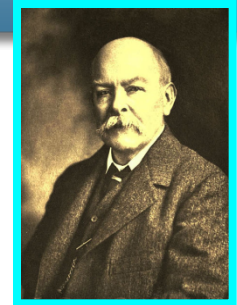


Your Comments



WHAT? "POYNTING" VECTOR NOT "POINTING" VECTOR? I THOUGHT I COUGHT A SPELLING MISTAKE!!!!!!!

actual question. so does the poynting vector always point in the direction of the wave?

Not too bad I just have difficulty understanding the correlation of equations to the graphs.

If you're in a car traveling at the speed of light and you turn your headlights on, does anything happen?

What is a plane wave? There can't possibly be a uniform electric/magnetic field at all points in that plane!

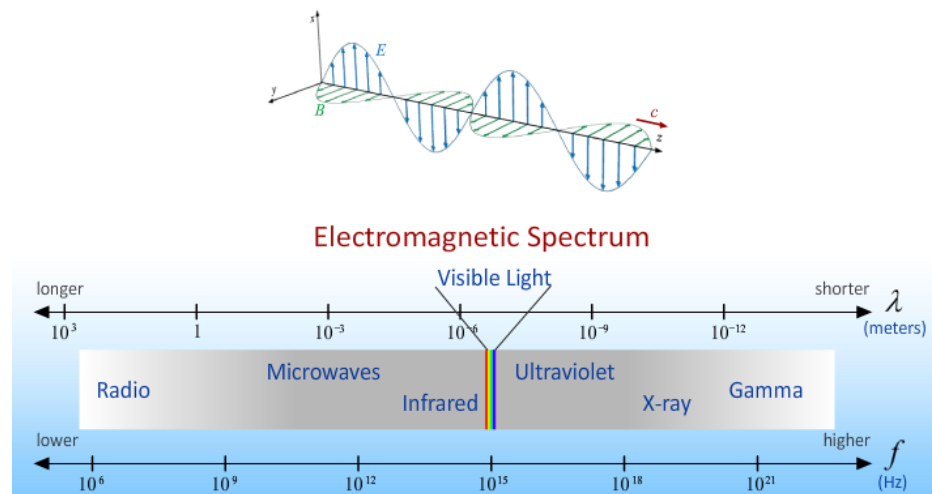
Is there any conflict between photoelectric effect (f decides energy) and the expression $p = \epsilon_0 E^2$?

I tried to come up with a witty waves pun, but I must poynt out that I don't have the energy.

Physics 212

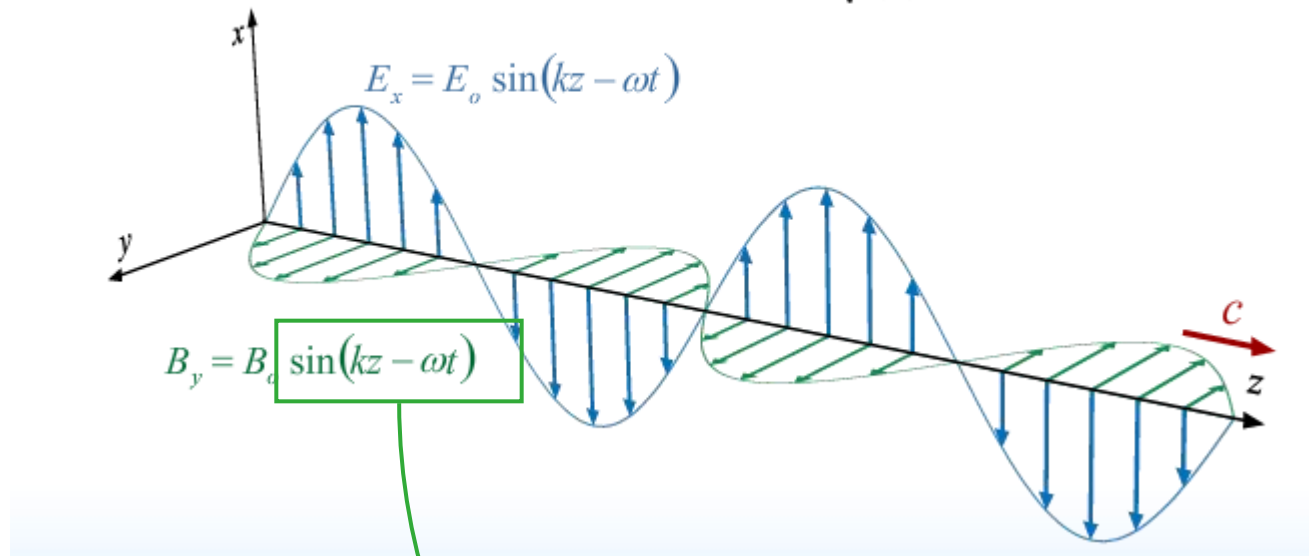
Lecture 23

PROPERTIES of ELECTROMAGNETIC WAVES



Plane Waves from Last Time

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_0}{B_0} = 3 \times 10^8 \text{ m/s}$



E and B are perpendicular and in phase

Oscillate in time and space

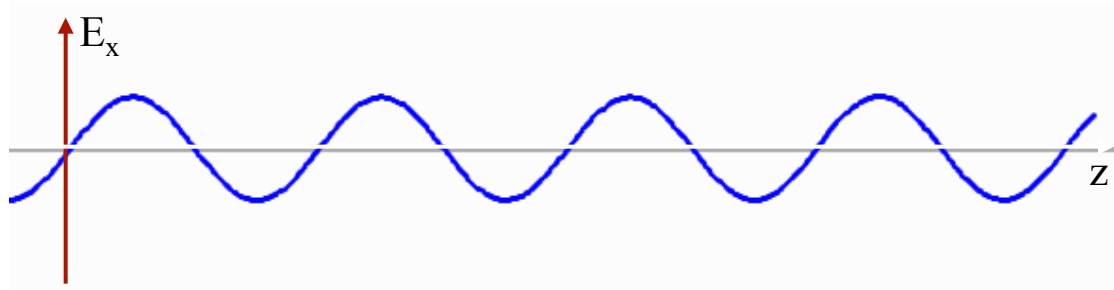
Direction of propagation given by $E \times B$

