Electricity & Magnetism
Lecture 3

Today's Concepts:

A) Electric Flux

B) Field Lines

→ Gauss' Law

Can you pronounce your name one more time in class?

Your Comments

"What is E subnot o, and what does it represent?"

IT'S JUST A CONSTANT

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi r^2}\frac{q}{\varepsilon_o}\hat{r}$$

$$k \equiv \frac{1}{4\pi\varepsilon_o}$$

$k = 9 \text{ x } 10^9 \text{ N m}^2 / \text{C}^2$

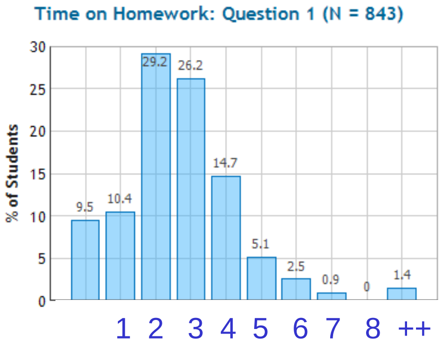
$\varepsilon_o = 8.85\ x\ 10^{-12}\ \text{C}^2 / \text{N}\cdot\text{m}^2$

The concept about Gauss law(how the formula was formed) was difficult for me. I would like to know more about the flux and Gauss law in depth.

What is the difference between flux and electric flux?

PLEASE explain flux better

the prelecture just vomited so many new equations at me and i'm scared now :(

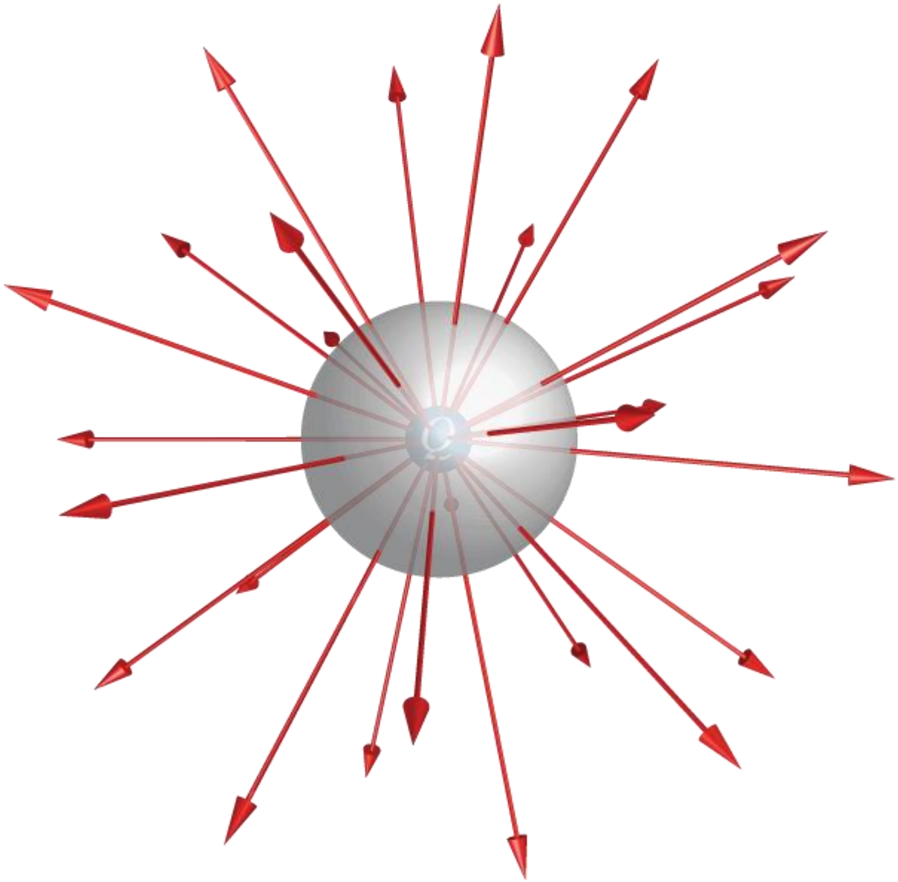


The homework takes a long time, but most of that time isn't actual thought. It's more a matter of plugging in way too many numbers.

Are we going to go more in-depth with how to calculate Gauss' Law, or should we be expected to apply it at this point?

Why do sometimes in the videos the equal signs have three lines instead of two?

Electric Field Lines



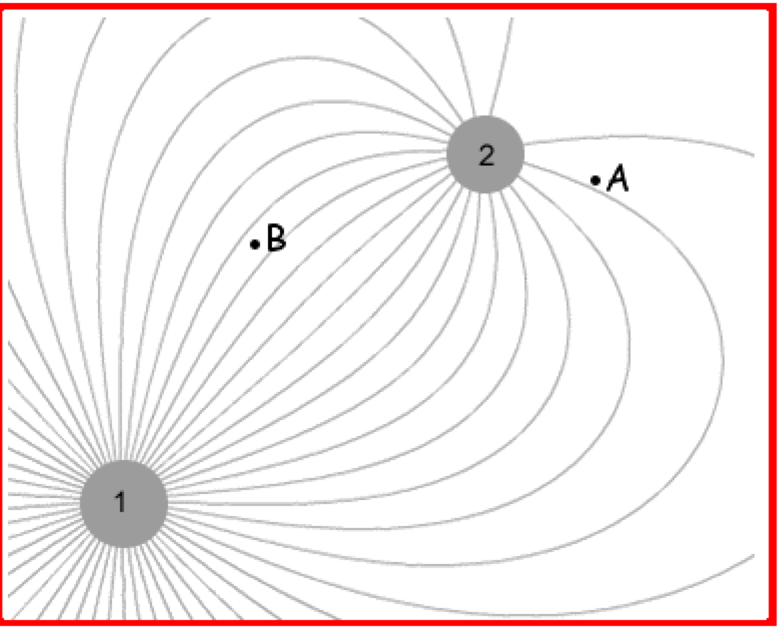
Point Charge:

