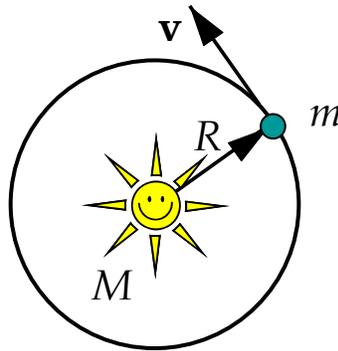


Discussion Question 1D
P212, Week 1
P211 Review: Uniform Circular Motion

$$\mathbf{F}_{1 \rightarrow 2} = -G \frac{m_1 m_2}{r_{12}^2} \hat{\mathbf{r}}_{12}, \quad 1 \xrightarrow{r_{12}} 2 \quad \hat{\mathbf{r}}_{12} \rightarrow$$

In P212 you will encounter problems where charged particles move in uniform circular motion. The forces involved may be electric or magnetic in nature. The answer to part(e) contains the secret of the cyclotron.

Kepler's Third Law (K-III) for planetary motion about the sun for circular orbits is $T^2 = CR^3$ where T is the period, R is the radius of the planet's orbit and C is a constant.



(a) **Derive K-III** for a circular orbit and in the process find an algebraic expression for C in terms the mass of the sun M_S , the universal gravitational constant G , and numerical factors.

(b) Using your answer from part (b), **re-express K-III as a relationship** between the angular frequency ω of the motion and the radius R of the form:
 $\omega^2 = f(R, G, M_S)$.

(c) Consider uniform circular motion of a body of mass m about a central force which depends on velocity: $F = Dv^a R^b$ where D is a constant and a and b are known exponents. Derive K-III for this force, again expressing your answer as a relationship between ω and R , (The constant D will necessarily appear in your final expression, but v must not.)

(d) For the case $a = 0$ and $b = -2$, verify that your answer to part(c) collapses to that for part (b) (this is an excellent **limiting behavior** check).

(e) Evaluate your answer to part (c) for the case $a = 1$ and $b = 0$. How does the angular frequency depend on radius for a force of this nature?