Physics 212
Lecture 7

Today's Concept: (Applications of Gauss, E and V)

A) Conductors

B) Capacitance

Your Comments

I didn't really get the point of the capacitors. How is the energy "stored"? What does that mean? And how could the energy be released? Also could you give an example of how the capacitor's abilities are useful in a real life situation, like a computer?

Can you go more into depth about what happens when you put the green conducting plate between a capacitor? I learned about this in high school too and it always confused me.

Please go over conductors more and how it changes with the addition of the plate and other situations we may come across. On top of that could you go more over finding the electric potential difference of different kinds of surfaces, I still have not fully grasped that subject. And will we be expected to know the derivations of the formulas they go over in the prelectures like for energy stored and energy density, etc.?

Can you explain what capacitance actually means? NOT just in types of equations C=Q/delta V but in some kind of real-life analogy, like cars and highway booths and etc? (circuitry stuff)

I am just memorizing a set of facts and plugging in equations.This feels like my grandparent's marriage after 25 years: just going through the motions and not understanding things at an intimate level. So sad.

I gotta say, I'm a bit disappointed in the in-lecture demos for 212. Where's all the crazy electrical stuff? Surely you can do better than balloons and bocce balls!"

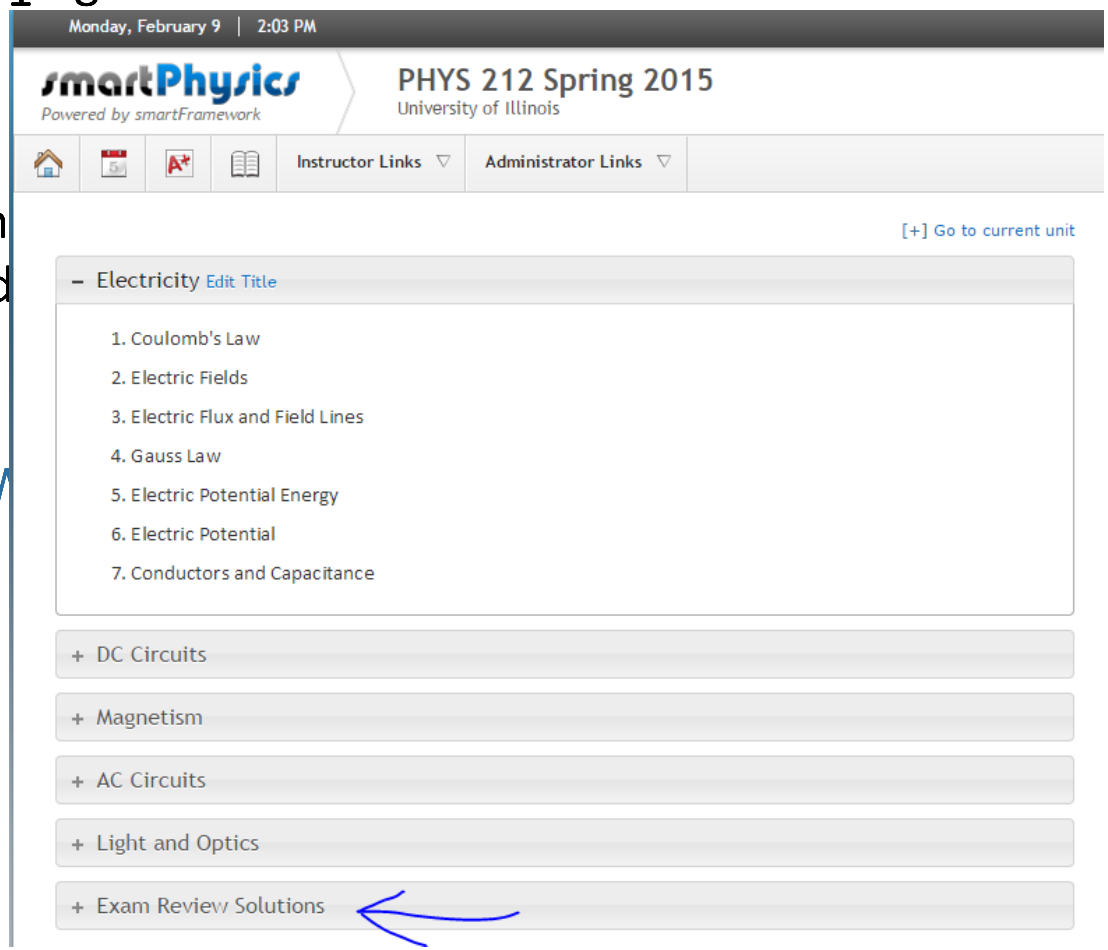
Just a friendly reminder to everyone about the coming midterm. And guys, it could be worse. At least we don't go to Purdue.

Exam Logistics

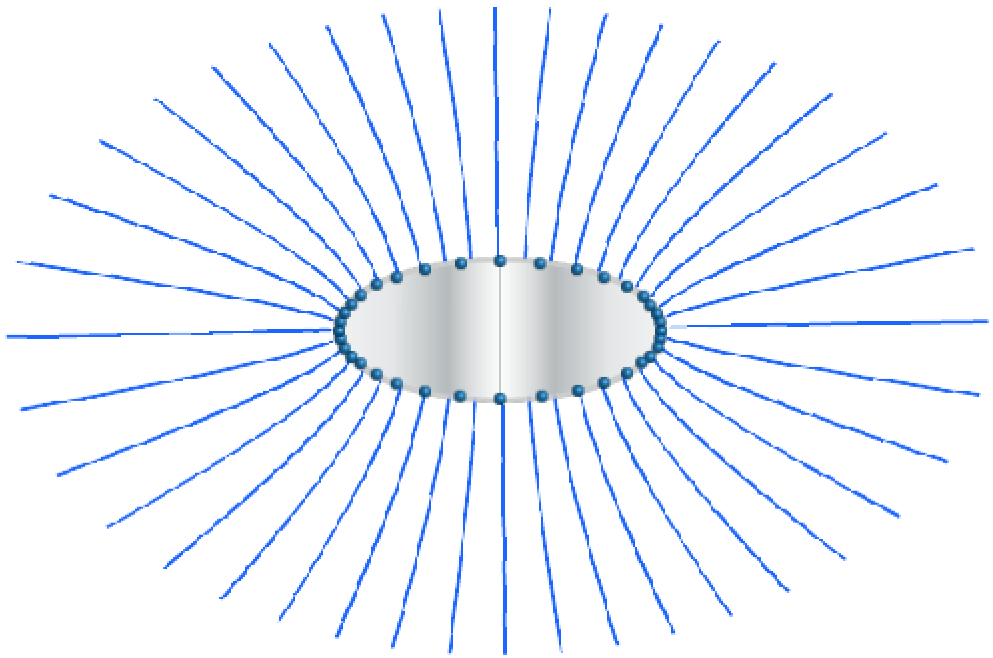
1) EXAM 1: WED February 18th at 7pm
   - Sign Up in Gradebook for Conflict Exam at 5:15pm if desired
   - If you have double conflict please email Prof. Ben Hooberman
   - MATERIAL: Lectures 1 - 8

