

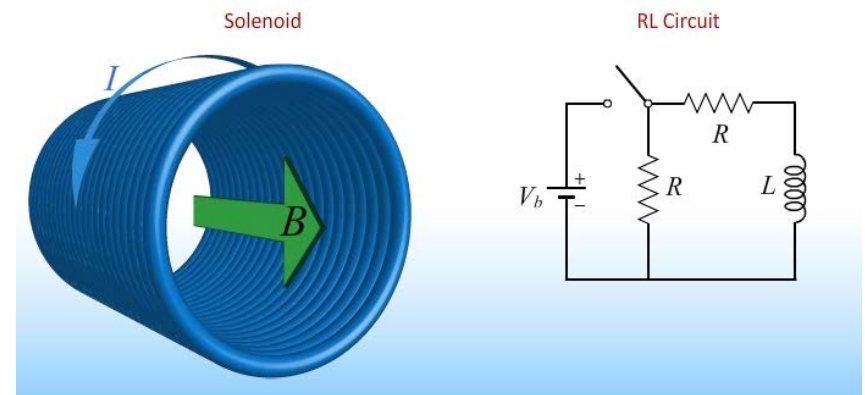
Physics 212

Lecture 18

INDUCTION and RL CIRCUITS

Today's Concepts:

- A) Induction
- B) RL Circuits



Your Comments

Can we go over how an inductor can double the battery's voltage please? This doesn't make sense to me...

Can we seriously not get one day off? I mean, its not like we have a midterms in this class in a few hours... JUST KIDDING, WE DO.

Thanks for making my birthday incredibly special with the gift of an exam!

I'm struggling with finding an intuitive way to think about inductance. Can you possibly provide one?

Okay, I know I breezed through this pre-lecture. This is just so inhumane. I've got three tests tomorrow and I almost forgot this pre-lecture. We are still going over new stuff while I am trying to solidify the knowledge for my exams. And I got one super hard homework for one of math classes on Friday morning which I haven't started yet and can't start until Thursday night. I'm just soooooo exhausted...Sorry for breezing through it but I promise I'll come back to read through the pre-lecture after I finish my second test at 3pm. I'm going to Florida for spring break, yay!!!

THANK YOU SO MUCH for making the homework due the Tuesday after break! I have to write cobalt

There are so many formulas in this class. I'm trying to understand exactly where everything is derived from instead of just memorizing them and their functions, but that is a lot of stuff to derive, and I often get one derivation confused with another.

Some Exam Stuff

Exam 2: Tonight March 19 at 7:00

- Covers material in Lectures 9 – 16 (NOT Today's material)

No Lab or Quiz this week

The exam is tonight!



Should you get the full bonus point just for choosing to come to today's lecture? (you still need to answer all questions to get the participation point)

- a) Yes
- b) No
- c) I forgot my clicker!



RC Circuit Review



An an RC circuit what do we know “immediately after...”?

- a) Charge on capacitor is same as immediately before
- b) Current through capacitor is same as immediately before
- c) Current through capacitor is zero
- d) Both a and b
- e) Both a and c

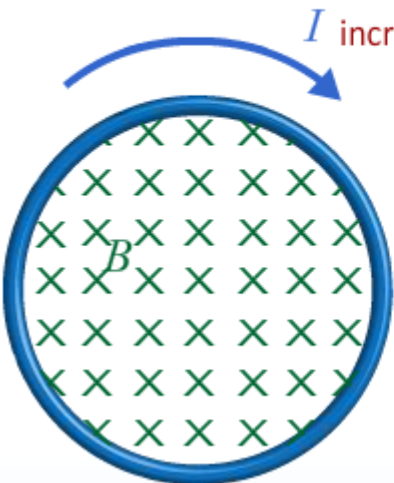
RC Circuit Review 2



An an RC charging circuit what do we know “after a long time...”?

- a) Voltage across capacitor is same as battery voltage
- b) Voltage across capacitor is zero
- c) Current through capacitor is zero
- d) Both a and b
- e) Both a and c

From the Prelecture: Self Inductance



I increases

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

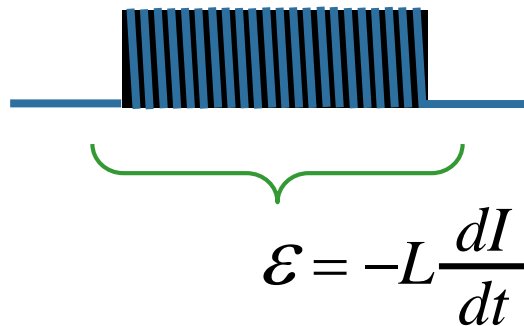
Self-Inductance

$$L \equiv \frac{\Phi_B}{I}$$

SI Unit

$$H = T \cdot m^2 / A$$

Wrap a wire into a coil to make an “inductor”...



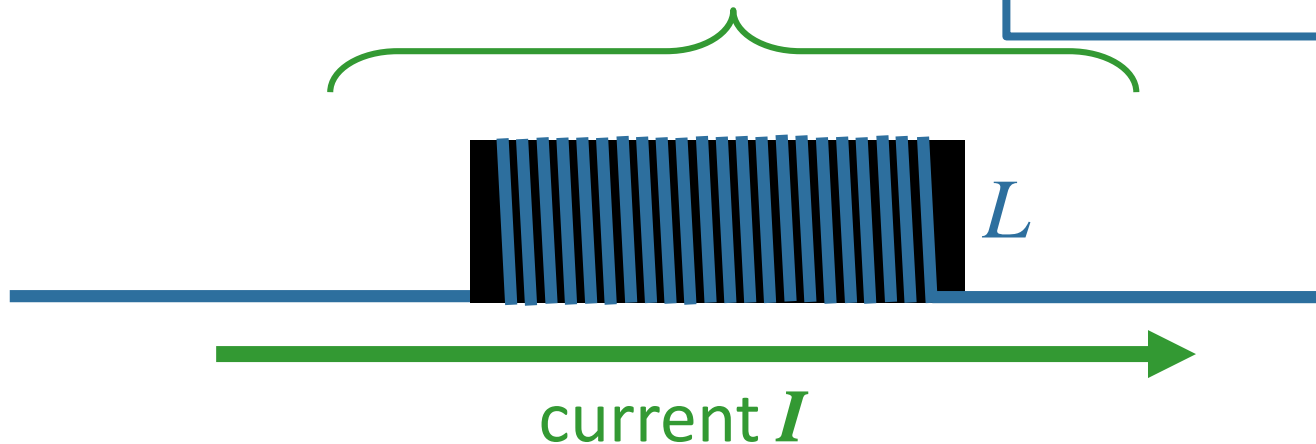
What this really means:

emf induced across L tries to keep I constant.

