

Solution

Physics 213

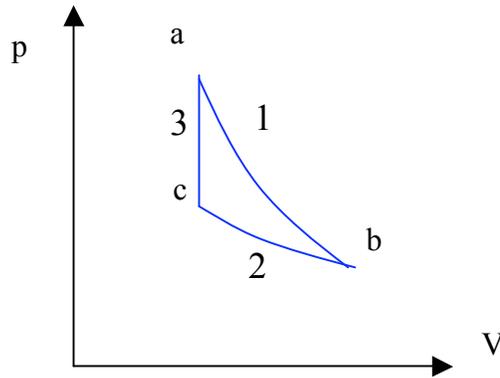
Problem 2
Closed Cycle

Week 3

Nitrogen gas initially at 300K and atmospheric pressure in a volume of 1 liter (state a) is adiabatically expanded to 2 liters (state b), then isothermally returned to 1 liter (state c). How much heat is required to restore the gas to its initial conditions?

1. Draw and label a pV diagram and complete the following tables.

Hints: Start by using the ideal gas law $pV = nRT$ and the known values of p, V and T to find nR. Remember the relationships for isothermal work and adiabatic work.



For a diatomic gas, $\alpha = 5/2$ and $\gamma = (\alpha + 1)/\alpha = 7/5$. $n = \# \text{ moles}$
 $U = \alpha nRT = \alpha pV$ $nR = pV/T = .00333 \text{ l-atm/K}$

state	p	V	T
a	1 A	1 ℓ	300 K
b	0.38 A	2 ℓ	227.4 K
c	0.76 A	1 ℓ	227.4 K

process	Q	$W_{\text{by gas}}$
1 (a→b)	0	0.60 ℓ-atm
2 (b→c)	-0.52 ℓ-atm	-0.52 ℓ-atm
3 (c→a)	0.60 ℓ-atm	0

The calculations:

Process 1: $V_b T_b^\alpha = V_a T_a^\alpha \rightarrow T_b = T_a (V_a/V_b)^{1/\alpha} = 227.4 \text{ K}$
 (Q = 0) $p_a V_a^\gamma = p_b V_b^\gamma \rightarrow p_b = p_a (V_a/V_b)^\gamma = .379 \text{ atm}$
 $W_{\text{by}} = -\Delta U = \alpha(p_a V_a - p_b V_b) = .605 \text{ l-atm}$
 or use $W_{\text{by}} = -\Delta U = \alpha nR(T_a - T_b) = .605 \text{ l-atm}$ (don't need to compute p)

Process 2: $p_b V_b/T_b = p_c V_c/T_c \rightarrow p_c = p_b (V_b/V_c) = .758 \text{ atm}$
 ($\Delta U = 0$) $W_{\text{by}} = nRT_b \ln(V_c/V_b) = -.524 \text{ l-atm} = Q$

Process 3: $Q = \Delta U = \alpha nR(T_a - T_c) = .605 \text{ l-atm}$ ($W = 0$)