

Your Comments

The more demos the better. I'm not just saying that because I like to watch things blow up, they legitimately help my understanding of the material.

I'm starting to get lost with all these circuit techniques. Can you show us any good methods or tricks?

i found the exam to be pretty straightforward and very similar to past exams... except i went and checked how i did and it was by a COUPLE letter grades lower than i expected...

I thought I did a lot better on the first exam than I actually did.....
seems like a lot of people thought so too. I really hope there's a curve!

Will we have to know how to solve those first order linear ordinary differential equations?

Can you go a bit slow during the lecture? This one is confusing...

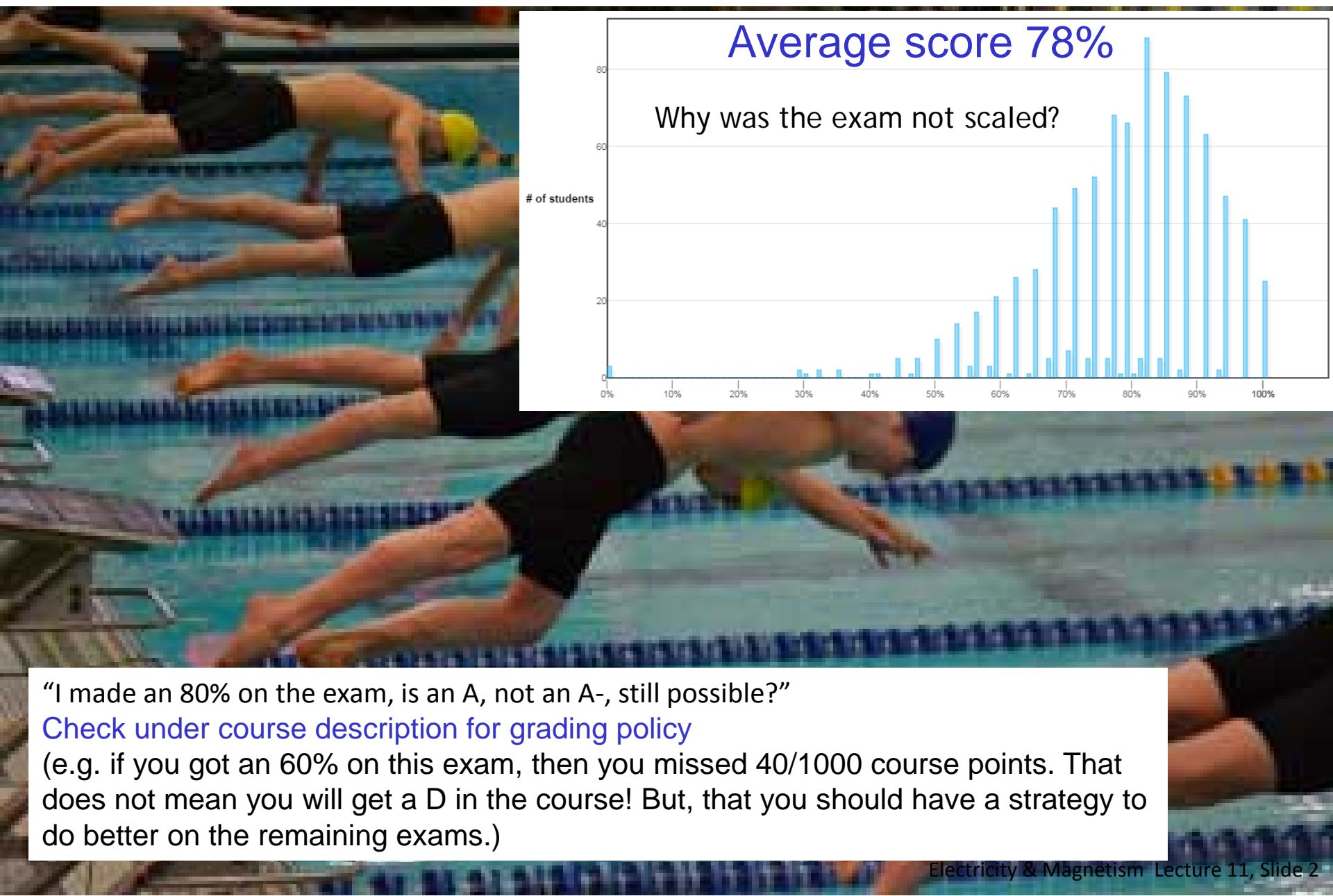
Non mi piace molto le sisteme delle resistenze e i condensatori. Non capisco gli equazioni che descrivono quando tu carichi e scarichi i condensatori.

I was admitted by Purdue Grad school, shall I go there?

Einstein invented a theory about space. And it was about time too.

Capacitor over time... yep. Resistor over time... yep. Capacitor and resistor in same circuit over time with battery... huh?

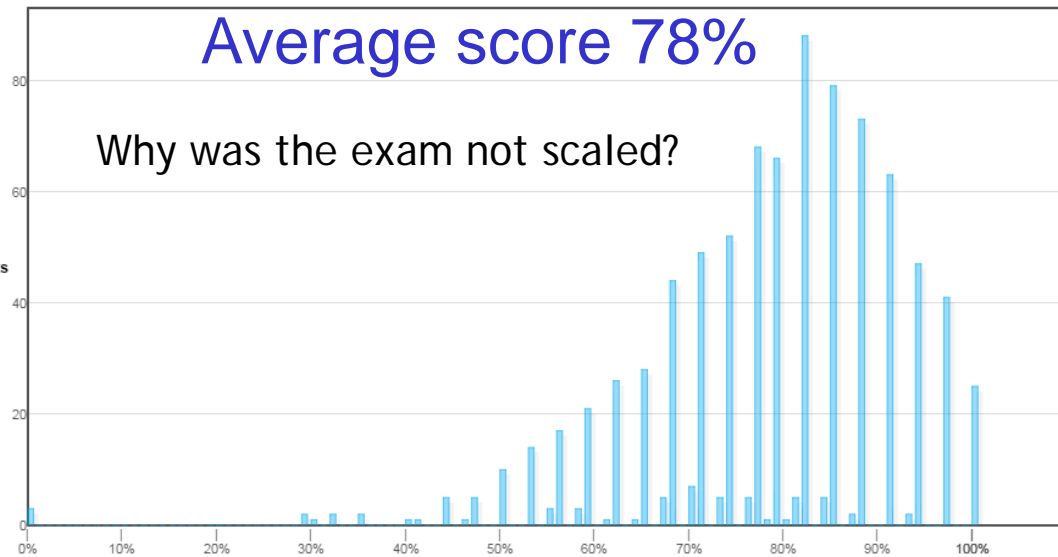
Great job on Hour Exam 1



Average score 78%

Why was the exam not scaled?

of students



"I made an 80% on the exam, is an A, not an A-, still possible?"

