

Your Comments

RC circuits were very hard for me, so I am scared of RL Circuits.

I thought this one was pretty tough, plus I am exhausted from my Diff Eq test...

This was difficult. I just need good conceptual understanding of inductors and then this topic will become much easier.

Why did the inductor date the resistor? He wanted a time-constant relationship.

Will RL circuits be on the test? Because I'm very confused by them. **Yes.**

It's very annoying to have the Smart Physics spell check flag inductor as a misspelled word.

Pretty proud of the fact that I have INDUCTed this prelecture into my physics 212 CAPACITANCE in my brain.

Yesterday was what my high school chem teacher called "Avagadro Day" because of the 10^{23} in Avagadro's number. She nerded out and made guacaMOLE and partied. We should too. Partying can still be described by physics, so I find it reasonable.

Some Exam Stuff

Exam 2: Wed. Oct. 29 at 7:00

- Covers material in Lectures 9 – 18
- Sign up in Gradebook for Conflict Exam at 5:15pm
- Link in Gradebook if you have a double-conflict

Exam Preparation:

- Study HW, Discussion, Checkpoints
- Old Exams are a good way to assess what you need to know
- Prelecture of Fall 2010 solutions available (10/29 “prelecture”)
- Video Solutions of Spring 2014 Hour Exam 2 (10/29 “optional HW”)

Extra Office Hours:

- Fri., Sun., Mon., Tue., Wed. (see website for schedule and rooms)

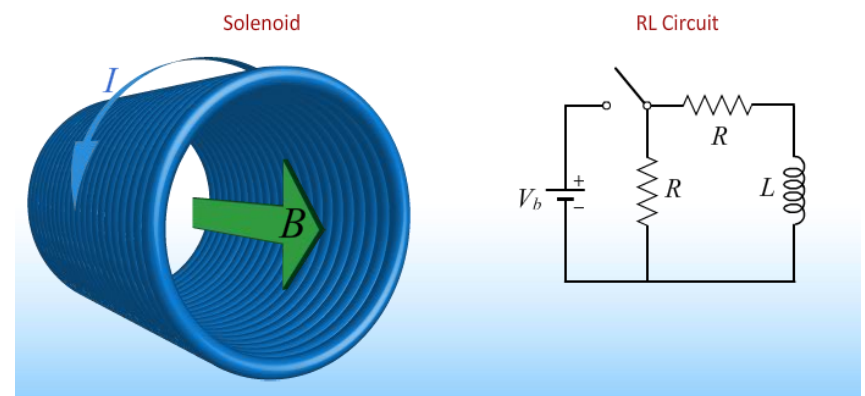
Physics 212

Lecture 18

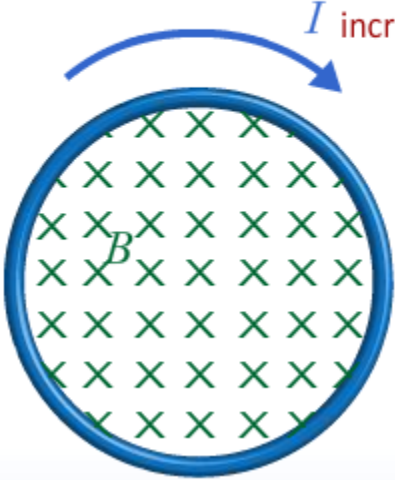
INDUCTION and RL CIRCUITS

Today's Concepts:

- A) Induction
- B) RL Circuits



From the Prelecture: Self Inductance



I increases

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

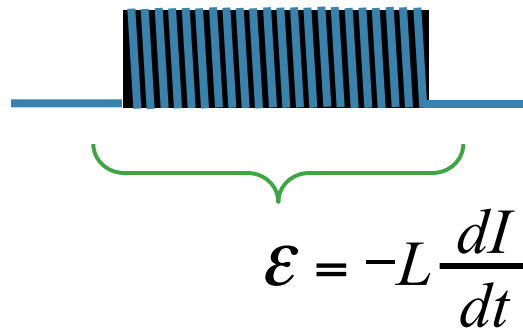
Self-Inductance

$$L \equiv \frac{\Phi_B}{I}$$

SI Unit

$$\text{H} = \text{T}\cdot\text{m}^2/\text{A}$$

Wrap a wire into a coil to make an “inductor”...



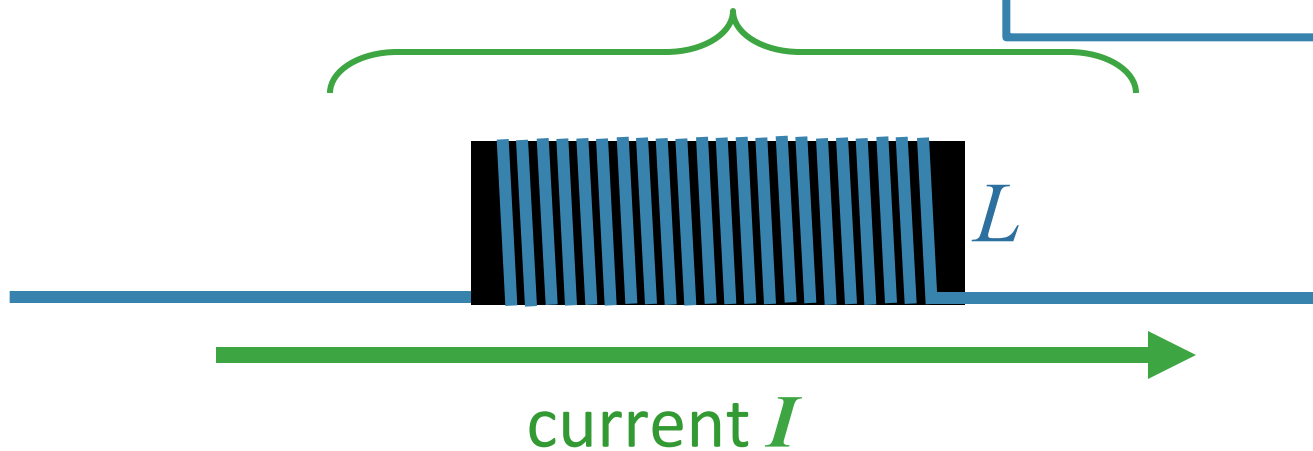
What this really means:

emf induced across L tries to keep I constant.

$$\mathcal{E}_L = -L \frac{dI}{dt}$$

Short Term $I_{\text{before}} = I_{\text{after}}$

Long Term $V_L = 0$



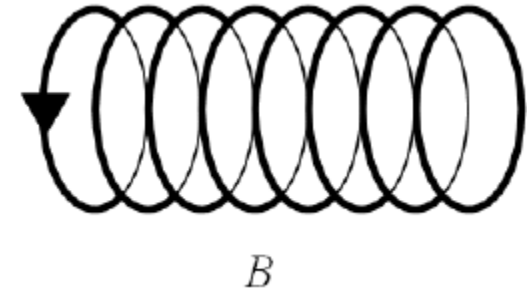
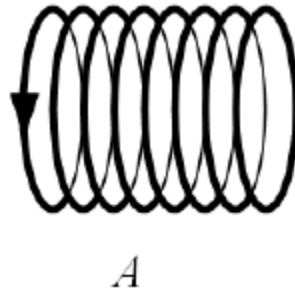
Inductors prevent discontinuous current changes!

