

## *Your Comments*

dead after the physics exam....

why do you give us prelectures in the same day as our exam?

I'm very confused about the real battery, and about the directions of current flow

Can we go over drops and gains, its confusing.

That question with the blue wire. I still don't fully understand what's going on.

The charges flowing in clockwise direction made my head spin for a while, the signs changing is confusing.

You know Watt? The Current amount of electricity puns is Shockingly low. We need to Amp it up. Yes, there will be Resistance to such tom-foolery, some may even be reVolved by its childishness, but there's no reason for assault and Battery on their part. Come on, I believe we Conduit.

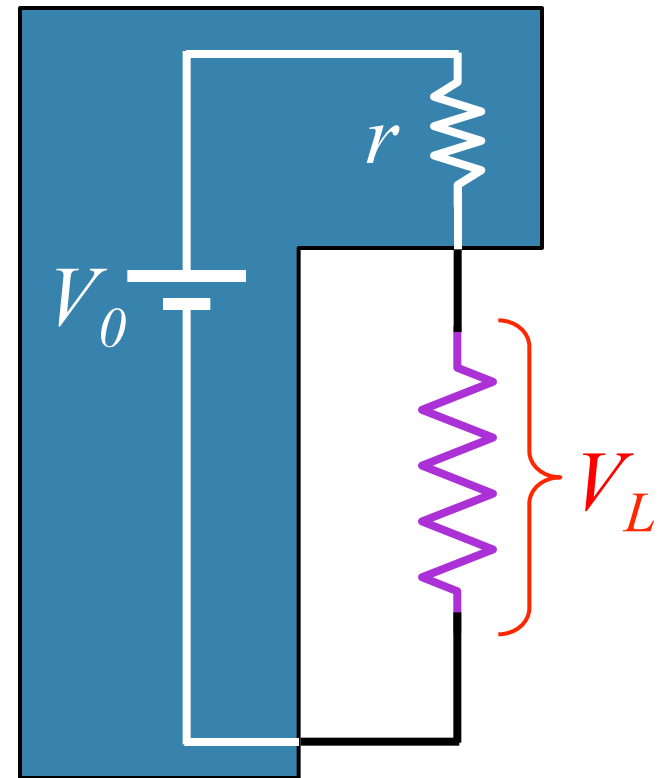
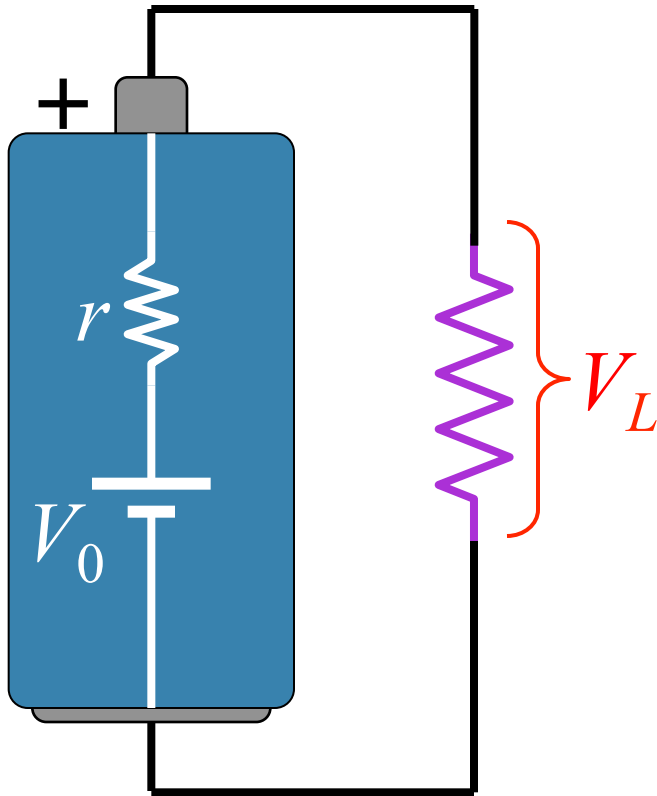
# *Physics 212*

## *Lecture 10*

Today's Concept:

Kirchhoff's Rules

# Model for Real Battery: Internal Resistance



Usually can't supply too much current to the load  
without voltage "sagging"

# Last Time

## Resistors in series:

Current through is same.

Voltage drop across is  $IR_i$

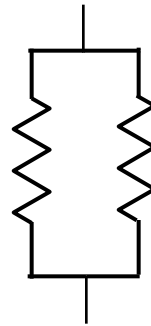


$$R_{\text{effective}} = R_1 + R_2 + R_3 + \dots$$

## Resistors in parallel:

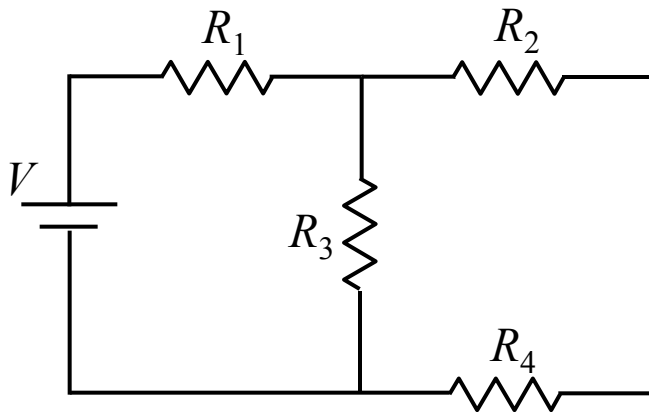
Voltage drop across is same.