2) EXAM 1 PREPARATION
   - Study HW, Discussion
   - Old Exams are a good
   - See SmartPhysics for

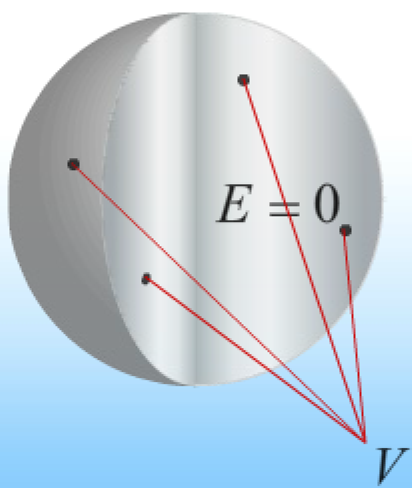
3) Extra Office Hours (Tuesday/ W
   rooms/times)



Main Point 1: (Conductors)



Conducting Sphere

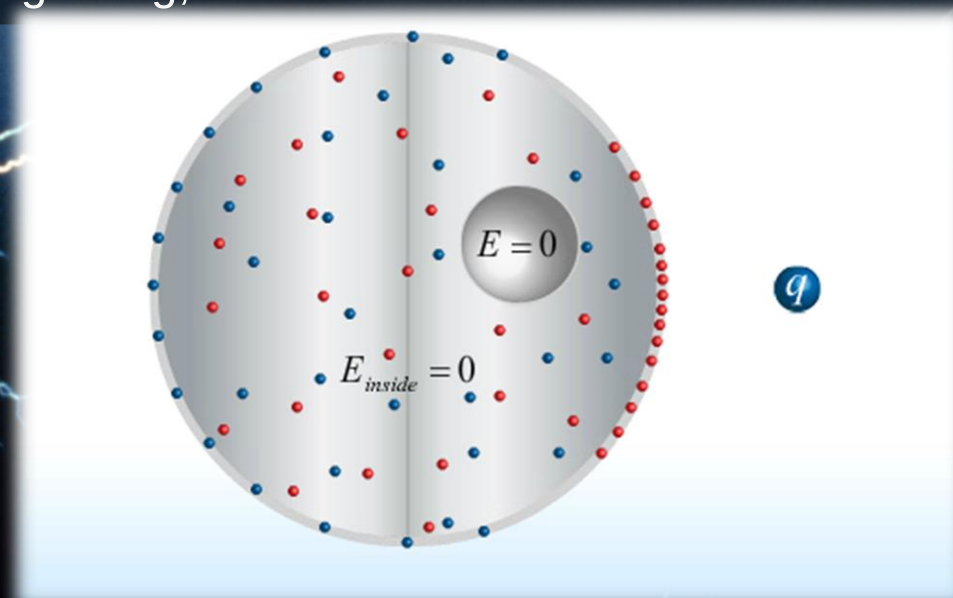


- Charges are free to move
- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

Storm Safety

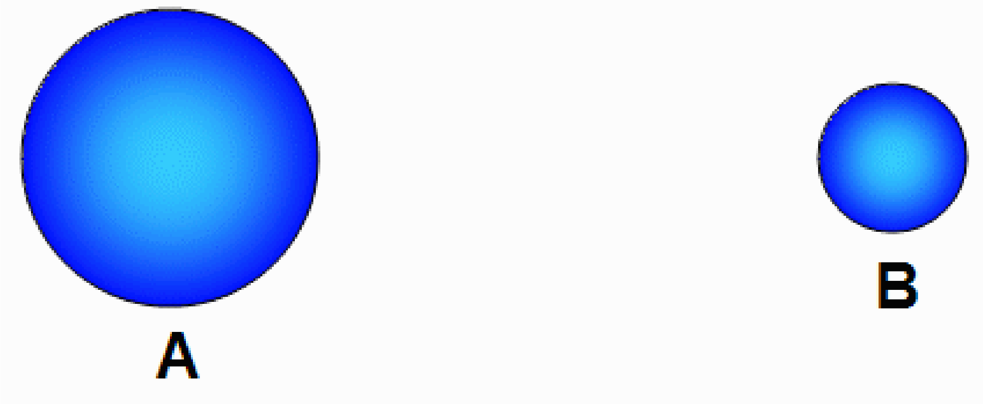
You are at the park when you see lightning. You decide to take shelter in a car, which car is safer, a (mainly steel) Volkswagen with thick rubber tires, or a (mainly fiberglass) Corvette with thin rubber tires

A) Corvette because it is fiberglass
B) Corvette because it is lower to ground
C) $\boxed{\text{Volkswagen because it is steel}}$
D) Volkswagen because tires are thicker
E) Corvette, because if I do get struck by lightning, I would rather be in a corvette.



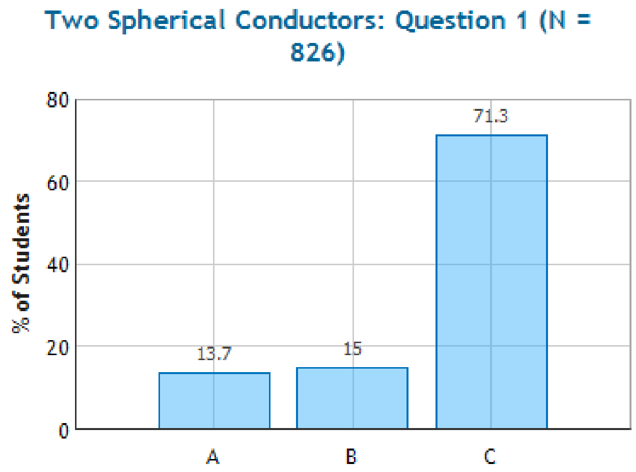
CheckPoint 1a

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B



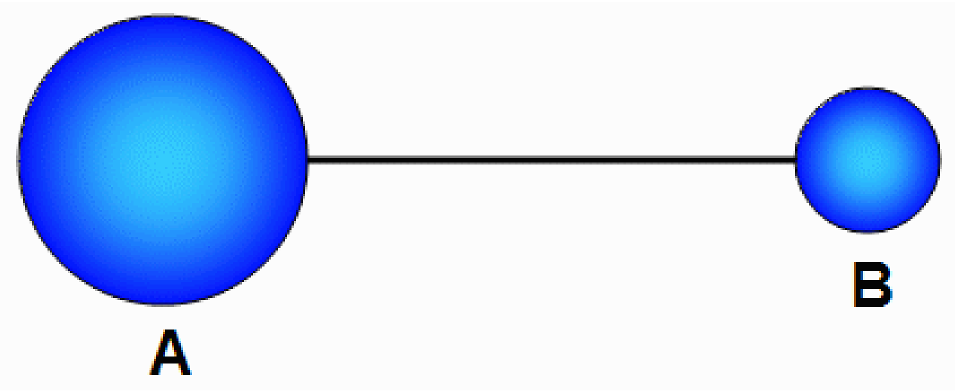
Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$



"Va=k*Q/Ra and Vb=k*Q/Rb. Then Ra>Rb, so Va<Vb"

The two conductors are now attached by a conducting wire.



