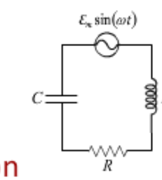
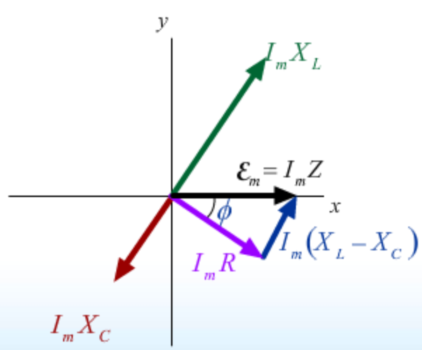
Physics 212
Lecture 21



Phase Relation

$$\tan\phi = \frac{X_L - X_C}{R}$$

Voltage Phasor Diagram



Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Maximum Current

$$I_m = \frac{\mathcal{E}_m}{Z}$$

Your Comments

I'm still really confused on phase diagrams and how the phase angle relates to the voltage and current of each component.

What's up with these Q and x substitutions? Usually substitutions are supposed to make formulas look prettier.

Is the voltage across the generator just represented as E? In the homework they used Vgenerator and it matched up with E, so just wondering if they were the same thing. When is the current in phase with the generator? Always? Or is there a way to tell?

I like this unit, resonance has lots of real world applications especially with the radio.

Can you please explain why we use root mean square voltage and current for the power equations instead of just the voltage and current?

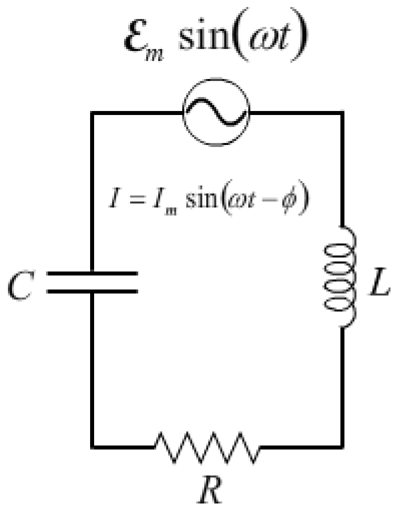
rms = peak/sqrt(2)

Advice for students feeling a little bit overwhelmed?

Get ready. 1. This content is coming WAY too fast. For the sake of next year's freshmen, take some material off the curriculum. 2. This prelecture seemed to introduce a lot of variables and formulas arbitrarily. For instance, x and Q were initially defined just to "simplify" formulas. Not only are the equations not any simpler, but the underlying relationships between the variables get obfuscated by our new terminology. 3. How did our tedious discussion on resonance help us understand transformers? 4. How do we know that the change in flux is equal across both coils of a transformer? 5. Why bother with flux at all? Doesn't it suffice to say that energy is conserved (after all, where would energy leak out)? 6. What's the point of rms? Also, what does it mean and why is Irms=Im/sqrt(2)? 7. You first introduced Q to simplify equations, then later re-defined it, and after that, showed that the first definition is a special case. What motivated you to introduce the topic in such an ass-backwards way? (Same applies for x.) 8. How can we infer how to maximize Im from the phasors? What sets apart emfm=ImZ as the equation to use? 9. At one point you showed that Z=R cos phi and proceeded to substitute into Im=emfm/R cos phi, only to substitute cos phi with R/Z. What was the logic in going through the loop? You could have just started by substituting Z=sqrt(R^2+(Xl-Xc)^2) directly and pulled the R out. 10. Is <P> to be the convention for average P? I thought the standard was an overbar. 11. Why do we call cos phi the power factor? 12. You said that Q is large when R<<Xl, Xc. What does the comma mean? Is Q large when R<<Xl or R<<Xc or R<<Xl+Xc or what? 13. I don't see anything at electrical substations that looks much like the transformer we discussed. Where are they? Do they have different geometries? 14. The prelecture mentioned that the multiplication factor Q is important when a circuit is used to pick up radio signals. How does our driven LRC circuit pick up radio signals? 15. Why is Vlmax = Vcmax = Qemfm? You just threw that at us. 16. The prelecture mentions that averaging sin*cos=0 and gives that as the reason why inductors and capacitors don't dissipate power. What's the deal with that average? And why do we care- don't we already know that capacitors and inductors don't dissipate energy? 17. The coils of a generator look like inductors to me. How does this affect their

Looks intimidating, but isn't bad!

