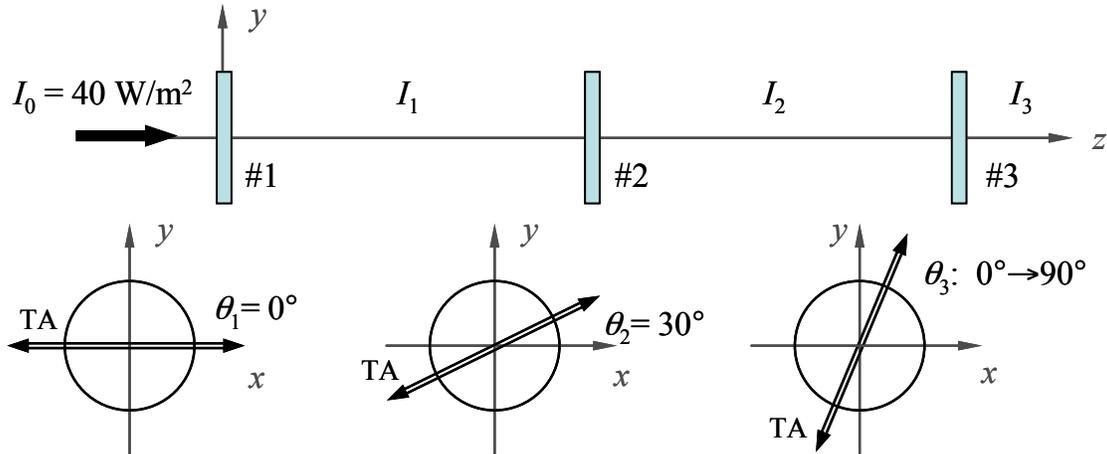


A beam of light of intensity $I_0 = 40 \text{ W/m}^2$ is incident in the $+z$ direction upon a set of three linear polarizers, each oriented perpendicular to the beam. The transmission axes (TA) of the first two polarizers make angles of $\theta_1 = 0^\circ$ and $\theta_2 = 30^\circ$ with respect to the x axis (see figure). The angle θ_3 of polarizer #3 can be varied from 0° to 90° .

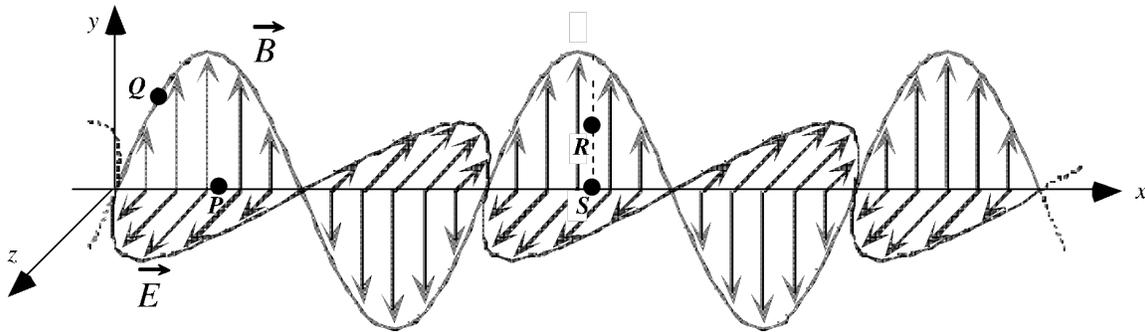


1. Calculate the intensity I_1 of the light after the first polarizer if the incident beam I_0 is linearly polarized at an angle $\theta = 60^\circ$ to the x -axis. [5]
2. The incident beam is now linearly polarized along the x -axis. Consider polarizer #3: at what angle θ_3 between 0° and 90° will the final intensity I_3 will be maximized? Determine both the angle and the maximum intensity. [5]

3. The incident beam is again linearly polarized along the x -axis. At what angle θ_3 between 0° and 90° will the final intensity I_3 be minimized? Determine both the angle and the minimum intensity. [5]

Shown below are mathematical and pictorial representations of an electromagnetic plane wave propagating through empty space. The electric field is parallel to the z -axis.

$$\vec{E}(x, y, z, t) = E_o \sin(kx + \omega t) \hat{z}$$



4. If $E_o = 150$ V/m, what is the average intensity of the plane wave? [5]

Q10B

1. $I_1 = I_0 \cos^2(60^\circ) = 10 \text{ W/m}^2$ (5)

Correct formula but numerical error (4)

Use $\sin\theta$ or $\cos\theta$ instead of $\cos^2\theta$ (2)

2. $\theta_3 = \theta_2 = 30^\circ$ $I_3 = I_0 \cos^2(30^\circ) = 30 \text{ W/m}^2$ (5)

Recognize $\theta_3 = \theta_2$ (3)

Correct intensity formula (2)

Use of θ_1 (-2)

Numerical error (-1)

3. True maximum would be obtained with $\theta_3 = 120^\circ$ but answer is restricted to $0^\circ \leq \theta_3 \leq 90^\circ$ so the best one can do is $\theta_3 = 90^\circ$.

$I_3 = I_0 \cos^2(\theta_2 = 30^\circ) \cos^2(\theta_3 - \theta_2 = 60^\circ) = (3/16)I_0 = 7.5 \text{ W/m}^2$ (5)

Numerical error (-1)

Correct attenuation by second polarizer (2)

Observe that true maximum would require $\theta_3 = \theta_2 = 120^\circ$ (2)

Everything else correct but incorrect θ_3 (3)

4. $I = \frac{c\epsilon_0 E_0^2}{2} = \frac{E_0^2}{2(Z_0 = 377\Omega)} = 29.8 \text{ W/m}^2$ (5)

Omit the $\frac{1}{2}$ (-1)

Numerical error (-1)

Quadratic in E_0 but some other error in the formula (2)