Compare the potential on surface A with the potential on surface B

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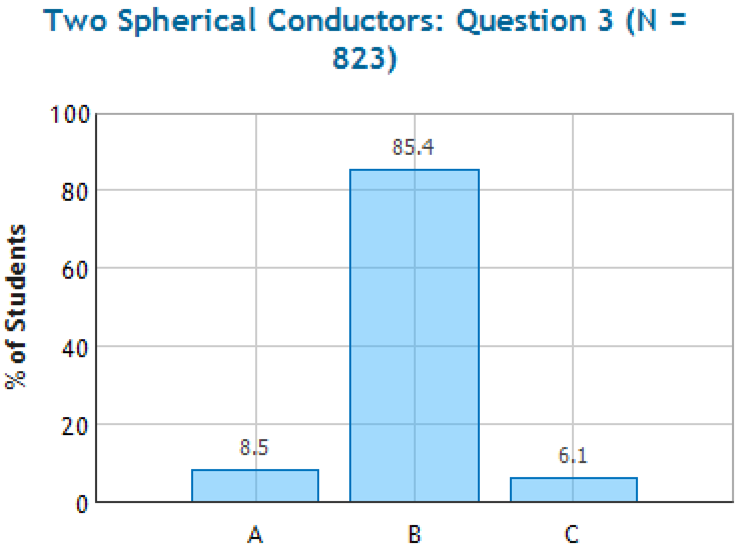
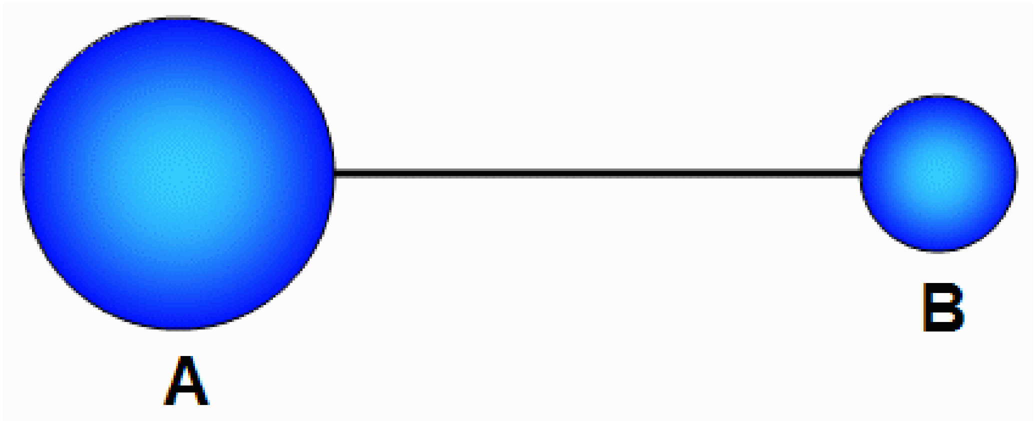


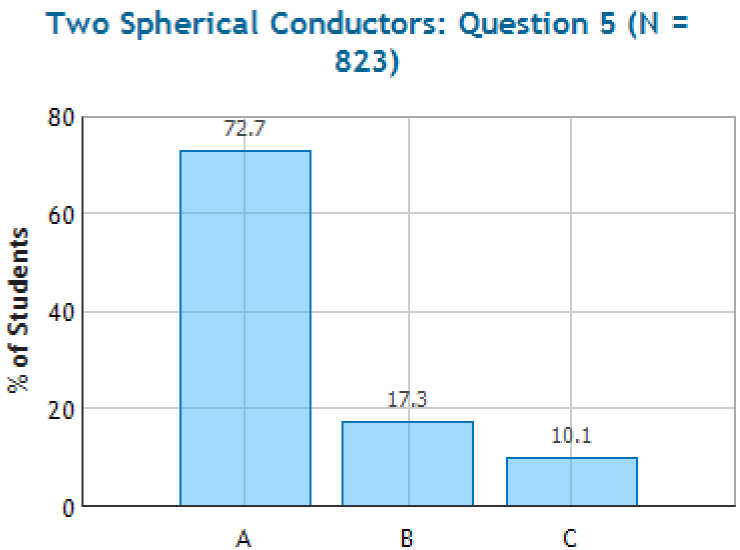
"By connecting the two conductors, we are effectively making them one conductor, and the potential on a conductor is the same everywhere, so the potential of A = the potential of B."