[Check under course description for grading policy](#)

(e.g. if you got an 60% on this exam, then you missed 40/1000 course points. That does not mean you will get a D in the course! But, that you should have a strategy to do better on the remaining exams.)

Physics 212

Lecture 11

Today's Concept:

RC Circuits

The 212 Differential Equations

We describe the world (electrical circuits, problems in heat transfer, control systems, financial markets, etc.) using differential equations

You only need to know the solutions of two basic differential equations

$$\frac{dq}{dt} + \frac{1}{\tau} q = 0 \quad q = q_{\text{const}} e^{-t/\tau}$$

$$\frac{d^2 q}{dt^2} + \omega^2 q = 0 \quad q = q_{\text{const}} \sin(\omega t + \phi)$$

Capacitors in RC Circuits

Solve by applying Kirchhoff's Rules to circuit.
Need to understand some key phrases.

IMMEDIATELY After === Charge on capacitor is same as immediately before

After a LONG TIME === Current through capacitor = 0

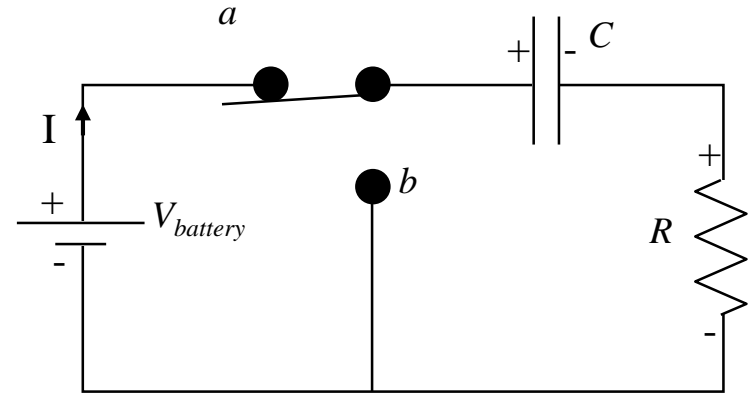
After xx seconds === *Exponentially* more difficult!

RC Circuit (Charging)

Capacitor uncharged, Switch is moved to position “a”

Kirchoff's Voltage Rule

$$-V_{battery} + \frac{q}{C} + IR = 0$$



Short Term ($q = q_0 = 0$)

$$-V_{battery} + 0 + I_0 R = 0$$

$$I_0 = \frac{V_{battery}}{R}$$

Long Term ($I_c = 0$)

$$-V_{battery} + \frac{q_\infty}{C} + 0 \cdot R = 0$$

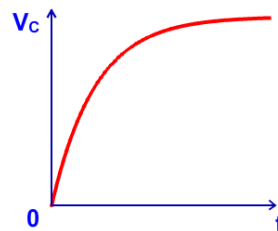
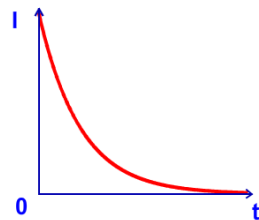
$$q_\infty = CV_{battery}$$

Intermediate

$$-V_{battery} + \frac{q}{C} + \frac{dq}{dt} R = 0$$

$$q(t) = q_\infty (1 - e^{-t/RC})$$

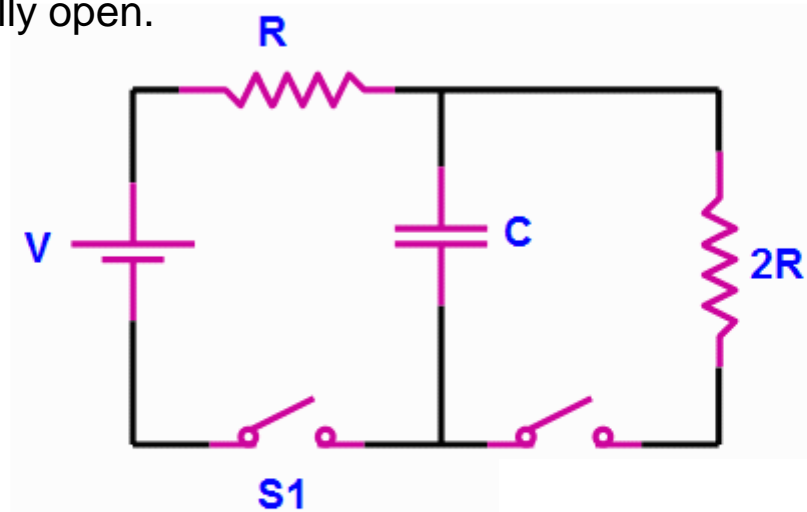
$$I(t) = \frac{dq}{dt} = I_0 e^{-t/RC}$$



CheckPoint 1



A circuit is wired up as shown below. The capacitor is initially uncharged and switches S_1 and S_2 are initially open.



Close S_1 ,

V_1 = voltage across C immediately after

V_2 = voltage across C a long time after

Immediately after the
switch S_1 is closed:

A) $V_1 = V$ $V_2 = V$

B) $V_1 = 0$ $V_2 = V$

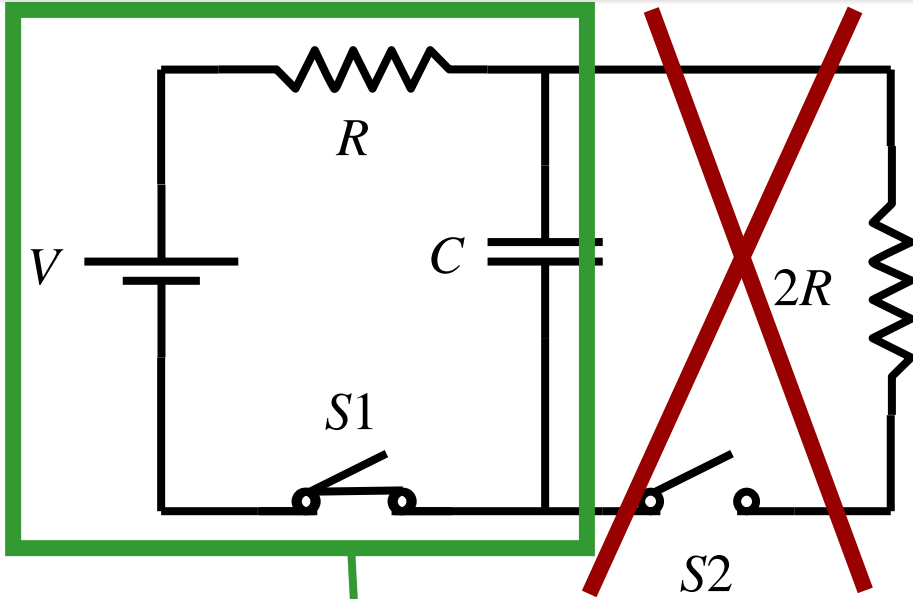
C) $V_1 = 0$ $V_2 = 0$

D) $V_1 = V$ $V_2 = 0$

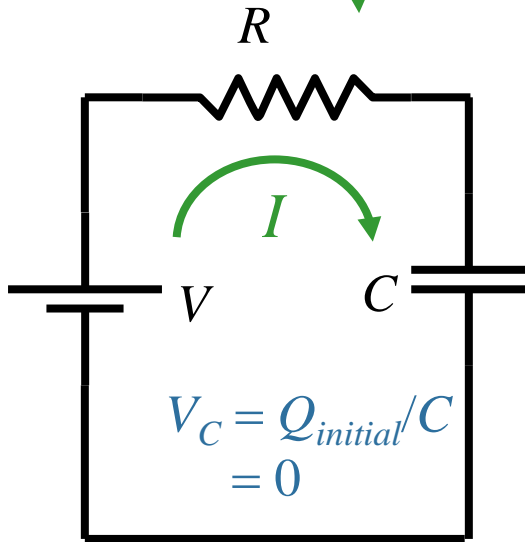
After the switch S_1 has been
closed for a long time

Q is same as immediately before

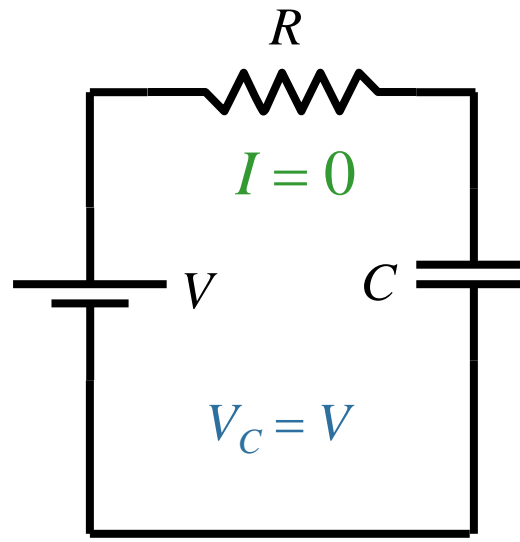
$$I_C = 0$$



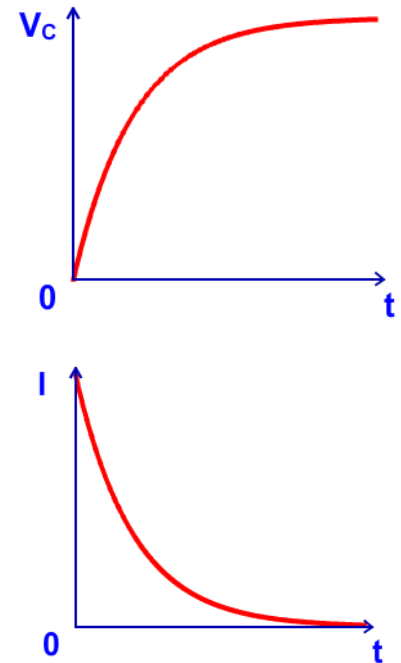
Close S_1 at $t = 0$
(leave S_2 open)



At $t = 0$



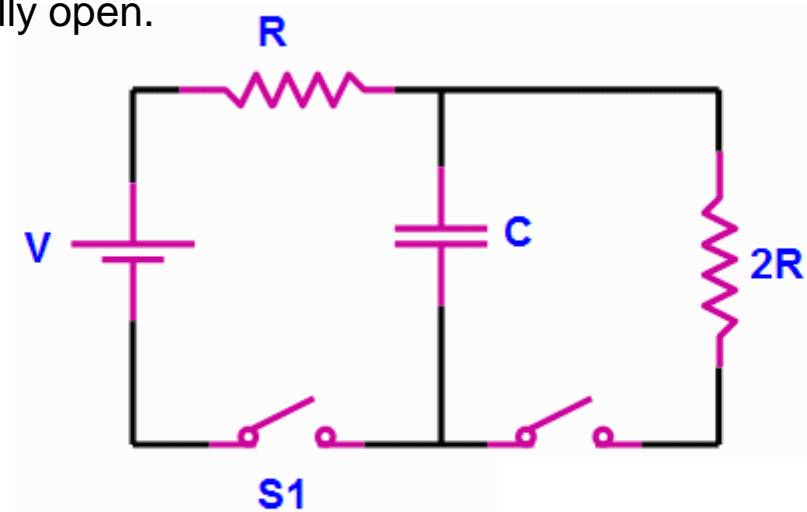
At $t = \text{big}$



CheckPoint 1



A circuit is wired up as shown below. The capacitor is initially uncharged and switches S_1 and S_2 are initially open.



Close S_1 ,

V_1 = voltage across C immediately after

V_2 = voltage across C a long time after

Immediately after the
switch S_1 is closed:

Q is same as immediately before

A) $V_1 = V$ $V_2 = V$

B) $V_1 = 0$ $V_2 = V$

C) $V_1 = 0$ $V_2 = 0$

D) $V_1 = V$ $V_2 = 0$

After the switch S_1 has been
closed for a long time

$$I_C = 0$$

RC Circuit (Discharging)

Capacitor has $q_0 = CV_{battery}$, Switch is moved to position “b”

Kirchoff's Voltage Rule

$$+\frac{q}{C} + IR = 0$$

Short Term ($q = q_0$)

