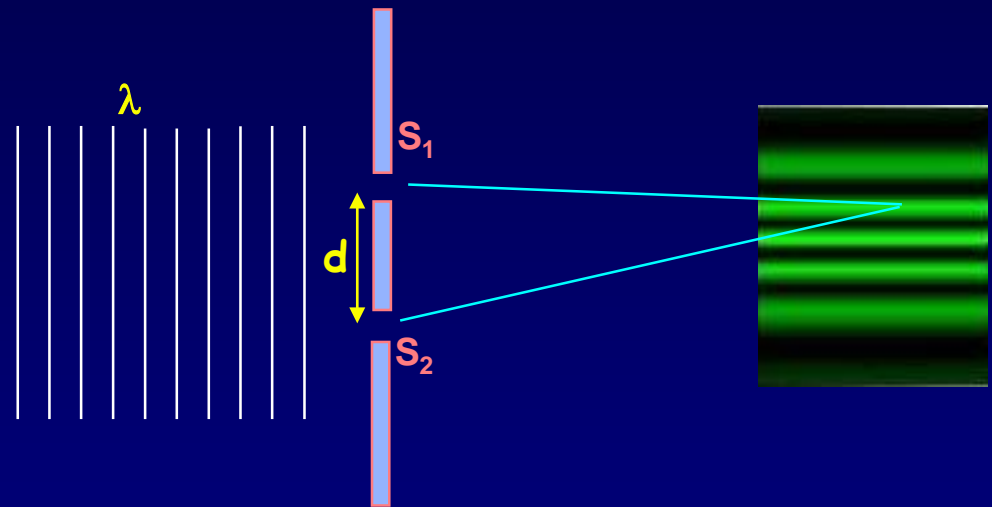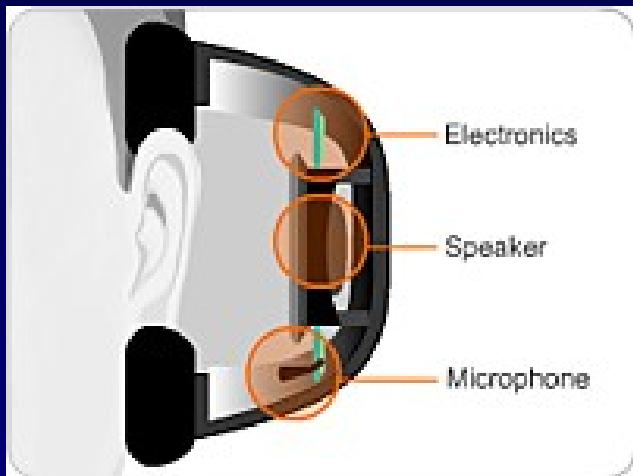# Lecture 2: Interference

EM Waves: Always add wave functions, then square for intensity.

QM waves: Always add wave functions, then square for relative probablility

# The Many "Fathers" of QM

- 1900 <u>Planck</u> "solves" the blackbody problem by postulating that the oscillators in the walls have quantized energy levels.

"Until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me…the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be."

- 1905 <u>Einstein</u> proposes that light energy is quantized – "photons"

- 1913 <u>Bohr</u> proposes that electron orbits are quantized

- 1923 <u>de Broglie</u> proposes that particles behave like waves

- 1925 <u>Pauli</u> introduces "exclusion principle" – only 2 electrons/orbital

- 1925 <u>Heisenberg</u> introduces matrix-formulation of QM

- 1926 <u>Schrödinger</u> introduces the wave-formulation of QM

Today

- Superposition review

- Interference of sound waves (phase differences)

- Two-slit interference

Review: Wave Summary

The formula $y(x,t) = A\cos(kx - \omega t)$
describes a harmonic plane wave
of amplitude *A* moving in the *+x* direction.
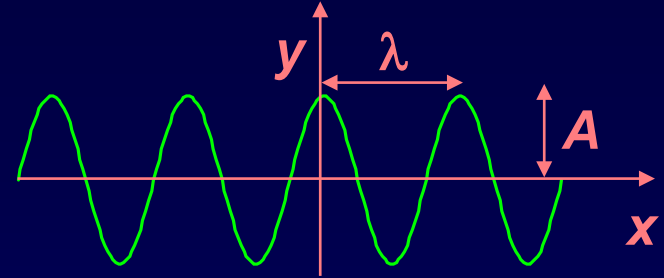
For a wave on a string, each point on the wave oscillates in the *y* direction with simple harmonic motion of angular frequency ω.

The wavelength is $\lambda = \dfrac{2\pi}{k}$; the speed is $v = \lambda f = \dfrac{\omega}{k}$

The intensity is proportional to the square of the amplitude: $I \propto A^2$

Superposition:
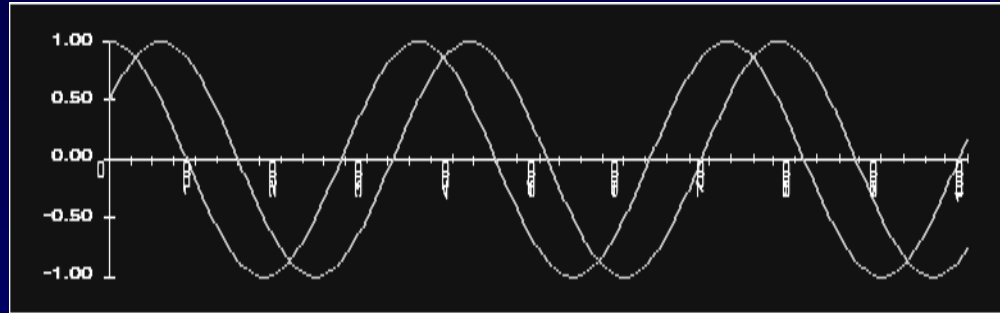
(ALWAYS ADD WAVE FUNCTIONS, THEN SQUARE FOR INTENSITY)

Because the wave equation is linear, arbitrary combinations of solutions will also be solutions. For unequal intensities, the maximum and minimum intensities are:

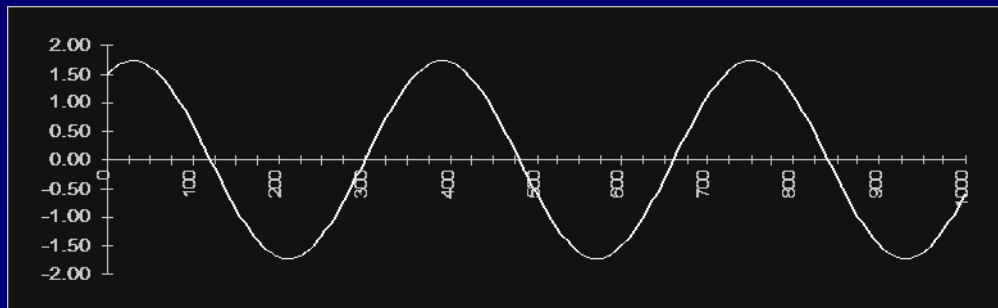$$I_{max} = |A_1 + A_2|^2$$

$$I_{min} = |A_1 - A_2|^2$$

Superposing sine waves

If you added the two sinusoidal waves shown, what would the result look like?

The sum of two sines having the same frequency is another sine with the same frequency.