$$E_0 = cB_0$$

Argument of \sin/\cos gives direction of propagation

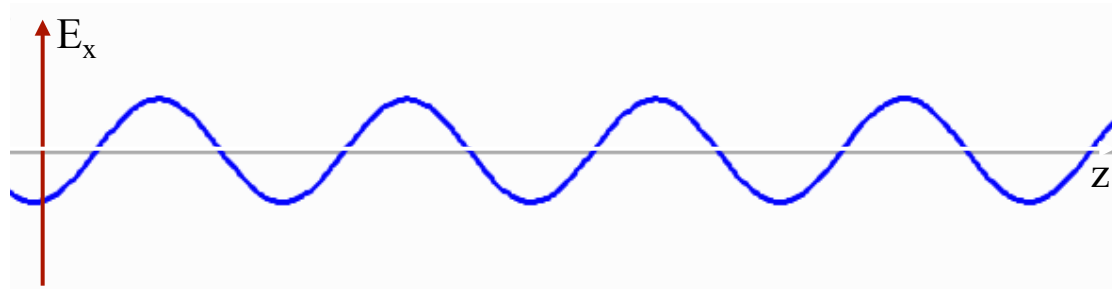
Understanding the speed and direction of the wave

$$E_x = E_0 \sin(kz - \omega t)$$



$t = 0$

$$\sin(kz - \frac{\pi}{2}) = -\cos(kz)$$

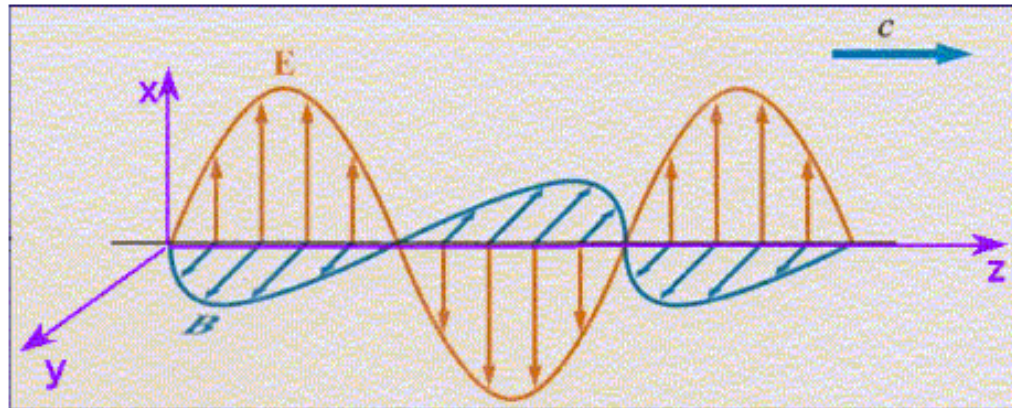


$t = \pi/(2\omega)$

What has happened to the wave form in this time interval?

It has MOVED TO THE RIGHT by $1/4 \lambda$

CheckPoint 1a



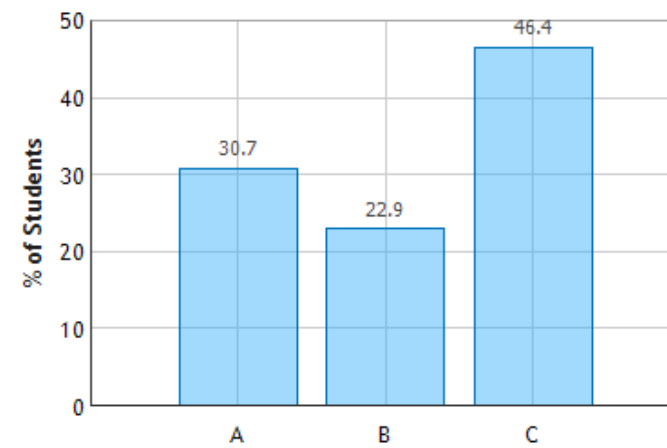
Which equation correctly describes this electromagnetic wave?

☐ $E_x = E_o \sin(kz \oplus \omega t)$ No – moving in the minus z direction

☐ $E_y = E_o \sin(kz - \omega t)$ No – has E_y rather than E_x

☒ $B_y = B_o \sin(kz - \omega t)$

Electromagnetic Waves: Question 1 (N = 828)



Checkpoint 2a



Your iclicker operates at a frequency of approximately 900 MHz (900×10^6 Hz). What is the approximately wavelength of the EM wave produced by your iclicker?

- A. ☐ 0.03 meters
- B. ☒ 0.3 meters
- C. ☐ 3.0 meters
- D. ☐ 30. meters

$$C = 3.0 \times 10^8 \text{ m/s}$$

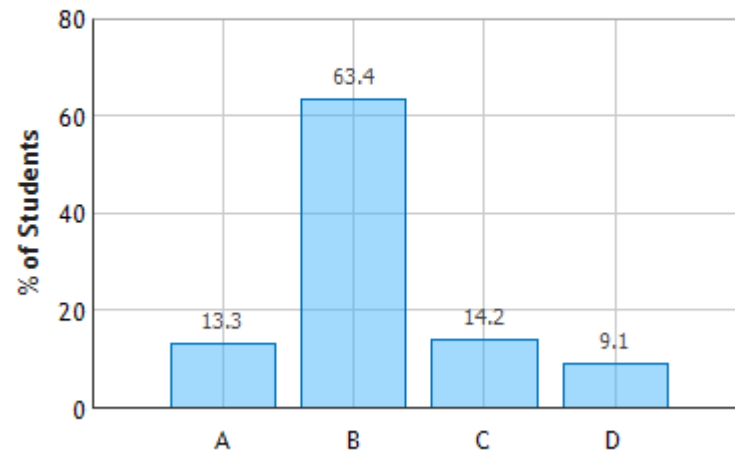
Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300,000,000}{900,000,000} = \frac{1}{3}$$

