# *Physics 212*
# *Lecture 10*

Today's Concept:

Kirchhoff's Rules

# Your Comments

# *Your Comments*

please go over the connecting a and b problem

current direction seems to arbitrary why does it sometimes come from the positive side of the battery and why does it sometimes go into the positive side

I like this new stuff so far :) please don't make me hate it...

I am really confused when there is a wire between a parallel circuit(CHECKPOINT 3), Can you explain?

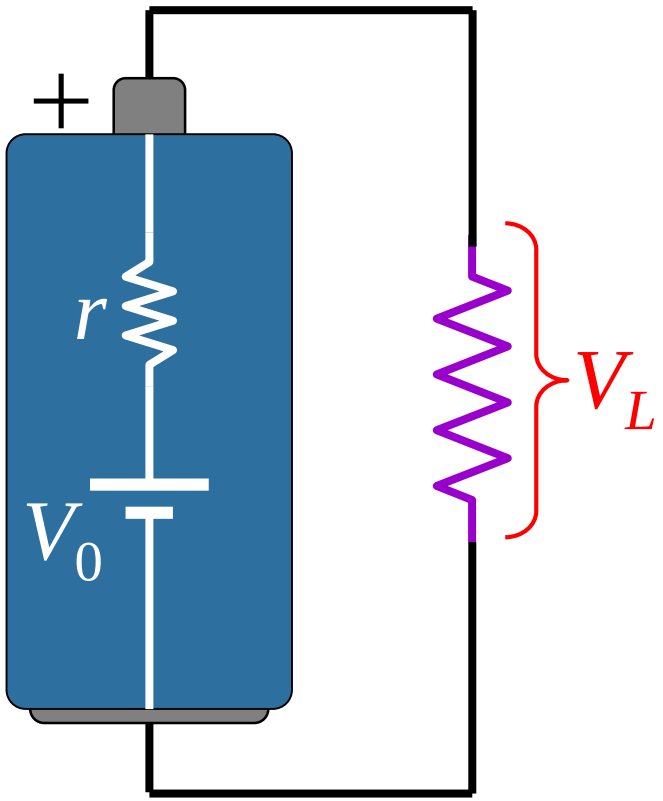
Woah...how exactly did they pick where the current flows in two-loop circuits?

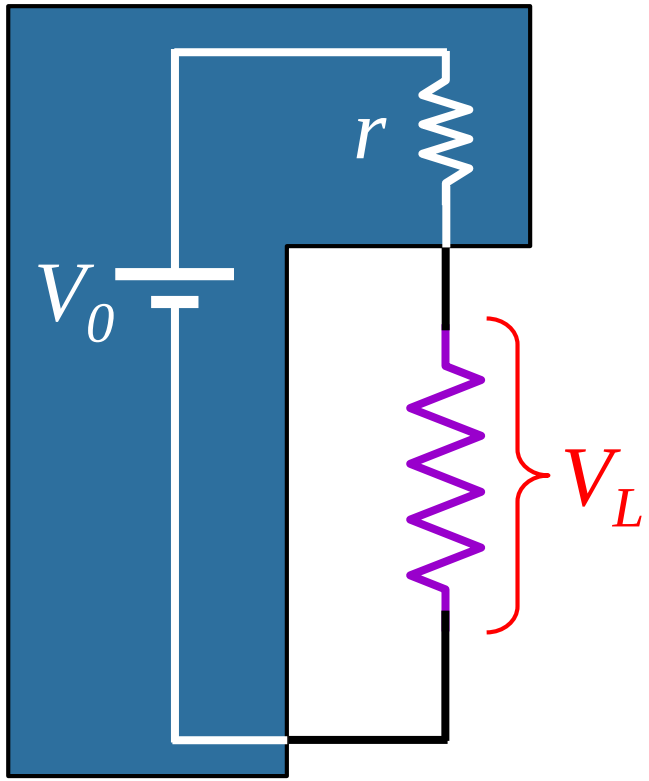
The objectification of batteries is all too real today. Physics Homework and tests portray what they claim to be the "ideal" battery by setting such unreasonable standards such as the "EMF". Its time to face the truth. Real batteries have resistance.

A homework assignment due after the midterm :((((((((((((((((((( (I guess bad planning on my part)

When will the test scores be posted? Any will the scores be scaled?

# Model for Real Battery: Internal Resistance



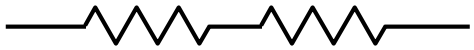

Usually can't supply too much current to the load without voltage "sagging"

# Last Time

## Resistors in series:

Current through is same.

Voltage drop across is $IR_i$

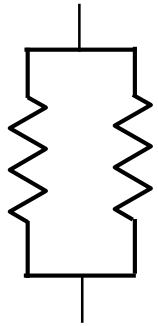


$$R_{effective} = R_1 + R_2 + R_3 + \ldots$$

## Resistors in parallel:

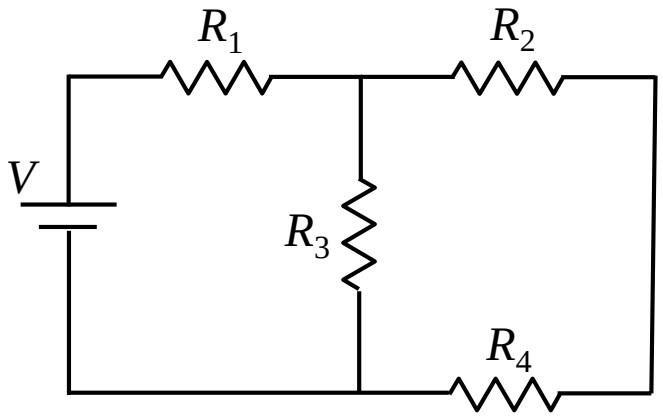
Voltage drop across is same.