It's like inertia!

Checkpoint 2

Two solenoids are made with the same cross sectional area and total number of turns. Inductor *B* is twice as long as inductor *A*

$$L_B = \mu_0 \underset{\substack{\uparrow \\ (1/2)^2}}{n^2} \pi r^2 \underset{\substack{\uparrow \\ 2}}{z}$$

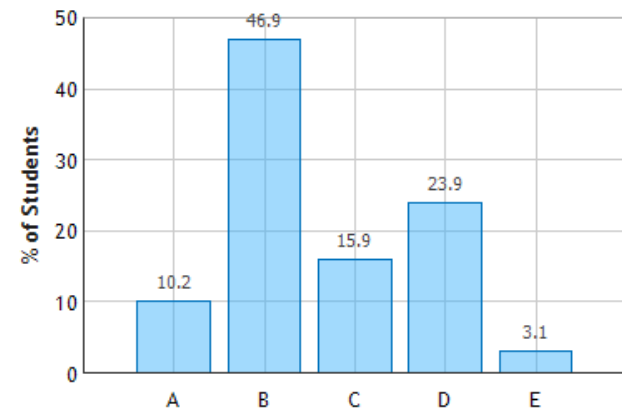


$$\longrightarrow L_B = \frac{1}{2} L_A$$

Compare the inductance of the two solenoids

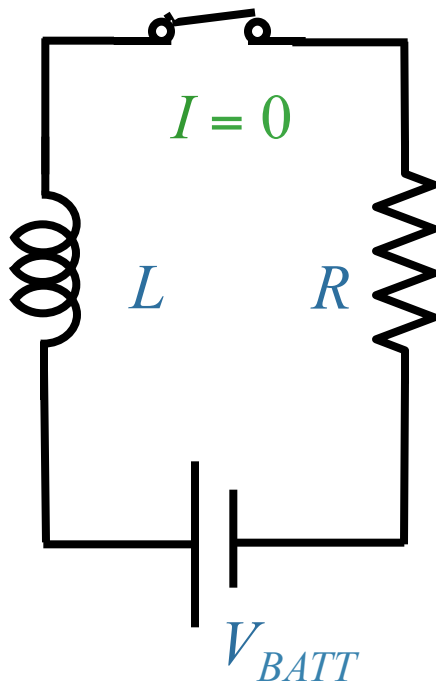
- A) $L_A = 4 L_B$
- B) $L_A = 2 L_B$
- C) $L_A = L_B$
- D) $L_A = (1/2) L_B$
- E) $L_A = (1/4) L_B$

Inductance of Solenoids: Question 1 (N = 762)



How to think about RL circuits Episode 1:

When no current is flowing initially:



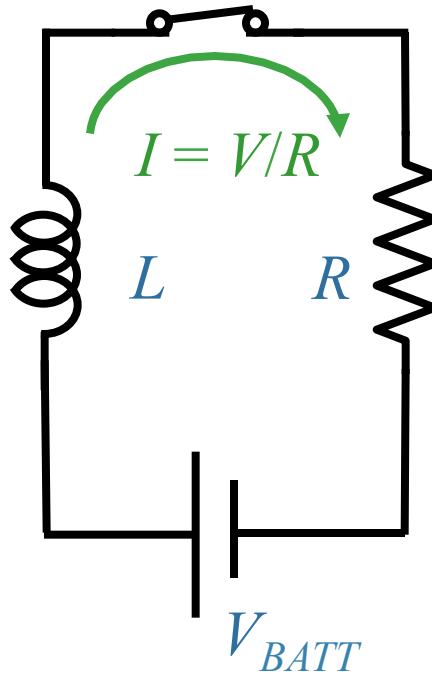
At $t = 0$: I_L unchanged

$$I_L = 0$$

$$V_R = 0$$

$$V_L = V_{BATT}$$

(L is like an open circuit)



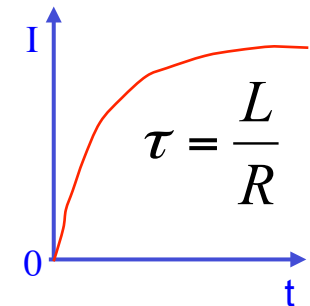
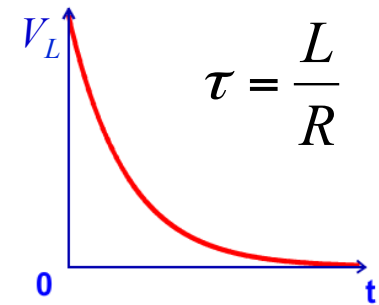
At $t \gg L/R$: $V_L = 0$

$$V_L = 0$$

$$V_R = V_{BATT}$$

$$I = V_{BATT}/R$$

(L is like a wire)



Checkpoint 2a

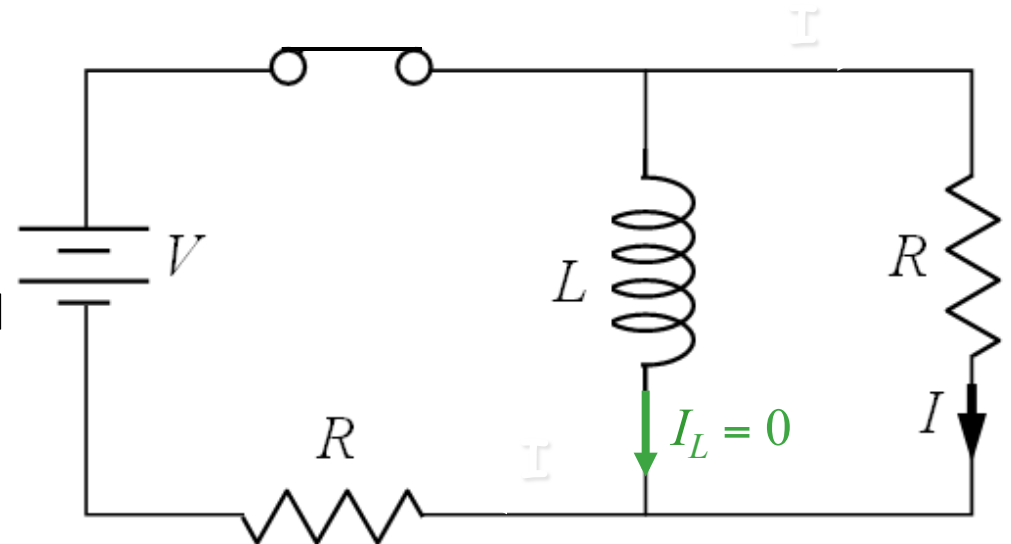


In the circuit, the switch has been open for a long time, and the current is zero everywhere.

