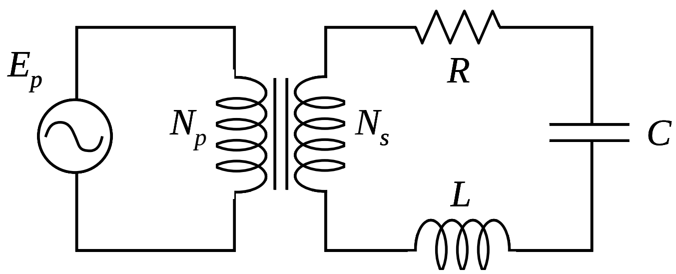
Discussion Question 12A
P212, Week 12
Power in AC Circuits

An electronic device, consisting of a simple RLC circuit, is designed to be connected to an American-standard power outlet delivering an EMF of 120 V_{rms} at 60 Hz frequency. However, you have transported this device to Guyana, where standard AC power outlets deliver a blazing 240 V_{rms}. To overcome this problem, you connect your device to the Guyanese wall outlet through a step-down transformer with N_p turns in the primary coil and N_s turns in the secondary. The values of the resistance, capacitance, and inductance in your device are given in the figure.



$E_{p,\mathrm{rms}}$ = 240 V
E_{rms} = 120 V
f = 60 Hz
R = 3.8 Ω
L = 0.05 H
C = 177 μF

(a) First, let's design the transformer. What relation must you impose on N_p and N_s to drop the Guyanese line voltage $\mathcal{E}_p$ down to $\mathcal{E}$ = 120 V_{rms}**?**

Try thinking physically: As is typical, your transformer has an iron core which ensures that the magnetic field produced in the primary coil is transferred exactly to the secondary coil. Now $\mathcal{E} = -d\Phi/dt$ … to step down the voltage, which coil needs to have more turns?

(b) Let the time-dependent EMF produced by the Euro generator be $\mathcal{E}_p(t) = \mathcal{E}_{p,\max}\sin(\omega t)$ which means $\mathcal{E}(t) = \left(120\sqrt{2}\right)\sin\omega t$ where $\mathcal{E}(t)$ EMF delivered to the secondary side of the transformer . What is the time-dependent current $I(t)$ produced in the load? Write your answer in the form $I(t) = I_{\max}\sin(\omega t - \phi)$, and determine the numerical values of $I_{\max}$ and ϕ.

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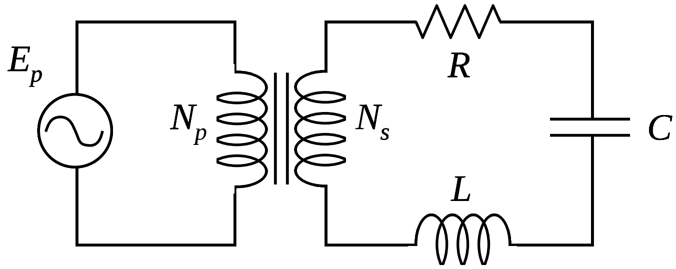

$E_{p,rms}$ = 240 V
E_{rms} = 120 V
f = 60 Hz
R = 3.8 Ω
L = 0.05 H
C = 177 μF

(c) What is the average power <P> delivered to the load?
Your formula sheet has a couple of "cookbook" formulas you can use.

(d) Obtain an algebraic expression for the <P> delivered to the load in terms of $\mathcal{E}_{rms}$, R, and $X \equiv X_L - X_C$. *Hint -write a phasor diagram for Z in terms of X and L and use trig to find the required phase angle.*

(e) Suppose your Guyanese AC generator has a *variable* frequency *f*, though the EMF it supplies is fixed at 240 V_{rms}. What *f* would you select so that the <u>maximum possible power</u> is delivered to your device? *Hint- sketch the expression for <P> you obtained in part (d) as a function of X. Which value of X maximizes <P>? What frequency gives you that value for X?*

(f) What *is* the maximum average power $<P>_{max}$ that you can deliver to your device?

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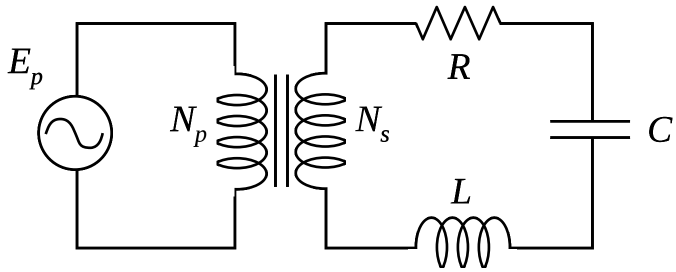
Consider a slightly different situation. Let's say you built your device to be purely inductive → it contains nothing more than a coil you wound yourself, giving L = 0.05 H. However, no device is perfect: *every* electronic circuit element contains *some* residual inductance, capacitance, and resistance. You connect your device to the Guyanese wall outlet and measure the response in the load. You find a current with an rms value of I_{rms} = 8.5 A and lags the transformer EMF by 30°.



$E_{p,rms}$ = 240 V
E_{rms} = 120 V
f = 60 Hz
L = 0.05 H
I_{rms} = 8.5 A
$|\phi|$ = 30º

(g) What is the resistance R of your device?

You will need a couple of equations to "reverse engineer" this circuit: you measured the voltage, current, and phase and now you have to figure out R. Big hint: obtain an equation for R in terms of the impedance Z and the phase ϕ. (It's a useful equation, you might want to remember it!)

(h) What is the capacitance C of your device?

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(i) In part (c), you probably used the average-power formula $<P>=\mathcal{E}_{rms}\, I_{rms} \cos(\phi)$ from your formula sheet (the others, like $<P>=I_{rms}^2\, R$, are equivalent). Let's <u>derive</u> that result! First, what is the instantaneous power P(t) delivered to the load? Give an algebraic answer (no numbers) in terms of $\mathcal{E}_{rms}$, I_{rms}, ϕ, ω, *and* t.

Instantaneous power is P = IV, forever and ever, under all circumstances. Remember the time-dependent expressions you found in part (b) … and in preparation for the next question, you should "massage" your result using $\sin(a-b) = \sin a \cos b - \cos a \sin b$.

(j) Now calculate the time-average of your result for P(t) over one cycle to obtain the average power $<P>$ delivered to the load.

Remember, the average of $\sin^2$ *and* $\cos^2$ *is 1/2, while the average of (sin·cos) is zero.*