Electricity & Magnetism
Lecture 4

Today's Concepts:

A) Conductors

B) Using Gauss' Law

Your Comments

Everything hurt. Please go over everything.

Will I survive this?.

Please go over everything... please I have a family

Could you please go slower through this lecture than you did last lecture? You went so fast through everything last lecture that I was very confused by the end of lecture.

Is it worthwhile to memorize any of the derived electric fields?

I could use an explanation of what happens in conducting and insulating spherical shells with a point charge inside.

Can you go over the difference between E dot dA and E times dA. Like what properties change between the two.

Gaussian surfaces are ridiculously hard to understand, despite the simple-seeming equation used. Also, how exactly did you arrive at the conclusion that E=0 inside a conducting sphere? That wasn't explained in the pre-lecture.

"I don't understand why charge MUST only gather on the surface of a solid sphere and can't build below the surface. Wouldn't that maximize the space between the charges?

Close laptops and place cell phones in airplane mode

Conductors and Insulators

Conductors = charges free to move

e.g. metals

Insulators = charges fixed

e.g. glass

Define: Conductors = Charges Free to Move

Still not convinced on why charges in conductor all go to surface. I see that thats a stable point (each charged point is actually pushed outwards by its neighbor ever so slightly, but is there no way that a charged spot could wind up in the middle where it couldnt escape?

Claim: $E = 0$ inside any conductor at equilibrium

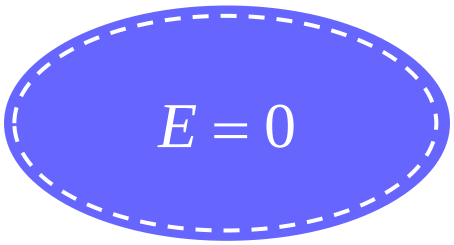
Charges in conductor move to make E field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge feels force and moves!

Claim: Excess charge on conductor only on surface at equilibrium

Why?

- Apply Gauss' Law
  - Take Gaussian surface to be just inside conductor surface
  - $E = 0$ everywhere inside conductor $\longrightarrow$ $\oint_{surface} \vec{E} \cdot d\vec{A} = 0$
  - Gauss' Law: $\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$ $\longrightarrow$ $\boxed{Q_{enc} = 0}$



SIMULATION 2

Gauss' Law + Conductors + Induced Charges

Could we go over how when there is a placed charge within a hollow conducting sphere the electric field is still zero with in that sphere. Wouldn't Gauss' Law say that because we are containing a charge there would have to be an electric field?

$$\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

ALWAYS TRUE!

If choose a Gaussian surface that is entirely in metal, then $E = 0$ so $Q_{enclosed}$ must also be zero!

$$0 = \frac{Q_{enc}}{\varepsilon_0}$$

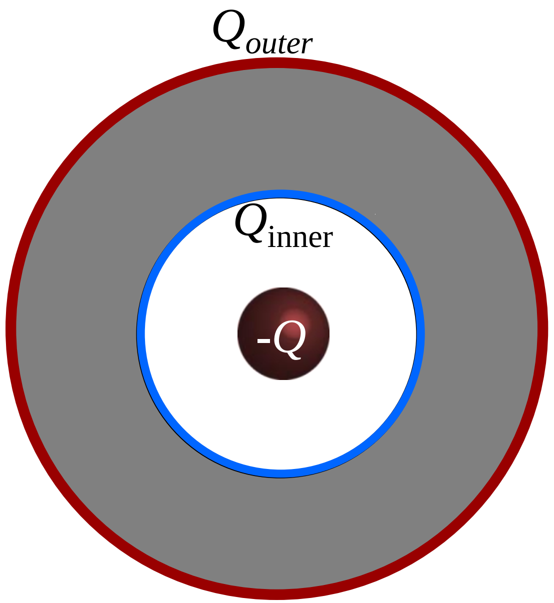
How Does This Work?

Charges in conductor move to surfaces to make $Q_{enclosed} = 0$.

We say charge is induced on the surfaces of conductors

Charge in Cavity of Conductor

A particle with charge $-Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?