Current through is  $V/R_i$

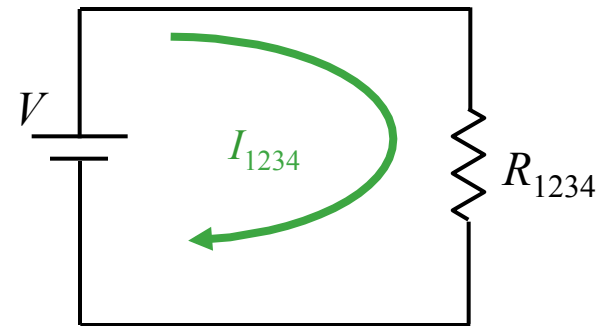


$$\frac{1}{R_{\text{effective}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

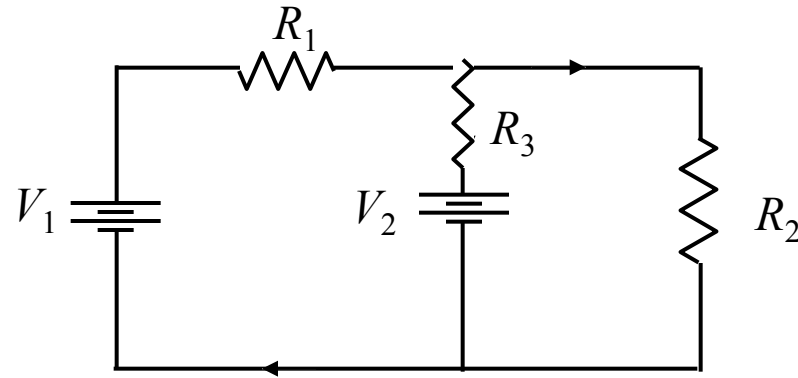
## Solved Circuits



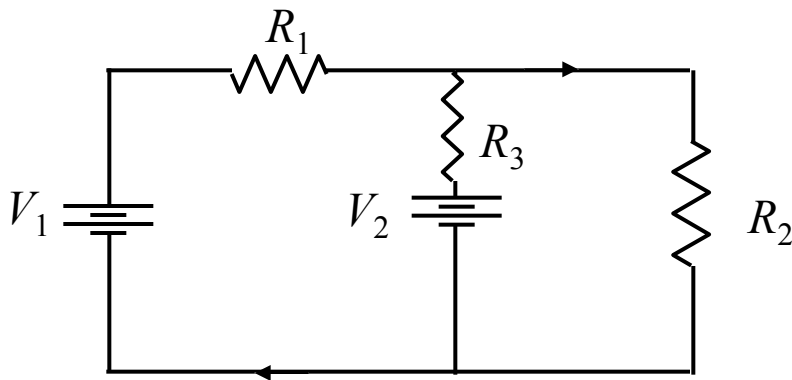
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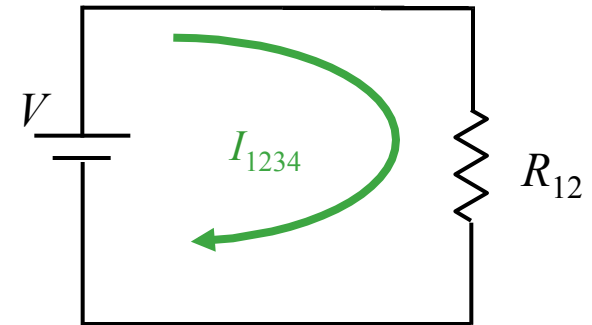
# New Circuit



How Can We Solve This One?



$\neq$



**THE ANSWER:** Kirchhoff's Rules

# *Kirchhoff's Voltage Rule*

$$\sum \Delta V_i = 0$$

Kirchhoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

WHY?

The potential difference between a point and itself is zero!

# Kirchhoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

WHY?

Electric Charge is Conserved

# Applying Kirchhoff's Laws in 5 easy steps

## 1) Label all currents

Choose any direction

## 2) Label +/– for all elements

Current goes +  $\Rightarrow$  – (for resistors)  
Long side is + for battery

## 3) Choose loop and direction

Must start on wire, not element.

## 4) Write down voltage drops

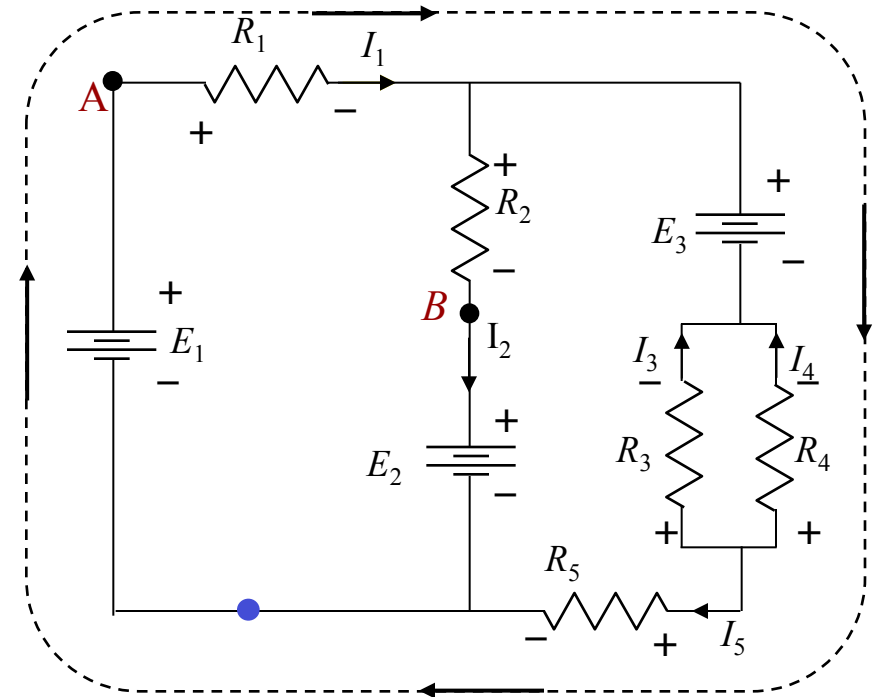
First sign you hit is sign to use.

