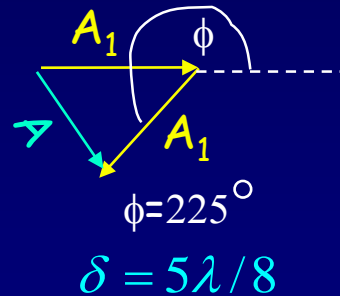
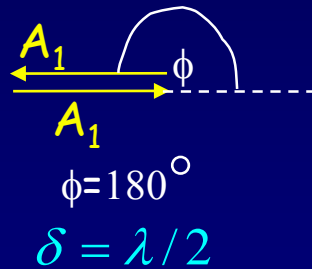
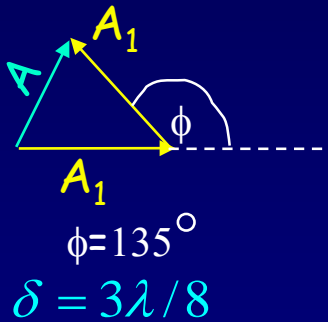
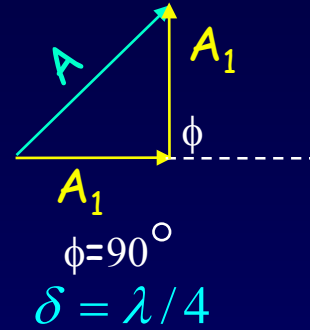
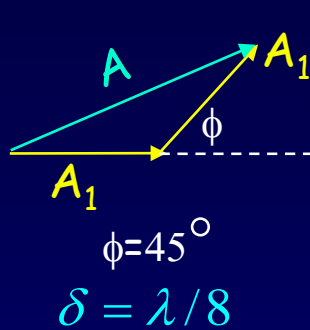
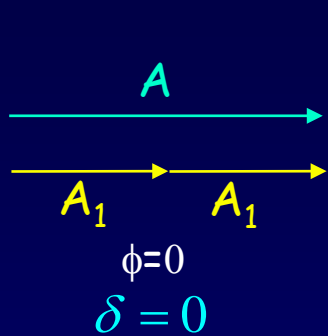
$$A \cos(kx - \omega t) + B \cos(kx - \omega t + \phi)$$

Here  $\phi$  is the external angle.

- See the supplementary slide.
- See text: 35.3, 36.3, 36.4.
- See Physics 212 lecture 20.
- Phasors make it easier to solve other problems later.

# Phasors for 2-Slits

- Plot the phasor diagram for different  $\phi$ :

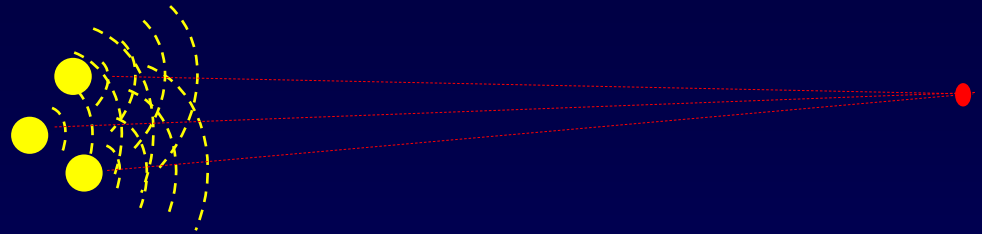


$$\phi = 2\pi \frac{r_2 - r_1}{\lambda} = 2\pi \frac{\delta}{\lambda}$$

\*Small-angle approx.  
assumed here

# Act 4: Multiple sources

Consider light now coming from *three* openings, arranged in an equilateral triangle.



At given distance far from the openings, and equidistant from each of them, the light from each independently has intensity  $I_1$ . What is the total intensity when all three openings are open?

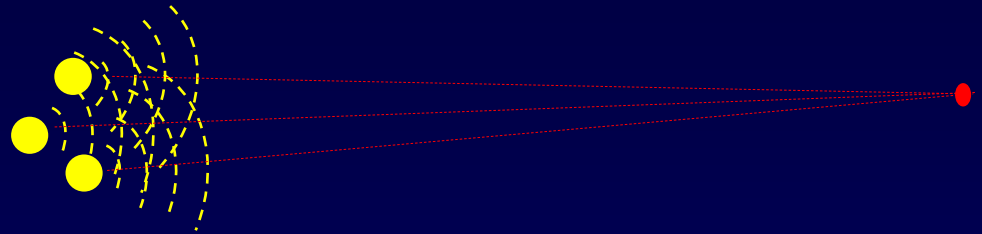
a.  $3I_1$

b.  $9I_1$

c. cannot be determined

# Act 4: Multiple sources

Consider light now coming from *three* openings, arranged in an equilateral triangle.