A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$, outer = $+Q/2$

C) inner = 0, outer = 0

D) inner = $+Q/2$, outer = $-Q/2$

E) inner = $+Q$, outer = $-Q$

- Gauss' Law: $\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$

Since $E = 0$ in conductor $\qquad 0 = \frac{Q_{enc}}{\varepsilon_o}$

$$0 = -Q + Q_{inner}$$

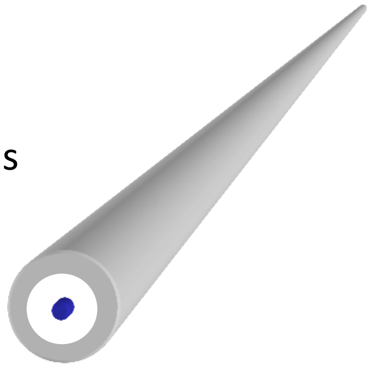
Since conductor is uncharged

$$Q_{inner} + Q_{outer} = 0$$

$$Q_{outer} = -Q_{inner}$$

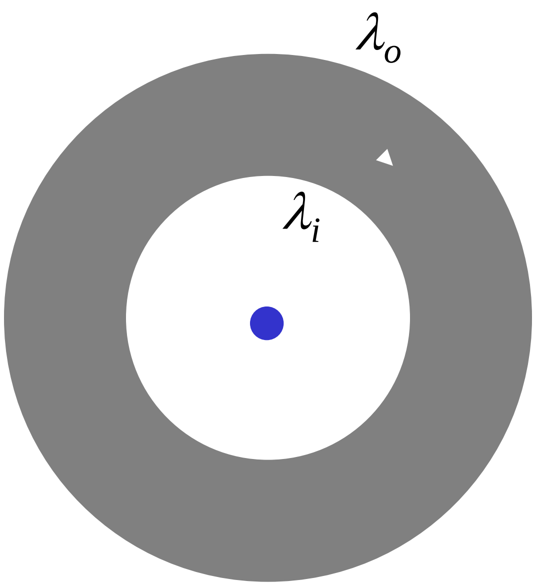
Infinite Cylinders

A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of -4C/m.



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder (λ_i) and on the outer surface (λ_o)?

	A	B	C	D	E
λ_i:	+2.5 C/m	–4 C/m	0	–2.5 C/m	–2.5 C/m
λ_o:	–6.5 C/m	0	-4 C/m	+2.5 C/m	–1.5 C/m



Gauss' Law

How do you choose the Gaussian surface???

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \dfrac{Q_{enc}}{A\varepsilon_0}$

Why can we not use Gauss's Law to find the electric field of a finite-length charged wire?

In General, integral to calculate flux is difficult…. and not useful!

To use Gauss' Law to calculate E, need to choose surface carefully!

1) Want E to be constant and equal to value at location of interest

OR

2) Want E dot $A = 0$ so doesn't add to integral

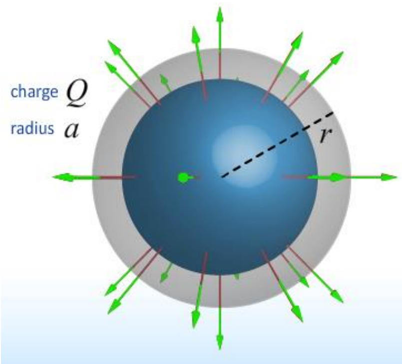
Gauss' Law Symmetries

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \dfrac{Q_{enc}}{A\varepsilon_0}$

