

# LAB 6 now after spring break!

## *Physics 212* *Lecture 15*

Tim Stelzer's office hour 4:00 March 10<sup>th</sup> is moved  
to 4:00 March 12<sup>th</sup>

Today's Concept:

Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

# *Your Comments*

Hmm, not so bad. That's what I said about Gauss though too. Really, the concepts are not hard, I just needed more practice. Maybe seeing the concept applied again to this context will help me do better with both Gauss and Ampere.

THANK YOU FOR PUTTING "SORRY PROF, I DIDN'T THINK ABOUT THIS" AS AN OPTION.

Why does this class and 211 have office hours on only two days out of seven?

Ampere meets Coulumb at a cafe. Coulumb decides to call over his family. Ampere runs away, since he can only manage one Coulumb per second.

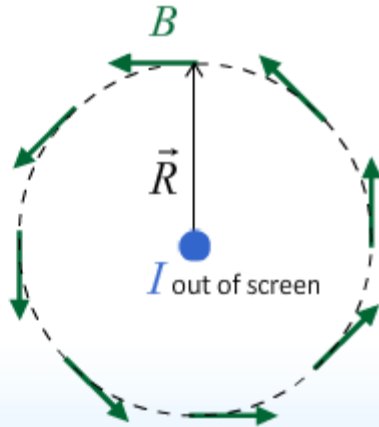
When the wire is wrapped around a tube, why is there  $B$  in the tube? Isn't it that the  $I_{\text{enc}}$  is 0?

All hail the mighty space lizard of physics!!!

Methinks that Mr. Ampere copied Mr. Gauss.... I don't see any works cited

## Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$



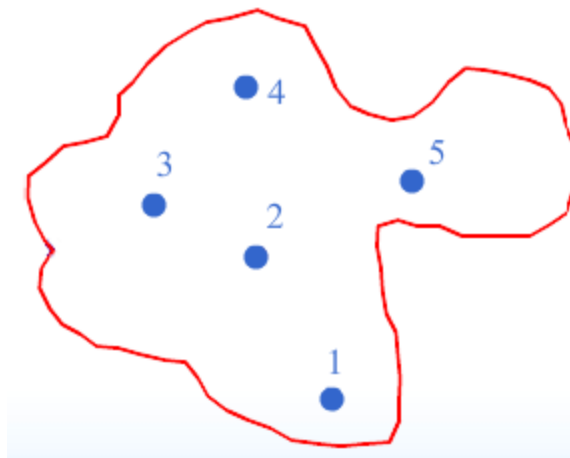
## Infinite current-carrying wire

$$\text{LHS: } \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B \cdot 2\pi R$$

$$\text{RHS: } I_{\text{enclosed}} = I$$

$$\longrightarrow B = \frac{\mu_0 I}{2\pi R}$$

## General Case



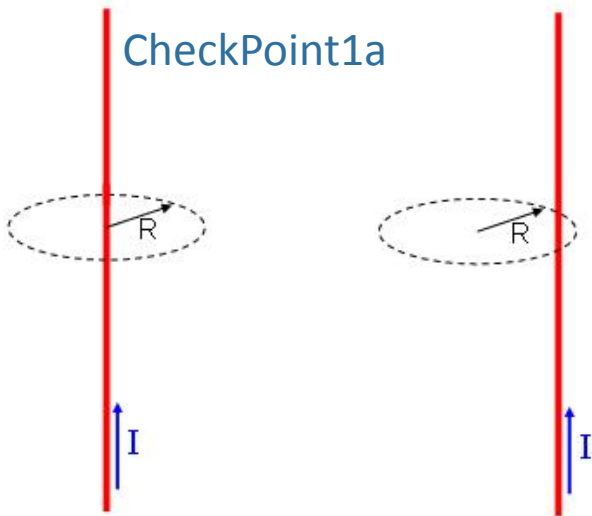
## Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

# Practice on Enclosed Currents

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{\text{enclosed}}$$

CheckPoint1a

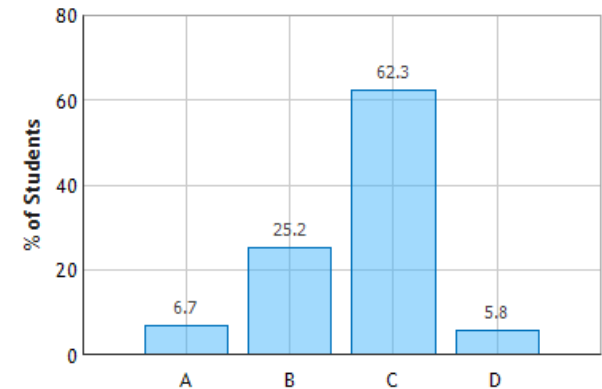
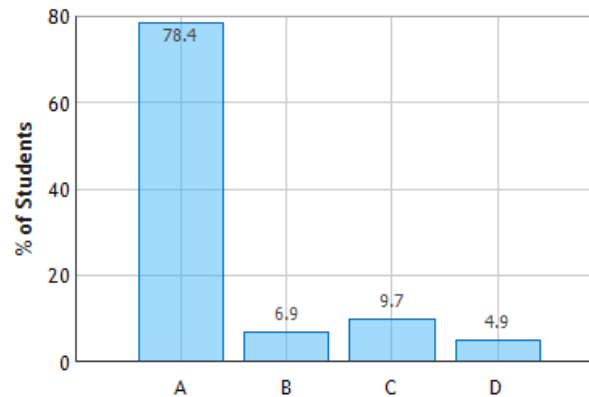
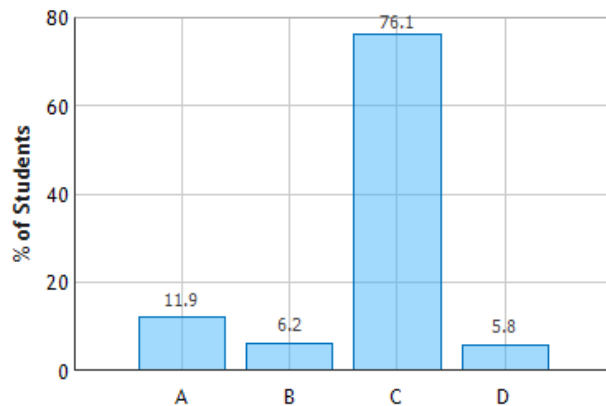


Case 1  
 $I_{\text{enclosed}} = I$

Case 2  
 $I_{\text{enclosed}} = I$

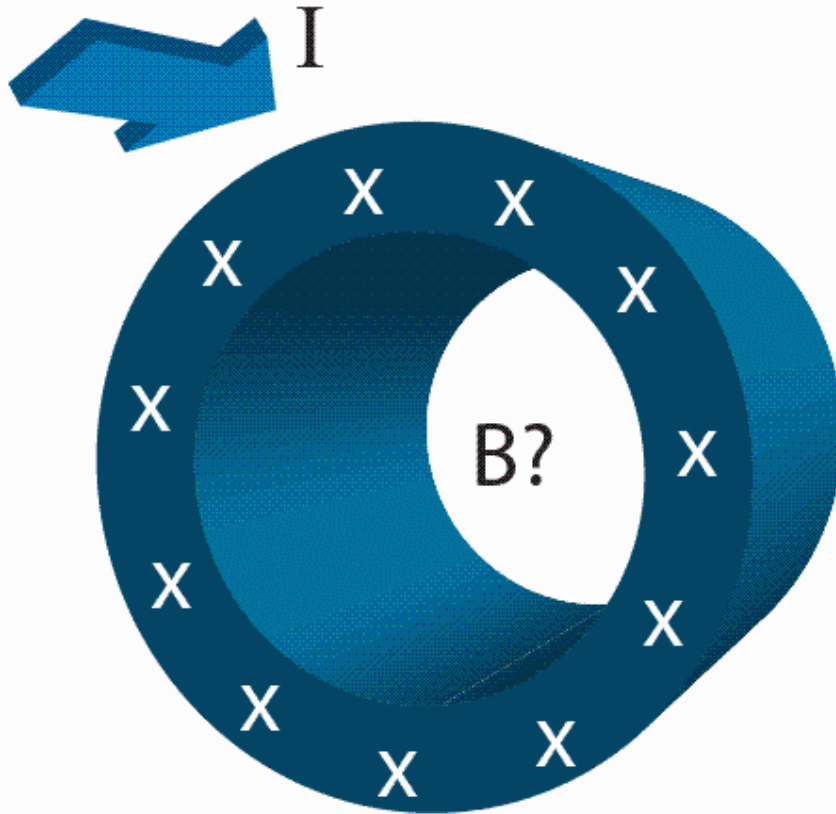
For which loop is  $\int \vec{B} \cdot d\vec{l}$  the greatest?

A. Case 1 B. Case 2 **C. Same**

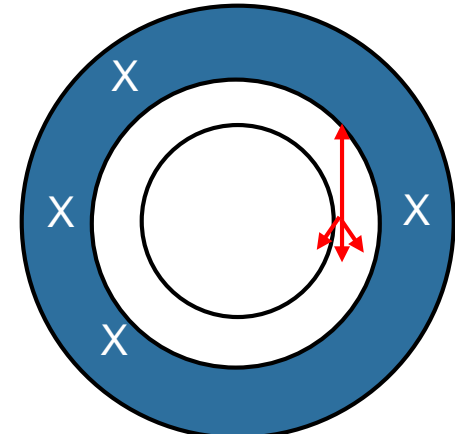


# CheckPoint 2a

An infinitely long hollow conducting tube carries current  $I$  in the direction shown.

