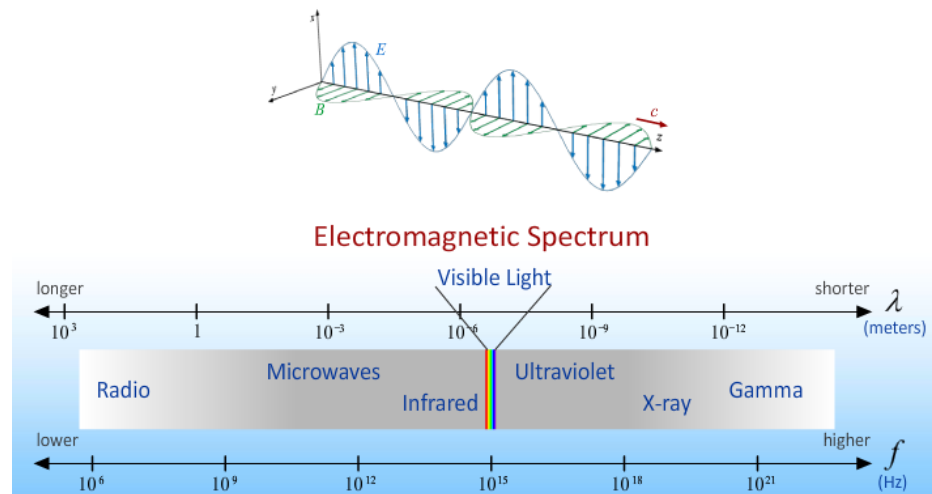


Tim Stelzer's office hour has been moved from Today at 4:00 to Thursday at 4:00.

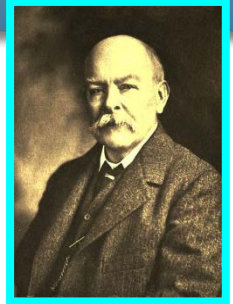
Physics 212

Lecture 23

PROPERTIES of ELECTROMAGNETIC WAVES



Your comments



Electromagnetic waves is honestly the coolest thing I have ever seen/studied before. Who studies the topic? Physicists? Engineers? I kind of want to switch majors to whatever does this.

What is this poynting vector, and why is it spelled wrong?

It is pretty hard to distinguish if the equation should be $(kz - \omega t)$ or $(kz + \omega t)$. Is there a good way to do this by looking at a given electromagnetic wave?

Is it possible for the plane electromagnetic field to be switched so E is in the y axis and B is in the x axis? Also is it possible for the E_0 and B_0 to equal their maximums at $\text{time}=0$, making them cosine functions instead of sin functions?

The frequency of bad physics jokes,,, hertz

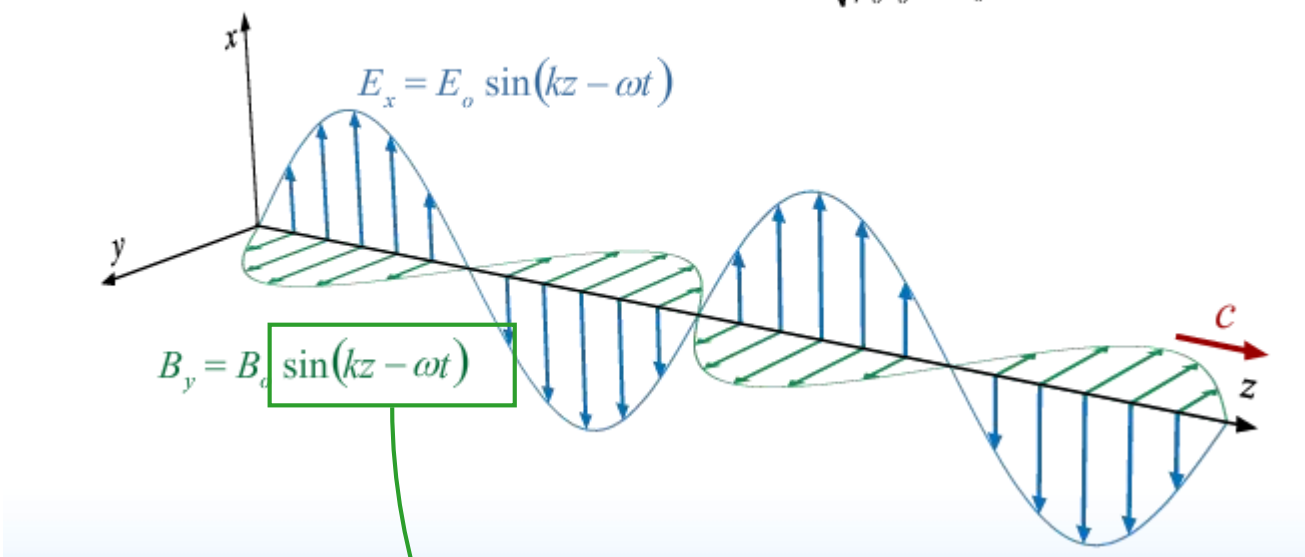
Where does the doppler shift equation come from? Why is time different for the source and the observer exactly? And how big is this difference?

If you shined two very slightly different frequencies of light toward an observer, how would the resulting "beat/pulsating" interference pattern be interpreted?

<https://www.youtube.com/watch?v=CuGEPkI117s>

Plane Waves from Last Time

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_0}{B_0} = 3 \times 10^8 \text{ m/s}$



E and B are perpendicular and in phase

Oscillate in time and space

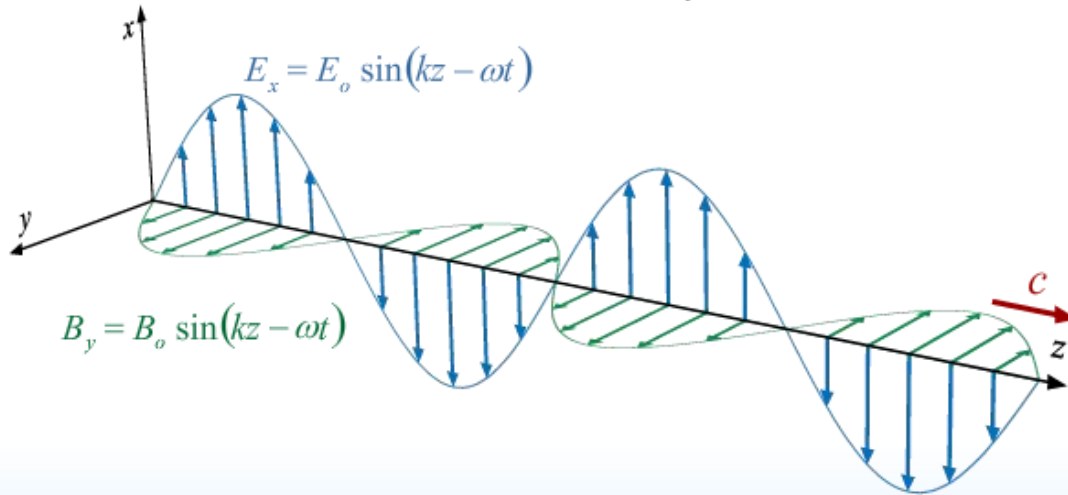
Direction of propagation given by $E \times B$