Current through is $V/R_i$



$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

## Solved Circuits

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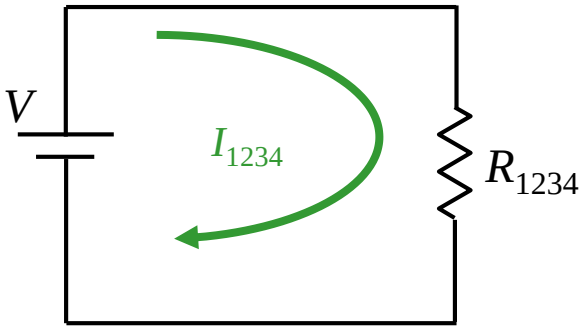
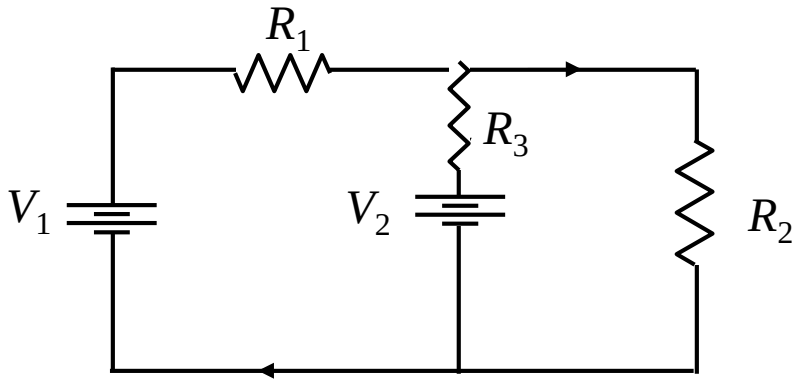
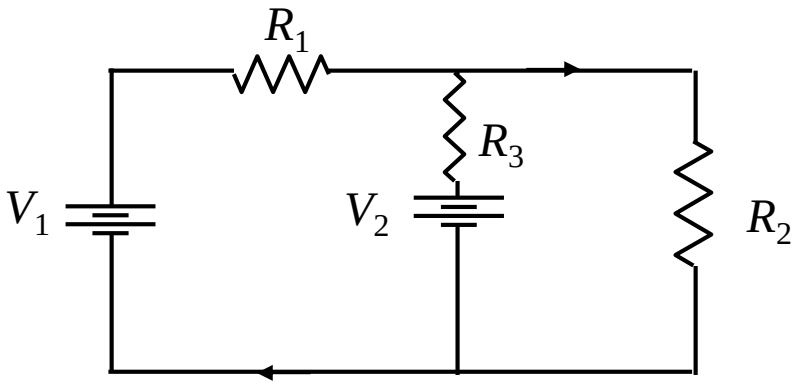


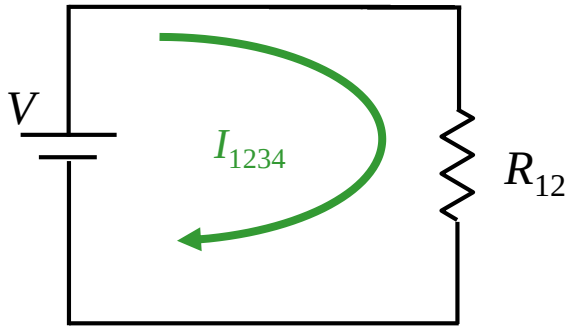
# New Circuit




How Can We Solve This One?




$\neq$




THE ANSWER: Kirchhoff's Rules

# Kirchhoff's Voltage Rule

$$\sum \Delta V_i = 0$$

Kirchhoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

WHY?

The potential difference between a point and itself is zero!

# Kirchhoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

WHY?

Electric Charge is Conserved

# Applying Kirchhoff's Laws in 5 easy steps

$-E_1 + I_1R_1 + E_3 - I_4R_4 + I_5R_5 = 0$

1) Label all currents

Choose any direction

2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

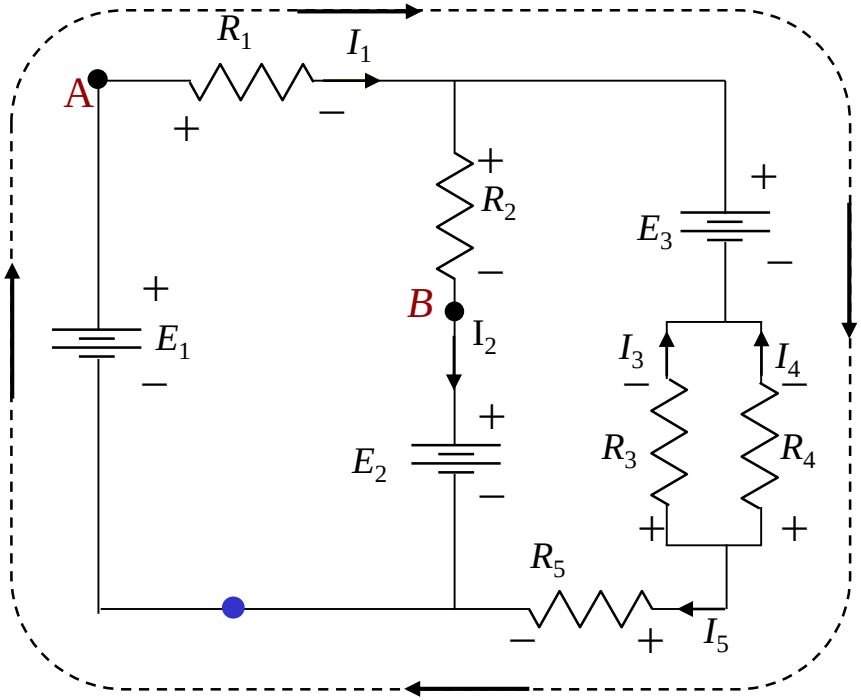
Long side is + for battery

3) Choose loop and direction

Must start on wire, not element.

4) Write down voltage drops

First sign you hit is sign to use.



