Last Name:________________First Name_________________NetID_______________
Discussion Section:________Discussion TA Name:______________________________

Instructions—
Turn off your cell phone and put it away.
Keep your calculator on your own desk. Calculators may not be shared.
This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil; do **not** use a mechanical pencil or a pen. Fill in completely (until there is no white space visible) the circle for each intended input – both on the identification side of your answer sheet and on the side on which you mark your answers. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner.

2. Print your last name in the **YOUR LAST NAME** boxes on your answer sheet and print the first letter of your first name in the **FIRST NAME INI** box. Mark (as described above) the corresponding circle below each of these letters.

3. Print your NetID in the **NETWORK ID** boxes, and then mark the corresponding circle below each of the letters or numerals. Note that there are different circles for the letter "I" and the numeral "1" and for the letter "O" and the numeral "0". **Do not** mark the hyphen circle at the bottom of any of these columns.

4. You may find the version of this Exam Booklet at the top of page 2. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**

5. Stop **now** and double-check that you have bubbled-in all the information requested in 2 through 4 above and that your marks meet the criteria in 1 above. Check that you do not have more than one circle marked in any of the columns.

6. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION block.

7. On the **SECTION** *line*, print your **DISCUSSION SECTION**. (You need not fill in the COURSE or INSTRUCTOR lines.)

8. Sign **(DO NOT PRINT)** your name on the **STUDENT SIGNATURE** *line*.

Before starting work, check to make sure that your test booklet is complete. You should have 15 ***numbered pages*** *plus two Formula Sheets at the end.*

Academic Integrity—**Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including expulsion.**

This Exam Booklet is Version A. Mark the **A** circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of **120** points, composed of two types of questions.

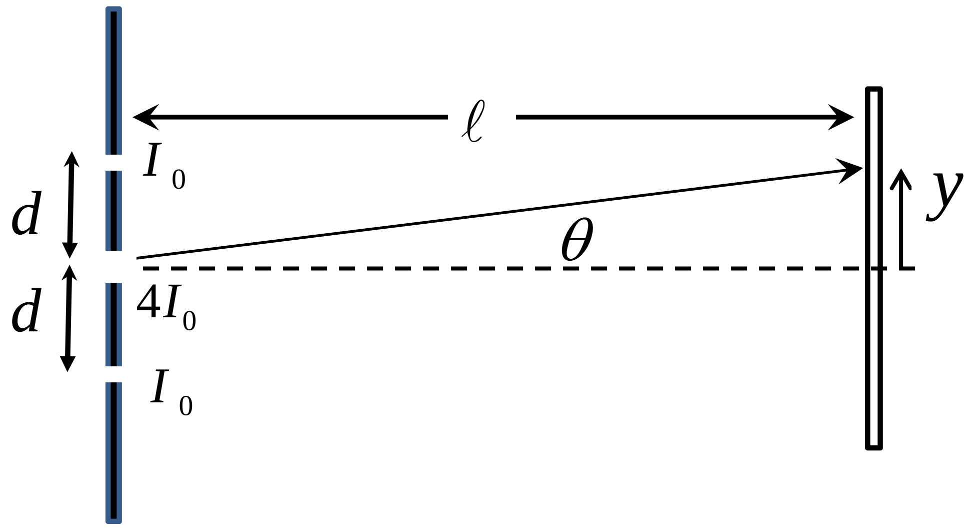
MC5: *multiple-choice-five-answer questions, each worth 6 points.*
Partial credit will be granted as follows.

(a) If you mark only one answer and it is the correct answer,
you earn **6** points.
(b) If you mark *two* answers, one of which is the correct answer,
you earn **3** points.
(c) If you mark *three* answers, one of which is the correct answer,
you earn **2** points.
(d) If you mark no answers, or more than *three*, you earn 0 points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*
No partial credit.

(a) If you mark only one answer and it is the correct answer,
you earn **3** points.
(b) If you mark a wrong answer or no answers, you earn **0** points.

The next four problems concern the 3-slit interference experiment illustrated below, on which a laser shines onto a screen with 3 slits.



The width of the center slit is larger than the other two slits such that the intensities due to each individual slit with the other two slits blocked are measured to be $I_0, 4I_0, I_0$ at the screen. Ignore diffraction throughout and let ϕ be the phase shift between adjacent slits. Assume all three slits are uncovered for the rest of this problem.