$$E_0 = cB_0$$

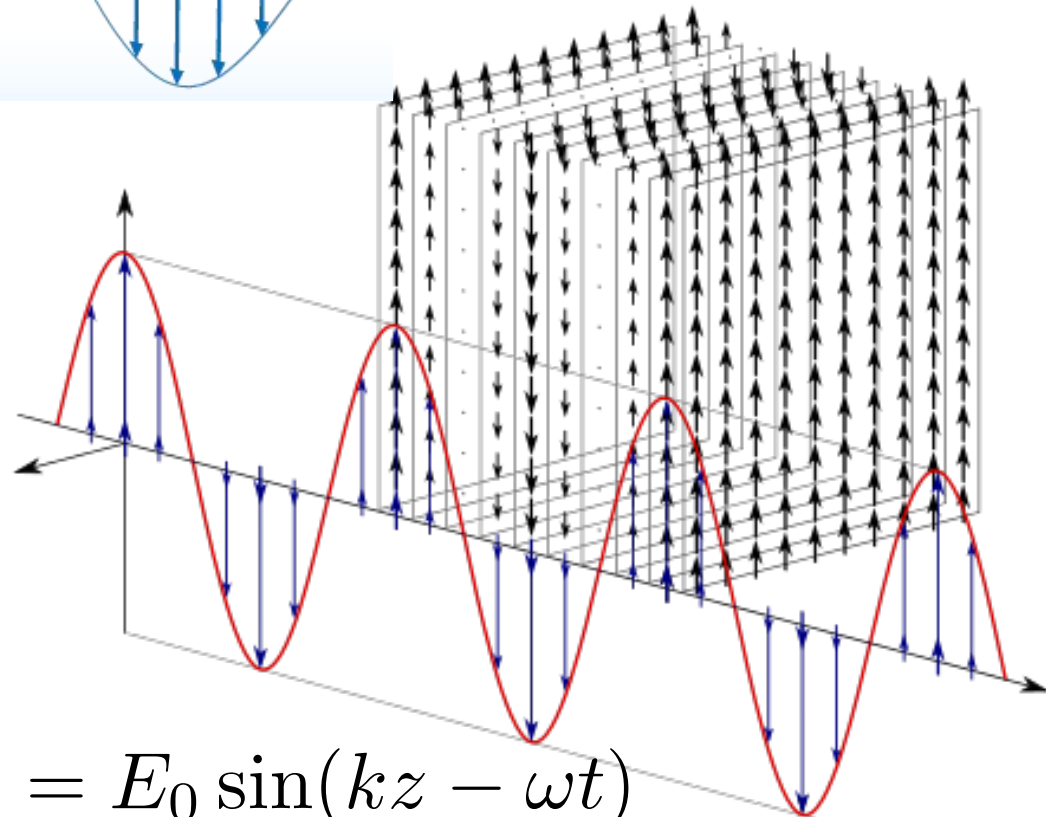
Argument of \sin/\cos gives direction of propagation

Plane Waves from Last Time

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \epsilon_o}} = \frac{E_o}{B_o} = 3 \times 10^8 \text{ m/s}$

