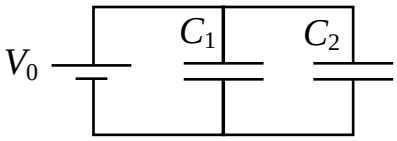
Discussion Question 5A

P212, Week 5

Analysis of Capacitor Networks

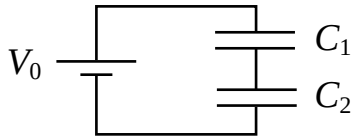
In this discussion question, we will go step-by-step through a classic problem of the type that you will encounter frequently in circuit analysis. But before we attack the full problem, we need to understand the basic concepts of how to analyze capacitors in **series** and in **parallel**.

(a) The diagram at right shows a battery of voltage V_0 hooked up to two capacitors C_1 and C_2 connected in **parallel**. Your task is to find the voltage (ΔV_1 and ΔV_2) across each capacitor and the charge (Q_1 and Q_2) on each capacitor. Then you know everything about the circuit! The way to solve this problem is the way to solve all circuit-analysis problems:



1. Combine the two capacitors into one *equivalent* capacitor C_{12}.
2. Find the voltage ΔV_{12} and charge Q_{12} on this equivalent capacitor.
3. Finally, break up C_{12} again, and solve the problem by realizing that the ***voltage*** across any elements in ***parallel*** is always the ***same***. (Can you explain why?)

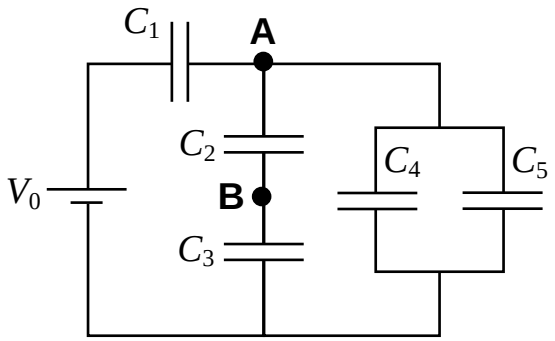
(b) The diagram at right shows a battery of voltage V_0 hooked up to two capacitors C_1 and C_2 connected in **series**. Your task is again to find the voltage (ΔV_1 and ΔV_2) across each capacitor and the charge (Q_1 and Q_2) on each capacitor. Here's how:



1. Combine the two capacitors into one *equivalent* capacitor C_{12}.
2. Find the voltage ΔV_{12} and charge Q_{12} on this equivalent capacitor.
3. Finally, break up C_{12} again, and solve the problem by realizing that the ***charge*** on the plates of any capacitors in ***series*** is always the ***same***. (Can you explain why?)

Now on to a more complex problem! A battery of voltage V_0 is hooked up to a network of 5 capacitors, connected together as shown. You are asked three questions:

1. What is the voltage V_A at the point A?
2. What is the voltage V_B at the point B?
3. What is the charge Q_5 on capacitor C_5?



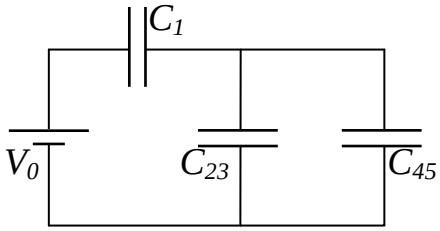
$V_0 = 12$ V
$C_1 = 5$ μF
$C_2 = 8$ μF
$C_3 = 15$ μF
$C_4 = 3$ μF
$C_5 = 10$ μF

4. What is the energy U_1 stored on capacitor C_1 ?

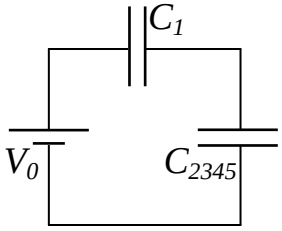
For questions 1 and 2, you are asked for the voltage at particular points in the circuit. But "voltage relative to *what*", you ask? Good question! It is traditional to consider the ***negative terminal of the battery*** as the zero of potential (i.e. as the reference point, or "ground"), and we will do so here.

Although there are more steps involved, the circuit-analysis procedure here is the same as before: first ***collapse*** the capacitors down to one *equivalent* capacitance … then determine the voltage and charge on that "capacitor" … and finally ***break it up*** step-by-step, determining the voltage and charge on each component as you go. Let's go through it in detail:

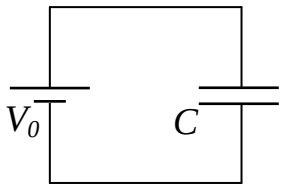
(a) ***Collapse*** the circuit down to three equivalent capacitors C_1, C_{23}, and C_{45}.
(Be sure to ***draw*** your new equivalent circuit at each step → you'll come back to it later!)



(b) Collapse the circuit further down to two equivalent capacitors C_1 and C_{2345}.



(c) Finally, collapse the circuit down to *one* equivalent capacitor C.



(d) This you can solve! Determine the charge Q and voltage ΔV on this one capacitor.

(e) Now we're ready for the second phase: ***breaking up*** our equivalent circuits, and solving for charges and voltages as we go. Using your answers to part (d), break up your 1-capacitor circuit into the 2-capacictor version with C_1 and C_{2345}, and find the voltages ΔV_1 and ΔV_{2345} and the charges Q_1 and Q_{2345}.

$\Rightarrow$ Once you're done, you should have the answer to question 1: the voltage V_A

(f) Break up the circuit further into the 3-capacitor version with C_1, C_{23}, and C_{45}. Find the voltages ΔV_{23} and ΔV_{45} and the charges Q_{23} and Q_{45}.

(g) Finally, break up the circuit into its original 5-capacitor form. All you need now are the answers to the last two questions: the voltage V_B and the charge Q_5.

4. Finally also find the energy U_1 stored on capacitor C_1 :