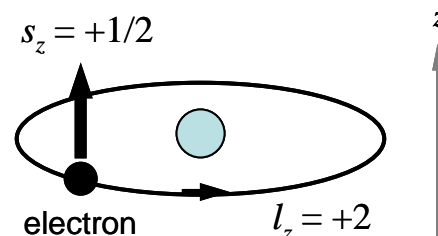


Discussion “Challenge” 8D
P212, Week 8
Magnetic Dipoles in Atomic Physics

Here is one reason that we are studying magnetic dipole moments: nature is *filled* with current loops at the atomic and subatomic levels! Magnetic dipoles appear constantly in those fields of physics.

Consider the hydrogen atom = one proton (charge $+e$) with an electron (charge $-e$) orbiting around it. As you know, the electron can live in a number of “orbitals”, which are states with different orbital angular momentum L . From quantum mechanics, we know that L of an electron in such an orbital is $L = l \hbar$ where l is a “quantum number” that takes only integer values: 0, 1, etc. Further, the electron has spin angular momentum S , where $S = s \hbar$. For an electron, s is always $1/2$. You may think of spin this way: the electron is constantly revolving around its own internal axis, producing a constant angular momentum $\hbar/2$.



The diagram shows a hydrogen atom in a “d-state” with $l = 2$. Both the spin S and orbital angular momentum L are pointing upwards ($+z$) in the figure, but that doesn’t have to be the case. We’ll consider 4 states: $\uparrow\uparrow$, $\downarrow\downarrow$, $\uparrow\downarrow$, and $\downarrow\uparrow$. We’ll use the first, thick arrow to refer to L (since it is bigger than S), so $\uparrow\uparrow$ means $l_z = +2$, $s_z = +1/2$, while $\downarrow\uparrow$ means $l_z = -2$, $s_z = +1/2$, etc.¹

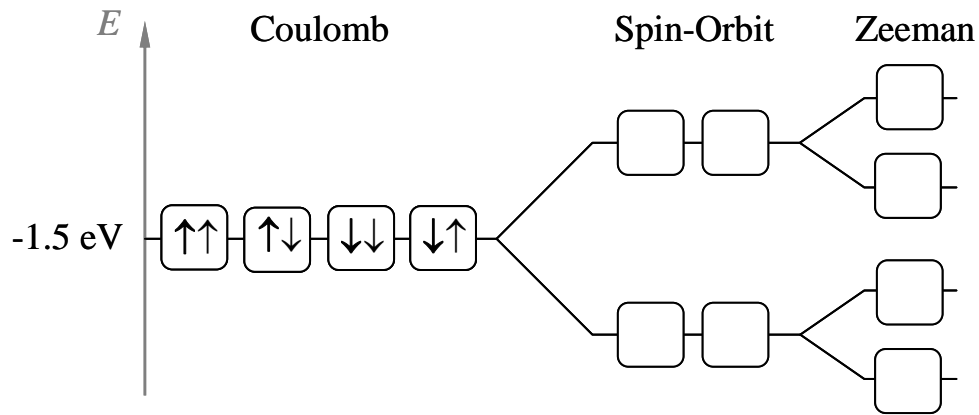
As it happens, any rigid charged body that is either orbiting or spinning has a **magnetic dipole moment** that is proportional to its angular momentum. The electron’s intrinsic magnetic moment (due to its spin) is $\mu_s = \mu_B$, where $\mu_B = 9.3 \times 10^{-24} \text{ J/T}$ is a physical constant called the “Bohr magneton”. The orbital motion of the electron also produces a magnetic moment: $\mu_l = l \mu_B$

(a) Given the spin and orbital directions shown in the figure, in which directions do the magnetic moments μ_s and μ_l point? Don’t forget the electron has a negative charge. Draw them on the figure.

(b) Can you explain why the magnetic moment of a spinning or orbiting charged object is proportional to its angular momentum? (e.g. as in $\mu_l = l \mu_B$)

¹ Quantum mechanics tells us that there are more states than these, but not many. To be precise, the z -projection of spin $s=1/2$ can *only* be $+1/2$ or $-1/2$, while the z -projection of $l=2$ can only be $+2$, $+1$, 0 , -1 or -2 . We’ll ignore the $l_z = 1$, 0 , and -1 possibilities here for brevity.

