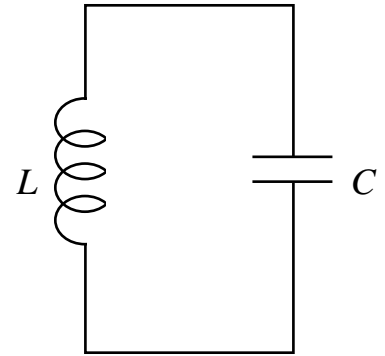


Discussion Question 11B
Physics 212 week 11
LC Circuits

At right, we see the classic **LC circuit**, consisting of an inductor in series with a capacitor. To be precise, this is an **undriven** LC circuit, since there is no battery driving the flow of current. Nevertheless, this simple circuit has an amazing and highly useful property: it supports a resonant oscillation of current. In the ideal case shown here where there is *no* resistance in the circuit, the oscillation can continue *indefinitely* without any external source of EMF to drive it. LC circuits often appear as parts of larger networks which are designed to operate at a particular frequency. The most familiar example is a radio, whose circuitry can be tuned to receive (i.e. respond to) incoming electromagnetic waves of a particular frequency.



$$L = 300 \text{ mH}$$
$$C = 0.025 \text{ }\mu\text{F}$$

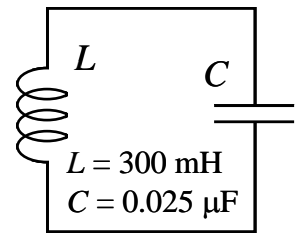
(a) What is the angular frequency ω_0 of the current maintained by the circuit? What is the linear frequency f_0 ?

An *undriven* LC circuit can *only* operate in a stable way at its natural resonant frequency. As for linear versus angular frequency, the conversion is easy if you remember the units: f is in $\text{Hz} = 1/\text{sec}$, ω is in rad/sec .

(b) Let's set our clock so that the current in the circuit is at its maximum value I_{max} at time $t = 0$. Write down an expression for the time-dependence of the current $I(t)$ in terms of I_{max} and the frequency ω_0 .

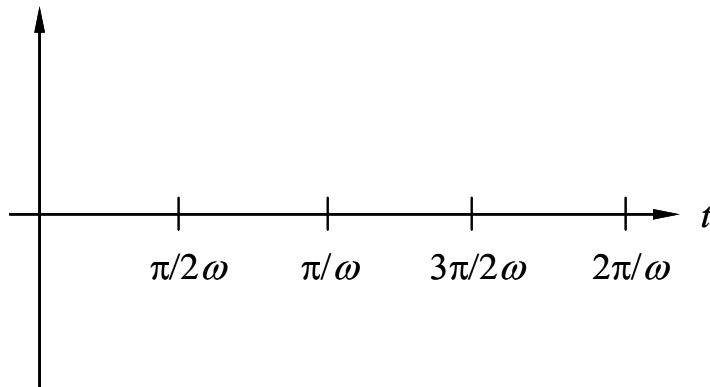
It's either a sine or a cosine ...

(c) Starting from your expression for $I(t)$, determine the voltages $V_L(t)$ and $V_C(t)$ across the inductor and capacitor. As part of your solution, find the peak values $V_{L,\max}$ and $V_{C,\max}$ in terms of I_{\max} .



You'll need your familiar formulas for the voltage across R , C , and L ... and remember that, by definition, $I = \frac{dQ}{dt}$ and so $Q = \int I dt$.

(d) To visualize what's going on, sketch your functions $I(t)$, $V_L(t)$, and $V_C(t)$. Don't worry about the amplitudes of your curves, just their shapes and phases.



e) Have a look at your plot → does V_L lead or lag the current? How about V_C ?

Leading and lagging can be tricky concepts. Think of it this way: which one “gets there” (i.e. reaches its maximum value) first, V or I ? The one that “gets there first” leads the other.

Congratulations! You've just derived the **master relations** between current and voltage for inductors and capacitors in an AC circuit!

Peak Values

$$V_{R,\max} = I_{\max} R$$

$$V_{L,\max} = I_{\max} X_L \rightarrow X_L = \omega L$$

$$V_{C,\max} = I_{\max} X_C \rightarrow X_C = 1/\omega C$$

Relative Phases

Across $R \rightarrow V$ in phase with I

Across $L \rightarrow V$ leads I by 90°

Across $C \rightarrow V$ lags I by 90°

Notice how all the peak-value formulas look like good old “ $V = IR$ ” \rightarrow the **reactances** X_L and X_C describe the effective resistance of inductors and capacitors in AC circuits.

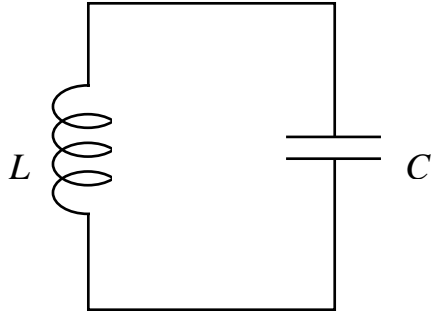
We now set the circuit into oscillation by “stimulating” it with a brief pulse from some external source of EMF. The result is that the peak voltage across the capacitor is $V_{C,\max} = 120 \text{ V}$.

(f) What is the peak current I_{\max} ?

$$L = 300 \text{ mH}$$

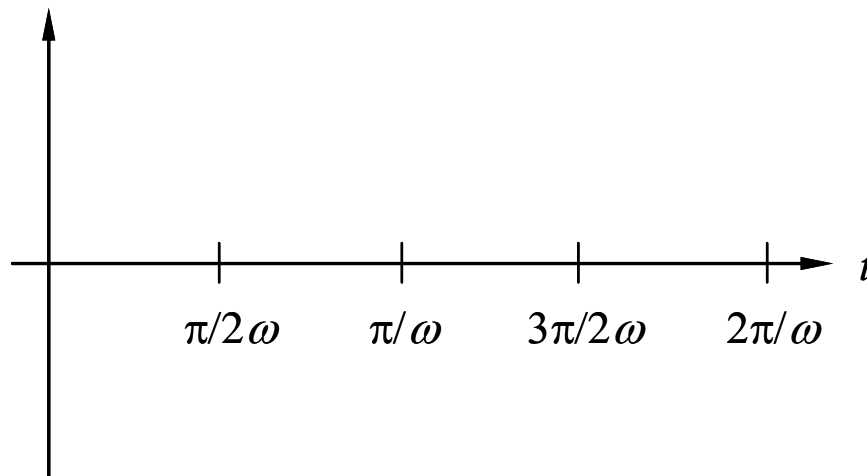
$$C = 0.025 \text{ } \mu\text{F}$$

$$V_{C,\max} = 120 \text{ V}$$



(g) What are the maximum energies $U_{L,\max}$ and $U_{C,\max}$ stored in the inductor and capacitor respectively?

(h) Determine time-dependent expressions $U_L(t)$ and $U_C(t)$ for the two stored energies, and add them together to find the total stored energy $U(t)$. Finally, plot your results for all three functions.



(i) How are $U_{L,\max}$, $U_{C,\max}$, and U_{\max} related to each other?

Knowing this relation is extremely helpful in solving LC circuit problems!