## 5) Write down node equation $I_{\text{in}} = I_{\text{out}}$

We'll do calculation today

It's actually the easiest thing to do!

$$-E_1 + I_1 R_1 + E_3 - I_4 R_4 + I_5 R_5 = 0$$

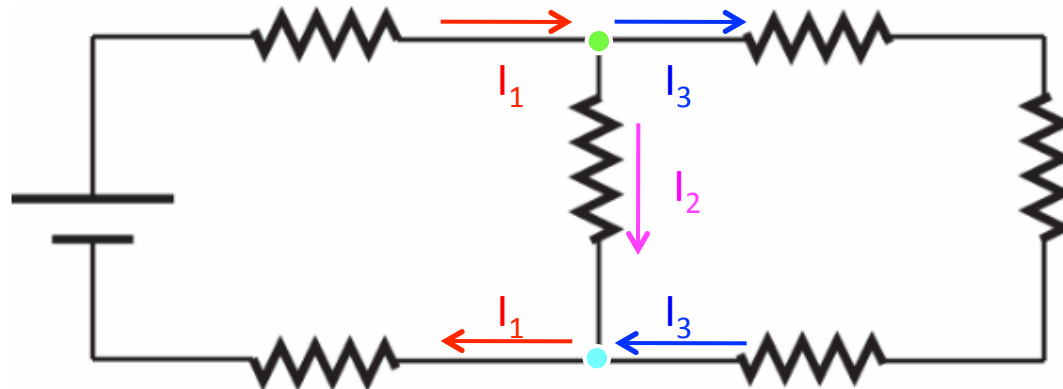




# Check Point 1



How many potentially different currents are there in the circuit shown?



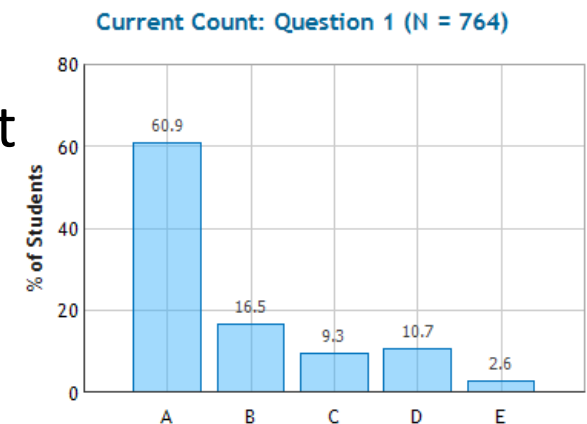
- A. 3**    B. 4    C. 5    D. 6    E. 7

Look at the nodes!

**Top node:**  $I_1$  flows in,  $I_2$  and  $I_3$  flow out

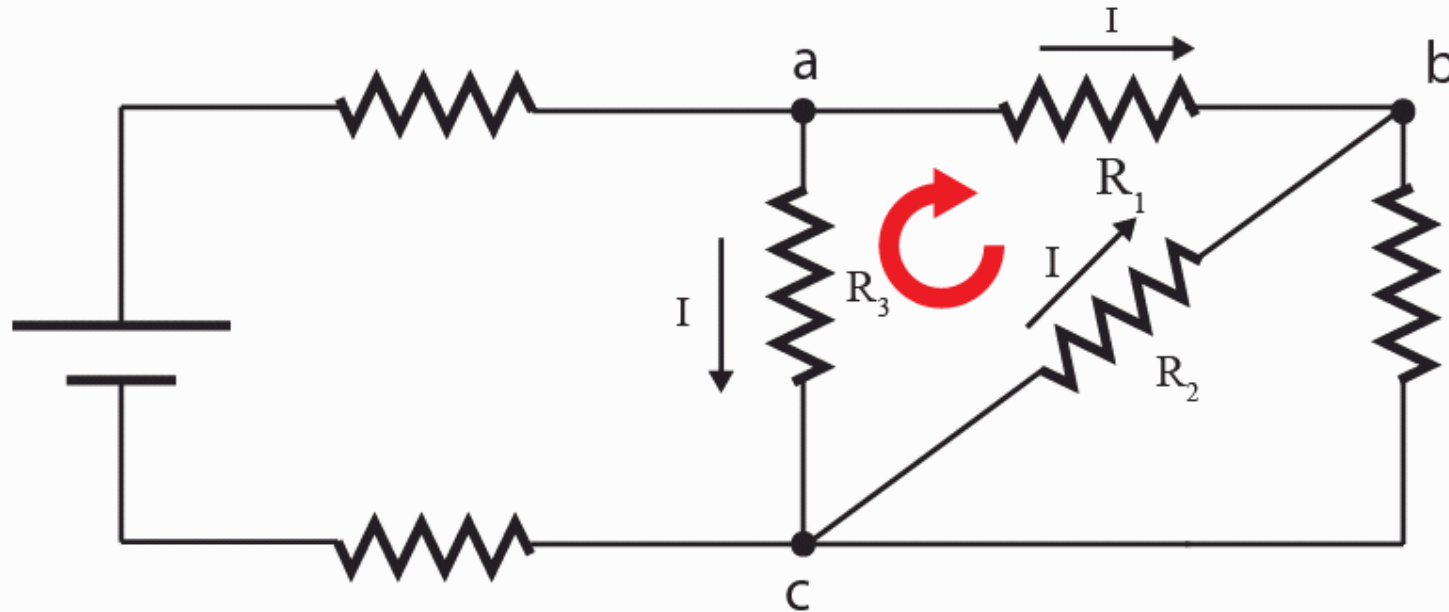
**Bottom node:**  $I_2$  and  $I_3$  flow in,  $I_1$  flows out

That's all of them!



## CheckPoint 2

In the following circuit, consider the loop abc. The direction of the current through each resistor is indicated by black arrows.



If we are to write Kirchhoff's voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors R1, R2 and R3?