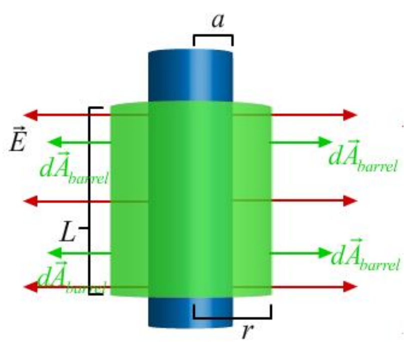
Spherical



$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0}$$

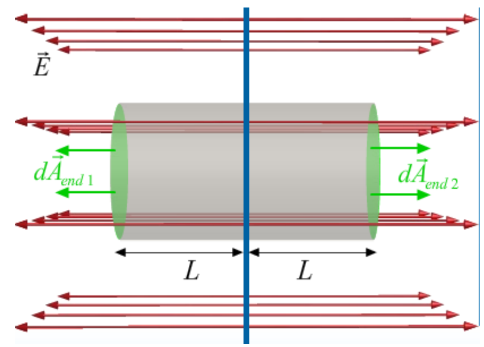
Cylindrical



$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

Planar

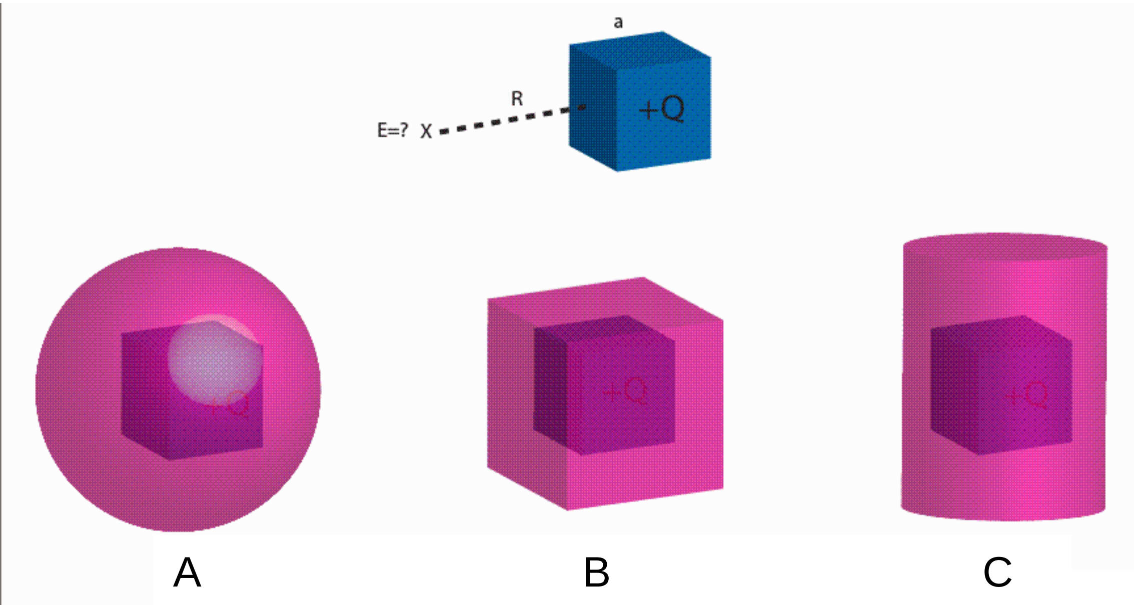


$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

CheckPoint 1

Which Gaussian Surface would you use to calculate E due to cube of charge?



D) The field cannot be calculated using Gauss’ Law

E) None of the above

THE CUBE HAS NO GLOBAL SYMMETRY!

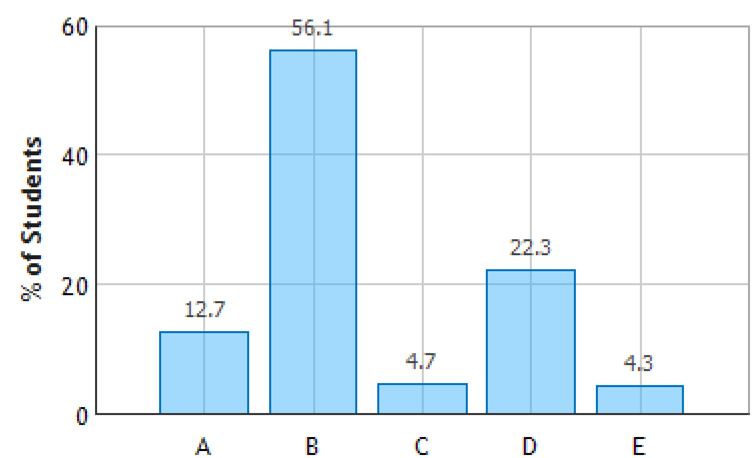
THE FIELD AT THE FACE OF THE CUBE

IS NOT

PERPENDICULAR OR PARALLEL

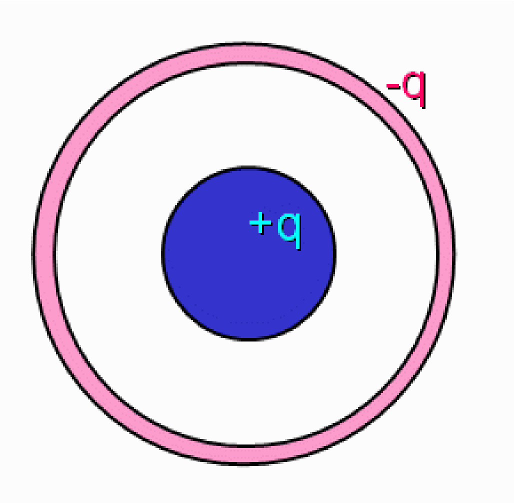
3D	POINT	→	SPHERICAL
2D	LINE	→	CYLINDRICAL
1D	PLANE	→	PLANAR