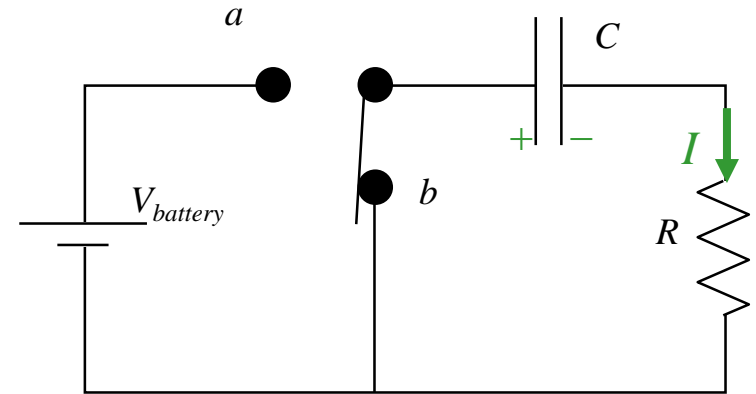
$$V_{battery} + IR = 0$$

$$I_0 = \frac{-V_{battery}}{R}$$

Long Term ($I_c = 0$)

$$\frac{q_\infty}{C} + 0 \cdot R = 0$$

$$q_\infty = 0$$

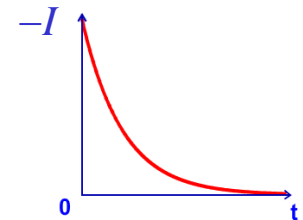
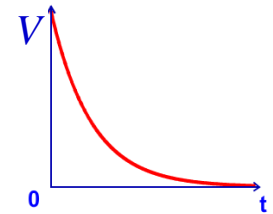


Intermediate

$$+\frac{q}{C} + \frac{dq}{dt} R = 0$$

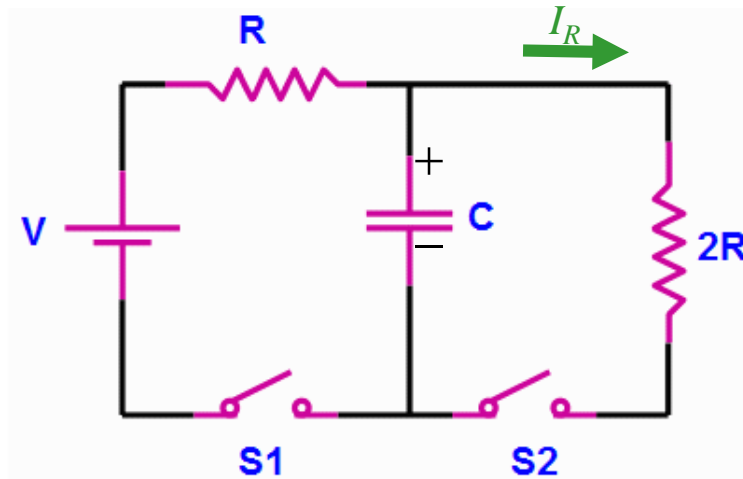
$$q(t) = q_0 e^{-t/RC}$$

$$I(t) = I_0 e^{-t/RC}$$



CheckPoint 1c

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



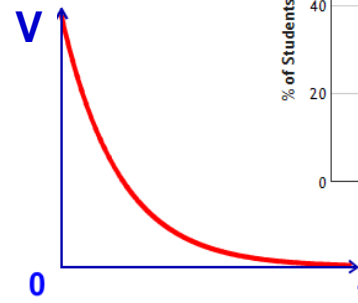
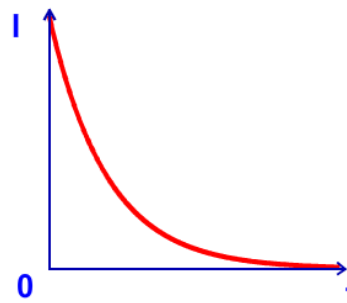
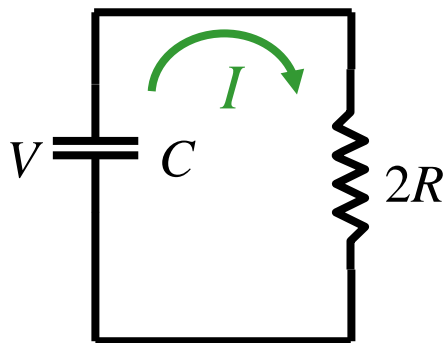
Switch 1 is closed for a long time. Then, switch 1 is opened and switch 2 is closed. What is the current through the right resistor immediately after switch 2 is closed?

A. $I_R = 0$

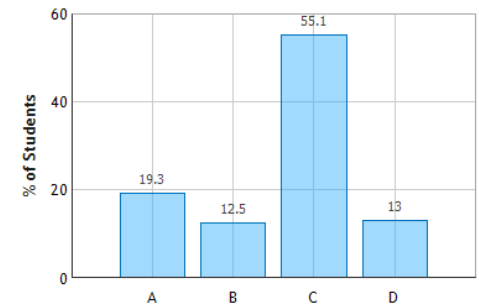
B. $I_R = V/3R$

C. $I_R = V/2R$

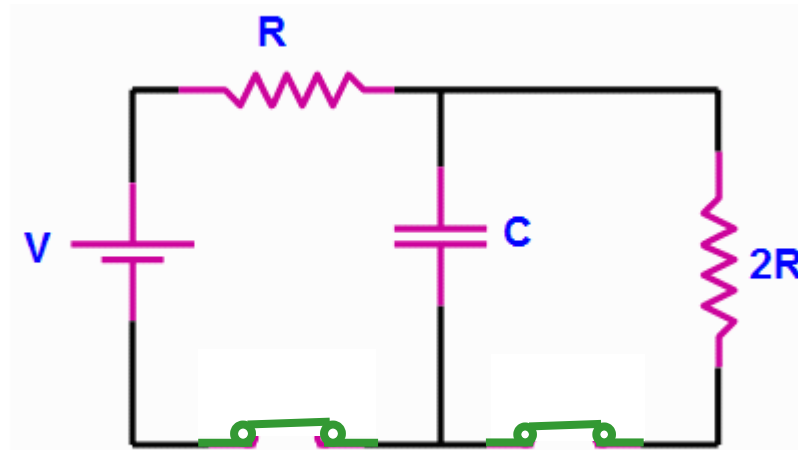
D. $I_R = V/R$



Two Loop RC Circuit: Question 5 (N = 813)



Checkpoint 1 d



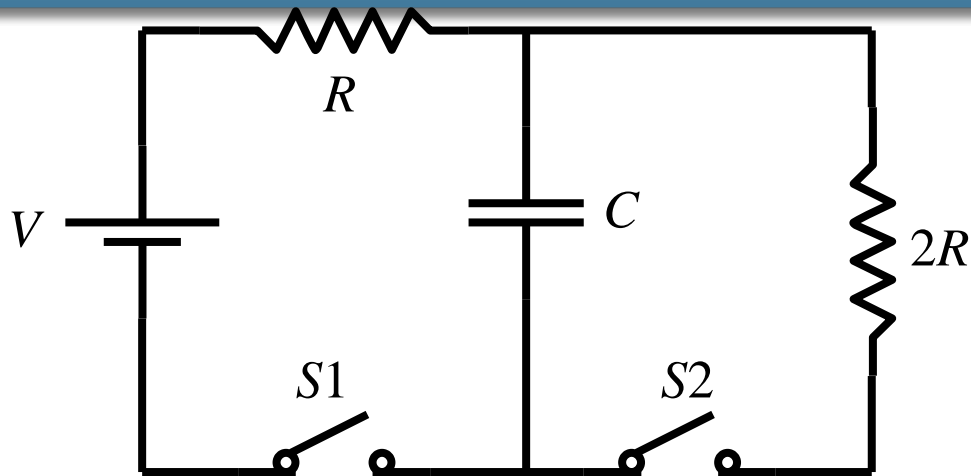
Now suppose both switches are closed. What is the voltage across the capacitor after a very long time?

