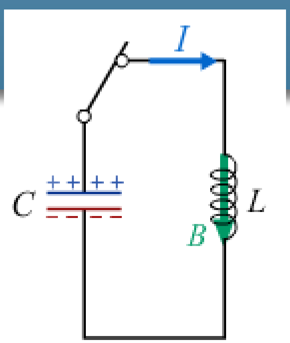
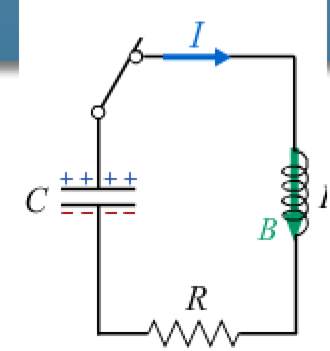
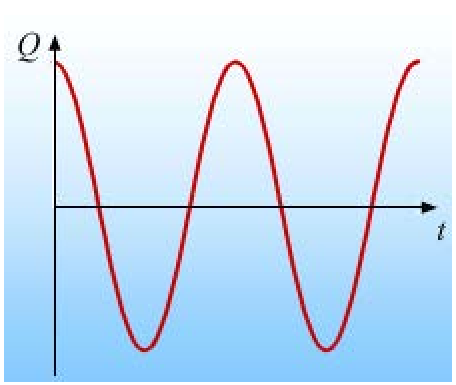
Physics 212
Lecture 19



RLC Circuit



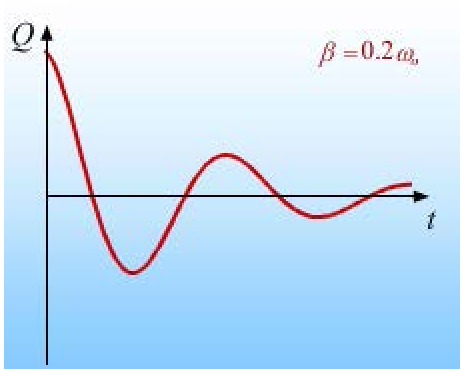
Today’s Concepts:



A) Oscillation Frequency

B) Energy

C) Damping



Your Comments

Please make sure we cover the calculation in lecture please, this is tricky stuff…

Will the solutions to hour exam 2 be posted on SmartPhysics? I want to go over the ones I got wrong.

If an inductor always hold the current still immediately after, what if I cut the wire and made an open circuit, would the current still flow and make the wire charged?

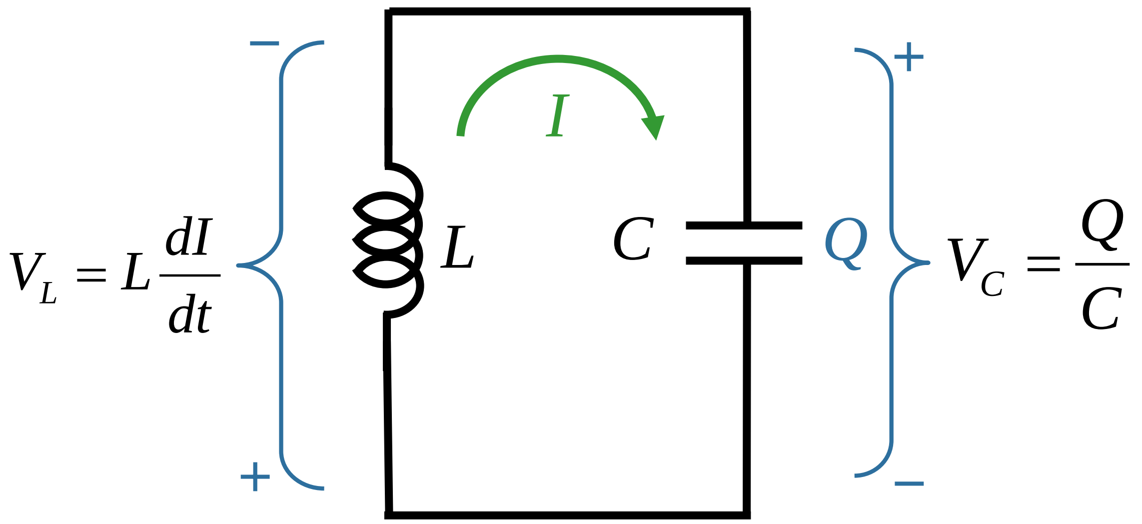
Hey. Hope you had a good break. Can you go over what the phase angle 'phi' is again? Like, not Alpha Phi, or Phi Kappa Psi, or Pi Beta Phi… but like the one in physics. I'm confused 'bout that concept.

When is a coil of wire an inductor, and when is it a solenoid? I don't understand why, when the capacitor is fully discharged, there is ANY magnetic field at all to convert back to current. Shouldn't zero current result in zero field?

It was a really hard prelecture (and homework)… I'm still really confused what physically an inductor does. I can memorize all its properties, sure, but it would be a lot easier if you clarified in lecture what physically happens to the individual charges as they move in the circuit.