The Driven LCR Circuit



Frequency Dependence of Maximum Current

$$I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1+Q^2\frac{(x^2-1)^2}{x^2}}}$$

Average Power per Cycle

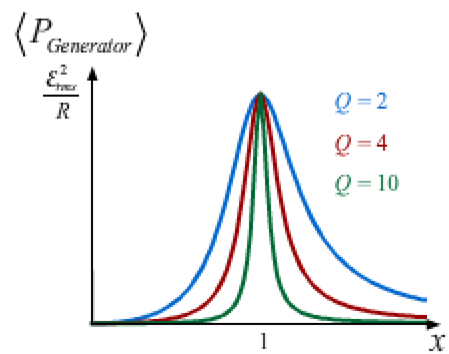
$$\langle P_{Generator}\rangle = \frac{\mathcal{E}_{rms}^2}{R}\frac{x^2}{x^2+Q^2(x^2-1)^2}$$

where $x \equiv \frac{\omega}{\omega_o}$ & $Q^2 = \frac{L}{R^2C}$

Quality Factor

$$Q \equiv 2\pi\left[\frac{U_{max}}{\Delta U}\right]_{cycle}$$

evaluate at $\omega = \omega_o$



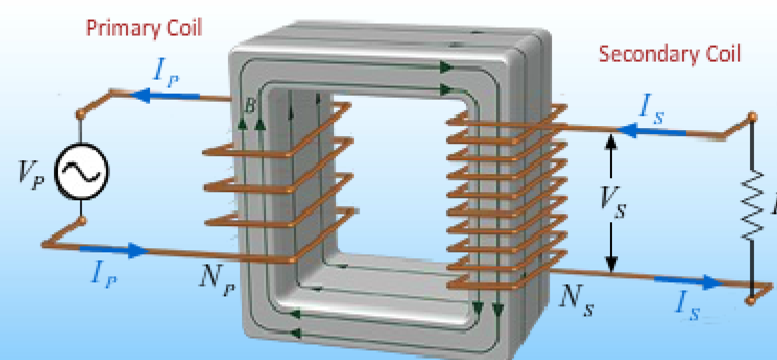
Transformers

Voltage Relation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Current Relation

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

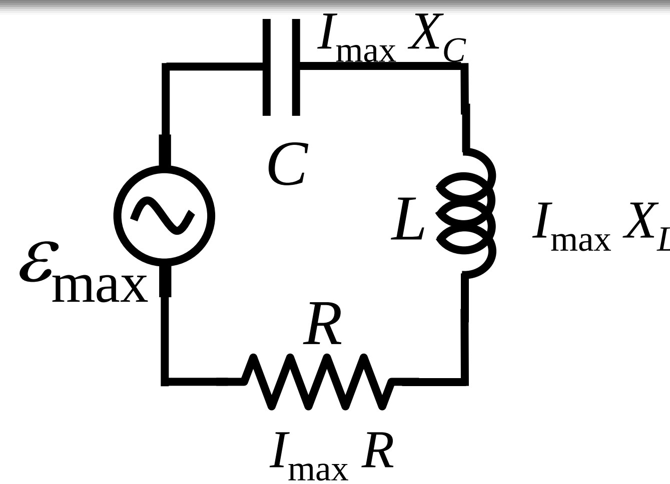


AC Circuits & Phasors

PHASORS ARE THE KEY !
FORMULAS ARE NOT !