At time $t = 0$ the switch is closed.

What is the current I through the vertical resistor immediately after the switch is closed?

(+ is in the direction of the arrow)



A) $I = V/R$

B) $I = V/2R$

C) $I = 0$

D) $I = -V/2R$

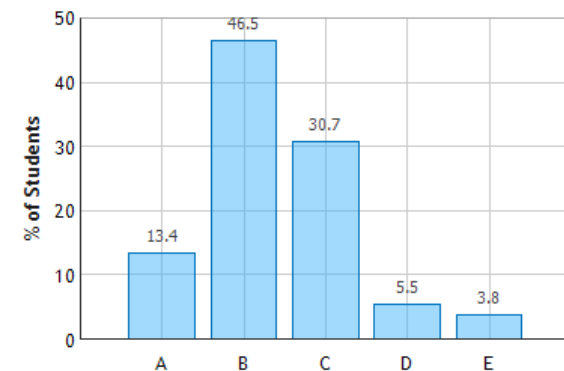
E) $I = -V/R$

Before: $I_L = 0$

After: $I_L = 0$

→ $I = + V/2R$

RL Circuit: Question 1 (N = 761)



RL Circuit (Long Time)



What is the current I through the vertical resistor after the switch has been closed for a long time?

(+ is in the direction of the arrow)

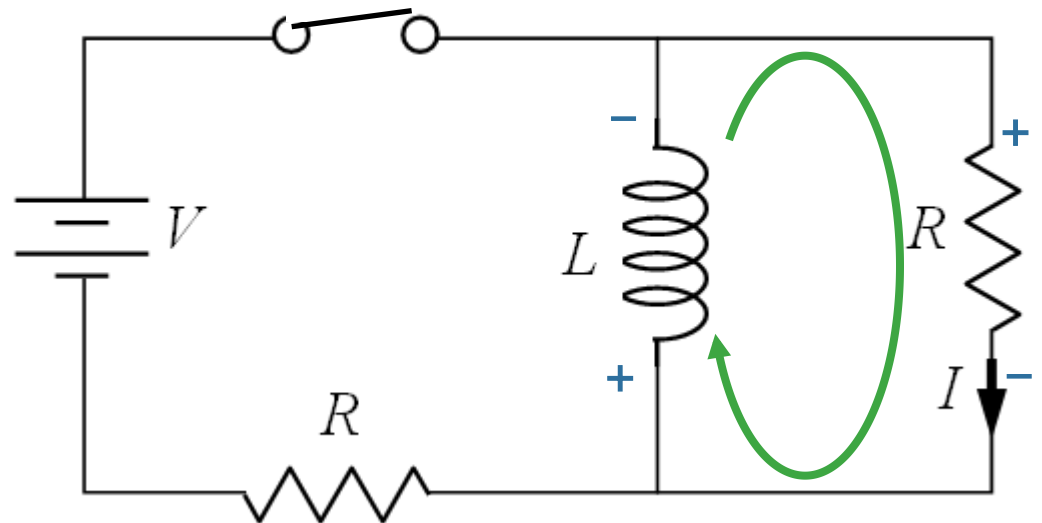
A) $I = V/R$

B) $I = V/2R$

C) $I = 0$

D) $I = -V/2R$

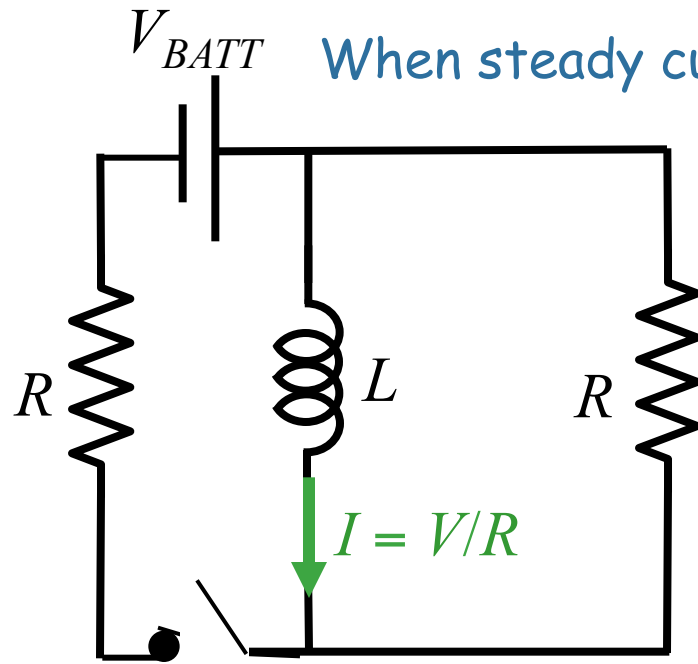
E) $I = -V/R$



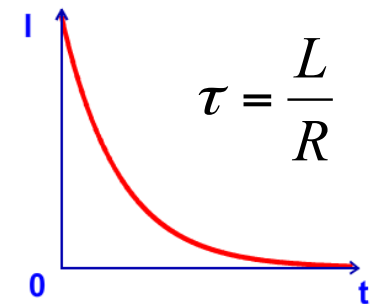
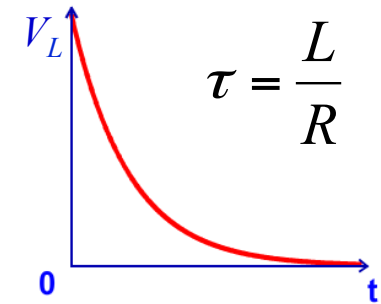
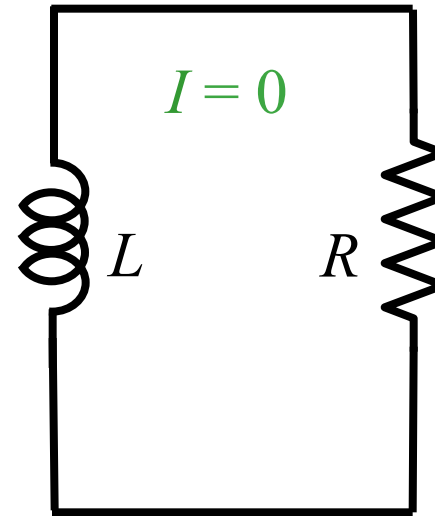
After a long time in any static circuit: $V_L = 0$

