Discussion Question 1C
P212, Week 1
Review: Potential Energy

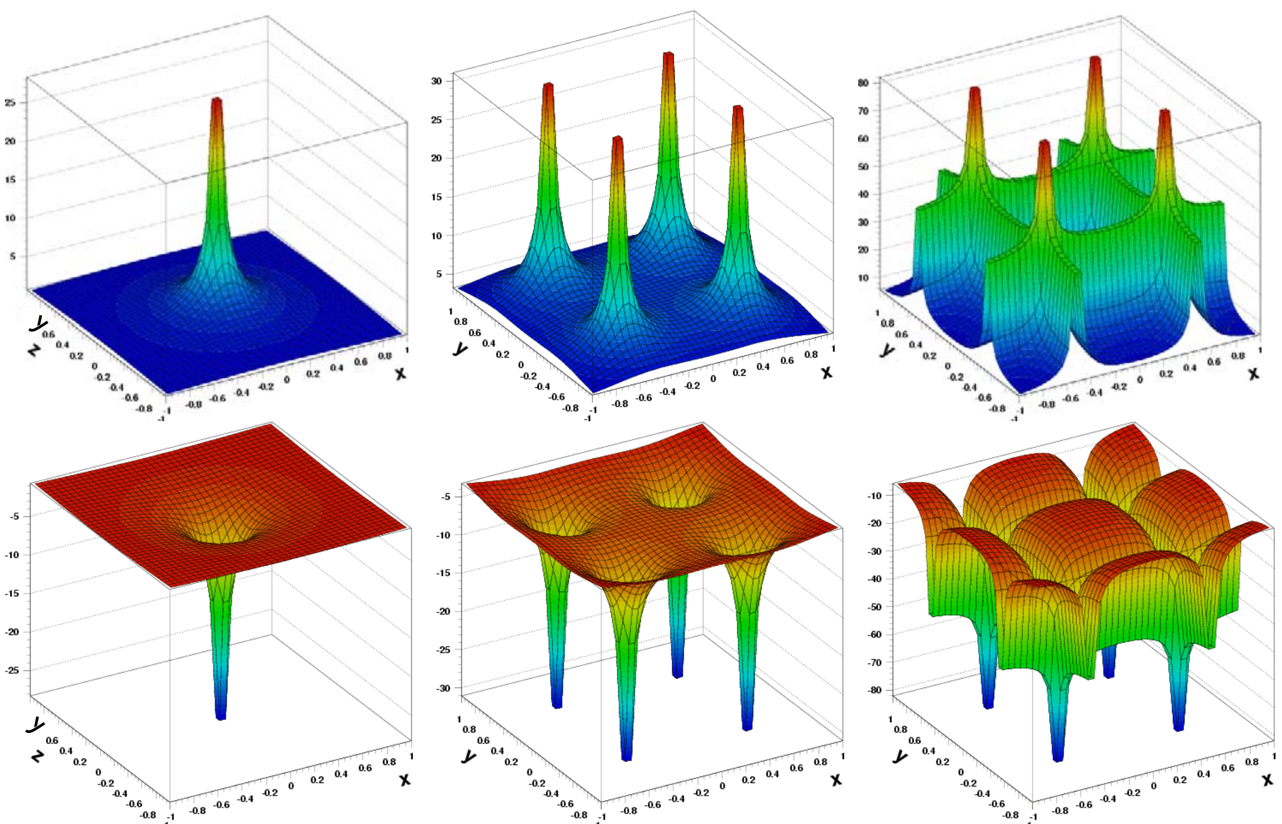
Four particles of equal mass M are fixed at the corners of a square with sides of length a. A fifth particle has mass m and moves under the gravitational forces of the other four.

Consider the gravitational potential energy U of the mass m as a function of its position (x,y). This *potential energy map* $U(x,y)$ is an extremely useful way of representing the effect of the gravitational force on the mass m:

- If m is held at rest at some point (x_1,y_1) and you let it go, it will always move toward a point of *lower* potential energy. You can therefore think of the function $U(x,y)$ as a topographical map: the particle m will always **"roll downhill"**.

- If the mass m moves from point (x_1,y_1) to point (x_2,y_2), the **potential energy difference** $U(x_1,y_1) – U(x_2,y_2)$ tells you exactly how much work was done by gravity, and exactly how much kinetic energy the particle gained as a result.

Let's visualize all this. The figures below show the potential energy map $U(x,y)$ for the square configuration of four masses M illustrated above. The height of the surfaces indicates the magnitude of U at each point. ***Which one is correct?***

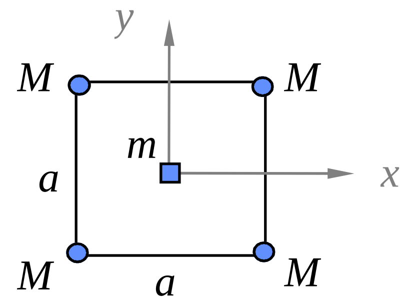


Now let's perform some calculations using potential energy. Here's the master formula for the gravitational potential energy between two masses m_1 and m_2 separated by a distance r_{12}:

$$U_{12} = -G\frac{m_1 m_2}{r_{12}}$$

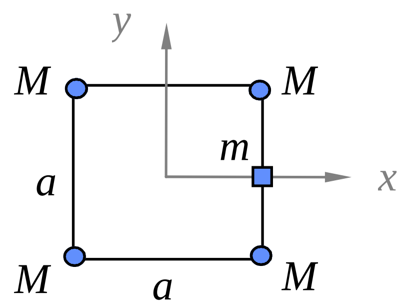
(a) Show that the net **gravitational potential energy** of particle m when at the origin is $U(0,0) = -4\sqrt{2}GMm / a$.

Hint: The equation above gives the gravitational potential energy of mass m_1 in the presence of mass m_2, or vice versa. Use it and superposition to find the net potential energy of mass m in the presence of the four fixed masses M. As long as the masses M remained fixed, it is not necessary to consider their mutual potential energies.



(b) If the particle m is released at rest from infinity and passes through the origin, calculate the **magnitude of its velocity** there.

(c) Calculate the **net potential energy** of *m* when it is at the mid-point of the right-hand side of the square.



(d) If the particle *m* is released at rest from the mid-point of the right-hand side of the square, does it **reach the origin**? Support your answers with physical arguments.

(e) In Discussion Question B, you calculated the *force* on the mass *m* when it is at the mid-point of the right-hand side of the square. If all went well, you found that this force points in the $-x$ direction, i.e. towards the origin. Given this fact, plus your answer to part (d), can you make a rough sketch of the potential energy function $U(x)$ for points along the x-axis? Give it a try!