Direction is radial

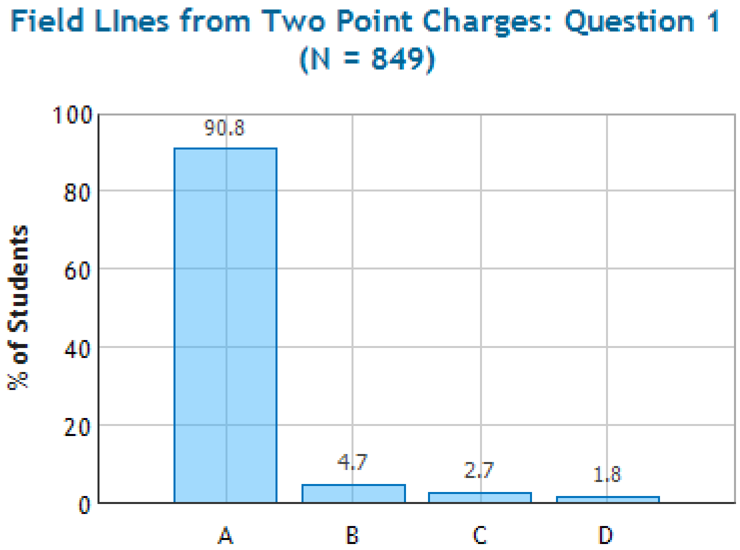
Density $\propto 1/R^2$

Direction & Density of Lines
represent
Direction & Magnitude of E

CheckPoint 3.1



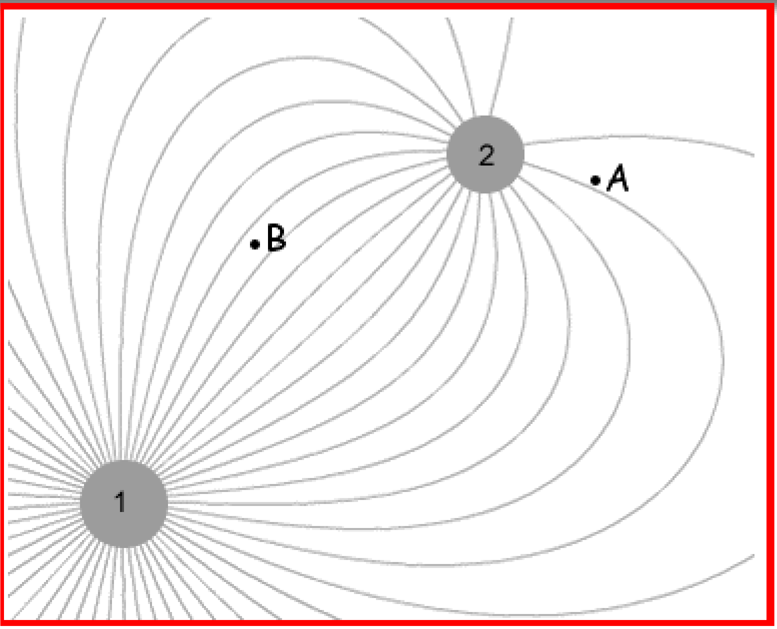
A. $|Q_1| > |Q_2|$
B. $|Q_1| = |Q_2|$
C. $|Q_1| < |Q_2|$
D. Not enough info



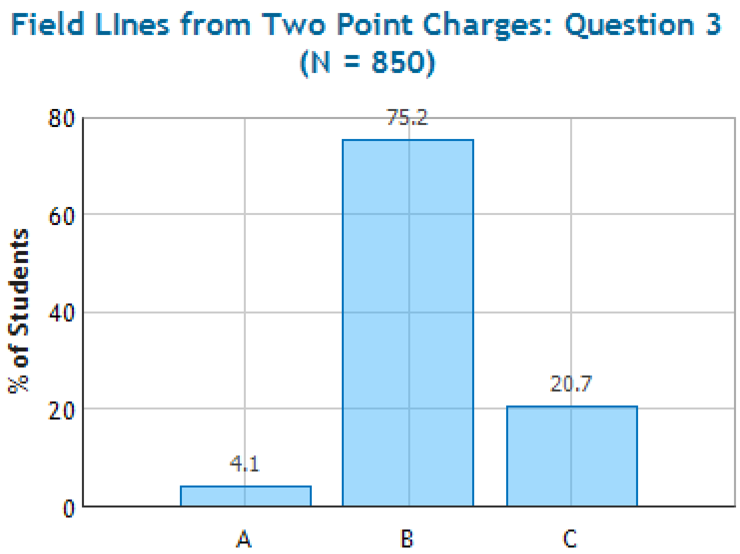
"q1 has a greater number of lines leaving the charge than q2."

Simulation

CheckPoint 3.3

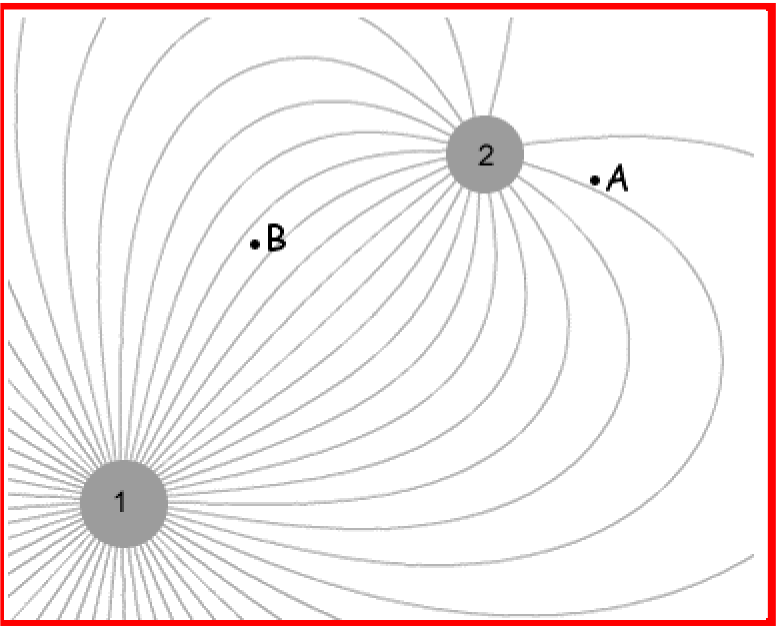


A. Q_1 and Q_2 have the same sign
B. Q_1 and Q_2 have opposite signs
C. Not enough info

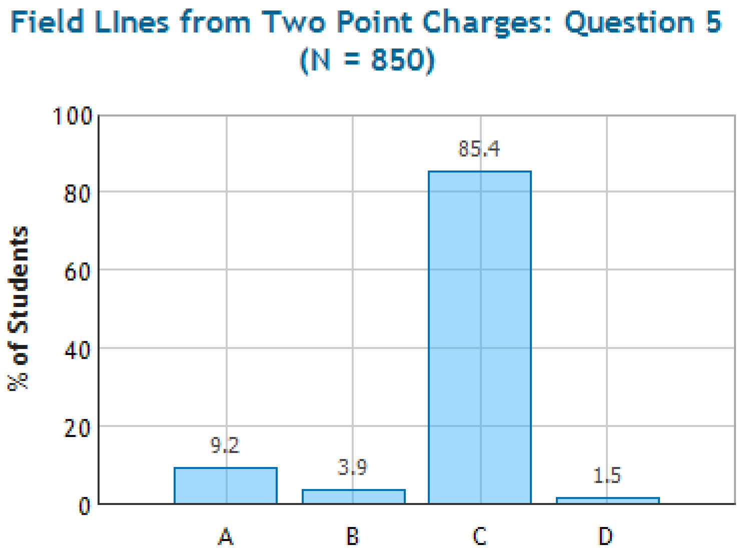


"Although we don't know the direction of the lines, we know lines must leave from one charge and enter in another."