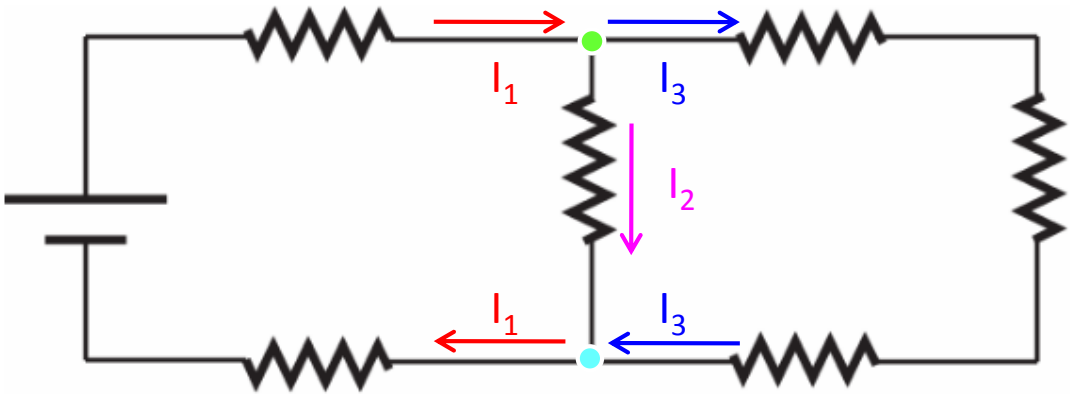

5) Write down node equation $I_{in} = I_{out}$

We'll do calculation today

It's actually the easiest thing to do!

# Check Point 1

How many potentially different currents are there in the circuit shown?



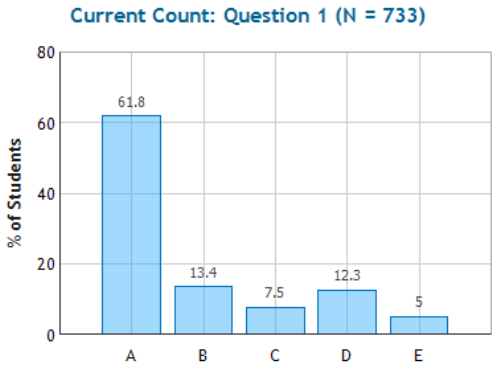

***A.*** 3 ***B.*** 4 ***C.*** 5 ***D.*** 6 ***E.*** 7

Look at the nodes!

Top node: $I_1$ flows in, $I_2$ and $I_3$ flow out

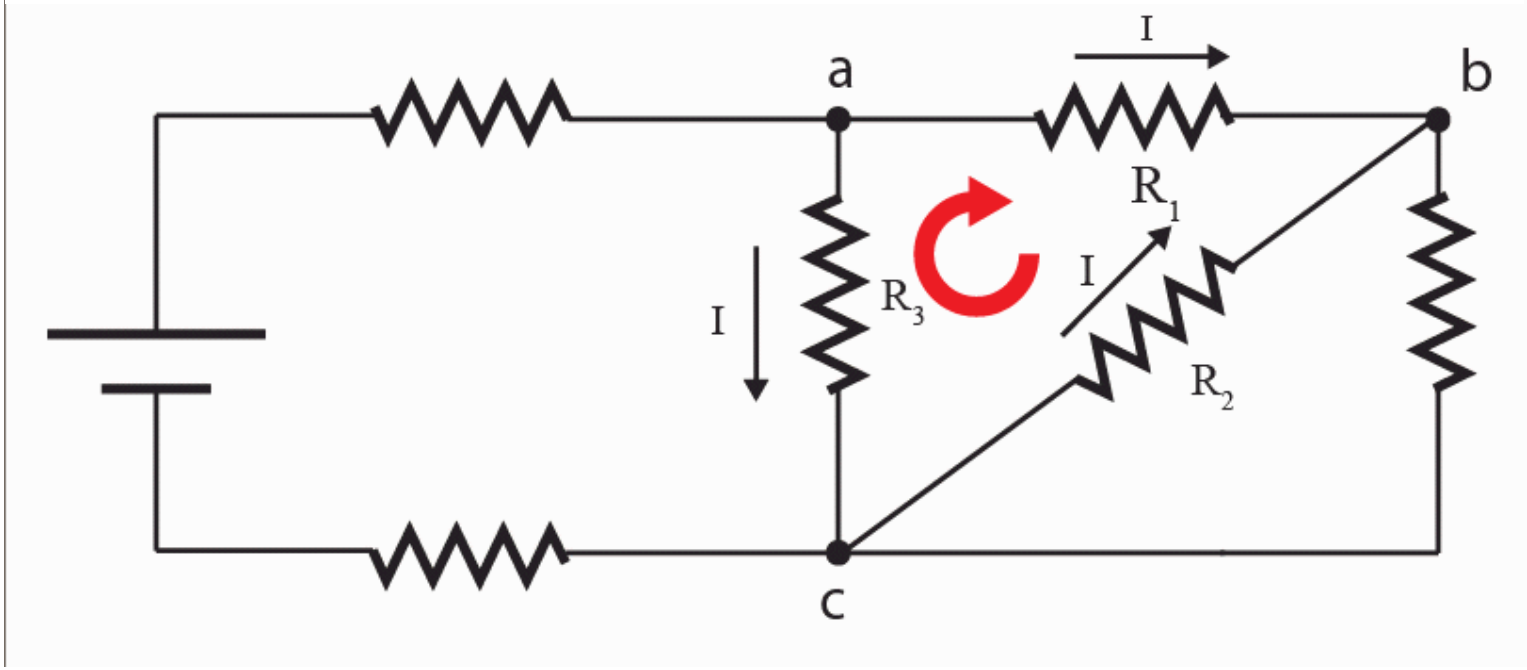
Bottom node: $I_2$ and $I_3$ flow in, $I_1$ flows out

That's all of them!



# CheckPoint 2

In the following circuit, consider the loop abc. The direction of the current through each resistor is indicated by black arrows.