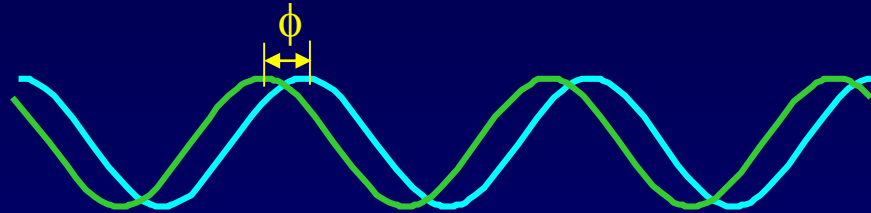Its amplitude depends on their relative phases.
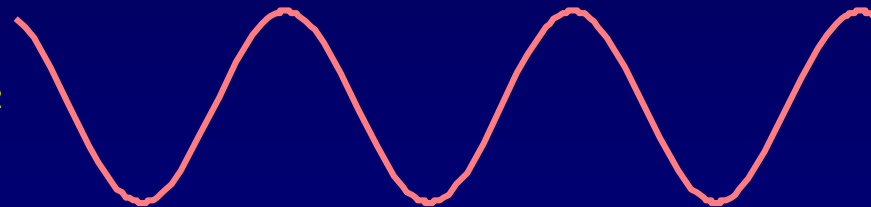

Now we'll see how this works.

Adding Sine Waves with Different Phases

Suppose we have two sinusoidal waves with the same A_1, ω, and k: $y_1 = A_1 \cos(kx - \omega t)$ and $y_2 = A_1 \cos(kx - \omega t + \phi)$
One starts at phase ϕ after the other:

Spatial dependence of 2 waves at t = 0:

Resultant wave: $y = y_1 + y_2$

Use this trig identity: $A_1 \left(\cos\alpha + \cos\beta \right) = 2A_1 \cos\left(\dfrac{\beta - \alpha}{2} \right) \cos\left(\dfrac{\beta + \alpha}{2} \right)$

α = kx-ωt

β = kx-ωt+ φ

$y_1 + y_2$ $\left(\phi / 2 \right)$ $\left(kx - \omega t + \phi / 2 \right)$

$y \;\; = \;\; 2A_1 \cos(\phi / 2) \; \cos(kx - \omega t + \phi / 2)$

Amplitude   Oscillation

Interference of Waves

What happens when two waves are present at the same place?

Always add amplitudes (pressures or electric fields).

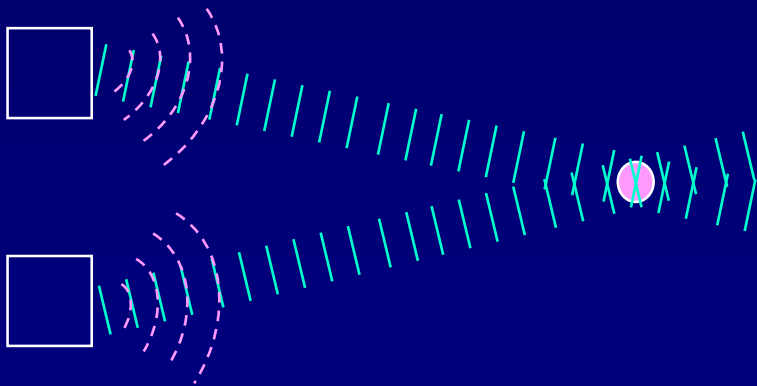However, we observe intensity (power).

For equal A and ω:

$$A = 2A_1 \cos(\phi/2) \Rightarrow I = 4I_1 \cos^2(\phi/2)$$

Example:

Stereo speakers: Listener:

Terminology: (two important cases)

Constructive interference:
waves are "in phase"
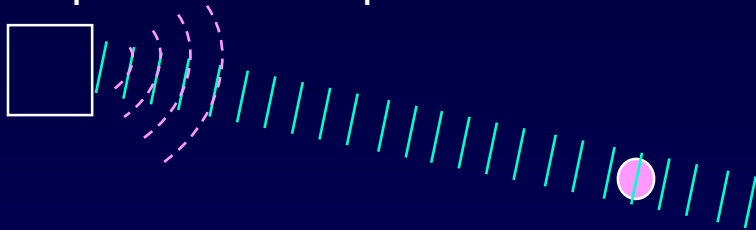(ϕ = 0, 2π, 4π, ..)

Destructive interference:
waves are "out of phase"
(ϕ = π, 3π, 5π, ...)

Of course, ϕ can take on an infinite number of values. We won't use terms like "mostly constructive" or "slightly destructive".

Example: Changing phase of the Source

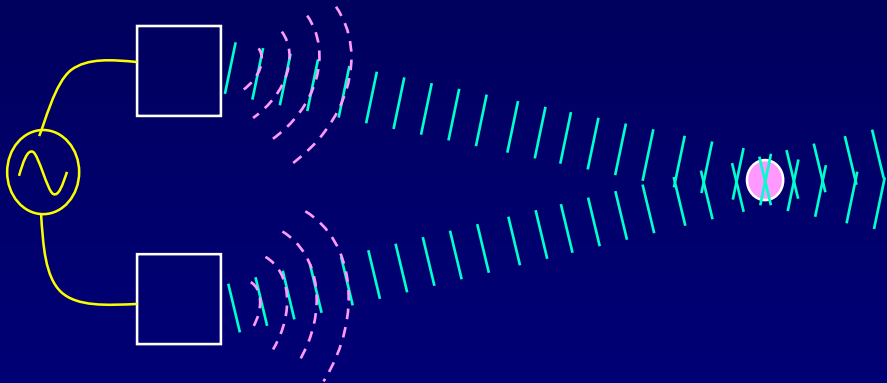Each speaker alone produces an intensity of $I_1 = 1$ W/m^2 at the listener:
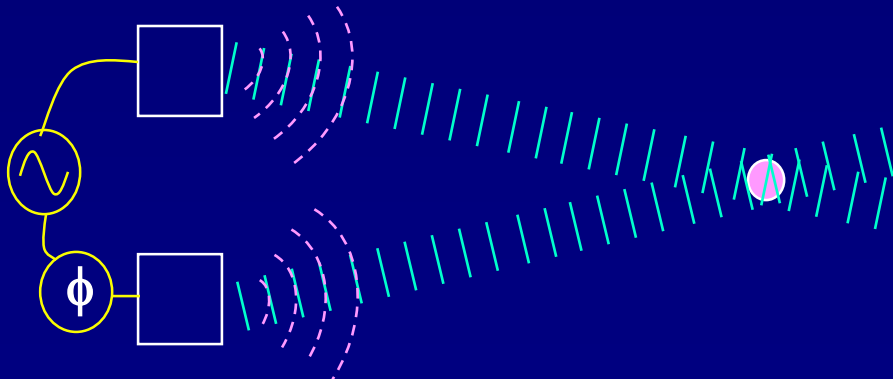
equal distances!

$$I = I_1 = A_1^2 = 1 \text{ W/m}^2$$

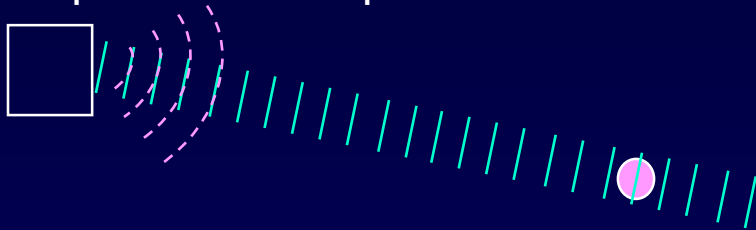Drive the speakers in phase. What is the intensity I at the listener?

$I =$

Now shift phase of one speaker by 90°. What is the intensity I at the listener?

$I =$

ϕ

Example: Changing phase of the Source

Each speaker alone produces an intensity of $I_1 = 1$ W/m^2 at the listener:

$I = I_1 = A_1^2 = 1$ W/m^2

Drive the speakers in phase. What is the intensity I at the listener?