A. drop, drop, drop

B. gain, gain, gain

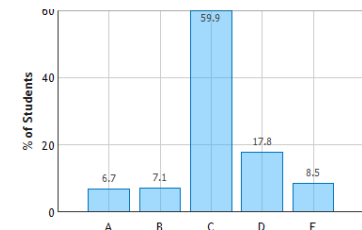
C. drop, gain, gain

D. gain, drop, drop

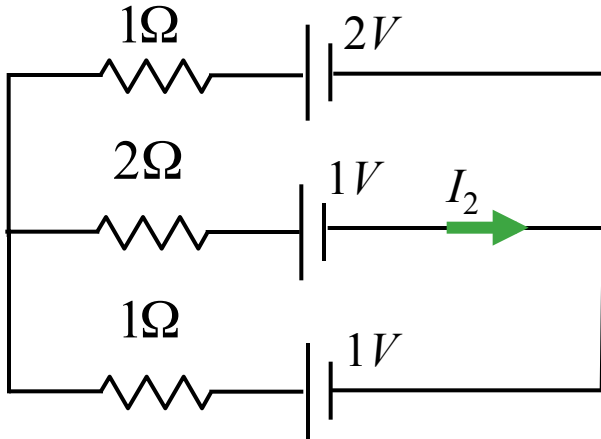
E. drop, drop, gain

With the current  VOLTAGE DROP

Against the current  VOLTAGE GAIN



# Calculation



In this circuit, assume  $V_i$  and  $R_i$  are known.

What is  $I_2$  ?

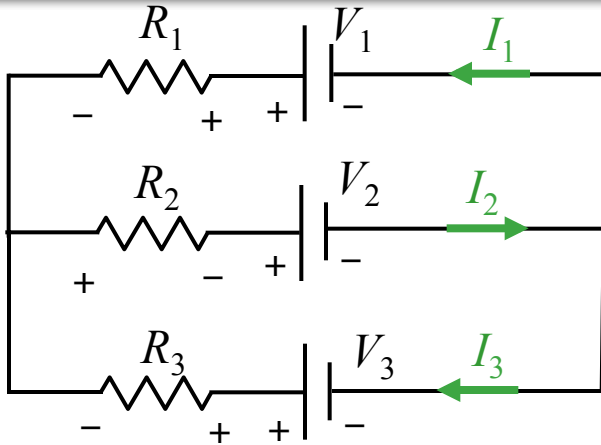
## Conceptual Analysis:

- Circuit behavior described by Kirchhoff's Rules:
  - KVR:  $\sum V_{drops} = 0$
  - KCR:  $\sum I_{in} = \sum I_{out}$

## Strategic Analysis

- Write down Loop Equations (KVR)
- Write down Node Equations (KCR)
- Solve

# Calculation



In this circuit, assume  $V_i$  and  $R_i$  are known.

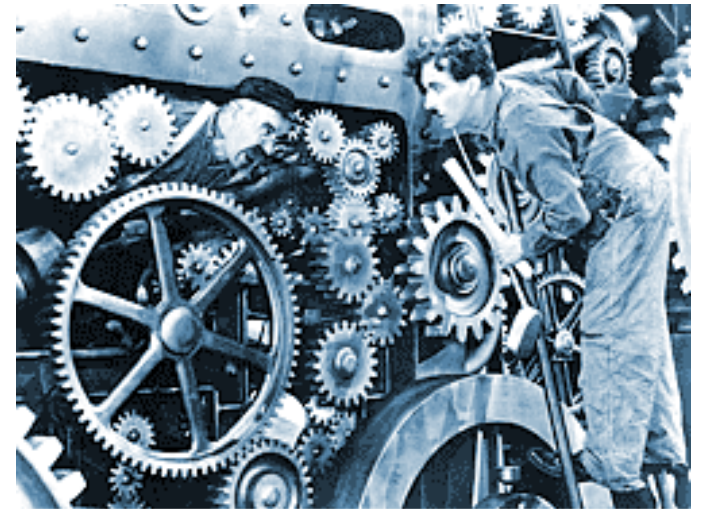
What is  $I_2$  ?

- 1) Label and pick directions for each current
- 2) Label the + and - side of each element

This is easy for batteries Long side is +

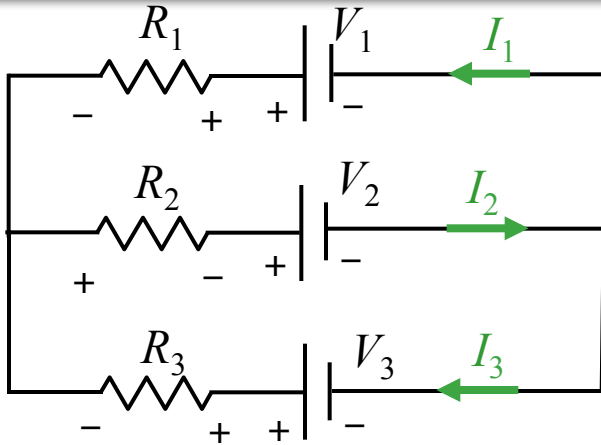
For resistors, the “upstream” side is +

Now write down loop and node equations



**Just turn the crank.**

# Calculation



In this circuit, assume  $V_i$  and  $R_i$  are known.

What is  $I_2$  ?

How many equations do we need to write down in order to solve for  $I_2$ ?

A) 1

B) 2

C) 3

D) 4

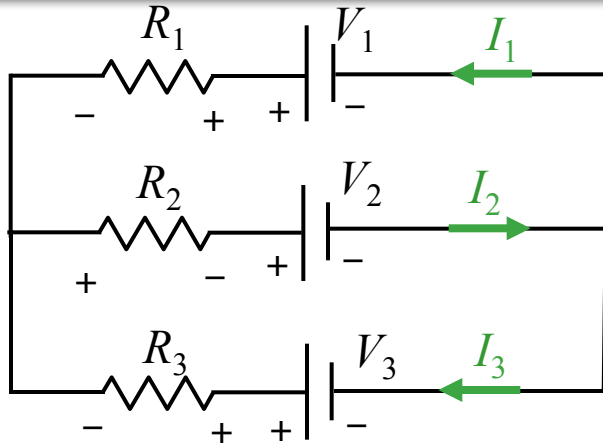
E) 5

Why?