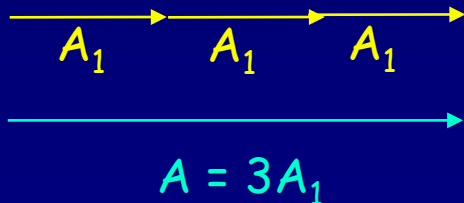


At given distance far from the openings, and equidistant from each of them, the light from each independently has intensity  $I_1$ . What is the total intensity when all three openings are open?

a.  $3I_1$

b.  $9I_1$

c. cannot be determined



$$\text{Therefore, } I_{\text{tot}} = A^2 = (3 A_1)^2 = 9 I_1$$

$$\text{In general, } I_{\text{tot}} = A^2 = (N A_1)^2 = N^2 I_1$$

# Multiple-Slit Interference

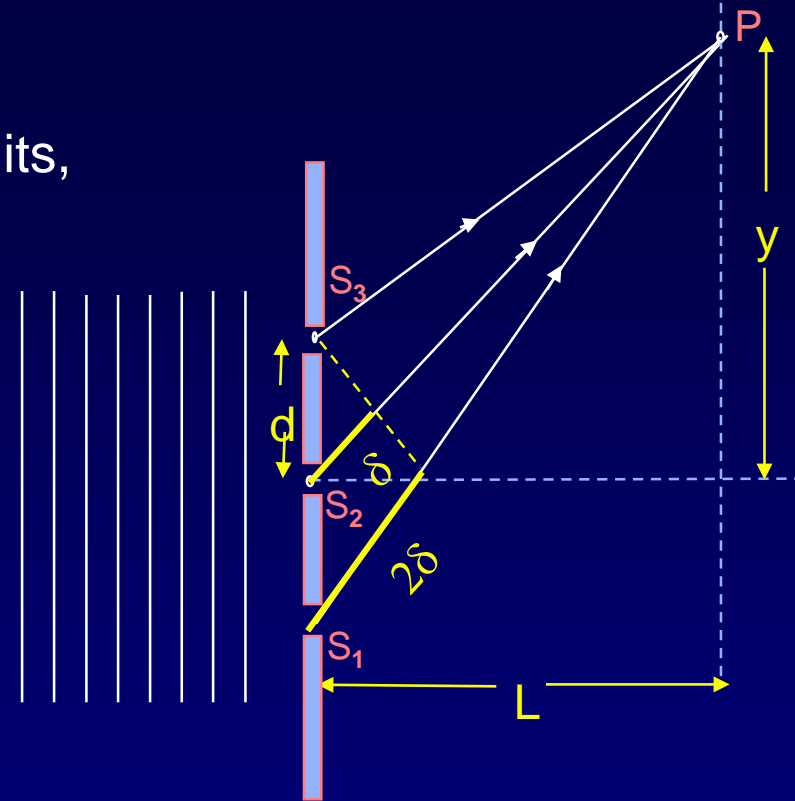
What changes if we increase the number of slits,  
e.g.,  $N = 3, 4, 1000, \dots$

(for now we'll go back to very small slits, so  
we can neglect diffraction from each of them)

First look at the principal maxima.

For equally spaced slits:

If slit 1 and 2 are in phase with each other,  
then slit 3 will also be in phase, etc.