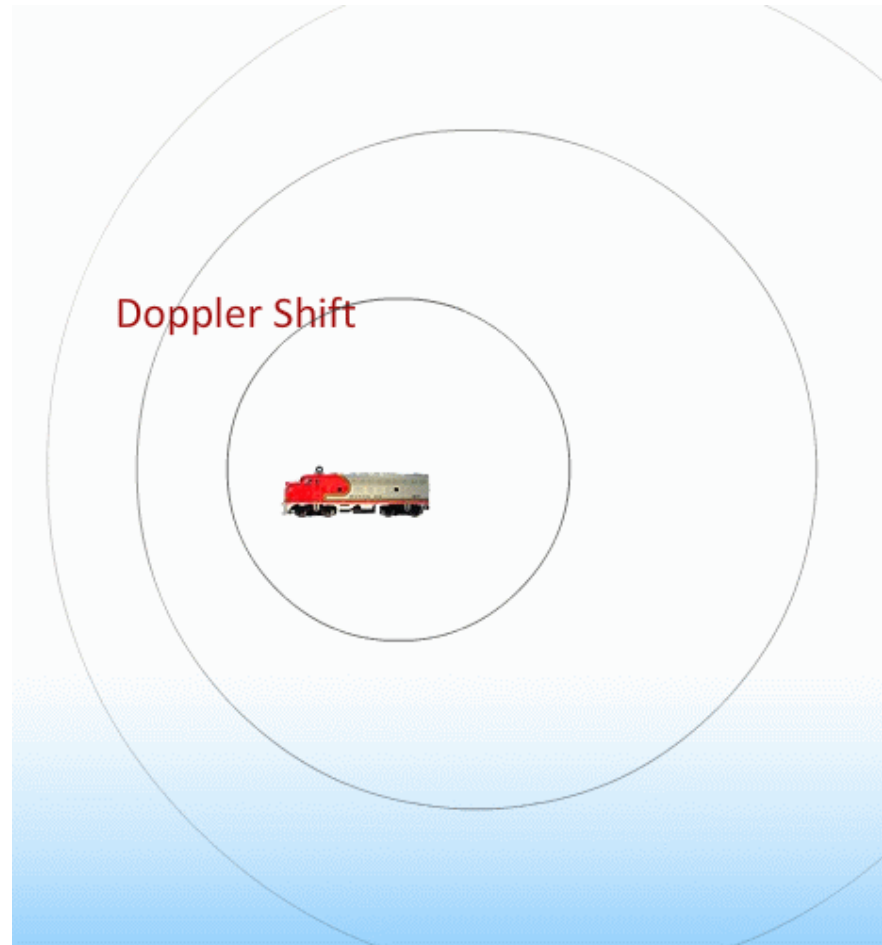
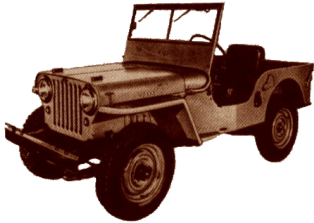
Check:

Look at size of antenna on base unit

EM waves from an iclicker: Question 1 (N = 825)



Doppler Shift



The Big Idea
As source approaches:
Wavelength decreases
Frequency Increases

Doppler Shift for E-M Waves

What's Different from Sound or Water Waves ?

Sound /Water Waves :

You can calculate (no relativity needed)

BUT

Result is somewhat complicated: is source or observer moving wrt medium?

Electromagnetic Waves :

You need relativity (time dilation) to calculate

BUT

Result is simple: only depends on relative motion of source & observer

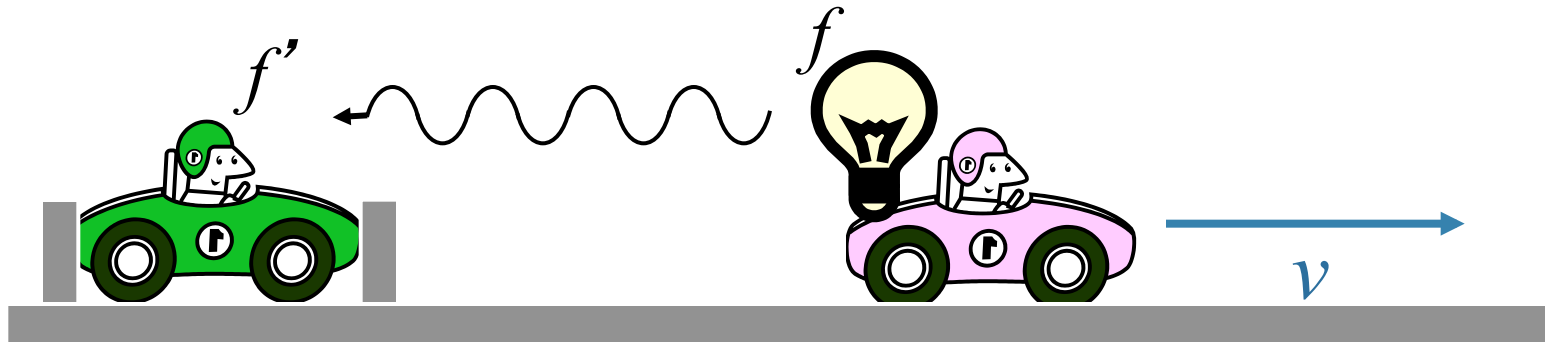
$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$$

$$\beta = v/c$$

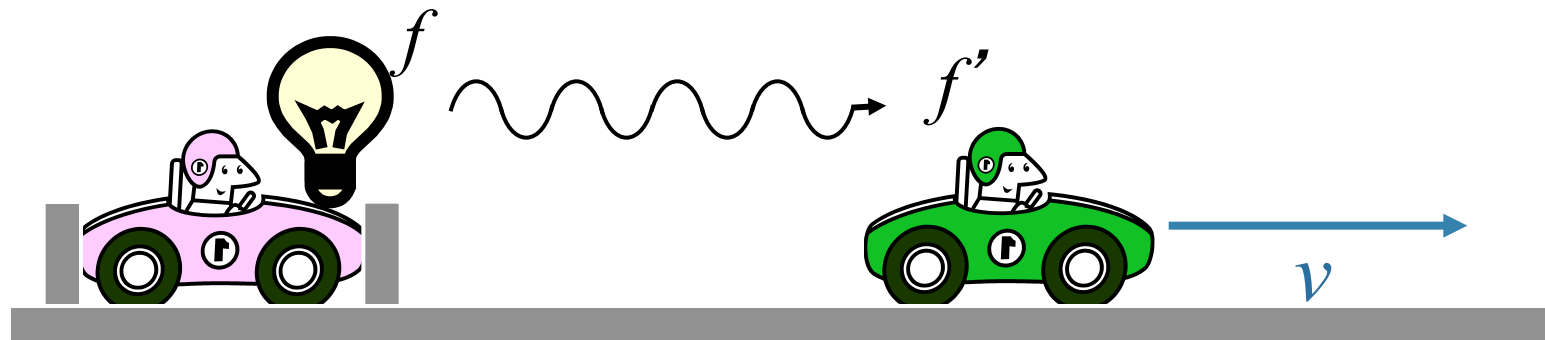
$\beta > 0$ if source & observer are approaching

$\beta < 0$ if source & observer are separating

Doppler Shift for E-M Waves



or



The Doppler Shift is the SAME for both cases!

f'/f only depends on the relative velocity

$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$$

Doppler Shift for E-M Waves

A Note on Approximations

$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}} \quad \xrightarrow{\beta \ll 1} \quad f' \approx f(1 + \beta)$$

why?