CheckPoint 3.5



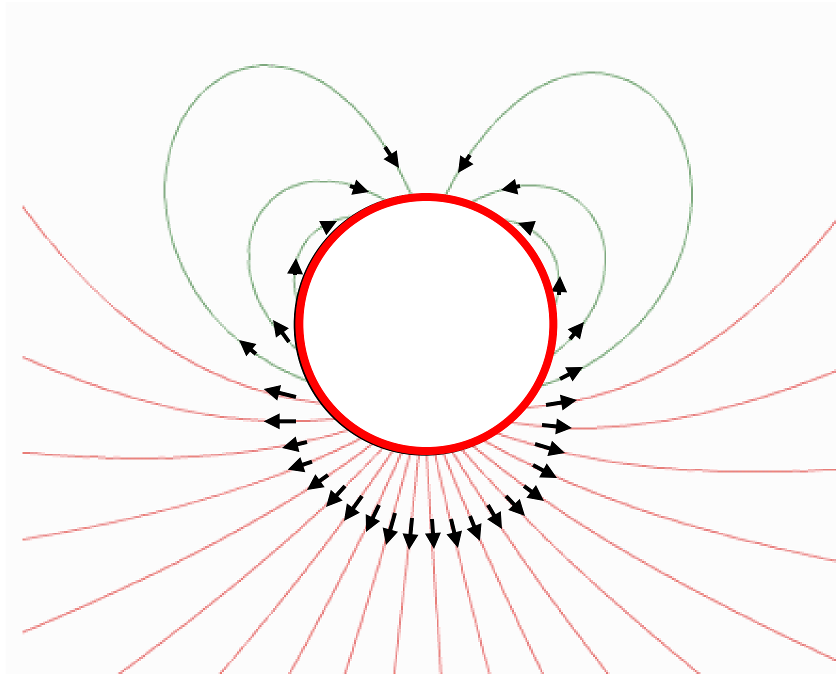
A. $|E_A| > |E_B|$
B. $|E_A| = |E_B|$
C. $|E_A| < |E_B|$
D. Not enough info



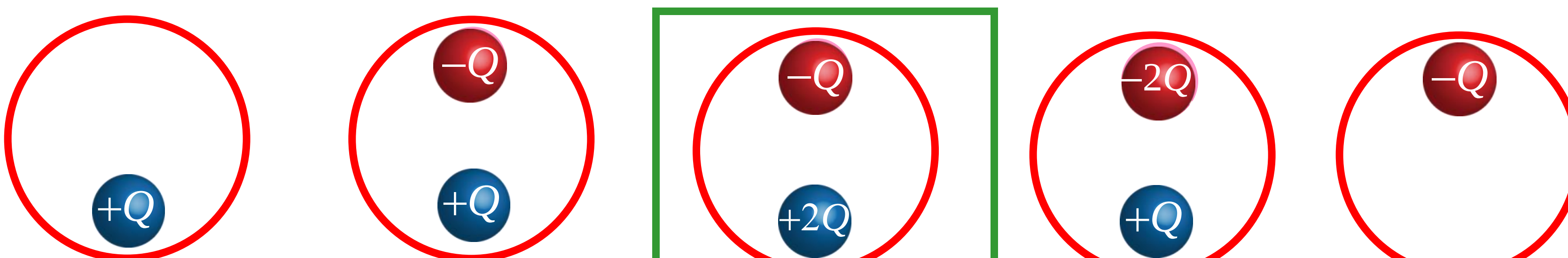
"b is greater than a because the lines are closer together at b."

Point Charges

Which configuration of charges inside the red circle match electric field pattern show?



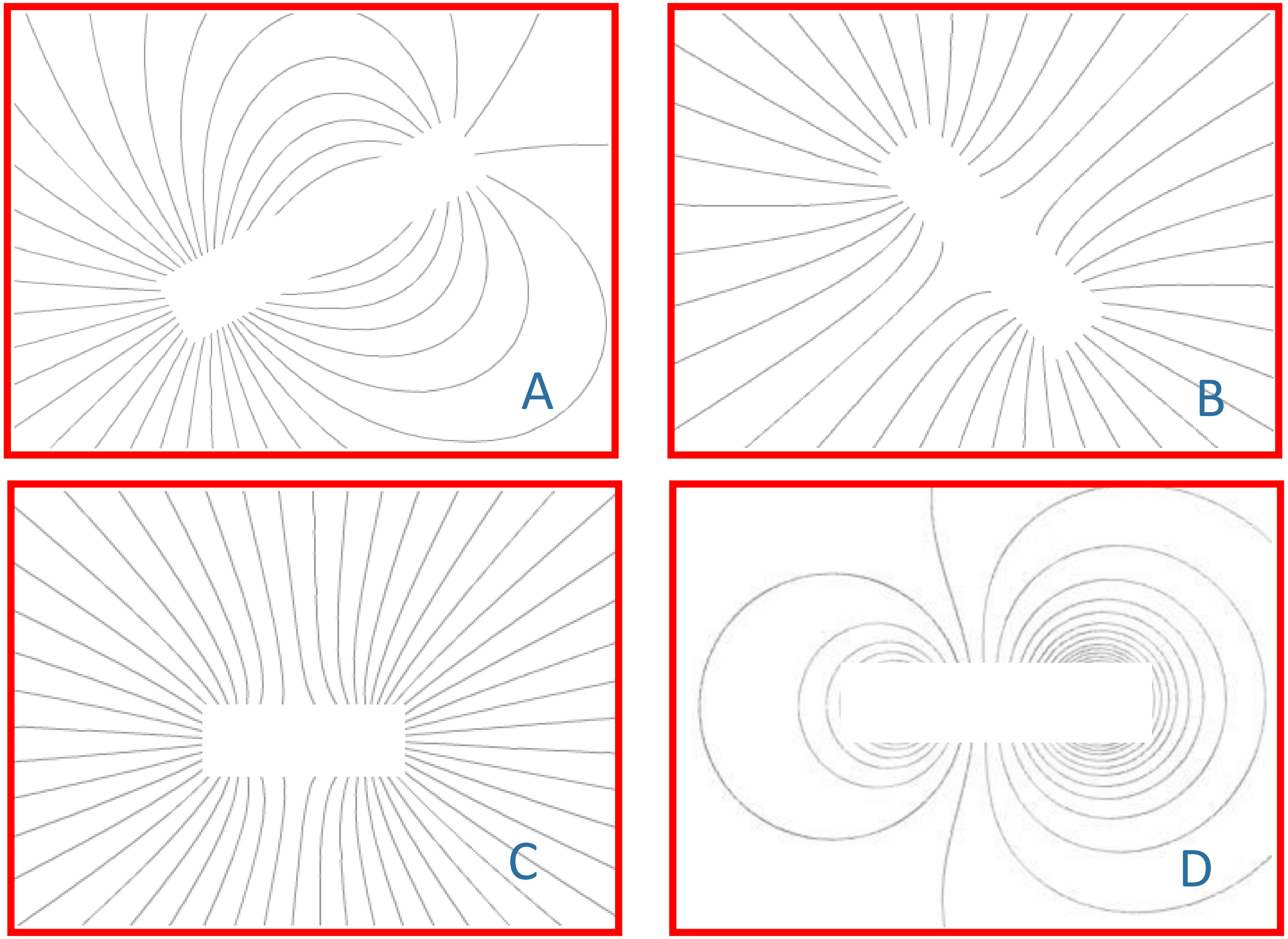
What charges are inside the red circle?



