Physics 213 **Discussion Notes** **Week 4**

This week, you will start to learn how the counting and probability calculations you've been doing relate to real physical systems.

- The probability of finding a system in a particular configuration (e.g., volume) is proportional to the number of microstates corresponding to that configuration. The number of microstates varies extremely rapidly with particle number and volume.

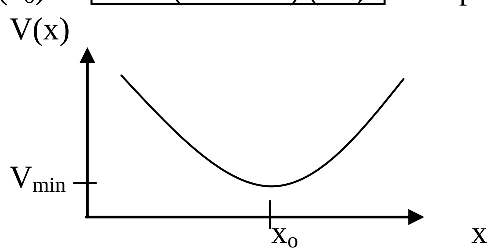
We will use a collection of harmonic oscillators to study energy exchange. There are two reasons:

- Harmonic oscillators have equal energy level spacing, which simplifies the math.

- Many systems are approximately harmonic oscillators. Every potential near a minimum value, $V_{min} = V(x_o)$, behaves like a parabolic harmonic-oscillator potential. To see this, do a Taylor series expansion, noticing that the slope of the potential $(dV/dx)_{xo} = 0$ at the minimum ($x = x_o$):

$$V(x_o + \Delta x) = V(x_o) + (dV/dx)_{xo}\,\Delta x + (1/2)(d^2V/dx^2)_{xo}\,(\Delta x)^2 + \ldots$$

$$\therefore\ V(x_o + \Delta x) - V(x_o) = \boxed{\Delta V = (\text{constant})\,(\Delta x)^2}$$ -- a parabola. (For a spring, $V = (1/2)kx^2$.)



This concept applies to vibrations in molecules and solids.

Important concepts:

- When two systems are in thermal contact, the number of microstates is the product of the numbers of the two systems' microstates.

- When one system is much larger than the other, its temperature is essentially constant regardless of how the energy distributes between the two. That's our model of a thermal reservoir.

- The probability that the small system is found in state "n" with energy E_n is proportional to $\exp(-E_n/kT)$. This is called the Boltzmann factor. Since the probabilities of all the states have to add up to 1, $P_n = (\exp(-E_n/kT))/Z$, where Z is the sum of the Boltzmann factors over *all* the states, $Z = \sum_{n=0}^{\infty} e^{-E_n/kT}$. You will use this in one problem to look at what fraction of spins are aligned with a magnetic field (with a lower energy), and what fraction aren't, a very important issue for magnetic resonance imaging.