- A.** $V_C = 0$ **B.** $V_C = V$ **C.** $V_C = 2V/3$

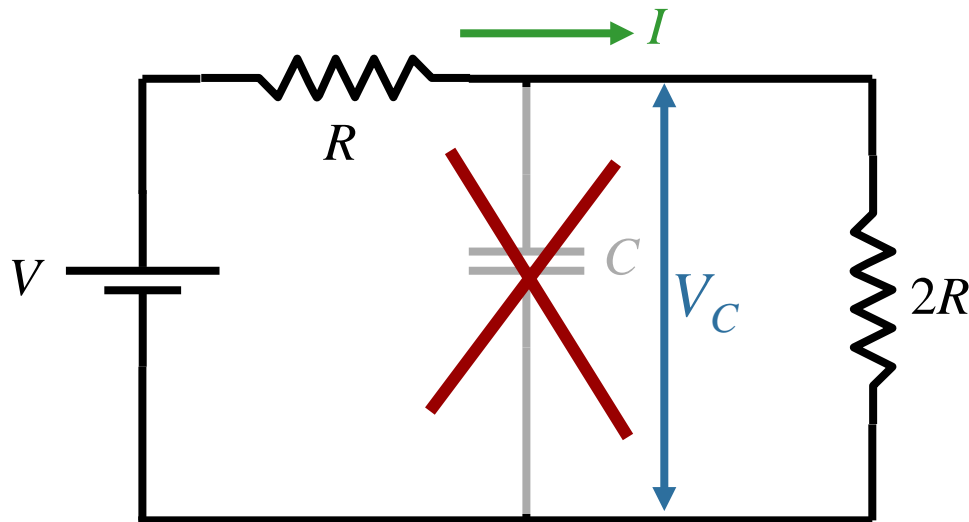
A) When time goes to infinity the current will stop and the voltage will go to zero.

B) As time approaches infinity, the capacitor V value approaches that of the battery

C) The current through the capacitor will be 0.



Close both $S1$ and $S2$ and wait a long time...



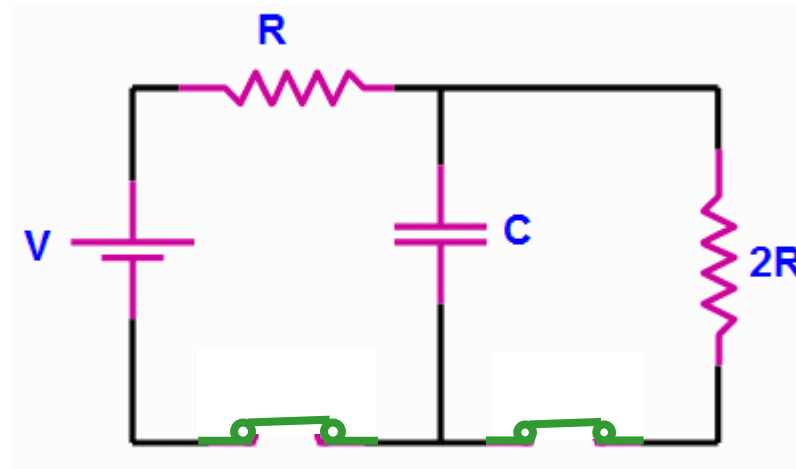
No current flows through the capacitor after a long time. **This will always be the case in any static circuit!!**

Outer Loop
 $IR + 2IR - V = 0$
 $I = V/(3R)$

Right Loop
 $+V_C - 2IR = 0$
 $V_C = 2IR$

→ $V_C = (2/3)V$

CheckPoint 1 d



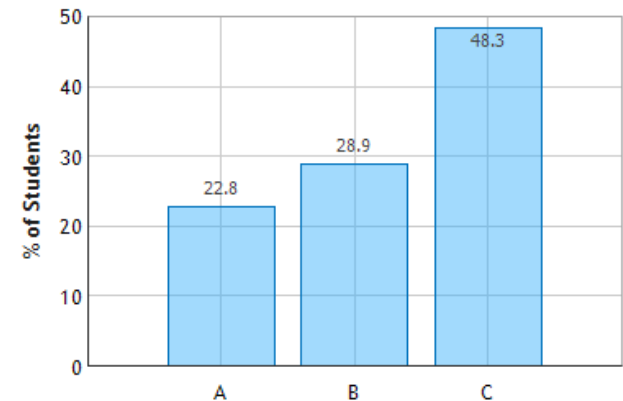
Now suppose both switches are closed. What is the voltage across the capacitor after a very long time?

A. $V_C = 0$

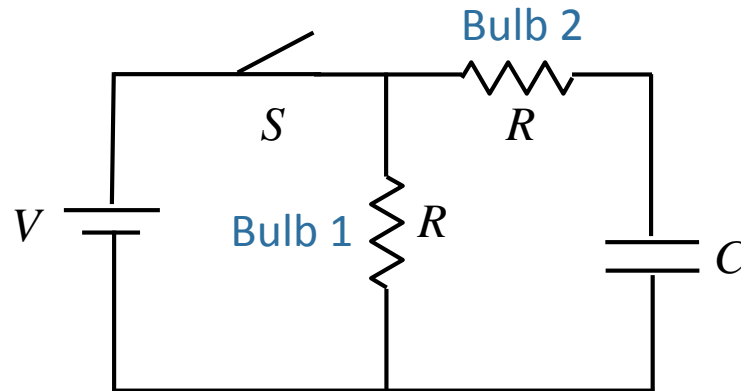
B. $V_C = V$

C. $V_C = 2V/3$

Two Loop RC Circuit: Question 7 (N = 813)



DEMO - Clicker Question 1



What will happen after I close the switch?