A B C D E

Electric Field lines

Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?



Simulation

Electric Flux "Counts Field Lines"

When the flux = 0. I don't understand why the flux through the surface of the rod is zero and the flux through the barrel = some number/ equation.

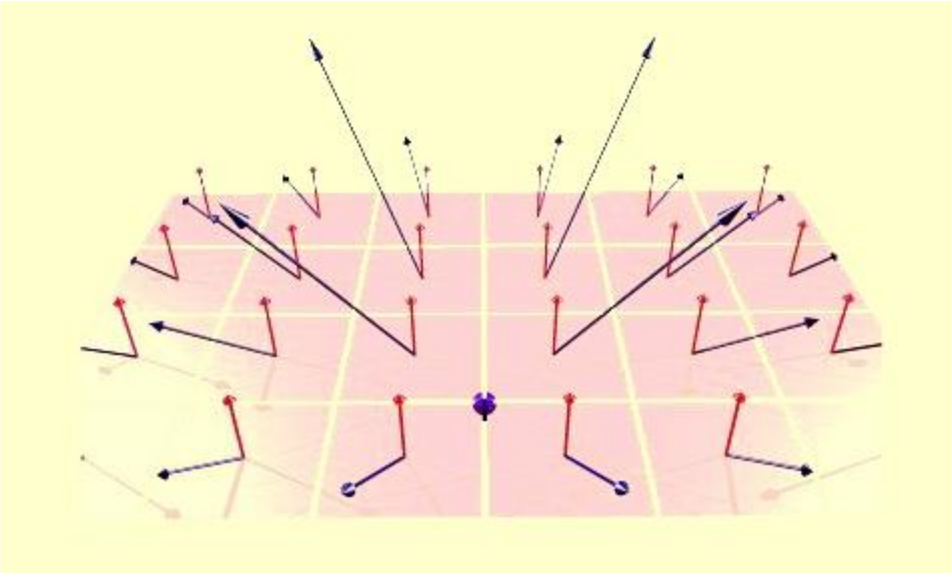
$$\Phi_S = \underbrace{\int_S \vec{E} \cdot d\vec{A}}$$

Can you give us a clear, simple definition of what flux is?

Representing the area of a surface as a vector in order to take the dot product.

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S

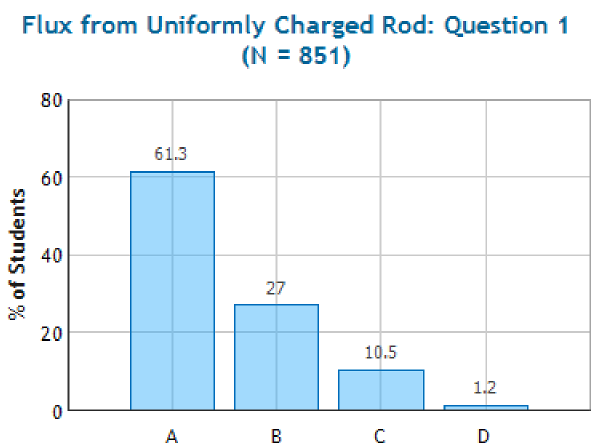


CheckPoint 1

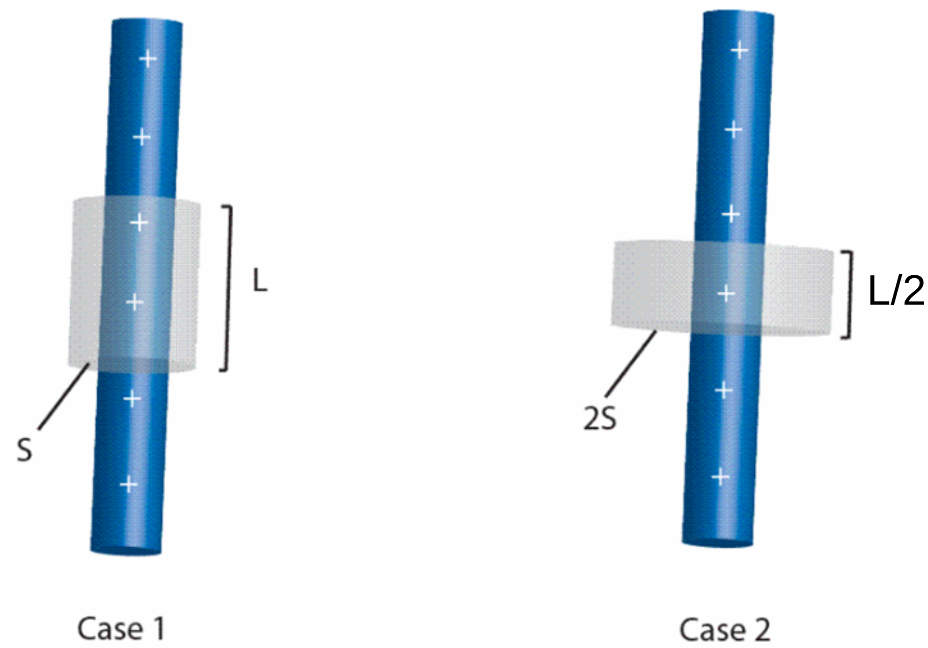
A) “twice the charge twice the flux”

B) “flux = E * 2pi * R * L. In case 1, R/L are normal; in case 2, we have 2R and 1/2 L. The product is the same in both cases, so the flux is the same.

C) “radius us squared so it would be 4 times .5 which is 2 which is greater than 1 so case 2 is twice the size of case 1”



An infinitely long charged rod has uniform charge density λ and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



$\Phi_1 = 2\Phi_2$ (A)

$\Phi_1 = \Phi_2$ (B)

$\Phi_1 = 1/2\Phi_2$ (C)

none (D)

CheckPoint 1 (Hard way)

"Can we PLEASE go through an example where we have to actually find the surface integral! I would like to see how that works."