$$\mathcal{E}_L = -L \frac{dI}{dt}$$

Short Term $I_{\text{before}} = I_{\text{after}}$

Long Term $V_L = 0$



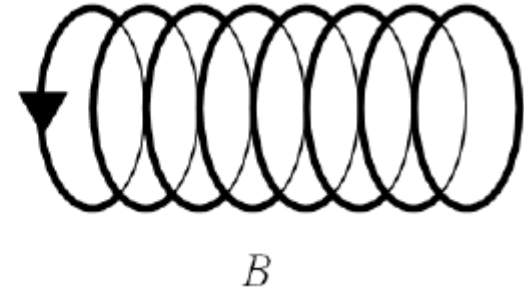
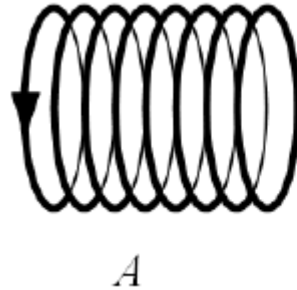
Inductors prevent discontinuous current changes!

It's like inertia!

CheckPoint 1

Two solenoids are made with the same cross sectional area and total number of turns. Inductor *B* is twice as long as inductor *A*

$$L_B = \mu_0 \underset{\substack{\uparrow \\ (1/2)^2}}{n^2} \pi r^2 \underset{\substack{\uparrow \\ 2}}{z}$$

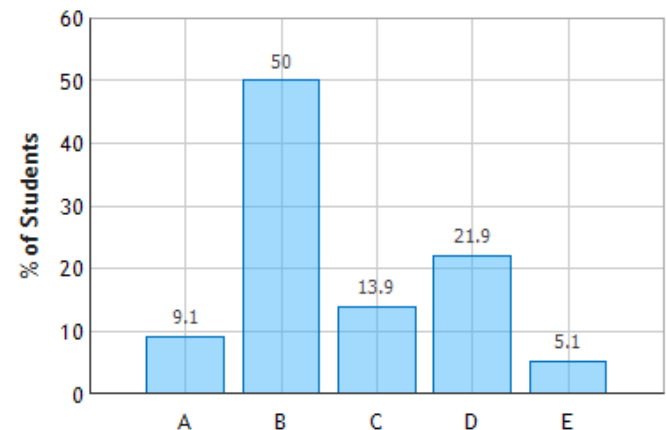


$$\longrightarrow L_B = \frac{1}{2} L_A$$

Compare the inductance of the two solenoids

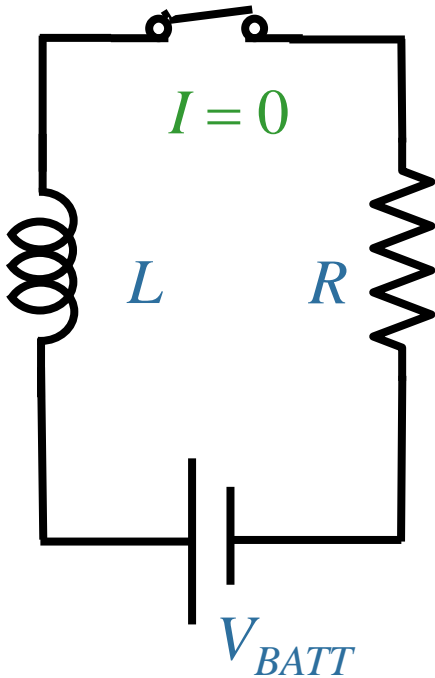
- A) $L_A = 4 L_B$
- B) $L_A = 2 L_B$
- C) $L_A = L_B$
- D) $L_A = (1/2) L_B$
- E) $L_A = (1/4) L_B$

Inductance of Solenoids: Question 1 (N = 690)



How to think about RL circuits Episode 1:

When no current is flowing initially:



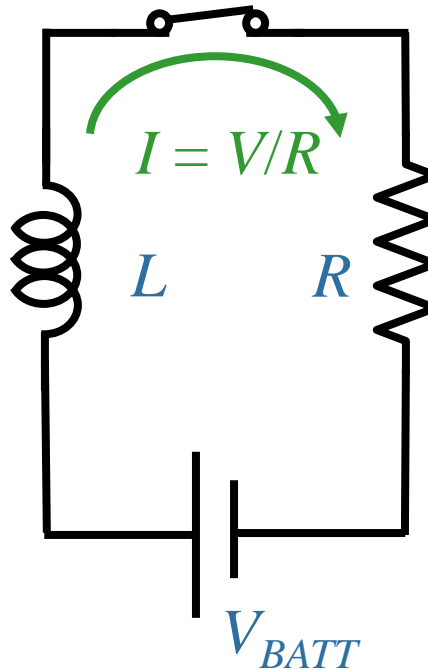
At $t = 0$: I_L unchanged

$$I_L = 0$$

$$V_R = 0$$

$$V_L = V_{BATT}$$

(L is like an open circuit)



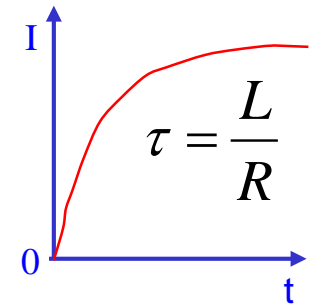
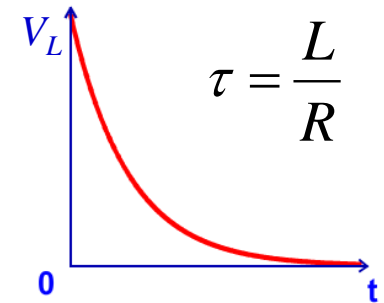
At $t \gg L/R$: $V_L = 0$

$$V_L = 0$$

$$V_R = V_{BATT}$$

$$I = V_{BATT}/R$$

(L is like a wire)



CheckPoint 2a

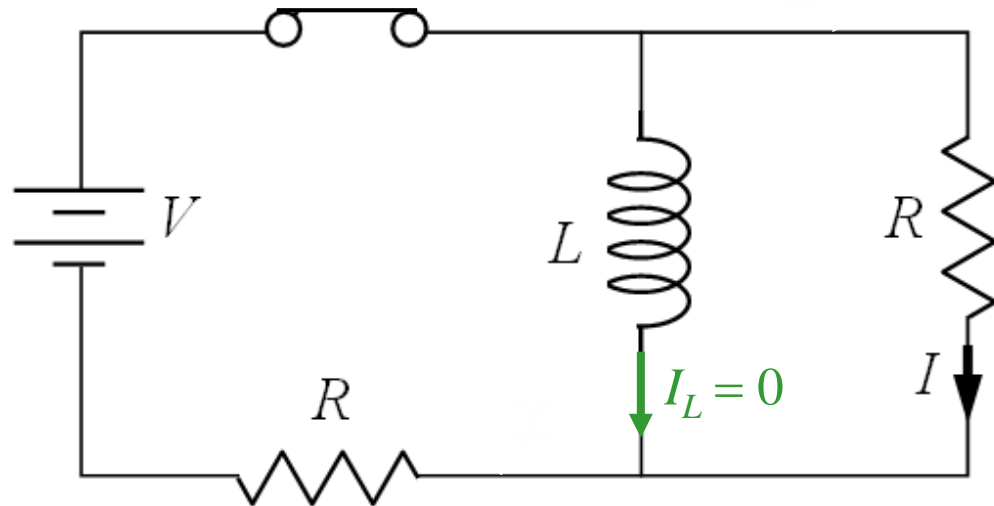


In the circuit, the switch has been open for a long time, and the current is zero everywhere.