1. What is the intensity measured by an observer located at $y = 0$?

   a. $16I_0$
   b. $9I_0$
   c. $6I_0$

2. Which expression most correctly gives the intensity that the observer will see when sitting at a y position where the phase shift is $\phi = \pi/2$ and find a y position where this occurs? This problem is easiest if you use phasers.

   a. I_0 , $y = \ell \tan\left(\sin^{-1}\left[\lambda/(2d)\right]\right)$
   b. $2I_0$, $y = \ell \tan\left(\sin^{-1}\left[\lambda/d\right]\right)$
   c. $4I_0$, $y = \ell \tan\left(\sin^{-1}\left[\lambda/(4d)\right]\right)$
   d. $2I_0$ $y = \ell \tan\left(\sin^{-1}\left[\lambda/(4d)\right]\right)$
   e. I_0, $y = \ell \tan\left(\sin^{-1}\left[\lambda/(2d)\right]\right)$

3. Now we block one of the 3 slits. Which of the following is a true statement?

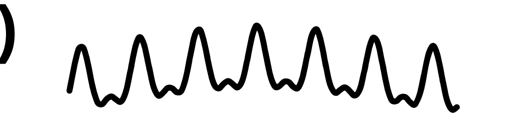
a. The intensity pattern is the same whether we block the center or the top slit.

b. The interference fringe spacing is smaller when we block the center slit than when we block the top slit.

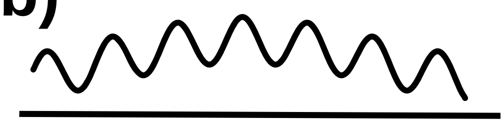
c. The interference fringe spacing is greater when we block the top slit than when we block the center slit.

4. Assume that the incident light is unpolarized. What pattern would be observed if a horizontal polarizer covers the center slit and vertical polarizers cover the top and bottom slits?

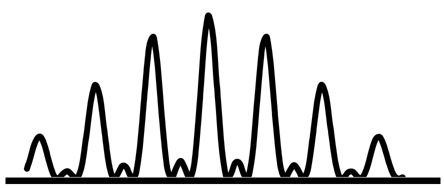
(a)



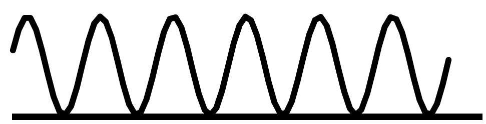
(b)



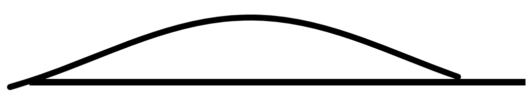
(c)



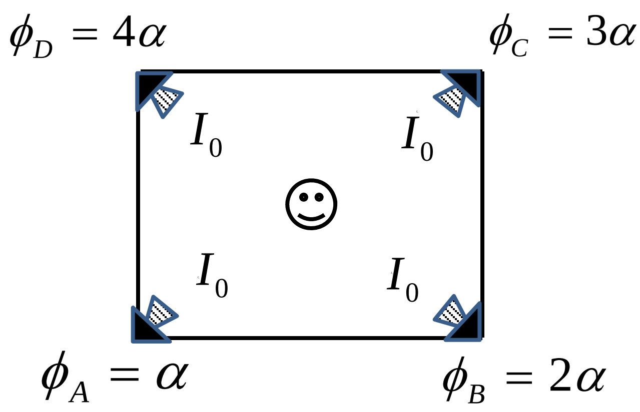
(d)



(e)



The next problem concerns the sound heard by an observer sitting in the center of a rectangular array of loudspeakers as shown.



The observer hears an intensity of I_0 from each of the four speakers when sounded individually. The initial phase of the pressure wave at each speaker is shown in the diagram.