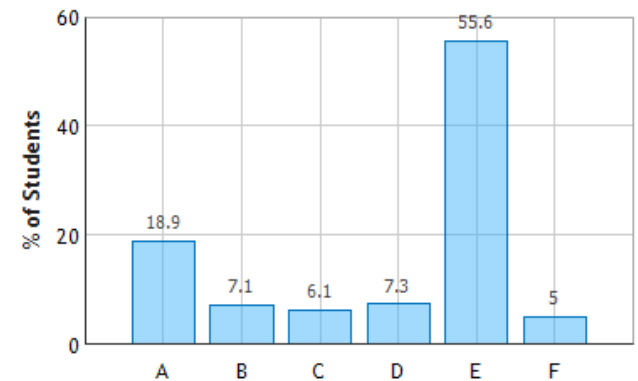


Cylindrical Symmetry



Enclosed Current = 0

Check cancellations

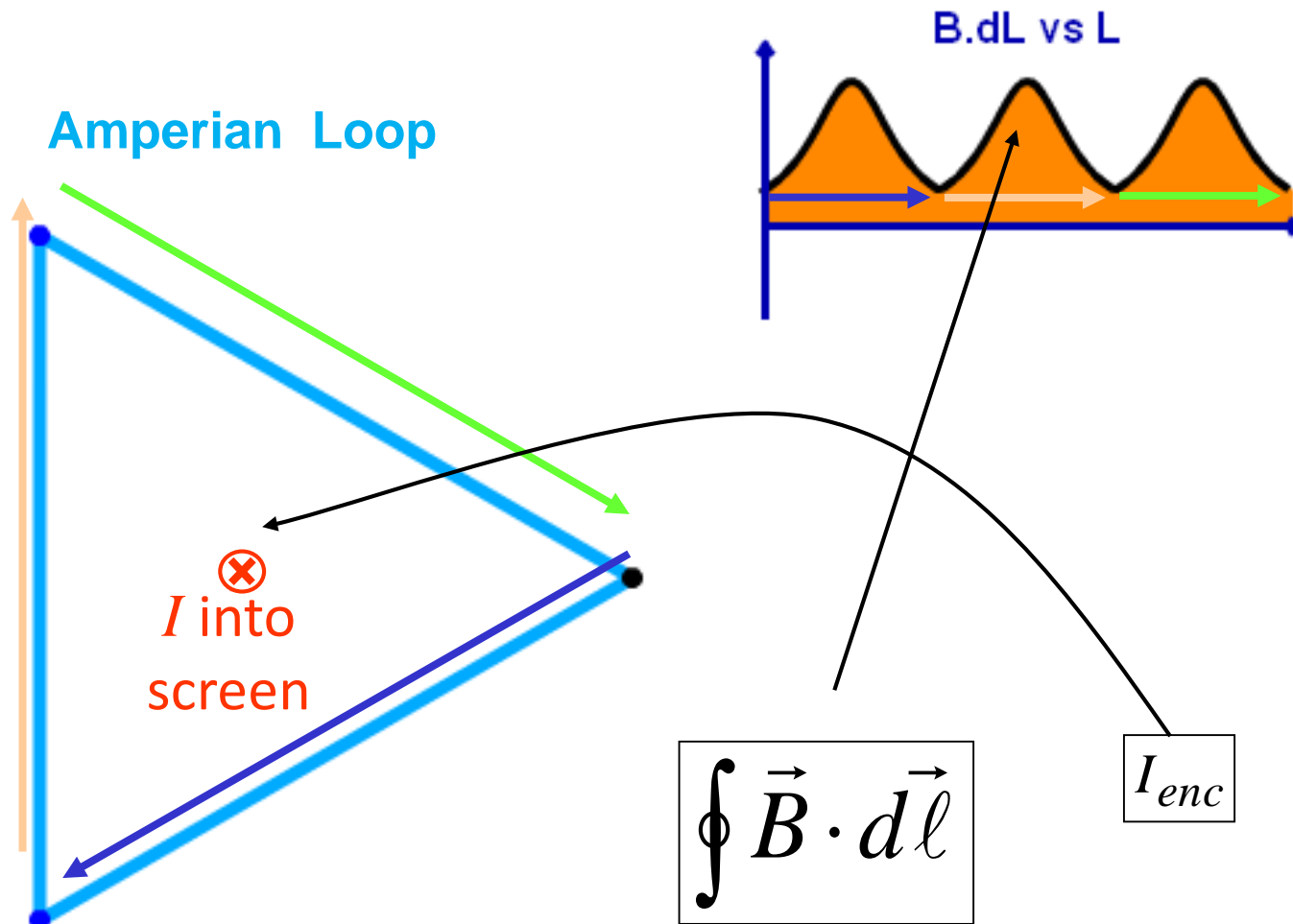


What is the direction of the magnetic field inside the tube?

- A. clockwise
- B. counterclockwise
- C. radially inward to the center
- D. radially outward from the center
- E. the magnetic field is zero

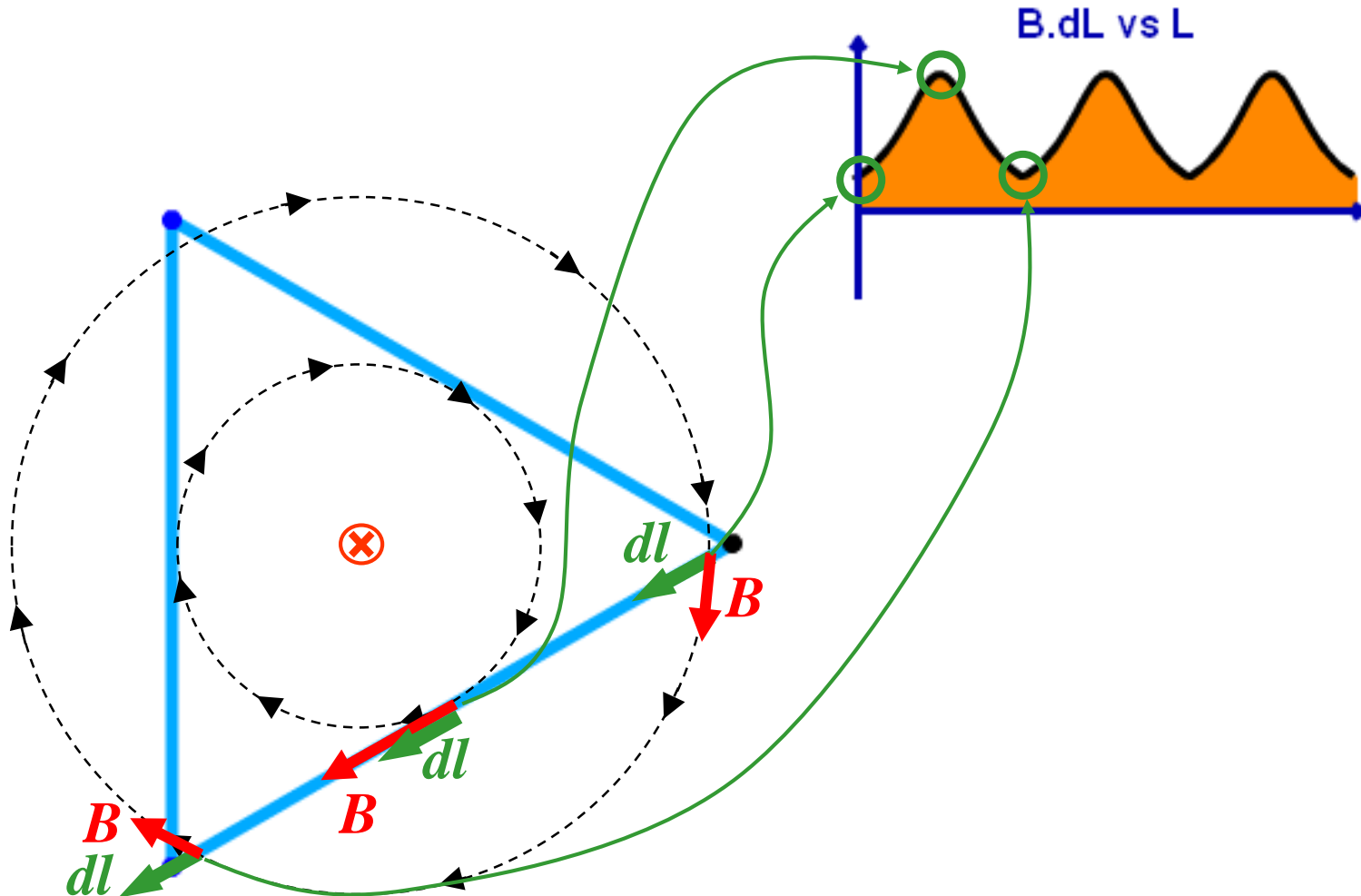
# Ampere's Law

+integrals + magnetic field directions



# Ampere's Law

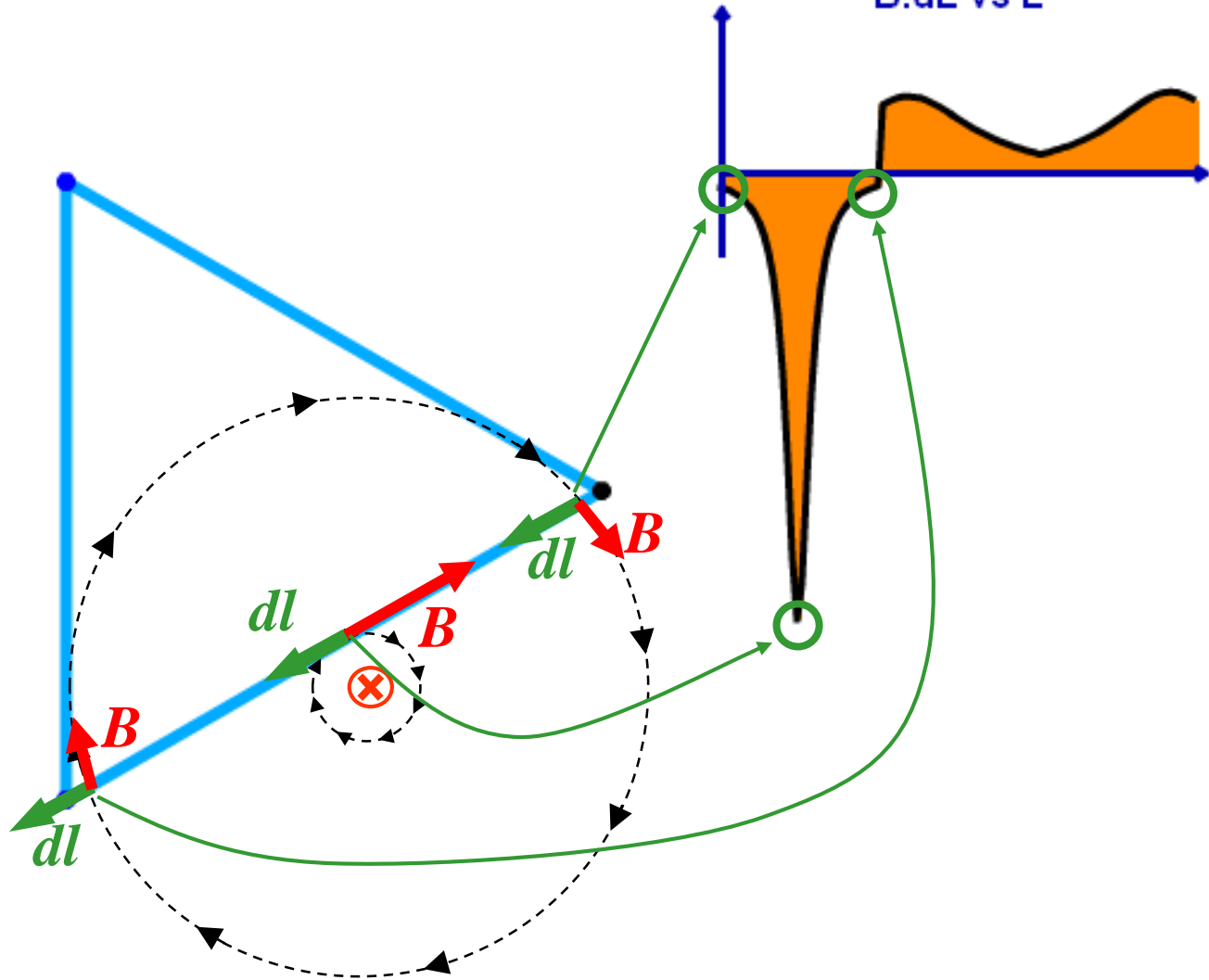
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$



# Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

B.dL vs L



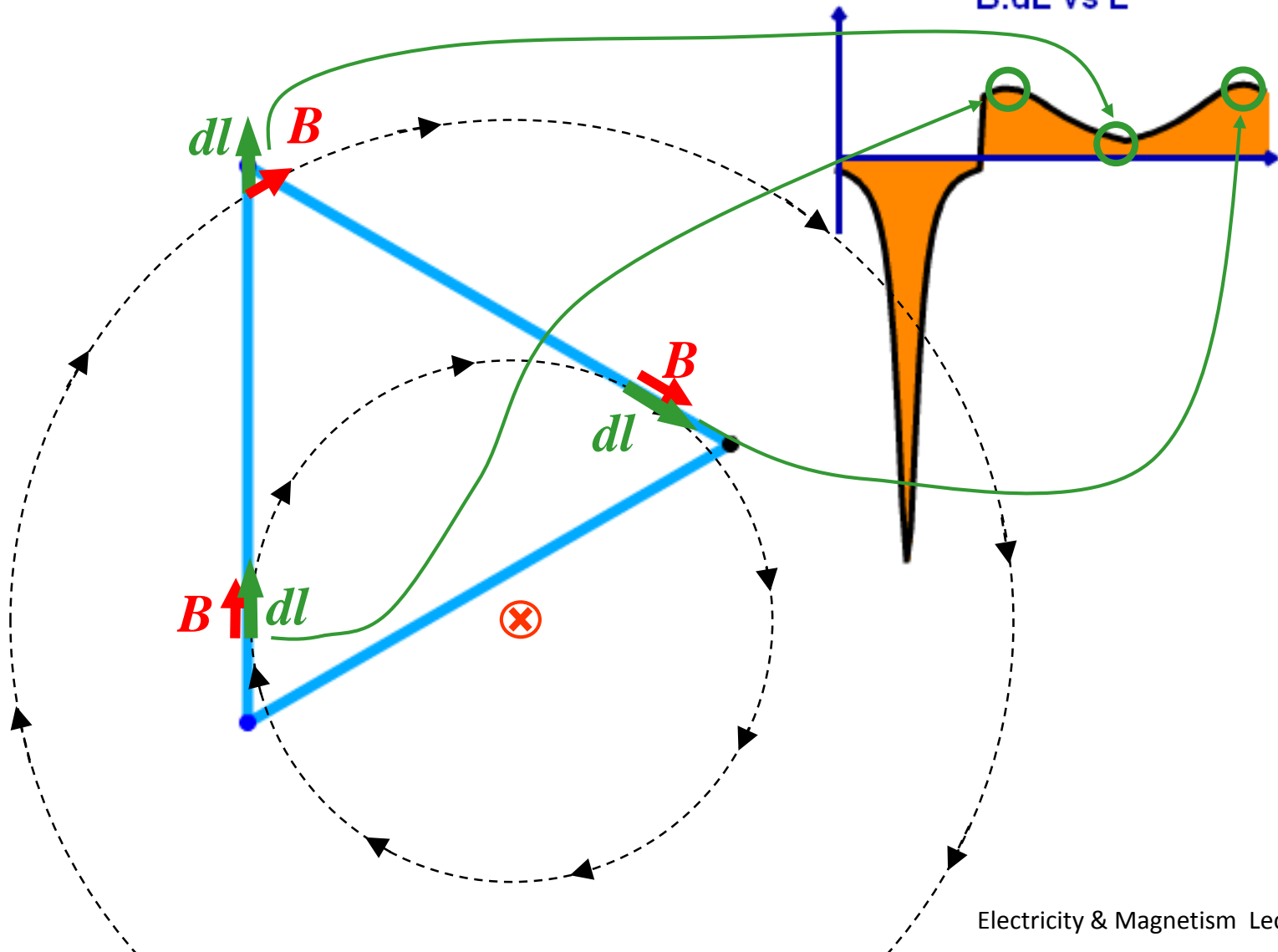


# Ampere's Law

$$I_{enc} = 0!$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$

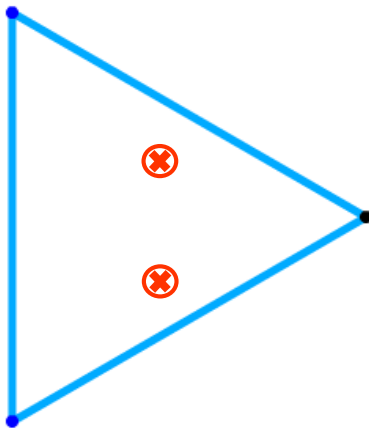
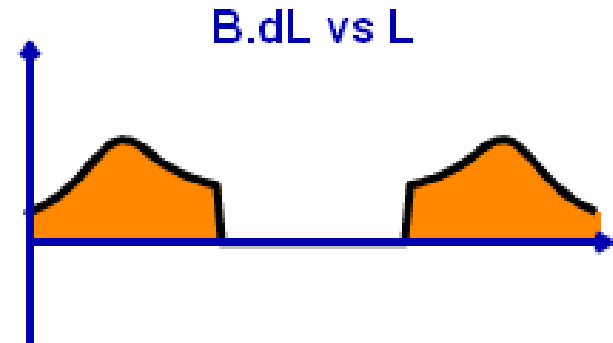
B.dL vs L



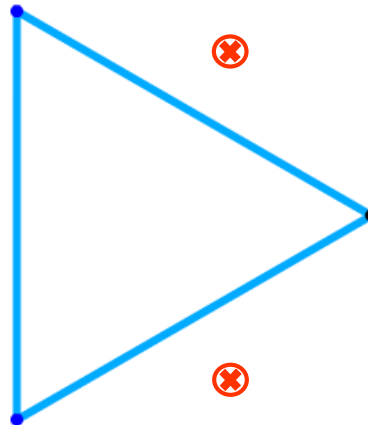
# Ampere's Law Clicker Question



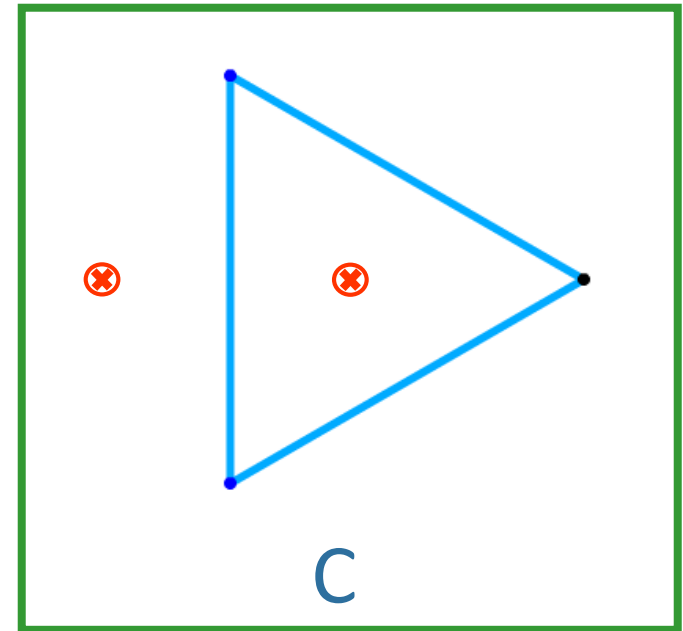
Which of the following current distributions would give rise to the  $B \cdot dL$  distribution at the right?



A



B

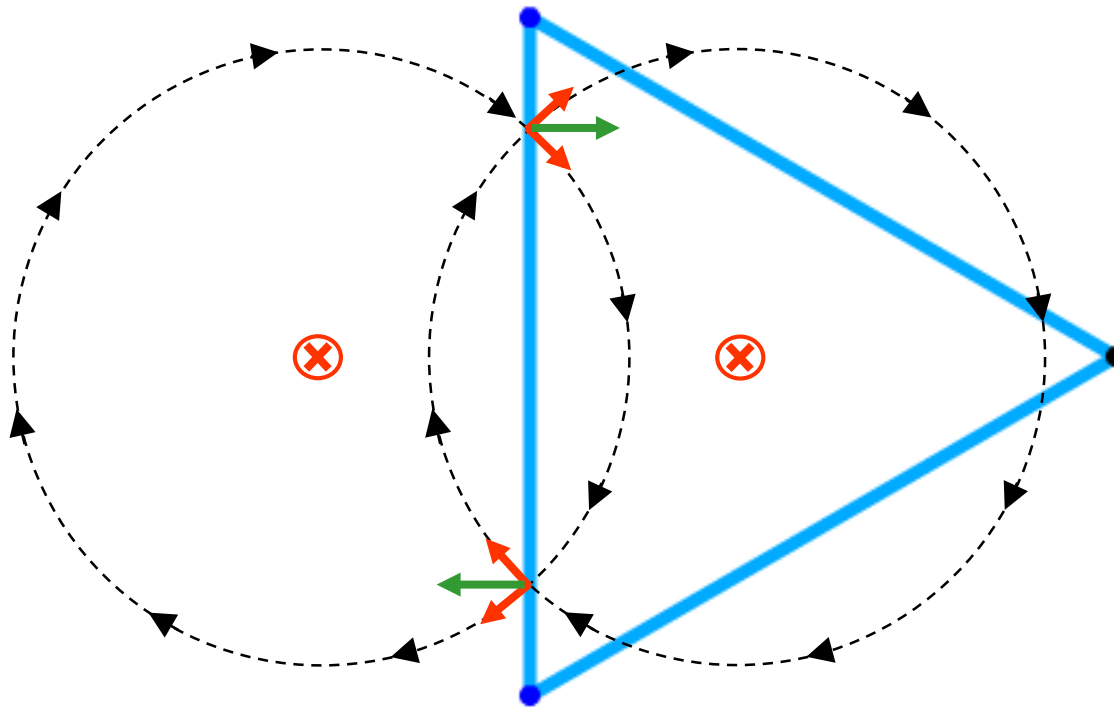
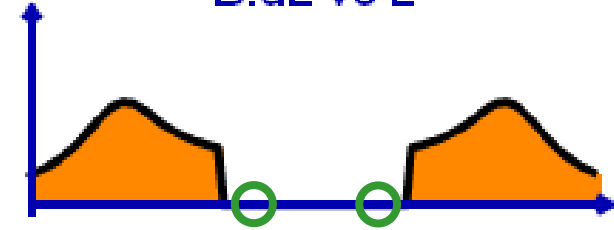