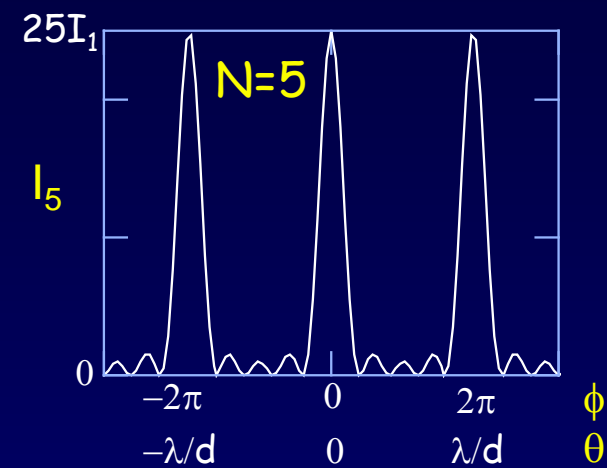
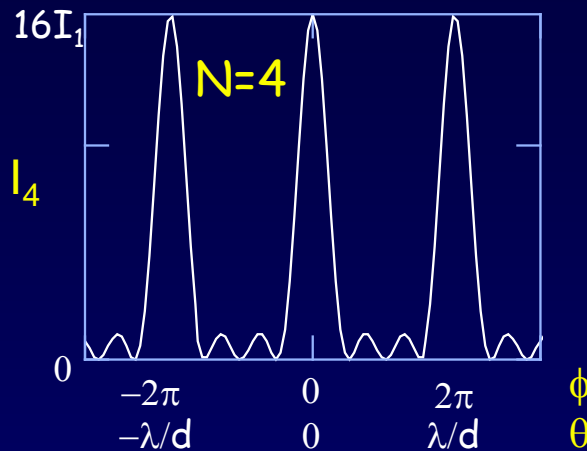
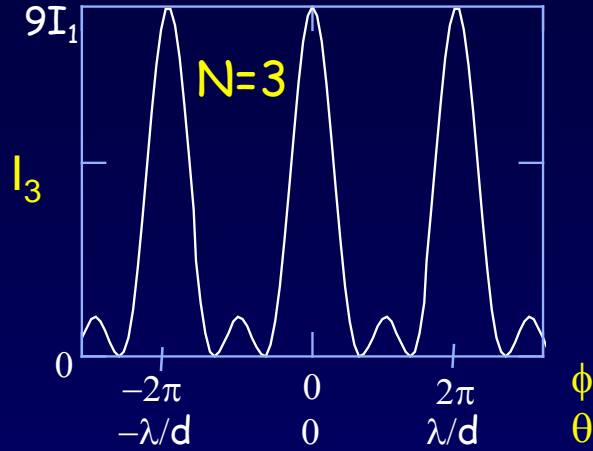


The positions of the principal interference maxima  
are the *same* for any number of slits!

$$d \sin \theta = m \lambda$$

We will almost always consider equally spaced slits.

# Multiple-Slit Interference (2)



The positions of the principal maxima occur at  $\phi = 0, \pm 2\pi, \pm 4\pi, \dots$   
 where  $\phi$  is the phase between adjacent slits.  $\theta = 0, \pm \lambda/d, \pm 2\lambda/d, \dots$

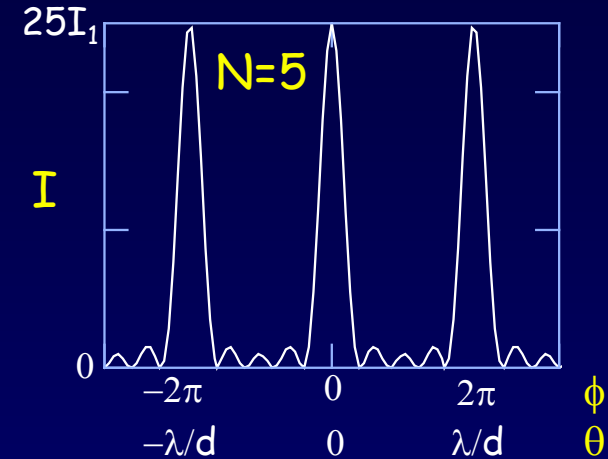
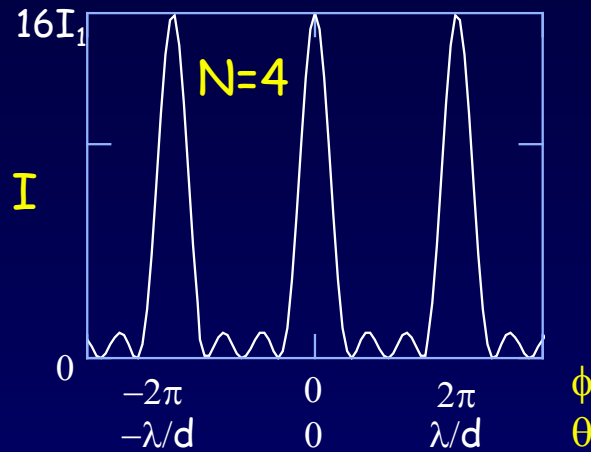
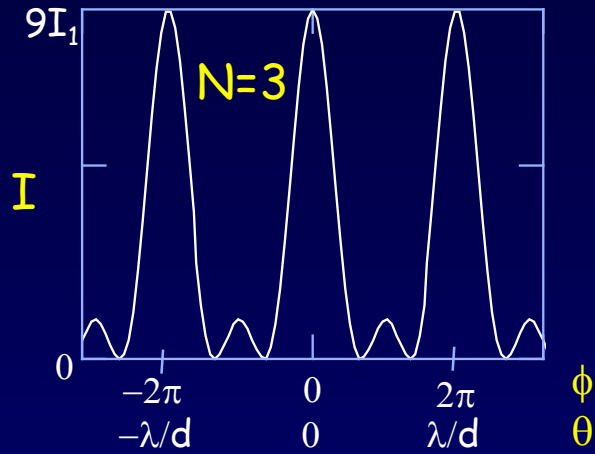
The intensity at the peak of a principal maximum goes as  $N^2$ .

