Solution

Physics 213 **Problem 5** **Week 7**

More Realistic Rubber Bands

You were introduced to an ultra simple picture of a rubber band: an object with no internal energy but whose entropy goes down as it's stretched: $S = S_0 - bL^2$, where S_0 is the entropy when the length, L, is zero, and b is a positive constant that depends on the particular rubber band.

1. At fixed temperature T, what is the spring constant of this rubber band, in terms of k_B, T, and b? (Think of a weight W hanging on the band, so the energy is -WL. For what L will this whole thing be in thermal equilibrium?)

The free energy of the band/weight system is: $F = U - TS = -WL-T(S_0-bL^2)$.
To find the equilibrium length, minimize F as a function of L:

$$\frac{dF}{dL} = -W + 2bTL = 0 \quad \Rightarrow \quad L = \frac{W}{2bT}.$$

The spring constant is $\frac{dW}{dL} = 2bT$.

Let's get a little more realistic and include some internal thermal energy, with some constant heat capacity C_L at fixed length.

2. If the band starting at T_i is stretched *adiabatically* (no heat flow) from length zero to L, what is its final temperature T_f?

An adiabatic process is reversible, so the total entropy does not change. Therefore:

$$S_{tot} = S_0 + C_L \ln T_i = S_0 + bL^2 + C_L \ln T_f$$

Therefore, $T_f = T_i e^{-bL^2/C_L}$.