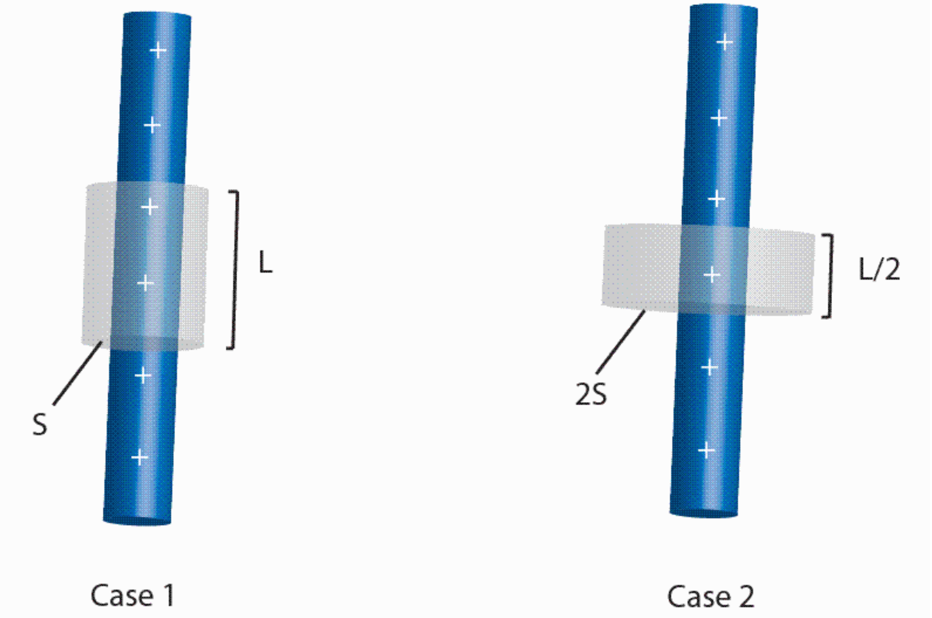
Definition of Flux:

$$\Phi \equiv \int_{surface} \vec{E} \cdot d\vec{A}$$

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

$$\Phi = E \int_{barrel} d\vec{A}$$

$$= EA_{barrel}$$



$\Phi_1 = 2\Phi_2$ (A)	$\Phi_1 = \Phi_2$ (B)	$\Phi_1 = 1/2\Phi_2$ (C)	none (D)

RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Case 1

$$A_{barrel} = 2\pi sL$$

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 s}$$

$$\boxed{\Phi_1 = \frac{\lambda L}{\varepsilon_0}}$$

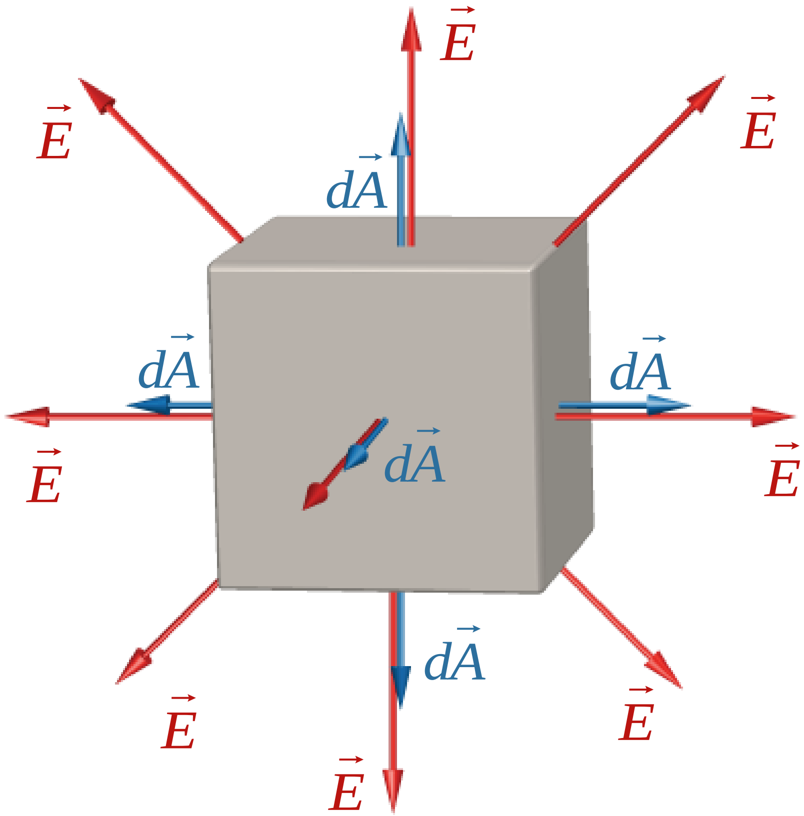
Case 2

$$A_2 = (2\pi(2s))L/2 = 2\pi sL$$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0(2s)}$$

$$\boxed{\Phi_2 = \frac{\lambda(L/2)}{\varepsilon_0}}$$

Direction Matters:

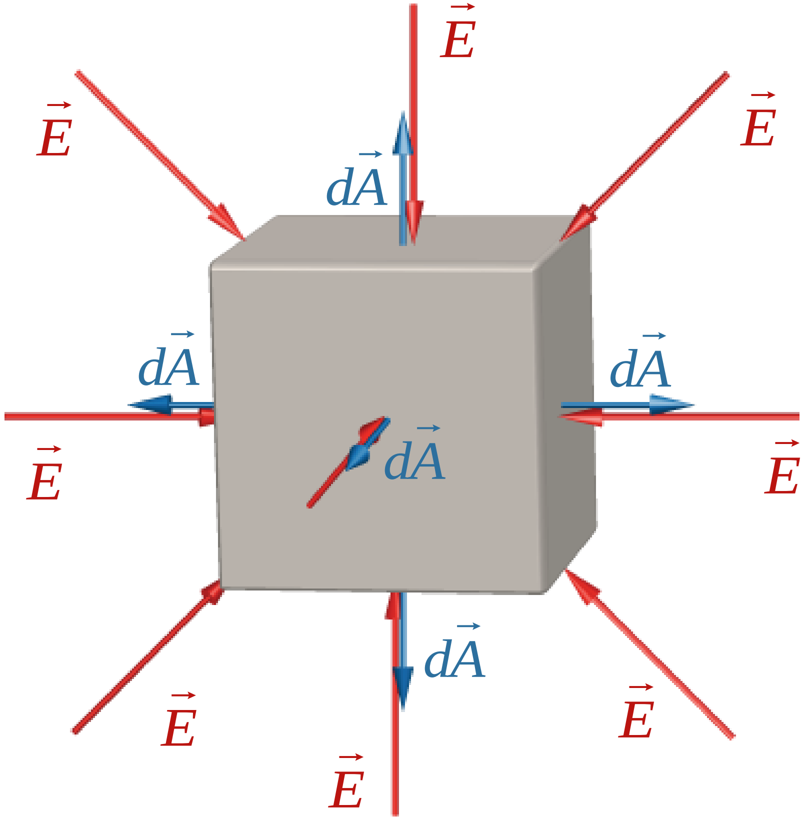


For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

Direction Matters:

Can flux through a flat plane be negative? What would that represent?