3 slits:  $A_{\text{tot}} = 3A_1 \Rightarrow I_{\text{tot}} = 9I_1$ .  $N$  slits:  $I_N = N^2I_1$ .

Between two principal maxima there are  $N-1$  zeros and  $N-2$  secondary maxima  $\Rightarrow$  The peak width  $\propto 1/N$ .

The total power in a principal maximum is proportional to  $N^2(1/N) = N$ .

# Phasors for N-Slit Interference

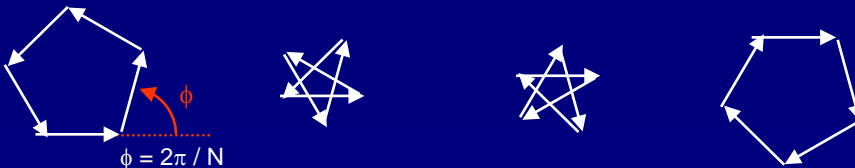


Drawn here for  $N = 5$ :

Principal maxima:  $\phi = 0, \pm 2\pi, \text{etc.}$



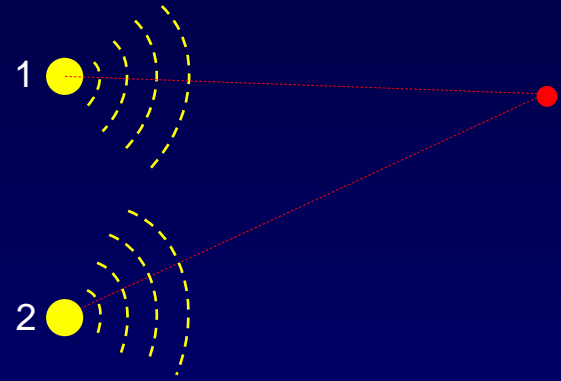
Zeros.  $\phi = m(2\pi/N)$ , for  $m = 1$  to  $N-1$ .



# At Home: Phasor Exercise

Two speakers emit equal intensity (call the amplitude  $A = 1$ ) sound of frequency  $f = 256$  Hz. The waves are in phase at the source. Suppose that the path difference to the observer is  $\delta = 0.3$  m (speaker 1 is closer).  $v = 330$  m/s.

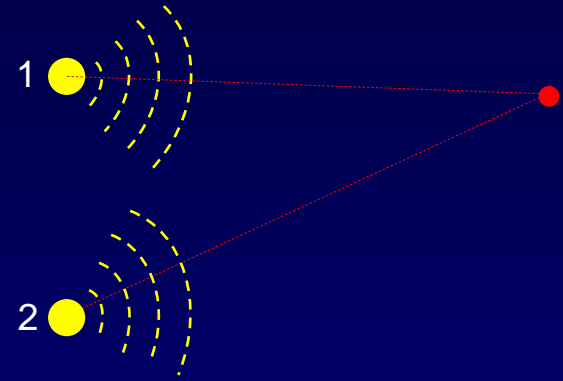
Draw a phasor diagram that describes the two waves at the observer and the resulting wave. What is the resulting amplitude?