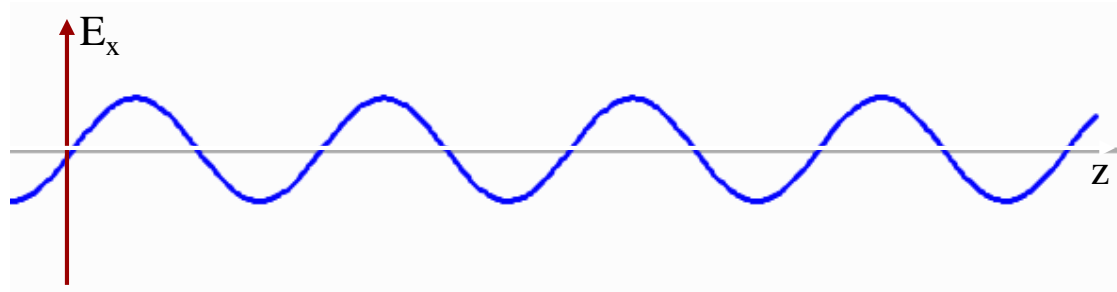


Note: E_x has no x or y dependence

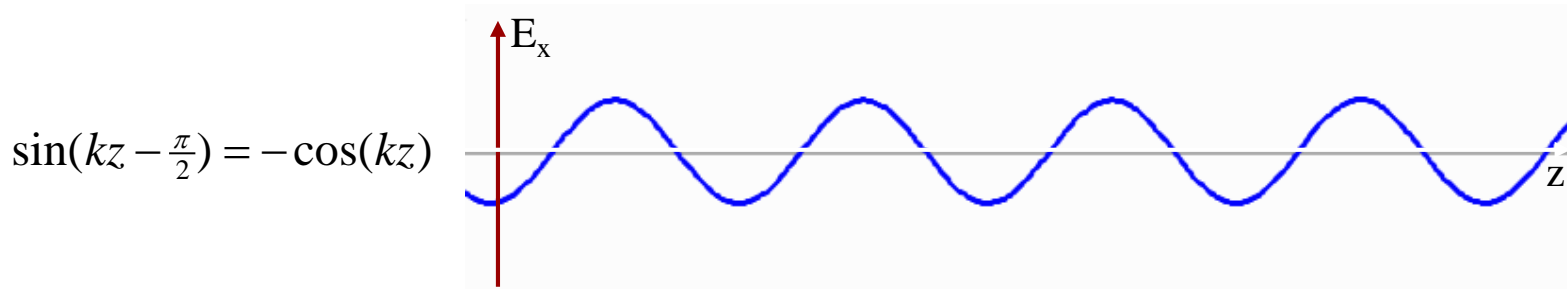


Understanding the speed and direction of the wave

$$E_x = E_0 \sin(kz - \omega t)$$



$t = 0$

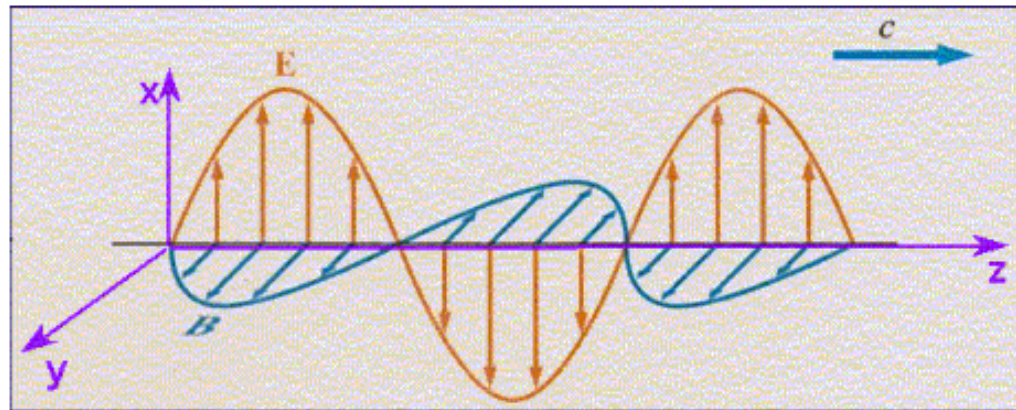


$t = \pi/(2\omega)$

What has happened to the wave form in this time interval?

It has MOVED TO THE RIGHT by $\lambda/4$

CheckPoint 1a



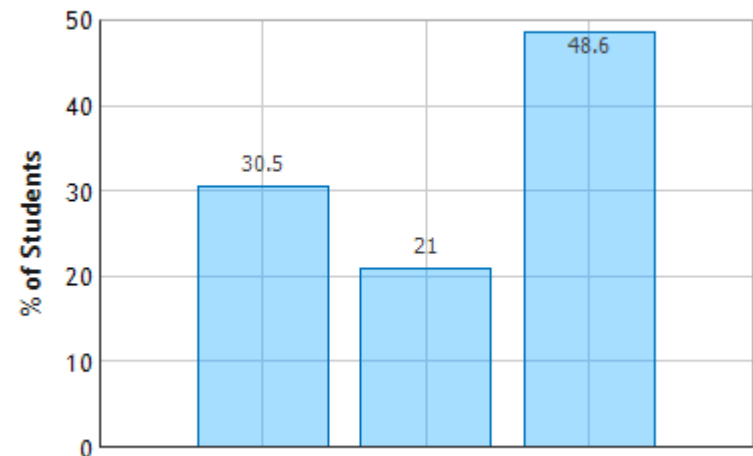
Which equation correctly describes this electromagnetic wave?

☐ $E_x = E_0 \sin(kz \oplus \omega t)$ No – moving in the minus z direction

☐ $E_y = E_0 \sin(kz - \omega t)$ No – has E_y rather than E_x

☒ $B_y = B_0 \sin(kz - \omega t)$

Electromagnetic Waves: Question 1 (N = 778)



CheckPoint 2a



Your iclicker operates at a frequency of approximately 900 MHz (900×10^6 Hz). What is the approximately wavelength of the EM wave produced by your iclicker?

- A. ☐ 0.03 meters
- B. ☒ 0.3 meters
- C. ☐ 3.0 meters
- D. ☐ 30. meters

$$C = 3.0 \times 10^8 \text{ m/s}$$

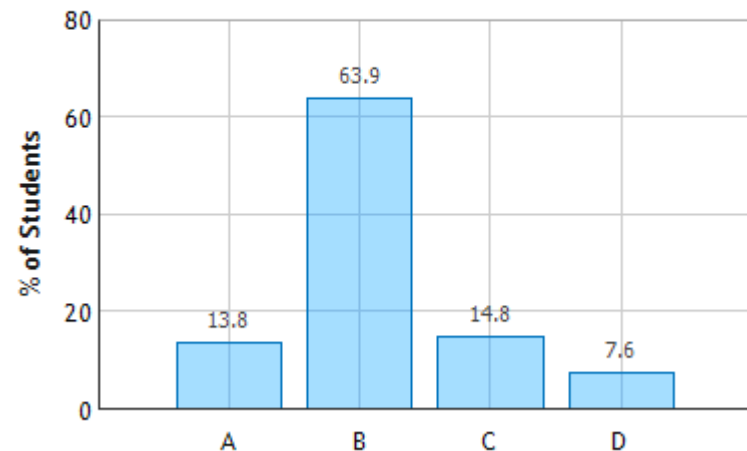
Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300,000,000}{900,000,000} = \frac{1}{3}$$

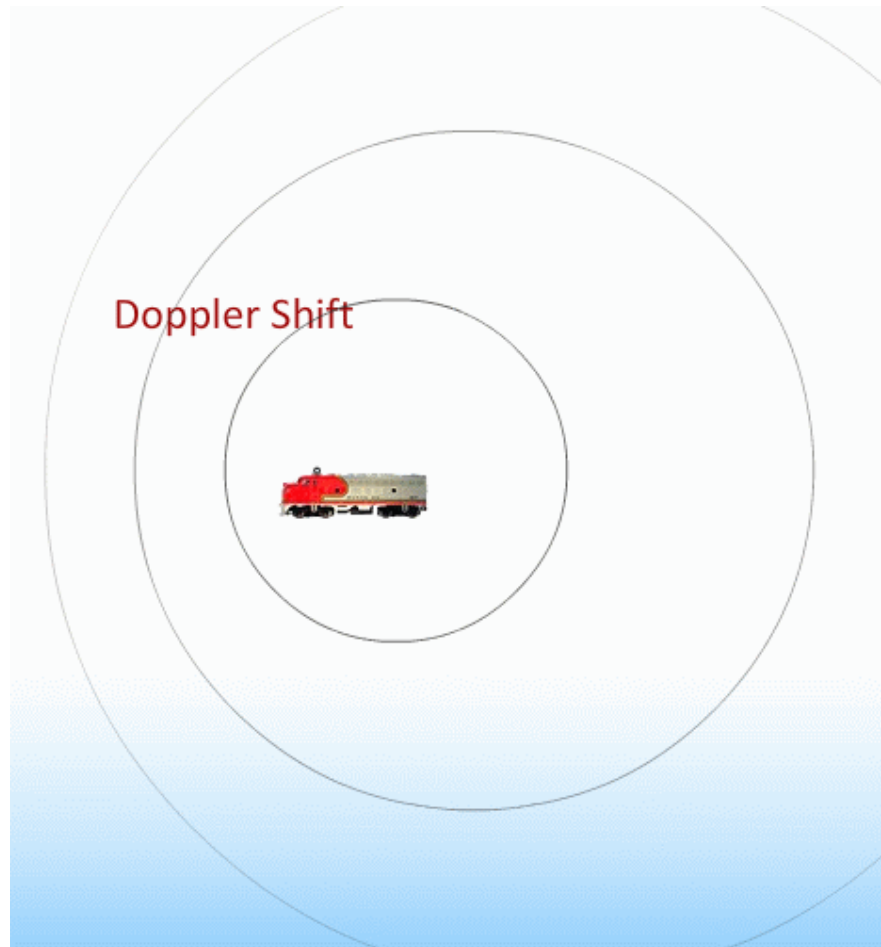
Check:

Look at size of antenna on base unit

EM waves from an iclicker: Question 1 (N = 778)



Doppler Shift



The Big Idea

As source approaches:
Wavelength decreases
Frequency Increases

Doppler Shift for E-M Waves

What's Different from Sound or Water Waves ?