At time $t = 0$ the switch is closed.

What is the current I through the vertical resistor immediately after the switch is closed?

(+ is in the direction of the arrow)



A) $I = V/R$

B) $I = V/2R$

C) $I = 0$

D) $I = -V/2R$

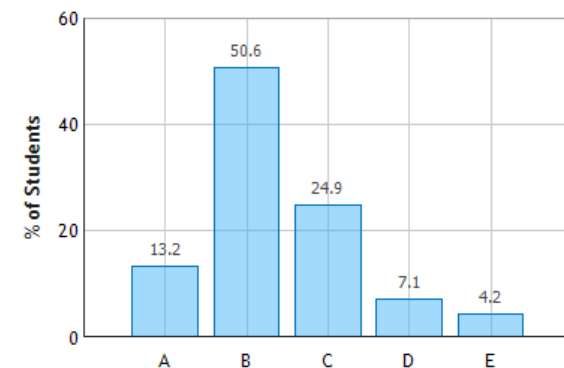
E) $I = -V/R$

Before: $I_L = 0$

After: $I_L = 0$

→ $I = + V/2R$

RL Circuit: Question 1 (N = 688)



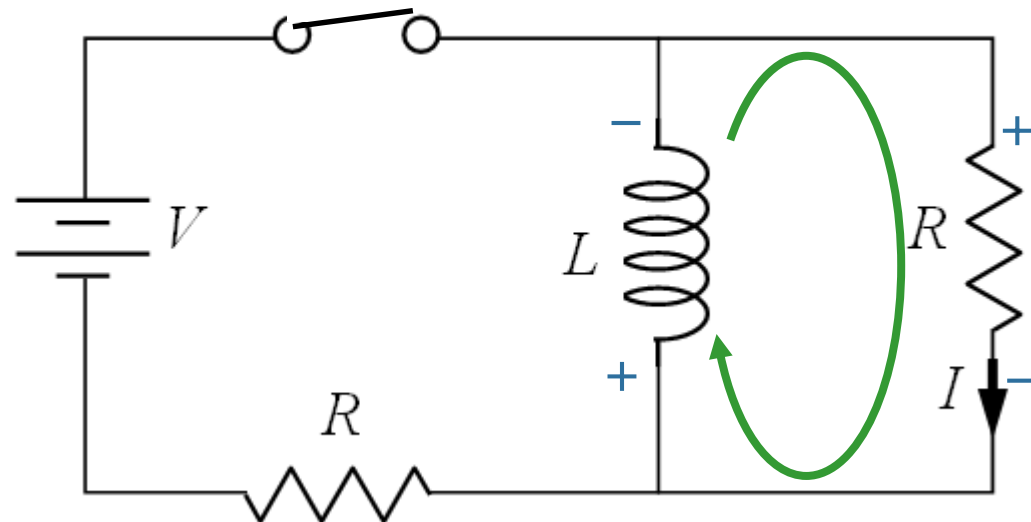
RL Circuit (Long Time)



What is the current I through the vertical resistor after the switch has been closed for a long time?

(+ is in the direction of the arrow)

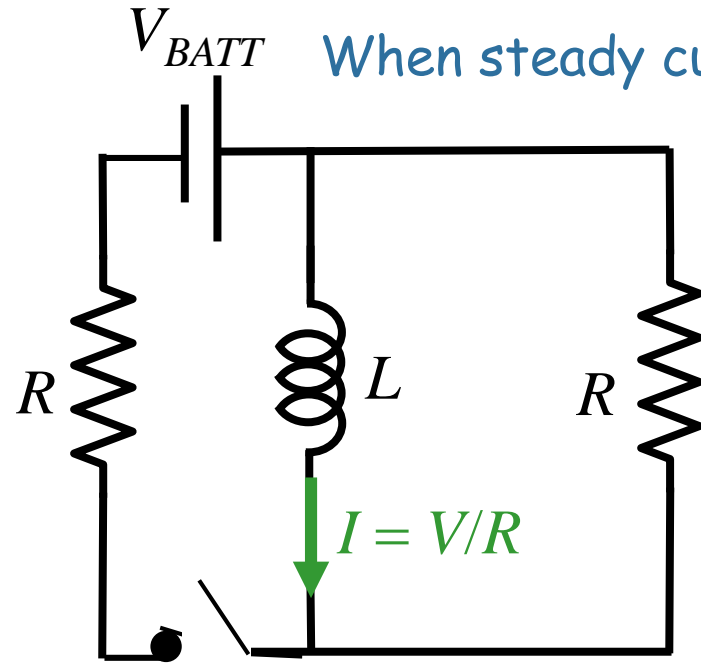
- A) $I = V/R$
- B) $I = V/2R$
- C) $I = 0$**
- D) $I = -V/2R$
- E) $I = -V/R$



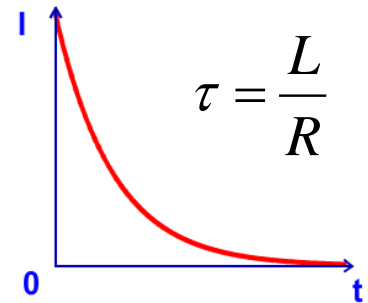
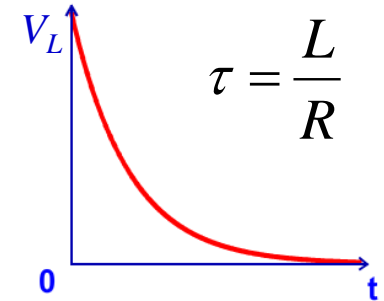
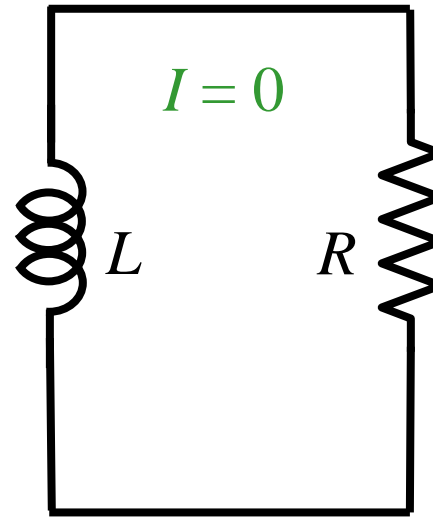
After a long time in any static circuit: $V_L = 0$