# Solution

Two speakers emit equal intensity (call the amplitude  $A = 1$ ) sound of frequency  $f = 256$  Hz. The waves are in phase at the source. Suppose that the path difference to the observer is  $\delta = 0.3$  m (speaker 1 is closer).  $v = 330$  m/s.

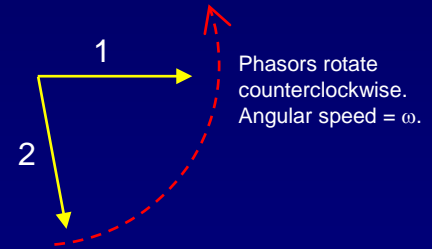
Draw a phasor diagram that describes the two waves at the observer and the resulting wave. What is the resulting amplitude?



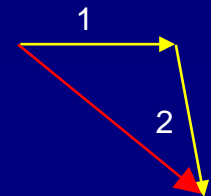
The wavelength is  $\lambda = v/f = 1.29$  m, so the phase difference is  $\phi = 2\pi(\delta/\lambda) = 1.46$  radians  $= 83.7^\circ$ .

Notes:

- The two phasors have the same length (amplitude).
- We can always pick one phasor to be horizontal.
- Source 2 is farther from the observer, so its phasor lags behind.

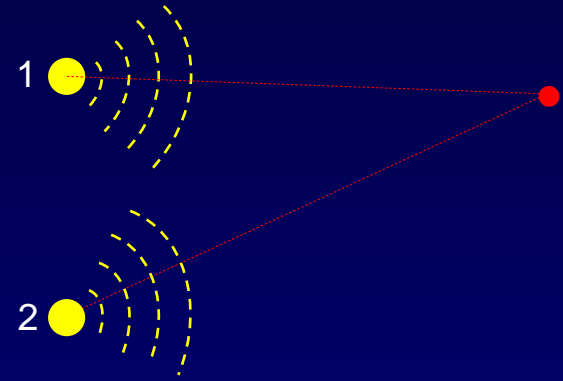


Find the resultant by adding the phasors. The resulting amplitude is approximately  $\sqrt{2}$ . You'll need to use the algebraic solution to get a more accurate answer.



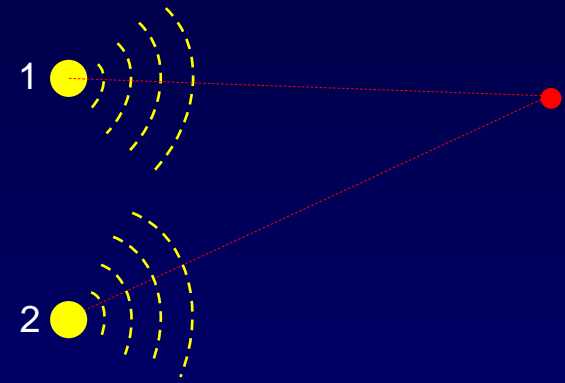
# At Home: Phasor Exercise 2

Suppose the intensity of speaker 2 is twice that of speaker 1. Everything else is the same as in the previous exercise. Draw the phasor diagram that describes this situation.

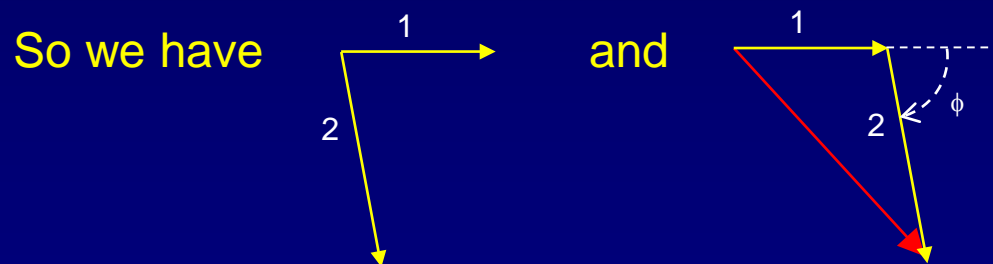


# Solution

Suppose the intensity of speaker 2 is twice that of speaker 1. Everything else is the same as in the previous exercise. Draw the phasor diagram that describes this situation.



The phase difference is unchanged:  $\phi = 83.7^\circ$ .  
Now, the length of phasor 2 is  $\sqrt{2}$  larger.  
(Remember that phasors are amplitudes.)