Sound /Water Waves :

You can calculate (no relativity needed)

BUT

Result is somewhat complicated: is source or observer moving wrt medium?

Electromagnetic Waves :

You need relativity (time dilation) to calculate

BUT

Result is simple: only depends on relative motion of source & observer

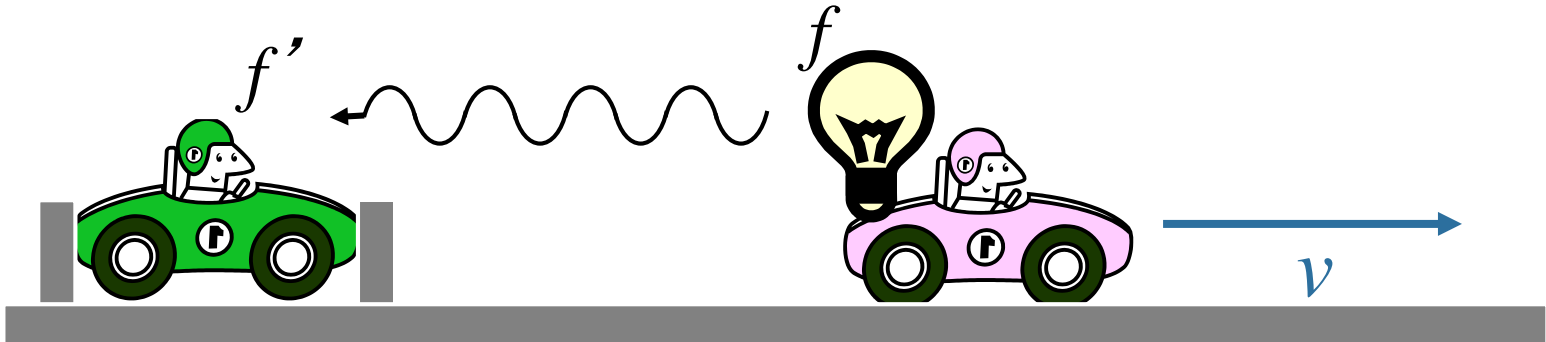
$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$$

$$\beta = v/c$$

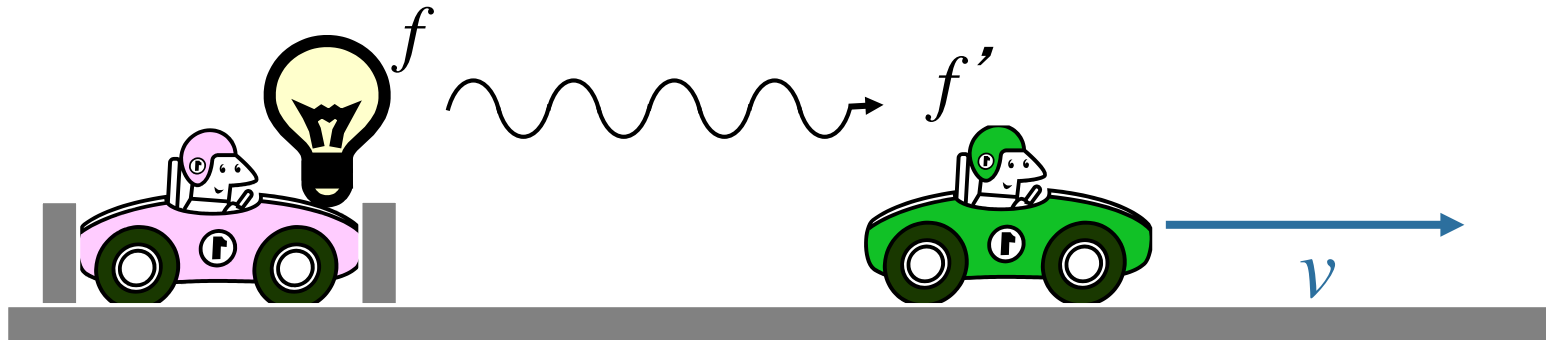
$\beta > 0$ if source & observer are approaching

$\beta < 0$ if source & observer are separating

Doppler Shift for E-M Waves



or



The Doppler Shift is the SAME for both cases!

f' / f only depends on the relative velocity

$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$$

Doppler Shift for E-M Waves

A Note on Approximations

$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}} \quad \xrightarrow{\beta \ll 1} \quad f' \approx f(1 + \beta)$$

why?

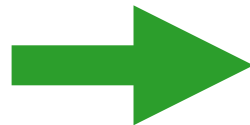
Taylor Series: Expand $F(\beta) = \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$ around $\beta = 0$

$$F(\beta) = F(0) + \frac{F'(0)}{1!} \beta + \frac{F''(0)}{2!} \beta^2 + \dots$$

Evaluate:

$$F(0) = 1$$

$$F'(0) = 1$$



$$F(\beta) \approx 1 + \beta$$

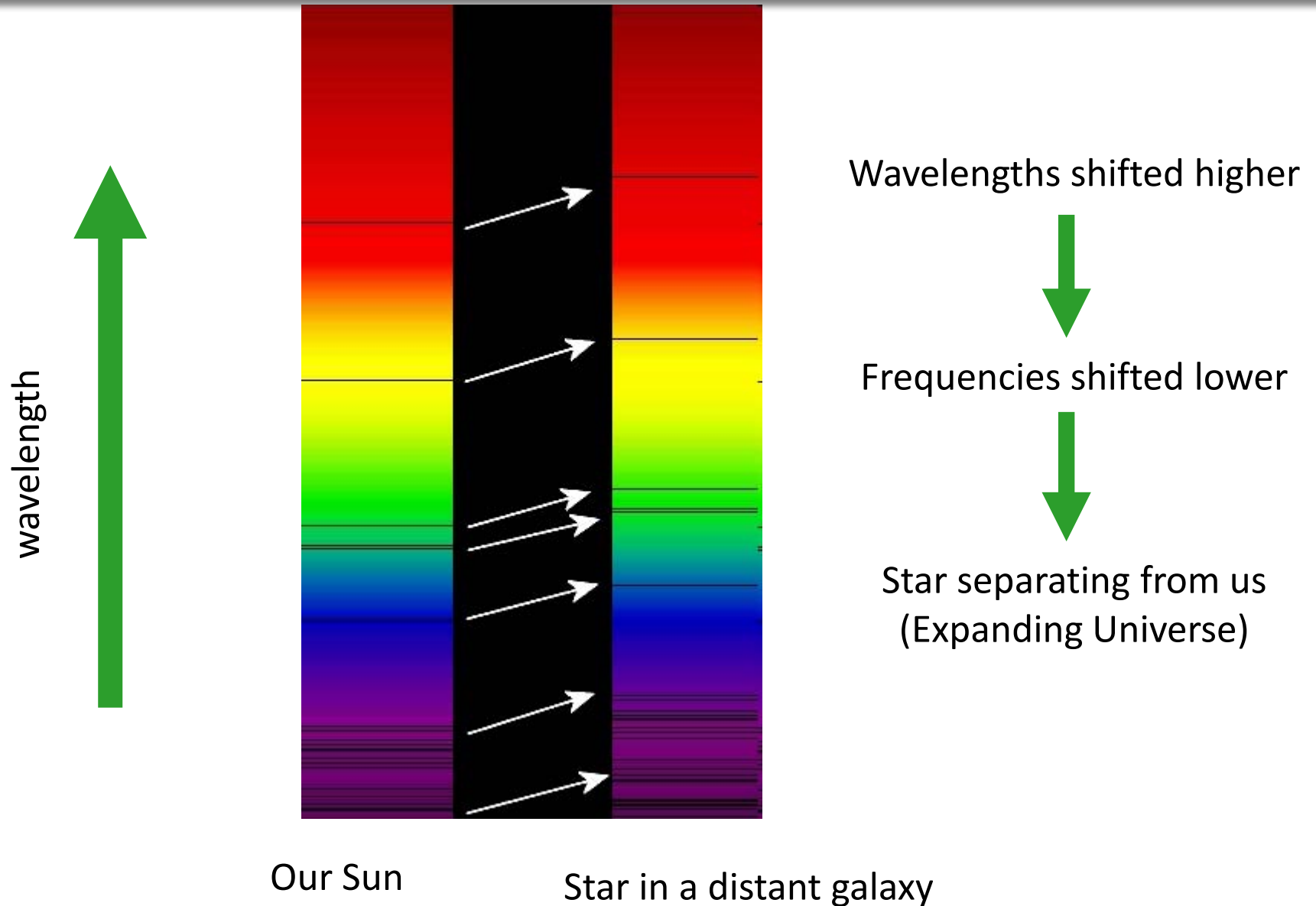
NOTE:

$$F(\beta) = (1 + \beta)^{1/2}$$

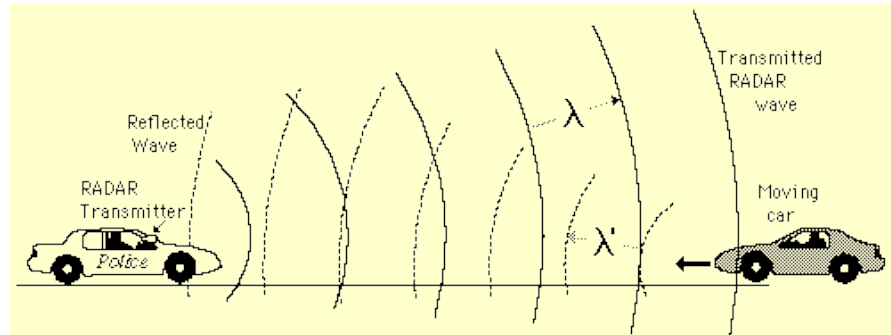


$$F(\beta) \approx 1 + \frac{1}{2} \beta$$

Red Shift



Example



Police radars get twice the effect since the EM waves make a round trip:

$$f' \approx f(1 + 2\beta)$$

If $f = 24,000,000,000 \text{ Hz}$ (k-band radar gun)

$$c = 300,000,000 \text{ m/s}$$

v	β	f'	$f' - f$
30 m/s (67 mph)	1.000×10^{-7}	24,000,004,800	4800 Hz
31 m/s (69 mph)	1.033×10^{-7}	24,000,004,959	4959 Hz

CheckPoint 2b



If you wanted to see the EM wave produced by the iclicker with your eyes, which of the following would work? (Note: Your eyes are sensitive to EM waves w/ frequency around 10^{14} Hz)

A) ☐ Run away from the iclicker when it is voting.

B) ☒ Run toward the iclicker when it is voting.

C) ☐ Neither will work, moving relative to the iclicker won't change the frequency reaching your eyes.

$$f_{\text{iclicker}} = 900 \text{ MHz}$$

Need to shift frequency UP



Need to approach iclicker ($\beta > 0$)

How fast would you need to run to see the iclicker radiation?

$$\frac{f'}{f} = \frac{10^{14}}{10^9} = 10^5 = \left(\frac{1+\beta}{1-\beta} \right)^{1/2}$$



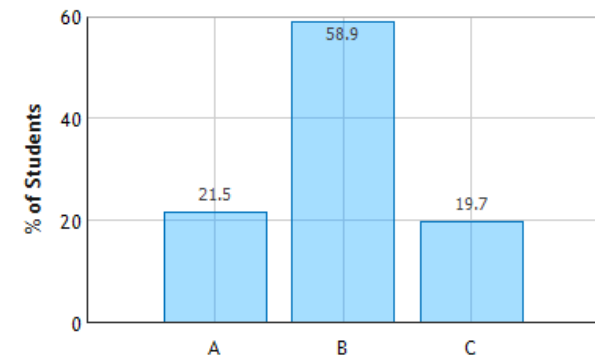
$$10^{10} = \left(\frac{1+\beta}{1-\beta} \right)$$



$$\beta = \frac{10^{10} - 1}{10^{10} + 1} = \frac{1 - 10^{-10}}{1 + 10^{-10}}$$

Approximation Exercise: $\beta \approx 1 - (2 \times 10^{-10})$

EM waves from an iclicker: Question 2 (N = 778)