WELCOME BACK FROM BREAK!! LESS THAN TWO MONTHS TO GO!! :D

LC Circuit



Circuit Equation: $\frac{Q}{C} + L\frac{dI}{dt} = 0$

$$I = \frac{dQ}{dt} \rightarrow \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \rightarrow \frac{d^2Q}{dt^2} = -\omega^2 Q$$

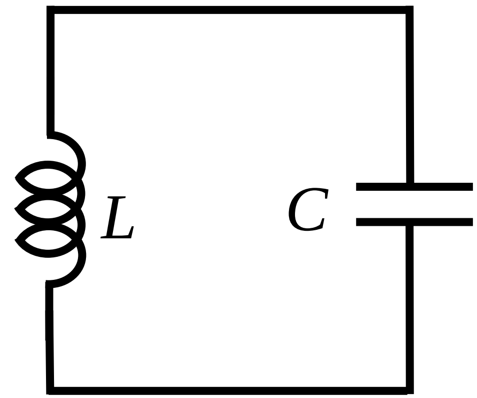
where

$$\omega = \frac{1}{\sqrt{LC}}$$

Pendulum.

CheckPoint 1a

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.

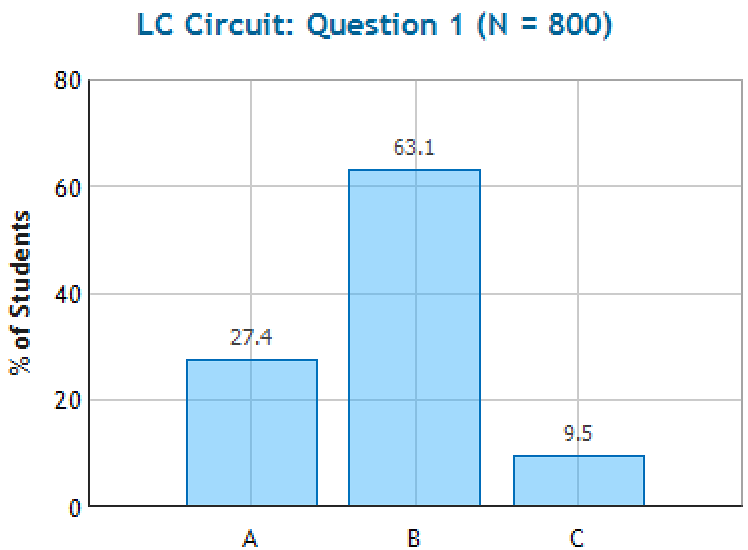


What is the potential difference across the inductor at $t = 0$?

A) $V_L = 0$
B) $V_L = Q_{max}/C$
C) $V_L = Q_{max}/2C$

since $V_L = V_C$

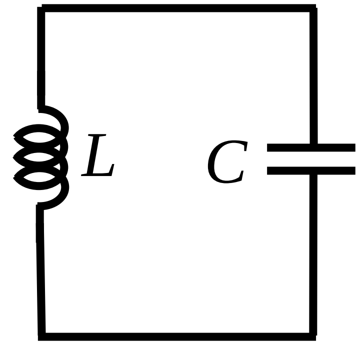
The two elements are in parallel, so VL=VC=Q/C



LC Circuits analogous to mass on spring

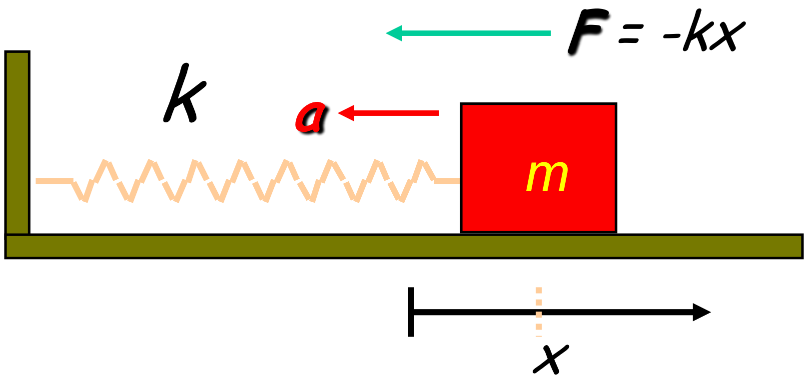
$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

$$\omega = \frac{1}{\sqrt{LC}}$$



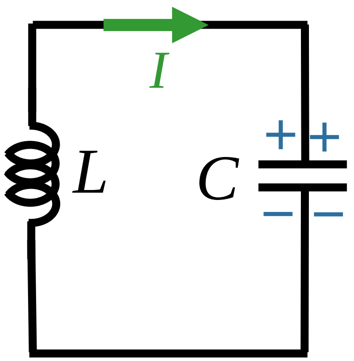
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