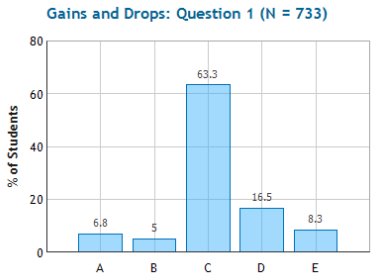



If we are to write Kirchoff's voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors R1, R2 and R3?

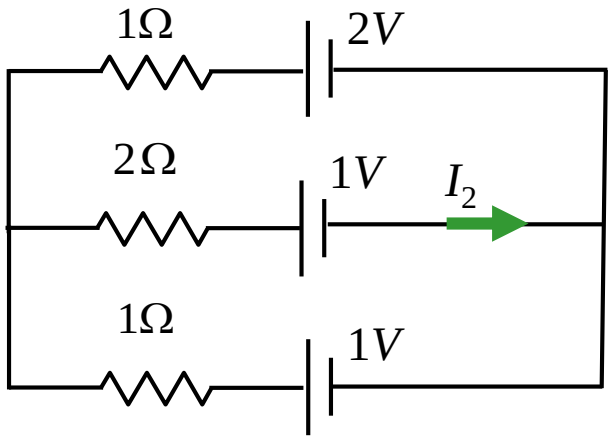
***A.*** drop, drop, drop
***B.*** gain, gain, gain
***C.*** drop, gain, gain
***D.*** gain, drop, drop
***E.*** drop, drop, gain

With the current → VOLTAGE DROP

Against the current → VOLTAGE GAIN



# Calculation




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ?

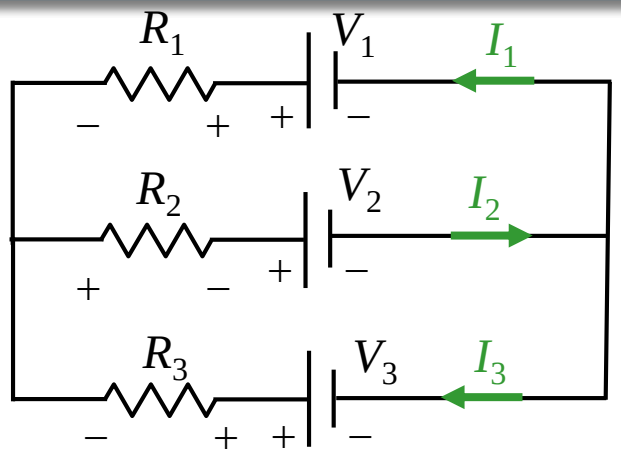
Conceptual Analysis:

- Circuit behavior described by Kirchhoff's Rules:
  - KVR: $\Sigma V_{drops} = 0$
  - KCR: $\Sigma I_{in} = \Sigma I_{out}$

Strategic Analysis

- Write down Loop Equations (KVR)
- Write down Node Equations (KCR)
- Solve

# Calculation




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ?

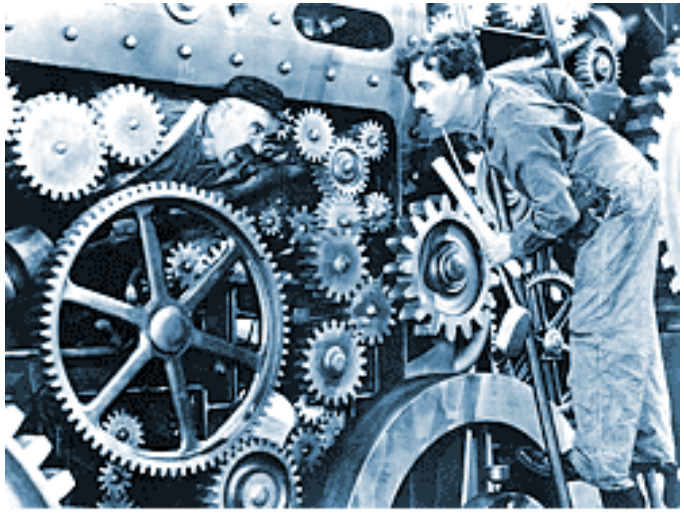
1) Label and pick directions for each current

2) Label the + and – side of each element

This is easy for batteries Long side is +

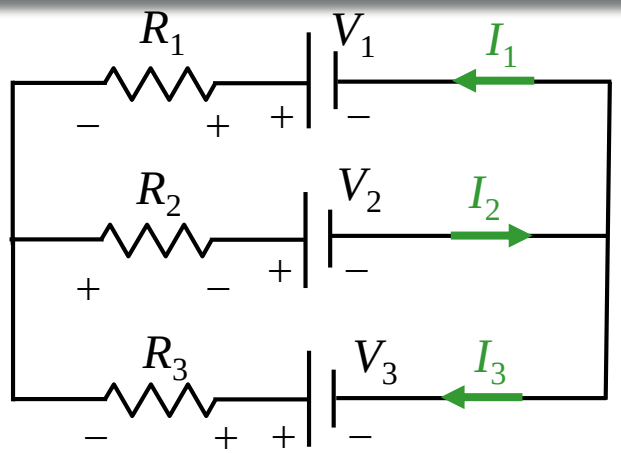
For resistors, the "upstream" side is +

Now write down loop and node equations



Just turn the crank.

# Calculation




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ?

How many equations do we need to write down in order to solve for $I_2$?

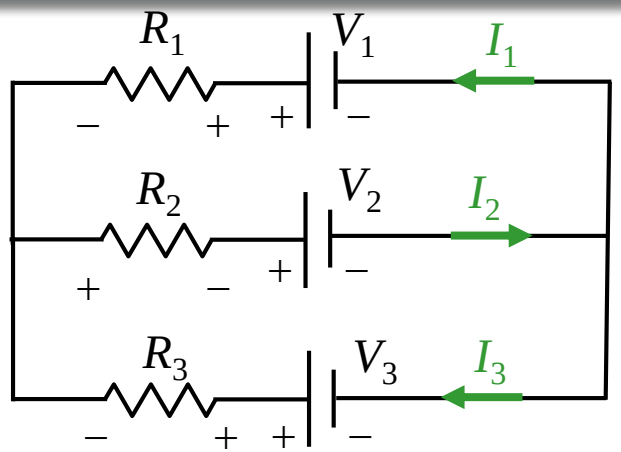
A) 1 B) 2 **C) 3** D) 4 E) 5

Why?