C

# Ampere's Law Clicker Question



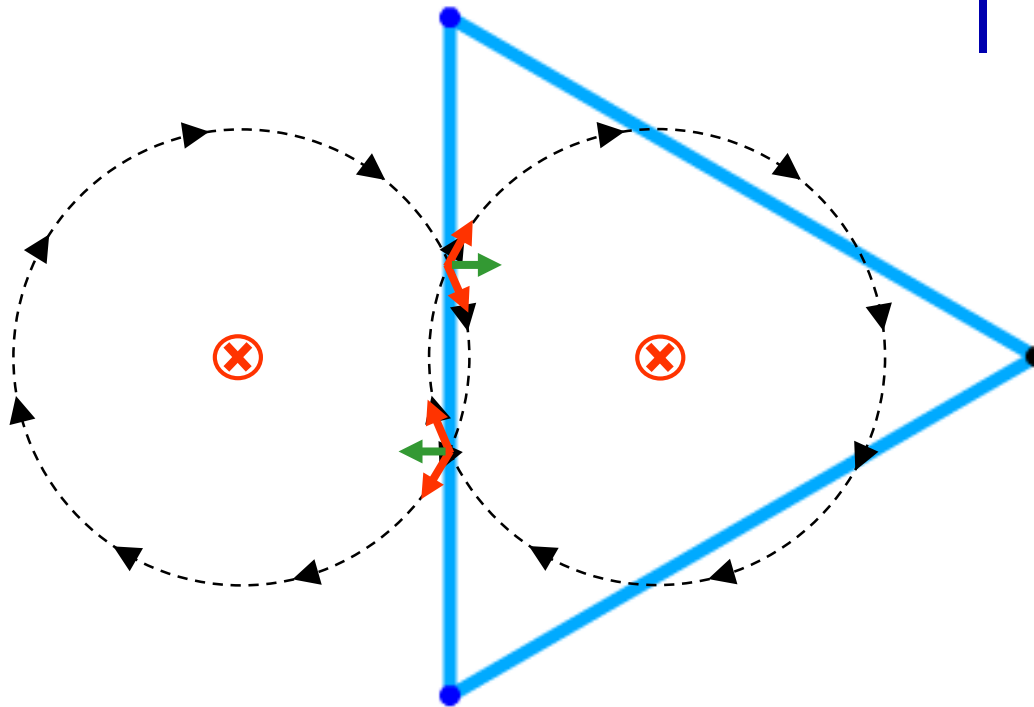
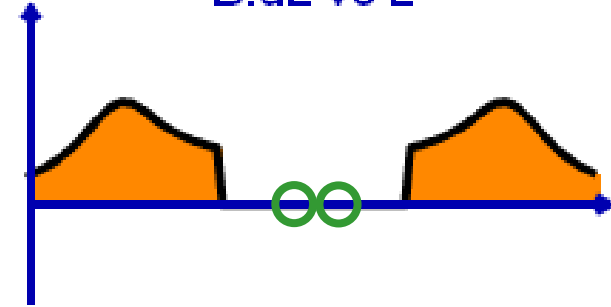
$B \cdot dL$  vs  $L$



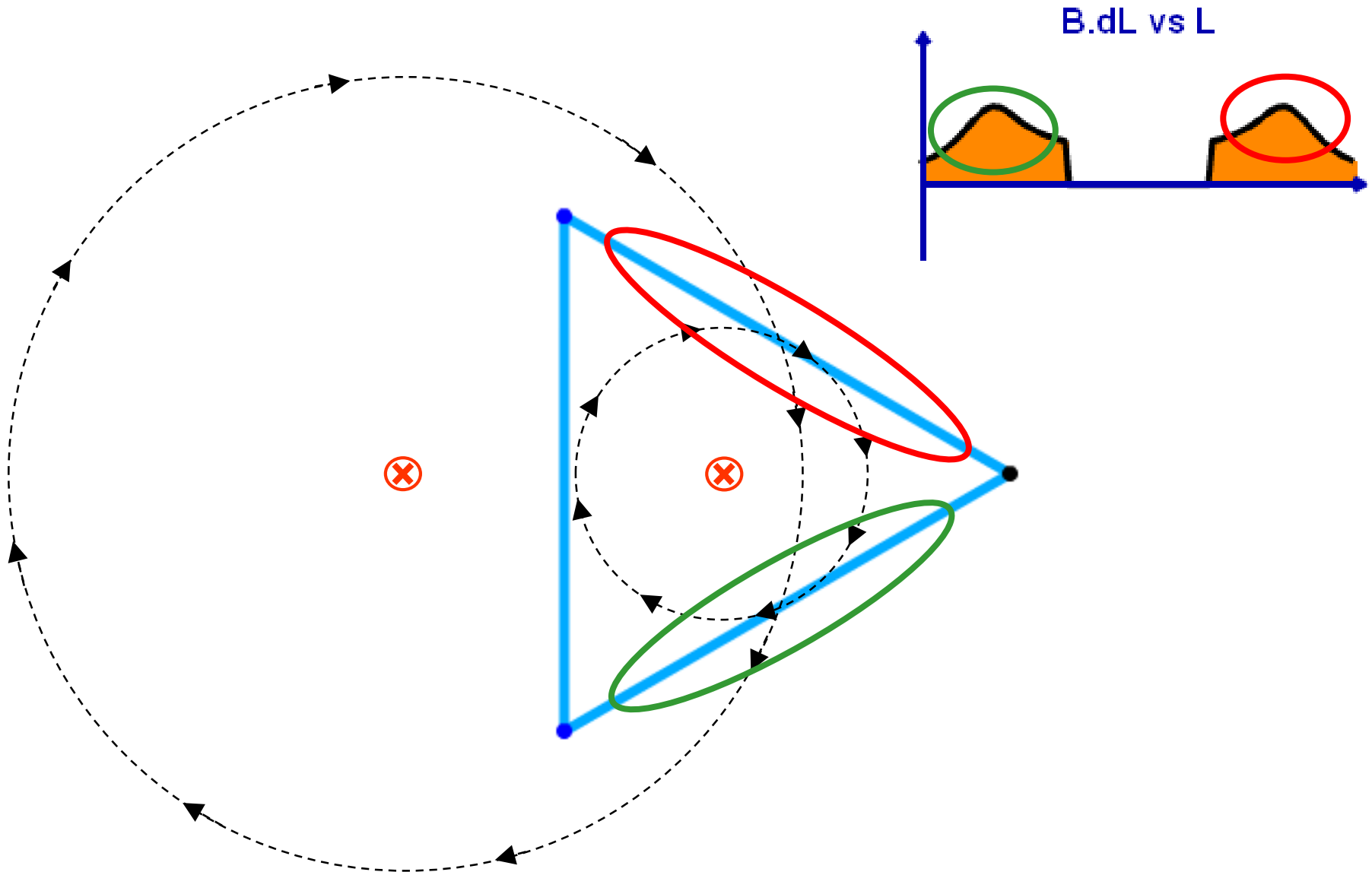
# Ampere's Law Clicker Question



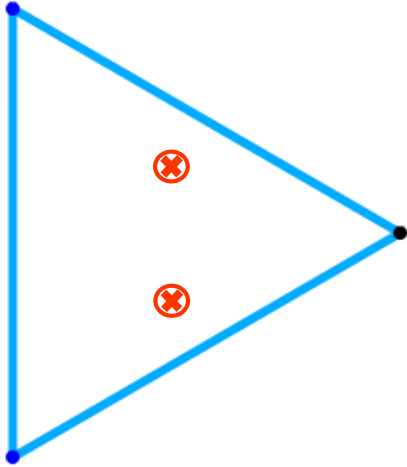
$B \cdot dL$  vs  $L$



# Ampere's Law Clicker Question

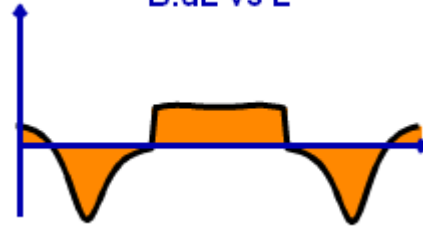


# Match the other two:



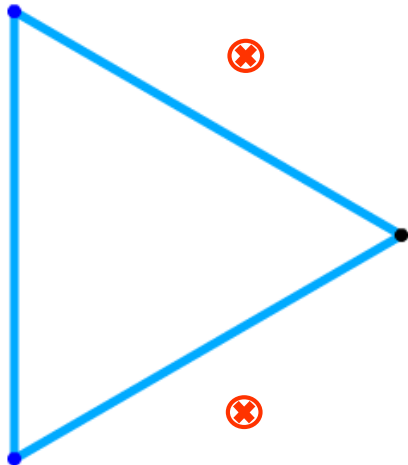
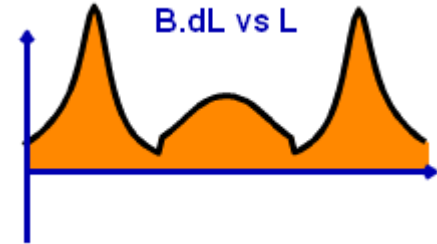
A

B.dL vs L

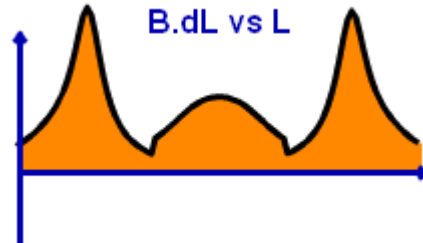


B

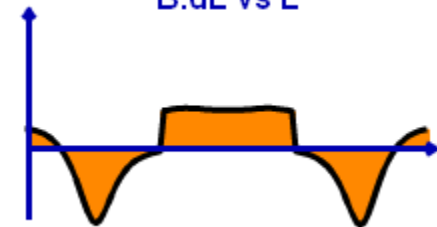
B.dL vs L



B.dL vs L

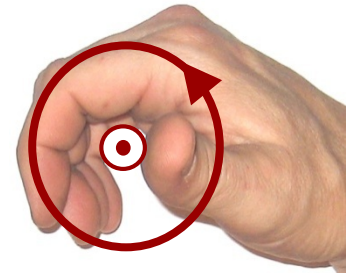
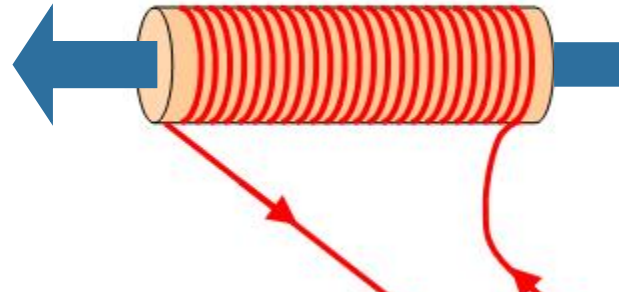


B.dL vs L



# CheckPoint 2b

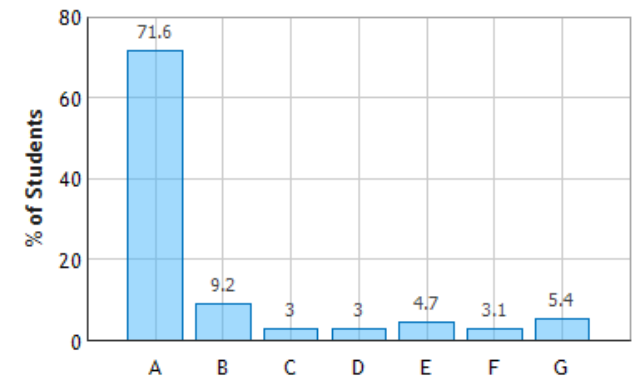
A current carrying wire is wrapped around cardboard tube as shown below.