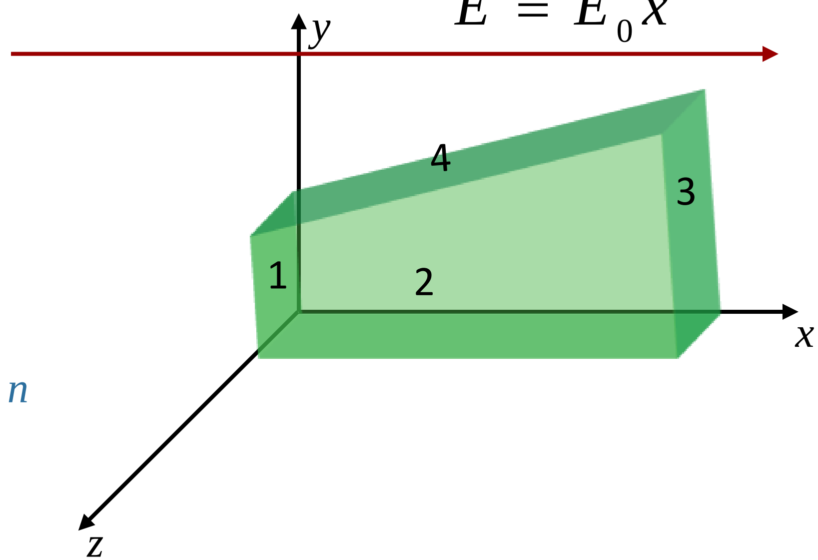


For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

Trapezoid in Constant Field

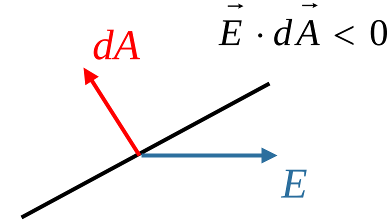
$$\vec{E} = E_0\hat{x}$$



Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted

Define $\Phi_n =$ Flux through Face n

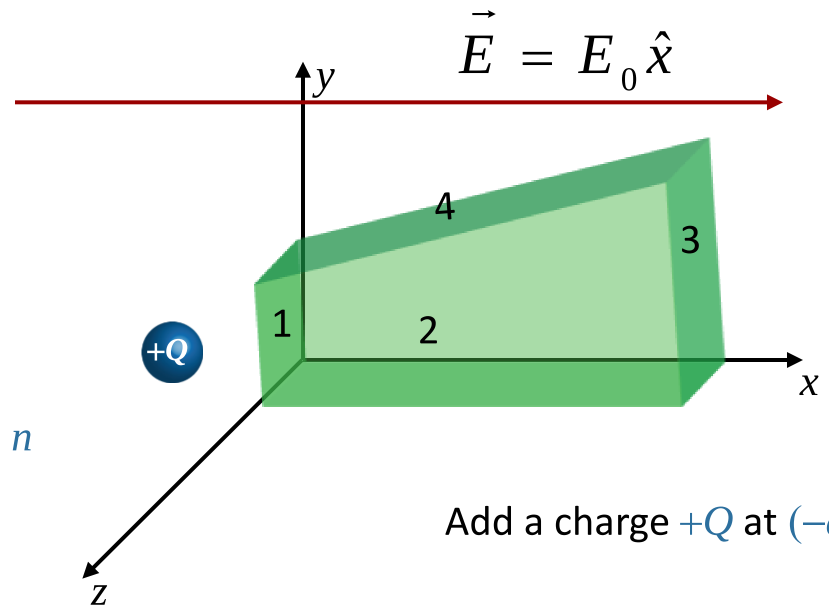


Face 1	Face 2	Face 3	Face 4
$\boxed{\text{A)}\ \Phi_1 < 0}$	A) $\Phi_2 < 0$	A) $\Phi_3 < 0$	$\boxed{\text{A)}\ \Phi_4 < 0}$
B) $\Phi_1 = 0$	$\boxed{\text{B)}\ \Phi_2 = 0}$	B) $\Phi_3 = 0$	B) $\Phi_4 = 0$
C) $\Phi_1 > 0$	C) $\Phi_2 > 0$	$\boxed{\text{C)}\ \Phi_3 > 0}$	C) $\Phi_4 > 0$

Trapezoid in Constant Field + Q

Label faces:
1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted

Define Φ_n = Flux through Face n
Φ = Flux through Trapezoid



Add a charge $+Q$ at $(-a, a/2, a/2)$

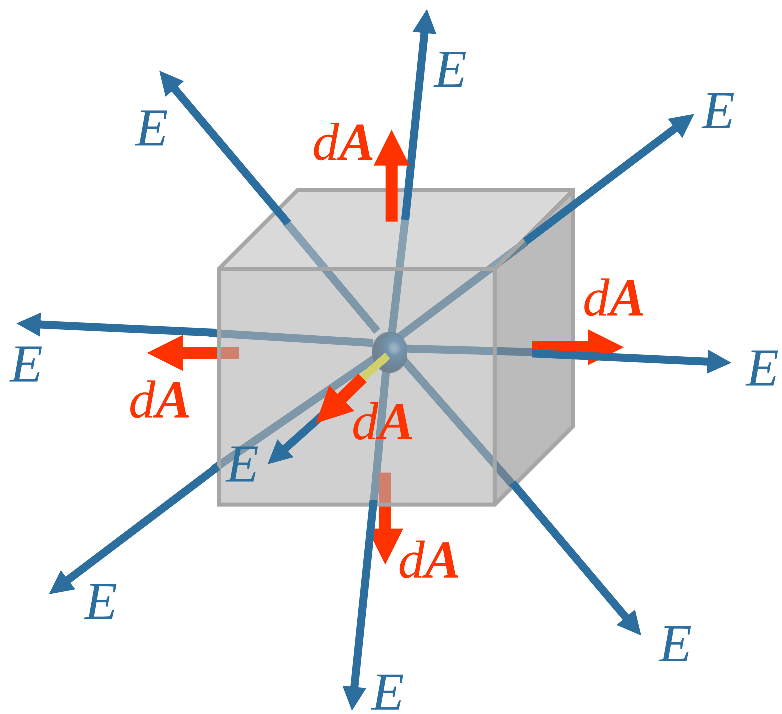
How does Flux change?
Note (-6 < -4) sign matters