Note that the algebraic solution we wrote before does not apply here, because the amplitudes aren't equal. You can use some trigonometry to calculate the length of the third side of the triangle.

$$\text{Law of cosines: } c^2 = a^2 + b^2 + 2ab \cos\phi = 1 + 2 + 2\sqrt{2} \times 0.11 = 3.31 \quad (c = 1.82)$$

# Supplement: Phase shift and Position or Time Shift

Because the wave is oscillating both in time and position, we can consider  $\phi$  to be either a time or position shift:

Time:

$$\begin{aligned}y &= A_1 \cos(kx - \omega t + \phi) \\&= A_1 \cos(kx - \omega(t - \phi/\omega)) \\&= A_1 \cos(kx - \omega(t - \phi T/2\pi)) \\&= A_1 \cos(kx - \omega(t - \delta t))\end{aligned}$$

The time shift:  $\delta t/T = \phi/2\pi$

Positive  $\phi$  shifts to later times.

Position:

$$\begin{aligned}y &= A_1 \cos(kx - \omega t + \phi) \\&= A_1 \cos(k(x + \phi/k) - \omega t) \\&= A_1 \cos(k(x + \phi\lambda/2\pi) - \omega t) \\&= A_1 \cos(k(x - \delta x) - \omega t)\end{aligned}$$

The position shift:  $\delta x/\lambda = -\phi/2\pi$

Positive  $\phi$  shifts to negative position.

# Supplement: Phasor Math

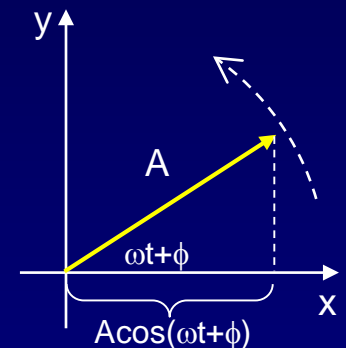
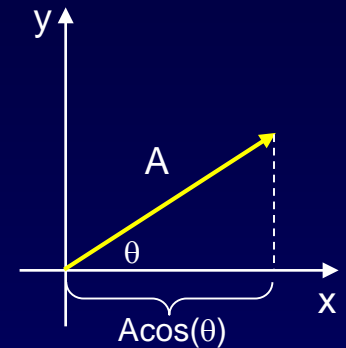
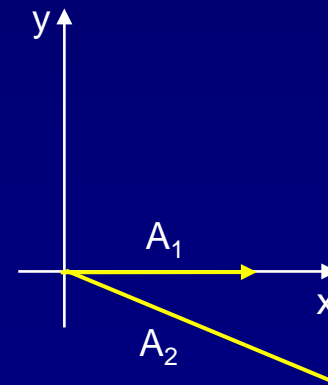
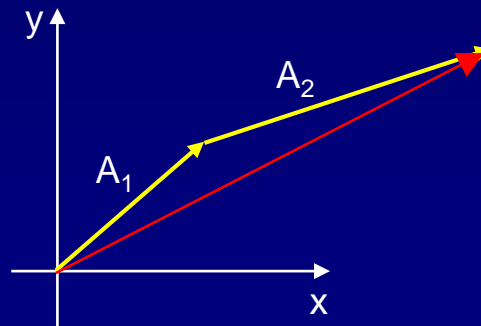
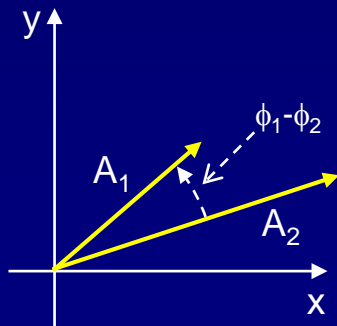
We want to manipulate  $A\cos(\omega t + \phi)$ . Use the fact that the x-component of a 2-dimensional vector is  $A\cos(\theta)$ .

If  $\theta$  is changing with time,  $\theta = \omega t$ , the vector is rotating, and the x component is  $A\cos(\omega t + \phi)$ . That's what we want.

If we have two quantities that have the same frequency, but different amplitudes and phases:

$$A_1\cos(\omega t + \phi_1) \text{ and } A_2\cos(\omega t + \phi_2)$$

we can use vector addition to calculate their superposition.



It is conventional to draw one phasor horizontal. Because the phasors are rotating, this merely means we are looking at them at a particular time.