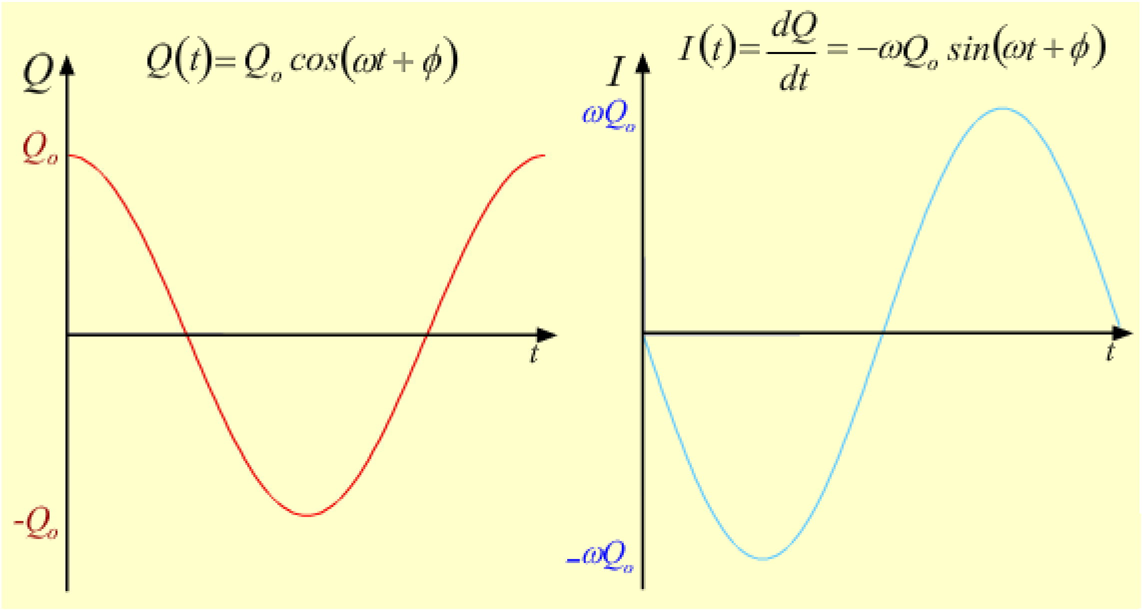
$$\omega = \sqrt{\frac{k}{m}}$$



Same thing if we notice that $k \leftrightarrow \frac{1}{C}$ and $m \leftrightarrow L$

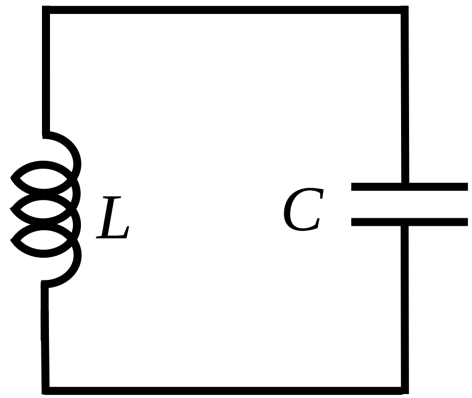
Time Dependence





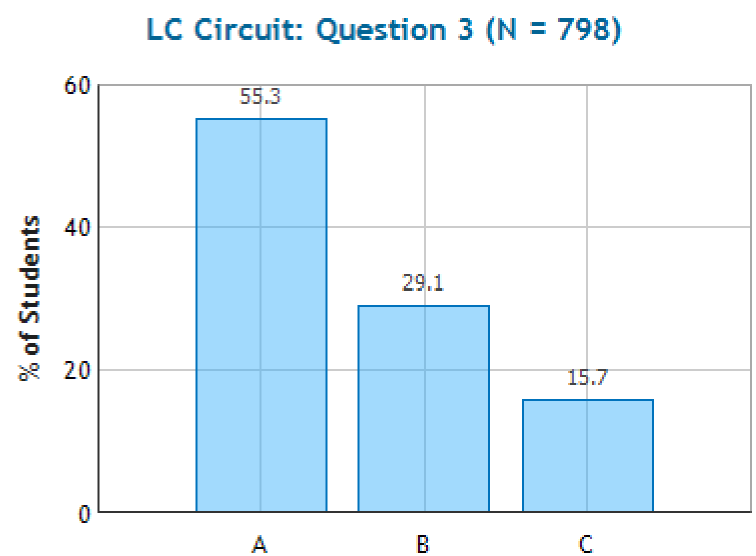
CheckPoint 1b

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



What is the potential difference across the inductor at when the current is maximum?