CheckPoint 1c



What happens to the charge on sphere A when the wire is attached

A) Q_A increases B) Q_A decreases C) Q_A does not change

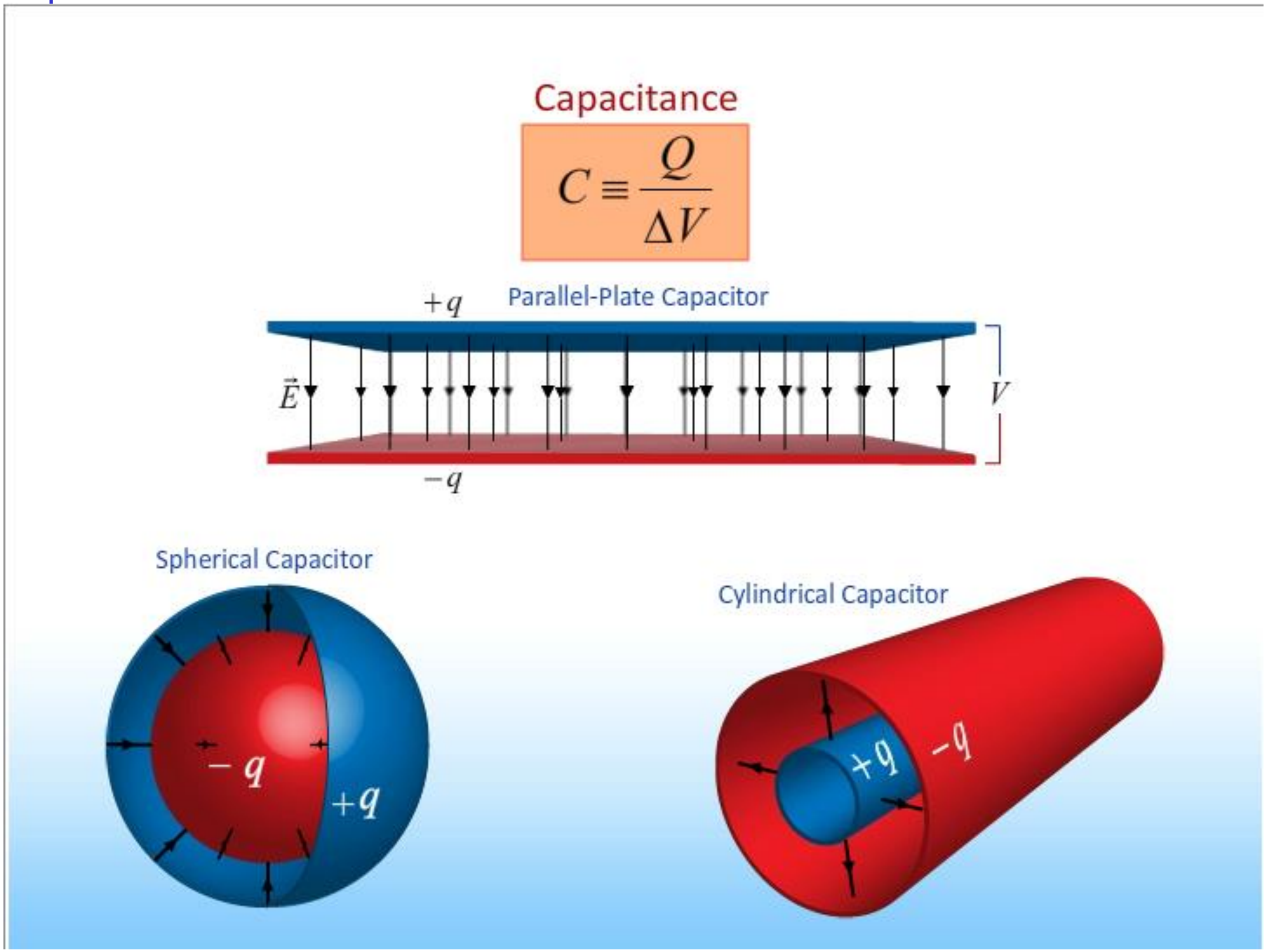


$$\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$$

$$\Rightarrow \quad Q_A = Q_B \frac{R_A}{R_B}$$

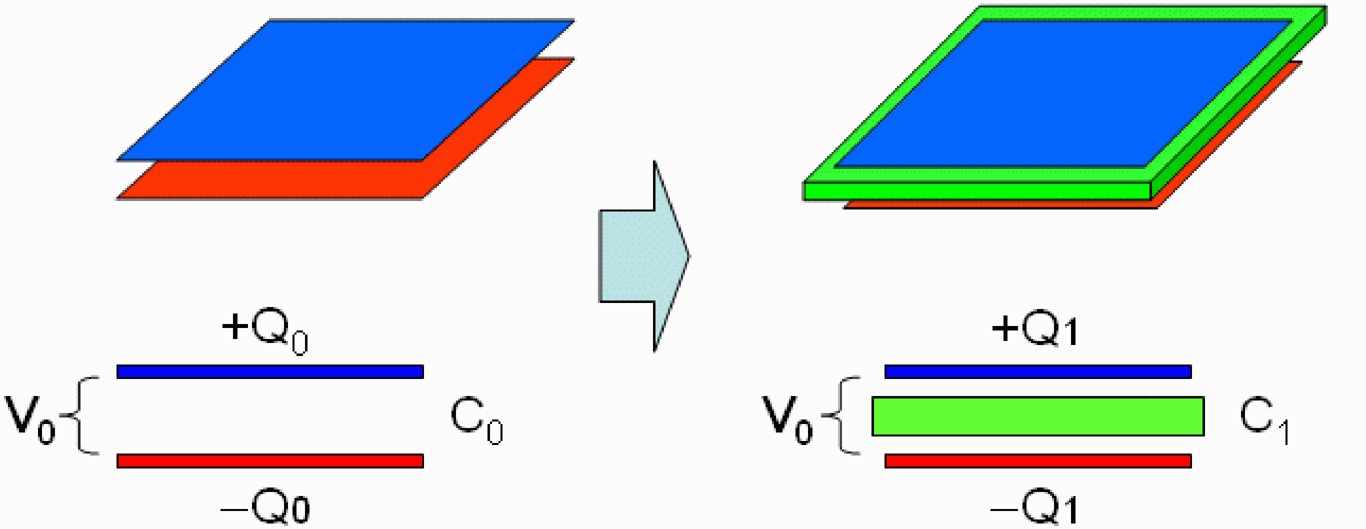
Main Point 2: Capacitance = Q/V

could you give an example of how the capacitor's abilities are useful in a real life situation, like a computer?



Parallel Plate Capacitor

Two parallel plates of area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same as before.



THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We'll work through the example in the prelecture and then do the preflight questions.

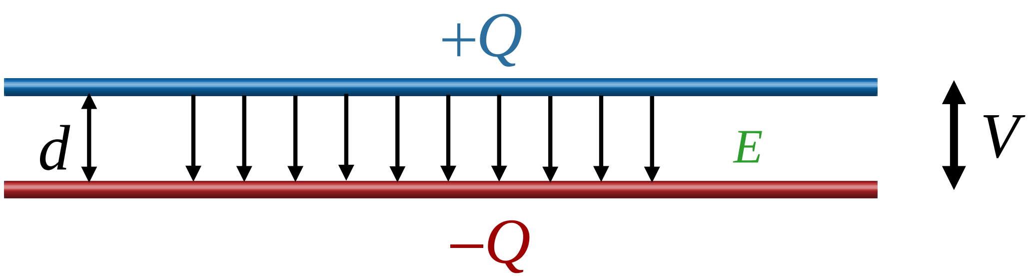
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

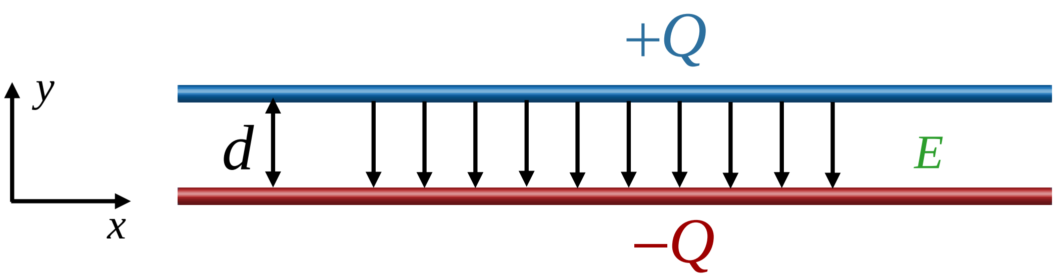
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)



What is σ ?

$$E = \frac{\sigma}{\varepsilon_o} \qquad \sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

$$V = -\int_0^d \vec{E} \cdot d\vec{y} \longrightarrow V = -\int_0^d (-E dy) = E\int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

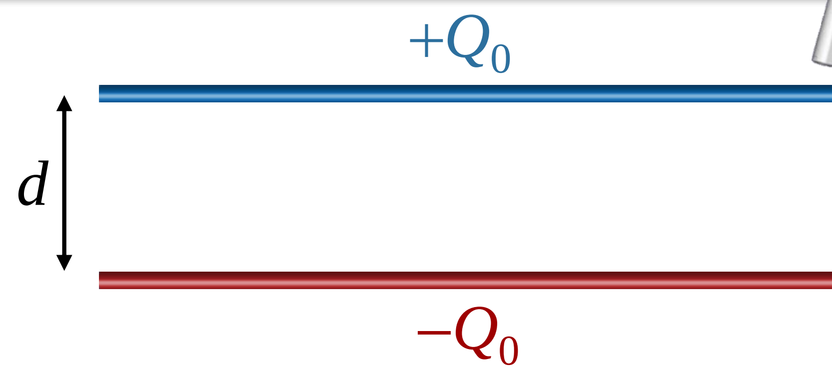
As promised, V is proportional to Q !

