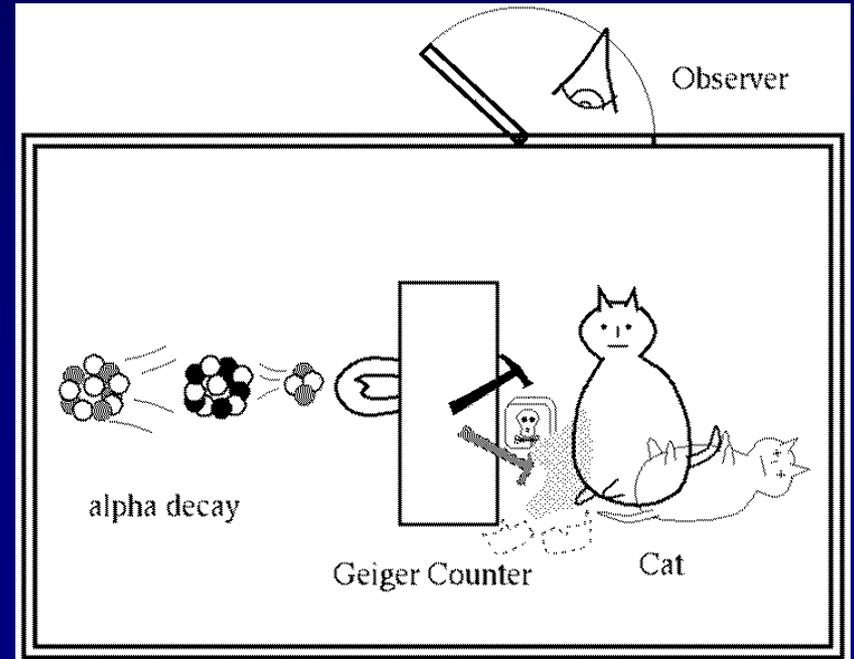
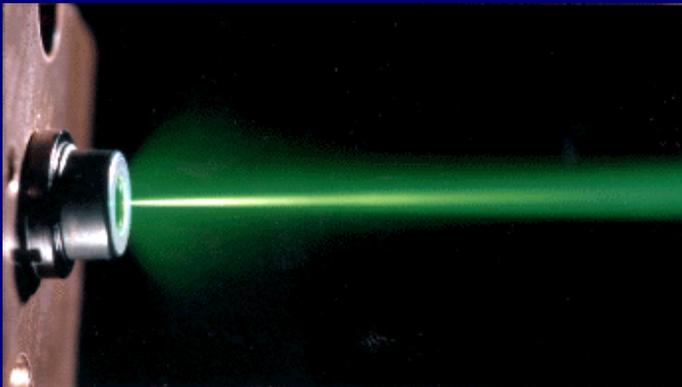


"When I hear of  
Schrödinger's cat, I  
reach for my gun."

--Stephen W.  
Hawking



# Lecture 21: Lasers, Schrödinger's Cat, Atoms, Molecules, Solids, etc. Review and Examples



# The five principles of quantum mechanics summarize what we've discovered

1. All information about a physical system is encoded into its wave function.
2. The wave function obeys the time-dependent Schrödinger's Equation.
3. Measurement of a property, say energy, momentum or location, puts the physical system into a state in which this quantity is absolutely known – the wavefunction “collapses” into the measured state, and all other information in the original state is lost.
4. Probabilities of possible measurement outcomes can be obtained from the wave function. For example, if we measure the energy, the only values of energy that can be obtained are the energy eigenvalues. They occur with a probability given by the **square of the amplitude** for the original state to be in various particular energy eigenstates. This also holds true for measurements of any physical variable: momentum, position, spin, angular momentum, etc. To measure the probability of finding a particle at  $\mathbf{r}$ , calculate  $|\psi(\mathbf{r})|^2$ .
5. Quantities like  $x$  and  $p$  (and  $E$  and  $t$ , and  $L_z$ ,  $L_x$  and  $L_y$ ) are ‘conjugate’. They obey an uncertainty principle that states that since they are incompatible measurements, complete knowledge of one means complete ignorance of the other.

# Act 1

The Pauli exclusion principle applies to all fermions in all situations (not just to electrons in atoms). Consider electrons in a 2-dimensional infinite square well potential.

1. How many electrons can be in the first excited energy level?

- a. 1    b. 2    c. 3    d. 4    e. 5

Hint: Remember the  $(n_x, n_y)$  quantum numbers.

2. If there are 4 electrons in the well, what is the energy of the most energetic one (ignoring e-e interactions, and assuming the total energy is as low as possible)?

- a.  $(h^2/8mL^2) \times 2$   
b.  $(h^2/8mL^2) \times 5$   
c.  $(h^2/8mL^2) \times 10$

# Solution

The Pauli exclusion principle applies to all fermions in all situations (not just to electrons in atoms). Consider electrons in a 2-dimensional infinite square well potential.

1. How many electrons can be in the first excited energy level?

- a. 1    b. 2    c. 3    **d. 4**    e. 5

The first excited energy level has  $(n_x, n_y) = (1, 2)$  or  $(2, 1)$ .