A) Φ_1 increases
B) Φ_1 decreases
C) Φ_1 remains same

A) Φ_3 increases
B) Φ_3 decreases
C) Φ_3 remains same

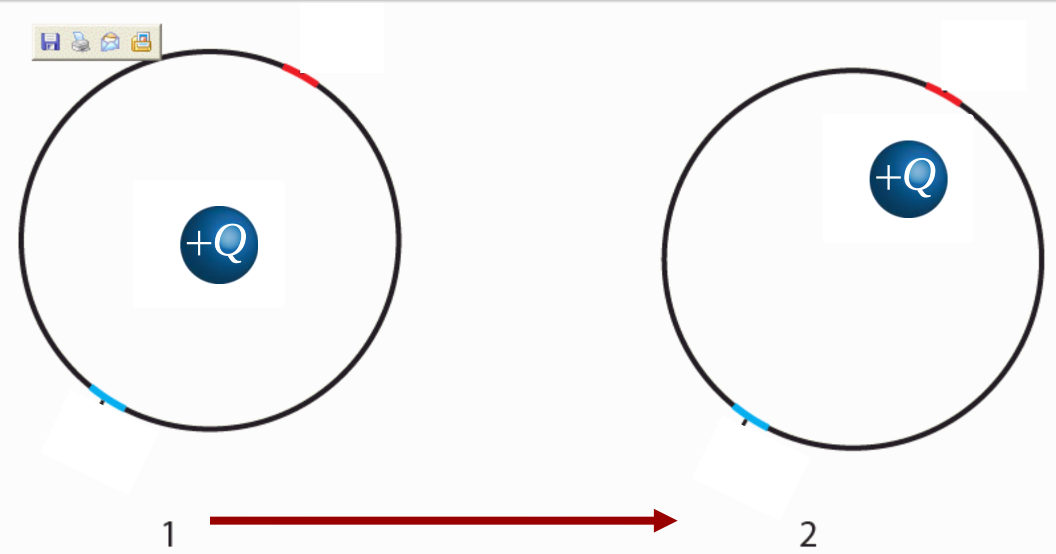
A) Φ increases
B) Φ decreases
C) Φ remains same

Gauss Law



$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

CheckPoint 2.3

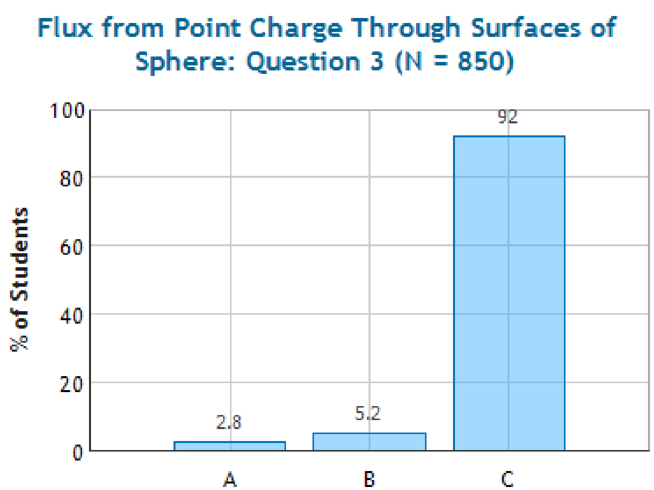


How does flux through entire surface change as charge is moved from center toward edge?

A
Φ_E increases

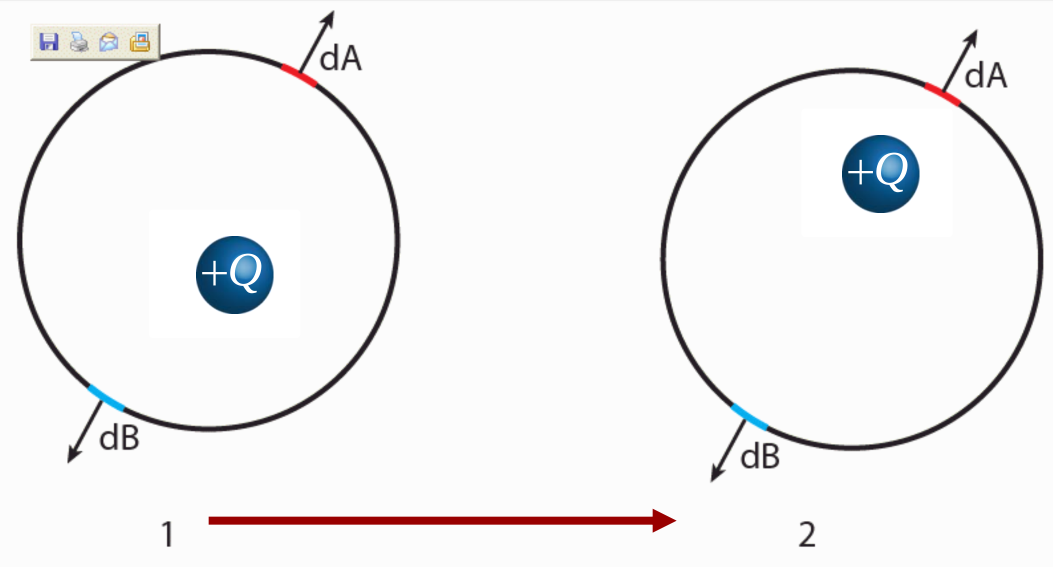
B
Φ_E decreases

C
Φ_E stays same



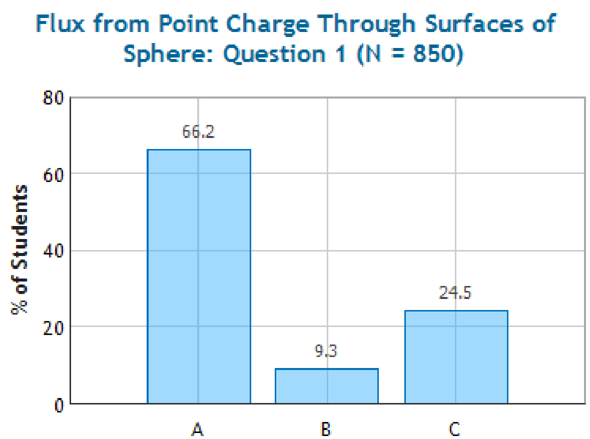
"The same amount of field lines are being produced and pass through the shell because the size of the charge did not change."

CheckPoint 2.1



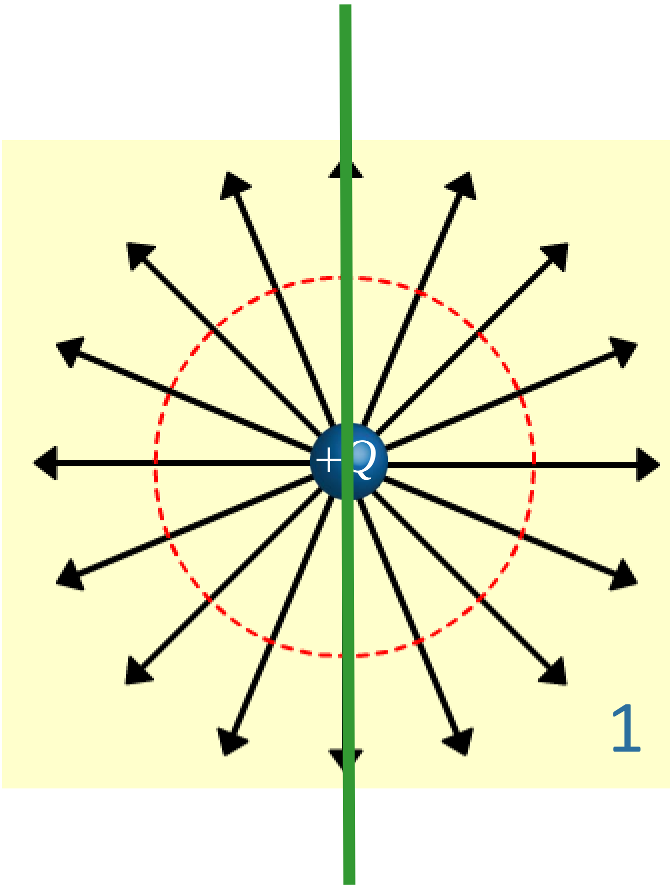
How does flux through red surface element dA change as charge is moved from center toward edge?

