Solution

Physics 213 — Problem 6 — Week 1

Logarithms and exponentials (required background)

Definitions:
$x = e^{\ln(x)}$
$x = 10^{\log(x)}$

Here we use "log" for log base 10, "ln" for log base e ("natural" logarithm).

The main math facts used in this review problem are:

I. $\ln(AB) = \ln(A) + \ln(B)$

II. $\ln(A^B) = B\ln(A)$

III. $\frac{d}{dx}e^u = e^u\frac{du}{dx}$

IV. $\frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$

V. Generalize III and IV: $\frac{d}{dx}f(u) = \frac{df}{du}\frac{du}{dx}$ (chain rule)

a) What is log(0.01) ? $10^{-2} = 0.01$, so $\log(0.01) = -2$

b) Let ln(x) = 3.2, and ln(y)= -7.2. What is ln(xy) ? $e^x e^y = e^{x+y}$, so $\ln(xy) = \ln(x) + \ln(y)$

c) Let $y = e^{306554}$. What is ln(y) ? $\ln(y) = 306544$

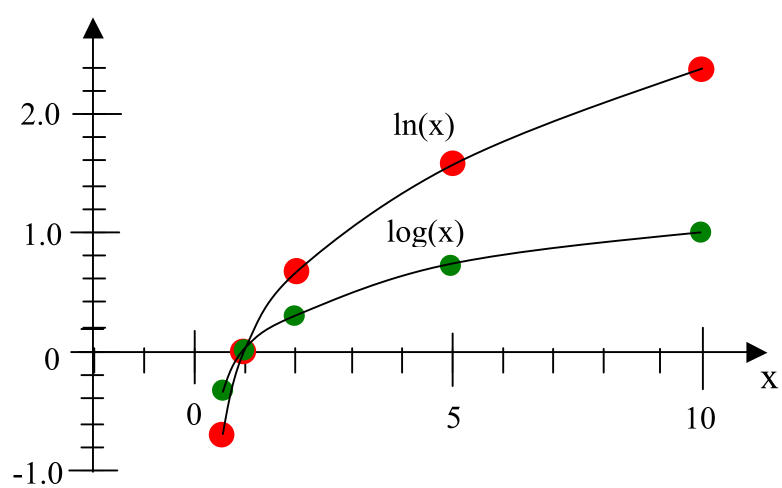
d) Give an expression for $\frac{d}{dx}e^{35x}$: $35e^{35x}$

e) Give an expression for $\frac{d}{dx}e^{-10x^2}$: $-10(2x)e^{-10x^2} = -20xe^{-10x^2}$

f) Give an expression for $\frac{d}{dx}\left(5\ln(3x)\right)$: $5*3/3x = 5/x$

g) Give an expression for $\frac{d}{dx}\left(5\ln(x^3)\right)$: $5(3x^2)/x^3 = 15/x$

h) Plot a few points for $\log(x)$ and $\ln(x)$:
(x = 0.5, 1, 2, 5, 10)
What about $x = 0$?
Logarithm not defined for $x \leq 0$.



Note $x = 10^{\log(x)}$ and $10 = e^{\ln(10)}$
so $x = e^{\ln(x)} = e^{\ln(10)\log(x)}$ and
ln(x) = ln(10) log(x) = **2.303 log(x)**
They are proportional.

i) Stirling's approximation says that ln N! is approximately N ln N – N

Fill out the table below to see how accurate this approximation is.

N	ln N!	N ln N – N
10	15.1044125731	13.0258509299
20	42.3356164608	39.9146454711
50	148.478	145.601
100	363.74	360.517
1000	5912.13	5907.76

j) Later in the course we will also need to know how to calculate:
d (ln N!)/dN

Use Stirling's approximation to rewrite this in a simpler form.

d (N ln N – N) = ln N + N/N -1 = ln N