- A) Both bulbs come on and stay on.
- B) Both bulbs come on but then bulb 2 fades out.**
- C) Both bulbs come on but then bulb 1 fades out.
- D) Both bulbs come on and then both fade out.

No initial charge
on capacitor



$$V(\text{bulb 1}) = V(\text{bulb 2}) = V$$



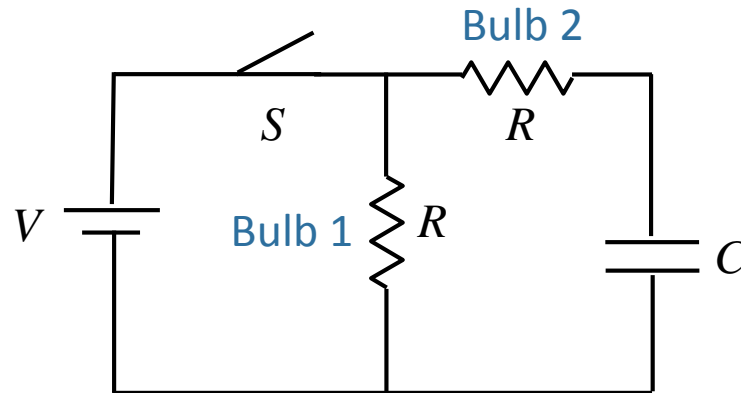
Both bulbs light

No final current
through capacitor



$$V(\text{bulb 2}) = 0$$

DEMO Clicker Question 2



Suppose the switch has been closed a long time.
Now what will happen after open the switch?

- A) Both bulbs come on and stay on.
- B) Both bulbs come on but then bulb 2 fades out.
- C) Both bulbs come on but then bulb 1 fades out.
- D) Both bulbs come on and then both fade out.

Capacitor has charge ($=CV$)  Capacitor discharges through both resistors

How do Exponentials Work?

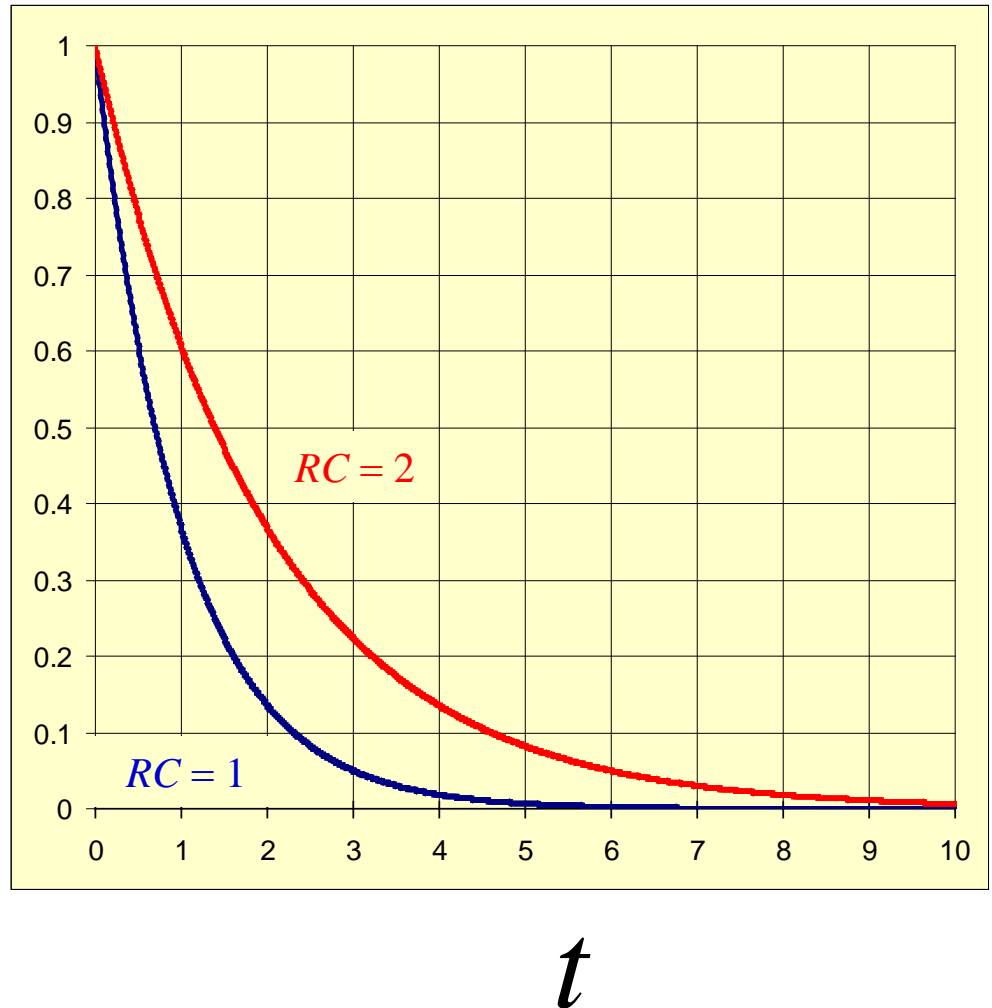
$$Q(t) = Q_0 e^{-t/RC}$$

$$\frac{Q(t)}{Q_0}$$

Time constant:

$$\tau = RC$$

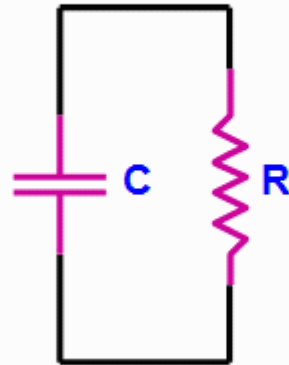
The bigger τ is,
the longer it takes to get
the same change...



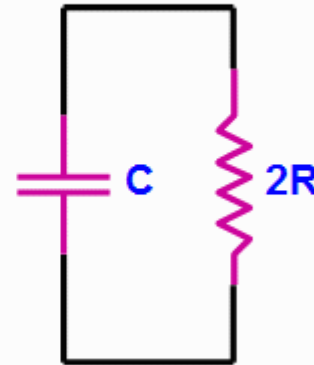
CheckPoint 2



The two circuits shown below contain identical capacitors that hold the same charge at $t = 0$. Circuit 2 has twice as much resistance as circuit 1.