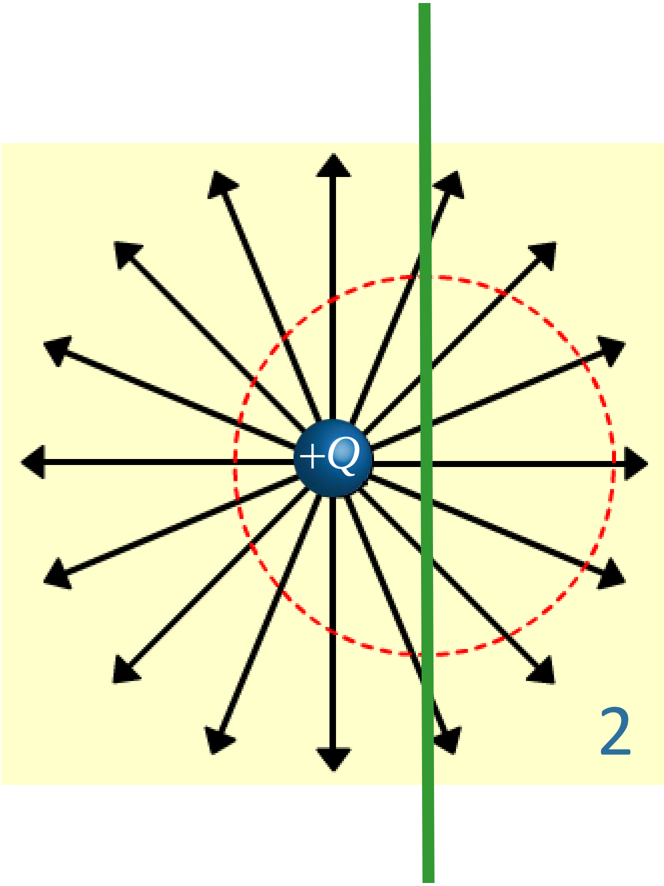
A	B	C
$d\Phi_A$ increases $d\Phi_B$ decreases	$d\Phi_A$ decreases $d\Phi_B$ increases	$d\Phi_A$ stays same $d\Phi_B$ stays same



"As dA is closer to the charge, the strength of the electric field at dA increases, thus making the flux increase.On the flip side, dB is moved further away from the charge, decreasing the strength of the electric field and flux at dB."

Think of it this way:

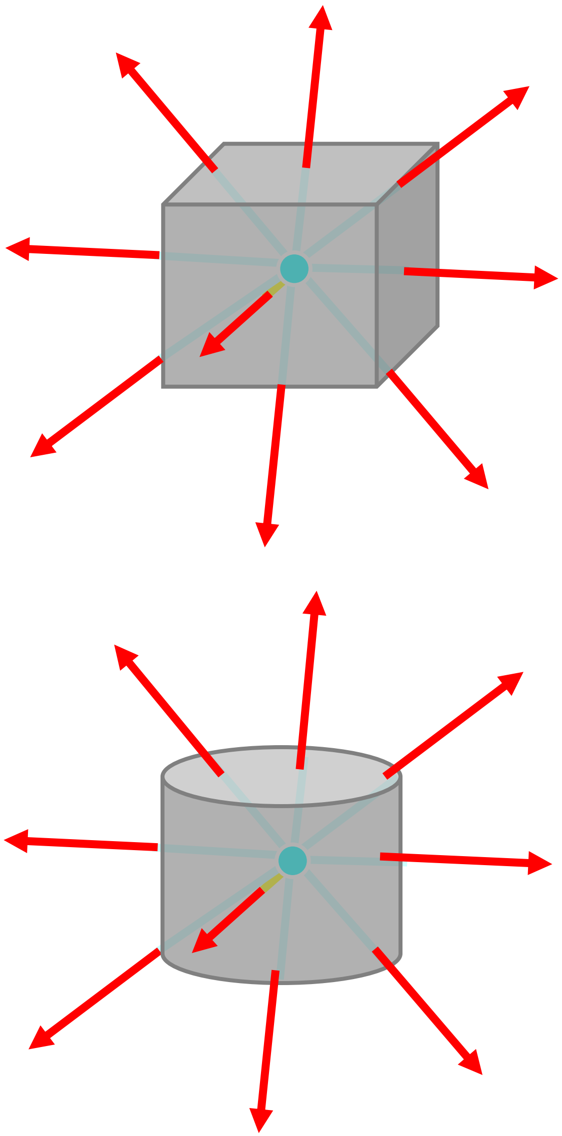




The total flux is the same in both cases (just the total number of lines)

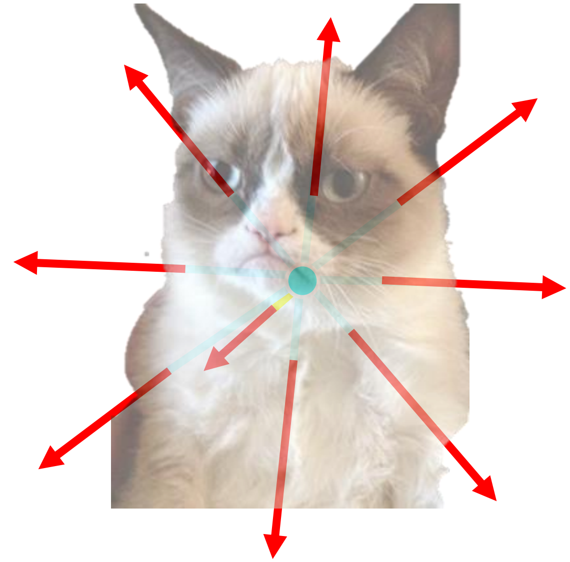
The flux through the right (left) hemisphere is smaller (bigger) for case 2.

Things to notice about Gauss Law



$$\Phi_S = \int_{\substack{closed \\ surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.



Things to notice about Gauss Law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

In cases of high symmetry it may be possible to bring E outside the integral. In these cases we can solve Gauss Law for E

$$E \int dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{Q_{enclosed}}{A \varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E!

This is the topic of the next lecture.