5. When $\alpha = 120^0$, the observer hears an intensity given by:

a. $16 I_0$

b. $4.83 I_0$

c. 0

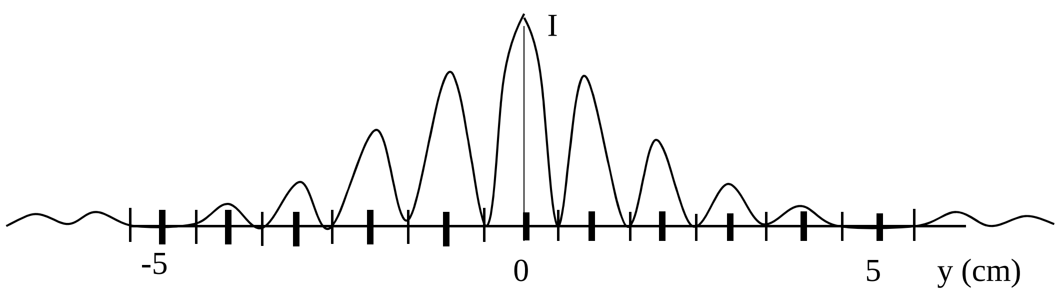
d. I_0

e. $2 I_0$

6. What is the greatest distance that the human eye can resolve the two headlights of a car, if they are blue (i.e., the wavelength is 450 nm)? Assume that the human eye can just resolve two headlights from a distance of 15 km in the case if the headlights are red (i.e., the wavelength is 650 nm). The distance between the lights is 2 m.

a. 10 km
b. 15 km
c. 22 km

The next problem refers to the following situation:



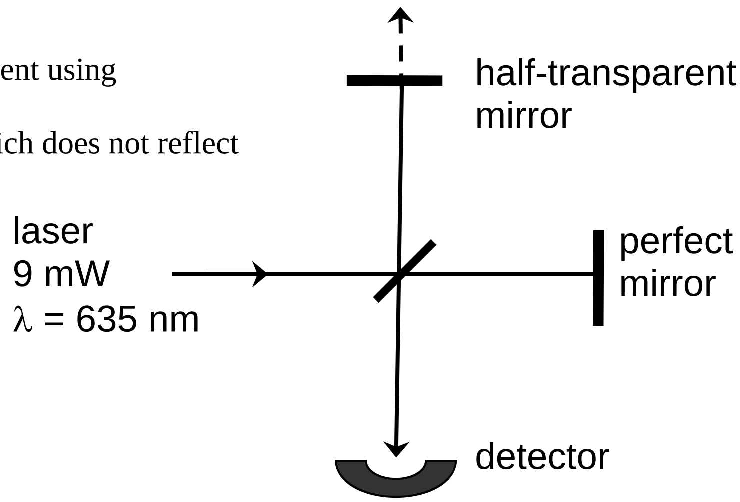
As in our laboratory experiment, the following interference pattern is observed on a screen located 2 m from slits illuminated by a coherent light source with the wavelength of 633 nm.

7. What is the width of the slits?

a. 33 μm
b. 23 μm
c. 15 μm
d. 12.7 μm
e. 127 μm

The next two questions are related.

You make a light interference experiment using
a 50-50 beam splitter and two mirrors;
one being a perfect mirror and one which does not reflect
all light.



Because the top mirror is not perfectly reflective (half the light is actually transmitted), the power measured at the detector when only the vertical arm is blocked is 2.25 mW, while the power measured at the detector when only the horizontal arm is blocked is only 1.125 mW. **Assume identical length arms.**

8. What is the power measured at the bottom detector when both arms are unblocked,

a. 1.5 mW
b. 3.375 mW
c. 6.56 mW
d. 6.75 mW
e. 9 mW

9. The above experiment was initially performed in vacuum; now we introduce air. What will happen to the intensity at the detector (neglect the minute scattering loss from the air)?

a. increase
b. decrease
c. stay the same

The following two problems pertain to the same situation.

10. Suppose you investigate atomic spectra using a diffraction grating with 9000 lines/cm. If you observe a spectral line at an angle of $\theta = 48°$ in the second-order spectrum, what is the wavelength λ of this spectral line?

a. $\lambda = 611$ nm
b. $\lambda = 413$ nm
c. $\lambda = 465$ nm
d. $\lambda = 825$ nm
e. $\lambda = 295$ nm

11. You now illuminate only 1/3 of the 1-cm wide grating with 500 nm light. What is the closest wavelength you can resolve (again using 2^{nd} order spectrum)?