$I = (2A_1)^2 = 4I_1 = 4$ W/m^2

Now shift phase of one speaker by 90°. What is the intensity I at the listener?
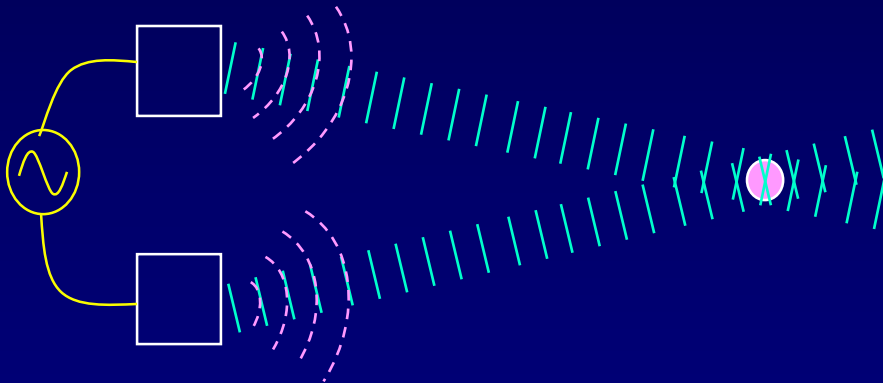
$I =$

Example: Changing phase of the Source

Each speaker alone produces an intensity of $I_1 = 1$ W/m^2 at the listener:

$$I = I_1 = A_1^2 = 1 \text{ W/m}^2$$
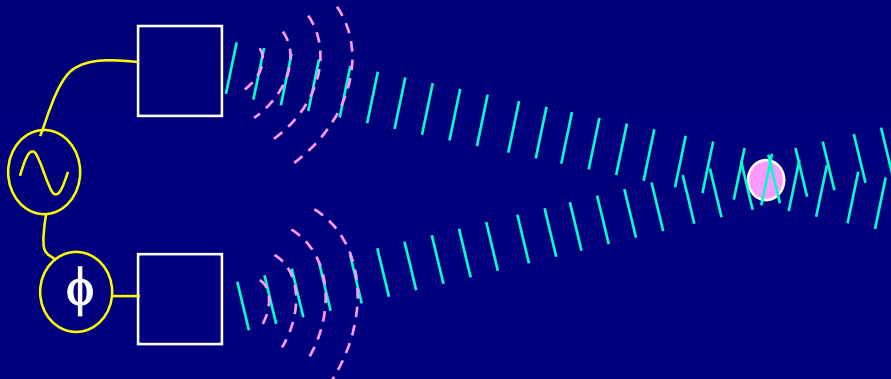
Drive the speakers <u>in phase</u>. What is the intensity I at the listener?

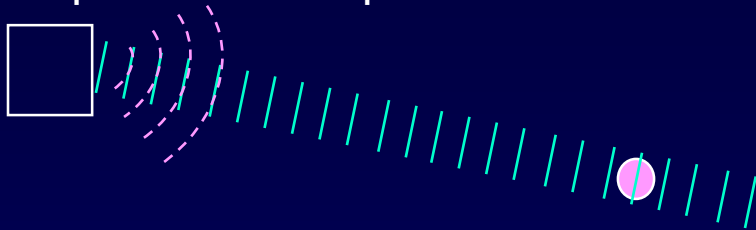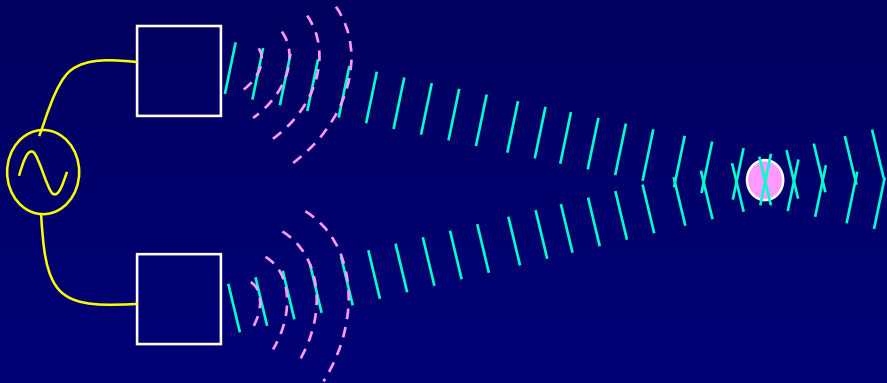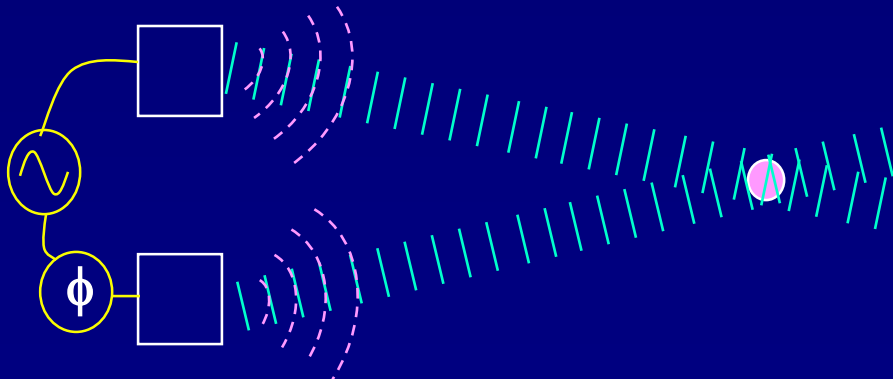$$I = (2A_1)^2 = 4I_1 = 4 \text{ W/m}^2$$

Now <u>shift phase</u> of one speaker *by* <u>90°</u>. What is the intensity I at the listener?

$$I = 4 I_1 \cos^2(45^0) = 2.0 I_1 = 2.0 \text{ W/m}^2$$

ACT 1:
Noise-cancelling Headphones

Noise-canceling headphones work using interference. A microphone on the earpiece monitors the instantaneous amplitude of the external sound wave, and a speaker on the inside of the earpiece produces a sound wave to cancel it.

Electronics

Speaker

Microphone

1. What must be the phase of the signal from the speaker relative to the external noise?

a. 0　　　　b. 90˚　　　c. π　　　d. -180˚　　e. 2π

2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n?

a. $I_s = I_n$　　b. $I_s < I_n$　　c. $I_s > I_n$

Solution

Noise-canceling headphones work using interference. A microphone on the earpiece monitors the instantaneous amplitude of the external sound wave, and a speaker on the inside of the earpiece produces a sound wave to cancel it.

1. What must be the phase of the signal from the speaker relative to the external noise?

   a. 0   b. 90°   c. π d. -180°   e. 2π

   Destructive interference occurs when the waves are ±180° out of phase.

   180⁰ = π radians!

2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n?

   a. $I_s = I_n$ b. $I_s < I_n$ c. $I_s > I_n$
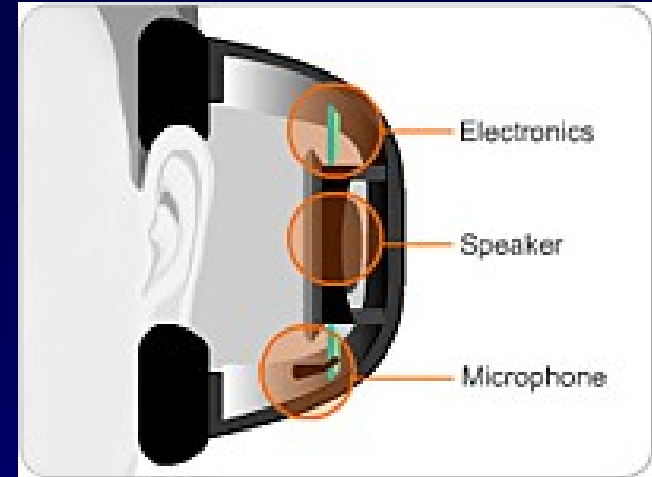
Solution

Noise-canceling headphones work using interference. A microphone on the earpiece monitors the instantaneous amplitude of the external sound wave, and a speaker on the inside of the earpiece produces a sound wave to cancel it.