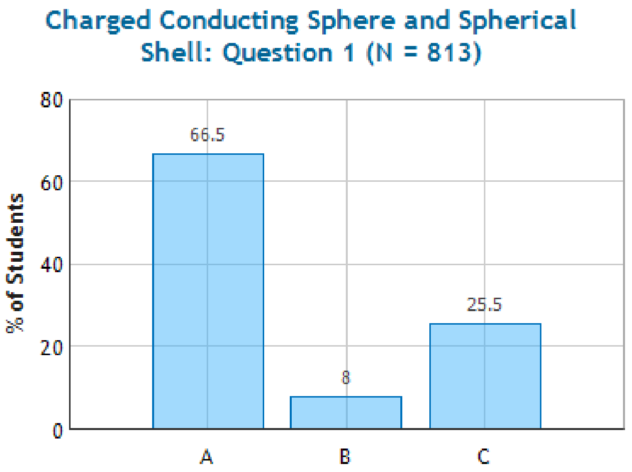
Gaussian Surface Choice: Question 1 (N = 817)



CheckPoint 3.1

4) A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.





What is direction of field between blue and red spheres?

- The field point radially outward
- The field point radially inward
- The field is zero

A) Outward B) Inward C) Zero

Careful: what does inside mean?
This is always true for a solid conductor (within the material of the conductor)
Here we have a charge "inside"

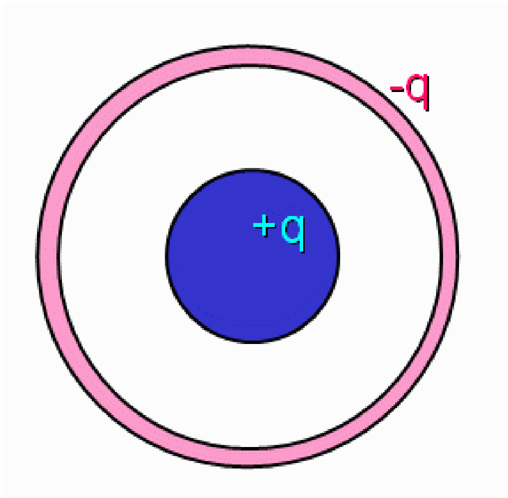
A) "Electric field lines point away from positive charges and since the q enclosed is +q, the electric field lines would point radially outward.

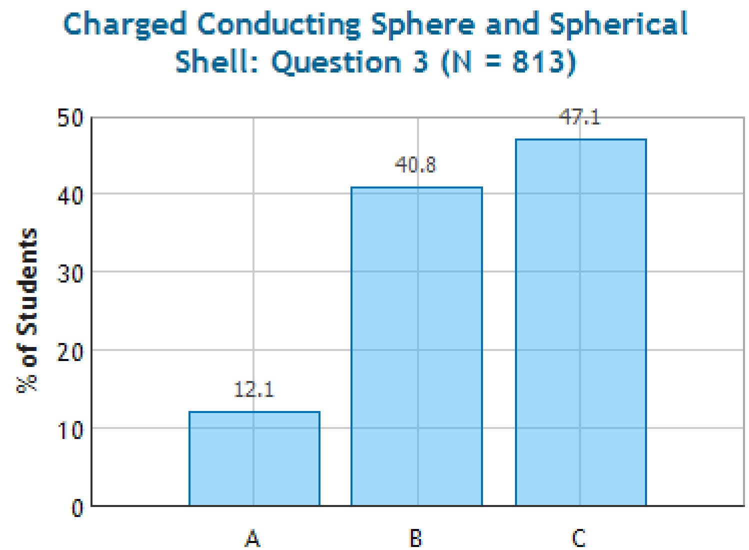
B) "Since the magnitude of electric field inside the solid conducting sphere is 0, so the electric field in the region between the spheres are -q and points radially inward."