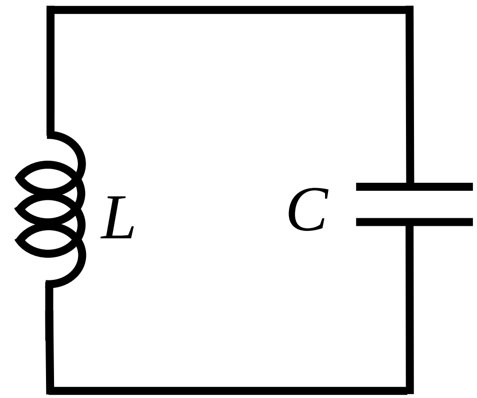
A) $V_L = 0$
B) $V_L = Q_{max}/C$
C) $V_L = Q_{max}/2C$



dI/dt is zero when current is maximum

CheckPoint 1c

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.

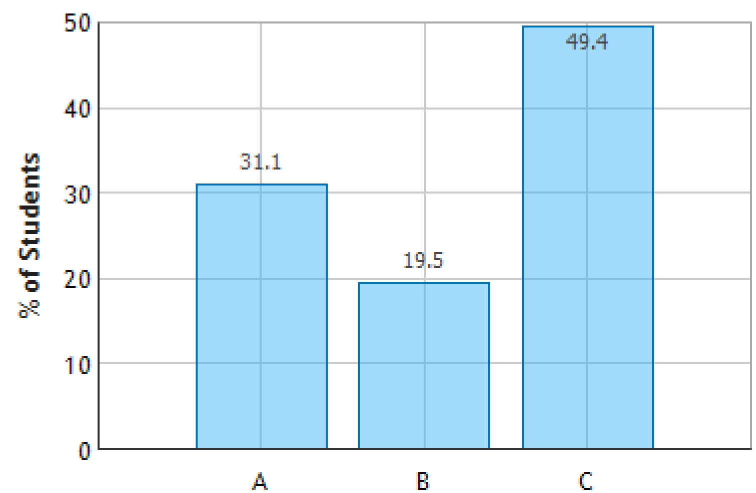


How much energy is stored in the capacitor when the current is a maximum ?

A) $U = Q_{max}^2/(2C)$
B) $U = Q_{max}^2/(4C)$
C) $U = 0$

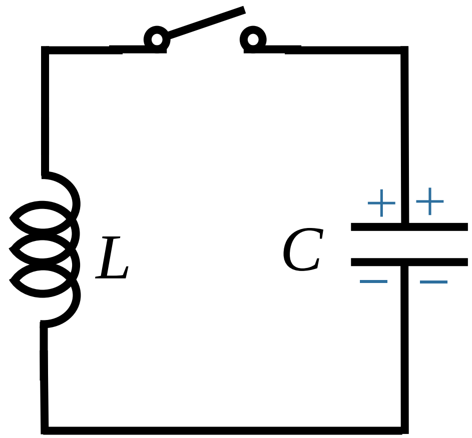
Total Energy is constant!

$U_{Lmax} = ½ LI_{max}^2$
$U_{Cmax} = Q_{max}^2/2C$
$I = max$ when $Q = 0$



CheckPoint 2a

The capacitor is charged such that the top plate has a charge $+Q_0$ and the bottom plate $-Q_0$. At time $t = 0$, the switch is closed and the circuit oscillates with frequency $\omega = 500$ radians/s.



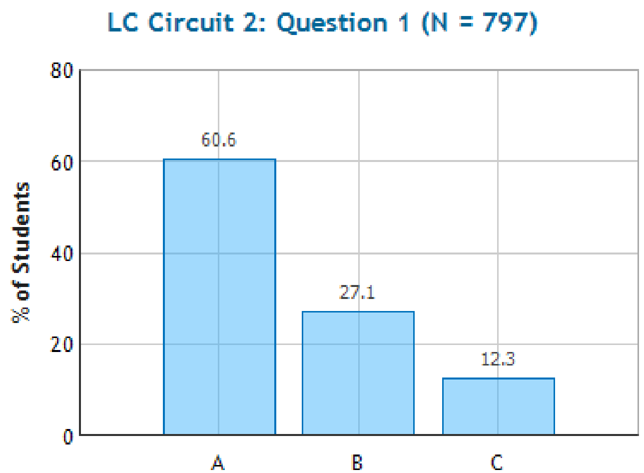
$L = 4 \times 10^{-3}\ H$

$\omega = 500$ rad/s

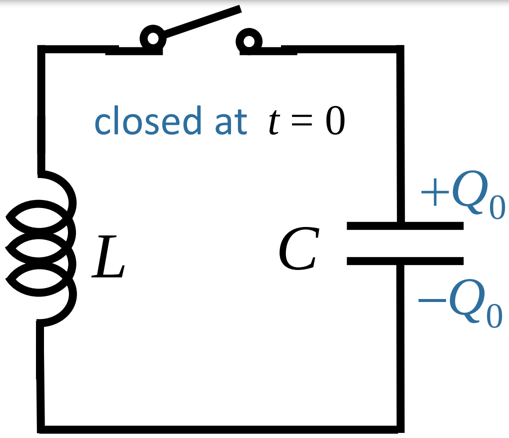
What is the value of the capacitor C?