KVR:
$$V_L + IR = 0$$

How to Think about RL Circuits Episode 2:



When steady current is flowing initially: then switch is opened



At $t = 0$: I_L unchanged

$$\begin{aligned} I &= V_{BATT}/R \\ V_R &= IR \\ V_L &= V_R \end{aligned}$$

At $t \gg L/R$: $V_L = 0$

$$\begin{aligned} V_L &= 0 \\ V_R &= 0 \\ I &= 0 \end{aligned}$$

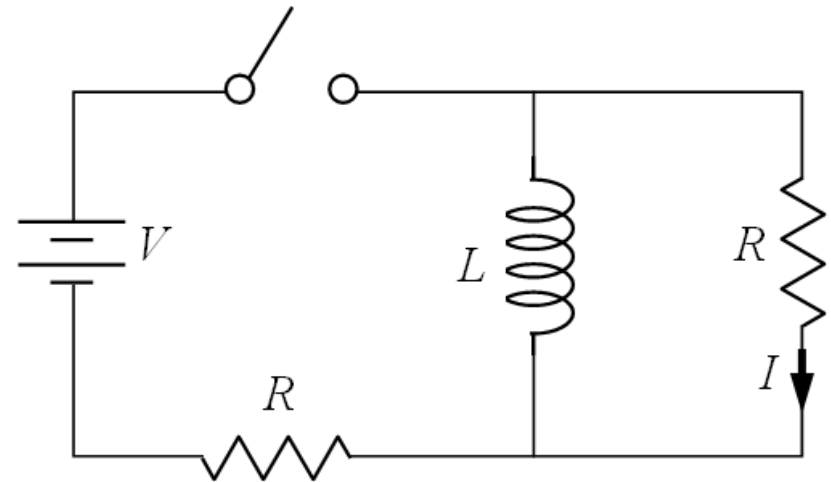
CheckPoint 2b



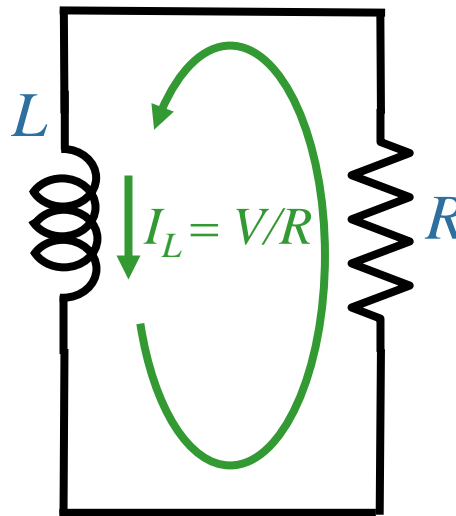
After a long time, the switch is opened, abruptly disconnecting the battery from the circuit.

What is the current I through the vertical resistor immediately after the switch is opened?

(+ is in the direction of the arrow)



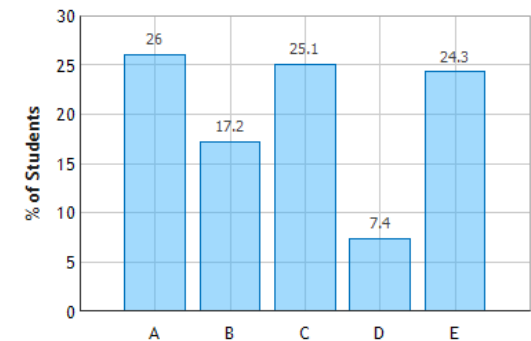
circuit when switch opened



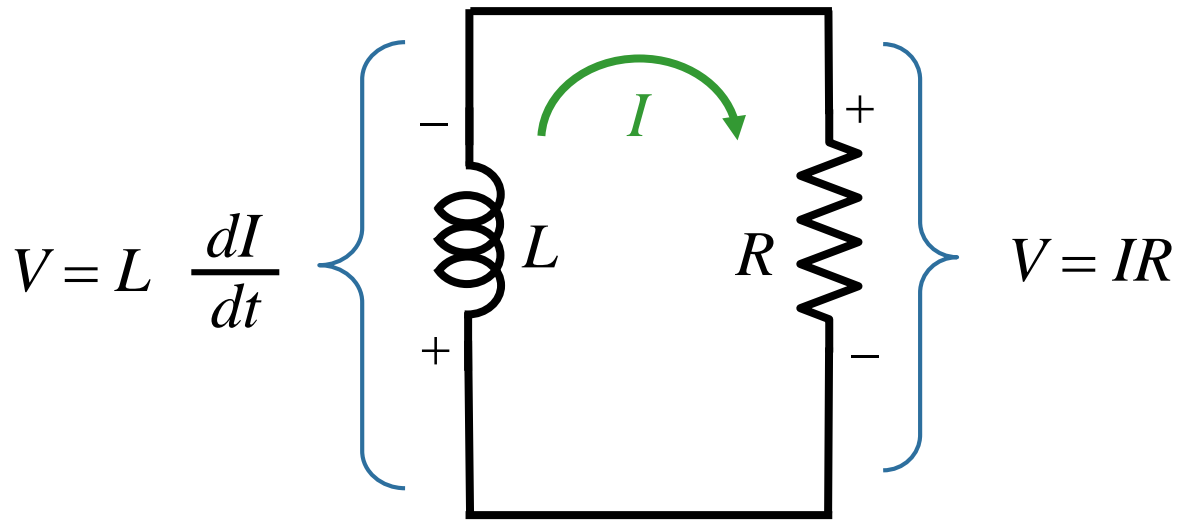
- A) $I = V/R$
- B) $I = V/2R$
- C) $I = 0$
- D) $I = -V/2R$
- E) $I = -V/R$

Current through inductor
cannot change
DISCONTINUOUSLY

RL Circuit: Question 3 (N = 688)



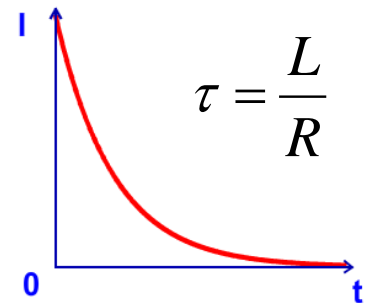
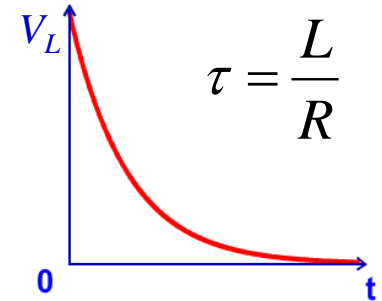
Why is there Exponential Behavior?