KVR:
 $V_L + IR = 0$

How to Think about RL Circuits Episode 2:



When steady current is flowing initially: then switch is opened



At $t = 0$: I_L unchanged

$$\begin{aligned} I &= V_{BATT}/R \\ V_R &= IR \\ V_L &= V_R \end{aligned}$$

At $t \gg L/R$: $V_L = 0$

$$\begin{aligned} V_L &= 0 \\ V_R &= 0 \\ I &= 0 \end{aligned}$$

Checkpoint 2b

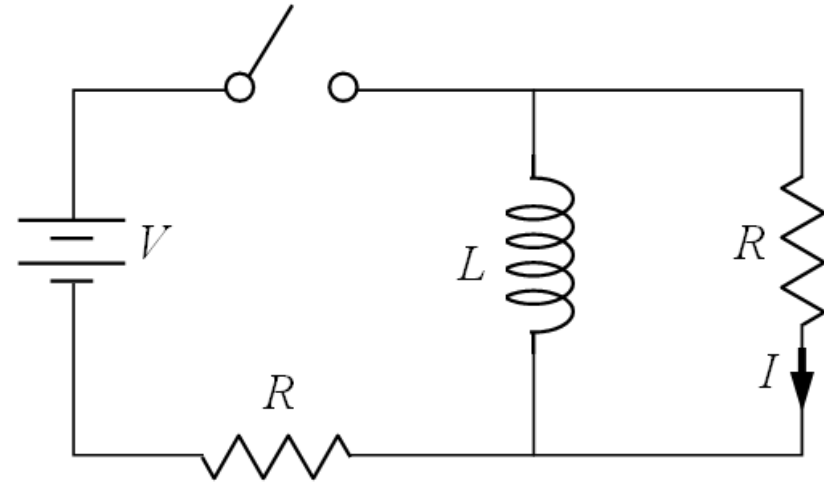
After a long time, the switch is opened, abruptly disconnecting the battery from the circuit.

What is the current I through the vertical resistor immediately after the switch is opened?

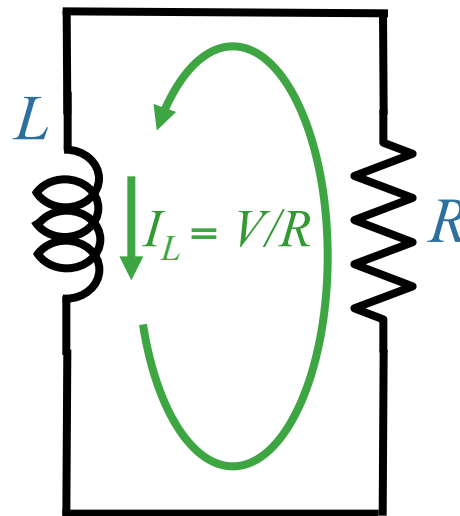
(+ is in the direction of the arrow)

- A) $I = V/R$
- B) $I = V/2R$
- C) $I = 0$
- D) $I = -V/2R$
- E) $I = -V/R$

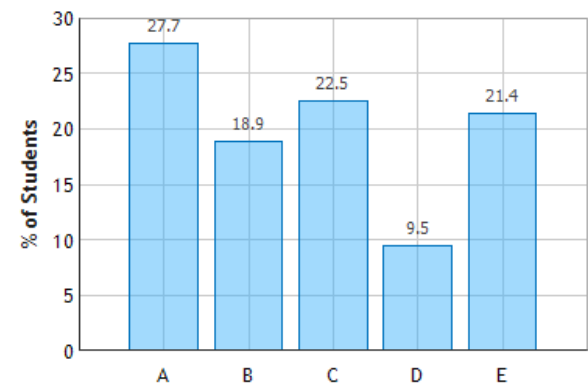
Current through inductor cannot change
DISCONTINUOUSLY



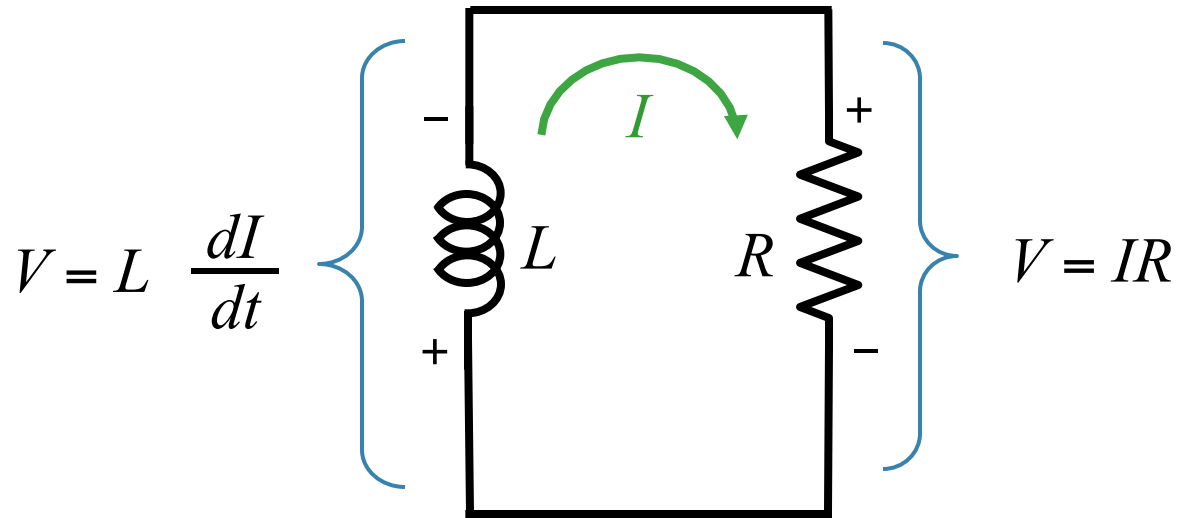
circuit when switch
opened



RL Circuit: Question 3 (N = 761)



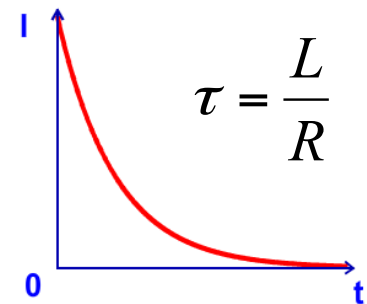
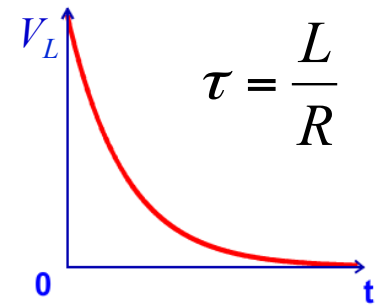
Why is there Exponential Behavior?