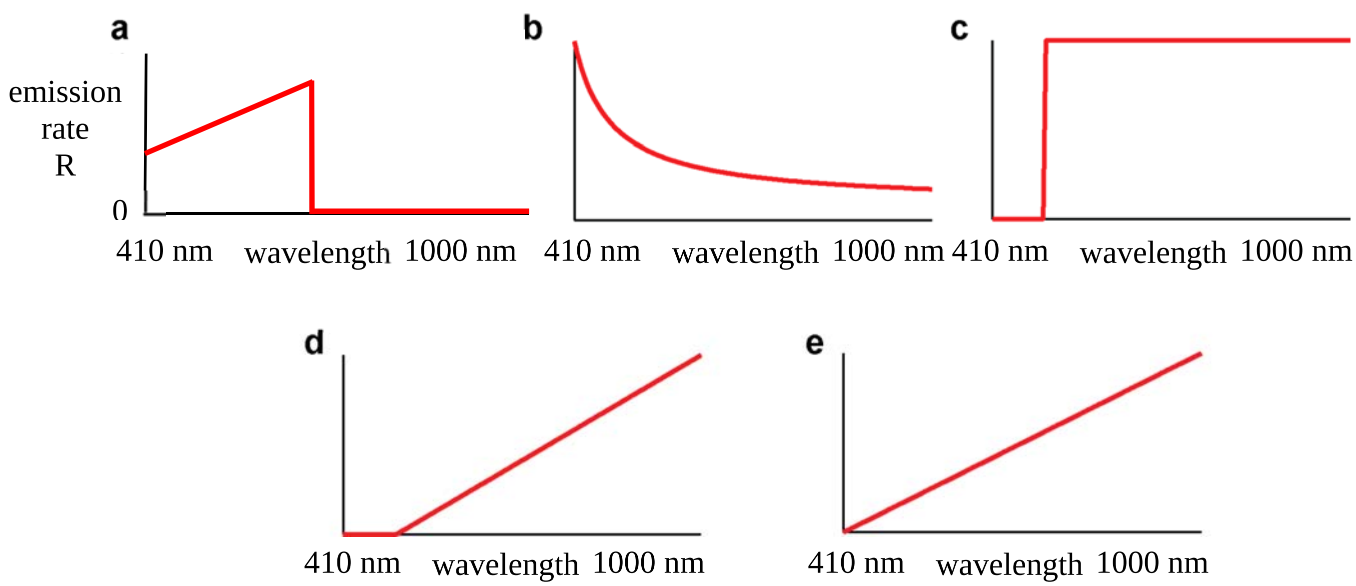
a. 500.03 nm
b. 500.06 nm
c. 500.08 nm

The following two problems pertain to the same situation.

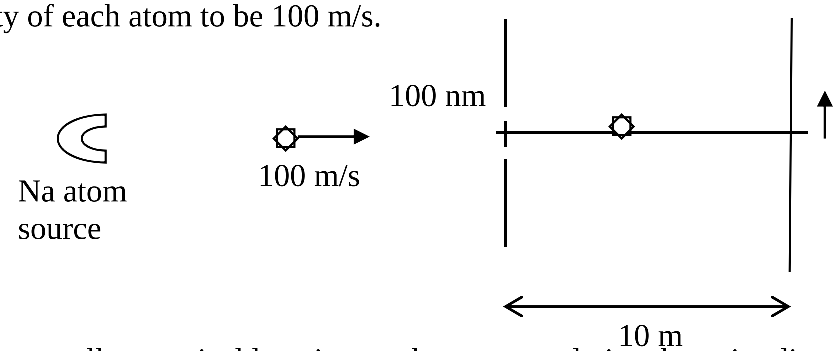
12. A laser of wavelength 410 nm is incident on a metal with a workfunction of 1.7 eV. There is no stopping voltage applied. Assuming the liberated electrons went straight up, how high could they go? (Assume earth's gravitational potential g = 9.8 m/s^2.)

a. 0.8 mm
b. 4.7 m
c. 3200 m
d. > 10^{10} m, i.e., into space
e. No electrons are ejected.

13. The laser beam incident on the metal surface has **FIXED INTENSITY** (and fixed diameter, so fixed power). If we increase the photon wavelength above 410 nm (while keeping the intensity the same), which of the graphs below best describes the electron emission rate R, *i.e.*, the number of electrons emitted per second, as a function of wavelength? (Assume that the photo-emission probability is either 0 or 1, i.e., it does not depend on photon wavelength as long as an electron can be emitted). Hint: Calculate the number of photons per second for a given intensity (power).



