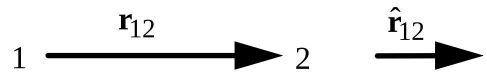
Discussion Question 1B

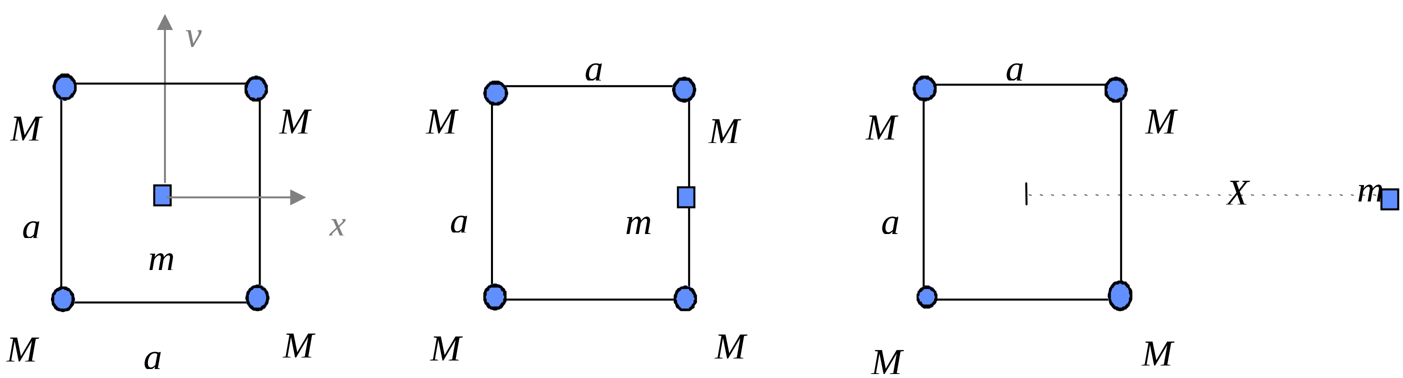
P212, Week 1

P211 Review: Gravitational Forces and Superposition

$$\mathbf{F}_{1\to 2} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12} ,$$



This problem develops skills you will need in P212 in finding the electric fields created by sets of point charges.



Four particles of equal mass M are fixed at the corners of a square with sides of length a. A fifth particle has mass m and moves under the gravitational forces of the other four.

(a) Find the ***x*- and *y*-components** of the net gravitational force on m due to the other four masses when m is located at the center of the square (left-hand figure).

Hint: <u>**Draw a sketch!**</u> Use superposition and draw a *vector diagram* consisting of four vectors, each representing the force exerted by one of the corner particles on m. For ease of reference, label the four (equal) corner masses M_1, M_2, M_3, and M_4. Label the corresponding force vectors $\mathbf{F}_1, \mathbf{F}_2$, etc. With the vector diagram in hand, it is vastly easier to calculate the requested components of the total force.

(b) Find the ***x*- and *y*-components** of the net gravitational force on m when it is located at the center point of the right-hand side of the square (middle figure). Use the same solution procedure that was recommended above for part (a).

(c) Check your answer to part (b) by testing at least 3 examples of **limiting behavior**. Do you get the results you expect?

Now, an important point: suppose you were given numerical values in this problem:

M = 3 kg, m = 1 kg, and a = 5 cm. If you had plugged those numbers into your equations right from the start, you'd get the final result $F_x = 1.15 \times 10^{-7}$ N. Would you be able to check the limiting behavior of this answer? No! We've learned an important lesson:

Never plug in numbers until the end of your calculation.

(d) When mass m is located on the x-axis a distance X large compared to a

(right-hand figure), one can use a *simple physical argument* to see that the net force on m due to the other four particles is approximately $F_x = CGMmX^{-2}$ and $F_y = 0$ where C is a numerical constant. (This is a long-distance approximation: it gets better and better as X increases.) What is the applicable **physical argument**? Use it to find the value of the dimensionless **constant** C.