$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Q}d / \varepsilon_o A} \longrightarrow C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry !

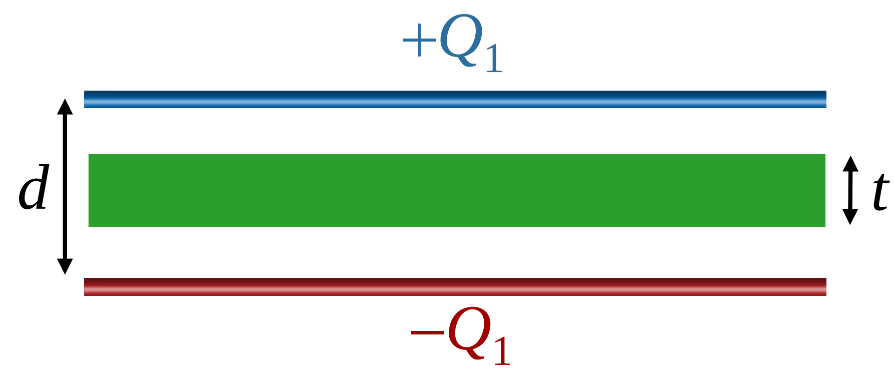
Question Related to CheckPoint

Initial charge on capacitor $= Q_0$



Insert uncharged conductor

Charge on capacitor now $= Q_1$



How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$

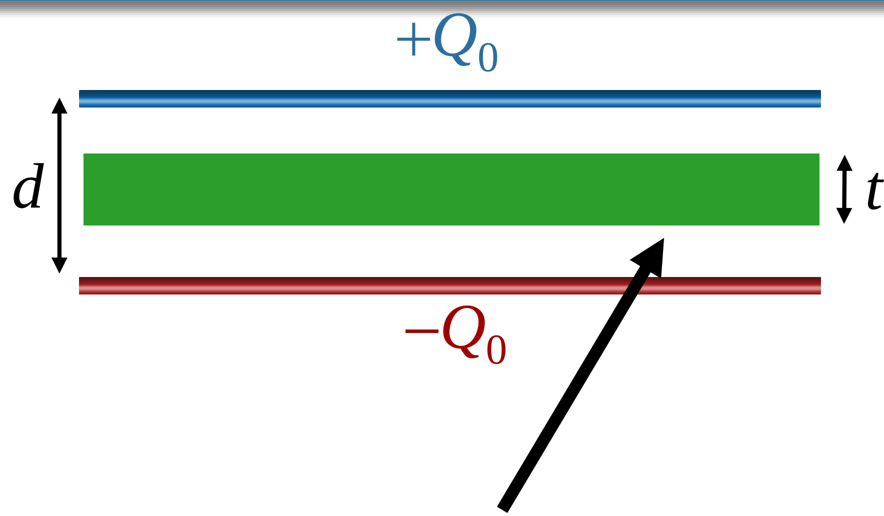
$\boxed{\text{B) } Q_1 = Q_0}$

C) $Q_1 > Q_0$

Plates not connected to anything

CHARGE CANNOT CHANGE !

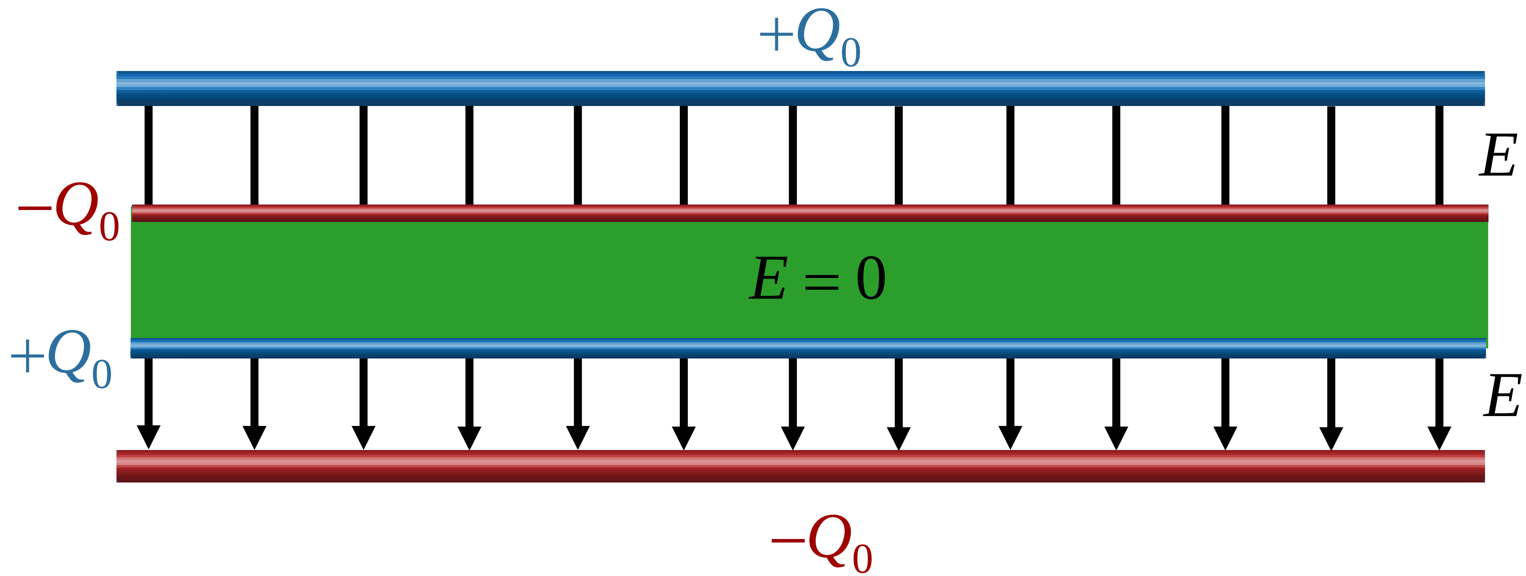
Where to Start ?



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$

Why ?



WHAT DO WE KNOW ?

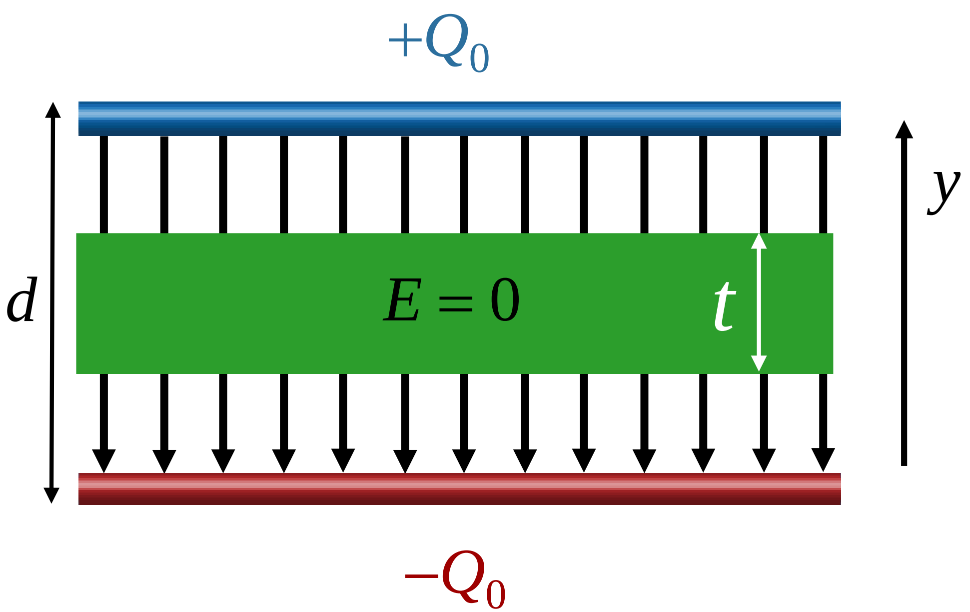
E must be $= 0$ in conductor !