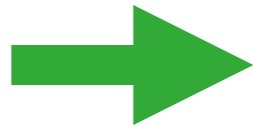
Taylor Series: Expand $F(\beta) = \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$ around $\beta = 0$

$$F(\beta) = F(0) + \frac{F'(0)}{1!} \beta + \frac{F''(0)}{2!} \beta^2 + \dots$$

Evaluate:

$$F(0) = 1$$

$$F'(0) = 1$$



$$F(\beta) \approx 1 + \beta$$

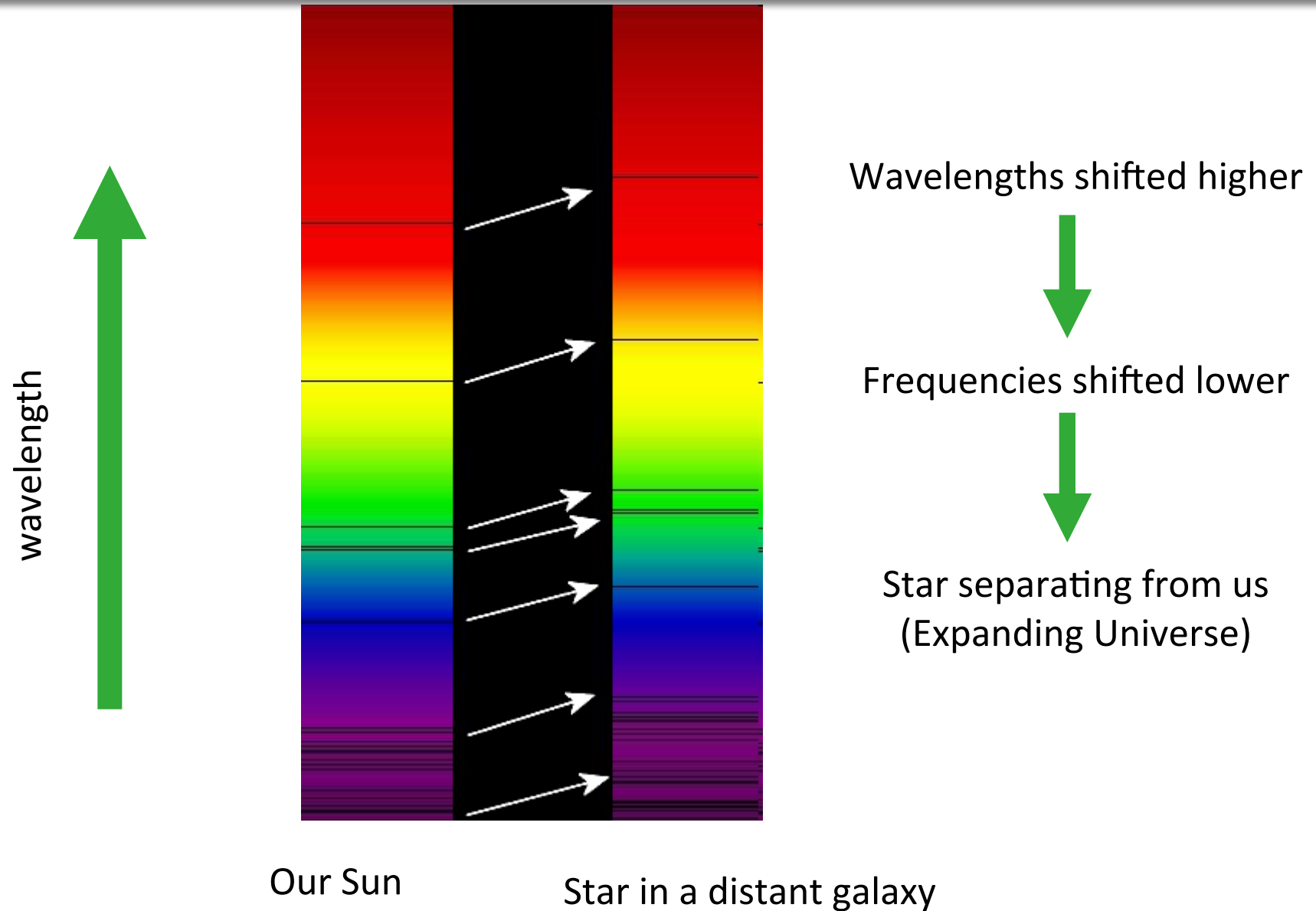
NOTE:

$$F(\beta) = (1 + \beta)^{1/2}$$

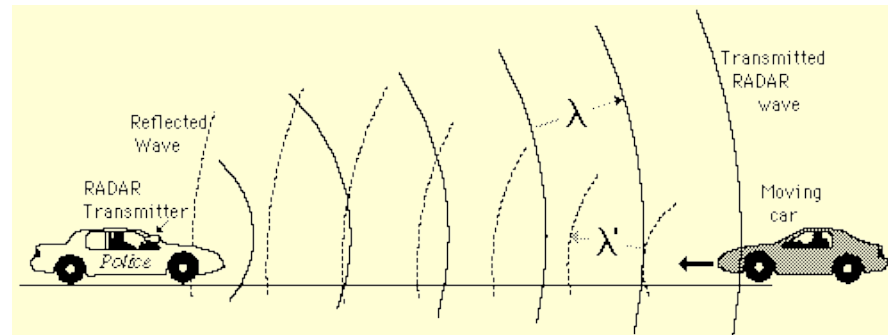


$$F(\beta) \approx 1 + \frac{1}{2} \beta$$

Red Shift



Example



Police radars get twice the effect since the EM waves make a round trip:

$$f' \approx f(1 + 2\beta)$$

If $f = 24,000,000,000 \text{ Hz}$ (k-band radar gun)

$c = 300,000,000 \text{ m/s}$

v	β	f'	$f' - f$
30 m/s (67 mph)	1.000×10^{-7}	24,000,004,800	4800 Hz
31 m/s (69 mph)	1.033×10^{-7}	24,000,004,959	4959 Hz

CheckPoint 2b



If you wanted to see the EM wave produced by the iclicker with your eyes, which of the following would work? (Note: Your eyes are sensitive to EM waves w/ frequency around 10^{14} Hz)

A) ☐ Run away from the iclicker when it is voting.

B) ☒ Run toward the iclicker when it is voting.

C) ☐ Neither will work, moving relative to the iclicker won't change the frequency reaching your eyes.

$$f_{\text{iclicker}} = 900 \text{ MHz}$$

Need to shift frequency UP



Need to approach iclicker ($\beta > 0$)

How fast would you need to run to see the iclicker radiation?