C) "The charges on the outside will position themselves to negate the field created from the inside as they can freely move

CheckPoint 3.3

4) A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.





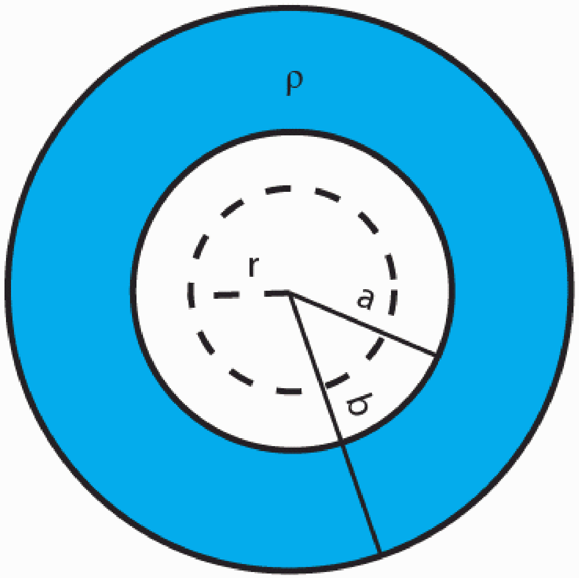
What is direction of field OUTSIDE the red sphere?

○ The field point radially outward ○ The field point radially inward ○ The field is zero

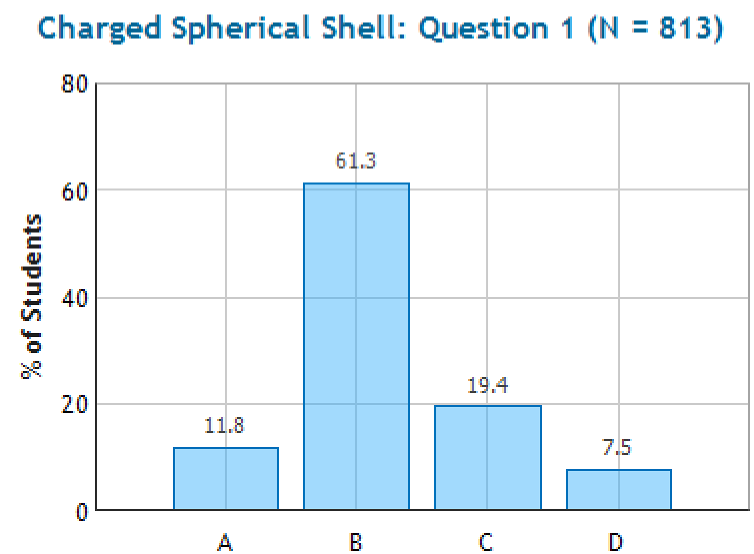
A) Outward B) Inward $\boxed{\text{C) Zero}}$

CheckPoint 2

4) A charged spherical insulator shell has inner radius a and outer radius b. The charge density on the shell is ρ.



What is magnitude of E at dashed line (r)?



A) $\dfrac{\rho}{\varepsilon_0}$

B) Zero

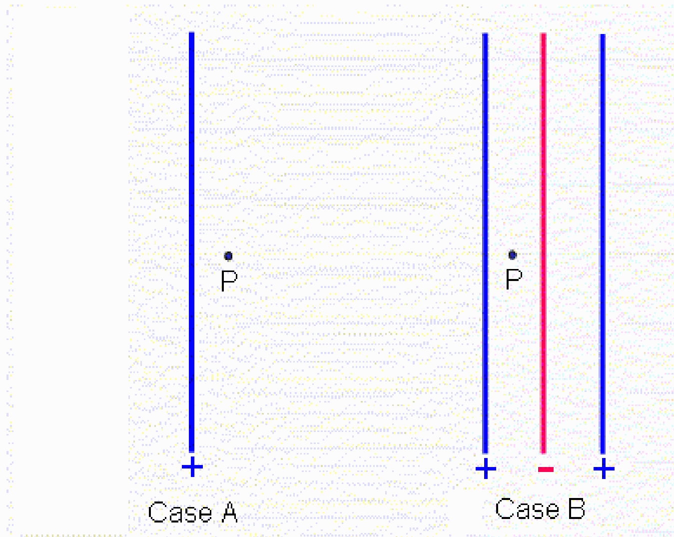
C) $\dfrac{\rho(b^3 - a^3)}{3\varepsilon_0 r^2}$