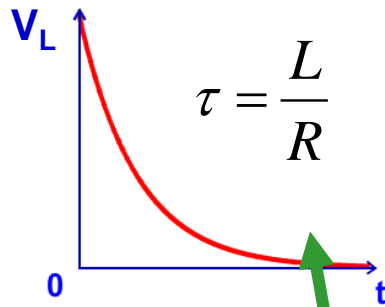
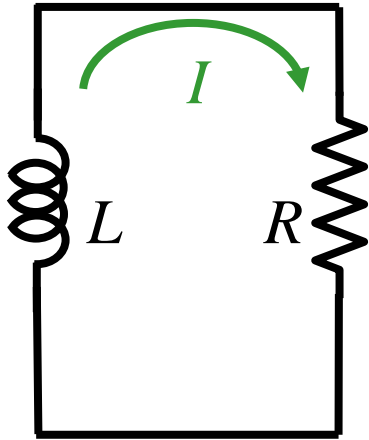


$$L \frac{dI}{dt} + IR = 0$$

$$I(t) = I_0 e^{-tR/L} = I_0 e^{-t/\tau}$$

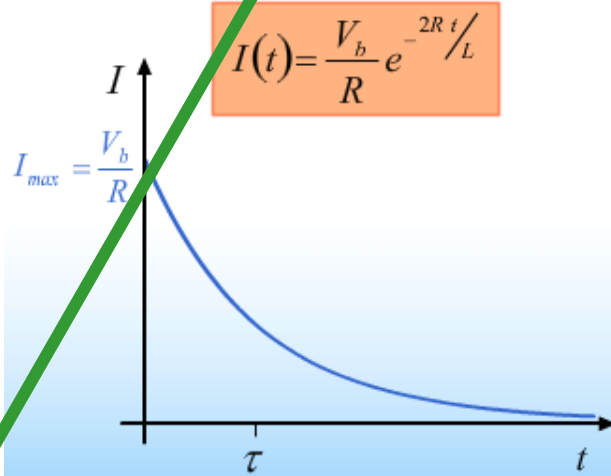
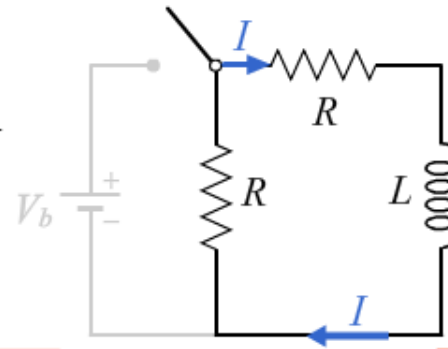
where $\tau = \frac{L}{R}$



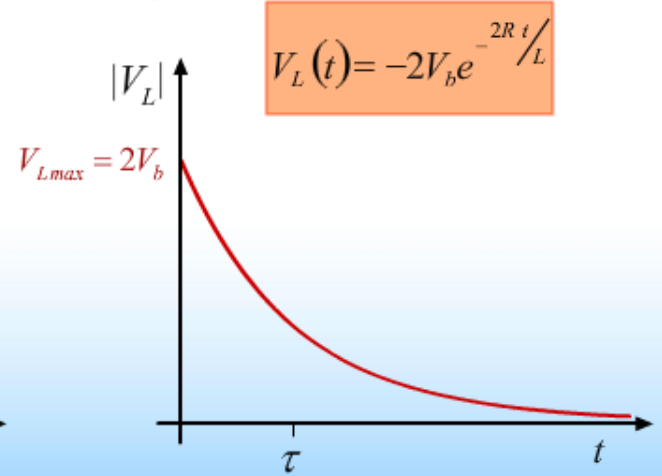


Lecture:

Time Constant $\tau = \frac{L}{2R}$



Prelecture:



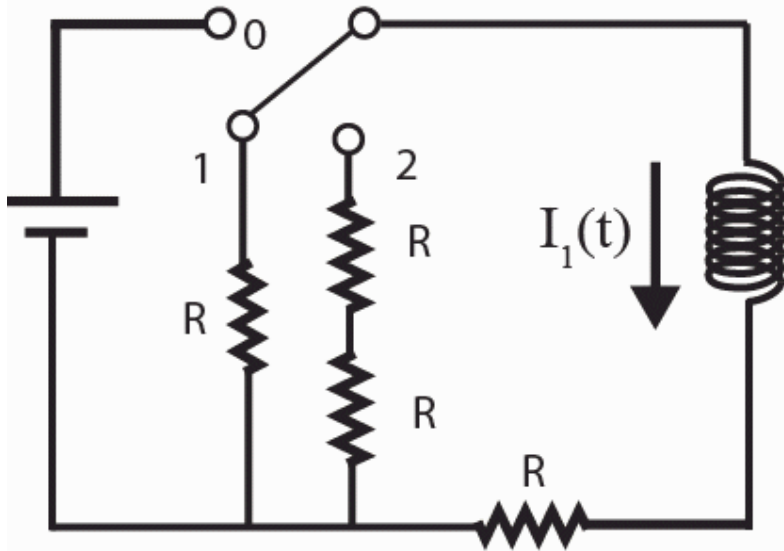
Did we mess up?

No: The resistance is simply twice as big in one case.