- We have 3 unknowns:  $I_1$ ,  $I_2$ , and  $I_3$
- We need 3 independent equations to solve for these unknowns

3) Choose Loops and Directions

# Calculation



In this circuit, assume  $V_i$  and  $R_i$  are known.

What is  $I_2$  ?

Which of the following equations is NOT correct?

A)  $I_2 = I_1 + I_3$

B)  $-V_1 + I_1 R_1 - I_3 R_3 + V_3 = 0$

C)  $-V_3 + I_3 R_3 + I_2 R_2 + V_2 = 0$

D)  $-V_2 - I_2 R_2 + I_1 R_1 + V_1 = 0$

4) Write down voltage drops

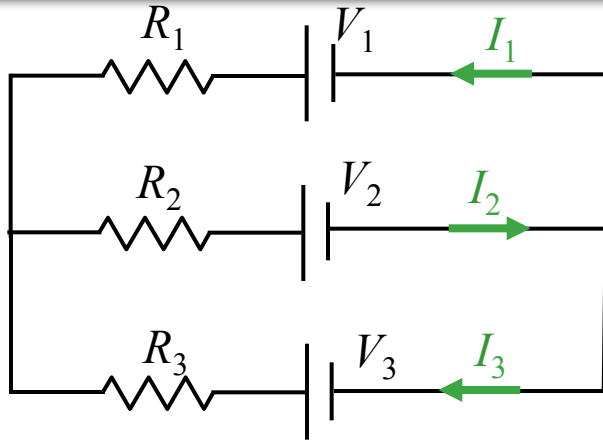
5) Write down node equation

Why?

- (D) is an attempt to write down *KVR* for the top loop
- Start at negative terminal of  $V_2$  and go clockwise

$V_{\text{gain}} (-V_2)$  then  $V_{\text{gain}} (-I_2 R_2)$  then  $V_{\text{gain}} (-I_1 R_1)$  then  $V_{\text{drop}} (+V_1)$

# Calculation



In this circuit, assume  $V_i$  and  $R_i$  are known.

What is  $I_2$  ?

We have the following 4 equations:

1.  $I_2 = I_1 + I_3$
2.  $-V_1 + I_1 R_1 - I_3 R_3 + V_3 = 0$
3.  $-V_3 + I_3 R_3 + I_2 R_2 + V_2 = 0$
4.  $-V_2 - I_2 R_2 - I_1 R_1 + V_1 = 0$

Why?

- We need 3 INDEPENDENT equations
- Equations 2, 3, and 4 are NOT INDEPENDENT

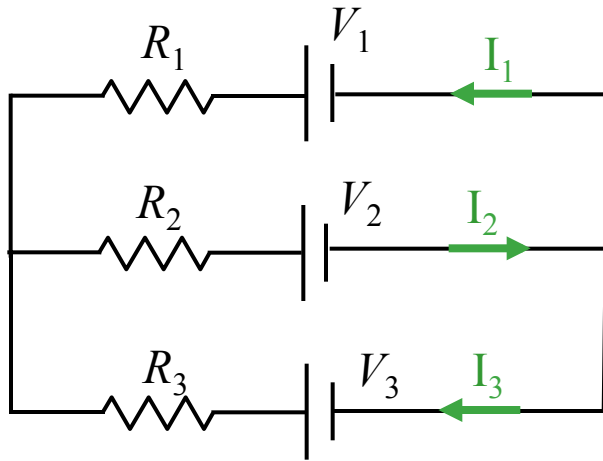
$$\text{Eqn 2} + \text{Eqn 3} = -\text{Eqn 4}$$

- We must choose Equation 1 and any two of the remaining ( 2, 3, and 4)

We need 3 equations:  
Which 3 should we use?

- A) Any 3 will do
- B) 1, 2, and 4**
- C) 2, 3, and 4

# Calculation



In this circuit, assume  $V_i$  and  $R_i$  are known.

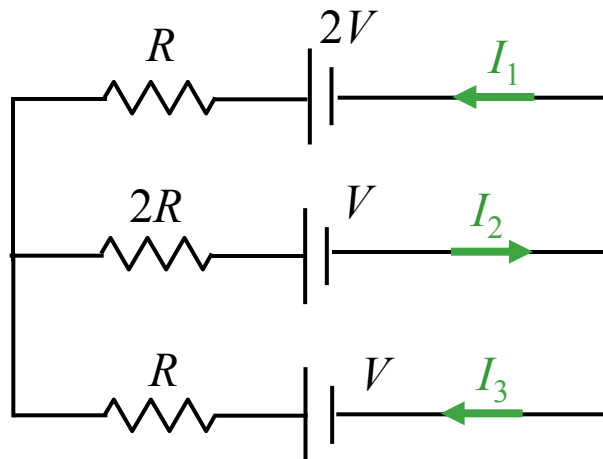
What is  $I_2$  ?

We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$V_1 + I_1 R_1 - I_3 R_3 + V_3 = 0$$

$$V_2 - I_2 R_2 - I_1 R_1 + V_1 = 0$$



Now just need to solve 😊

The solution will get very messy!

Simplify: assume  $V_2 = V_3 = V$

$$V_1 = 2V$$

$$R_1 = R_3 = R$$

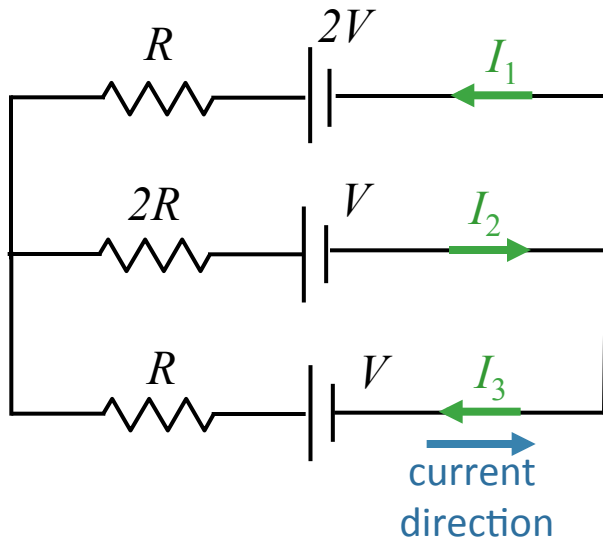
$$R_2 = 2R$$



# Calculation: Simplify

In this circuit, assume  $V$  and  $R$  are known.