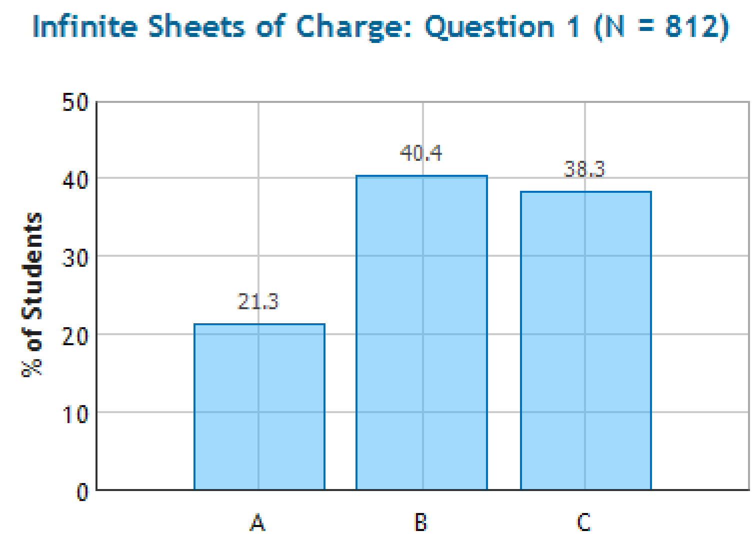
D) None of above

“The Gaussian surface has r<a. There will be no enclosed charge, and EdA = enclosed charge / epsilon0. So, E is 0”

CheckPoint 4

10) In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same.





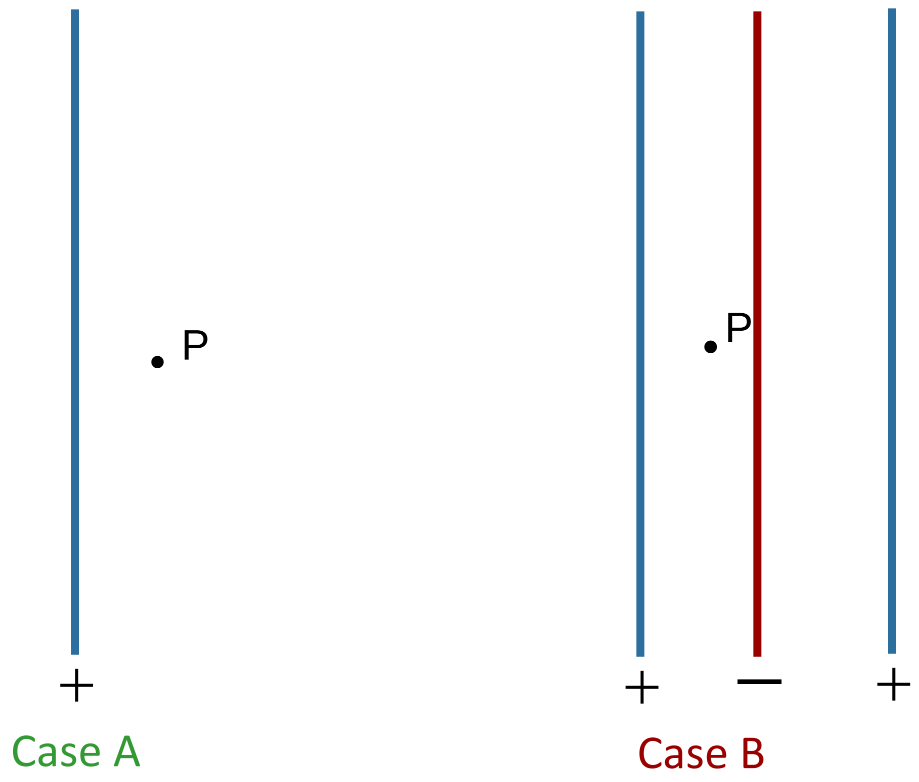
In which case is E at point P the biggest?