CheckPoint 3a

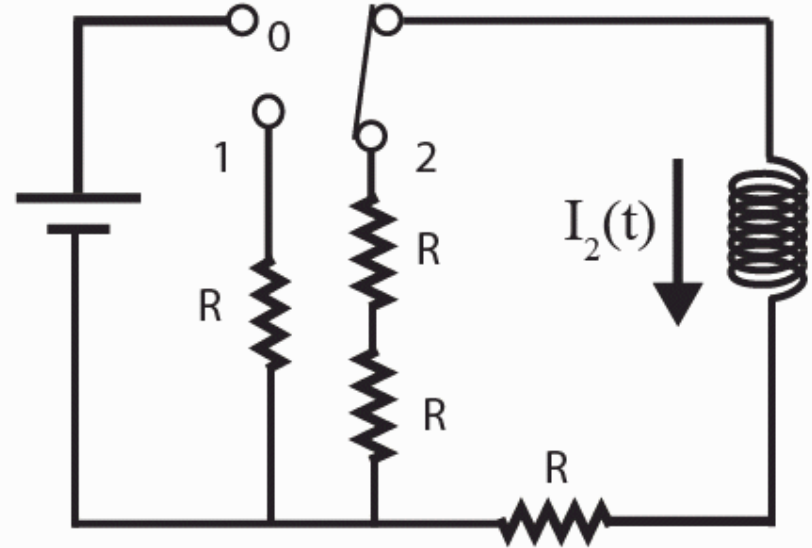


After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



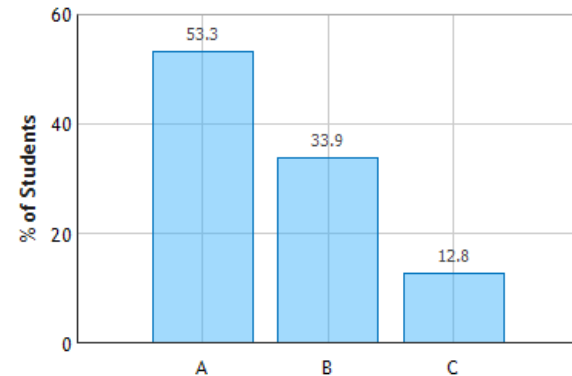
Case 2

After switch moved, which case has larger time constant?

- ☒ A) Case 1
- ☐ B) Case 2
- ☐ C) The same

$$\tau_1 = \frac{L}{2R} \quad \tau_2 = \frac{L}{3R}$$

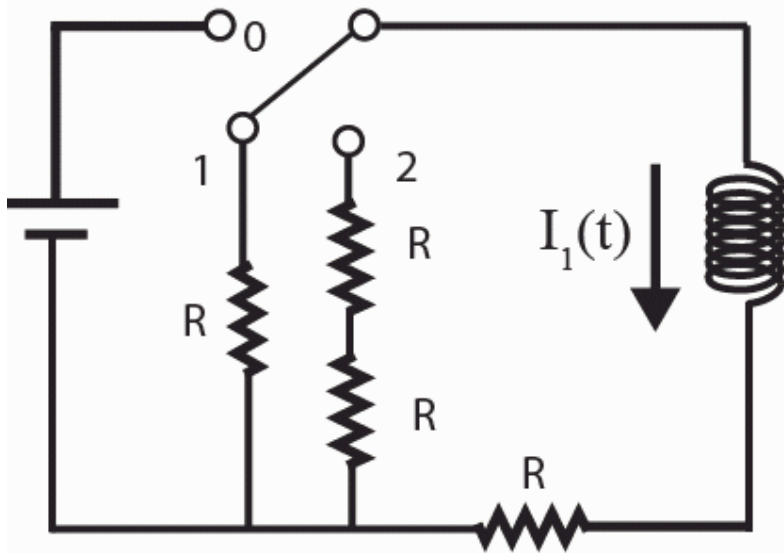
Compare RL Circuits: Question 1 (N = 758)



CheckPoint 3b

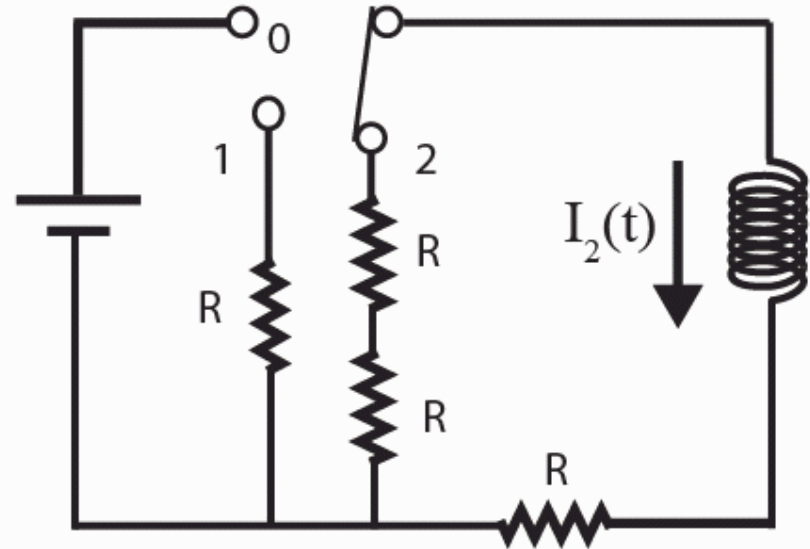


After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

Immediately after switch moved,
in which case is the voltage
across the inductor larger?

- A) Case 1
- B) Case 2**
- C) The same

Before switch moved:

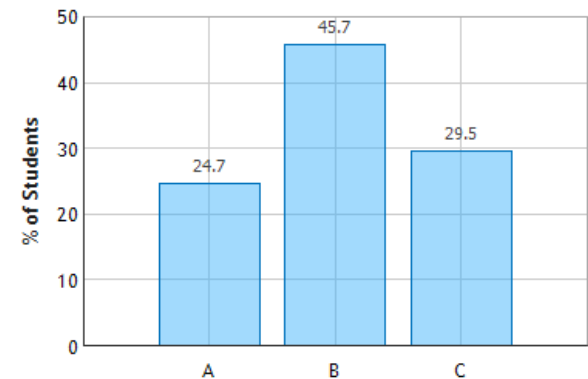
$$I = \frac{V}{R}$$

After switch moved:

$$V_{L1} = \frac{V}{R} 2R$$

$$V_{L2} = \frac{V}{R} 3R$$

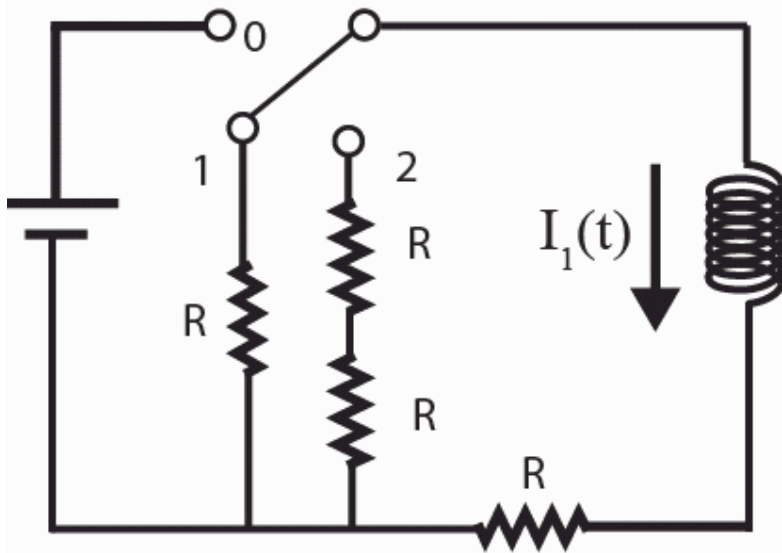
Compare RL Circuits: Question 3 (N = 687)