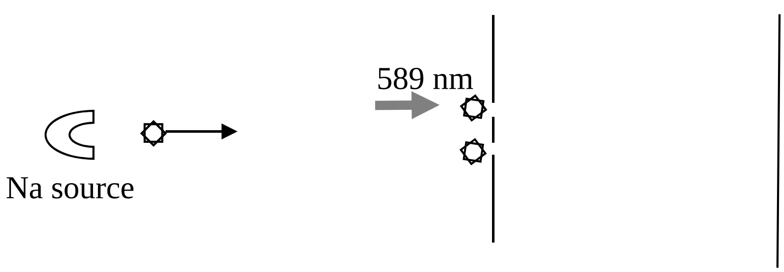
The following four problems pertain to the same situation.
A collection of sodium atoms (m_{Na} = 3.84 x 10^{-26} kg) are directed *one at a time* from a (small) source to a pair of nano-fabricated slits, separated by 100 nm, and 10 m from the detection screen (figure below not to scale). A "velocity selector" at the source defines the velocity of each atom to be 100 m/s.



14. At what smallest vertical location on the screen, relative the point directly opposite the midpoint of the slits, will the probability of finding any sodium atoms fall to zero?

a. 0 mm (i.e., on the center line indicated in the figure)
b. 1.2 mm
c. 8.6 mm
d. 11.2 mm
e. There is *no* location where the probability falls to zero, unless you have more than one atom passing through the slits at a time.

15. Next we shine laser light pulse (wavelength 589 nm) onto atoms passing through the top slit, so that *any atom passing through the top slit will have its speed increased to 101 m/s (due to the absorption and reemission of the photons); atoms passing through the bottom slit still have 100 m/s*:



Which of the following best describes what happens to the interference pattern on the detection screen?

a. There is no change – the slight velocity shift is insufficient to disturb any possible interference effects.
b. The interference pattern will now move vertically up the screen, at 1m/s.
c. The interference pattern will disappear, because the interfering processes are not indistinguishable, due to the availability of information on which slit a given atom passed through.

16. In the previous problem, the change in velocity arose from the transfer of momentum from the 589-nm photons to the atom (assume here that the entire photon momentum goes to speeding up the atom toward the slits, and that every photon hits the atom). Approximately how many photons need to hit each atom in order to produce the desired velocity boost of 1 m/s?

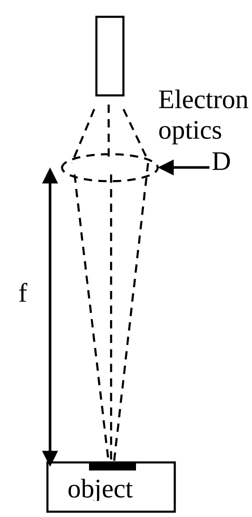
a. 3.
b. 34
c. 340

17. Consider the 100-m/s sodium atoms passing through the bottom slit. What should be done to the initial speed of the atoms (i.e., before the slit) in order to result in a broader pattern on the detection screen, i.e., the central peak is wider.

a. reduce the speed
b. increase the speed
c. the speed of the atoms doesn't affect the spread of detections on the screen

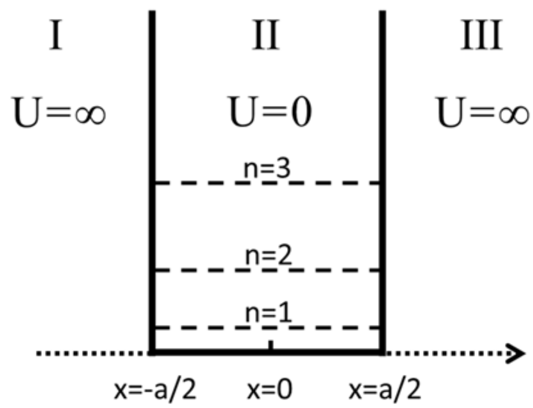
18. An electron microscope accelerates electrons through a 4-kV potential for imaging. The diameter of the electrostatic lens is 5 cm with a focal length of 100 cm. What is the diffraction limited resolution of this instrument? The resolution is defined as the minimum distance between two points on the surface that can be clearly resolved.

a. 0.02 nm
b. 0.1 nm
c. 0.5 nm
d. 2 nm
e. 8 nm



The next three problems refer to the following situation:

The figure shows an infinite one-dimensional quantum well of width *a*, centered at the origin. An electron occupies one of the energy levels shown.