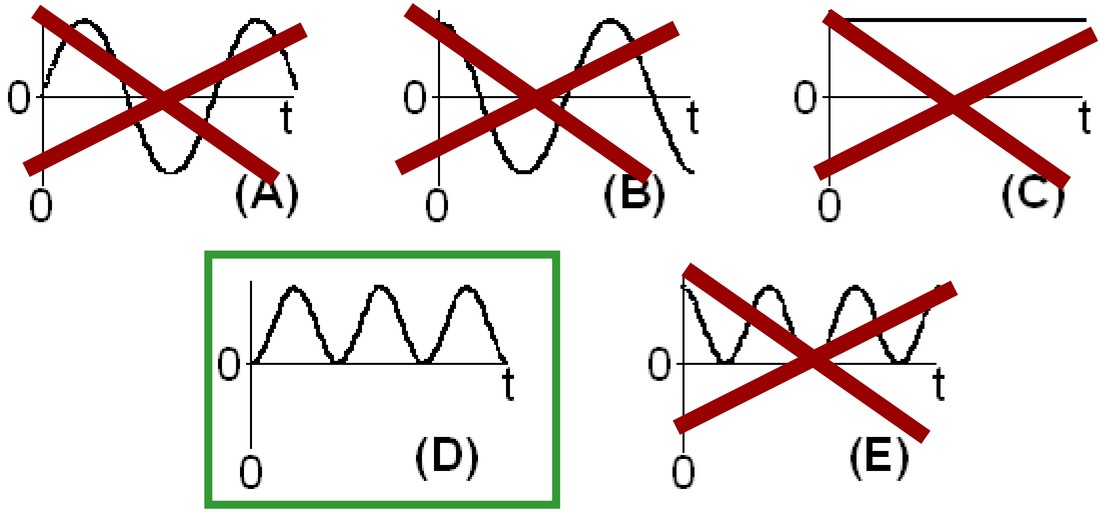
A) $C = 1 \times 10^{-3}\ F$
B) $C = 2 \times 10^{-3}\ F$
C) $C = 4 \times 10^{-3}\ F$

$$\omega = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(25\times10^4)(4\times10^{-3})} = 10^{-3}$$



CheckPoint 2b





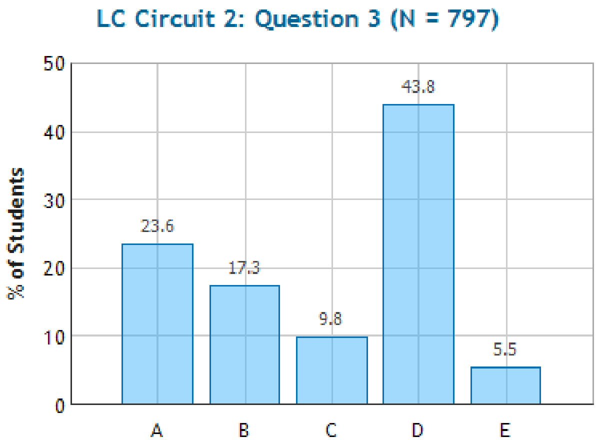
Which plot best represents the energy in the inductor as a function of time starting just after the switch is closed?

$$U_L = \frac{1}{2} L I^2$$

Energy proportional to I^2 Þ U_L cannot be negative

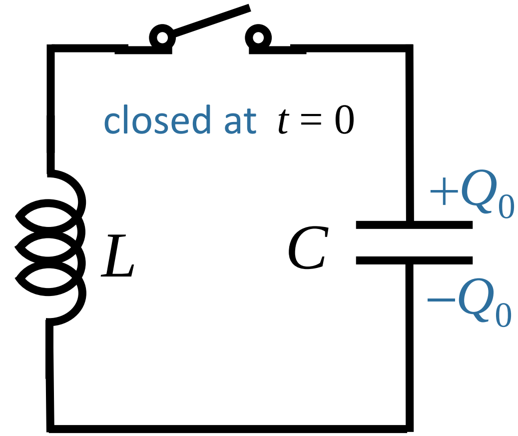
Current is changing Þ U_L is not constant

Initial current is zero

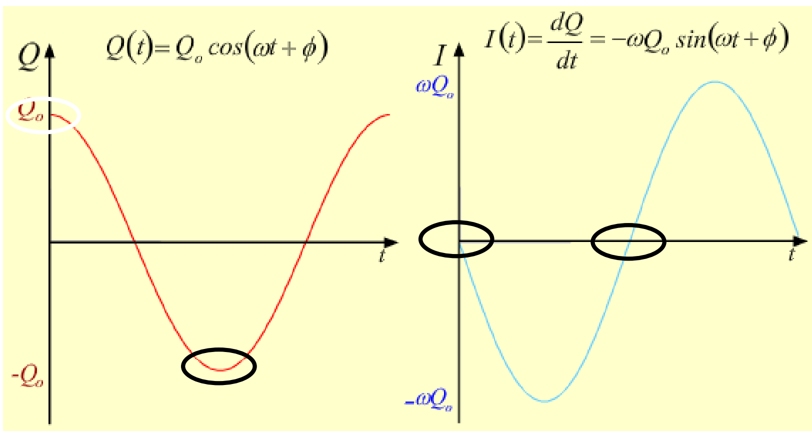


CheckPoint 2c

When the energy stored in the capacitor reaches its maximum again for the first time after $t = 0$, how much charge is stored on the top plate of the capacitor?