CheckPoint 3c

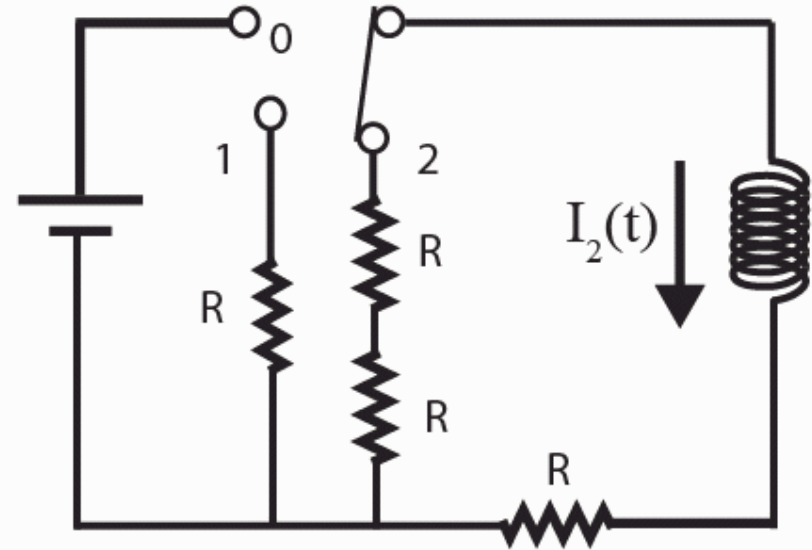


After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

After switch moved for finite time, in which case is the current through the inductor larger?

- ☒ A) Case 1
- ☐ B) Case 2
- ☐ C) The same

After awhile

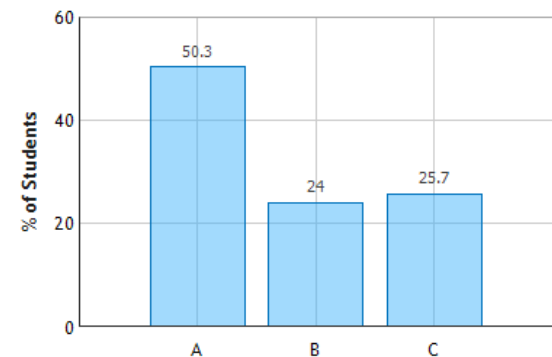
$$I_1 = Ie^{-t/\tau_1}$$

$$I_2 = Ie^{-t/\tau_2}$$

$$\tau_1 > \tau_2$$

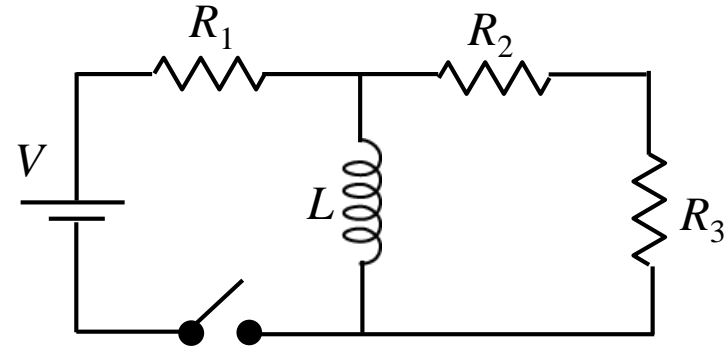
Immediately after: $I_1 = I_2$

Compare RL Circuits: Question 5 (N = 688)



Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is dI_L/dt , the time rate of change of the current through the inductor immediately after switch is closed

Conceptual Analysis

Once switch is closed, currents will flow through this 2-loop circuit.

KVR and KCR can be used to determine currents as a function of time.

Strategic Analysis

Determine currents immediately after switch is closed.

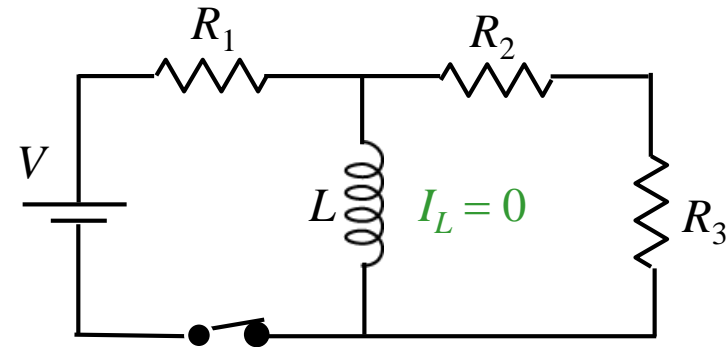
Determine voltage across inductor immediately after switch is closed.

Determine dI_L/dt immediately after switch is closed.

Calculation



The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is I_L , the current in the inductor, immediately after the switch is closed?

A) $I_L = V/R_1$ up

B) $I_L = V/R_1$ down

C) $I_L = 0$

INDUCTORS: Current cannot change discontinuously !



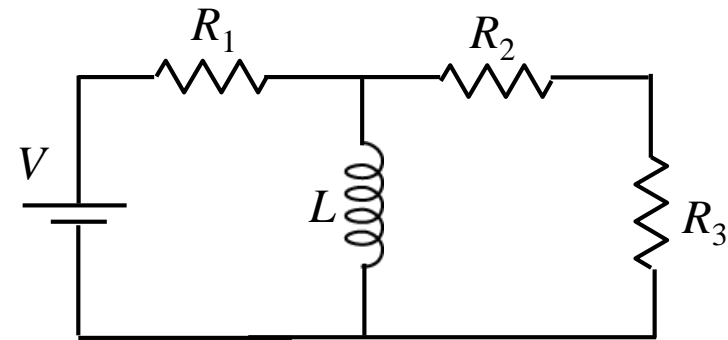
Current through inductor immediately **after** switch is closed
is the same as
the current through inductor immediately **before** switch is closed

Immediately **before** switch is closed: $I_L = 0$ since no battery in loop

Calculation



The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$$I_L(t = 0+) = 0$$

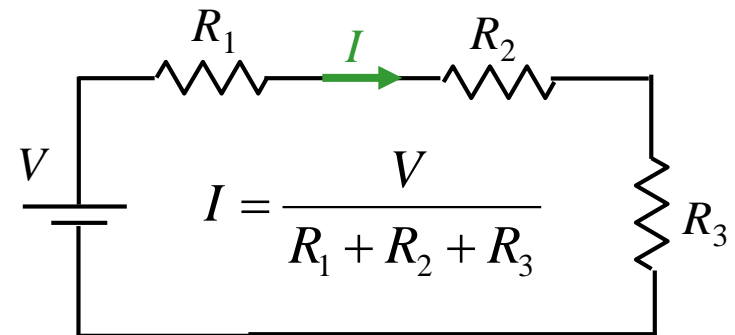
What is the magnitude of I_2 , the current in R_2 , immediately after the switch is closed?