Waves Carry Energy

Total Energy Density

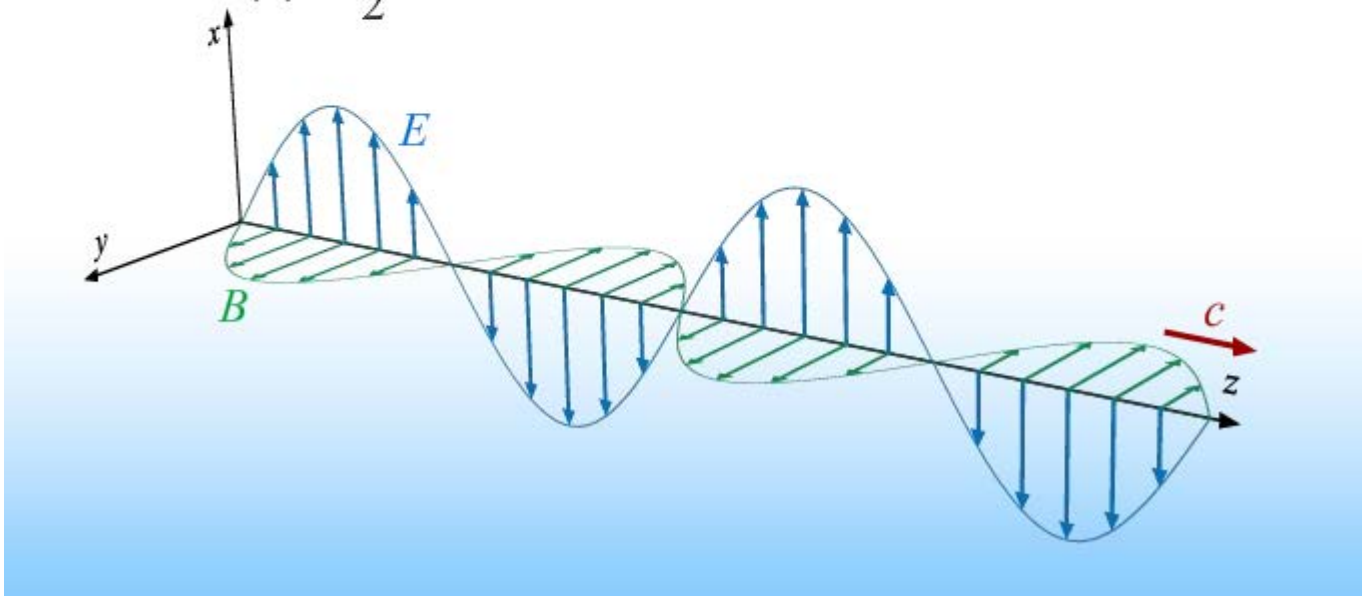
$$u = \epsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_o E_o^2$$

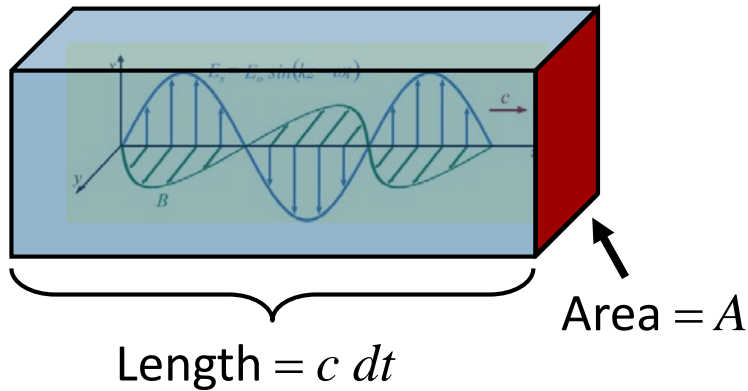
Intensity

$$I = \frac{1}{2} c \epsilon_o E_o^2 = c \langle u \rangle$$



Intensity

Intensity = Average energy delivered per unit time, per unit area



$$\rightarrow I \equiv \frac{1}{A} \left\langle \frac{dU}{dt} \right\rangle$$

$$\rightarrow \langle dU \rangle = \langle u \rangle \cdot \text{volume} = \langle u \rangle A c dt$$

Total Energy Density

$$u = \epsilon_0 E^2$$

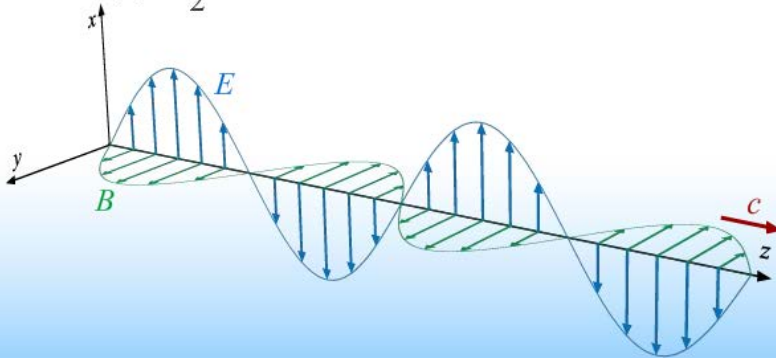
Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_o^2$$

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_o^2 = c \langle u \rangle$$

$$\rightarrow I = c \langle u \rangle$$



Sunlight on Earth:

$$I \sim 1000 \text{ J/s/m}^2$$

$$\sim 1 \text{ kW/m}^2$$

Waves Carry Energy

Total Energy Density

$$u = \epsilon_o E^2$$

Intensity

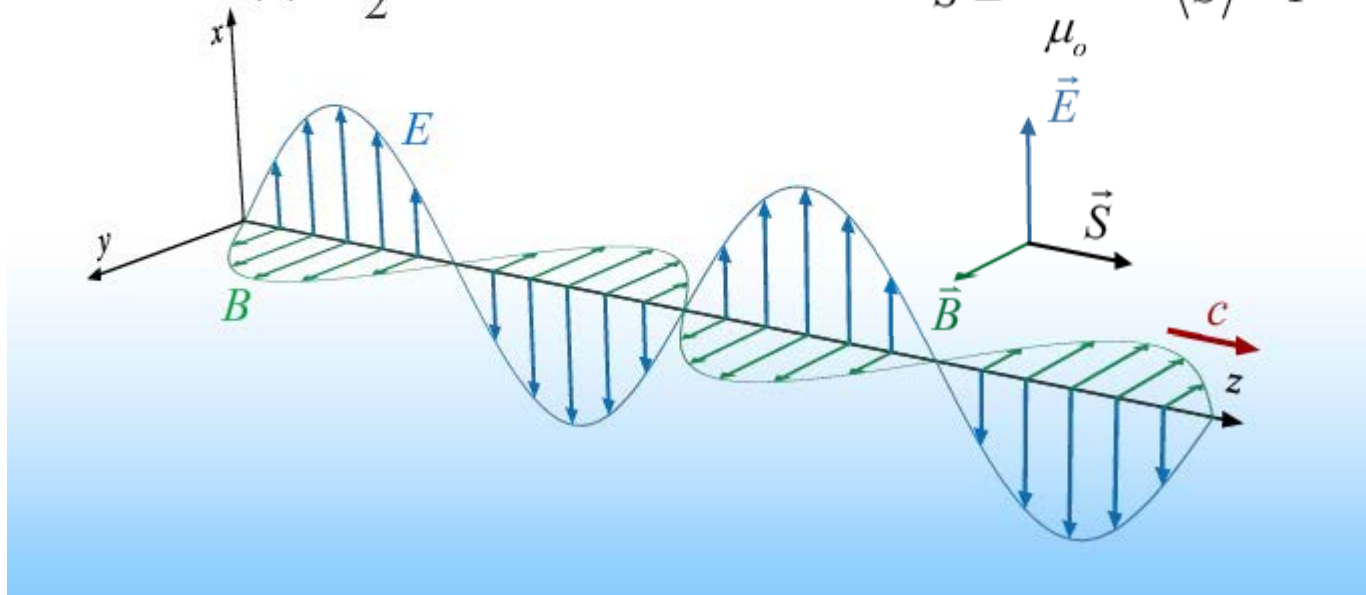
$$I = \frac{1}{2} c \epsilon_o E_o^2 = c \langle u \rangle$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_o E_o^2$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \quad \langle S \rangle = I$$