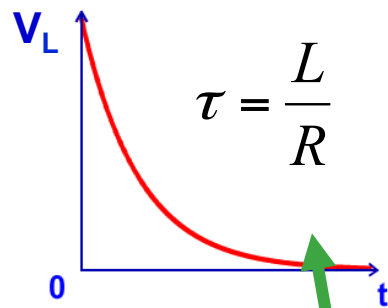
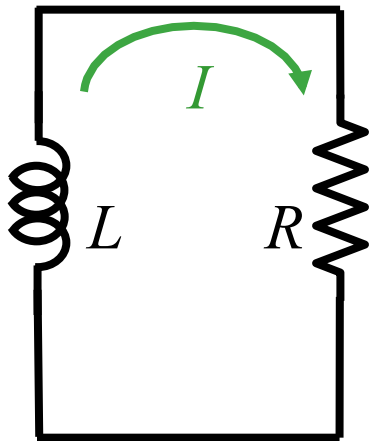


$$L \frac{dI}{dt} + IR = 0$$

$$I(t) = I_0 e^{-tR/L} = I_0 e^{-t/\tau}$$

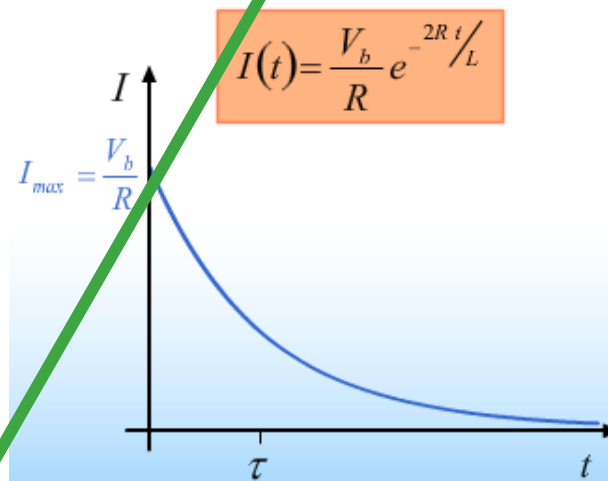
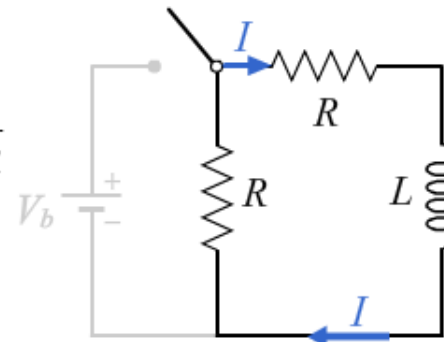
where $\tau = \frac{L}{R}$



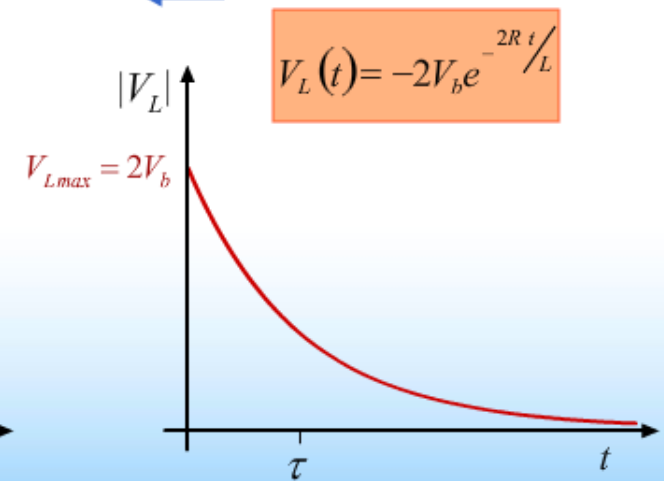


Lecture:

Time Constant $\tau = \frac{L}{2R}$



Prelecture:



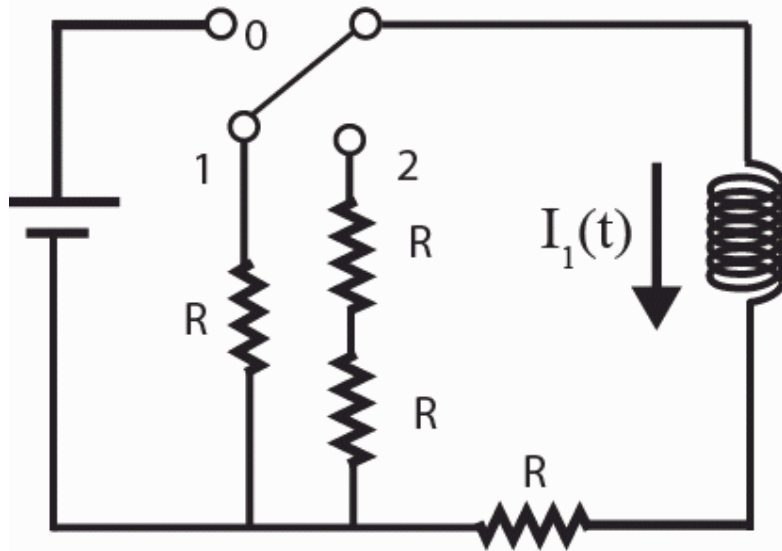
Did we mess up?

No: The resistance is simply twice as big in one case.

Checkpoint 3a

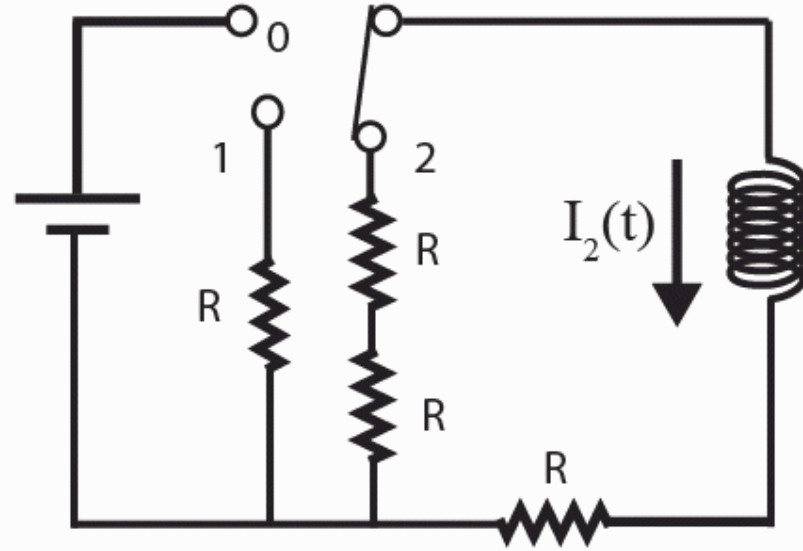


After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



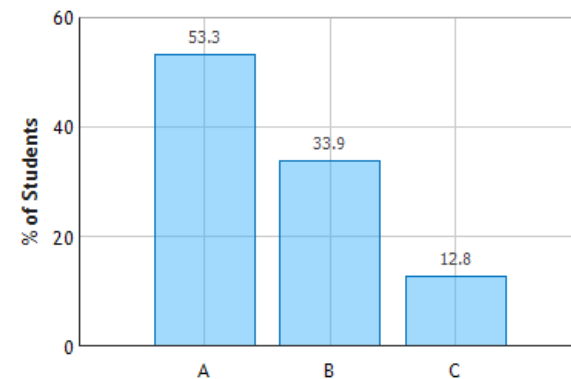
Case 2

After switch moved, which case has larger time constant?