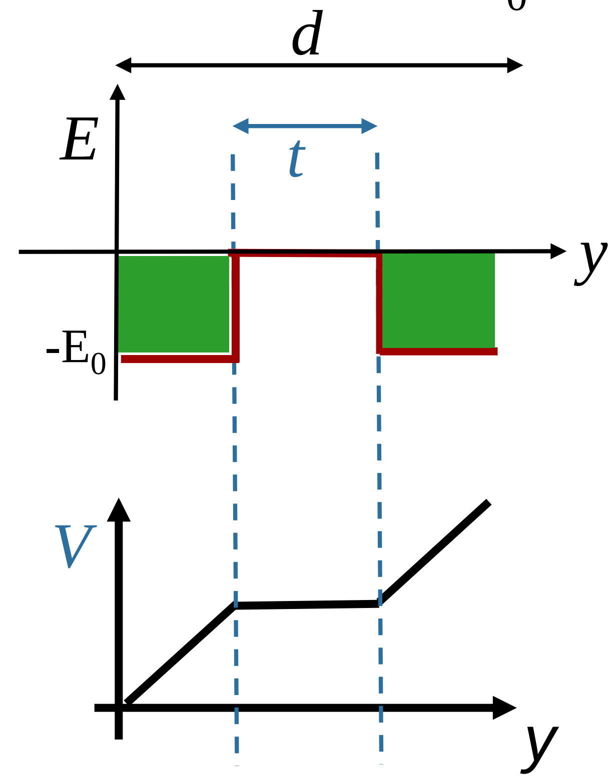
Charges inside conductor move to cancel E field from top & bottom plates.

Calculate V

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$





What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

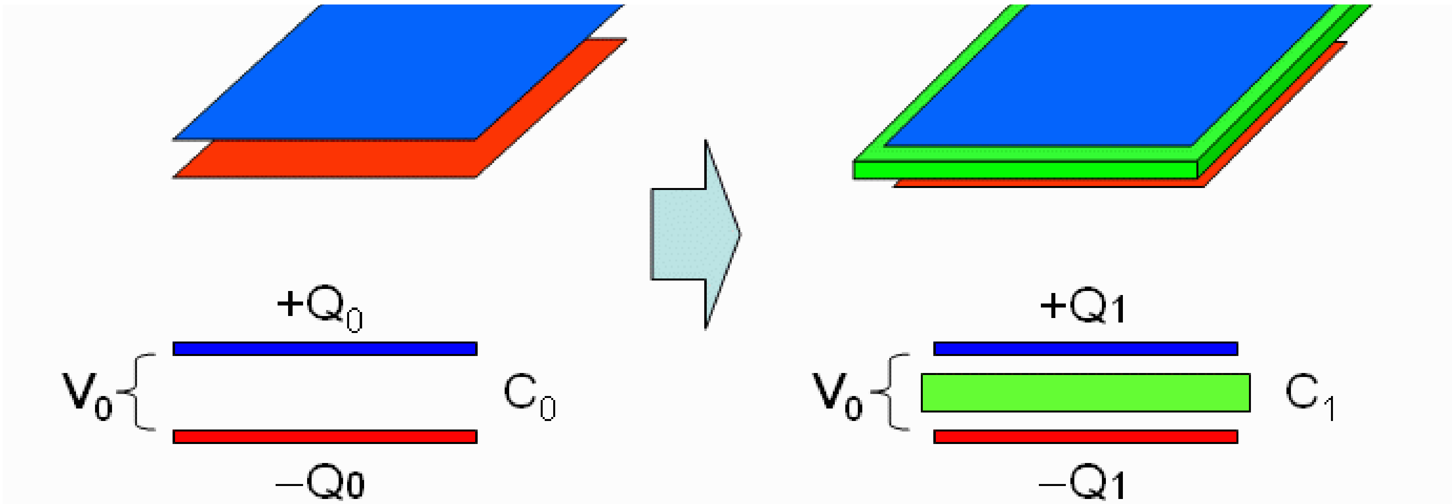
B) $\boxed{\Delta V = E_0(d - t)}$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

Back to CheckPoint 2a

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, how would charge need to be adjusted to keep same potential difference?

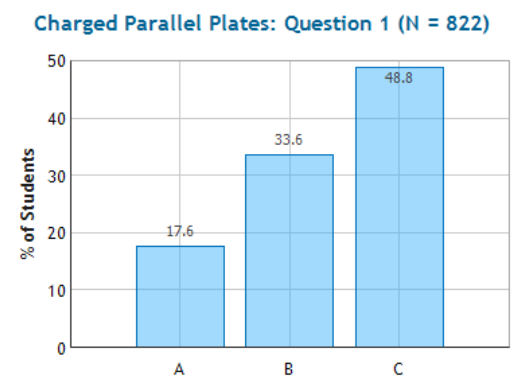


A) $Q_1 < Q_o$ B) $Q_1 = Q_o$ C) $\boxed{Q_1 > Q_o}$

"The charge Qo is greater than Q1 because the electric field, which is represented as Q/EoA is greater when the conducting plate is not placed in between the conductors."

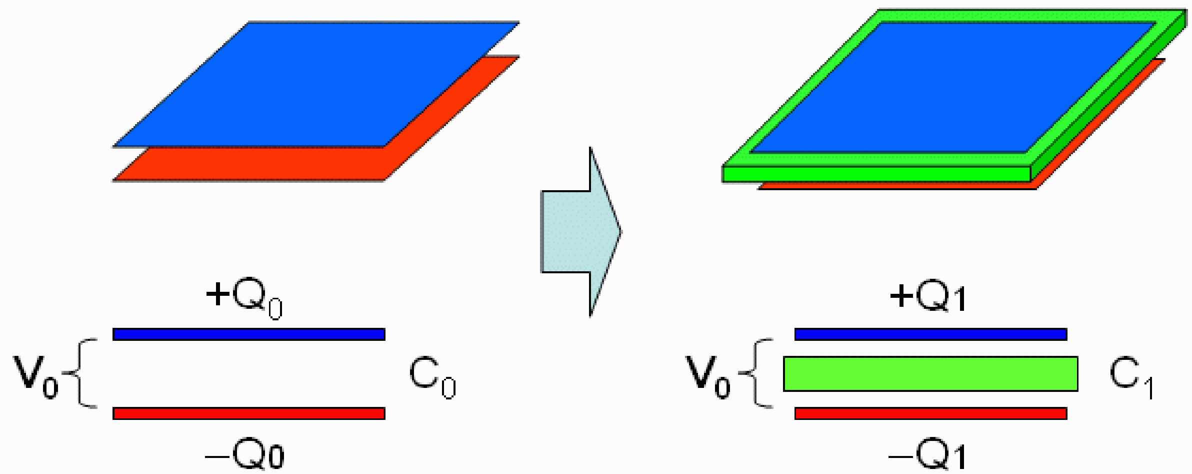
"The conducting plate does not touch either plate so the charges should remain the same."