That is, it is degenerate.

Each of these can hold two electrons (spin up and down).

(Note: On an exam, I'd word this question a bit more carefully.)

2. If there are 4 electrons in the well, what is the energy of the most energetic one (ignoring e-e interactions, and assuming the total energy is as low as possible)?

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c.  $(h^2/8mL^2) \times 10$

Two electrons are in the  $(1, 1)$  state, and two are in the  $(2, 1)$  or  $(1, 2)$  state.

So,  $E_{\max} = (1^2 + 2^2) \times (h^2/8mL^2)$ .

Here the 1 and 2 are from the two independent dimensions

# Insulators, Semiconductors and Metals

Energy bands and the gaps between them determine the conductivity and other properties of solids.

## Insulators

Have a full valence band and a large energy gap (a few eV). Higher energy states are not available.

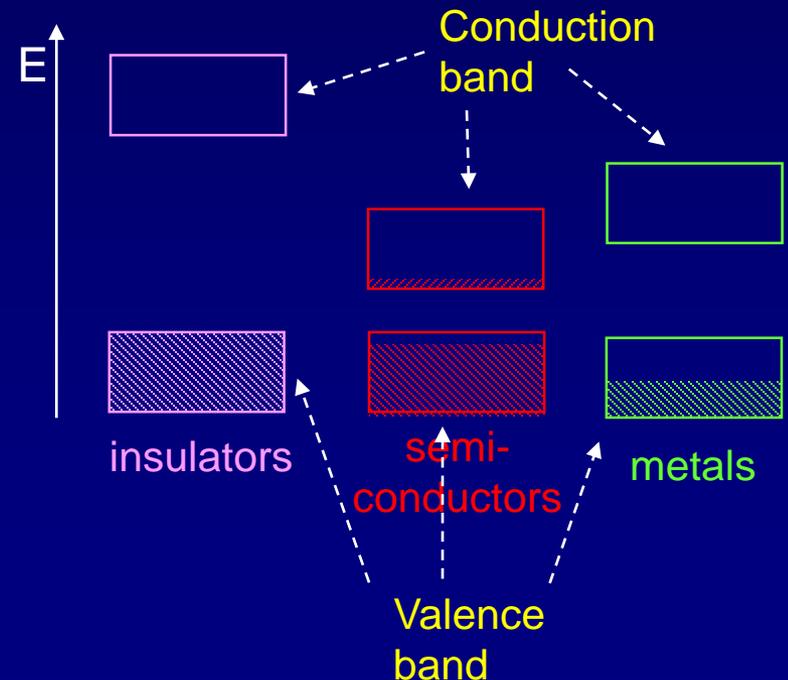
In order to conduct, an electron must have an available state at slightly higher energy.

## Semiconductors

Are insulators at  $T = 0$ .  
Have a small energy gap ( $\sim 1$  eV) between valence and conduction bands. Higher energy states become available (due to  $kT$ ) as  $T$  increases.