1. What must be the phase of the signal from the speaker relative to the external noise?

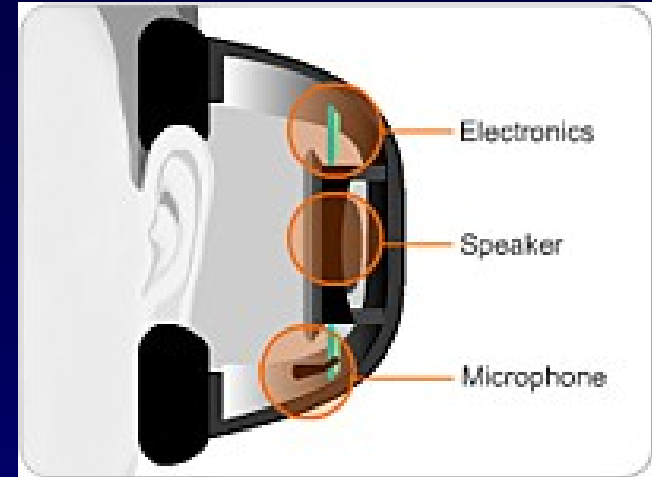a. 0   b. 90˚   c. π d. -180˚   e. 2π

Destructive interference occurs when the waves are ±180˚ out of phase.

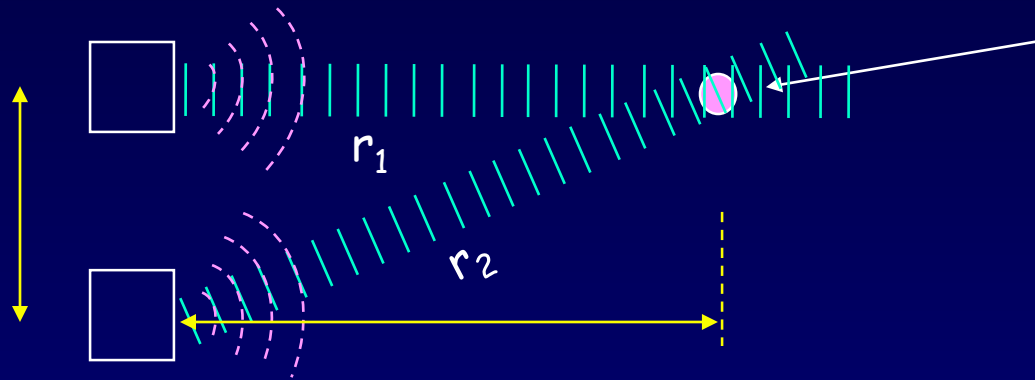180⁰ = π radians!

2. What must be the intensity I_s of the signal from the speaker relative to the external noise I_n?

a. $I_s = I_n$ b. $I_s < I_n$ c. $I_s > I_n$

We want $A = A_s - A_n = 0$.
Note that I is never negative.

Interference Exercise

The relative phase of two waves also depends on the relative distances to the sources:

r_1

r_2

The two waves at this point are "out of phase". Their phase difference ϕ depends on the path difference $\delta \equiv r_2 - r_1$.

Each fraction of a wavelength of path difference gives that fraction of 360° (or 2π) of phase difference:

$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$$

Path difference	Phase difference		
δ	ϕ	$A = 2A_1\cos(\phi/2)$	I
0			
$\lambda/4$			
$\lambda/2$			
λ			

Solution

The relative phase of two waves also depends on the relative distances to the sources:

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The two waves at this point are "out of phase". Their phase difference ϕ depends on the path difference $\delta \equiv r_2 - r_1$.

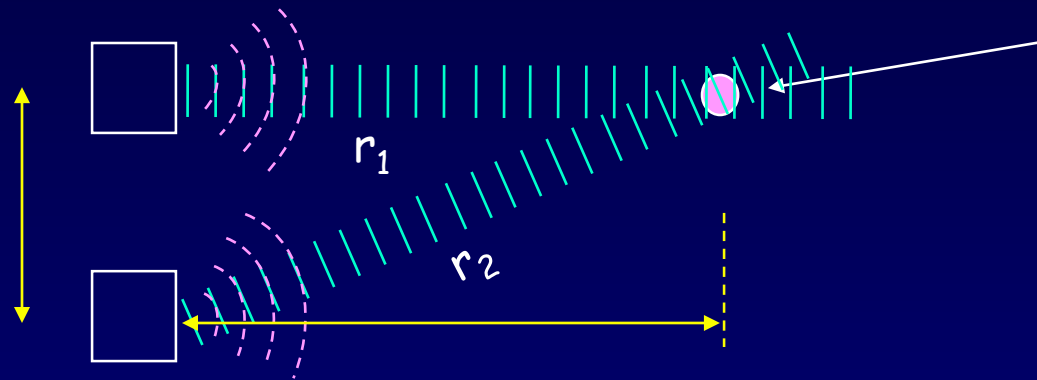Reminder: A can be negative. "Amplitude" is the absolute value.

Each fraction of a wavelength of path difference gives that fraction of 360º (or 2π) of phase difference:

$$\frac{\phi}{2\pi} = \frac{\delta}{\lambda}$$

Path difference δ	Phase difference ϕ	$A = 2A_1\cos(\phi/2)$	I
0	0	$2A_1$	$4I_1$
$\lambda/4$	$\pi/2$	$\sqrt{2}A_1$	$2I_1$
$\lambda/2$	π	0	0
λ	2π	$2A_1$	$4I_1$

Amplitude vs. Intensity for 2 Equal (A_1) Interfering Waves

Plot $2A_1\cos(\phi/2)$ and $4A_1^2\cos^2(\phi/2)$ as a function of ϕ.