What is  $I_2$  ?



We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$-2V + I_1R - I_3R + V = 0 \quad (\text{outside})$$

$$-V - I_2(2R) - I_1R + 2V = 0 \quad (\text{top})$$

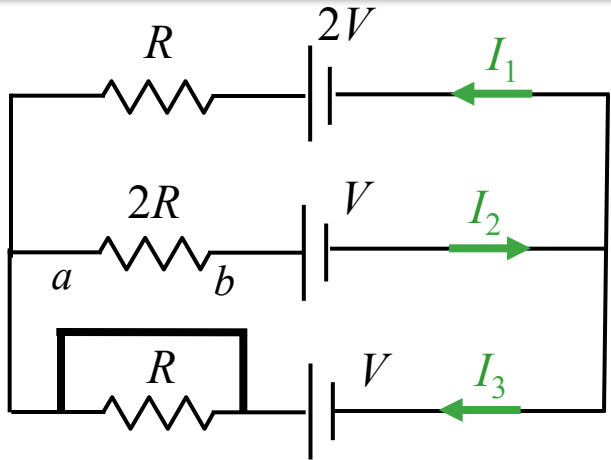
With this simplification, you can verify:

$$I_2 = (1/5) V/R$$

$$I_1 = (3/5) V/R$$

$$I_3 = (-2/5) V/R$$

# Follow Up



We know:

$$I_2 = (1/5) V/R$$

$$I_1 = (3/5) V/R$$

$$I_3 = (-2/5) V/R$$

Suppose we short  $R_3$ : What happens to  $V_{ab}$  (voltage across  $R_2$ )?

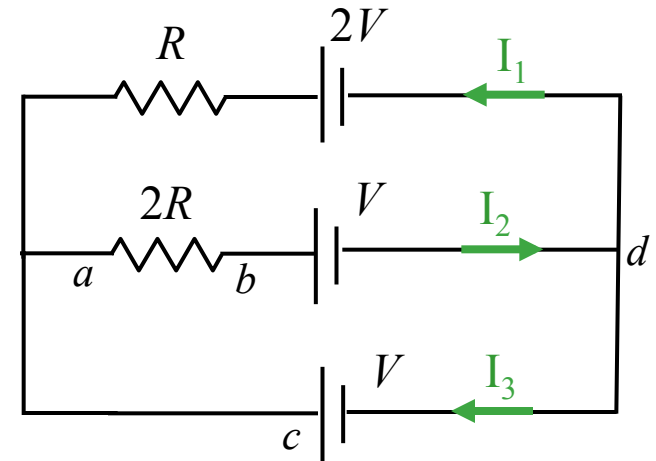
- A)  $V_{ab}$  remains the same
- B)  $V_{ab}$  changes sign
- C)  $V_{ab}$  increases
- D)  $V_{ab}$  goes to zero**

Bottom Loop Equation:

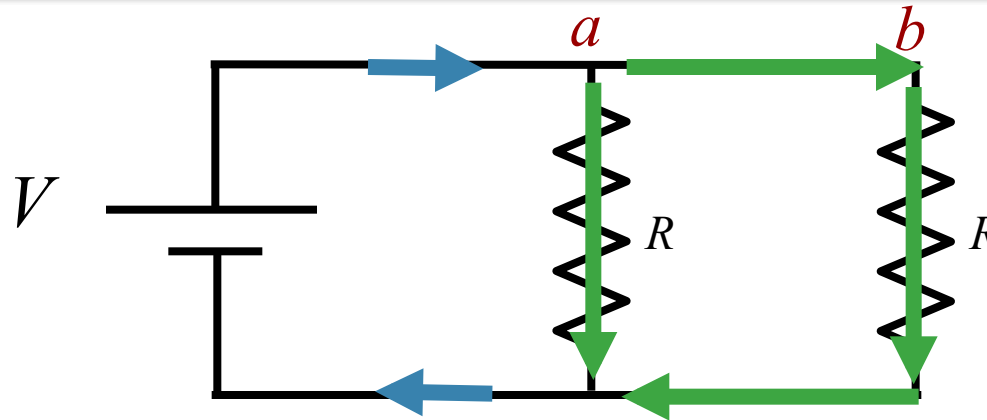
$$V_{ab} + V - V = 0$$

↓  
 $V_{ab} = 0$

Why?  
Redraw:



# CheckPoint 3 Warm up



Is there a current flowing between *a* and *b* ?