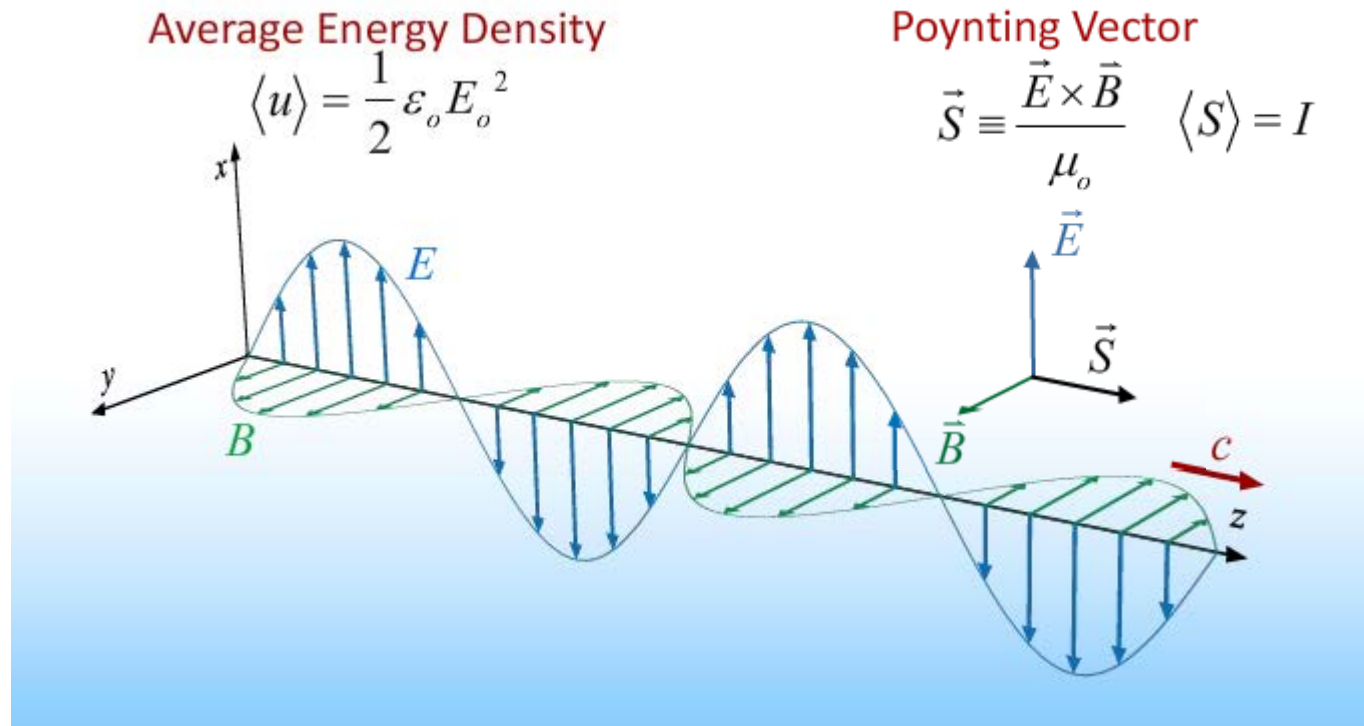


Comment on Poynting Vector

Just another way to keep track of all this:

Its magnitude is equal to I

Its direction is the direction of propagation of the wave



Power in EM Waves: Example

A cell phone tower has a transmitter with a power of 100 W. What is the magnitude of the peak electric field a distance 1500 m (~ 1 mile) from the tower? Assume the transmitter is a point source.

What is the intensity of the wave 1500 m from the tower?

A) 1.5 nW/m²

B) 3.5 μW/m²

C) 6 mW/m²

$$I = \frac{P}{4\pi r^2} = \frac{100 \text{ W}}{4\pi (1500 \text{ m})^2} = 3.5 \frac{\mu\text{W}}{\text{m}^2}$$

What is the average (rms) value of the electric field?

$$I = \left\langle \left| \vec{S} \right| \right\rangle = \left\langle \frac{|\vec{E} \times \vec{B}|}{\mu_0} \right\rangle = \left\langle \frac{E}{\mu_0} \frac{E}{c} \right\rangle = \frac{1}{\mu_0 c} \frac{E_0^2}{2} \Rightarrow E_0 = \sqrt{2\mu_0 c I}$$

$$E_0 = \left(2 \cdot 4\pi \times 10^{-7} \cdot 3 \times 10^8 \cdot 3.5 \times 10^{-6} \right)^{1/2} = 51 \frac{\text{mV}}{\text{m}}$$

Checkpoint 1 b



Which of the following actions will increase the energy carried by an electromagnetic wave?

A. Increase E keeping ω constant

C. Both of the above will increase the energy

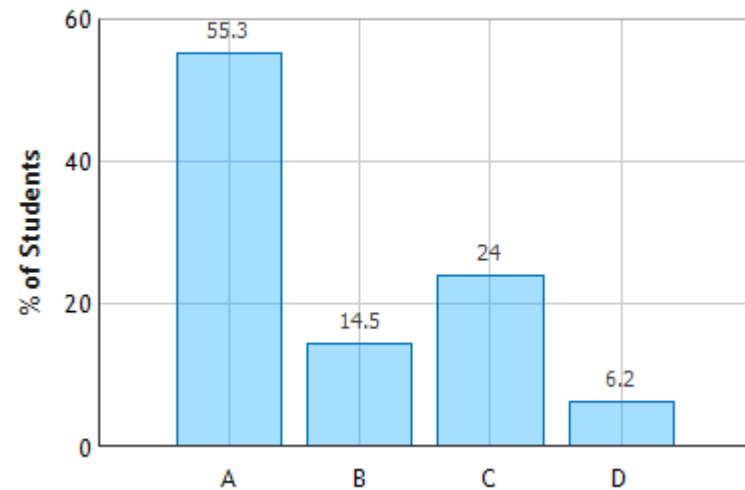
B. Increase ω keeping E constant

D. Neither of the above will increase the energy

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_o^2$$

Electromagnetic Waves: Question 3 (N = 779)



Photons

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength ($f\lambda = c$)

Connections seen in equations:

$$E = hf$$

$$p = h/\lambda$$

Planck's constant

$$h = 6.63e^{-34} \text{ J} \cdot \text{s}$$

Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

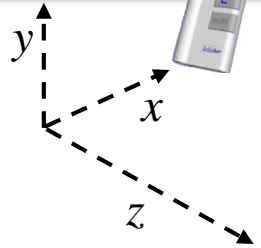
What we see depends on how we choose to measure it!