19. Suppose that the energy of the ground state is 10 meV. What is the well width, *a*?

    a. *a* = 0.21 nm
    b. *a* = 0.93 nm
    c. *a* = 3.49 nm
    d. *a* = 6.13 nm
    e. *a* = 14.80 nm

20. Let $\psi_n(x)$ be the wavefunction for the nth state. Which statements are correct?

    a. $\psi_1(x)$ is a symmetric function and $\psi_2(x)$ is an antisymmetric function of *x*.
    b. $\psi_2(x)$ is a symmetric function and $\psi_3(x)$ is an antisymmetric function of *x*.
    c. $\psi_1(x)$ is a symmetric function and $\psi_3(x)$ is an antisymmetric function of *x*.

21. The electron is prepared in the first excited state. What wavelength light can induce a transition to the second excited state?

a. $\lambda = \frac{8}{5}\frac{mca^2}{h}$

b. $\lambda = \frac{2}{3}\frac{mca^2}{h}$

c. $\lambda = 8\frac{mca^2}{h}$

d. $\lambda = \frac{1}{2}\frac{mca^2}{h}$

e. $\lambda = 3\frac{mca^2}{h}$

22. This problem compares the probability of finding an electron prepared in the ground state between x=-a/6 and x=a/6, P_1, with the probability of finding an electron prepared in the second excited state over the same interval, P_3.

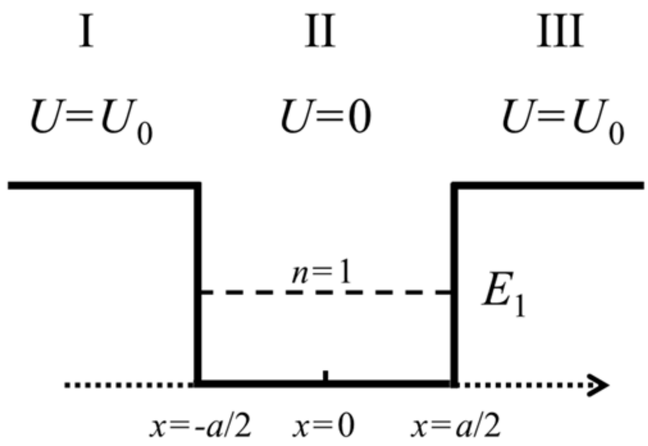
a. $P_1 > P_3$
b. $P_1 < P_3$
c. $P_1 = P_3$

23. If an electron is in the second excited state, where is the probability of finding it the greatest?

a. At x=a/4 and x=-a/4
b. At x=-a/2, x=0 and x=+a/2
c. Only at x=0
d. At x= -a/3, x=0, and x=+a/3
e. At x=-3a/8, x=-a/8, x=a/8, and x=+3a/8

The next three problems refer to the following situation:

A one dimensional quantum particle occupies the ground state of a finite potential well of width *a* shown in the figure. The energy depth of the well is U_0 Joules and the energy of the state is E_1 as shown in the figure. **Note that the well extends from –a/2 to a/2.**



24. What is the particle's wavefunction, $\psi(x)$ for $|x| < a/2$?

a. $\psi(x) = A\left(\cos kx + e^{\gamma x}\right)$
b. $\psi(x) = -A\sin kx$
c. $\psi(x) = Ae^{\gamma x}$
d. $\psi(x) = A\cos kx$
e. $\psi(x) = Ae^{-\gamma x}$

25. Which of the following statements is true?

a. At –a/2 and +a/2 the wavefunction must be continuous, but its first derivative must be discontinuous
b. At –a/2 and +a/2 the wavefunction and its first derivative must be continuous.
c. $d\psi/dx$ is a maximum at x=0.

