Discussion Question 13C
P212, Week 13
Quarter Wave Plates

A **quarter wave plate** (**QWP**) is an example of a **birefringent** device: its refractive index n depends on the polarization of incoming light. If light is polarized along the *fast* axis, it will experience a smaller refractive index … but if it is polarized along the *slow* axis, the refractive index it sees will be *larger*. Recall that the index of refraction describes the degree to which light slows down in a material: $v = c / n$. So here is how a birefringent element works: if linearly polarized light passes through the element, the *component* of the light polarized along the slow axis will be slowed down *more* that the component along the fast axis. Thus, when the light emerges from the element, its two components will be out of phase with each other. A QWP is a special case of such a device, where the phase shift between the fast and slow components is exactly 90°. Further, if the incoming light is polarized at 45° to both the fast and slow axes (so that the fast and slow components are of equal amplitude), the outgoing light will be circularly polarized.

Consider a single QWP that lies in the xy plane. It is oriented so that its fast axis lies along the y direction and its slow axis lies along the x axis. We will send light of various types at this QWP, but the light will always be traveling in the +z direction.

(a) Find the polarization state of the light transmitted by the QWP in each of the following cases. If your answer is "linearly polarized", be sure to specify the direction of linear polarization.

1. The incident light is linearly polarized in the y direction.
2. The incident light is linearly polarized in the direction $(\hat{x} - \hat{y})/\sqrt{2}$
3. The incident light is unpolarized.
4. The incident light is circularly polarized.

(b) Given your answer to part 4 above, what sort of device would you have if you glued two identical QWP's together? And what if you glued *four* of them together?

(c) Suppose the incident light has intensity I_0 = 5 W/m^2. Find the intensity I of the transmitted light in each of the following cases:

1. The incident light is linearly polarized in the y direction.
2. The incident light is linearly polarized in the direction $(\hat{x}-\hat{y})/\sqrt{2}$
3. The incident light is unpolarized.
4. The incident light is circularly polarized.

Notice a trend? ☺

(d) Now for a challenge: let's design a QWP! You will build your QWP from crystalline quartz (SiO_2), a common birefringent material. Your task is to deterime how thick your plate of quartz needs to be to make a QWP. As you will quickly discover, you need one more parameter: the frequency of the incident light → QWP's *only* function properly at certain frequencies. So let's use the blue-green line at λ = 488 nm produced by an Argon-Ion laser. At this wavelength, the fast and slow indices of refraction of crystalline quartz are n_f = 1.54955 and n_s = 1.55885 respectively. How thick should your QWP be?

Note: the given λ = 488 nm is the laser's wavelength in vacuum, *not* in the material.

…additional hint: set $\Delta\phi = \phi_f - \phi_s = \pi/2$, where ϕ is the total angle the waves travels through in the material

(e) I suspect that you came up with a very *small* thickness. In practice, optical elements cannot be machined too thin, or they will be mechanically unstable (i.e. they break too easily to be practical!). A conservative minimum thickness is 1 mm. Is there some other thickness that you could use for your QWP that is ≥ 1 mm but still works at λ = 488 nm?

In fact, there is a whole *series* of thicknesses that you can use in your design …

hint: change $\pi/2$ to $\pi/2 + 2\pi N$