- A) Case 1
- B) Case 2
- C) The same

$$\tau_1 = \frac{L}{2R} \quad \tau_2 = \frac{L}{3R}$$

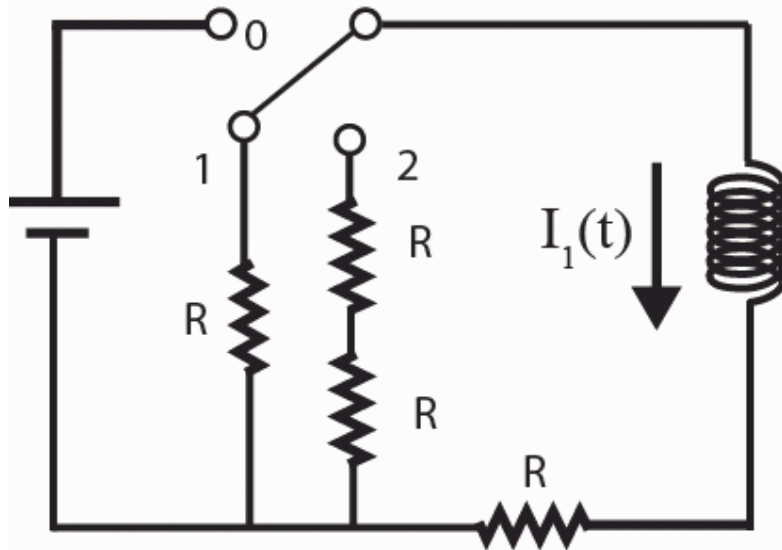
Compare RL Circuits: Question 1 (N = 758)



Checkpoint 3b

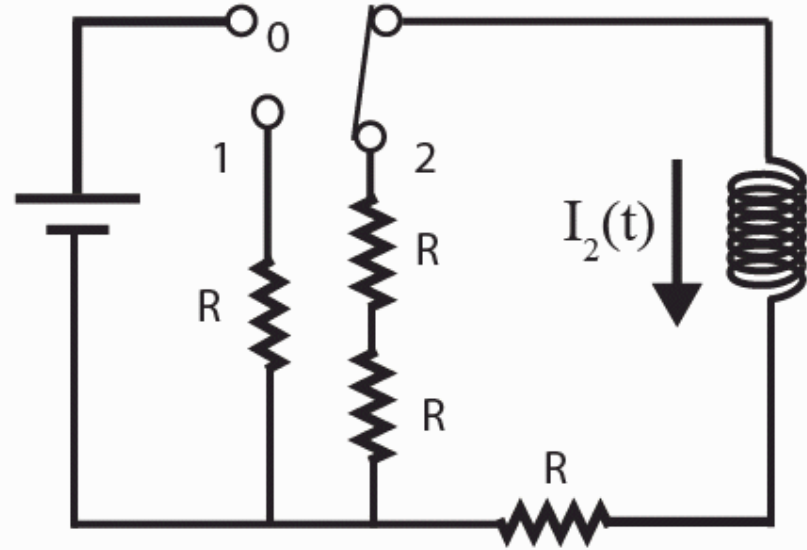


After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

Immediately after switch moved,
in which case is the voltage
across the inductor larger?

- A) Case 1
- ☒ B) Case 2
- C) The same

Before switch moved:

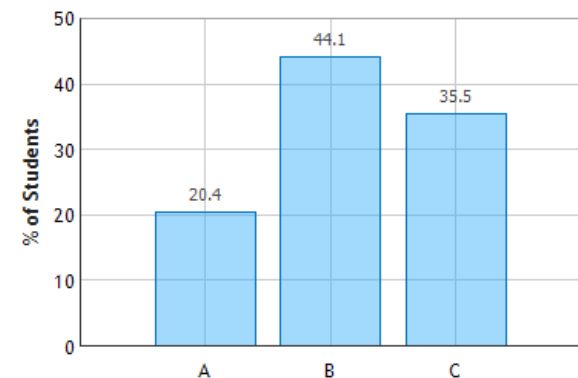
$$I = \frac{V}{R}$$

After switch moved:

$$V_{L1} = \frac{V}{R} 2R$$

$$V_{L2} = \frac{V}{R} 3R$$

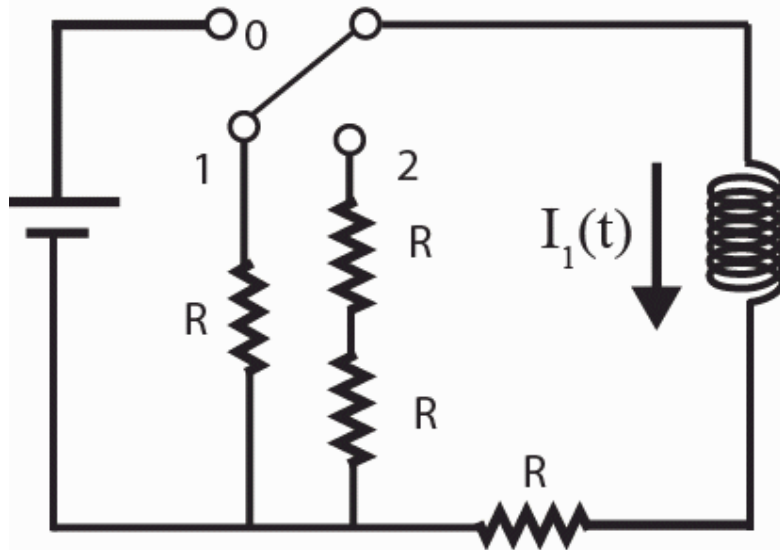
Compare RL Circuits: Question 3 (N = 755)