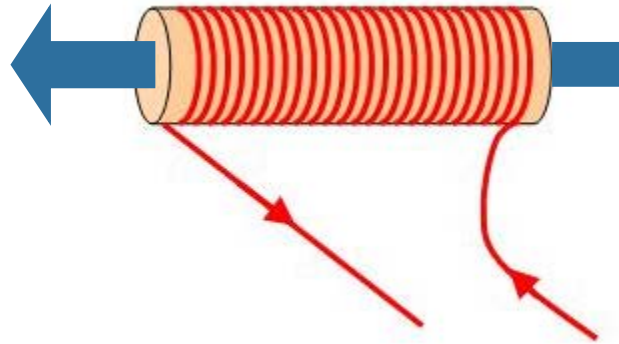
In which direction does the magnetic field point inside the tube?

- A. Left**      **B. Right**      **C. Up**      **D. Down**      **E. Out of screen**

Use the right hand rule and curl your fingers along the direction of the current.



"When the wire is wrapped around a tube, why is there  $B$  in the tube? Isn't it that the  $I_{\text{enc}}$  is 0?"



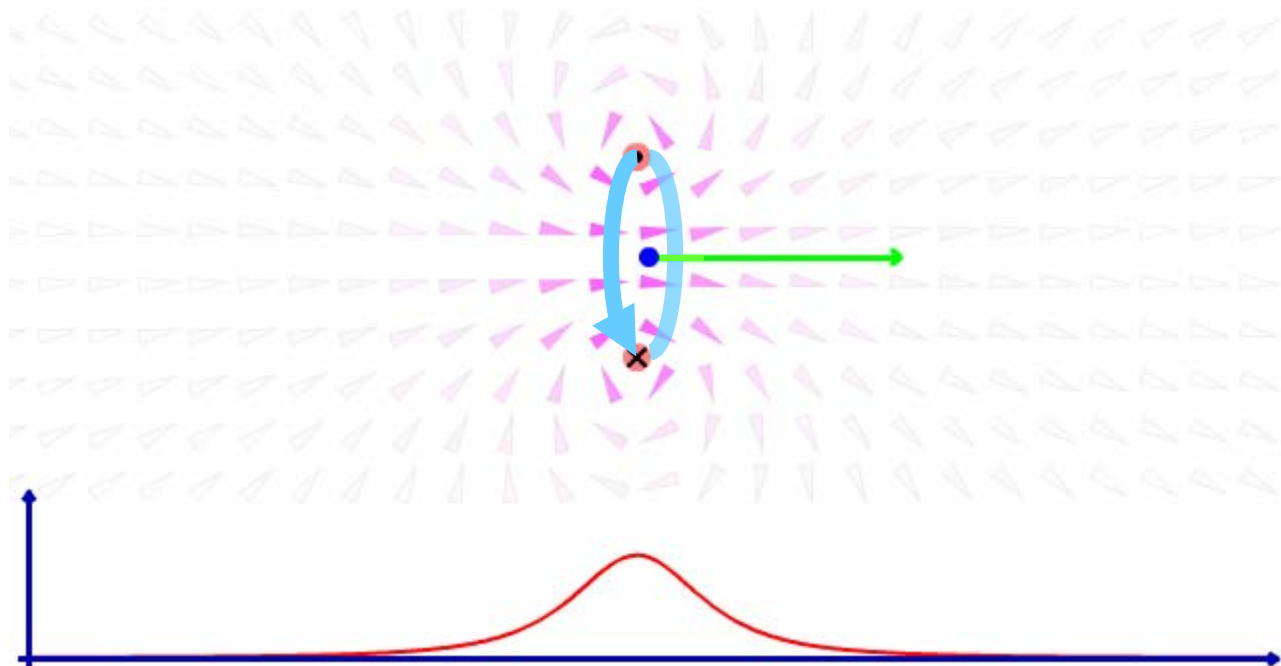


# Simulation

**1** 5 10 20 40  
n-loops

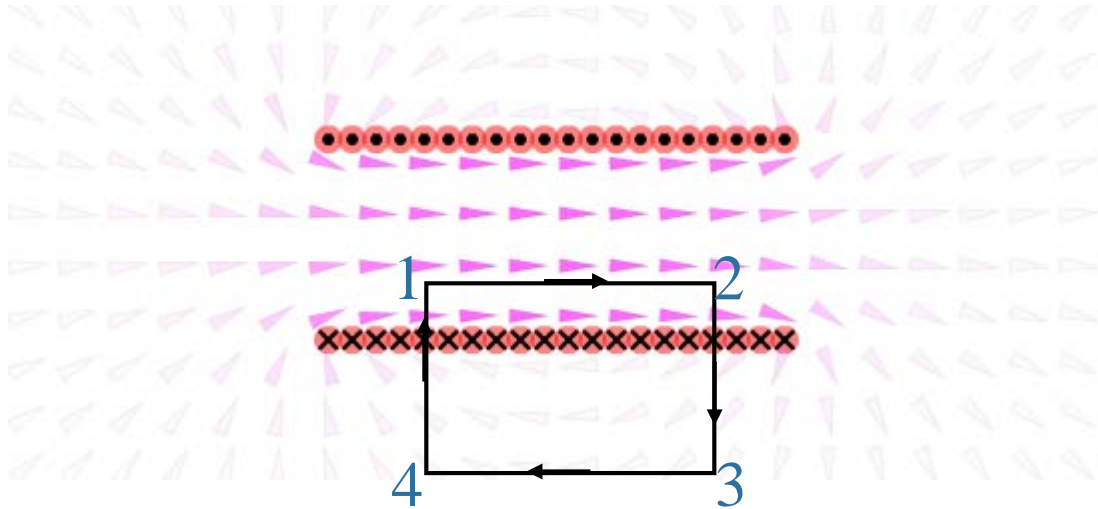
1 10  
current

$B_z = 125.565$   
 $B_y = 0$



# Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



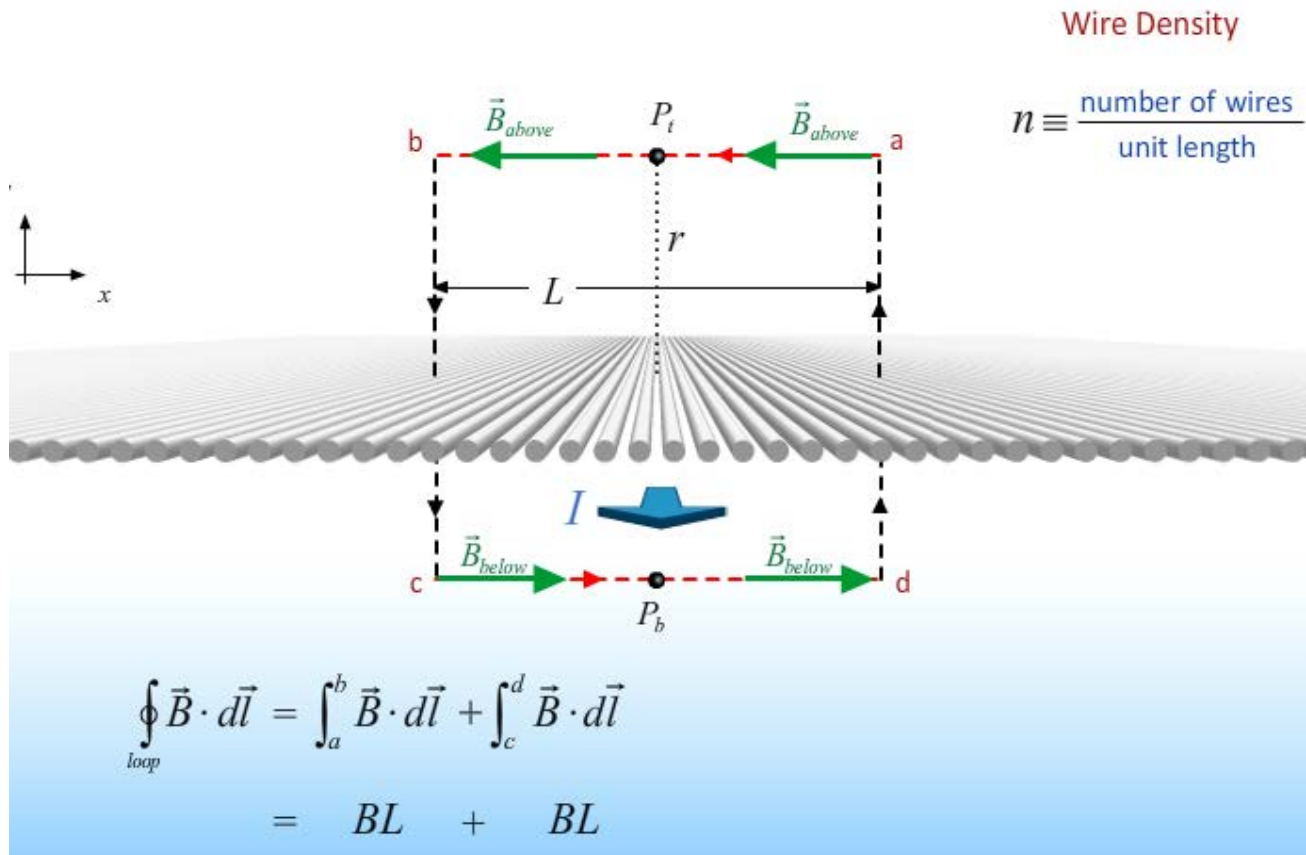
From this simulation, we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid!