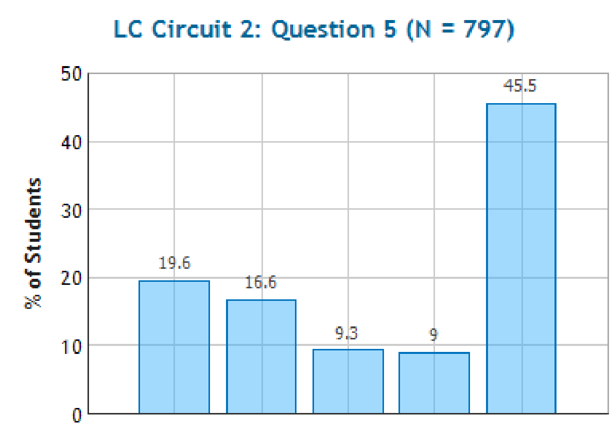
A) $+Q_0$
B) $+Q_0 /2$
C) 0
D) $-Q_0/2$
E) $-Q_0$



Q is maximum when current goes to zero

$$I = \frac{dQ}{dt}$$

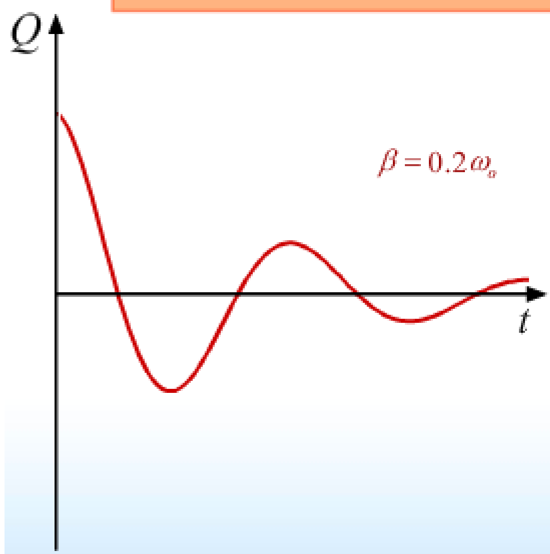
Current goes to zero twice during one cycle

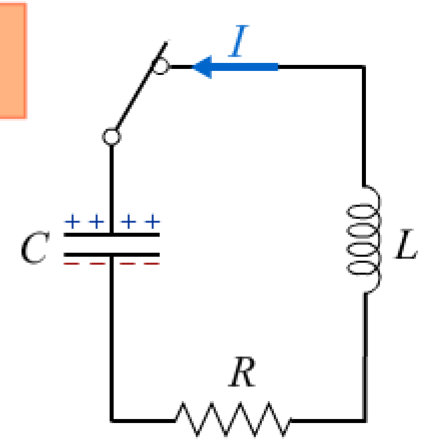


Add R: Damping

Just like LC circuit but energy but the oscillations get smaller because of R

$$Q(t) = Q_o e^{-\beta t}\cos(\omega' t + \phi)$$





Damping Factor $\beta = \frac{R}{2L}$

Oscillation Frequency $\omega'^2 = \omega_o^2 - \beta^2$

Natural Frequency $\omega_o = \frac{1}{\sqrt{LC}}$

Concept makes sense… …but answer looks kind of complicated

Physics Truth #1:

Even though the answer sometimes looks complicated…

$$Q(t) = Q_o \cos(\omega t - \phi)$$

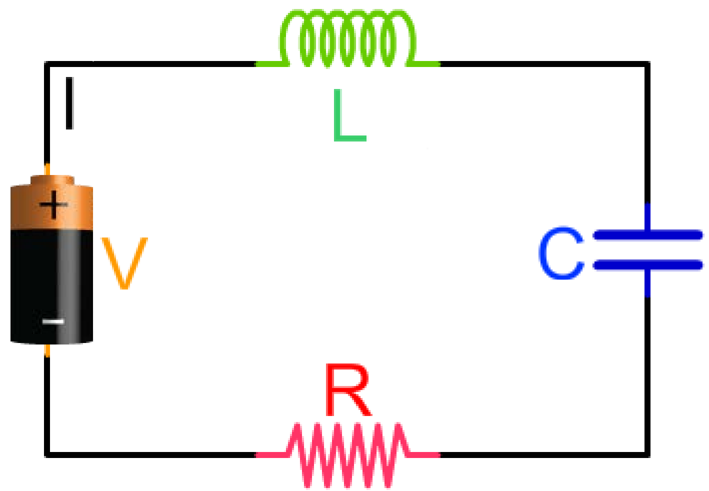
the physics under the hood is still very simple!

