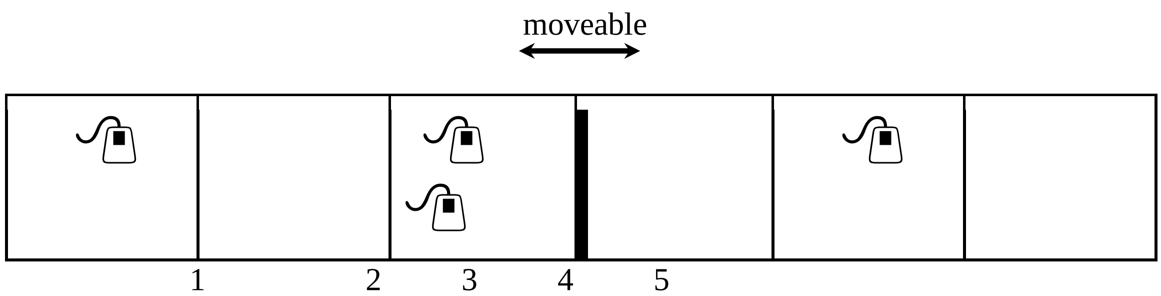
Physics 213 Quiz 3 Solution Week 4

In this problem we consider the equilibration of volume in the system pictured below. This chamber contains four **indistinguishable** particles and is composed of six discreet bins, each of which may hold many particles. There is a moveable barrier dividing the chamber (shown at position three below) through which the particles can not pass. The barrier may be found at positions m=1, 2, 3, 4, or 5, each of which corresponds to an observable macrostate of the system.



What is the dimensionless entropy of the most likely macrostate?

$\Omega_n = \dfrac{(N+M-1)!}{(M-1)N!}$ Gives the number of microstates for N particles and M bins.

$\Omega_{total} = \Omega_{left} \cdot \Omega_{right}$ This gives the total multiplicity of a given macrostate.

There are five possible macrostates:

M=1	$\wedge_L = 1$	$\wedge_R = 5$	$\wedge_m = 5$
M=2	$\wedge_L = 4$	$\wedge_R = 4$	$\wedge_m = 16$
M=3	$\wedge_L = 10$	$\wedge_R = 3$	$\wedge_m = 30$
M=4	$\wedge_L = 20$	$\wedge_R = 2$	$\wedge_m = 40$
M=5	$\wedge_L = 35$	$\wedge_R = 1$	$\wedge_m = 35$

The macrostate m=4 has the largest number of microstates.

Entropy is defined as $\sigma = \ln(\Omega)$.

σ= **3.69**

What is the most likely position of the barrier?

Position 4 is most likely since the entropy is maximal for the corresponding macrostate

What is the probability of finding the barrier at its most likely position?

P= 40/126= **0.317**

4. What is the average barrier position?

$$< m > \ = \sum_{m=1}^{4} m P_m = \ 1\times0.04 + 2\times0.13 + 3\times0.24 + 4\times0.32 + 5\times0.28 = \mathbf{3.7}$$