Discussion Question 6A

P212, Week 6
Two Methods for Circuit Analysis

Method 1: Progressive "collapsing" of circuit elements

In last week's discussion, we learned how to analyse circuits involving batteries and **capacitors**. Our method was to progressively *collapse* groups of capacitors (connected in series or in parallel) into *effective* capacitors. Once the circuit became simple enough, we could calculate everything about it: charge and voltage. Then we worked backwards, breaking up each combination and calculating the charge and voltage on the individual capacitors.

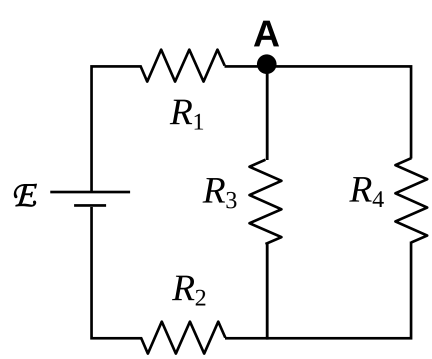
This week, we will analyze circuits involving **resistors**. As you know, the formulas for combining resitors in series and in parallel are "opposite" to those for capacitors. We also need an expanded set of rules for "breaking up" combinations of devices:

- For devices connected in ***parallel***: the <u>*voltage*</u> across them is always the <u>*same*</u>.
- For devices connected in ***series***: the <u>*charge*</u> and <u>*current*</u> is always the <u>*same*</u>.

But otherwise, the procedure is the same!

(a) Can you explain the rules stated above in terms of physical principles? They have simple physical origins, so they're easy to remember.

(b) Consider the circuit shown at right. All the resistors R_i have the same value R. Find the **current I_4** through resistor R_4 (including its direction) and the electric **potential V_A** at the indicated point **A**. Be sure to express your answers in terms of the given parameters $\mathcal{E}$ (battery "EMF" = voltage) and R (resistance of each resistor).



Method 2: Kirchhoff's Laws

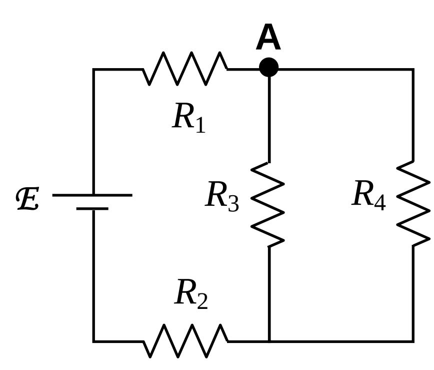
This week, we will also practice a *new* and more powerful method for circuit analysis. The new method is based on Kirchoff's two Laws:

- **Kirchhoff's Current Law (KCL)**: The sum of the currents flowing into any ***node*** (junction) in the circuit equals the sum of the currents flowing out of that node.
- **Kirchhoff's Voltage Law (KVL)**: The sum of the potential changes ΔV around any ***loop*** is zero. And here are the rules for how you sum up those ΔV's as you go around a loop in the circuit:
  - ➢ Crossing a battery from the negative to the positive terminal = a rise → add $+\mathcal{E}$
  - ➢ Crossing a battery from the positive to the negative terminal = a drop → add $-\mathcal{E}$
  - ➢ Crossing a resistor in the same direction as the current flow = a drop → add $-IR$
  - ➢ Crossing a resistor against the direction of the current flow = a rise → add $+IR$

(c) Can you explain these rules on physical principles? (They're relatively obvious, actually …)

Now, let's apply these laws to analyze the same circuit as on the previous page. Once more, your job is to find the **current I_4** and the **potential V_A**. Let's go through the procedure in detail:

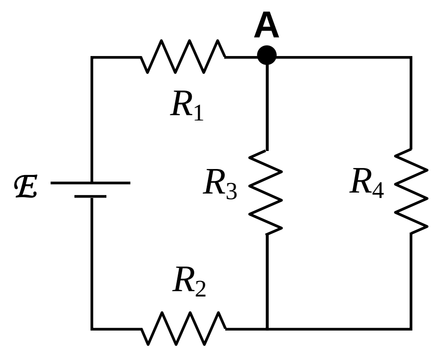
(d) Identify how many *different currents* there are in the circuit. These are often called ***branch currents***. Label them on the figure, from I_1 to I_n. Be sure to label them with *arrows*, indicating the *direction* of each current! If the directions you chose are wrong, no problem, your currents will simply come out with minus signs.



(e) Next, apply *KCL* to every *node* in the circuit. How many nodes are there in this circuit? Write down a KCL equation for each one.

(f) Examine the equations you just wrote down … how many of them are *independent*? Here's the rule: if you have N_{nodes} nodes in a circuit, KCL will only give you **N_{nodes}-1 independent equations**. In other words, one of the equations gives you *no new information*. Use all but one of your equations to 'get rid' of as many of the unknown currents as possible, by writing expressions for them in terms of the remaining currents.

(g) How many unknown currents are left? Let's call this number n. To determine these n currents, we will use KVL. Apply *KVL* to n *loops* in the circuit … this will give you the n equations you need to determine all the branch currents in the circuit.



(h) *Solve* your complete set of KCL and KVL equations! Remember, your job is to find I_4 and V_A.

For reference, here is a **<u>summary</u>** of the steps to take in analyzing a circuit using Kirchhoff's Laws:

1. Identify all the different *<u>branch currents</u>* in the circuit, and label them with *<u>arrows</u>* to indicate direction. Let N_I be this number of currents.
2. Apply *<u>KCL</u>* to *all but one* of the *<u>nodes</u>* in the circuit. This will give you (N_{nodes}-1) equations.
3. Apply *<u>KVL</u>* to N_I-(N_{nodes}-1) *<u>loops</u>* in the circuit. Together with your KCL equations, this gives you a total of N_I equations → enough to determine all the different currents.
4. *<u>Solve</u>* your complete set of N_I equations to determine all the branch currents. Knowing that, and good old "V = IR", you can calculate *everything* about your resistor network!