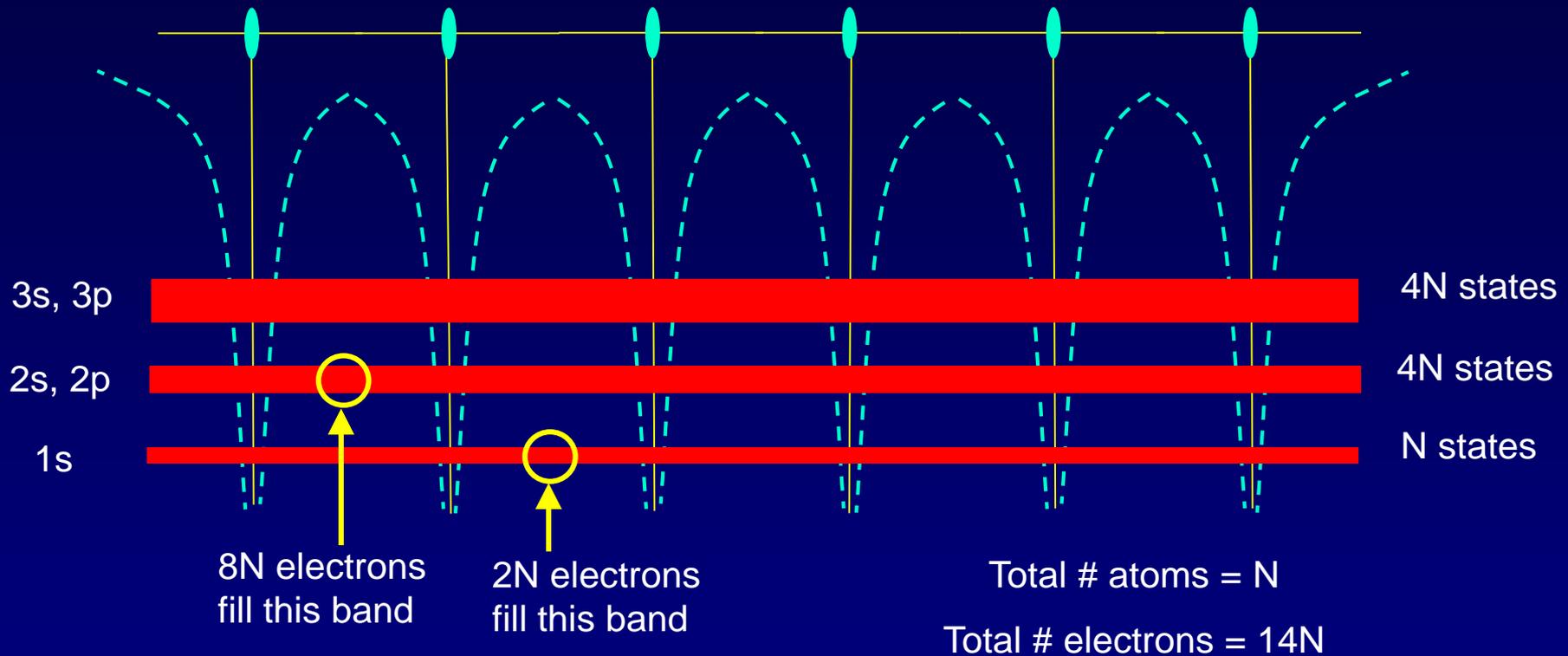
## Metals

Have a partly filled band. Higher energy states are available, even at  $T = 0$ .

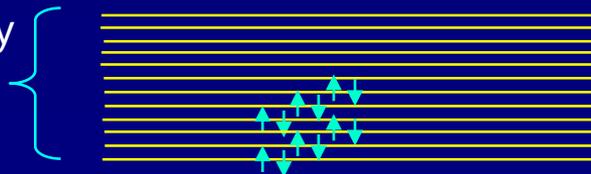


# Semiconductors: How about Silicon?

Silicon:  $Z = 14$  ( $1s^2 2s^2 2p^6 3s^2 3p^2$ )



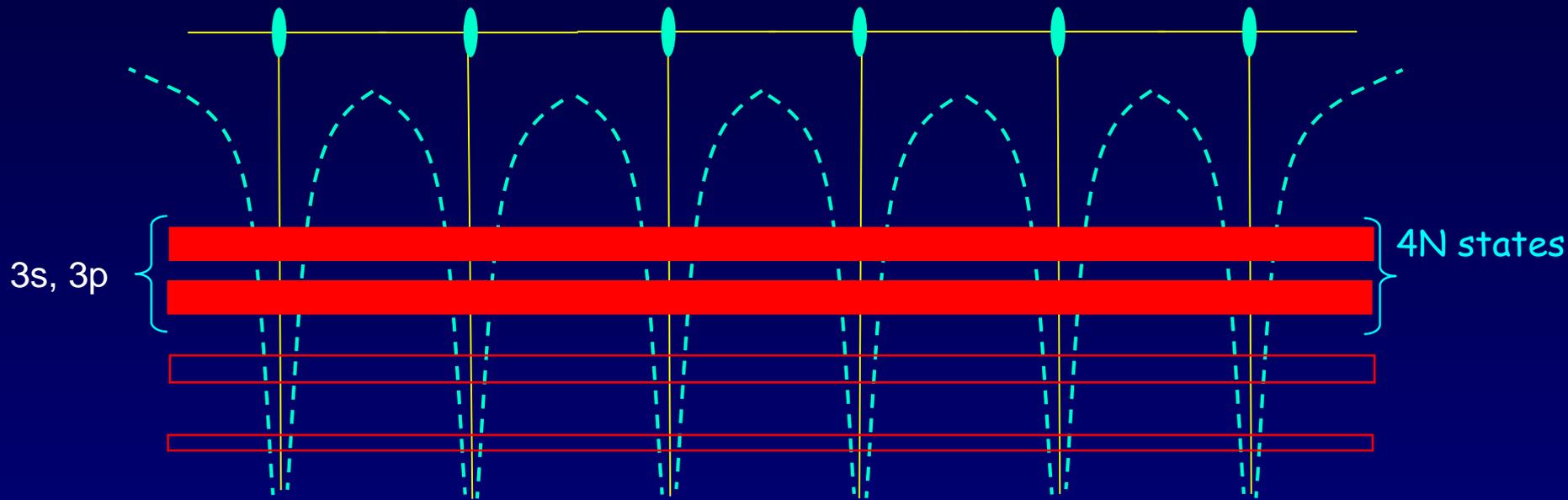
The 3s/3p band is only half filled ( $4N$  states and  $4N$  electrons)