26. If the well is made deeper by increasing U_0, what happens to the probability of finding the particle at $|x| > a/2$

a. The probability of finding at $|x| > a/2$ decreases.

b. The probability of finding at $|x| > a/2$ remains the same.

c. The probability of finding at $|x| > a/2$ increases.1, 6d

Physics 214 Common Formulae

SI Prefixes		
Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0		
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Physical Data and Conversion Constants	
speed of light	$c = 2.998 \times 10^8$ m/s
Planck constant	$h = 6.626 \times 10^{-34}$ J·s $= 4.135 \times 10^{-15}$ eV·s
Planck constant / 2π	$\hbar = 1.054 \times 10^{-34}$ J·s $= 0.658 \times 10^{-15}$ eV·s
electron charge	$e = 1.602 \times 10^{-19}$ C
energy conversion	1 eV = 1.602×10^{-19} J
conversion constant	hc = 1240 eV·nm= 1.986×10^{-25} J-m
useful combination	$h^2/2m_e = 1.505$ eV nm^2
Bohr radius	$a_o = (4\pi\varepsilon_o)\hbar^2 / m_e e^2 = 0.05292$ nm
Rydberg energy	$hcR_\infty = m_e e^4 / 2(4\pi\varepsilon_o)^2 \hbar^2 = 13.606$ eV
Coulomb constant	$\kappa = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N·m^2 / C^2
Avagadro constant	$N_A = 6.022 \times 10^{23}$ / mole
electron mass	$m_e = 9.109 \times 10^{-31}$ kg = 0.511 MeV/c^2
proton mass	$m_p = 1.673 \times 10^{-27}$ kg = 938.3 MeV/c^2
neutron mass	$m_n = 1.675 \times 10^{-27}$ kg = 939.6 MeV/c^2
hydrogen atom mass	$m_H = 1.674 \times 10^{-27}$ kg
Electron magnetic moment	$\mu_\varepsilon = 9.2848 \times 10^{-24}$ J/T $= 5.795 \times 10^{-5}$ eV/T
Proton magnetic moment	$\mu_p = 1.4106 \times 10^{-26}$ J/T $= 8.804 \times 10^{-8}$ eV/T

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$A_1\sin(\omega t+\phi_1) + A_2\sin(\omega t+\phi_2) = A_3\sin(\omega t+\phi_3)$$

$A^2 + B^2 + 2AB\cos\phi = C^2$ (ϕ here is the *external* angle)

Waves, Superposition

$$k \equiv \frac{2\pi}{\lambda} \quad \omega \equiv 2\pi f \quad T \equiv \frac{1}{f} \quad v = \lambda f = \frac{\omega}{k}$$

General relation for I and A: $I \propto A^2$, $A = A_1 + A_2 + \ldots$

Two sources: $I_{max} = |A_1 + A_2|^2$, $I_{min} = |A_1 - A_2|^2$

Two sources, same I_1: $I = 4I_1\cos^2(\phi/2)$ where $\phi = 2\pi\delta/\lambda$

Interference: Slits, holes, etc.

Far-field path-length difference: $\delta \equiv r_1 - r_2 \approx d\sin\theta$

Phase difference: $\dfrac{\phi}{2\pi} \equiv \dfrac{\delta}{\lambda} = \dfrac{d\sin\theta}{\lambda} \approx \dfrac{d\,\theta}{\lambda} \approx \dfrac{d}{\lambda}\dfrac{y}{L}$ if θ small

Principal maxima: $d\sin\theta_{max} = \pm m\lambda \quad m = 0, 1, 2, \ldots$

N slit: $I_N = I_1\left\{\dfrac{\sin(N\phi/2)}{\sin(\phi/2)}\right\}^2$ where $\phi = 2\pi d\sin\theta/\lambda$

Single slit: $\delta_a = a\sin\theta \quad a\sin\theta_{min} = \pm m\lambda$ with $m = 1, 2, 3\ldots$

$$\frac{\beta}{2\pi} \equiv \frac{\delta_a}{\lambda} = \frac{a\sin\theta}{\lambda} \approx \frac{a\,\theta}{\lambda} \approx \frac{a}{\lambda}\frac{y}{L}$$

Single slit: $I_1 = I_0\left\{\dfrac{\sin(\beta/2)}{\beta/2}\right\}^2$ with $\beta = 2\pi a\sin\theta/\lambda$

slit: $\theta_0 \approx \lambda/a$ *or* hole: $\theta_0 \approx 1.22\lambda/D \approx \alpha_c$

Approx. grating resolution: $\dfrac{\Delta\lambda}{\lambda} \geq \dfrac{1}{Nm}$

Quantum laws, facts….