$2A_1$

$4A_1^2$

Constructive Interference

Destructive Interference

0   2π   4π   6π   8π   10π

φ

δ

0   λ   2λ   3λ   4λ   5λ
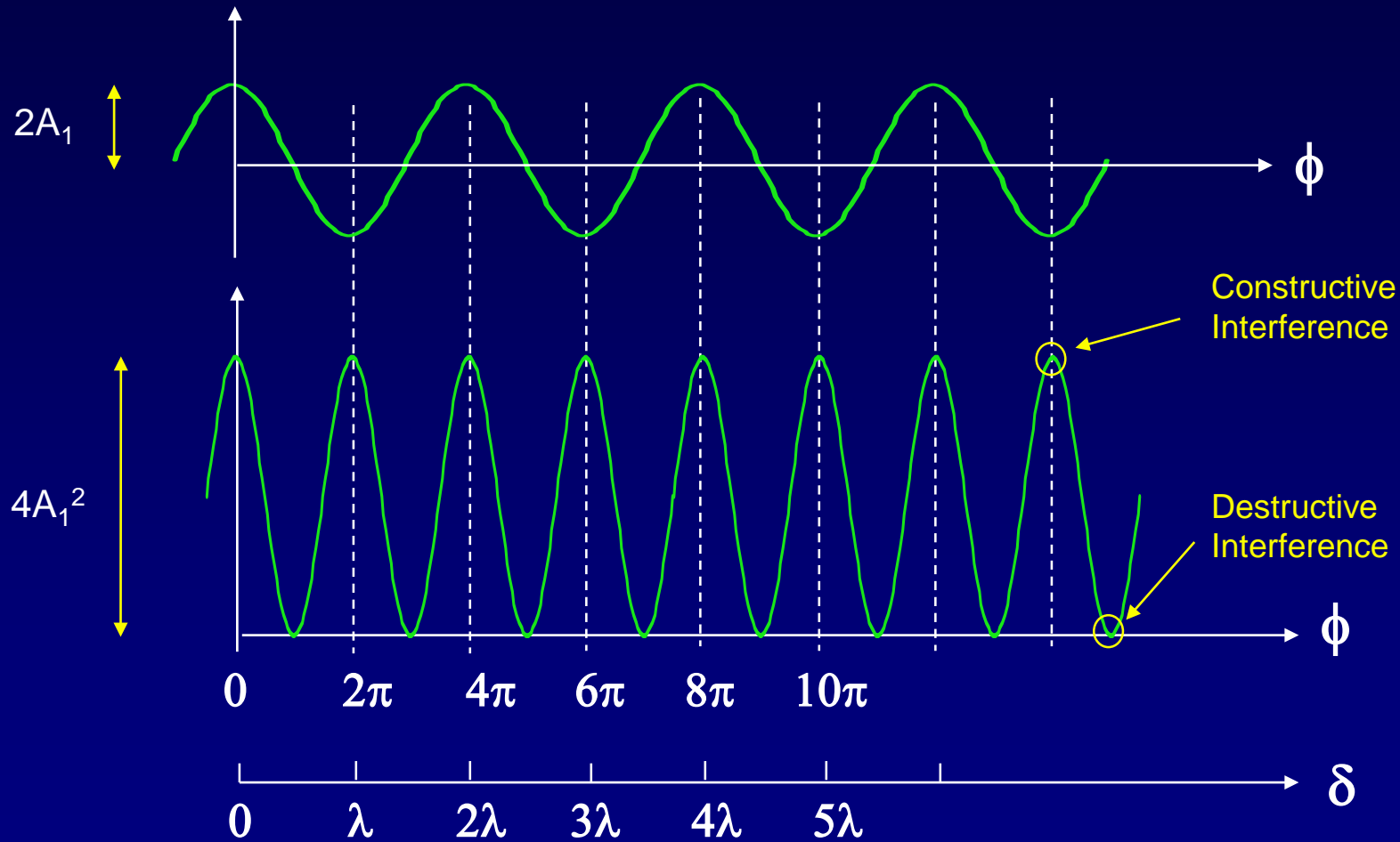
Q: What is the spatial average intensity?

A: $I_{av} = 4I_1 * 0.5 = 2I_1$ Does this make sense?

Summary
Interference of coherent waves

Resultant intensity of two equal-intensity waves of the same wavelength at the same point in space:

$$I = 4\,I_1\cos^2(\phi/2)$$

In order to calculate I, we need to know ϕ.

For unequal intensities, the maximum and minimum intensities are

$$I_{max} = |A_1 + A_2|^2$$
$$I_{min} = |A_1 - A_2|^2$$

The phase difference between the two waves may be due to a difference in their **source phases** or in the **path difference** to the observer, or both. The difference due to path difference is:

$$\phi = 2\pi(\delta/\lambda)$$

where $\delta = r_2 - r_1$

Note: The phase difference can also be due to an index of refraction, because that will change the wavelength.

Light - Particle or Wave?

Diffraction of light played an important historical role.

- 1818: French Academy held a science competition

- Fresnel proposed the diffraction of light.

- One judge, Poisson, knew light was made of particles, and thought Fresnel's ideas ridiculous; he argued that if Fresnel ideas were correct, one would see a bright spot in the middle of the shadow of a disk.

- Another judge, Arago, decided to actually do the experiment…
(our lecture demo)

Light - Particle or Wave?

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- Another judge, Arago, decided to actually do the experiment…
  (our lecture demo)

- **Conclusion in 1818:**
  Light *must* be a wave, since particles don't diffract!

- **But in 1905 Einstein found that light also acts like a particle! (Photoelectric Effect)**
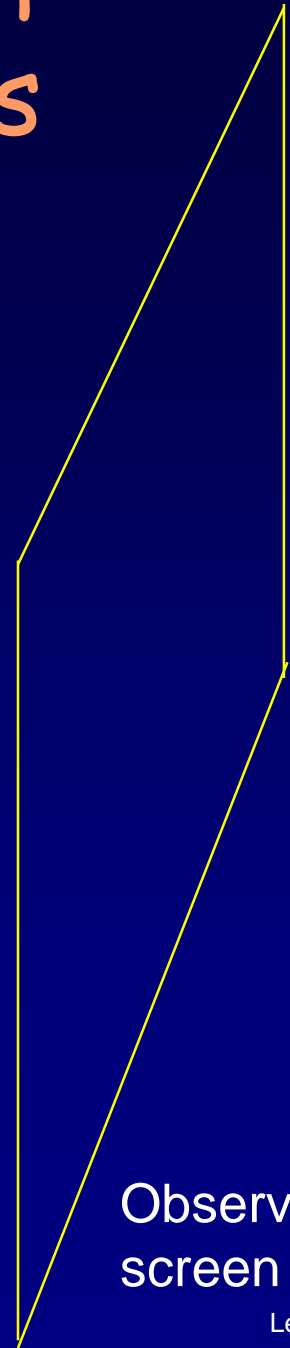
Transmission of Light through Narrow Slits
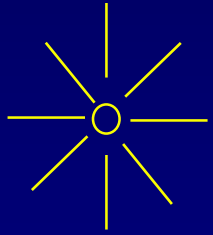
Monochromatic
light source
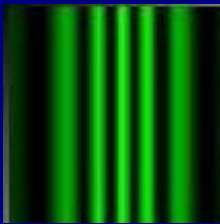at a great distance,
or a laser.
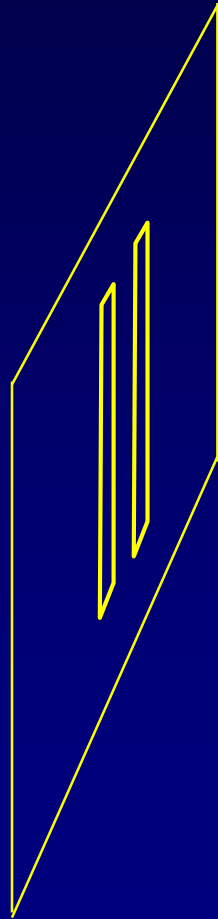
Slit pattern

Observation
screen

Double-slit interference

Light (wavelength λ) is incident on a two-slit (two narrow, rectangular openings) apparatus:

If either one of the slits is closed, a spread-out image of the open slit will appear on the screen. (The image is spread due to diffraction. We will discuss diffraction in more detail later.)
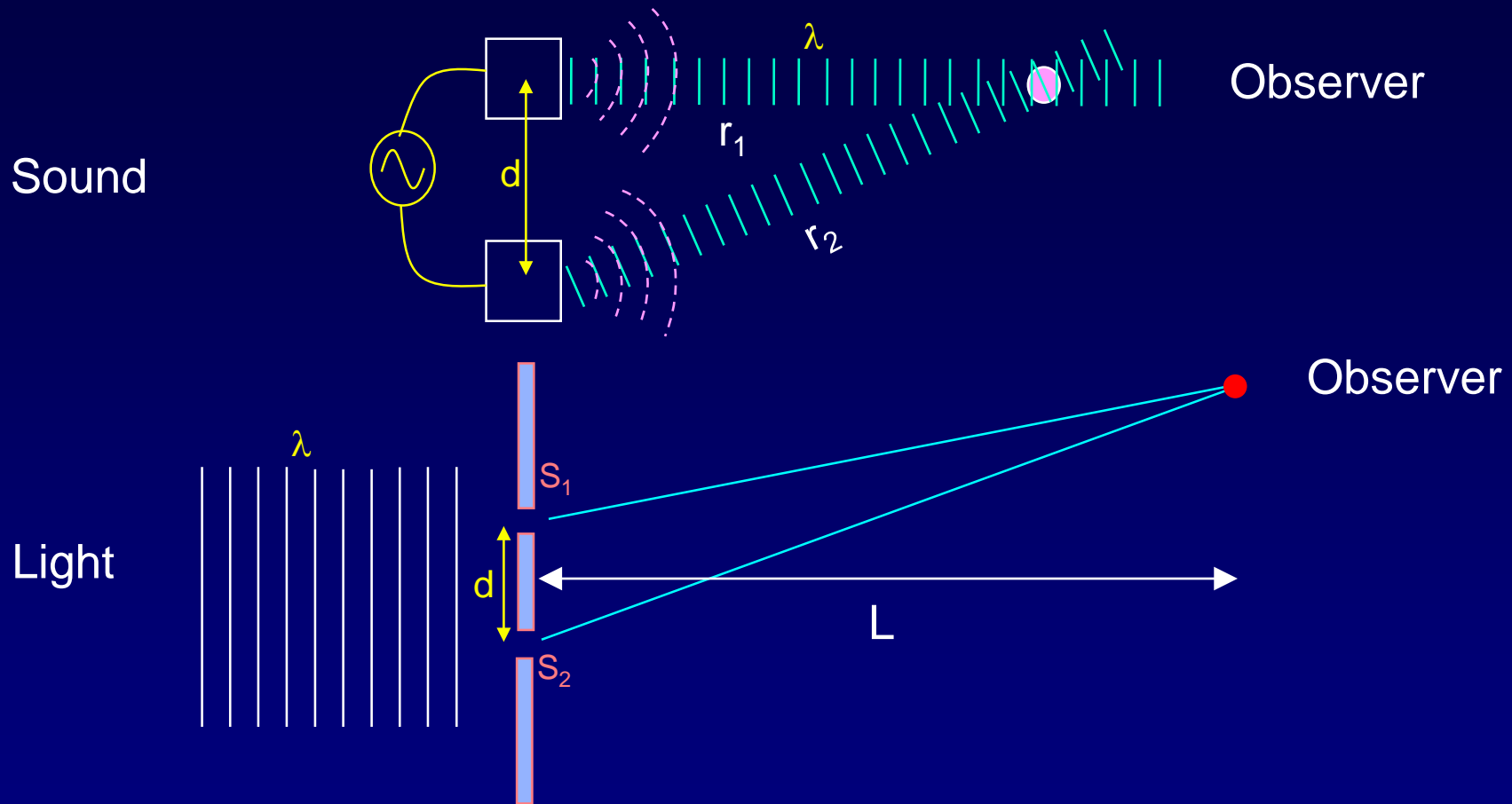
If both slits are open, we see interference "fringes" (light and dark bands), corresponding to constructive and destructive interference of the wave passing through the two slits.

Note: In the laser demo, there is little vertical spread, because the laser spot is small in that direction.

Monochromatic light (wavelength λ)

S_1

S_2

I_1

screen

Diffraction profile

Interference fringes

S_1

S_2

screen

I
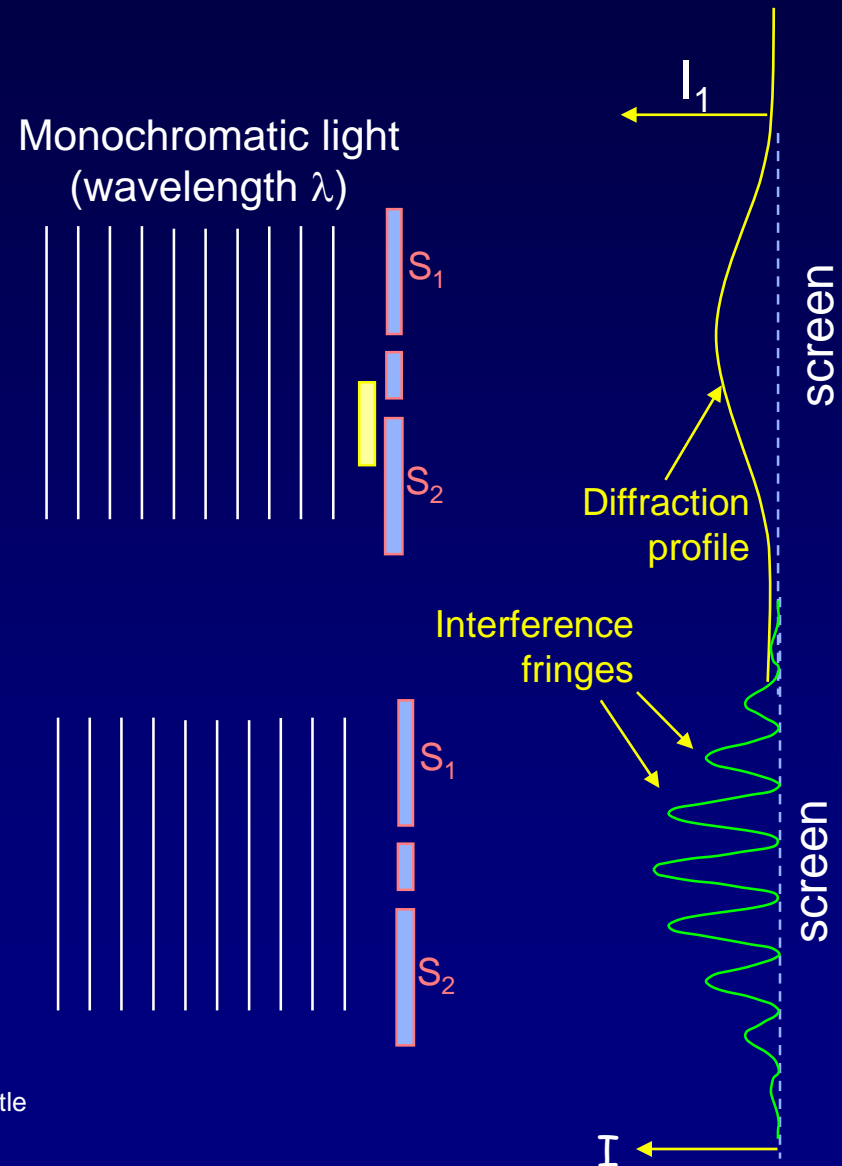
Sound and Light Waves Interfere the Same Way