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc} \quad \longrightarrow \quad \int_1^2 \vec{B} \cdot d\vec{\ell} + \int_2^3 \vec{B} \cdot d\vec{\ell} + \int_3^4 \vec{B} \cdot d\vec{\ell} + \int_4^1 \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$
$$BL + 0 + 0 + 0 = \mu_o I_{enc}$$

$$\longrightarrow BL = \mu_o nLI \quad \longrightarrow B = \mu_o nI$$

$n = \# \text{ turns/length}$

# Similar to the Current Sheet



Total integral around the loop

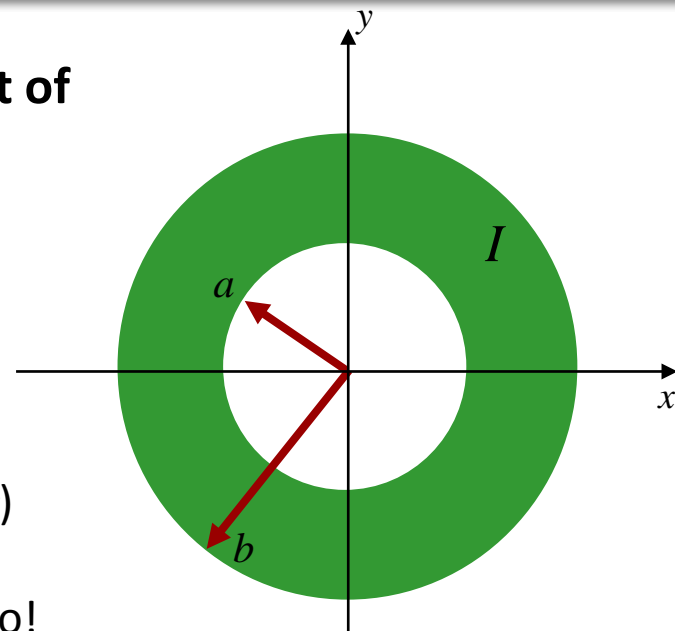
$$\oint_{\text{loop}} \vec{B} \cdot d\vec{\ell} = 2BL = \mu_0 I_{\text{enclosed}}$$

$$\therefore B = \frac{\mu_0 N I}{2L} = \frac{\mu_0 n I}{2}$$

# Example Problem

An infinitely long cylindrical shell with inner radius  $a$  and outer radius  $b$  carries a uniformly distributed current  $I$  **out of the screen**.

Sketch  $|B|$  as a function of  $r$ .



## Conceptual Analysis

Complete cylindrical symmetry (can only depend on  $r$ )

$\Rightarrow$  can use Ampere's law to calculate  $B$

$B$  field can only be clockwise, counterclockwise or zero!

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



$$B \oint d\ell = \mu_o I_{enc} \quad \text{For circular path concentric with shell.}$$

## Strategic Analysis

Calculate  $B$  for the three regions separately:

- 1)  $r < a$
- 2)  $a < r < b$
- 3)  $r > b$

# Example Problem

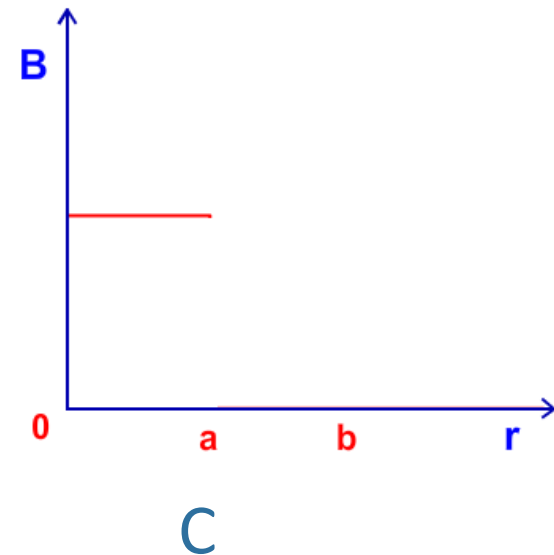
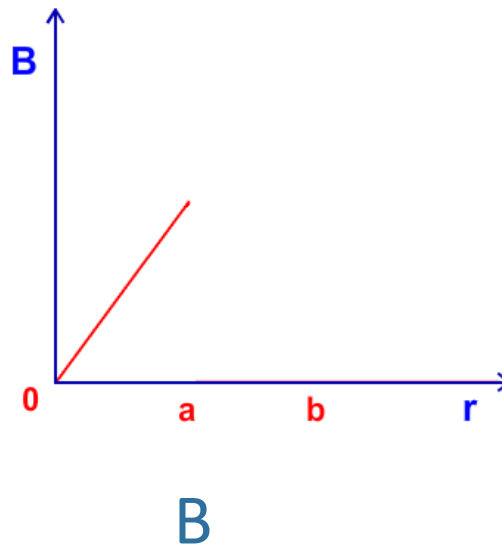
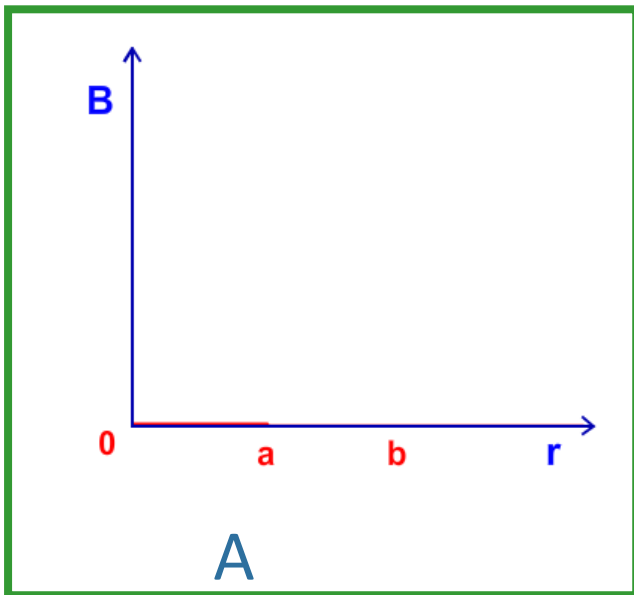
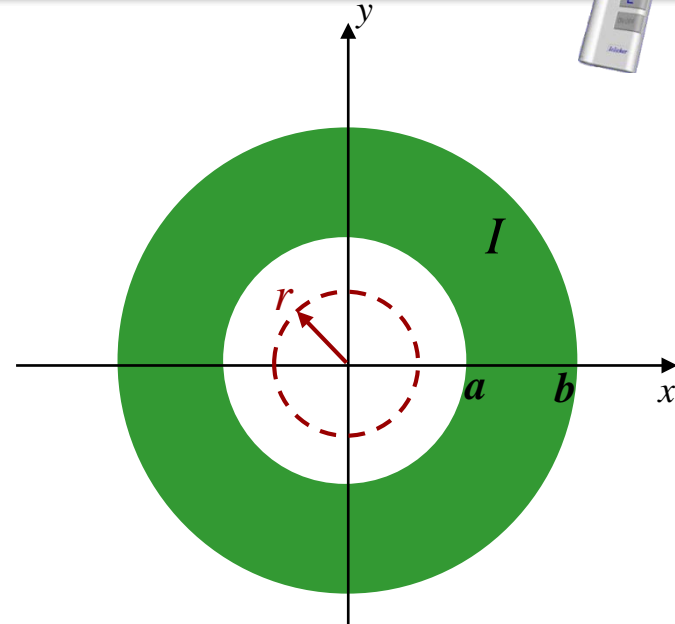


What does  $|B|$  look like for  $r < a$ ?

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{nc}$$

$\downarrow$   
 $0$

so  $\vec{B} = 0$



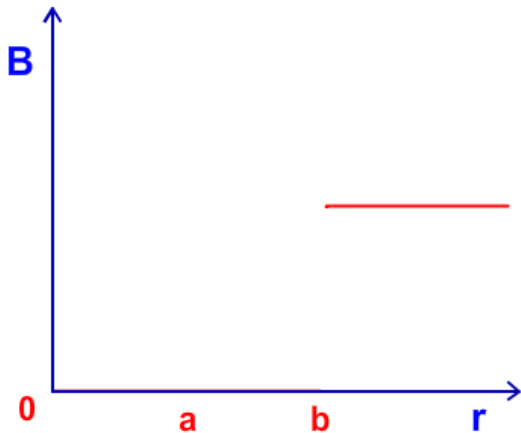
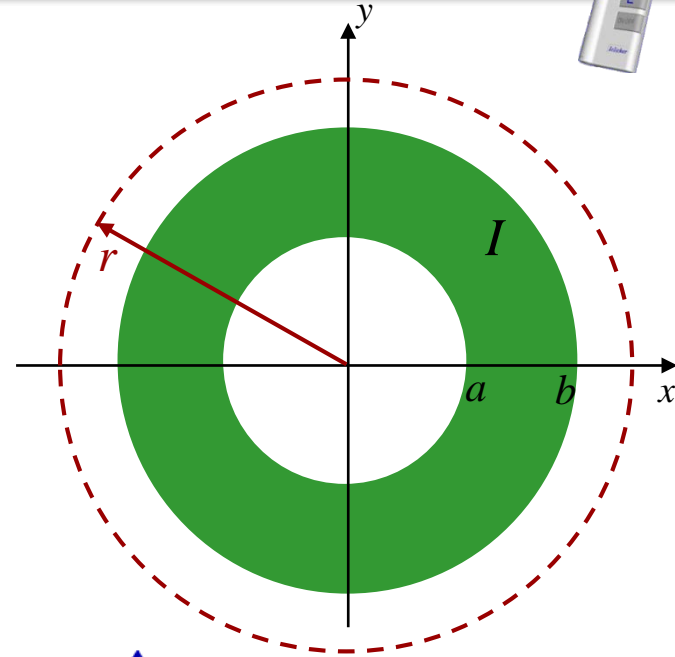
# Example Problem



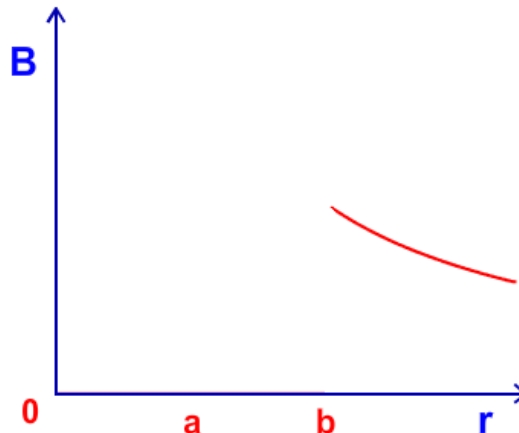
What does  $|B|$  look like for  $r > b$ ?