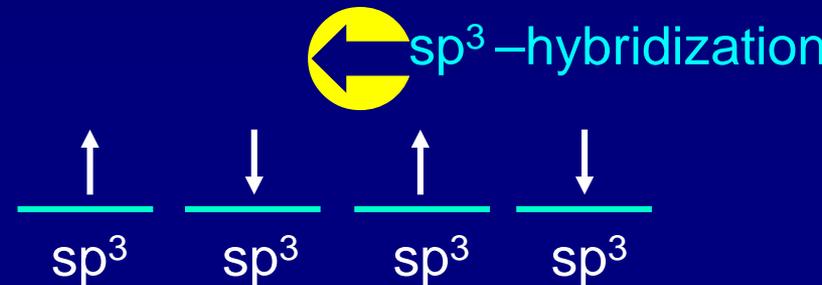
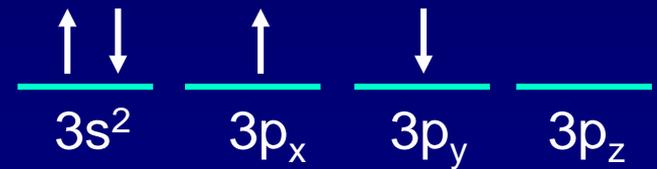
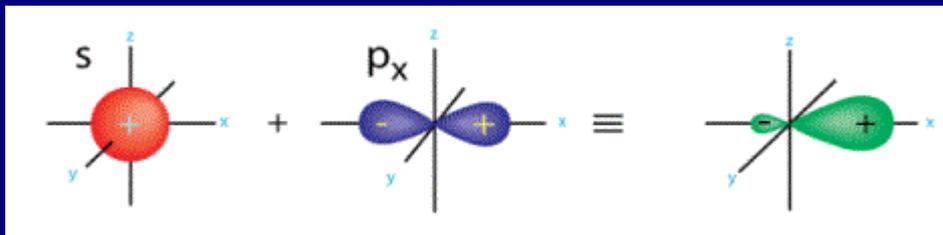
Why isn't Silicon a metallic conductor, like sodium?

# Energy Bands in Si

$$Z = 14: 1s^2 2s^2 2p^6 3s^2 3p^2$$

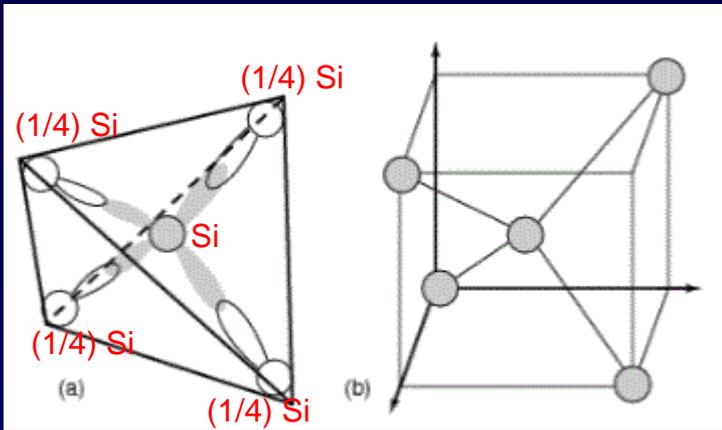


In Si, the 3s and 3p hybridize to form four equivalent  $sp^3$ -hybrid states. These eigenstates are linear combinations of s and p states:



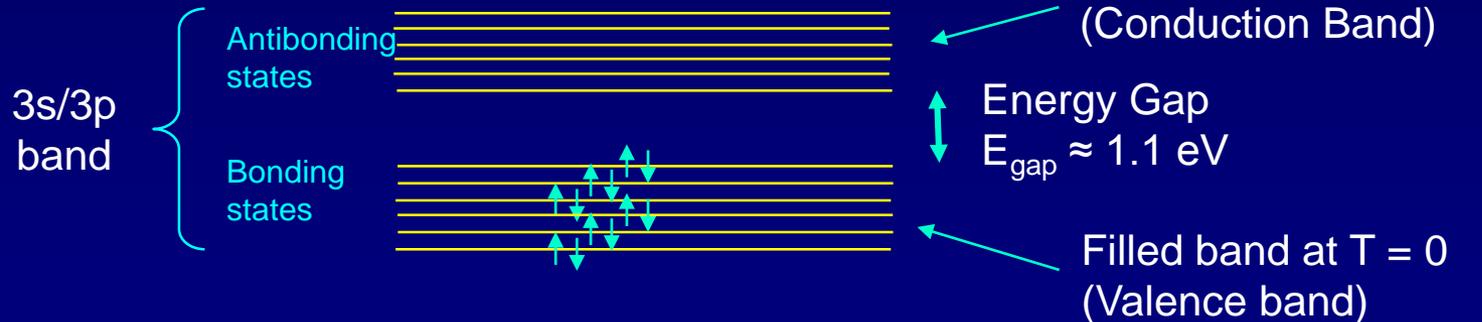
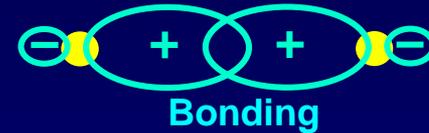
# Silicon (2)