CheckPoint 3c

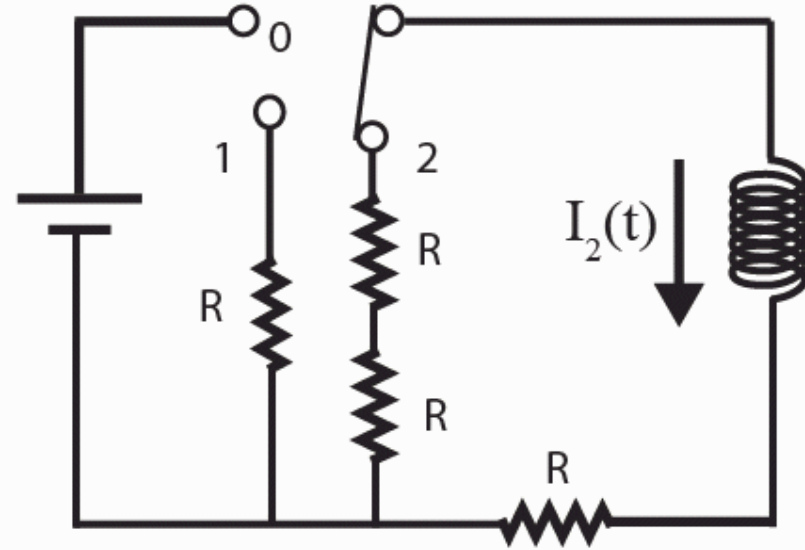


After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

After switch moved for finite time, in which case is the current through the inductor larger?

- A) Case 1
- B) Case 2
- C) The same

Immediately after: $I_1 = I_2$

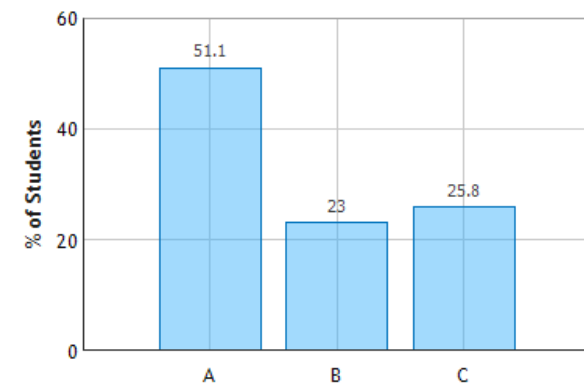
After awhile

$$I_1 = Ie^{-t/\tau_1}$$

$$I_2 = Ie^{-t/\tau_2}$$

$$\tau_1 > \tau_2$$

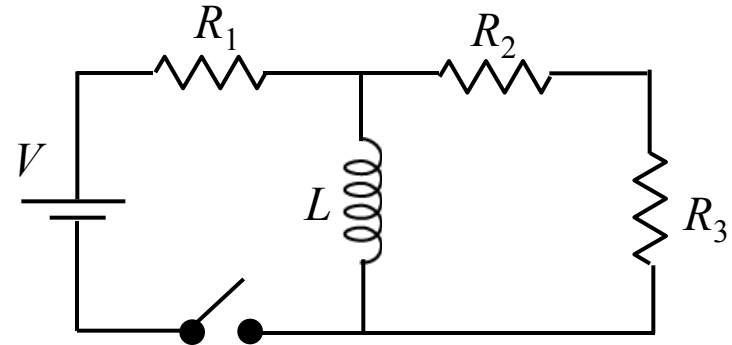
Compare RL Circuits: Question 5 (N = 755)



Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.

What is dI_L/dt , the time rate of change of the current through the inductor immediately after switch is closed



Conceptual Analysis

Once switch is closed, currents will flow through this 2-loop circuit.

KVR and KCR can be used to determine currents as a function of time.

Strategic Analysis

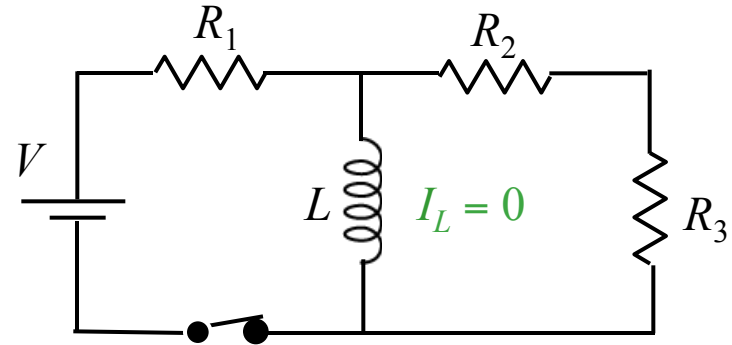
Determine currents immediately after switch is closed.

Determine voltage across inductor immediately after switch is closed.

Determine dI_L/dt immediately after switch is closed.

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is I_L , the current in the inductor, immediately after the switch is closed?

A) $I_L = V/R_1$ up

B) $I_L = V/R_1$ down

C) $I_L = 0$

INDUCTORS: Current cannot change discontinuously !

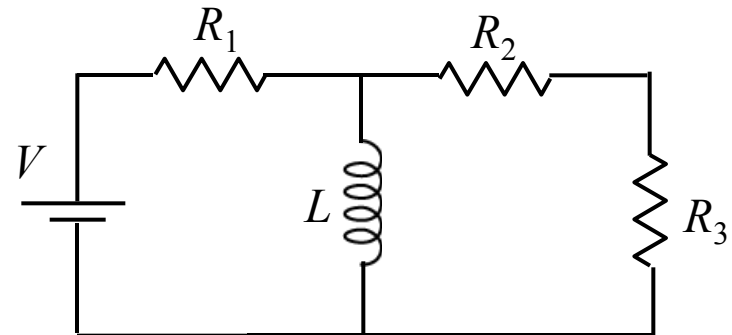


Current through inductor immediately **after** switch is closed
is the same as
the current through inductor immediately **before** switch is closed

Immediately **before** switch is closed: $I_L = 0$ since no battery in loop

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$$I_L(t = 0+) = 0$$

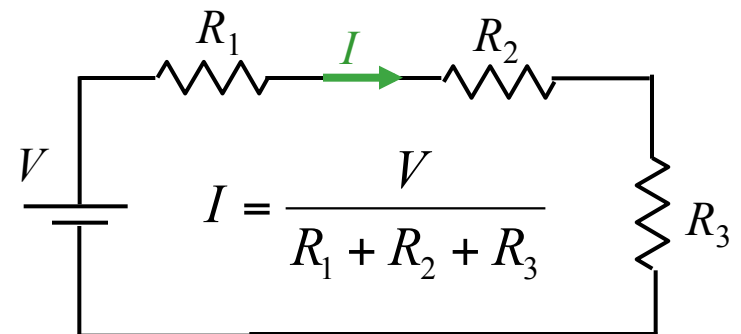
What is the magnitude of I_2 , the current in R_2 , immediately after the switch is closed?

