Physics 213 **Quiz 3a SOLUTION** **Week 4**

No calculators!

In this quiz, we assume that the number of states available to a gas atom is proportional to the volume of its container: M=bV, for some constant b.

1) N distinguishable gas atoms are placed in a container of volume V. Calculate the number of microstates available, assuming that the gas is dilute, in terms of b, V, and N.

   $\Omega=M^N=(bV)^N$ *(this is just to make sure they're starting with the right equation)*

2) Now consider a cylinder of volume V, broken into two spaces V_L and V_R by a frictionless piston, containing N_L and N_R gas atoms respectively. Write the total multiplicity $\Omega=\Omega_L\Omega_R$ in terms of the parameters given.

   $\Omega=(bV_L)^{NL}(bV_R)^{NR}$ *(this question is mainly to set them up for (3))*

3) The system will reach equilibrium when the multiplicity is highest. By maximizing Ω, find the equilibrium relation between N_L, N_R, V_L, and V_R. *Hint: Substituting $V_R=V-V_L$ may help! Remember, maximizing Ω has the same effect as maximizing the entropy, ln Ω, which is sometimes easier to do.*

   $\Omega=(bV_L)^{NL}(bV-bV_L)^{NR}$

   $d\Omega/dV_L=N_L(bV_L)^{NL-1}(bV-bV_L)^{NR}-N_R(bV_L)^{NL}(bV-bV_L)^{NR-1}=0$

   $N_L(bV_L)^{NL-1}(bV-bV_L)^{NR}=N_R(bV_L)^{NL}(bV-bV_L)^{NR-1}$

   $N_L(bV_L)^{NL-1}(bV_R)^{NR}=N_R(bV_L)^{NL}(bV_R)^{NR-1}$

   $N_L\,(bV_R)=N_R(bV_L)$

   $N_L/N_R=V_L/V_R$

4) Is this the same result we would predict from the Ideal Gas Law? You must justify your answer!!

   Yes! Holding P and T constant,

   $P_LV_L/N_LT_L=P_RV_R/N_RT_R$

   $V_L/N_L=V_R/N_R$