## Silicon Unit Cell



2 - Si atoms/unit cell

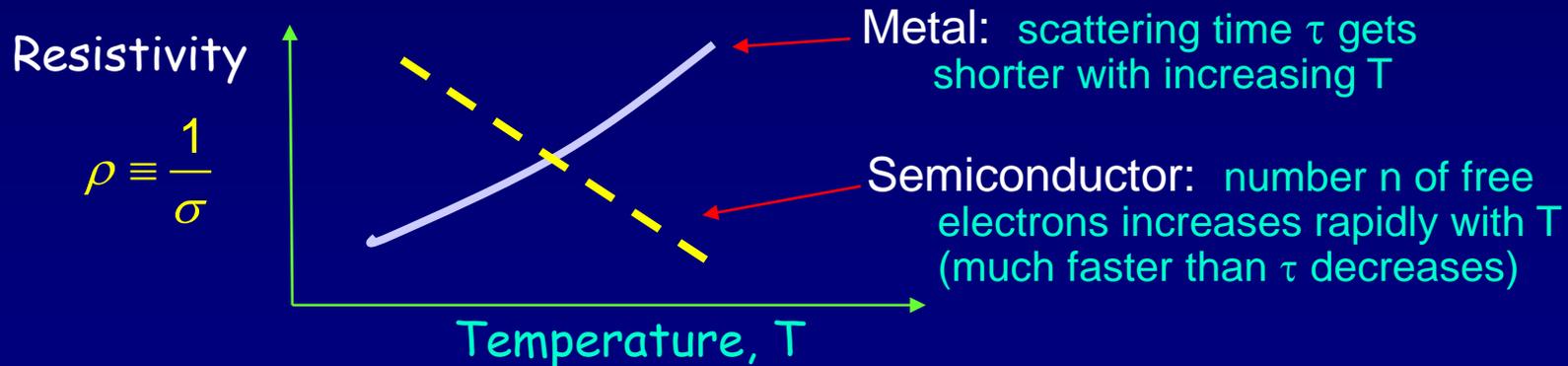
There are 2 types of superpositions between neighbors:



- ❑ 8  $sp^3$ -orbitals in one unit cell
- ❑ 8 orbitals form superpositions (4 bonding and 4 anti-bonding combinations)
- ❑ 8 orbitals  $\times$  2 spin = 16 states
- ❑ 8 electrons fill the 4 lower energy (bonding) states

# Silicon (3)

- At  $T = 0$ , the bonding states in Si are completely filled, and the anti-bonding states are completely empty. A small (1.1 eV) energy gap separates the bonding and anti-bonding states: Si is a semiconductor.
- The electrons in a filled band cannot contribute to conduction, because with reasonable E fields they cannot be promoted to a higher kinetic energy. Therefore, at  $T = 0$ , Si is an insulator. At higher temperatures, however, electrons are thermally promoted into the conduction band:



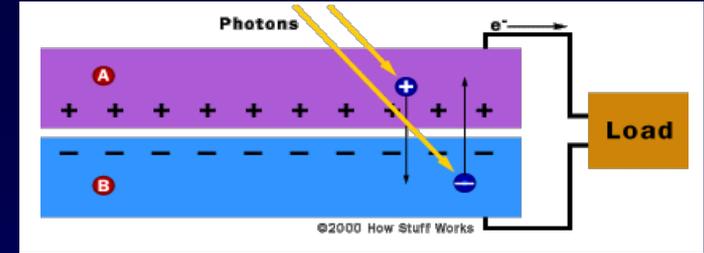
This graph only shows trends. A semiconductor has much higher resistance than a metal.

# Photodetectors

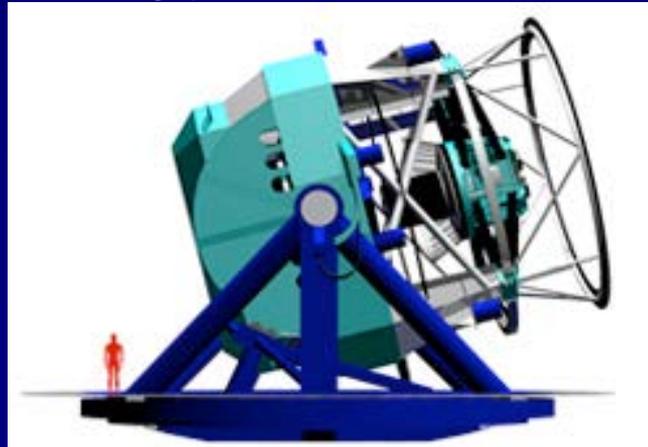
Shining light onto a semiconductor can excite electrons out of the valence band into the conduction band. The change in resistance can then be used to monitor the light intensity.

Examples:

- photodiode
- optical power meter
- barcode scanner
- digital cameras (each pixel)
- photovoltaic “solar cell”



A 3.2 Gigapixel camera



# Act 2

The band gap in Si is 1.1 eV at room temperature. What is the reddest color (*i.e.*, the longest wavelength) that you could use to excite an electron to the conduction band?

Hint: Si is used in the pixels of your digital camera.

a. 500 nm

b. 700 nm

c. 1100 nm

# Solution

The band gap in Si is 1.1 eV at room temperature. What is the reddest color (*i.e.*, the longest wavelength) that you could use to excite an electron to the conduction band?

