Solution

Physics 213 | **Problem 2** | **Week 7**

Semiconductor donor

In a tiny (simplified) semiconductor sample, an electron can either be in a single trapped state at a donor site or in a many-state band. The energy in the many-state band is 4.2×10^{-20} J higher than in the trap. Let T = 300 K. Ignore electron spin.

a) What is the ratio of the probability the electron is trapped to the probability it is in a particular single state in the band? (P_{trap}/P_{b1})

The ratio of the probabilities of two *states* is just the ratio of the Boltzmann factors for those states.

$$\frac{P(trap)}{P(b1)} = \frac{e^{-E_{trap}/kT}}{e^{-E_{band}/kT}} = e^{-\frac{E_{trap}-E_{band}}{kT}} = e^{\frac{4.2\times10^{-20}\,J}{(1.38\times10^{-23}\,J/K)(300\,K)}} = 25{,}500$$

The important thing to realize here is that P(b1) denotes the probability of *one* of the band states. There is only one trapped state, but were there more, P(trap) would be the probability of one of them. See b).

b) If there are 10^5 states in the band, what is the ratio of the probability that the electron is trapped to the probability it is somewhere in the collection of band states? (P_{trap}/P_{band})

Since there can be more than one state with the same energy, the probability of having an energy E_A is

$$P(E_A) = \Omega(E_A)\, P(A),$$

where P(A) is the probability of *one* of the states with energy E_A and $\Omega(E_A)$ is the number of states with energy E_A.

$$\frac{P(trap)}{P(band)} = \frac{P(E_{trap})}{P(E_{band})} = \frac{\Omega(E_{trap})P(trap)}{\Omega(E_{band})P(b1)} = \frac{1\cdot P(trap)}{10^5\cdot P(b1)} = \frac{1}{10^5}25{,}500 = 0.255$$

c) What is the probability the electron is trapped? (P_{trap})

We know from b) that $P(E_{band}) = (1 / 0.255)\, P(E_{trap}) = 3.92\, P(E_{trap})$.

We need a second equation in order to find $P(trap) = P(E_{trap})$, and we know that the total probability must be 1. That is, since the electron must have an energy equal to some value.

$$\sum_{All(E)} P(E) = 1; \quad P(E_{trap}) + P(E_{band}) = 1; \quad P(E_{trap}) + 3.92\, P(E_{trap}) = 1; \quad P(E_{trap}) = 1 / 4.92 = \mathbf{0.203}$$