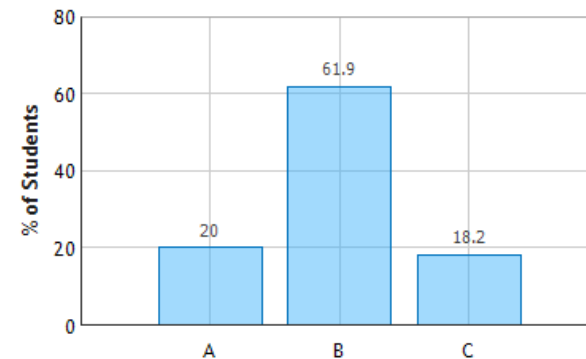
$$\frac{f'}{f} = \frac{10^{14}}{10^9} = 10^5 = \left(\frac{1+\beta}{1-\beta} \right)^{1/2}$$



$$10^{10} = \left(\frac{1+\beta}{1-\beta} \right) \rightarrow \beta = \frac{10^{10} - 1}{10^{10} + 1} = \frac{1 - 10^{-10}}{1 + 10^{-10}}$$

Approximation Exercise: $\beta \approx 1 - (2 \times 10^{-10})$

EM waves from an iclicker: Question 2 (N = 826)



Waves Carry Energy

Total Energy Density

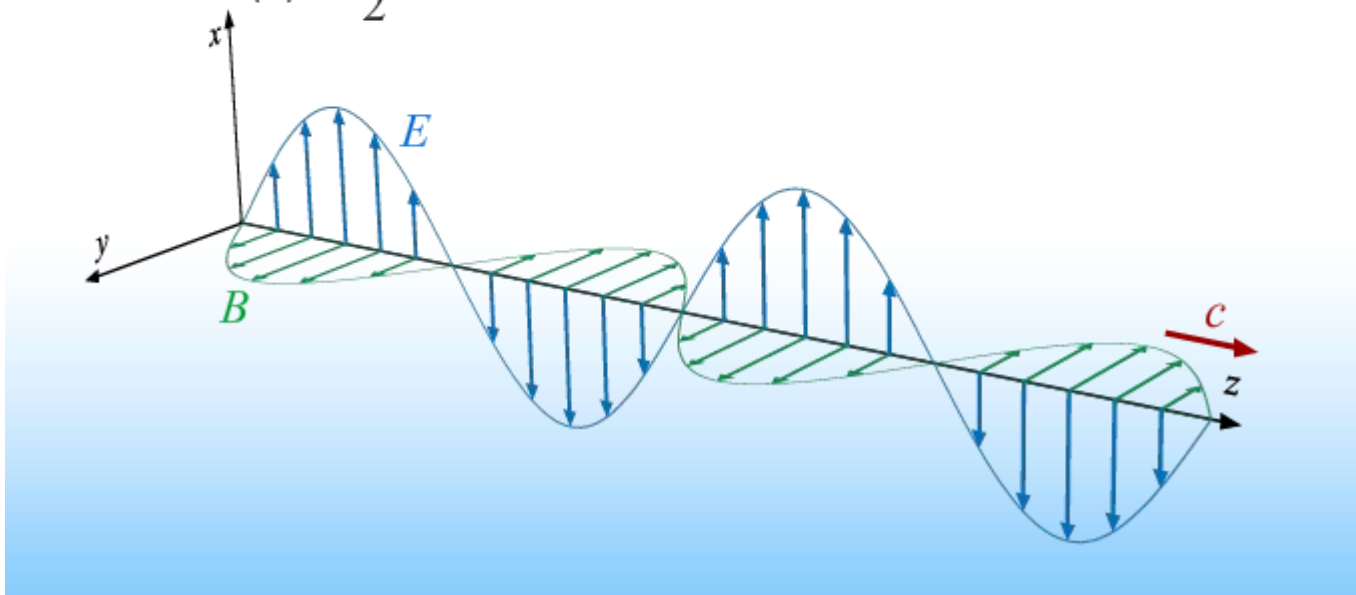
$$u = \epsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_o E_o^2$$

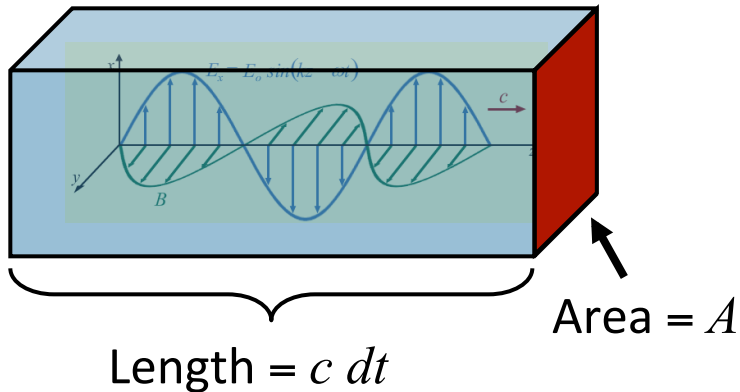
Intensity

$$I = \frac{1}{2} c \epsilon_o E_o^2 = c \langle u \rangle$$



Intensity

Intensity = Average energy delivered per unit time, per unit area



$$\rightarrow I \equiv \frac{1}{A} \left\langle \frac{dU}{dt} \right\rangle$$

$$\rightarrow \langle dU \rangle = \langle u \rangle \cdot \text{volume} = \langle u \rangle A c dt$$

Total Energy Density

$$u = \epsilon_0 E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_o^2$$

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_o^2 = c \langle u \rangle$$

$$\rightarrow I = c \langle u \rangle$$

Sunlight on Earth:

