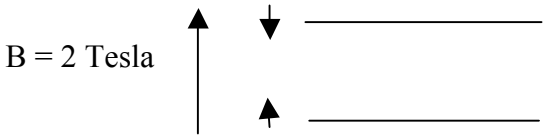
Solution

Physics 213 **Problem 5** **Week 4**

Polarization of Spins

Magnetic Resonance Imaging (MRI) detects the spins of protons (hydrogen nuclei) in organic matter and water. A proton in a magnetic field B has an energy $E = \pm\mu B$, where $\mu = 1.4\cdot10^{-26}$J/Tesla is the magnetic moment of the proton.

a) Compute the energies in Joules:



$E_{down} = \mu B = (1.4\cdot10^{-26})(2) = 2.8\text{x}10^{-26}$ J

$E_{up} = -\mu B = -2.8\text{x}10^{-26}$ J

Note where the + and – signs go.

b) At body temperature (310 K), what fraction of the spins are in the low-energy (up) state? The definition of the 'fraction of up-spins" *f* is given below. [Hint: You can calculate the ratio $r = N_{up}/N_{down}$ using the Boltzmann factor, and *f* can be written as a function of r.]

$r = N_{up}/N_{down} = e^{-Eup/kT}/e^{-Edown/kT} = e^{2\mu B/kT} = 1 + 1.31\cdot10^{-5}$

$f = N_{up}/(N_{up}+N_{down}) = r/(r+1) = 0.5000033$

f is only slightly different from 1/2.

c) At what temperature are 3/4 of the spins up?

We want $r/(r+1) = 3/4 \Rightarrow r = 3 = e^{2\mu B/kT}$.
$T = 2\mu B / k \ln 3 = 2 \cdot 2 \cdot 1.4\cdot10^{-26}$ J / $(1.381\cdot10^{-23}$ J/K $\cdot$ 1.099) = 0.0036 K
This is a very low temperature!

d) The MRI signal is proportional to $N_{up} - N_{down}$, which is relatively small at T = 310 K but increases significantly as the temperature is lowered. Is it practical to improve the signal of a medical MRI device by cooling the sample to line up more of the spins?
Yes__ **No_X_**

Not if you want the patient to stay alive.