Solution

Physics 213 **Problem 1** **Week 6**

Heat Pump

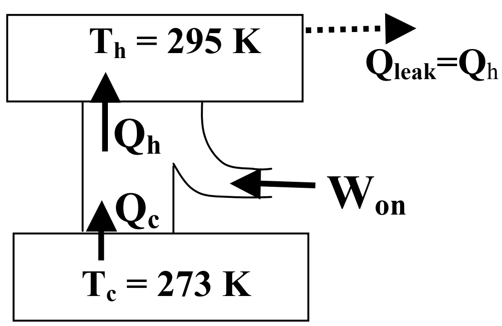
In the winter you probably want to heat your home. Although we traditionally think of heat as only flowing from hot to cold (e.g., from inside your cozy home to the wintry outside), we also know that we can get heat to flow the other way, by doing work. The device that does this is known as a "heat pump".

Consider that your house is at 22°C, and the temperature outside is 0°C. Previously in the Window problem, we showed that if you have a poorly insulated window, you could have a heat leak (under these temperature conditions) of 1500 watts. Here we will explore how much work you would need to do to keep your home at 22°C.

0. First, guess the answer (it's always important to know when we can trust our intuitions, and when we can't).

Most people would probably guess that it takes ~1500 watts to combat a 1500-watt leak (and that's true, if you simply "create" the heat in the house, e.g., by turning on an electric heater). We'll see that in fact one can do *much* better by transferring heat from the outside.

1. Draw an appropriate heat-flow diagram to describe this situation:



2. How much heat is leaked each second from the house? This is the amount of heat Q_h that the heat pump has to deposit into the house each second.

Since the heat (power) loss is 1500 W, the house is losing 1500 J each second.

3. Write an expression using the First Law relating the W_{on}, the work done *on* the heat pump; Q_c, the heat flow out of the outside cold air; and Q_h, the heat flow into the hot house?

The diagram makes this trivial: $W_{on} + Q_c = Q_h = Q_{leak}$.

4. Assuming your heat pump was an ideal heat engine, what is the minimum amount of work that would keep the house at 22°C (for one second), and thus the power needed.

Solving the previous equation, we have:

$$W_{on} = Q_h - Q_c = Q_h (1 - Q_c/Q_h) .$$

But for a Carnot engine (the most ideal heat engine), we showed that $Q_c/Q_h = T_c/T_h$.

Thus we have:

$$W_{on} = Q_{leak} (1 - T_c/T_h) = 1500 \text{ J} (1 - 273/295) = 1500 \text{ J} * 0.0746 = 112 \text{ J}.$$

The power then is 112 W, surprisingly low (but it would be still be better to fix the poorly insulated window!).

5. One figure of merit to characterize a heat pump is the amount of work required to pump a given amount heat: W_{on}/Q_{leak} (or alternatively, the amount of power needed to counter a given heat loss rate). Obviously we want this to be as low as possible. If you want to have the lowest required input power, are you better having the heat source for your heat pump be the outside air (which in the winter may drop to below -20°C), or the ground (which has a more or less stable temperature of ~5°C)? Explain your answer.

Because the $W_{on}/Q_{leak} = (1 - T_c/T_h)$ (and remembering that we need to convert all temperatures into Kelvin), we see that the required work will be lowest when T_c is closer to T_h. Therefore, you are better off connecting to the ground, or to some underground water as the source of heat.

Bonus: The coefficient of performance* (CP) for a heat pump is the ratio of the power transferred to the input power used in the process. Write an expression for this in terms of Q_c and Q_h. Assuming a Carnot cycle (best case), write a formula in terms of T_c and T_h. Evaluate this for the case considered in this problem. For comparison, "real" heat pumps typically have CP ~ 4-7 for this temperature differential.

$$CP = Q_h/W = Q_h/(Q_h - Q_c) = 1/(1 - Q_c/Q_h)$$

For a Carnot cycle this becomes: $CP = 1/(1 - T_c/T_h)$

For our problem, we then have $CP = 1/(1 - 273/295) = 13.4$.

* Note: In the US, manufacturers instead quote the "Energy Efficiency Ratio" (EER), which is conceptually the same thing as CP, except that the numerator is given in British Thermal Units (BTU) per hour, where 1 BTU/hour = 0.292 watt. Therefore CP = EER x 0.292.