Circuit 1



Circuit 2

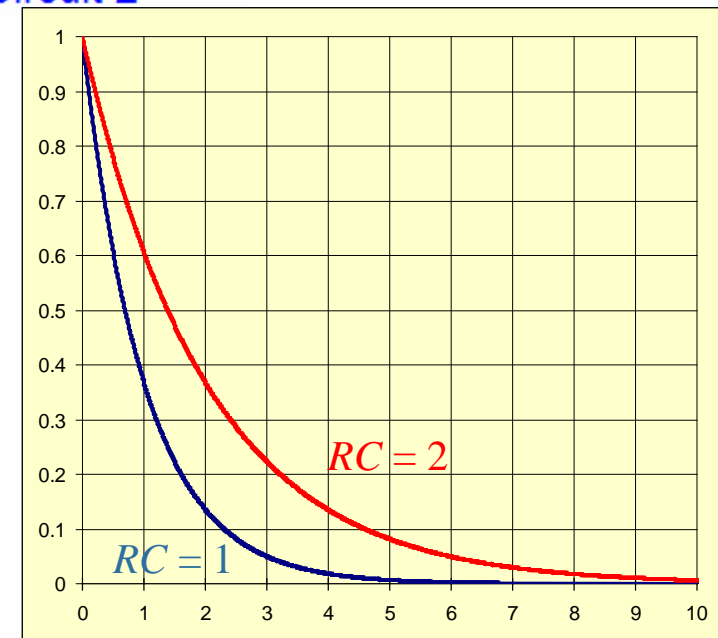
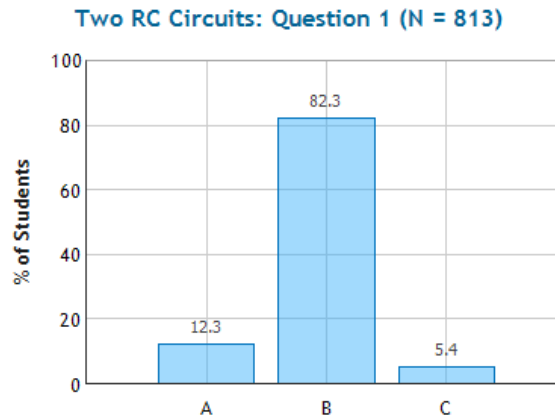
Which circuit has the largest time constant?

A) Circuit 1

B) Circuit 2

C) Same

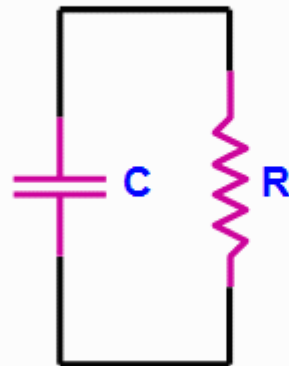
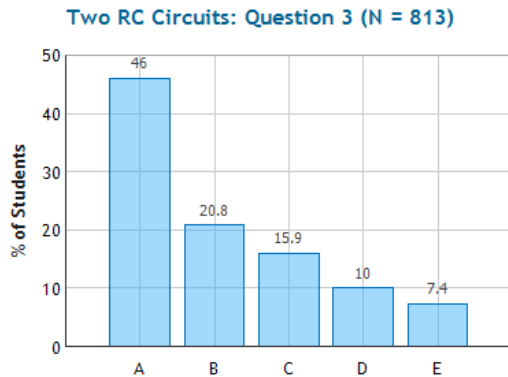
$$\tau = R_{equiv} C$$



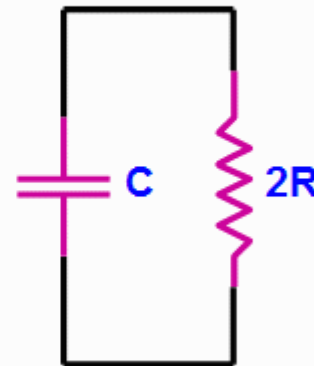
CheckPoint 2



The two circuits shown below contain identical capacitors that hold the same charge at $t = 0$. Circuit 2 has twice as much resistance as circuit 1.



Circuit 1



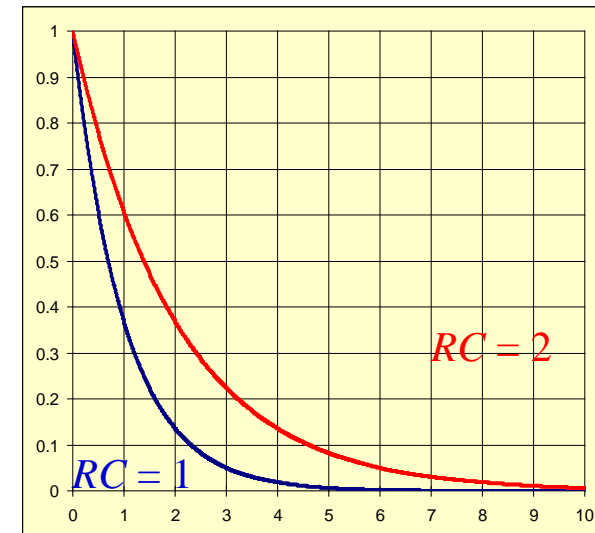
Circuit 2

Which of the following statements best describes the charge remaining on each of the two capacitors for any time after $t = 0$?

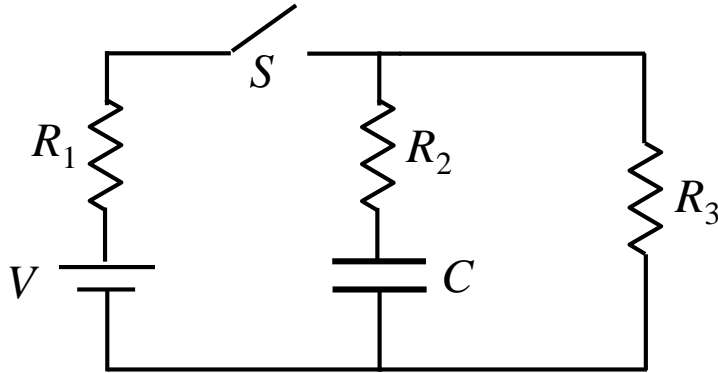
- ☒ $Q_1 < Q_2$
- ☐ $Q_1 > Q_2$
- ☐ $Q_1 = Q_2$
- ☐ $Q_1 < Q_2$ at first and then $Q_1 > Q_2$ after a long time
- ☐ $Q_1 > Q_2$ at first and then $Q_1 < Q_2$ after a long time

$$Q = Q_0 e^{-t/RC}$$

Look at plot!



Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Conceptual Analysis:

Circuit behavior described by Kirchhoff's Rules:

$$\sum V_{drops} = 0$$

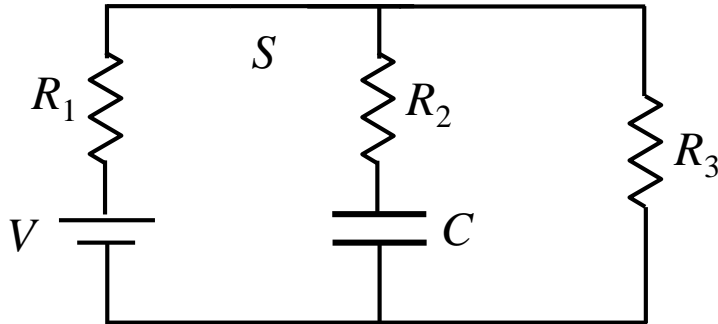
$$\sum I_{in} = \sum I_{out}$$

S closed and C charges to some voltage with some time constant

Strategic Analysis

Determine currents and voltages in circuit a long time after S closed

Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed:

what is I_2 , the current through C

what is V_C , the voltage across C ?

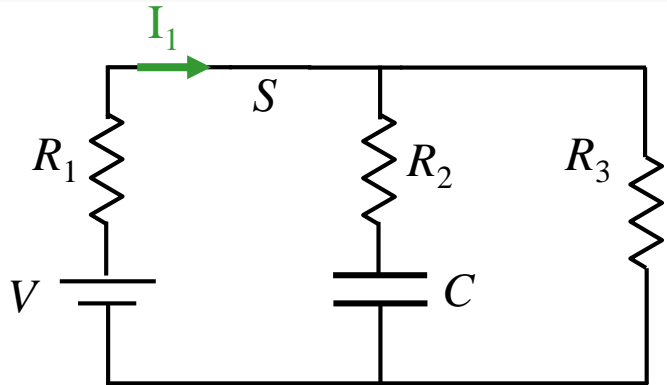
- A) Only $I_2 = 0$ B) Only $V_C = 0$ C) Both I_2 and $V_C = 0$ D) Neither I_2 nor $V_C = 0$

Why?

We are told that C is initially uncharged ($V = Q/C$)

I_2 cannot be zero because charge must flow in order to charge C

Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed, what is I_1 , the current through R_1 ?

$$\frac{V}{R_1}$$

A

$$\frac{V}{R_1 + R_3}$$

B

$$\frac{V}{R_1 + R_2 + R_3}$$

C

$$\frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

D

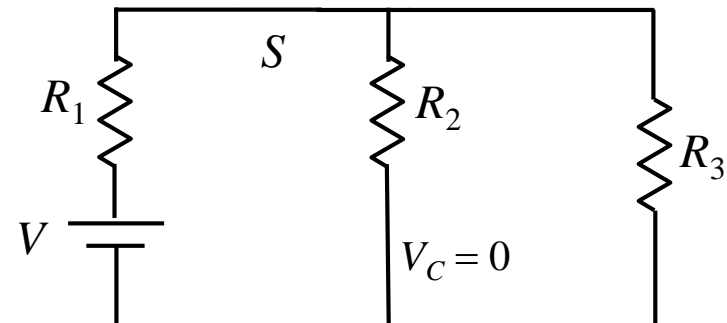
$$V \frac{R_1 + R_2 + R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

E

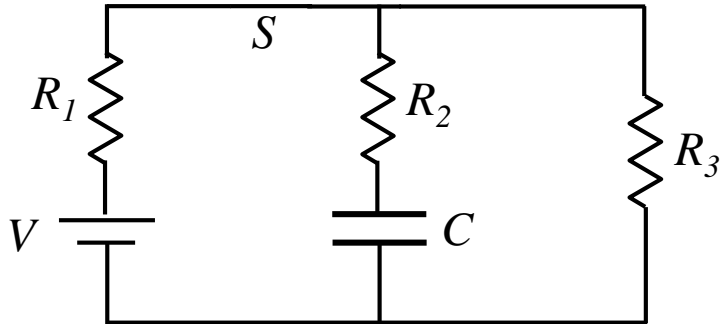
Why?

Draw circuit just after S closed (knowing $V_C = 0$)

R_1 is in series with the parallel combination of R_2 and R_3



Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed “for a long time”, what is I_2 , the current through R_2 ?

$$\frac{V}{R_2}$$

A

$$\frac{V}{R_1}$$

B

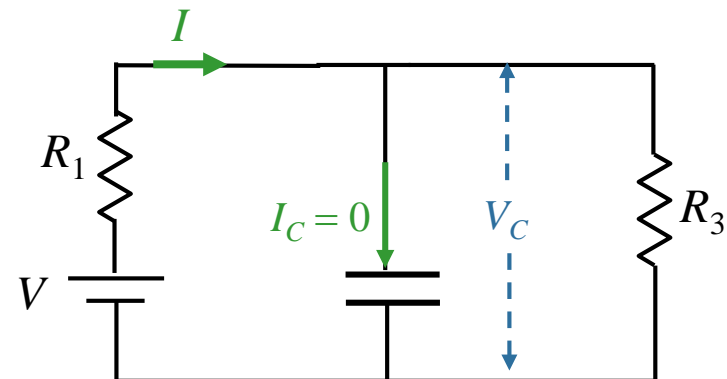
$$0$$

C

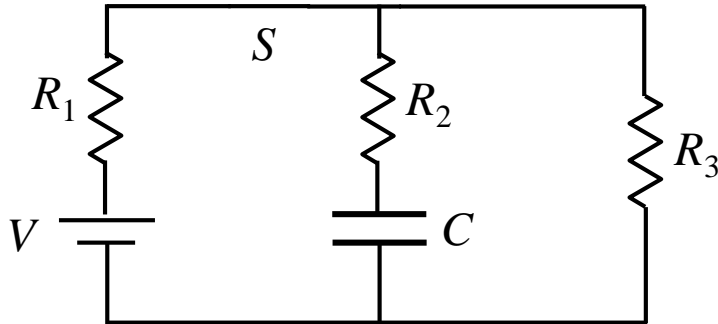
Why?

After a long time in a static circuit, the current through any capacitor approaches 0 !

This means we Redraw circuit with open circuit in middle leg



Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time?

After S has been closed “for a long time”, what is V_C , the voltage across C ?

$$V \frac{R_3}{R_1 + R_3}$$

A

$$V \frac{R_2}{R_1 + R_2}$$

B

V

C

$$V \frac{R_2}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

D

0

E

Why?

$$- V_C = V_3 = IR_3 = (V/(R_1 + R_3))R_3$$