Hint: Si is used in the pixels of your digital camera.

a. 500 nm

b. 700 nm

c. 1100 nm

Remember the relation between photon energy and wavelength:

$$\lambda = hc/E = 1240 \text{ eV}\cdot\text{nm} / 1.1 \text{ eV} = 1127 \text{ nm}$$

# Lasers

Photons are emitted when the electrons in atoms go from a higher state to a lower state

Conversely, photons are absorbed when the electrons in atoms go from a lower state to a higher state



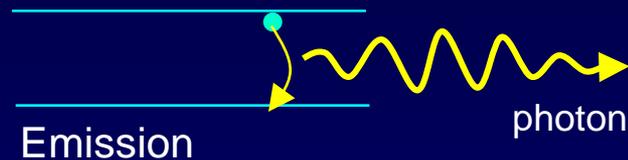
Fermions and bosons:

Electrons, protons, and neutrons are fermions. Two identical fermions cannot occupy the same quantum state. (exclusion principle)

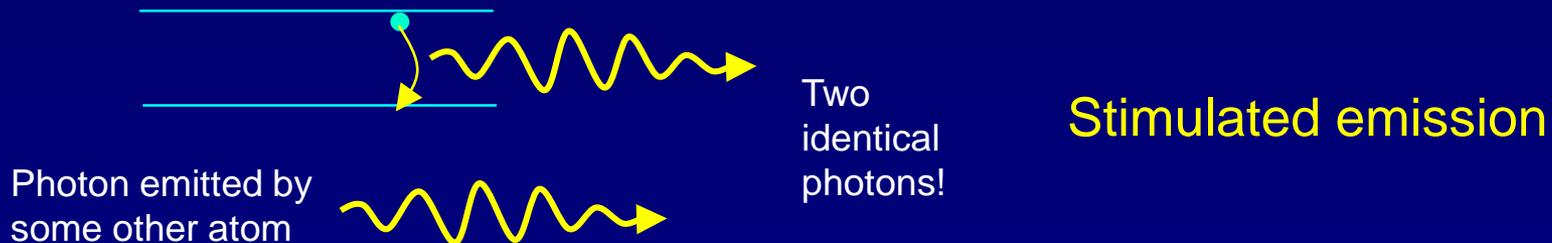
Photons (and many atoms) are bosons. Unlike fermions, bosons actually “prefer” (to be explained soon) to be in the same quantum state. This is the physical principle on which lasers are based.

# Lasers

Suppose we have an atom in an excited state. Eventually (at some random time) it will emit a photon and fall to the lower state. The emitted photon will go in a random direction. This is called “spontaneous emission”.



Suppose, however, that before the spontaneous emission occurs, another photon of the same energy (emitted by another atom) comes by. Its presence will stimulate the atom to emit its photon.



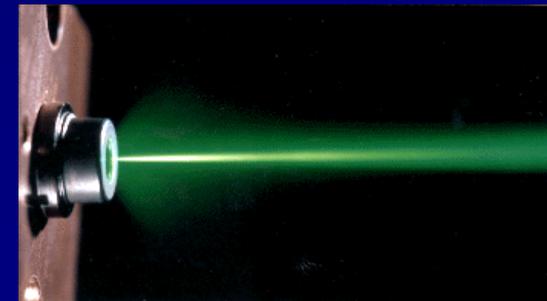
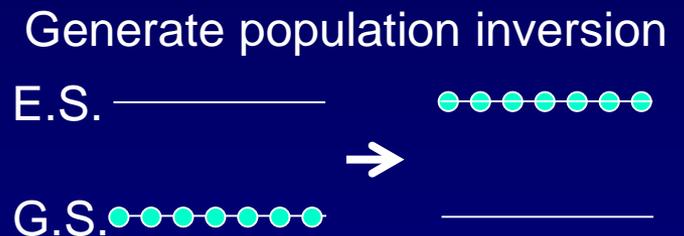
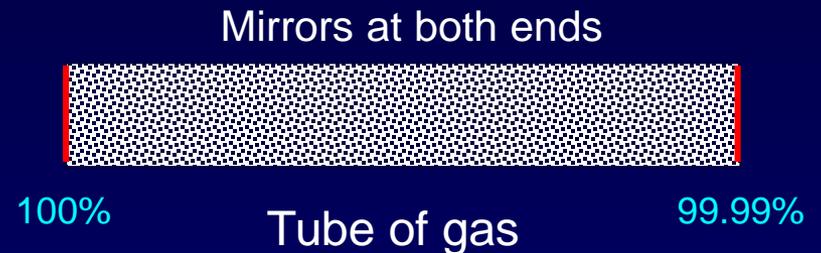
We now have two photons in the same quantum state: the same frequency, the same direction, and the same polarization. As they travel, they will stimulate even more emission.

# Lasers

## Light Amplification by Stimulated Emission of Radiation

### Laser operation (one kind):

- Tube of gas with mirrored ends.
- Excite the atoms to the upper state (mechanism not shown).
- Spontaneous emission generates some photons.
- Photons that travel along the axis are reflected. Other directions leak out the sides.
- Because the amplification process is exponential, the axial beam quickly dominates the intensity.
- One mirror allows a small fraction of the beam out.

