Discussion Question 3B

P212, Week 3

Application of Gauss' Law

Gauss' Law relates the electrical flux Φ through any closed surface to the total charge enclosed by that surface:

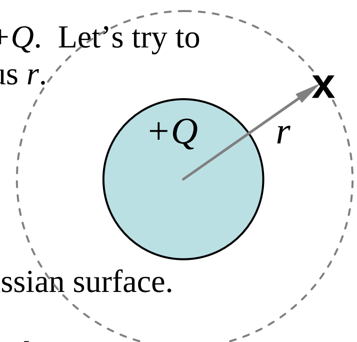
$$\Phi \equiv \int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Since flux is related to electric field, this Law provides a very useful way of *finding* the electric field of a charge distribution! But here's the thing about Gauss' Law:

It's always <u>true</u>, but it isn't always <u>useful</u>.

The trick is knowing when and how to apply Gauss' Law! It all boils down to choosing a suitable Gaussian surface. Let's illustrate. In the following three examples, our task is to find the electric field ***E*** at the point **x** marked on the figure. The dashed lines indicate the Gaussian surface we will attempt to use …

(a) The object here is a solid sphere carrying a uniformly-distributed charge $+Q$. Let's try to find the electric field at the point **x** using a spherical Gaussian surface of radius r.

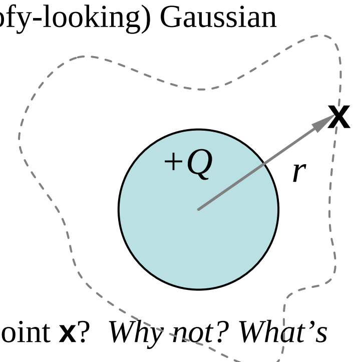


(i) First, visualize the field by drawing some electric field lines.

(ii) Next, use Gauss' Law to find the total flux Φ passing through our Gaussian surface.

(iii) Now that we know the flux Φ, can we determine the electric field ***E*** at the point **x**? Yes we can, go for it!

(b) Let's revisit the same problem, but using a different (and rather goofy-looking) Gaussian surface.

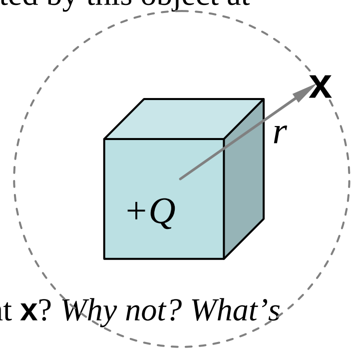


(i) Visualize the field by drawing some field lines.

(ii) What is the total flux Φ passing through this Gaussian surface?

(iii) Using this flux Φ, can we determine the electric field ***E*** at the point **x**? *Why not? What's different about this case than the previous one?*

(c) Finally, consider a solid charged cube. We'll try to find the field created by this object at the point **x** using a spherical Gaussian surface, as in part (a).



(i) Visualize the field of the cube by drawing some field lines.

(ii) What is the total flux Φ passing through our Gaussian surface?

(iii) Using this flux Φ, can we determine the electric field ***E*** at the point **x**? *Why not? What's different about this case than the first one?*

(iv) Can you solve this problem <u>*at all*</u> using Gauss' Law?

Summary

The Gaussian Surface

Gauss' Law is the most elegant way there is to calculate electric fields, but we can only *use* it in a practical way if we can find a suitable Gaussian surface. The mathematical surface we choose has to have these properties:

- The electric field must have a *constant magnitude* on our surface.

- The electric field must make a *constant angle* with our surface (or portions thereof).

Otherwise, we'll never be able to extract the electric field we want from that flux integral!

Procedure

Here's a step-by-step procedure for how to calculate the electric field at some *field point* of interest using Gauss' Law:

1. Visualize and **sketch the electric field**, using all the symmetry that the problem offers. Without knowing the direction and spatial behavior of the field *beforehand*, it is ***impossible*** to solve for the field using Gauss's Law!

2. Based on the field geometry, choose a suitable **Gaussian surface** that passes through the field point of interest.

3. Determine the **total charge enclosed** by your surface: that plus Gauss' Law gives you the **total flux** through the surface.

4. Finally, evaluate the flux integral to **determine *E***.

Charge Densities

Last week we studied the electric field produced by collections of point charges. With Gauss' Law in hand, we can now deal with *extended objects* that carry charge, not just points. To describe the charge on these points, we introduce three universal symbols for charge density:

- ρ = charge / unit volume
- σ = charge / unit area
- λ = charge / unit length

Problem Classes

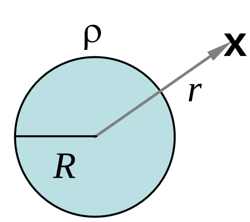
As it happens, there are ***exactly three*** classes of charge distribution that have enough *symmetry* to be analyzed easily with Gauss' Law:

- Spheres
- Infinite lines and cylinders
- Infinite planes and slabs

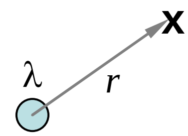
That's it! Knowing how to obtain the electric field caused by these three object-classes is like knowing how to play scales on the piano: the Gauss' Law problems you will encounter (like the music you play) are always constructed from these basic systems. So let's go through them!

Follow the steps outlined on the previous page, find the electric field ***E*** at the indicated field point **x** due to the three charged objects shown below. In all cases, the variable *r* denotes the distance from the center of the object to the field point.

<u>Basic Object #1</u>: A solid sphere of radius R carrying a volume charge density ρ $\Rightarrow$ find ***E*** at the indicated field point.



<u>Basic Object #2</u>: An infinitely long line carrying a line charge density λ $\Rightarrow$ find ***E*** at the indicated field point.



<u>Basic Object #3</u>: A plane of infinite area carrying a surface charge density σ $\Rightarrow$ find ***E*** at the indicated field point.