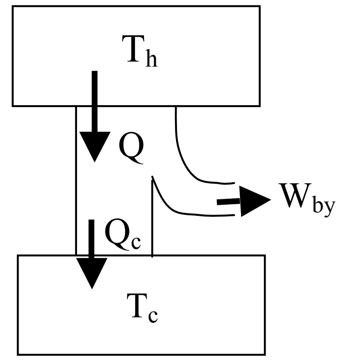
Solution

Physics 213 **Problem 2** **Week 6**

Energy sources: Coal

A power plant burns coal to heat a boiler to 450 °C. The waste heat leaves via a cooling tower at 60 °C.

a) What is the highest percentage of the chemical energy (released by burning the coal) that can be converted into electrical energy, if it all works as well as possible?

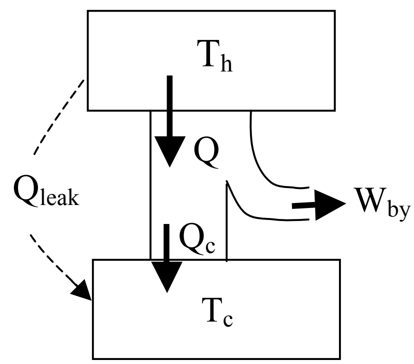


A power plant is a heat engine, generating work that is used at remote locations, such as your home. Thus, the electrical energy is the work, W_{by}, and the coal chemical energy is Q_H. The highest percentage of coal chemical energy that can be converted to work is the Carnot efficiency. (The symbols Q_C, Q_H and W_{by} are defined as positive.)

$$W_{by} / Q_H = [Q_H - Q_C] / Q_H = 1 - (Q_C / Q_H)$$
$$= 1 - (T_C / T_H) = 1 - ((60 + 273) / (450 + 273)) = \mathbf{0.539 = 53.9\%}$$

Of course, the real efficiency is quite a bit less than this, typically about 30%.

b) If 100 J of heat leaks directly from the boiler to the cooling system (i.e., lost energy), how much does the total entropy (engine plus reservoirs) increase?



The differential of the entropy in any quasistatic process is $dS = \delta Q/T$. The boiler and cooling system temperatures remain constant, so for each of them, the entropy change is $\Delta S = Q/T$, where Q is the total heat flow. (When the temperature changes, ΔS is found by replacing δQ with C dT, and then integrating.) For the boiler, $Q = - Q_H$ and for the cooler $Q = Q_C$. To find the total entropy change, add the changes in the boiler and in the cooling system.

$$\Delta S_{total} = \Delta S_H + \Delta S_C = (-Q_H / T_H) + (Q_C / T_C)$$
$$= (-100\text{ J} / 723\text{ K}) + (100\text{ J} / 333\text{ K}) = \mathbf{0.162\ J / K}$$

Note that the total entropy change is positive. Heat flows from hot to cold, because that makes S increase.

c) If for each 1 kJ of energy entering the boiler, 100 J leaks directly to the cooling system, but no other processes increase the net entropy, how much work can be obtained from the 1 kJ?

With 100 J leaking to the cooling system, 900 J remain to generate work. We saw in part a) that the Carnot efficiency is 0.539. Thus, the maximum work that can be extracted from 900 J is **900 J × 0.539 = 485 J**.