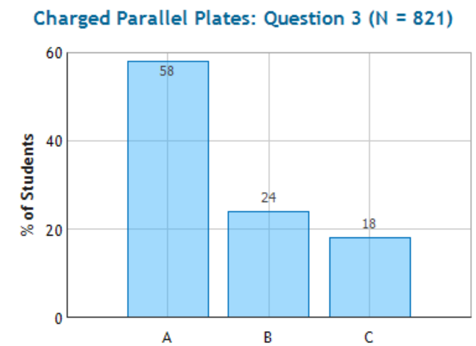
"Because the distance is smaller, E has to be bigger, therefore, density has to be bigger, Q has to be bigger.."



CheckPoint 2b

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, does C change?





$\boxed{\text{A) } C_1 > C_o}$ B) $C_1 = C_o$ C) $C_1 < C_o$

We can determine C from either case

same V (preflight)

same Q (lecture)

C depends only on geometry !

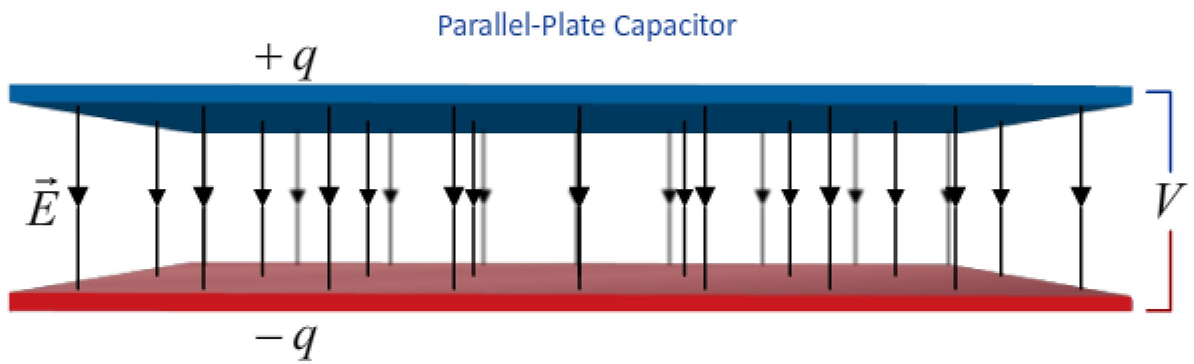
$$E_0 = Q_0 / \varepsilon_0 A$$

Same Q:

$$V_0 = E_0 d \qquad C_0 = Q_0 / E_0 d \qquad C_0 = \varepsilon_0 A / d$$

$$V_1 = E_0 (d - t) \qquad C_1 = Q_0 / (E_0 (d - t)) \qquad C_1 = \varepsilon_0 A / (d - t)$$

Main Point 3: Capacitors Store Energy in E



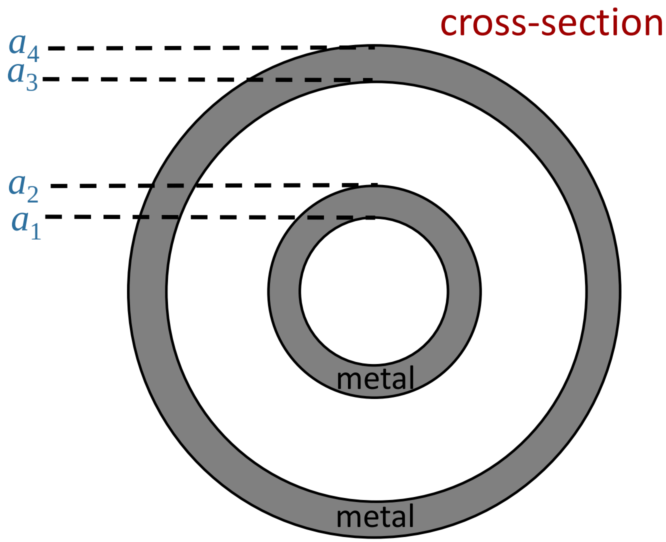
BANG!

$$u = \frac{1}{2}\varepsilon_o E^2$$ Energy Density

Energy Stored in Capacitors

$$U = \frac{1}{2}QV \quad \text{or} \quad U = \frac{1}{2}\frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2}CV^2$$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

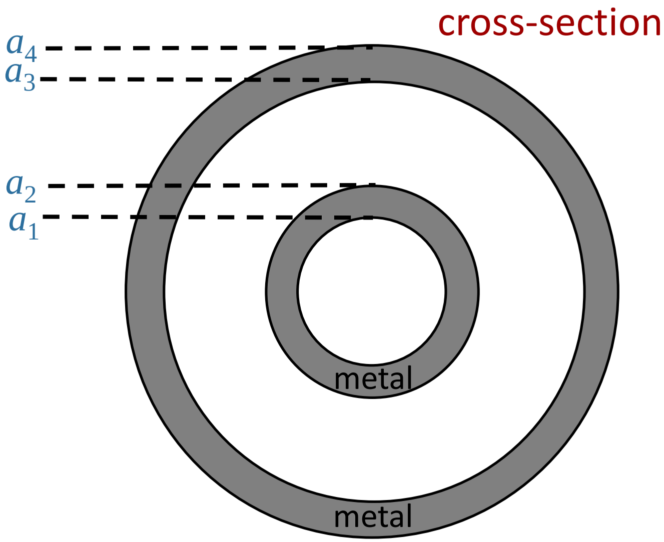
- Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

But what is Q and what is V ? They are not given?

- Important Point: C is a property of the object! (concentric cylinders here)
  - Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
  - These charges create E field in region between conductors
  - This E field determines a potential difference V between the conductors
  - V should be proportional to Q; the ratio Q/V is the capacitance.