$$I \sim 1000 \text{ J/s/m}^2$$

$$\sim 1 \text{ kW/m}^2$$

Waves Carry Energy

Total Energy Density

$$u = \epsilon_o E^2$$

Intensity

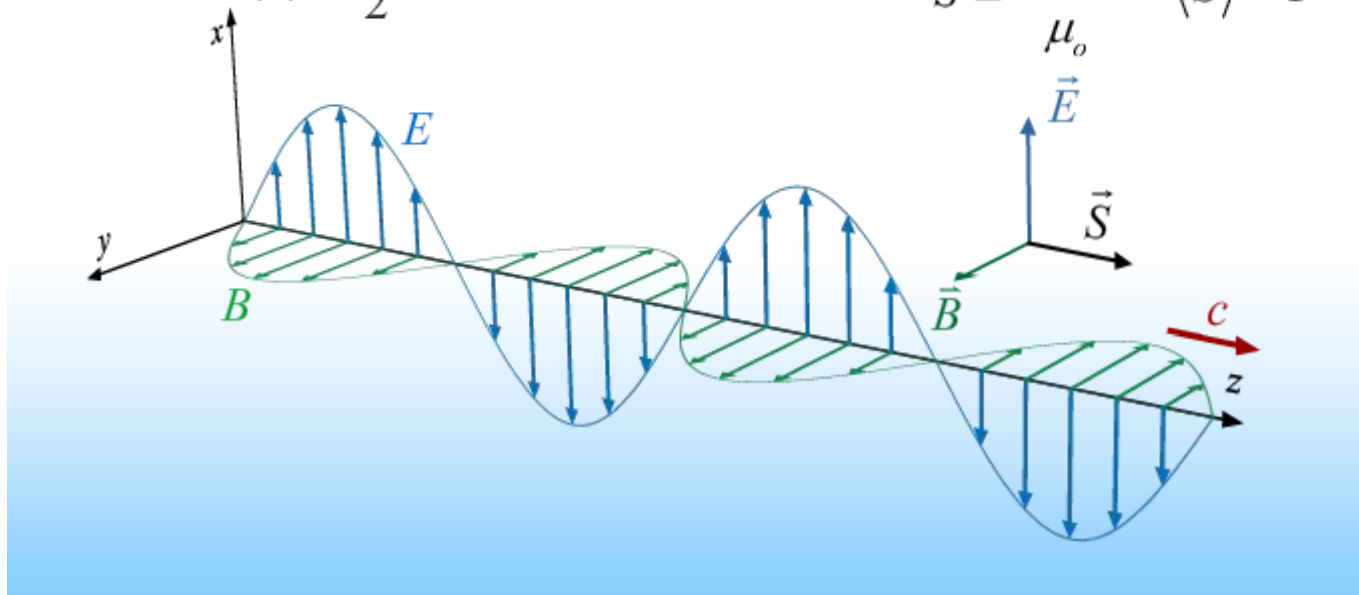
$$I = \frac{1}{2} c \epsilon_o E_o^2 = c \langle u \rangle$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_o E_o^2$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \quad \langle S \rangle = I$$

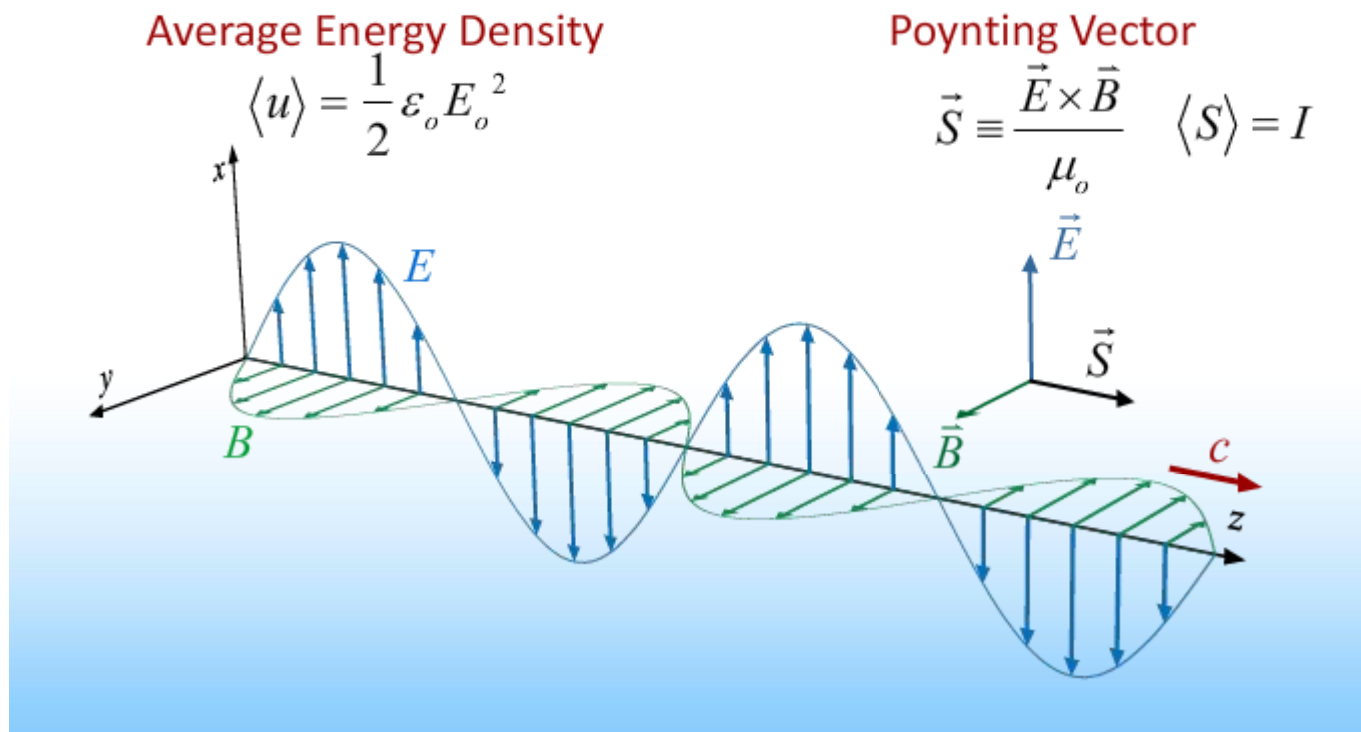


Comment on Poynting Vector

Just another way to keep track of all this:

Its magnitude is equal to I

Its direction is the direction of propagation of the wave



Power in EM Waves: Example

A cell phone tower has a transmitter with a power of 100 W. What is the magnitude of the peak electric field a distance 1500 m (~ 1 mile) from the tower? Assume the transmitter is a point source.

What is the intensity of the wave 1500 m from the tower?

A) 1.5 nW/m²

B) 3.5 μW/m²

C) 6 mW/m²

$$I = \frac{P}{4\pi r^2} = \frac{100 \text{ W}}{4\pi (1500 \text{ m})^2} = 3.5 \frac{\mu\text{W}}{\text{m}^2}$$

What is the peak value of the electric field?

$$I = \left\langle \left| \frac{\mathbf{r}}{S} \right| \right\rangle = \left\langle \frac{|\mathbf{E} \times \mathbf{B}|}{\mu_0} \right\rangle = \left\langle \frac{E}{\mu_0} \frac{E}{c} \right\rangle = \frac{1}{\mu_0 c} \frac{E_0^2}{2} \Rightarrow E_0 = \sqrt{2\mu_0 c I}$$

$$E_0 = \left(2 \cdot 4\pi \times 10^{-7} \cdot 3 \times 10^8 \cdot 3.5 \times 10^{-6} \right)^{1/2} = 51 \frac{\text{mV}}{\text{m}}$$

Light has Momentum!

