Solution

Physics 213 **Problem 4** **Week 3**

Two-State System: Spins

Consider a set of 7 spins which each point either up or down.

a) What is the total number of microstates for this 7-spin system?

Each spin, independent of the others, has two possible states. Thus, the number of microstates of the system is $\Omega = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^7 = \mathbf{128}$.

b) How many microstates have 3 spins up, 4 spins down?

Here we simply choose exactly 3 of the spins to be up, which leaves the other 4 spins down. The number of ways to choose 3 things out of 7 when order of choice doesn't matter is given by the binomial function (also called n-choose-m, or N-choose-N_{up} here):

$$\Omega(N, N_{up}) = \frac{N!}{N_{up}!\,N_{down}!} = \frac{7!}{3!4!} = 35$$

It is useful to notice that we could as well have chosen the four spins to be down instead of the three to be up. Common sense tells us that the number of ways to do this would still have to be 35. In general, choosing M things out of N is equivalent to choosing the N-M things one does not want.

c) What is the probability in equilibrium that 3 spins are up, 4 down?

Since all of the microstates (particular arrangements of up and down for the 7 spins) are equally likely, the probability is **P(3) = 35/128 = 0.273.**

d) The entropy σ is simply the (natural) logarithm of the number of states. What is the entropy for the 3-up, 4-down configuration?

$\sigma_{3up} = \ln(\Omega_{3up}) = \ln(35) = \mathbf{3.55}$

e) Plot σ for all 7 configurations (macrostates): (Notice that σ has only 4 different values.)