Sound

Observer

λ

r_1

d

r_2

Light

Observer
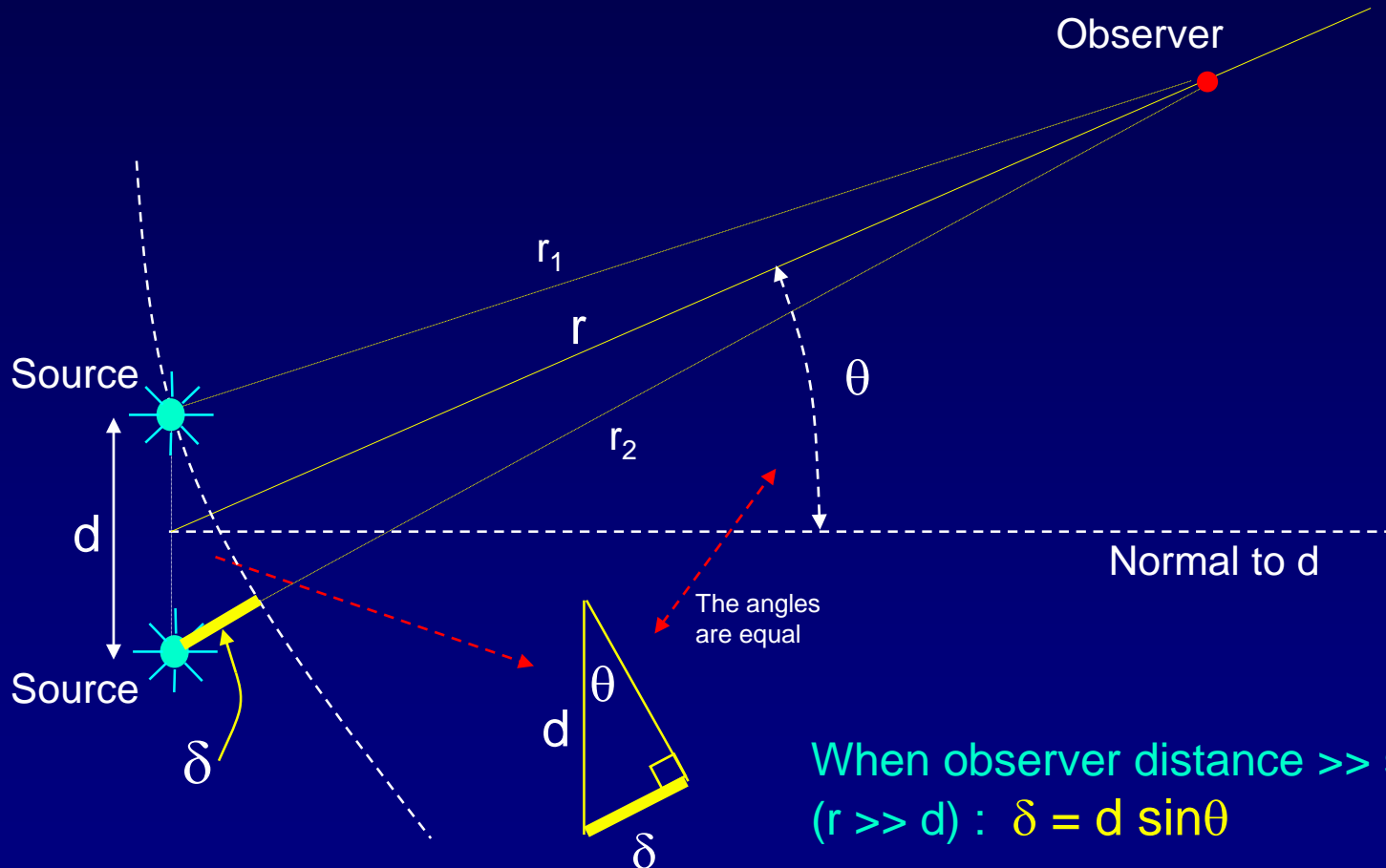
λ

S_1

d

L

S_2

In both cases, $I = 4\, I_1 \cos^2(\phi/2)$ with $\phi = 2\pi(\delta/\lambda)$, $\delta = r_2 - r_1$

However, for light, the distance L is generally much greater than the wavelength λ and the slit spacing d: L >> λ, L >> d.

Simple formula for the path difference, δ, when the observer is far from sources.

Assume 2 sources radiating in phase:

When observer distance >> slit spacing (r >> d) : $\delta = d \sin\theta$

Therefore: $\phi = 2\pi(\delta/\lambda) = 2\pi(d \sin\theta / \lambda)$

Two-Slit Interference
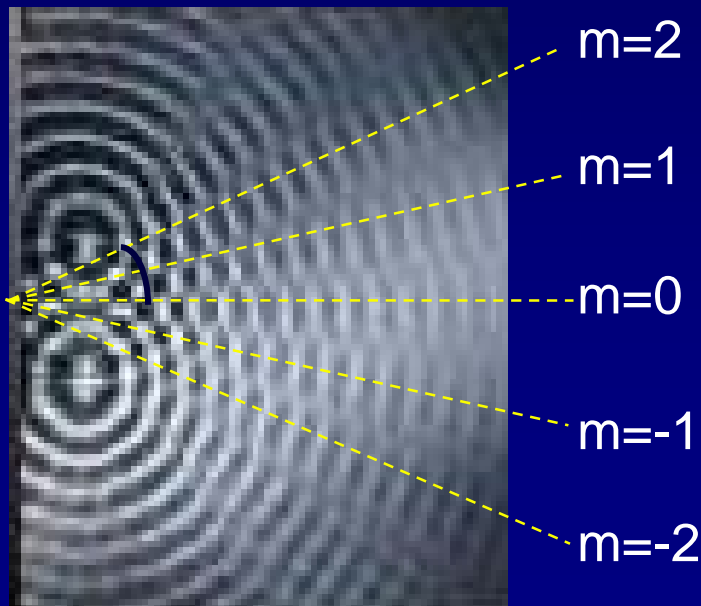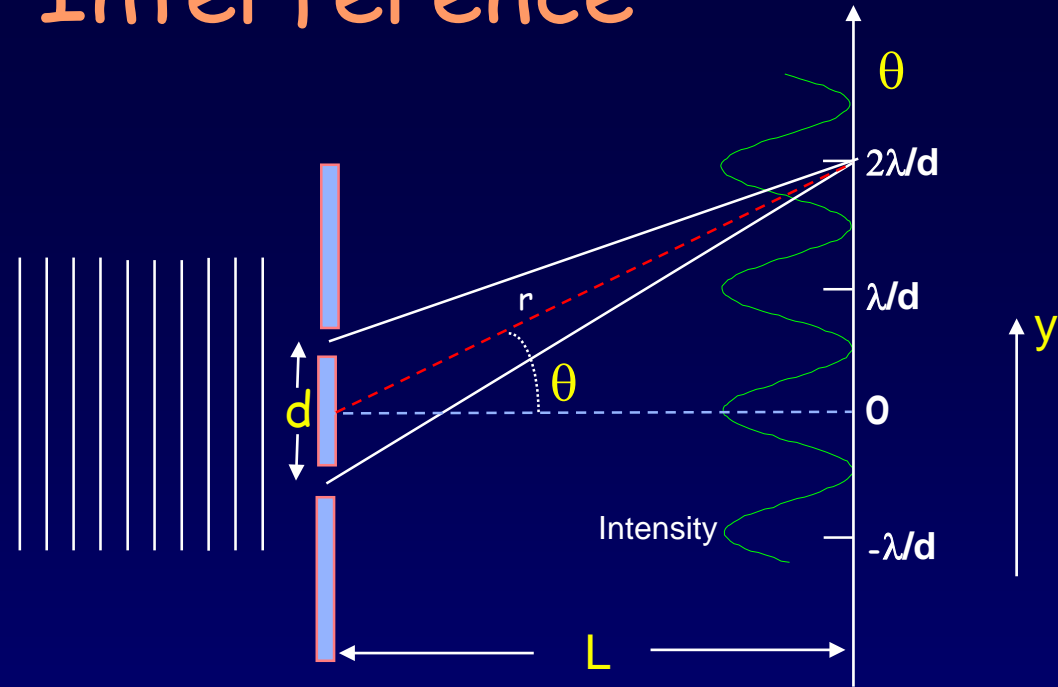
Constructive interference:
$$\delta = d\sin\theta = m\lambda$$
$$m = 0, \pm1, \pm2, ...$$

Destructive Interference:
$$\delta = d\sin\theta = (m+\tfrac{1}{2})\lambda$$
$$m = 0, \pm1, \pm2, ...$$

θ

$2\lambda/d$

λ/d

y

O

Intensity

$-\lambda/d$

L

m=2

m=1

m=0

m=-1

m=-2

Lines of constructive interference

Usually we care about the linear displacement y of the pattern (because our screens are flat):

$$y = L\tan\theta$$

Two-Slit Interference, small angles:

Often, d >>λ, so that θ is small.