A) Yes

B) No

*a* & *b* have the same potential

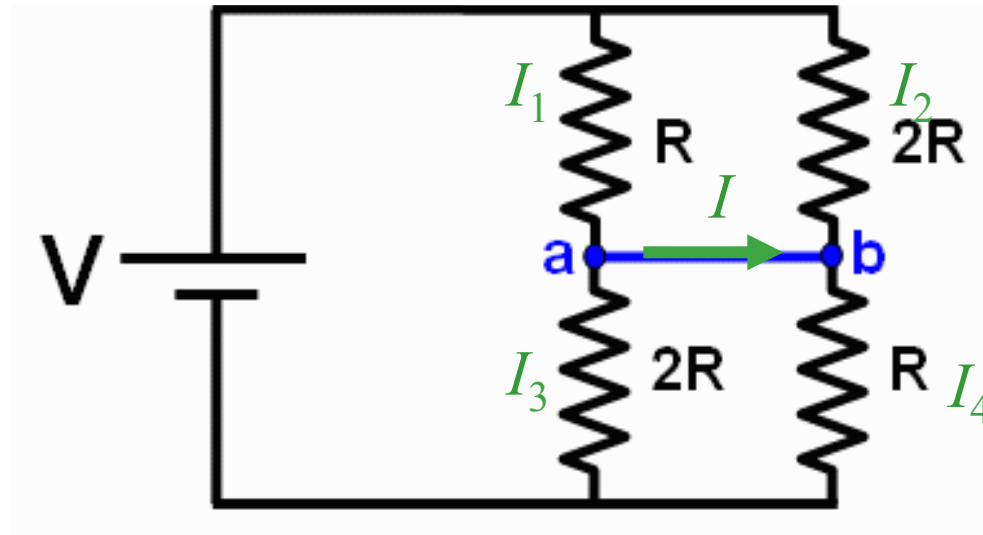
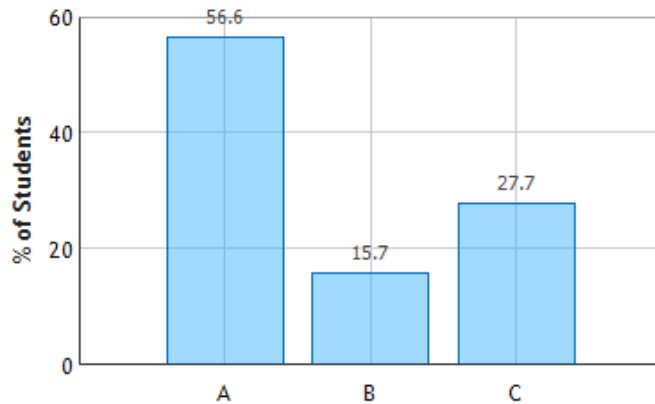
No current flows between *a* & *b*

Current flows from battery and splits at *a*

Some current flows down  
Some current flows right

# CheckPoint 3a

Consider the circuit shown below. Note that this question is *not* identical to the similar looking one you answered in the prelecture.



Which of the following best describes the current flowing in the blue wire connecting points **a** and **b**?

- A. Positive current flows from a to b**  
**C. No current flows between a and b**

**B. Positive current flows from b to a**

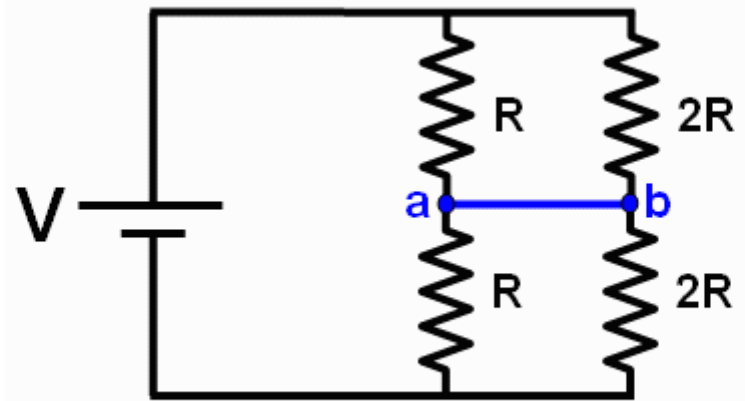
$$I_1 R - I_2 (2R) = 0 \quad \rightarrow \quad I_2 = \frac{1}{2} I_1$$

$$I_4 R - I_3 (2R) = 0 \quad \rightarrow \quad I_4 = 2 I_3$$

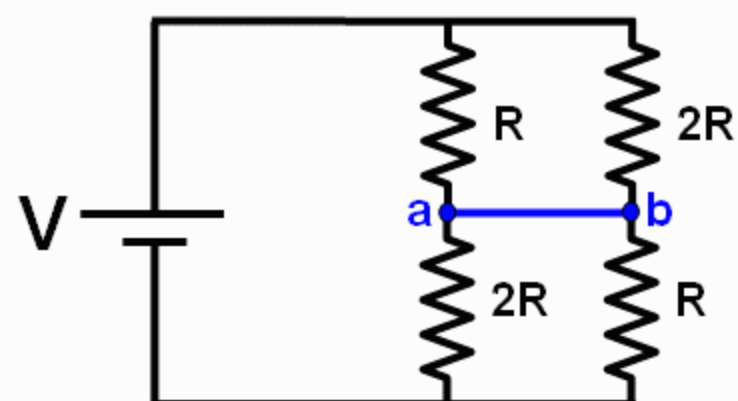
$$I = I_1 - I_3$$

$$I + I_2 = I_4 \quad \rightarrow \quad I_1 - I_3 + \frac{1}{2} I_1 = 2 I_3 \quad \rightarrow \quad I_1 = 2 I_3 \quad \rightarrow \quad I = +I_3$$