- We have 3 unknowns: $I_1$, $I_2$, and $I_3$
- We need 3 independent equations to solve for these unknowns

3) Choose Loops and Directions

# Calculation




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ?

Which of the following equations is NOT correct?

A) $I_2 = I_1 + I_3$

B) $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$

C) $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$

D) $-V_2 - I_2R_2 + I_1R_1 + V_1 = 0$

4) Write down voltage drops

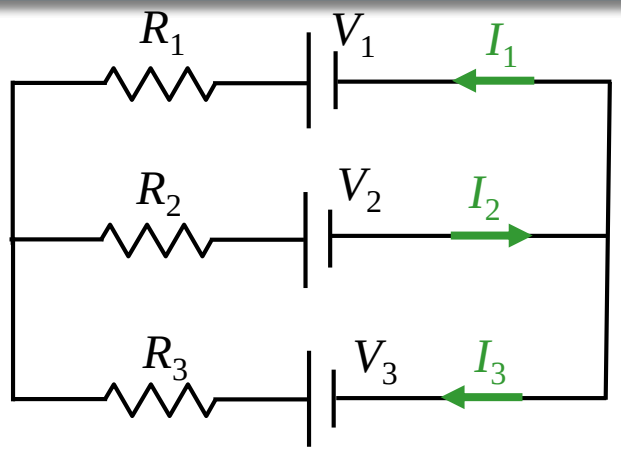
5) Write down node equation

Why?

- $(D)$ is an attempt to write down $KVR$ for the top loop
- Start at negative terminal of $V_2$ and go clockwise

  $V_{gain}\,(-V_2)$ then $V_{gain}\,(-I_2R_2)$ then $V_{gain}\,(-I_1R_1)$ then $V_{drop}\,(+V_1)$

# Calculation




In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ?

We need 3 equations:
Which 3 should we use?

We have the following 4 equations:

1. $I_2 = I_1 + I_3$
2. $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
3. $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
4. $-V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

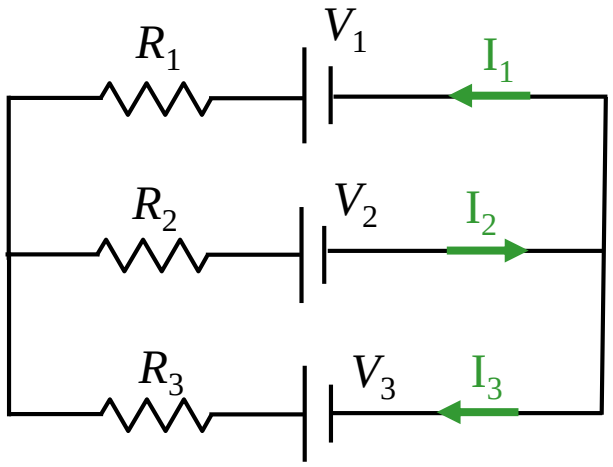
A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4

Why?

- We need 3 INDEPENDENT equations
- Equations 2, 3, and 4 are NOT INDEPENDENT

  Eqn 2 + Eqn 3 = – Eqn 4
- We must choose Equation 1 and any two of the remaining ( 2, 3, and 4)

# Calculation



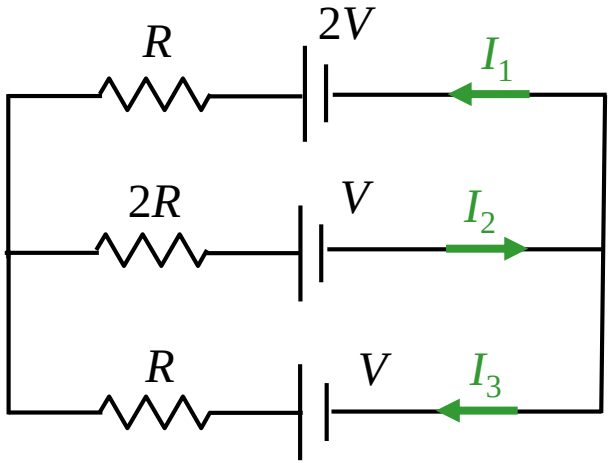

In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$ ?

We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$V_1 + I_1 R_1 - I_3 R_3 + V_3 = 0$$
$$V_2 - I_2 R_2 - I_1 R_1 + V_1 = 0$$

Now just need to solve ☺



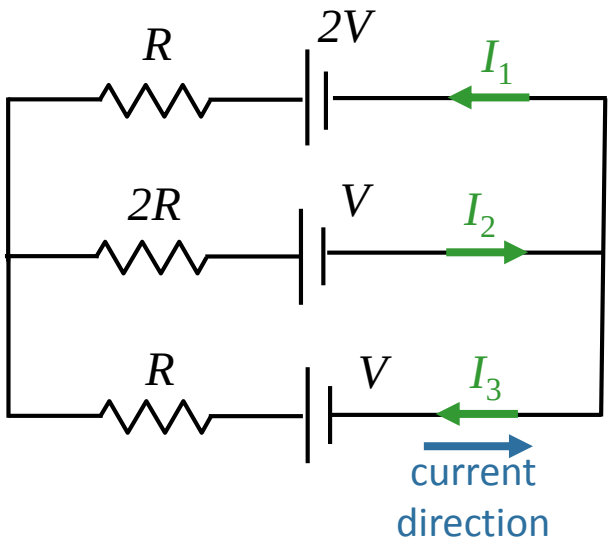

The solution will get very messy!

Simplify: assume $V_2 = V_3 = V$

$$V_1 = 2V$$
$$R_1 = R_3 = R$$
$$R_2 = 2R$$

# Calculation: Simplify

In this circuit, assume $V$ and $R$ are known. What is $I_2$ ?