The mystery of quantum mechanics: More on this in PHYS 214

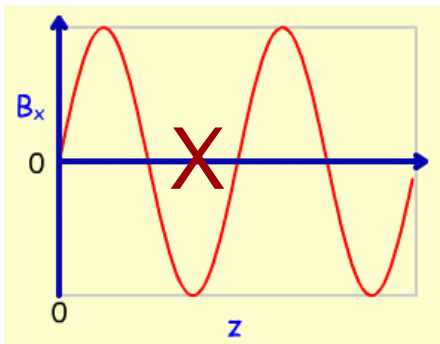
Exercise

An electromagnetic wave is described by:
where \hat{j} is the unit vector in the $+y$ direction.

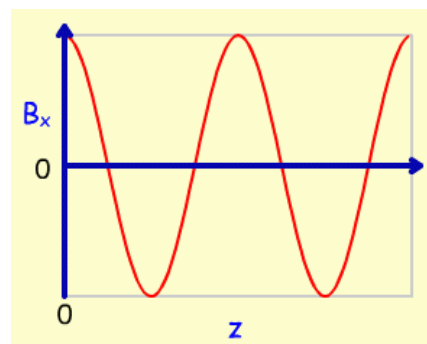
$$\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$$



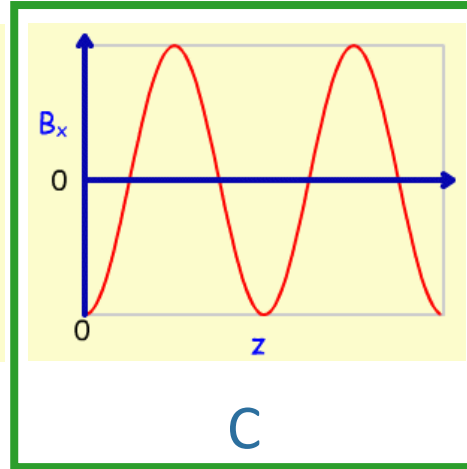
Which of the following graphs represents the z – dependence of B_x at $t = 0$?



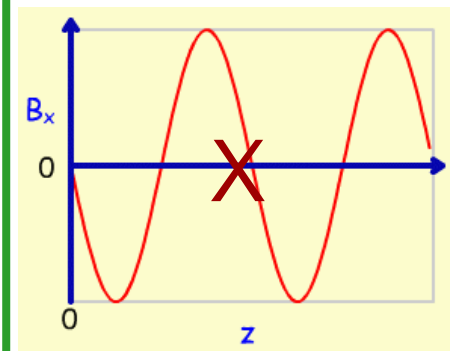
A



B



C

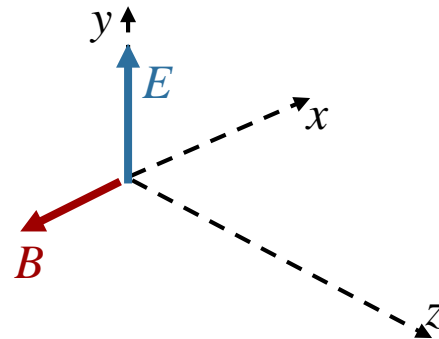


D

E and B are “in phase” (or 180° out of phase)

$\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$ Wave moves in $+z$ direction

$\vec{E} \times \vec{B}$ Points in direction of propagation



$$\vec{B} = -\hat{i}B_0 \cos(kz - \omega t)$$

Exercise

An electromagnetic wave is described by:

$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$

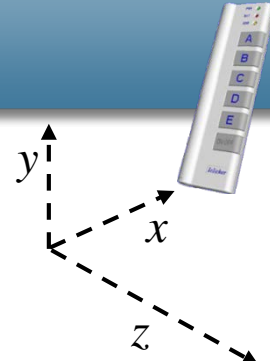
What is the form of B for this wave?

A) $\vec{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

B) $\vec{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

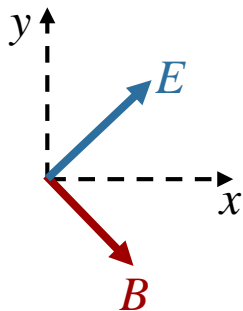
D) $\vec{B} = \frac{-\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$



$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$



Wave moves in $-z$ direction



$+z$ points out of screen

$-z$ points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation

Exercise



A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

Is it possible that the professor's argument is correct?

$$(\lambda_{\text{green}} = 500 \text{ nm}, \lambda_{\text{red}} = 600 \text{ nm})$$

A) YES

B) NO

As professor approaches stoplight, the frequency of its emitted light will be shifted UP

The speed of light does not change

Therefore, the wavelength (c/f) would be shifted DOWN

If he goes fast enough, he could observe a green light !

Follow-Up



A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

How fast would the professor have to go to see the light as green?

($\lambda_{\text{green}} = 500 \text{ nm}$, $\lambda_{\text{red}} = 600 \text{ nm}$)

- A) 540 m/s B) $5.4 \times 10^4 \text{ m/s}$ **C) $5.4 \times 10^7 \text{ m/s}$** D) $5.4 \times 10^8 \text{ m/s}$

Relativistic Doppler effect: $f' = f \sqrt{\frac{1+\beta}{1-\beta}}$

$$\frac{f'}{f} = \frac{600}{500} = \sqrt{\frac{1+\beta}{1-\beta}} \quad \longrightarrow \quad 36(1-\beta) = 25(1+\beta) \quad \longrightarrow \quad \beta = \frac{11}{61} = 0.18$$

Note approximation for small β is not bad: $f' = f(1+\beta) \quad \longrightarrow \quad \beta = \frac{1}{5} = 0.2$

$c = 3 \times 10^8 \text{ m/s} \rightarrow v = 5.4 \times 10^7 \text{ m/s} \quad \longrightarrow \quad \text{Change the charge to speeding!}$