If it has energy and its moving, then it also has momentum:

Analogy from mechanics:

$$E = \frac{p^2}{2m}$$

$$\frac{dE}{dt} = \frac{\cancel{2}p}{\cancel{2}m} \frac{dp}{dt} = \frac{\cancel{m}v}{\cancel{m}} \frac{dp}{dt} = vF$$

For $E - M$ waves:

$$\frac{dE}{dt} \rightarrow \frac{dU}{dt} = IA$$

$$v \rightarrow c$$

$$IA = cF$$

$$P = \frac{I}{c}$$

Radiation pressure

$$\frac{I}{c} = \frac{F}{A} \text{ pressure}$$

CheckPoint 4

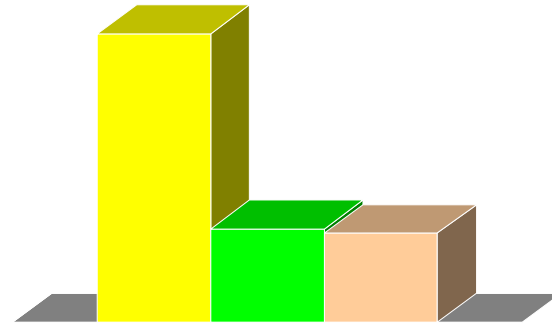


An electromagnetic wave has electric field amplitude E , wavelength λ , and frequency ω . Which should we increase if we want the energy carried by the wave to increase (you can mark more than one answer).

☒ E ☐ λ ☐ ω

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_o^2$$



But then again, what are we keeping constant here?

WHAT ABOUT PHOTONS?

Checkpoint 1 b



Which of the following actions will increase the energy carried by an electromagnetic wave?

A. Increase E keeping ω constant

B. Increase ω keeping E constant

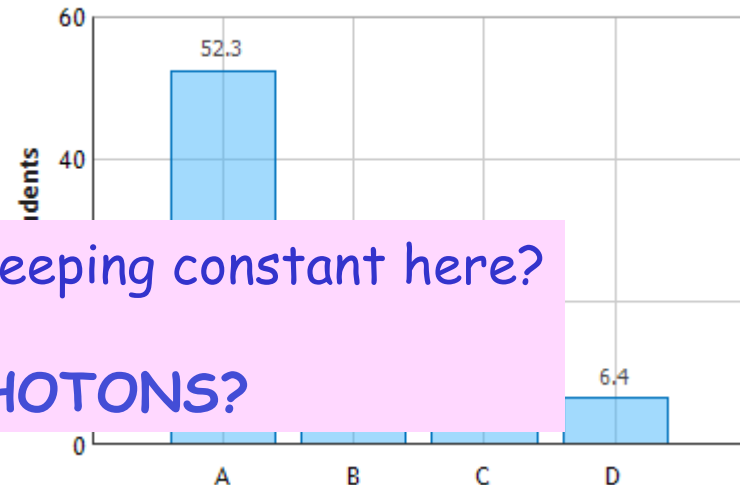
C. Both of the above will increase the energy

D. Neither of the above will increase the energy

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_0^2$$

Electromagnetic Waves: Question 3 (N = 826)



But then again, what are we keeping constant here?

WHAT ABOUT PHOTONS?

The energy of one photon is

$$\mathcal{E}_{\text{photon}} = hf = h\omega/2\pi$$

$$U_{\text{wave}} = N_{\text{photons}} \times \mathcal{E}_{\text{photon}}$$

$$\mathcal{E}_{\text{photon}} / \text{Volume} = 1/2 \epsilon_0 E_0^2$$

Photons

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength ($f\lambda = c$)

Connections seen in equations:

$$E = hf$$

$$p = h/\lambda$$

Planck's constant

$$h = 6.63e^{-34} \text{ J} \cdot \text{s}$$

Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

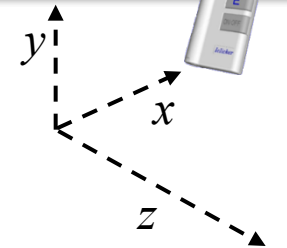
What we see depends on how we choose to measure it!

The mystery of quantum mechanics: More on this in PHYS 214

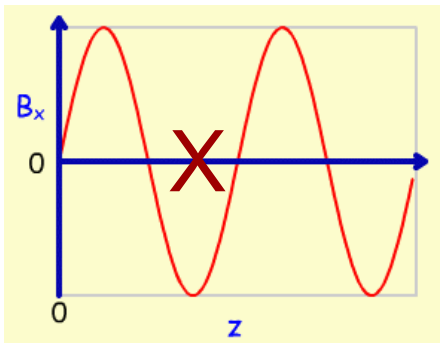
Exercise

An electromagnetic wave is described by:
where \hat{j} is the unit vector in the $+y$ direction.

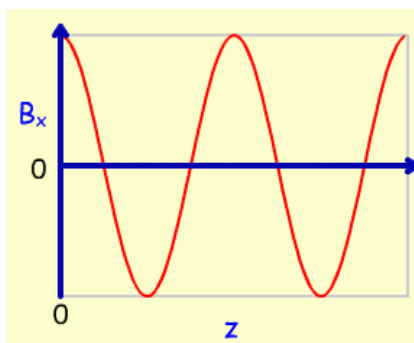
$$\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$$



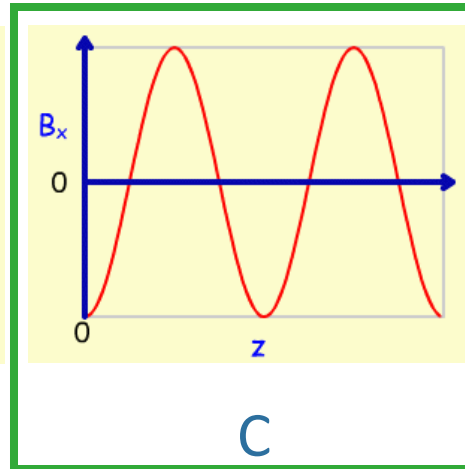
Which of the following graphs represents the z - dependence of B_x at $t = 0$?