We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$-2V + I_1R - I_3R + V = 0 \quad \text{(outside)}$$

$$-V - I_2(2R) - I_1R + 2V = 0 \quad \text{(top)}$$

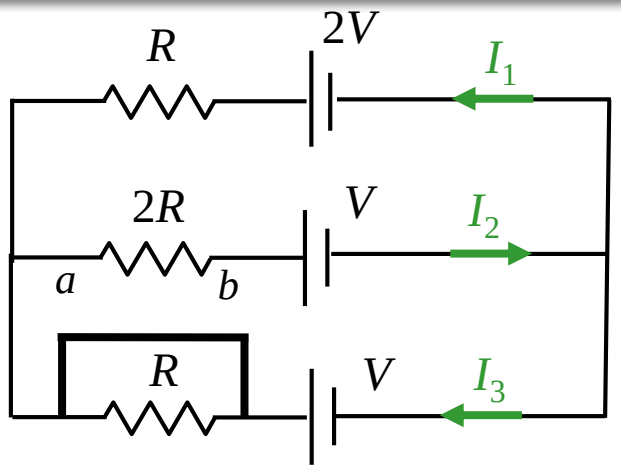
With this simplification, you can verify:

$$I_2 = (1/5)\, V/R$$

$$I_1 = (3/5)\, V/R$$

$$I_3 = (-2/5)\, V/R$$

# Follow Up




We know:

$I_2 = (1/5)\ V/R$

$I_1 = (3/5)\ V/R$

$I_3 = (-2/5)\ V/R$

Suppose we short $R_3$: What happens to $V_{ab}$ (voltage across $R_2$?)

A) $V_{ab}$ remains the same

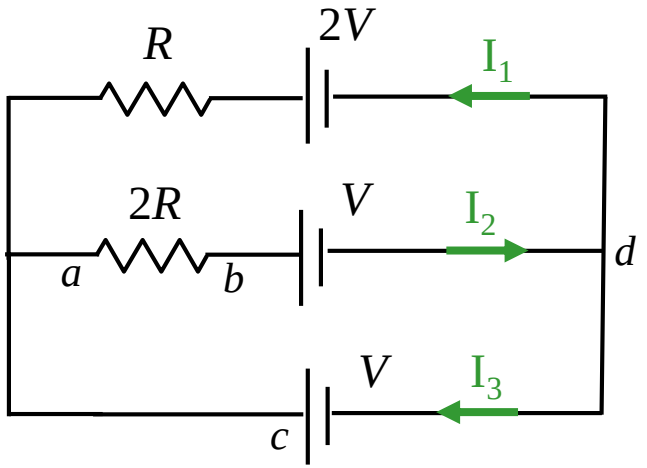
B) $V_{ab}$ changes sign

C) $V_{ab}$ increases

D) $V_{ab}$ goes to zero

Why?

Redraw:



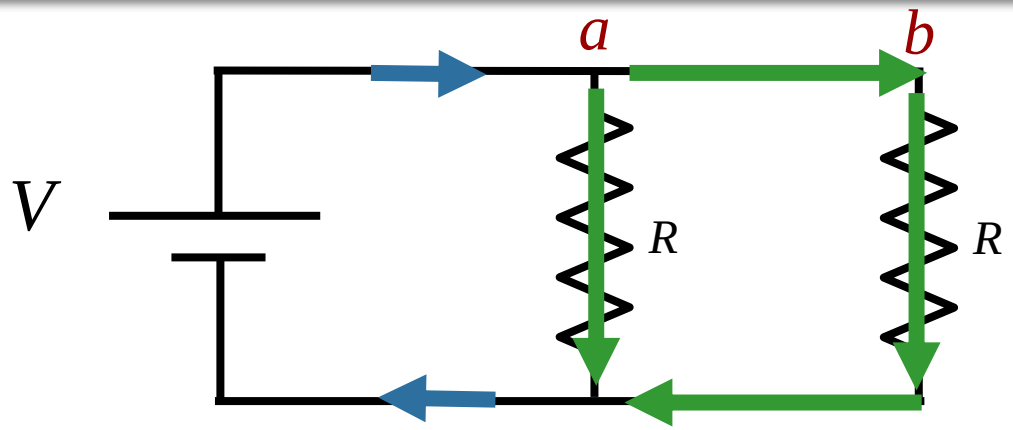

Bottom Loop Equation:

$$V_{ab} + V - V = 0$$

$$\downarrow$$

$$V_{ab} = 0$$

# CheckPoint 3 Warm up




Is there a current flowing between $a$ and $b$ ?

A) Yes
B) No

$a$ & $b$ have the same potential

No current flows between $a$ & $b$

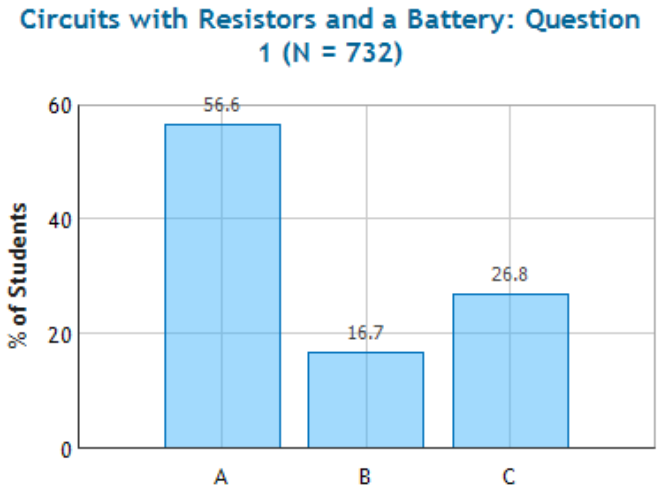
Current flows from battery and splits at $a$