$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

The elements of a circuit are very simple:

$$V_L = L\frac{dI}{dt}$$

$$V = V_L + V_C + V_R$$



$$V_C = \frac{Q}{C}$$

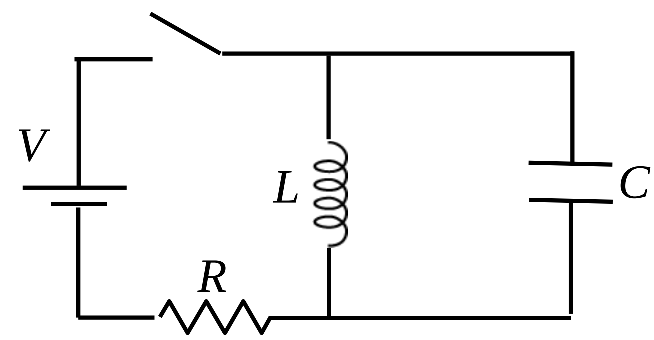
$$I = \frac{dQ}{dt}$$

$$V_R = IR$$

This is all we need to know to solve for anything!

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is Q_{MAX}, the maximum charge on the capacitor?

Conceptual Analysis

Once switch is opened, we have an LC circuit

Current will oscillate with natural frequency ω_0

Strategic Analysis

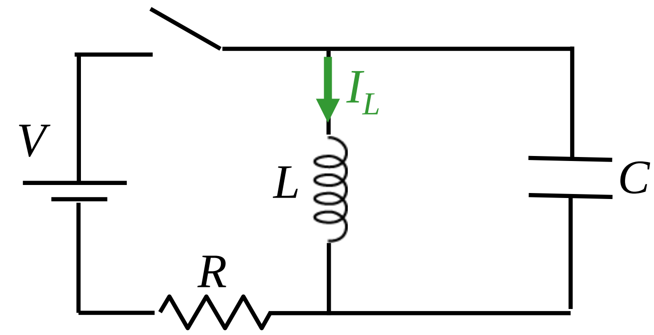
Determine initial current

Determine oscillation frequency ω_0

Find maximum charge on capacitor

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is I_L, the current in the inductor, immediately after the switch is opened? Take positive direction as shown.

A) $I_L < 0$ B) $I_L = 0$ C) $I_L > 0$

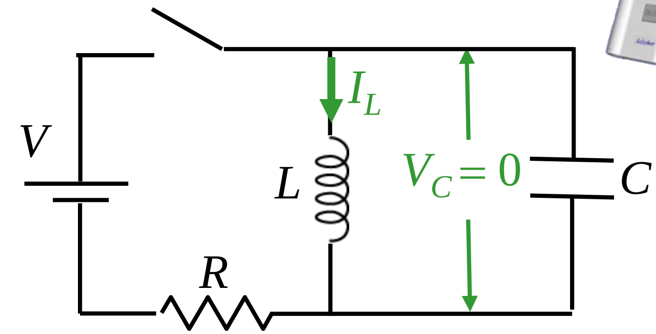
Current through inductor immediately after switch is opened
is the same as
the current through inductor immediately before switch is opened

before switch is opened:

all current goes through inductor in direction shown

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$

The energy stored in the capacitor immediately after the switch is opened is zero.

A) TRUE B) FALSE

before switch is opened:

$dI_L/dt \sim 0 \Rightarrow V_L = 0$

BUT: $V_L = V_C$
since they are in parallel

→ $V_C = 0$

after switch is opened:

V_C cannot change abruptly

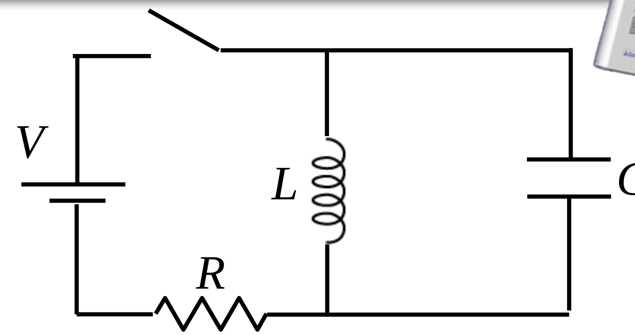
→ $V_C = 0$

→ $U_C = ½\, CV_C^2 = 0$!

IMPORTANT: NOTE DIFFERENT CONSTRAINTS AFTER SWITCH OPENED
CURRENT through INDUCTOR cannot change abruptly
VOLTAGE across CAPACITOR cannot change abruptly

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$ $\qquad V_C(t = 0+) = 0$

What is the magnitude of the current right after the switch is opened?