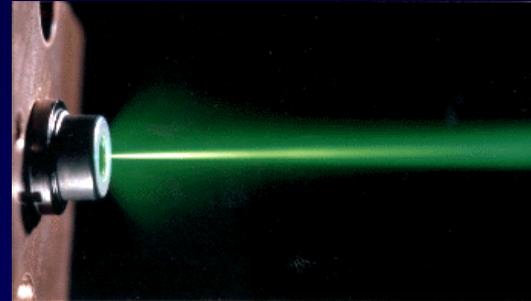


# Lasers

LASER: “Light Amplification by Stimulated Emission of Radiation”

What if you don't have an atom that emits the color you want?

“Make” one (e.g., by stressing the crystal lattice).



Did you know:

Semiconductors don't naturally emit in the red. Charles Henry (UIUC '65) discovered how to combine layers of different semiconductors to make a 'quantum well' (essentially a 1-D 'box' for electrons). By adjusting the materials, one could shift the emission wavelengths into the visible. Nick Holonyak (UIUC ECE Prof) used this to create the first visible LEDs & visible laser diodes.

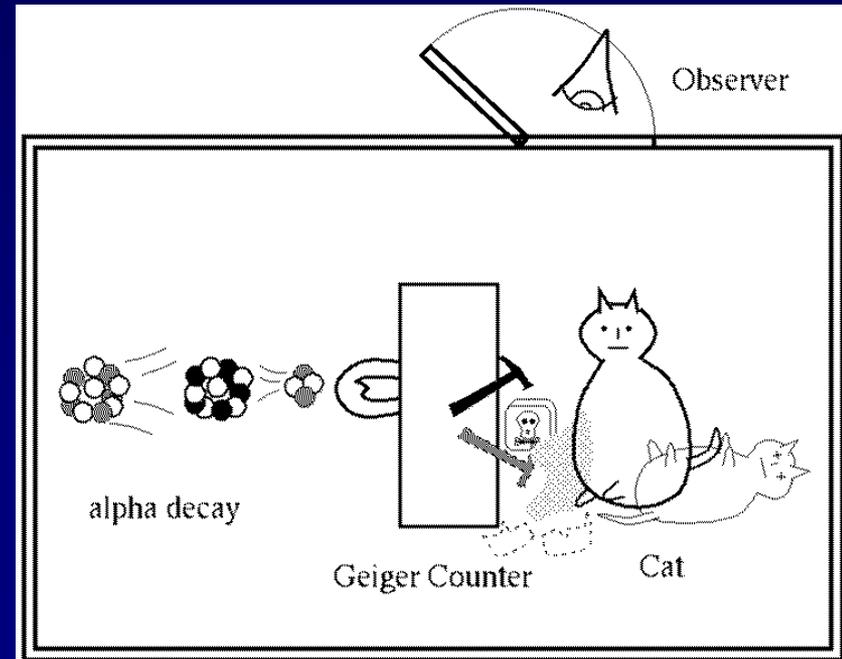
**The Nobel Prize in Physics 2014 was awarded jointly to Isamu Akasaki, Hiroshi Amano and Shuji Nakamura “for the invention of efficient blue light-emitting diodes which has enabled bright and energy-saving white light sources” → Blue laser diodes → BlueRay**

# Schrödinger's Cat:

## How seriously do we take superpositions?

- We now know that we can put a quantum object into a superposition of states.
- But if quantum mechanics is completely correct, shouldn't macroscopic objects end up in a superposition state too?
- This puzzle is best exemplified by the famous "Schrödinger's cat" paradox:
  - A radioactive nucleus can decay, emitting an alpha particle. This is a quantum mechanical process.
  - The alpha particle is detected with a Geiger counter, which releases a hammer, which breaks a bottle, which releases cyanide, which kills a cat.
  - Suppose we wait until there is a 50:50 chance that the nucleus has decayed.

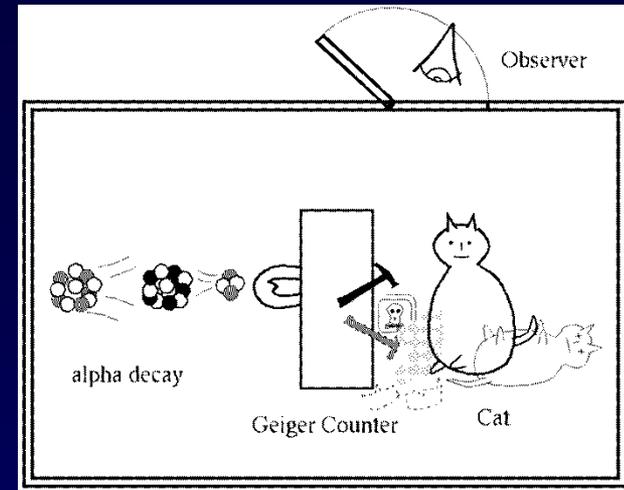
Is the cat alive or dead?



# Schrödinger's Cat

According to QM, because we can't know (*i.e.*, predict), until we look inside the box, the cat is in a superposition\* of being both alive and dead!

$$\Psi_{\text{cat}} = \frac{1}{\sqrt{2}} \{ \psi(\text{alive}) + \psi(\text{dead}) \}$$



And in fact, according to QM, when we look, we are put into a quantum superposition\* of having seen a live and a dead cat!!