Calculation



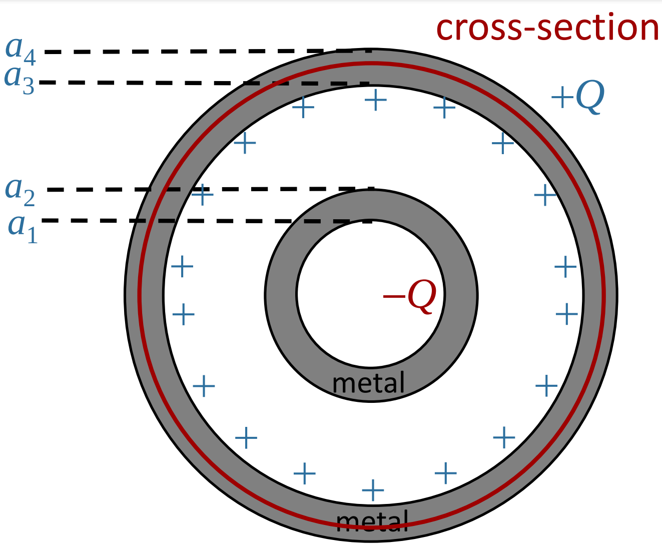
A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

- **Strategic Analysis:**
  - Put $+Q$ on outer shell and $-Q$ on inner shell
  - Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
  - Integrate E to get V
  - Take ratio Q/V: should get expression only using geometric parameters (a_i, L)

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$ B) at $r = a_3$ C) both surfaces D) throughout shell

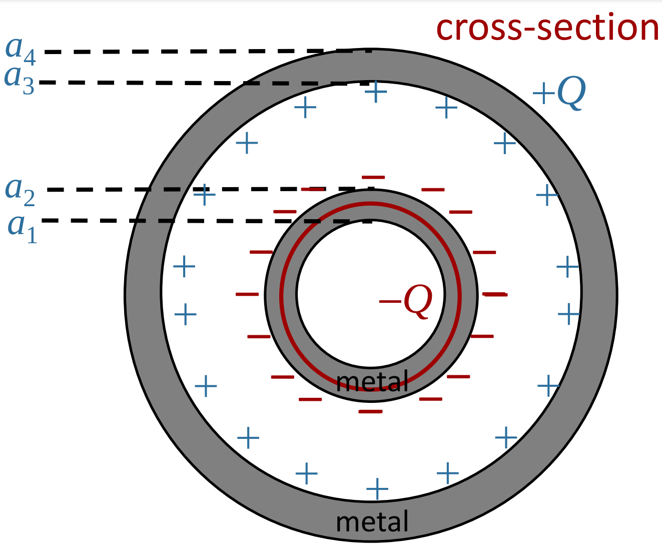
Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ $\longrightarrow$ $Q_{enclosed} = 0$

We know that $E = 0$ in conductor (between a_3 and a_4)

$Q_{enclosed} = 0$ $\longrightarrow$ $+Q$ must be on inside surface (a_3), so that $Q_{enclosed} = +Q - Q = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $-Q$ on inner conductor located?

A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout shell

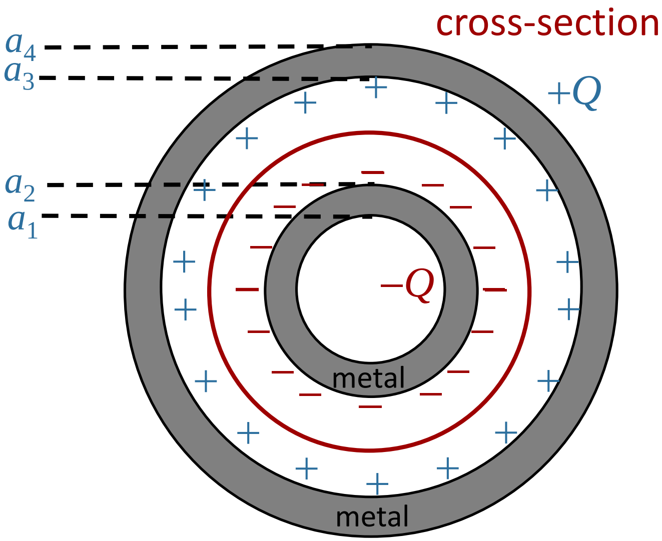
Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ $\longrightarrow$ $Q_{enclosed} = 0$

We know that $E = 0$ in conductor (between a_1 and a_2)

$Q_{enclosed} = 0$ $\longrightarrow$ $+Q$ must be on outer surface (a_2), so that $Q_{enclosed} = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $|E(r)|$?

A) 0

B) $\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$

C) $\boxed{\frac{1}{2\pi\varepsilon_o}\frac{Q}{Lr}}$

D) $\frac{1}{2\pi\varepsilon_o}\frac{2Q}{Lr}$

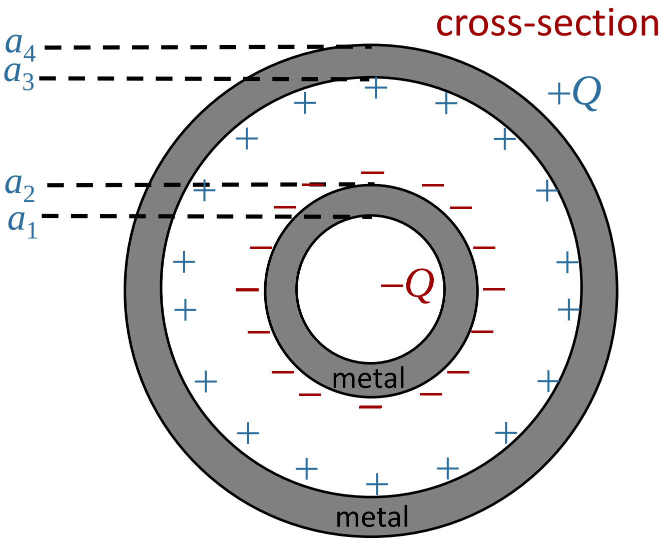
E) $\frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2}$

Why?

Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ $\rightarrow$ $E \cdot 2\pi r L = \frac{Q}{\varepsilon_o}$ $\rightarrow$ $E = \frac{1}{2\pi\varepsilon_o}\frac{Q}{Lr}$

Direction: Radially In

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

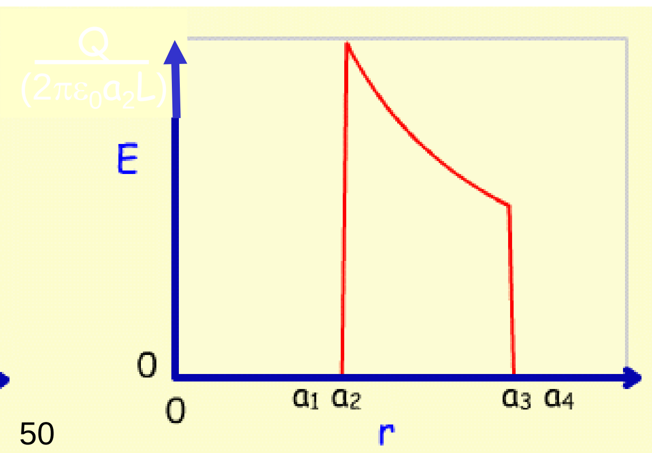
What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

What is $V \equiv V_{outer} - V_{inner}$?

$$\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_1}{a_4} \qquad \frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_4}{a_1} \qquad \boxed{\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_3}{a_2}} \qquad \frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_2}{a_3}$$

(A) (B) (C) (D)



$$V = -\int_{a_2}^{a_3} \frac{-Q}{2\pi\varepsilon_o L}\frac{dr}{r} \longrightarrow V = \frac{Q}{2\pi\varepsilon_o L}\int_{a_2}^{a_3}\frac{dr}{r} \longrightarrow V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{a_3}{a_2}$$

V proportional to Q, as promised

$$\longrightarrow \quad C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(a_3/a_2)}$$