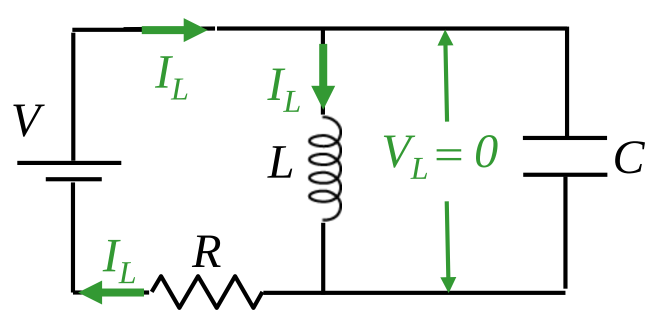
A) $I_o = V\sqrt{\frac{C}{L}}$ $\qquad$ B) $I_o = \frac{V}{R^2}\sqrt{\frac{L}{C}}$ $\qquad$ C) $I_o = \frac{V}{R}$ $\qquad$ D) $I_o = \frac{V}{2R}$

Current through inductor immediately after switch is opened

is the same as

the current through inductor immediately before switch is opened

Before switch is opened:



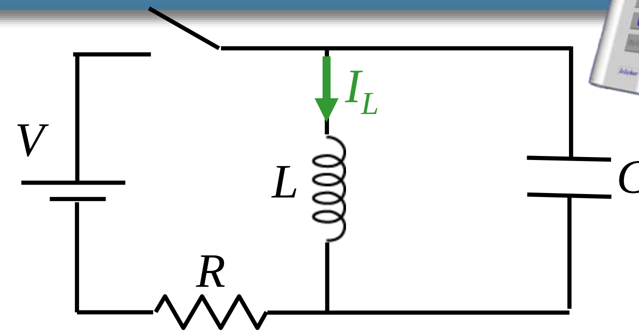
$V_L = 0$

↓

$V = I_L R$

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



Hint: Energy is conserved

$I_L(t = 0+) = V/R$ $\quad V_C(t = 0+) = 0$

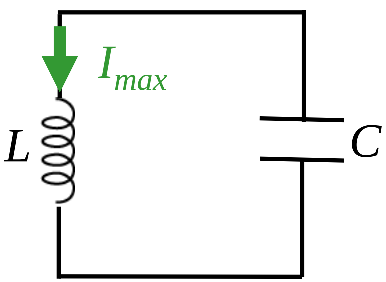
What is Q_{max}, the maximum charge on the capacitor during the oscillations?

A) $Q_{\max} = \frac{V}{R}\sqrt{LC}$

B) $Q_{\max} = \frac{1}{2}CV$

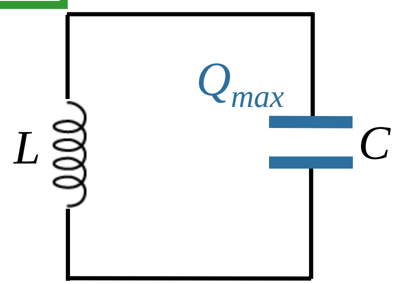
C) $Q_{\max} = CV$

D) $Q_{\max} = \frac{V}{R\sqrt{LC}}$



When I is max
(and Q is 0)

$$U = \frac{1}{2}LI_{\max}^2$$



When Q is max
(and I is 0)

$$U = \frac{1}{2}\frac{Q_{\max}^2}{C}$$

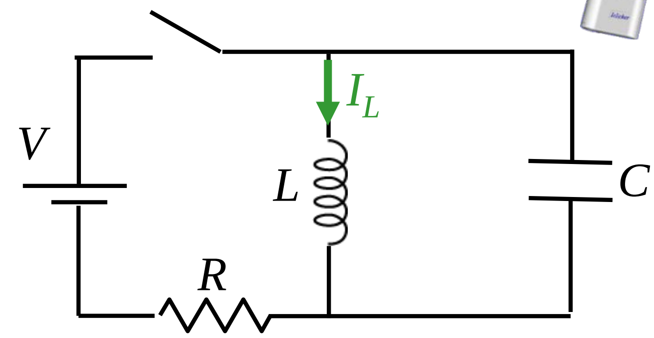
$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}\frac{Q_{\max}^2}{C}$$

$$Q_{\max} = I_{\max}\sqrt{LC}$$

$$= \frac{V}{R}\sqrt{LC}$$

Follow-Up

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



Is it possible for the maximum voltage on the capacitor to be greater than V ?

A) YES

B) NO

$I_{max} = V/R$ $\qquad$ $Q_{max} = \frac{V}{R}\sqrt{LC}$

$Q_{max} = \frac{V}{R}\sqrt{LC}$ → $V_{max} = \frac{V}{R}\sqrt{\frac{L}{C}}$ → V_{max} can be greater than V IF: $\sqrt{\frac{L}{C}} > R$

We can rewrite this condition in terms of the resonant frequency:

$$\omega_0 L > R \quad \text{OR} \quad \frac{1}{\omega_0 C} > R$$

We will see these forms again when we study AC circuits!