A) A B) B C) the same

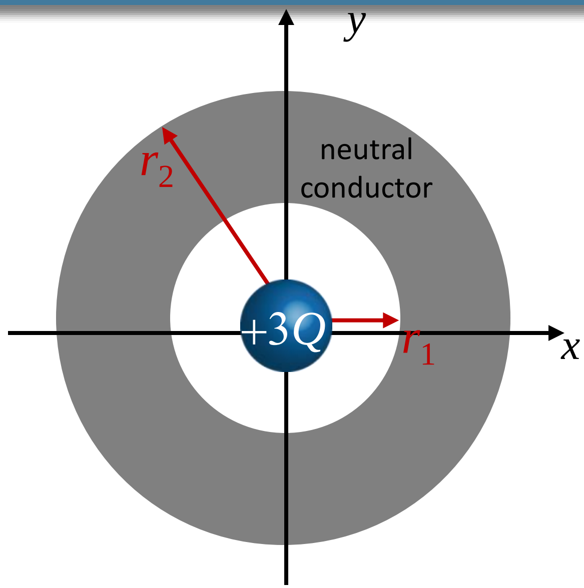
"The two positive plates in B have equal but opposite electric field values and will cancel out. So only the negative plate has an effect on P and since we are considering magnitude then 1 negative or positive plate will have the same electric field on P.

Superposition:

Lets do calculation!



Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

a) What is E everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine E?

Yes, Spherical Symmetry (what does this mean???)

A) Direction of E is along $\hat{x}$

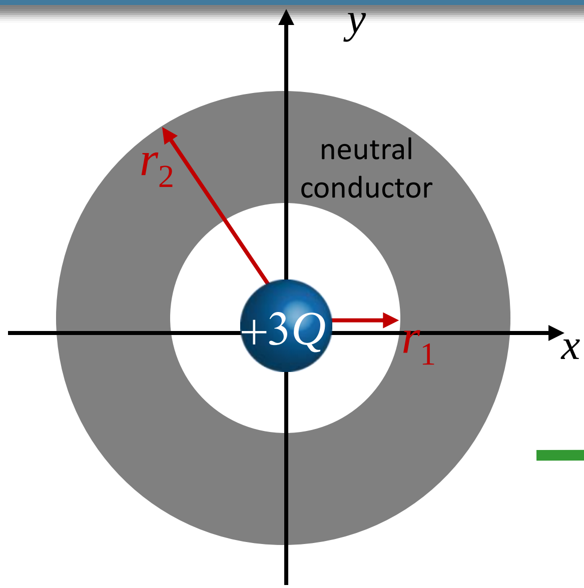
B) Direction of E is along $\hat{y}$

C) Direction of E is along $\hat{r}$

D) None of the above

Magnitude of E depends only on R

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

A) What is E everywhere?

We know:

magnitude of E is *fcn* of r

direction of E is along $\hat{r}$

We can use Gauss' Law to determine E

Use Gaussian surface = sphere centered on origin

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$r < r_1$

$$\int E dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{+3Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

$r_1 < r < r_2$

A) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

B) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r_1^2}$

C) $\boxed{E = 0}$

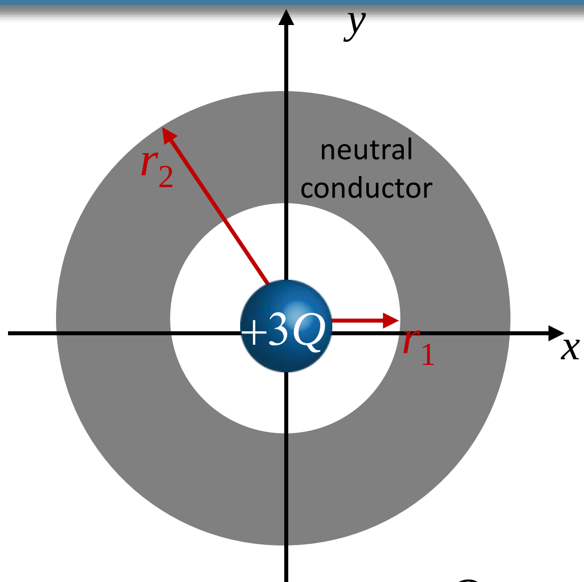
$r > r_2$

A) $\boxed{E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}}$

B) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{(r-r_2)^2}$

C) $E = 0$

Calculation



$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

A) What is E everywhere?