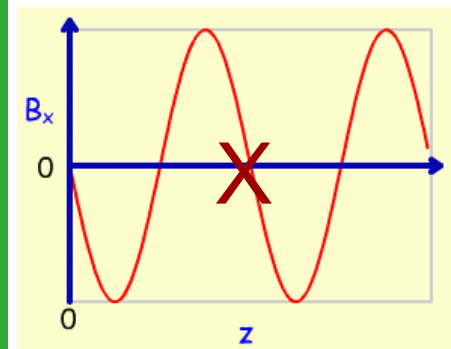
A



B



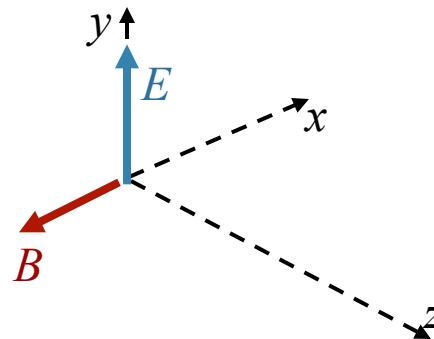
C



E and B are “in phase” (or 180° out of phase)

$\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$ Wave moves in $+z$ direction

$\vec{E} \times \vec{B}$ Points in direction of propagation

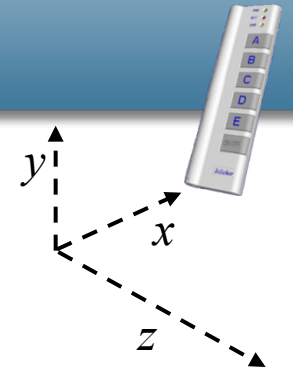


$$\vec{B} = -\hat{i}B_0 \cos(kz - \omega t)$$

Exercise

An electromagnetic wave is described by:

$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$



What is the form of B for this wave?

A) $\vec{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

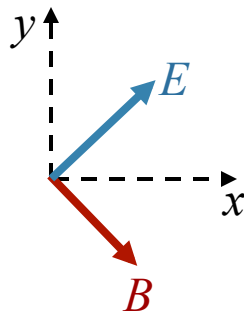
B) $\vec{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

D) $\vec{B} = \frac{-\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$



Wave moves in $-z$ direction



$+z$ points out of screen

$-z$ points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation

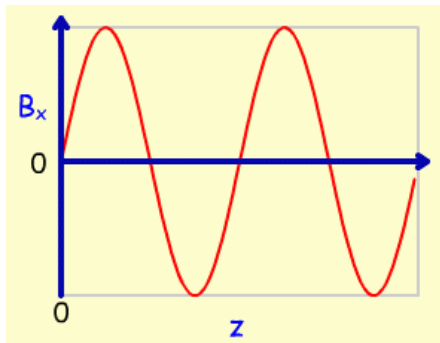
Exercise



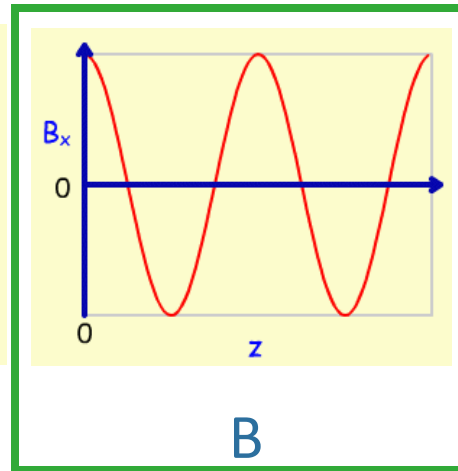
An electromagnetic wave is described by:

$$\vec{E} = \hat{j}E_0 \sin(kz + \omega t)$$

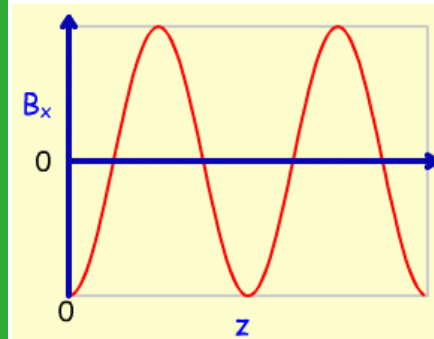
Which of the following plots represents $B_x(z)$ at time $t = \pi/2\omega$?



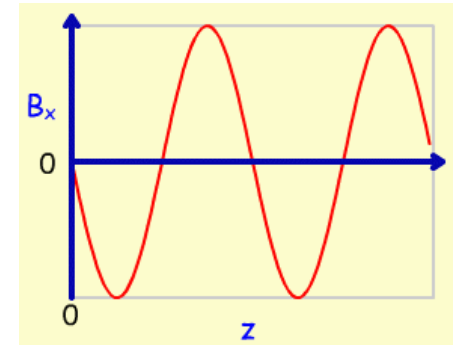
A
D



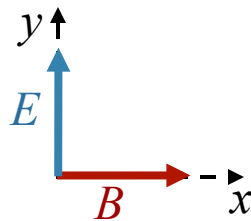
B



C



Wave moves in negative z - direction



+ z points out of screen
- z points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation



$$\vec{B} = \hat{i}(E_0/c) \sin(kz + \omega t)$$

at $\omega t = \pi/2$:

$$B_x = (E_0/c) \sin(kz + \pi/2)$$

$$B_x = (E_0/c) \{ \sin kz \cos(\pi/2) + \cos kz \sin(\pi/2) \}$$

$$B_x = (E_0/c) \cos(kz)$$

Exercise



A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

Is it possible that the professor's argument is correct?

$$(\lambda_{\text{green}} = 500 \text{ nm}, \lambda_{\text{red}} = 600 \text{ nm})$$

A) YES

B) NO

As professor approaches stoplight, the frequency of its emitted light will be shifted **UP**

The speed of light does not change

Therefore, the wavelength (c/f) would be shifted **DOWN**

If he goes fast enough, he could observe a green light !

Follow-Up



A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

How fast would the professor have to go to see the light as green?

$$(\lambda_{\text{green}} = 500 \text{ nm}, \lambda_{\text{red}} = 600 \text{ nm})$$

- A) 540 m/s B) $5.4 \times 10^4 \text{ m/s}$ **C) $5.4 \times 10^7 \text{ m/s}$** D) $5.4 \times 10^8 \text{ m/s}$

Relativistic Doppler effect: $f' = f \sqrt{\frac{1+\beta}{1-\beta}}$

$$\frac{f'}{f} = \frac{600}{500} = \sqrt{\frac{1+\beta}{1-\beta}} \quad \longrightarrow \quad 36(1-\beta) = 25(1+\beta) \quad \longrightarrow \quad \beta = \frac{11}{61} = 0.18$$

Note approximation for small β is not bad: $f' = f(1+\beta) \quad \longrightarrow \quad \beta = \frac{1}{5} = 0.2$

$$c = 3 \times 10^8 \text{ m/s} \rightarrow v = 5.4 \times 10^7 \text{ m/s} \quad \longrightarrow \quad \text{Change the charge to speeding!}$$