Then we can use the small angle approximation to simplify our results:
For small angles: (θ << 1 radian):
sinθ ≈ θ ≈ tanθ (only in radians!)

y = L tanθ ≈ Lθ

Constructive interference:
   θ ≈ m(λ/d)
   y ≈ m(λ/d)L
   m = 0, ±1, ±2, …

Destructive interference:
   θ ≈ (m+½)(λ/d)
   y ≈ (m+½)(λ/d)L
   m = 0, ±1, ±2, …
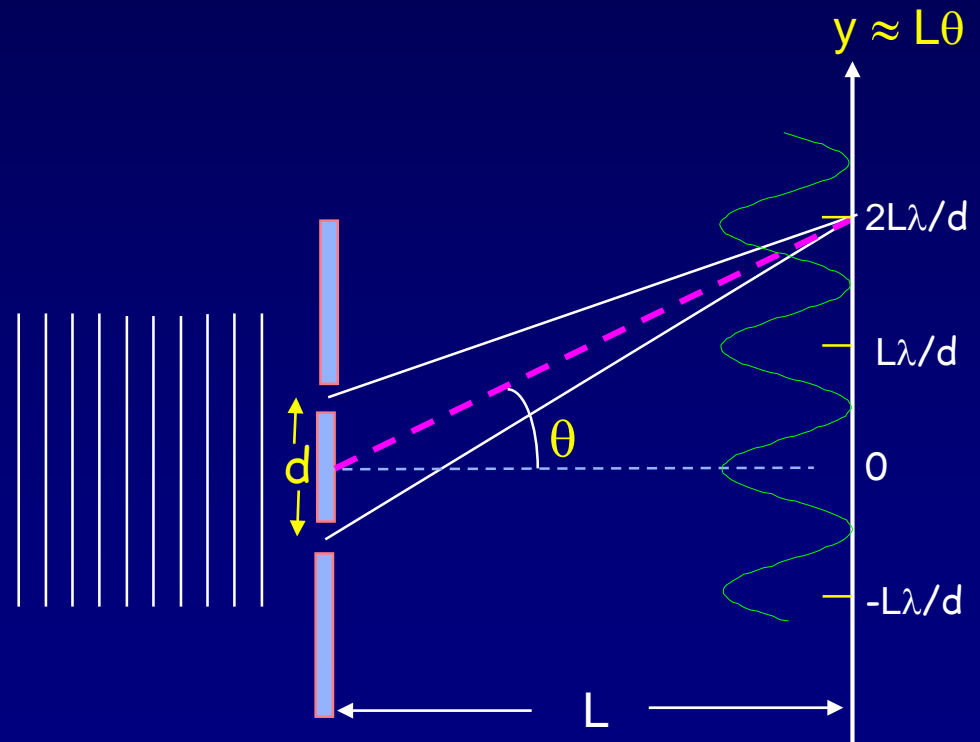
Act 2: 2-slit interference

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.
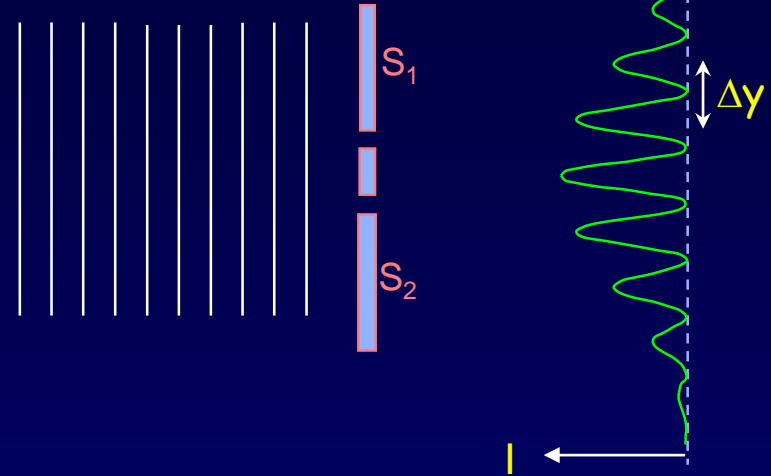
S_1

S_2

Δy

I

1. If we increase the spacing between the slits, what will happen to Δy?

    a. decrease   b. stay the same   c. increase

2. If we instead use a green laser (smaller λ), Δy will?

    a. decrease   b. stay the same   c. increase

Solution

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.

S_1

S_2

Δy

I

1. If we increase the spacing between the slits, what will happen to Δy?

   a. decrease   b. stay the same   c. increase

   $\Delta y \propto 1/d$, so it decreases. This is a general phenomenon: the more spread out the sources are, the narrower the interference pattern is.

2. If we instead use a green laser (smaller λ), Δy will?

   a. decrease   b. stay the same   c. increase

Solution

A laser of wavelength 633 nm is incident on two slits separated by 0.125 mm.

S$_1$

S$_2$

Δy

I

1. If we increase the spacing between the slits, what will happen to Δy?
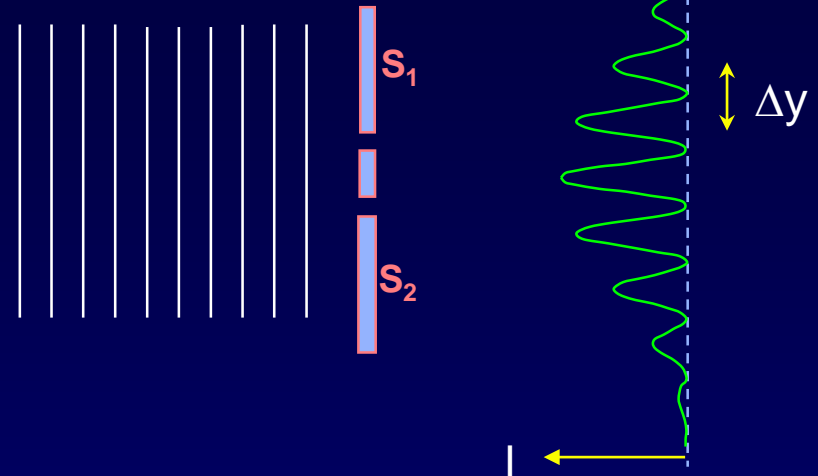
   a. decrease   b. stay the same   c. increase

$\Delta y \propto 1/d$, so it decreases. This is a general phenomenon: the more spread out the sources are, the narrower the interference pattern is.

2. If we instead use a green laser (smaller λ), Δy will?

   a. decrease   b. stay the same   c. increase

$\Delta y \propto \lambda$, so it decreases.

Next time

Phasors, review, examples, examples, examples

Next week

Diffraction from a single slit

Multiple-slit interference

Diffraction and Spectroscopy
   Text – Ch. 36 + added material

Applications – resolution of telescopes and
   microscopes, interferometers, crystallography, etc...