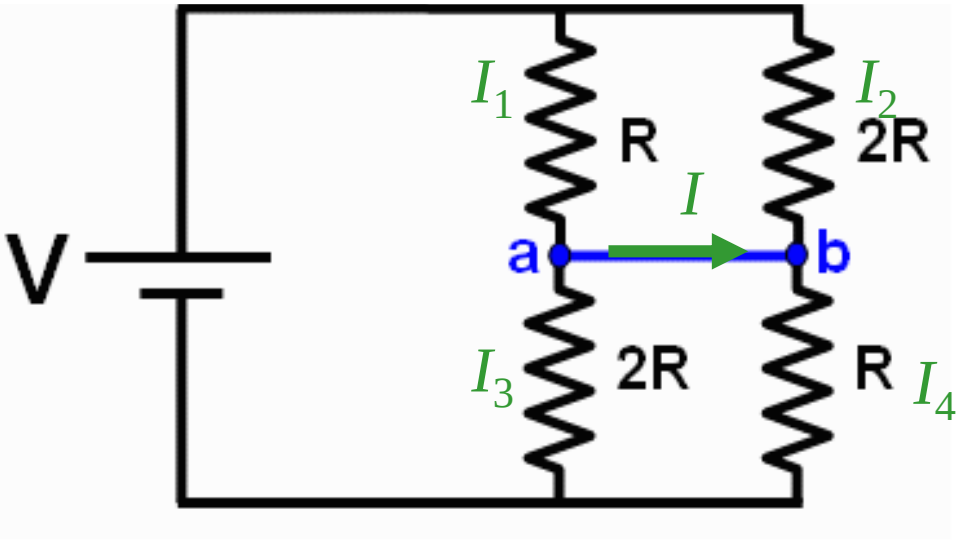
Some current flows down

Some current flows right

# CheckPoint 3a

Consider the circuit shown below. Note that this question is *not* identical to the similar looking one you answered in the prelecture.




Which of the following best describes the current flowing in the blue wire connecting points **a** and **b**?

**A.** Positive current flows from *a* to *b*

**B.** Positive current flows from *b* to *a*

**C.** No current flows between *a* and *b*

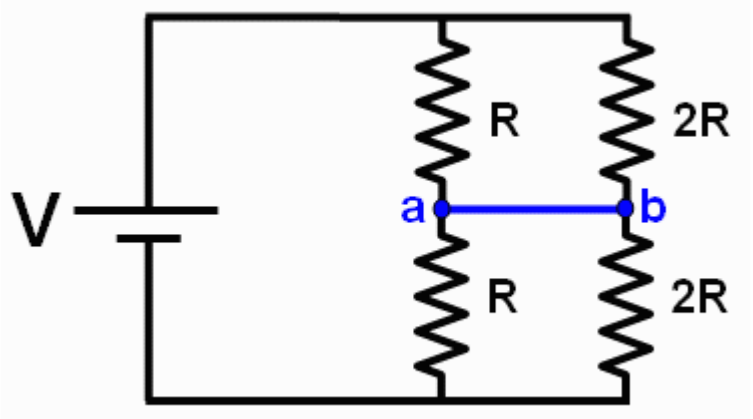
$I_1 R - I_2 (2R) = 0 \rightarrow I_2 = \frac{1}{2} I_1$

$I_4 R - I_3 (2R) = 0 \rightarrow I_4 = 2 I_3$

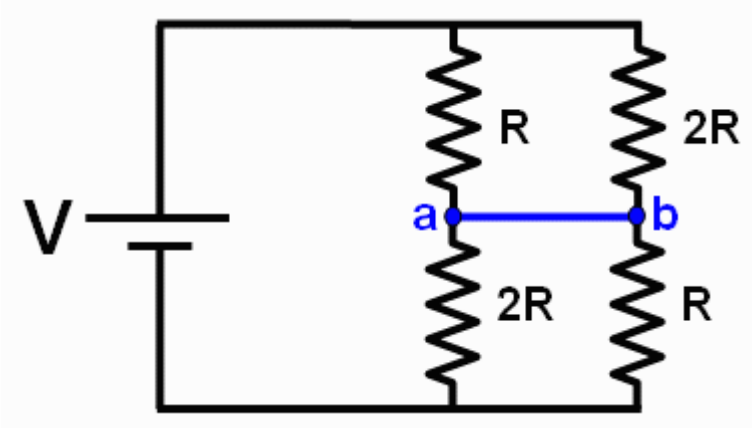
$I = I_1 - I_3$

$I + I_2 = I_4 \rightarrow I_1 - I_3 + \frac{1}{2} I_1 = 2I_3 \rightarrow I_1 = 2I_3 \rightarrow I = +I_3$