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{nc}$$

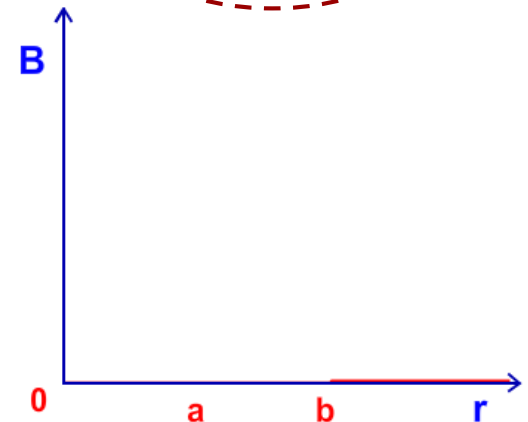
$I$



A



B



C

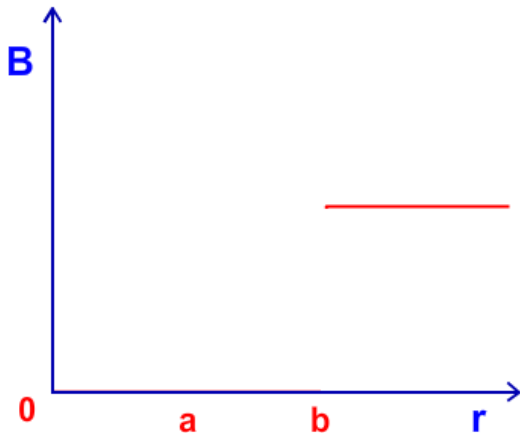
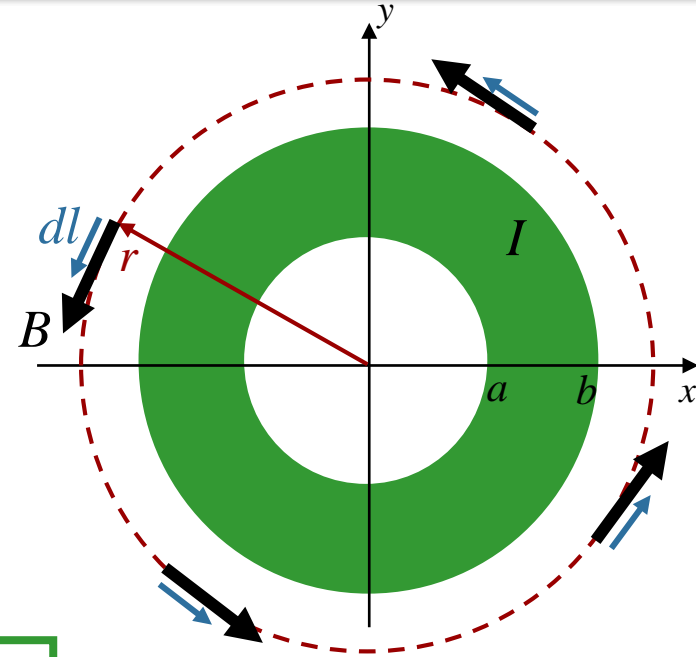
# Example Problem

What does  $|B|$  look like for  $r > b$ ?

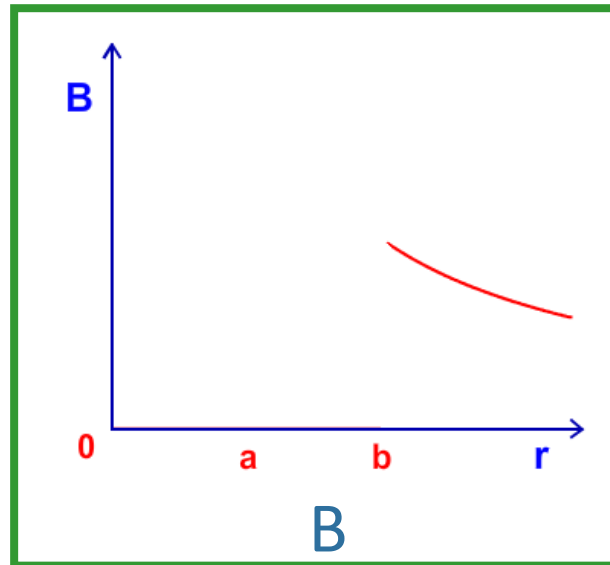
$$\text{LHS: } \oint \vec{B} \cdot d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi r$$

$$\text{RHS: } I_{\text{enclosed}} = I$$

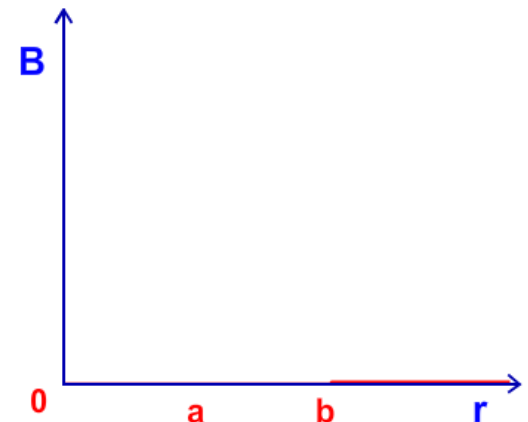
$$\longrightarrow B = \frac{\mu_o I}{2\pi r}$$



A



B

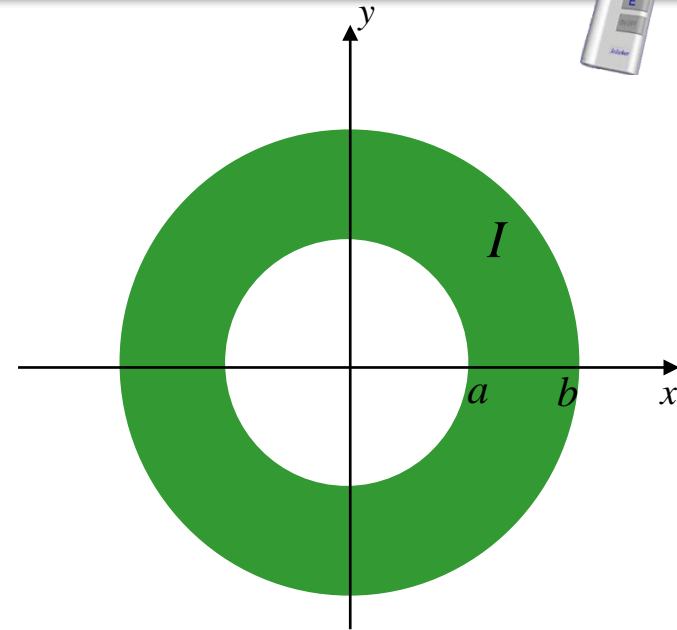


C

# Example Problem



What is the current density  $j$  ( $Amp/m^2$ ) in the conductor?



A)  $j = \frac{I}{\pi b^2}$

B)  $j = \frac{I}{\pi b^2 + \pi a^2}$

C)  $j = \frac{I}{\pi b^2 - \pi a^2}$

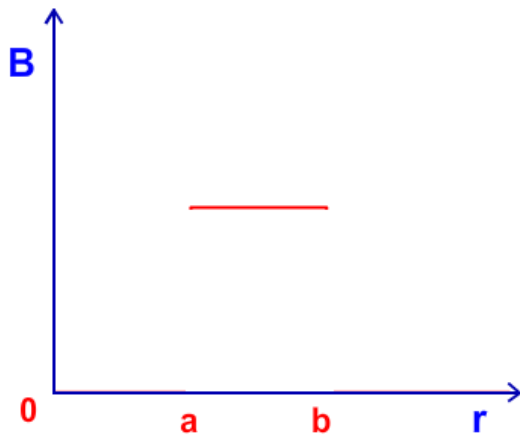
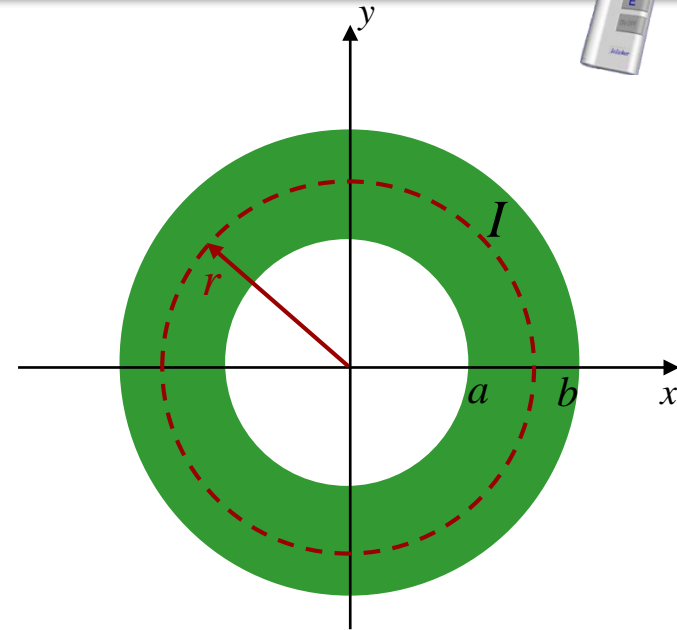
$$\underbrace{j = I / \text{area}}_{\text{area} = \pi b^2 - \pi a^2} \quad j = \frac{I}{\pi b^2 - \pi a^2}$$



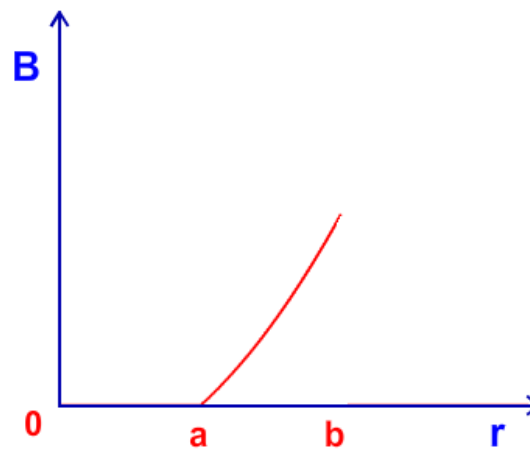
# Example Problem



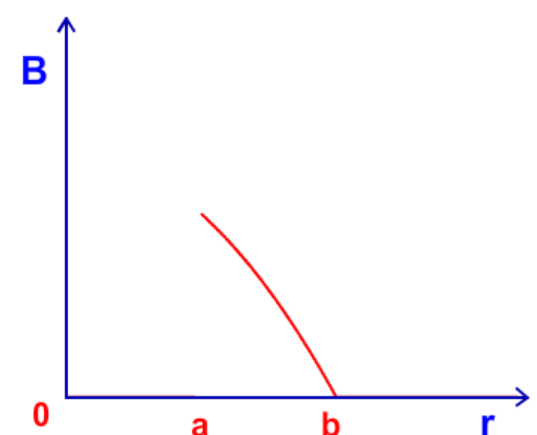
What does  $|B|$  look like for  $a < r < b$  ?