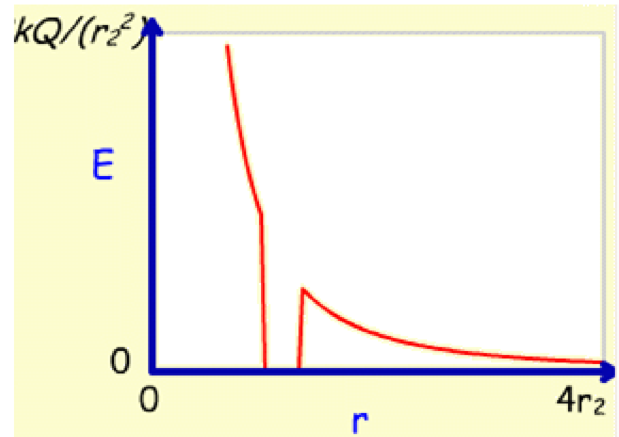
We know:

$r < r_1$

$r > r_2$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$$

$r_1 < r < r_2 \qquad E = 0$

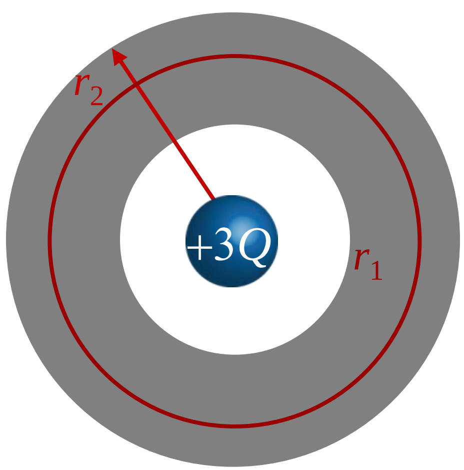


B) What is charge distribution at r_1?

$\boxed{\text{A)} \quad \sigma < 0}$

B) $\sigma = 0$

C) $\sigma > 0$



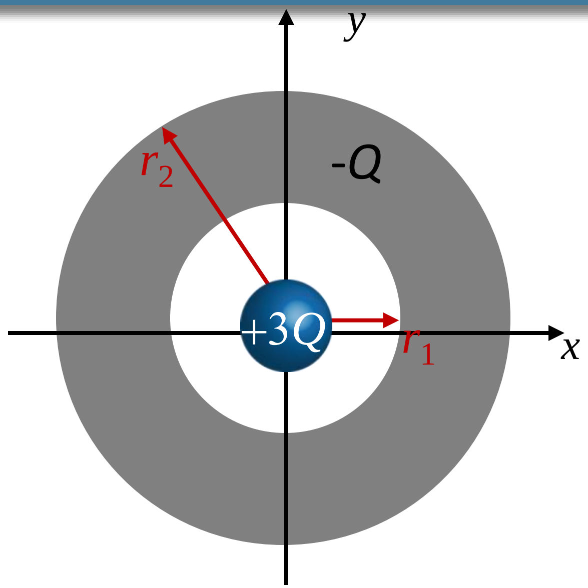
Gauss' Law:

$$E = 0 \rightarrow Q_{enc} = 0 \rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

Calculation

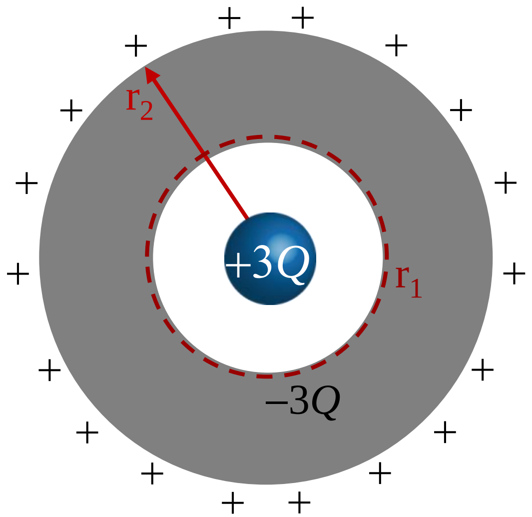


Suppose give conductor a charge of $-Q$

A) What is E everywhere?

B) What are charge distributions at r_1 and r_2?

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$



$r < r_1$

A) $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}}$

B) $E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}$

C) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$r > r_2$

A) $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$

B) $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}}$

C) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$r_1 < r < r_2$

$$E = 0$$