(c) Next, we will explore the **energy levels** of a d-state electron in the hydrogen atom. We'll express these energies in units of "electron-volts": $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The largest contributions to the electron's total energy E are its *kinetic* and *electric potential* energy (due to the Coulomb field of the proton). Adding these gives $E = -1.5 \text{ eV}$. Can you explain why E is negative?



(d) Next we consider magnetic sources of potential energy. Imagine *yourself* as the electron ... you would see the proton orbiting around you! This orbiting proton creates a magnetic field at your location, with magnitude $B = 22 \text{ mT}$ or so for the d-state. The field exerts a torque on your spin-magnetic moment μ_s , which changes your potential energy. This effect is called the **spin-orbit interaction**, and serves to split the energy levels of the atom according to the orientation of s and l . On the level diagram above, fill in the "Spin-Orbit" boxes to show which of our 4 states have higher energy and which have lower energy. Also, calculate the *magnitude* of the energy change using the numbers given above.

(e) Finally, let's split up the energy of the two pairs of "degenerate" (equal-energy) states we have left. To do so, we will place our hydrogen atom in an external B field. The field will interact with *both* the spin and orbital magnetic moments of the electron, causing another change in energy. This is called the **Zeeman effect**. Let's use the earth's magnetic field, with a strength of about $50 \mu\text{T}$, and arrange our coordinate system so that B_{earth} points in the $+z$ direction. How will the energy levels shift? Fill in the rest of the level diagram with your conclusions, including the size of this final energy change.

And now, an extra challenge!

To calculate the properties of the hydrogen atom, you need **quantum mechanics**. But believe it or not, you can get amazingly close to the right answers using only the classical physics that you have studied here and in Physics 111. On the preceding pages, I threw some numbers at you: the Bohr magneton ($\mu_B = 9.3 \times 10^{-24}$ J/T), the Coulomb + kinetic energy of the d-state (-1.5 eV), and the B field seen by the d-state electron due to the proton ($B = 22$ mT, spin-orbit effect). How close can you get to these values? The *only* information you need is the charge on the electron and proton, and the angular momentum of the electron: $L = l \hbar$ where $l = 2$ in the d-state and $\hbar = 1.055 \times 10^{-34}$ J·s.

(f) For several of these calculations, you'll need the orbital radius R of the d-state electron. Calculate it using the electric force exerted on the electron by the proton. (Ans: $R = 4.8 \times 10^{-10}$ m) ²

(g) Next, calculate the spin-orbit B field of the proton as viewed by the electron. You'll need your formula from question B, and you'll have to think a bit about what current to use. (Ans: $B = 22$ mT) ³

(h) The Bohr magneton μ_B relates the orbital angular momentum $L = r \times p = l \hbar$ of the electron to its magnetic moment: $\mu_l = l \mu_B$. It's not hard to calculate its value! (Ans: $\mu_B = e\hbar / 2m_e = 9.3 \times 10^{-24}$ J/T) ⁴

(i) Finally, determine the energy of the d-state *before* the magnetic effects were added. (Ans: -1.5 eV) ⁵

² You cannot get the right answer without quantum mechanics ... unless you try the following wacky substitution: change l to $l+1$. (I'm serious ☺ Note that this substitution prevents the radius of the $l=0$ "s-state" from going to zero.) And anyway, the "radius" of an orbital is not really a well-defined quantity in quantum mechanics ...

³ You'll be off by a factor of 2 due to "Thomas precession", a relativistic effect that accounts for the fact that you changed to a non-inertial coordinate system without blinking.

⁴ What about the spin magnetic moment μ_s ? Since the rotational angular momentum $S = s \hbar$, just like the orbital angular momentum $L = l \hbar$, you'd expect that $\mu_s = s \mu_B = \mu_B/2$, right? That's what everyone else expected too! One of the early triumphs of quantum mechanics was the non-classical prediction $\mu_s = \mu_B$.

⁵ You may remember from Physics 111 that the kinetic energy T of an orbiting body in a $1/r$ potential (like gravity or electricity) is half its potential energy U . You need both contributions to get the answer.