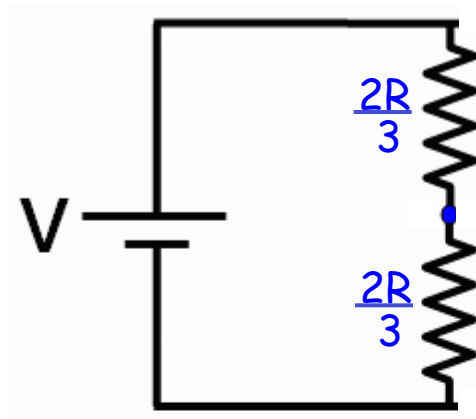
## Prelecture



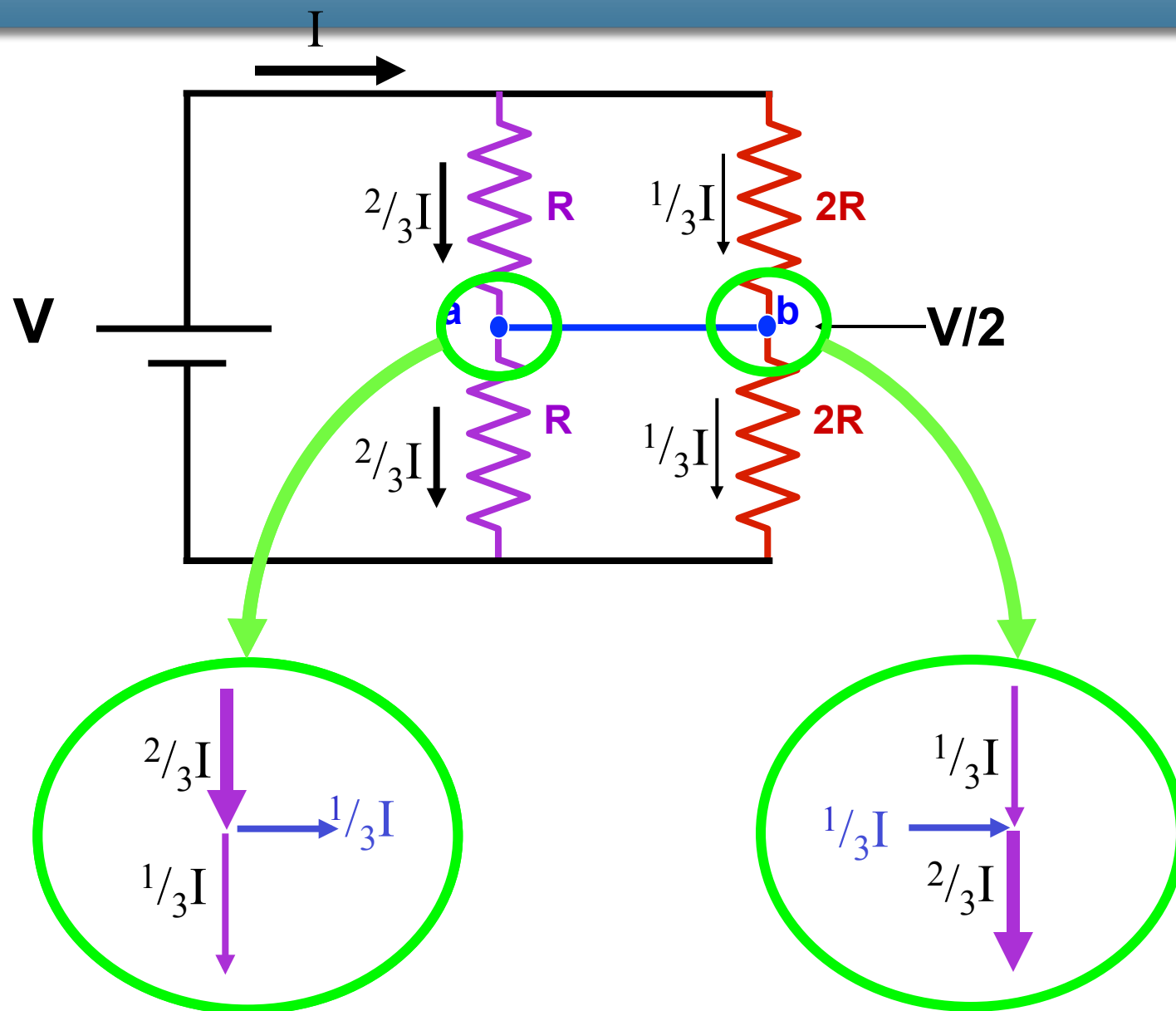
## CheckPoint



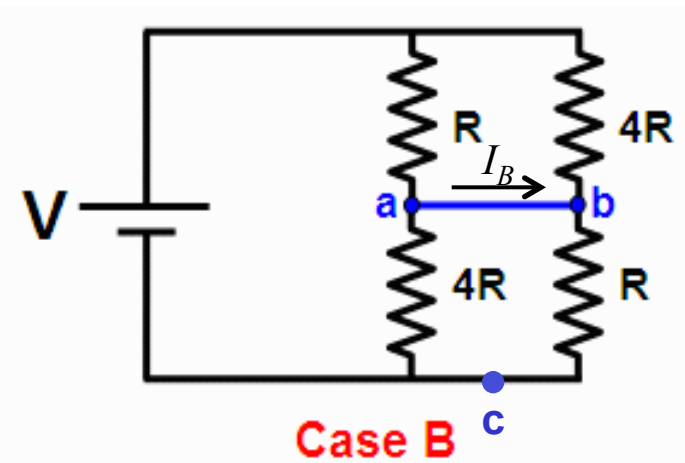
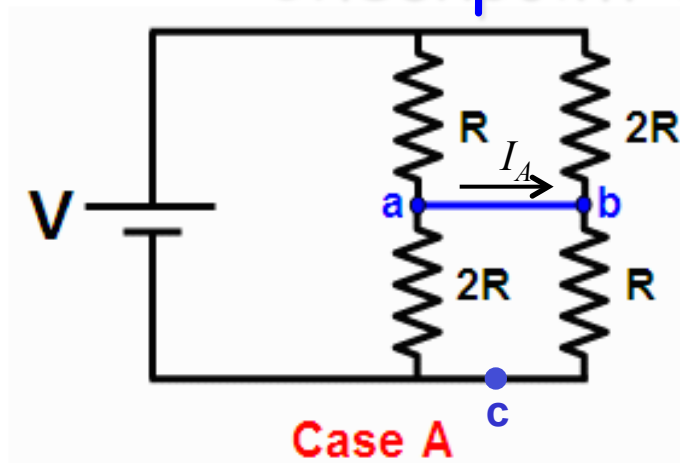
What is the same? Current flowing in and out of the battery.



What is different? Current flowing from a to b.



## Checkpoint 3b



which case is the current flowing in the blue wire connecting points **a** and **b** the largest?

**A.** Case A

**B.** Case B

**C.** They are both the same

Current will flow from left to right in both cases.

In both cases,  $V_{ac} = V/2$



$$I_{2R} = 2I_{4R}$$

$$\begin{aligned} I_A &= I_R - I_{2R} \\ &= I_R - 2I_{4R} \end{aligned}$$

$$I_B = I_R - I_{4R}$$