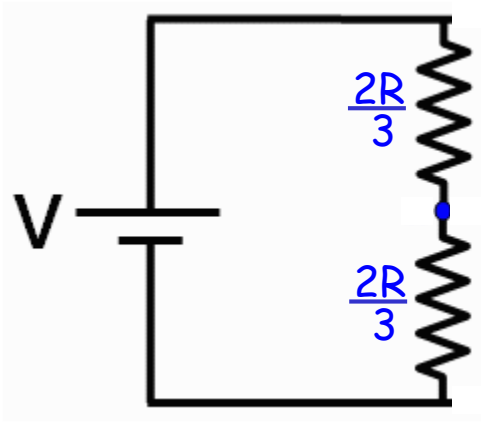
# Prelecture



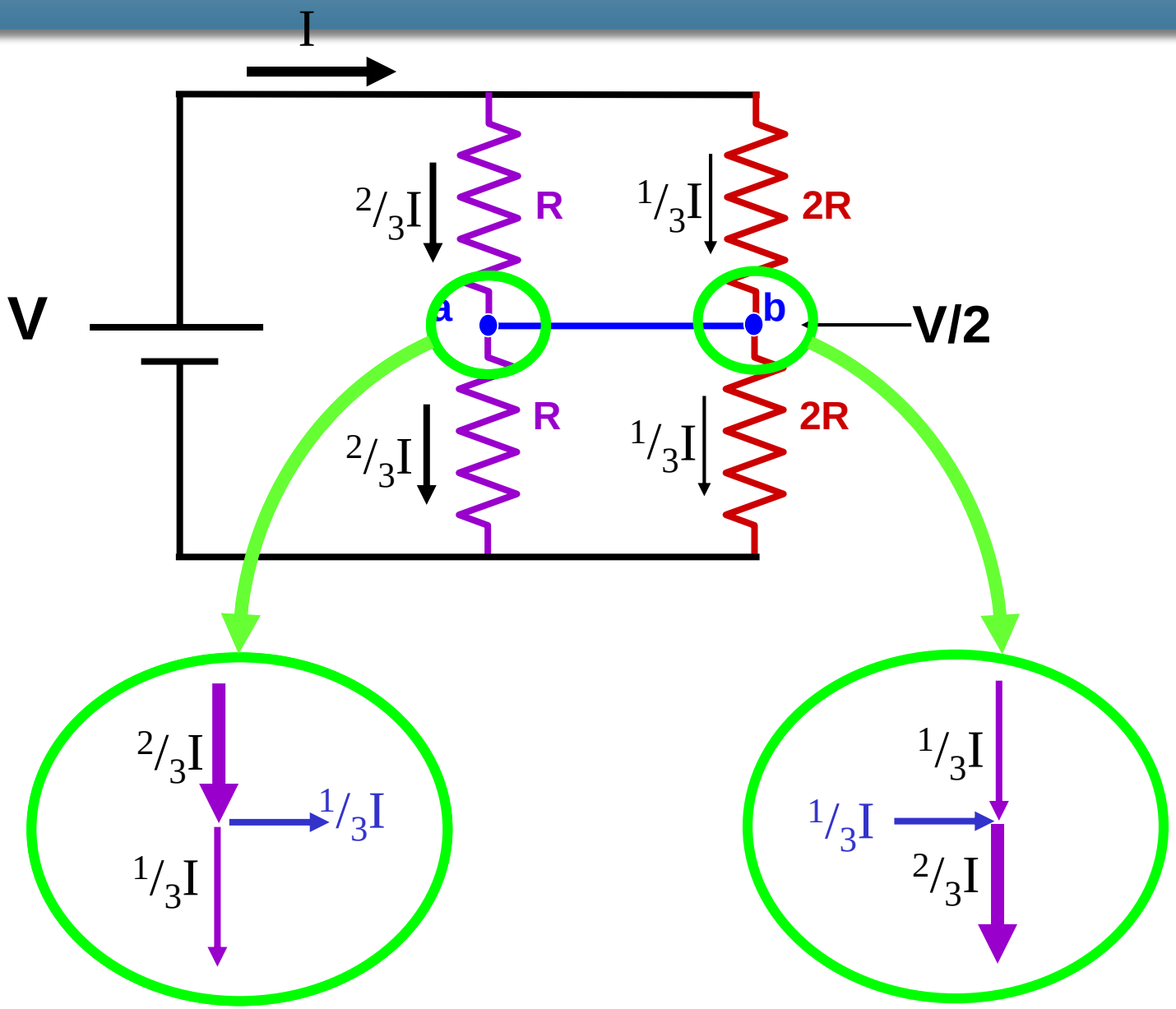

# CheckPoint



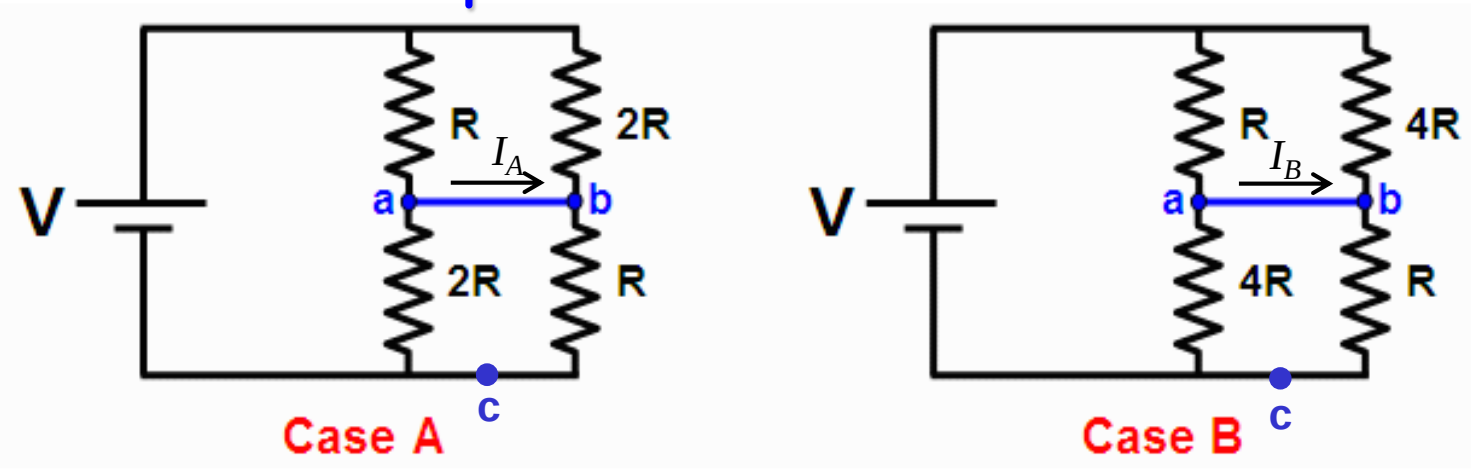

What is the same? Current flowing in and out of the battery.




What is different? Current flowing from a to b.


I
V
R
2R
R
2R
a
b
V/2
2/3I
1/3I
2/3I
1/3I
2/3I
1/3I
1/3I
2/3I
1/3I

# Checkpoint 3b




which case is the current flowing in the blue wire connecting points **a** and **b** the largest?

***A.*** Case A ***B.*** Case B ***C.*** They are both the same

Current will flow from left to right in both cases.

In both cases, $V_{ac} = V/2$

$$I_{2R} = 2I_{4R}$$

$$I_A = I_R - I_{2R} = I_R - 2I_{4R} \qquad I_B = I_R - I_{4R}$$