A) $I_2 = \frac{V}{R_1}$ B) $I_2 = \frac{V}{R_2 + R_3}$ **C) $I_2 = \frac{V}{R_1 + R_2 + R_3}$** D) $I_2 = \frac{VR_2 R_3}{R_2 + R_3}$

We know $I_L = 0$ immediately after switch is closed



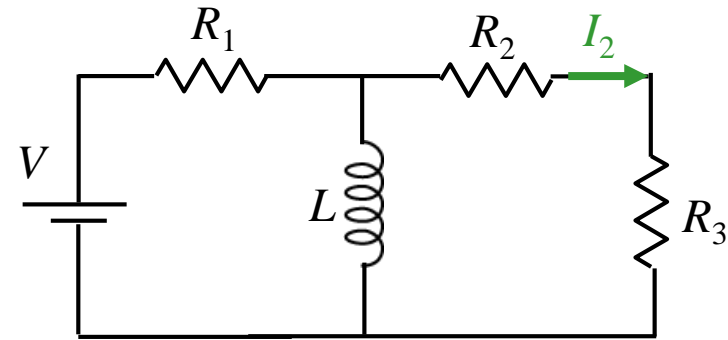
Immediately after switch is closed, circuit looks like:



Calculation



The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$$I_L(t = 0+) = 0 \quad I_2(t = 0+) = V/(R_1 + R_2 + R_3)$$

What is the magnitude of V_L , the voltage across the inductor, immediately after the switch is closed?

A) $V_L = V \frac{R_2 R_3}{R_1}$ B) $V_L = V$ C) $V_L = 0$ D) $V_L = V \frac{R_2 R_3}{R_1 (R_2 + R_3)}$ E) $V_L = V \frac{R_2 + R_3}{R_1 + R_2 + R_3}$

Kirchhoff's Voltage Law,

$$V_L - I_2 R_2 - I_2 R_3 = 0 \quad V_L = I_2 (R_2 + R_3)$$

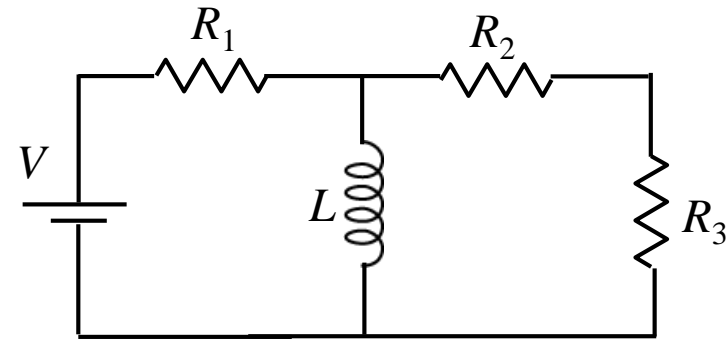
$$V_L = \frac{V}{R_1 + R_2 + R_3} (R_2 + R_3)$$

Calculation



The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.

What is dI_L/dt , the time rate of change of the current through the inductor immediately after switch is closed



$$V_L(t = 0+) = V(R_2 + R_3)/(R_1 + R_2 + R_3)$$

A) $\frac{dI_L}{dt} = \frac{V}{L} \frac{R_2 + R_3}{R_1}$ B) $\frac{dI_L}{dt} = 0$

C) $\frac{dI_L}{dt} = \frac{V}{L} \frac{R_2 + R_3}{R_1 + R_2 + R_3}$

D) $\frac{dI_L}{dt} = \frac{V}{L}$

The time rate of change of current through the inductor $(dI_L/dt) = V_L/L$

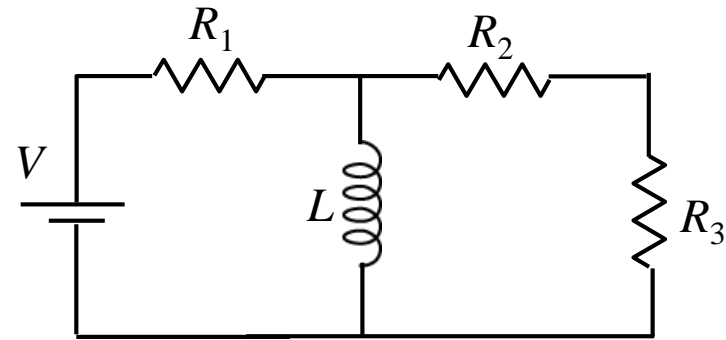
→ $\frac{dI_L}{dt} = \frac{V}{L} \frac{R_2 + R_3}{R_1 + R_2 + R_3}$

Follow Up



The switch in the circuit shown has been closed for a long time.

What is I_2 , the current through R_2 ?
(Positive values indicate current flows to the right)



A) $I_2 = +\frac{V}{R_2 + R_3}$ B) $I_2 = +\frac{V(R_2 R_3)}{R_1 + R_2 + R_3}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_2 + R_3}$

After a long time, $dI/dt = 0$

Therefore, the voltage across $L = 0$

Therefore the voltage across $R_2 + R_3 = 0$

Therefore the current through $R_2 + R_3$ must be zero!

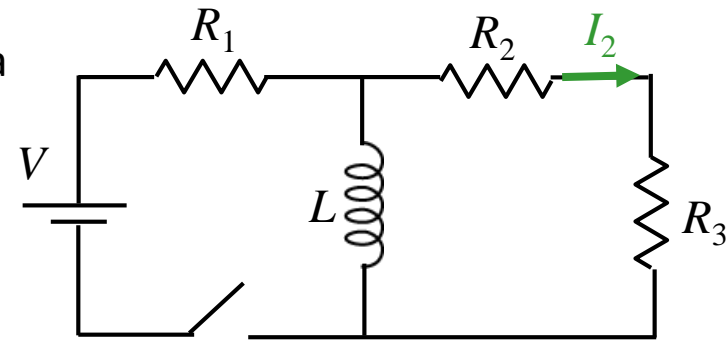
Follow Up 2



The switch in the circuit shown has been closed for a long time at which point, the switch is opened.

What is I_2 , the current through R_2 immediately after switch is opened ?

(Positive values indicate current flows to the right)



A) $I_2 = +\frac{V}{R_1 + R_2 + R_3}$

B) $I_2 = +\frac{V}{R_1}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_1}$

E) $I_2 = -\frac{V}{R_1 + R_2 + R_3}$

Current through inductor immediately **after** switch is opened
is the same as
the current through inductor immediately **before** switch is opened

Immediately **before** switch is opened: $I_L = V/R_1$

Immediately **after** switch is opened: I_L flows in right loop

Therefore, $I_L = -V/R_1$