A) $I_2 = \frac{V}{R_1}$ B) $I_2 = \frac{V}{R_2 + R_3}$ **C) $I_2 = \frac{V}{R_1 + R_2 + R_3}$** D) $I_2 = \frac{VR_2R_3}{R_2 + R_3}$

We know $I_L = 0$ immediately after switch is closed

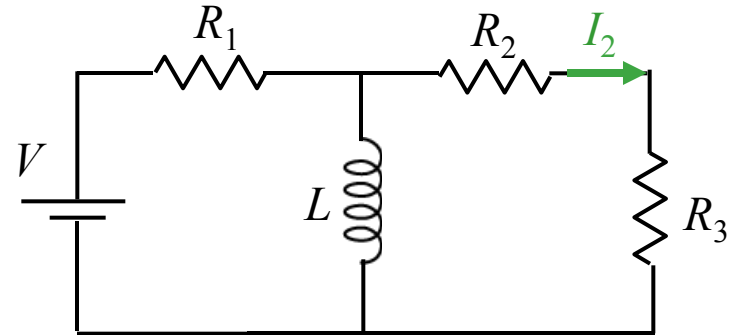


Immediately after switch is closed, circuit looks like:



Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$$I_L(t = 0+) = 0 \quad I_2(t = 0+) = V/(R_1 + R_2 + R_3)$$

What is the magnitude of V_L , the voltage across the inductor, immediately after the switch is closed?

A) $V_L = V \frac{R_2 R_3}{R_1}$
 B) $V_L = V$
 C) $V_L = 0$
 D) $V_L = V \frac{R_2 R_3}{R_1 (R_2 + R_3)}$
 E) $V_L = V \frac{R_2 + R_3}{R_1 + R_2 + R_3}$

Kirchhoff's Voltage Law,

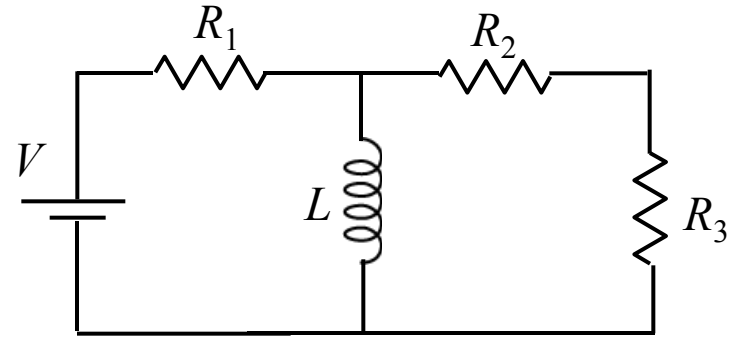
$$V_L - I_2 R_2 - I_2 R_3 = 0 \quad V_L = I_2 (R_2 + R_3)$$

$$V_L = \frac{V}{R_1 + R_2 + R_3} (R_2 + R_3)$$

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.

What is dI_L/dt , the time rate of change of the current through the inductor immediately after switch is closed



$$V_L(t = 0+) = V(R_2 + R_3)/(R_1 + R_2 + R_3)$$

A) $\frac{dI_L}{dt} = \frac{V}{L} \frac{R_2 + R_3}{R_1}$ B) $\frac{dI_L}{dt} = 0$ **C) $\frac{dI_L}{dt} = \frac{V}{L} \frac{R_2 + R_3}{R_1 + R_2 + R_3}$** D) $\frac{dI_L}{dt} = \frac{V}{L}$

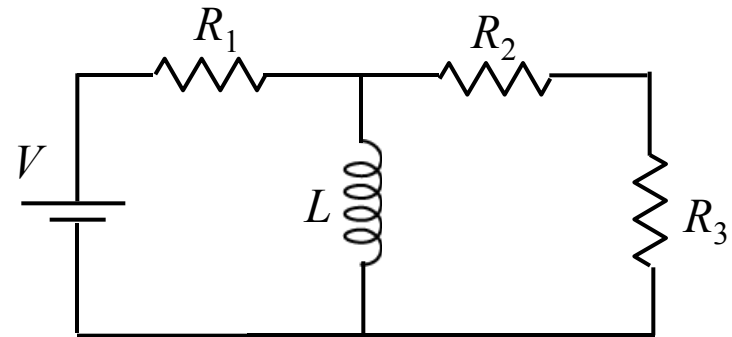
The time rate of change of current through the inductor $(dI_L/dt) = V_L/L$

$$\longrightarrow \frac{dI_L}{dt} = \frac{V}{L} \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

Follow Up

The switch in the circuit shown has been closed for a long time.

What is I_2 , the current through R_2 ?
(Positive values indicate current flows to the right)



A) $I_2 = +\frac{V}{R_2 + R_3}$ B) $I_2 = +\frac{V(R_2 R_3)}{R_1 + R_2 + R_3}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_2 + R_3}$

After a long time, $dI/dt = 0$

Therefore, the voltage across $L = 0$

Therefore the voltage across $R_2 + R_3 = 0$

Therefore the current through $R_2 + R_3$ must be zero!

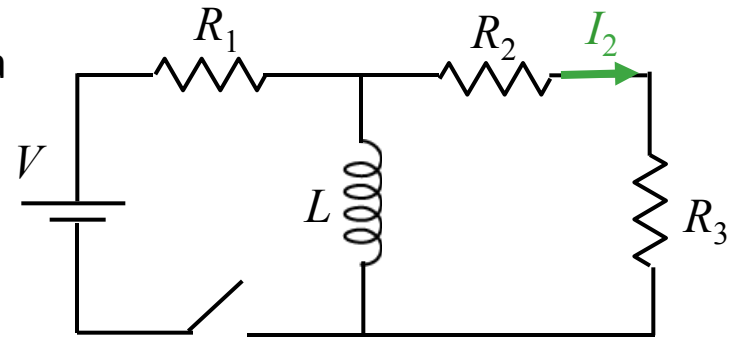
Follow Up 2



The switch in the circuit shown has been closed for a long time at which point, the switch is opened.

What is I_2 , the current through R_2 immediately after switch is opened ?

(Positive values indicate current flows to the right)



A) $I_2 = +\frac{V}{R_1 + R_2 + R_3}$

B) $I_2 = +\frac{V}{R_1}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_1}$

E) $I_2 = -\frac{V}{R_1 + R_2 + R_3}$

Current through inductor immediately **after** switch is opened
is the same as
the current through inductor immediately **before** switch is opened

Immediately **before** switch is opened: $I_L = V/R_1$

Immediately **after** switch is opened: I_L flows in right loop
Therefore, $I_L = -V/R_1$