UNIVERSAL: $p = \hbar k = h/\lambda \qquad E = hf = \hbar\omega$

Light: $E = hf = \hbar\omega = hc/\lambda = pc$

Slow particle: $KE = mv^2/2 = p^2/2m = h^2/2m\lambda^2$

Photoelectric effect: $KE_{max} = eV_{stop} = hf - \Phi$

UNIVERSAL: $\Delta x\,\Delta p_x \geq \hbar \qquad \Delta E\,\Delta t \geq \hbar$

$$\psi^*(x)\psi(x) \equiv |\psi(x)|^2$$

$$P_{ab} = \int_a^b |\psi(x)|^2\,dx, \quad a \leq x \leq b$$

(Slow) particle in fixed potential U:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U(x)\psi(x) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Physics 214 Common Formulae

Quantum stationary states (energy eigenstates):

$$\Psi(x,t)=\psi(x)e^{-i\omega t} \qquad \text{where } E=\hbar\omega$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=\hbar\omega\,\psi(x)=E\,\psi(x)$$

In 1-D box: $n\lambda=2L$ where $n=1,2...$

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \quad \text{for} \quad 0\le x\le L$$

$$E_n=\frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2=\left(\frac{h^2}{8mL^2}\right)n^2=E_1n^2 \quad \text{(*last part*)}$$

Box, 3-D:

$$\psi(x,y,z)=\sqrt{\frac{8}{abc}}\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi}{b}y\right)\sin\left(\frac{n_3\pi}{c}z\right)$$

$$E(n_1,n_2,n_3)=\frac{h^2}{8m}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}+\frac{n_3^2}{c^2}\right)$$

Simple Harmonic Oscillator (SHO):

$$E_n=\left(n+\tfrac{1}{2}\right)\hbar\omega \quad \text{where} \quad n=0,1,2...$$

$$\omega=\sqrt{k/m}$$

Free slow particle with definite p:

$$\Psi(x,t)=Ae^{i(kx-\omega t)} \text{ with } \hbar\omega=\hbar^2k^2/2m$$

H-like atom

potential $U(r)=-\dfrac{\kappa Ze^2}{r}$

$$E_n=\frac{-1}{4\pi\varepsilon_o}\frac{(Ze)^2}{2a_o}\frac{1}{n^2}=-\frac{1}{(4\pi\varepsilon_o)^2}\frac{me^4Z^2}{2\hbar^2n^2}$$

$$=-13.606\,\text{eV}\,\frac{Z^2}{n^2}$$

Ground state: $\psi_{1s}(r,\theta,\phi)=\dfrac{1}{\sqrt{\pi a_o^3}}e^{-r/a_o}$

Radial density for s-state: $P(r)\,dr=4\pi r^2\,|\psi(r)|^2\,dr$

Form of *n, l, m* eigenstate:

$$\psi_{n\ell m}(r,\theta,\phi)=R_{n\ell}(r)\,Y_{\ell m}(\theta,\phi)$$

$$Y_{00}=\frac{1}{\sqrt{4\pi}}, \quad Y_{10}=\sqrt{\frac{3}{4\pi}}\cos\theta,$$

$$Y_{1\pm1}=\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

Tunneling

$$T\approx Ge^{-2KL} \qquad \text{where} \qquad G=16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$$

$$K=\sqrt{\frac{2m}{\hbar^2}\left(U_0-E\right)}=2\pi\sqrt{\frac{2m}{h^2}\left(U_0-E\right)}$$

Angular momentum and magnetism

Orbital: $L_z=m\hbar$ where $m=0,\pm1,\pm2,...\pm\ell$

$L^2=\ell(\ell+1)\hbar^2$ where $\ell=0,1,2,...$

Spin: $S_z=m_s\hbar$ where $m_s=\pm 1/2$

Magnetic energy: $U=-\vec{\mu}\cdot\vec{B}$

Force: $F_z=\mu_z\dfrac{dB_z}{dz}$ where $\mu_z\approx-\dfrac{e}{m_e}S_z$

Atomic orbital filling