A



B

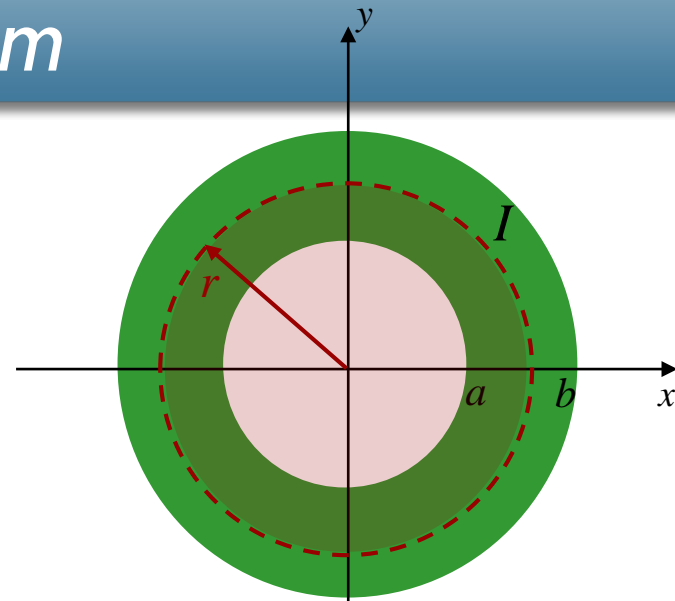


C

# Example Problem

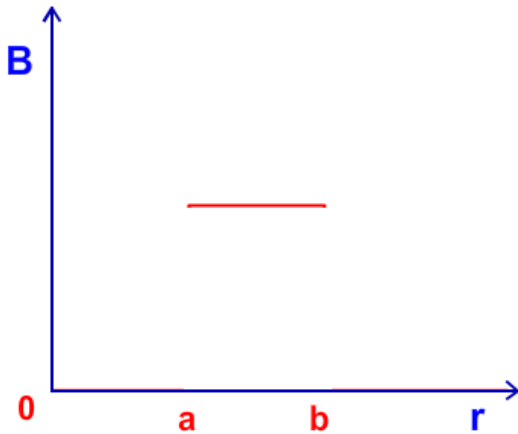
What does  $|B|$  look like for  $a < r < b$  ?

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc} \quad \longrightarrow \quad B \cdot 2\pi r = \mu_o \cdot jA_{enc}$$

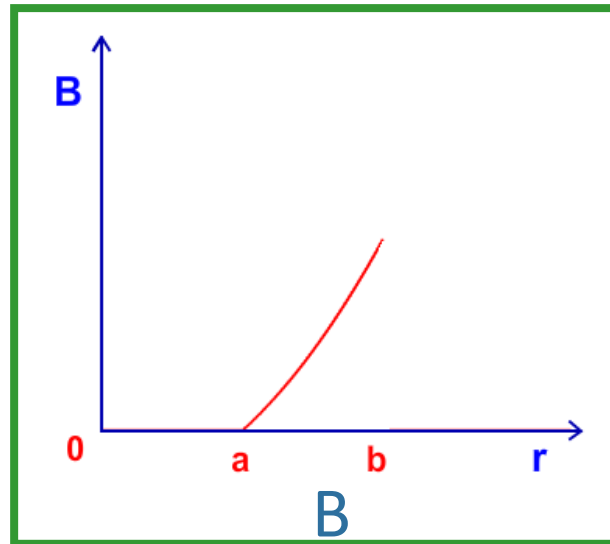


$$B \cdot 2\pi r = \mu_o \cdot \frac{I}{\pi(b^2 - a^2)} \cdot \pi(r^2 - a^2) \quad \longrightarrow \quad B = \frac{\mu_o I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

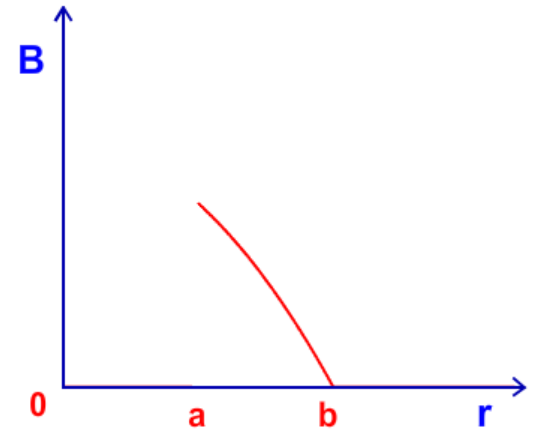
Starts at 0 and increases almost linearly



A



B

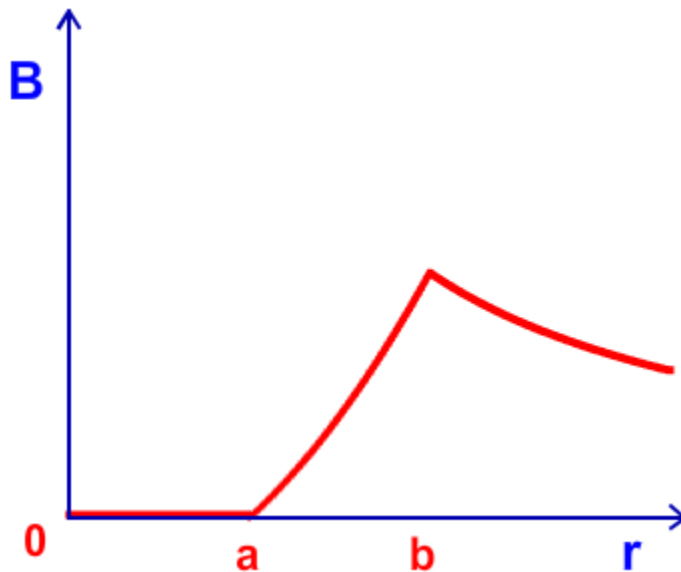
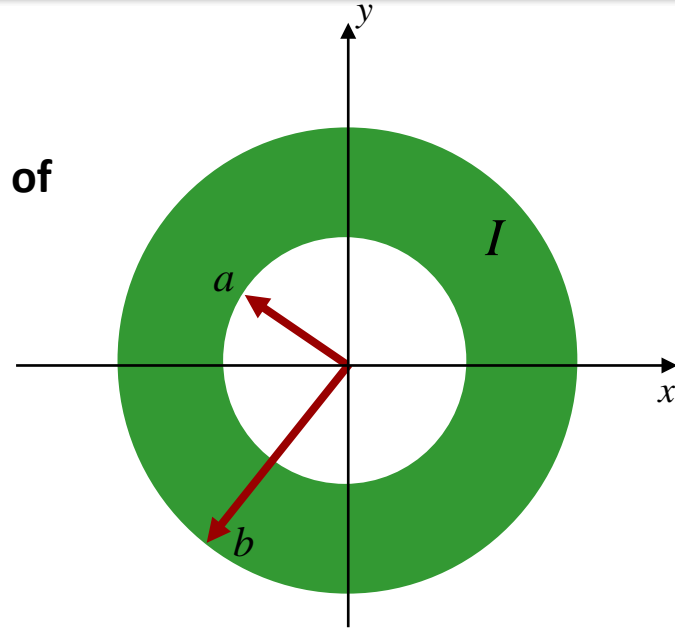


C

# Example Problem

An infinitely long cylindrical shell with inner radius  $a$  and outer radius  $b$  carries a uniformly distributed current  $I$  **out of the screen**.

Sketch  $|B|$  as a function of  $r$ .



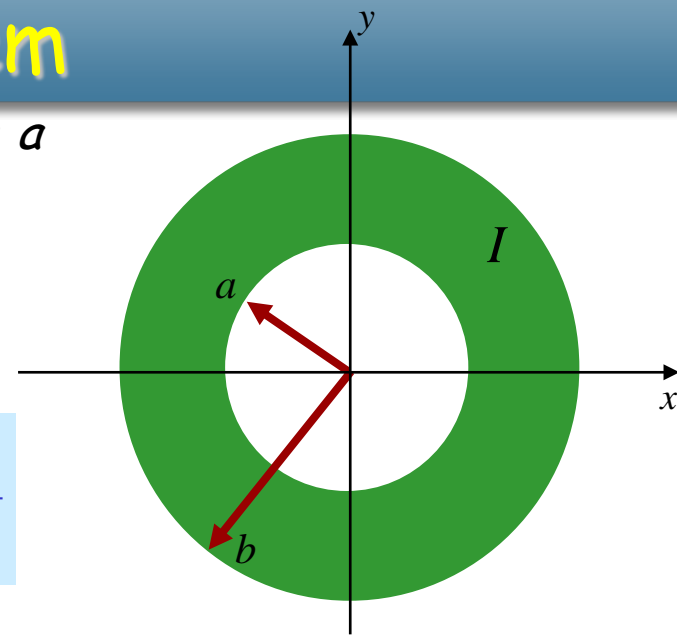
# Example Problem

An infinitely long cylindrical shell with inner radius  $a$  and outer radius  $b$  carries a uniformly distributed current  $I$  out of the screen.

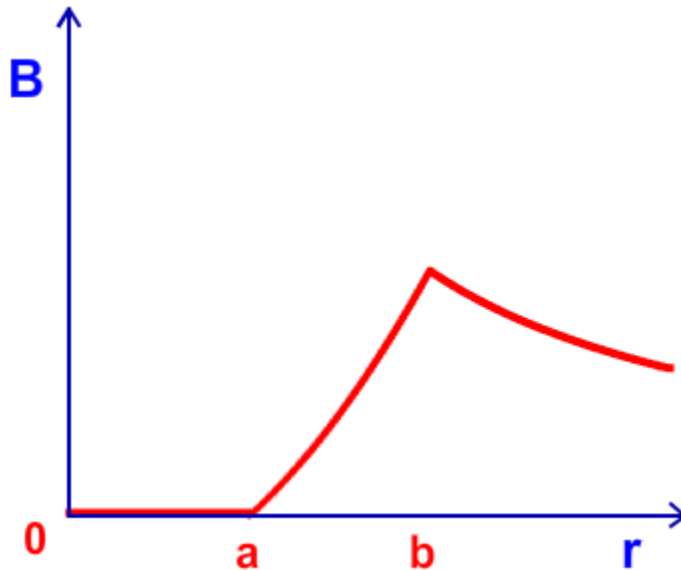
Sketch  $|B|$  as a function of  $r$ .

How big is  $B$  at  $r = b$ ?

$$B = \frac{\mu_0 I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$



Let  $I = 10 \text{ A}$ ,  $b = 1 \text{ mm}$



$$\begin{aligned} B(b) &= \frac{\mu_0 I}{2\pi b} \\ &= \frac{4\pi \times 10^{-7} \text{ Tm/A} \cdot 10 \text{ A}}{2\pi \cdot 0.001 \text{ m}} \\ &= 2 \times 10^{-3} \text{ T} \end{aligned}$$

# Follow-Up



Add an infinite wire along the  $z$  axis carrying current  $I_0$ .

What must be true about  $I_0$  such that there is some value of  $r$ ,  $a < r < b$ , such that  $B(r) = 0$ ?

- A)  $|I_0| > |I|$  AND  $I_0$  into screen
- B)  $|I_0| > |I|$  AND  $I_0$  out of screen
- C)  $|I_0| < |I|$  AND  $I_0$  into screen
- D)  $|I_0| < |I|$  AND  $I_0$  out of screen
- E) There is no current  $I_0$  that can produce  $B = 0$  there

$B$  will be zero if total current enclosed  $= 0$