Did you ever perceive yourself as being in a superposition? (probably not ...)

This paradox leads to some dispute over the applicability of QM to large objects. The experiments are difficult, but so far there is no evidence of a “size limit”.

Where does it end?!?

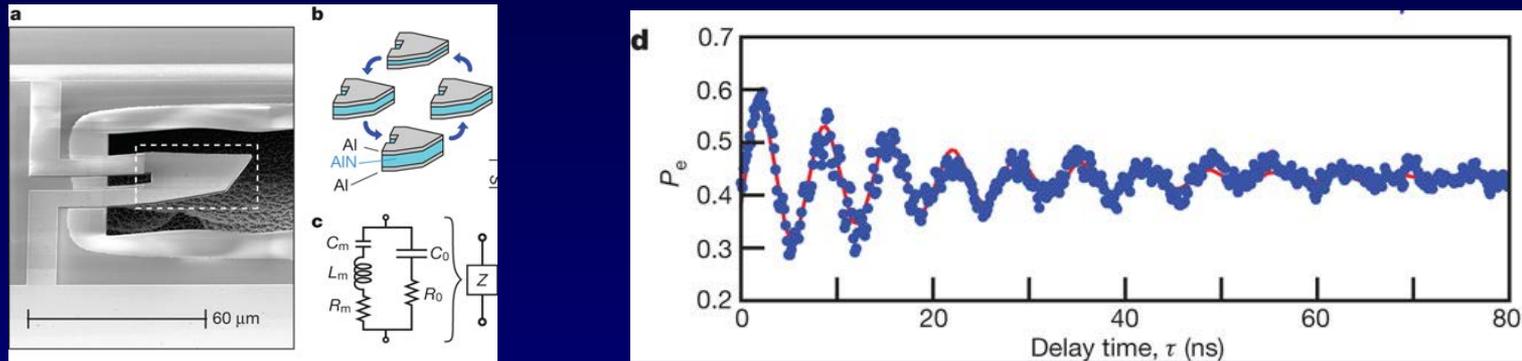
- it doesn't end (“wavefunction of the universe”)
- there is some change in physics (quantum → classical)
- “many-worlds” interpretation...

In any event, the correlations to the rest of the system cause *decoherence* and the *appearance of “collapse”*.

\*More correctly, we, the atom, and the cat become “entangled”.

# FYI: Recent Physics Milestone!

There has been a race over the past ~20 years to put a ~macroscopic object into a quantum superposition. The first step is getting the object into the ground state, below all thermal excitations. This was achieved for the first time in 2010, using vibrations in a small “drum” :



“Quantum ground state and single-phonon control of a mechanical resonator”,  
A. D. O’Connell, et al., *Nature* **464**, 697-703 (1 April 2010)

Quantum mechanics provides a highly accurate description of a wide variety of physical systems. However, a demonstration that quantum mechanics applies equally to macroscopic mechanical systems has been a long-standing challenge... Here, using conventional cryogenic refrigeration, we show that **we can cool a mechanical mode to its quantum ground state** by using a microwave-frequency mechanical oscillator—a ‘quantum drum’... We further show that **we can controllably create single quantum excitations (phonons) in the resonator**, thus taking the first steps to complete quantum control of a mechanical system.

# Entanglement

If we have two quantum systems, there are two possibilities for the total quantum state:

1. can be written as a product of the two individual systems:

$$\psi_{total} = \psi_1 \psi_2$$

2. cannot be written as a product:

$$\psi_{total} \neq \psi_1 \psi_2$$

We've seen several examples before in this course:

Atom that just emitted a photon:

$$\psi_{total} = \int d\vec{k} \psi_{atom}(\vec{k}) \psi_{photon}(-\vec{k})$$

Photon emitted in all directions; atom must recoil in opposite direction.

S. cat:  $\psi_{total} = \psi_{atom}(\text{excited})\psi_{cyanide\ vial}(\text{"unbroken"})\psi_{cat}(\text{"alive"}) + \psi_{atom}(\text{decayed})\psi_{cyanide\ vial}(\text{"broken"})\psi_{cat}(\text{"dead"})$

Double slit with quantum which-path detector:

$$\psi_{total} = \psi_{photon}(\text{"upper slit"})\psi_{detector\ 1}(\text{"yes"})\psi_{detector\ 2}(\text{"no"}) + \psi_{photon}(\text{"lower slit"})\psi_{detector\ 1}(\text{"no"})\psi_{detector\ 2}(\text{"yes"})$$

Entanglement is *“the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought”*

E. Schrödinger, 1935

Entangle: To become mixed up in something such that separation is difficult:

*“He became entangled with a lady whose looks were much better than her morals.”* F. Hume, 1888

Entanglement: [military] An extensive barrier or obstruction –formed of trees, branches, barbed wire, etc. arranged to impede the enemy.

*“They’ve all died on the entanglements.”*

Shepherd, 1917

THE END

Best wishes in Physics 213  
and  
all your other courses!