START WITH PHASOR DIAGRAM

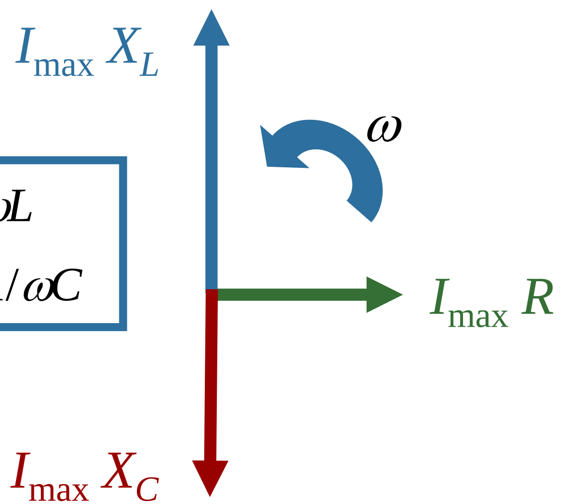
DEVELOP FORMULAS FROM THE DIAGRAM !!



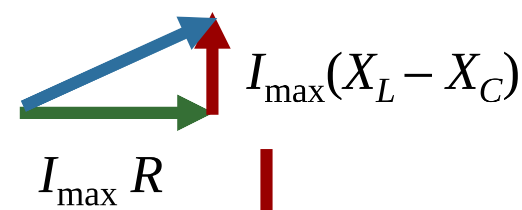
V = Projection along Vertical

$X_L = \omega L$

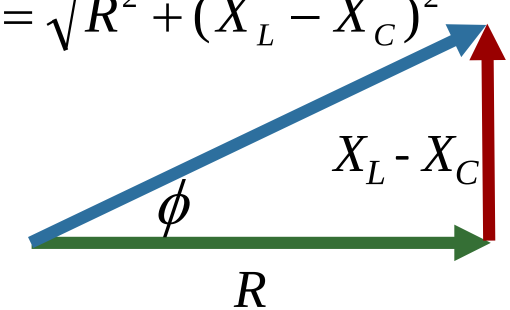
$X_C = 1/\omega C$



$$\mathcal{E}_{max} = I_{max}\, Z$$



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Peak AC Problems

"Ohms" Law for each element

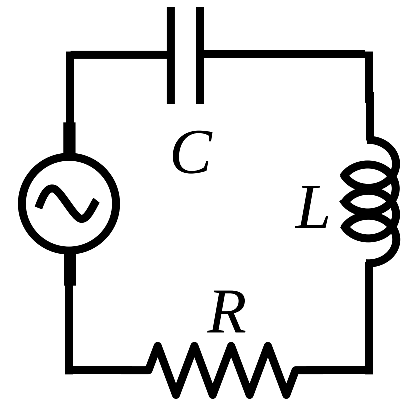
NOTE: Good for PEAK values only)

$$V_{gen} = I_{\max} Z$$
$$V_{Resistor} = I_{\max} R$$
$$V_{inductor} = I_{\max} X_L$$
$$V_{Capacitor} = I_{\max} X_C$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

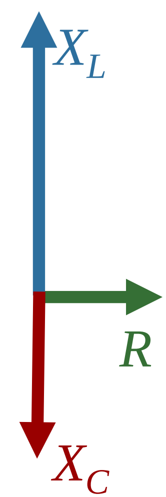
$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$



Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?



$$X_L = \omega L = 200\,\Omega \qquad Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = 122\,\Omega$$

$$X_C = \frac{1}{\omega C} = 100\,\Omega \qquad I_{\max} = \frac{V_{gen}}{Z} = 0.13A$$

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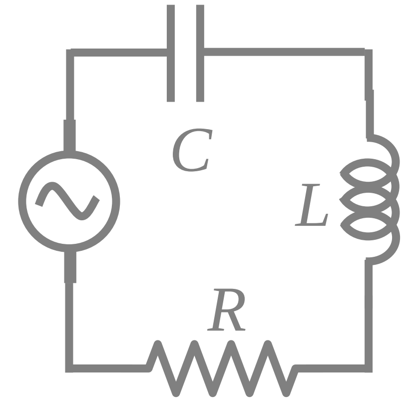


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

Which element has the largest peak voltage across it?

A) Generator

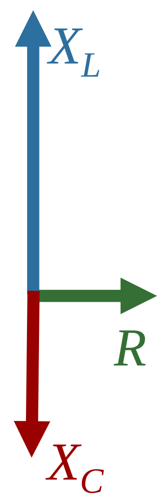
B) Inductor

C) Resistor

D) Capacitor

E) All the same.

$$V_{\max} = I_{\max} X$$



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Peak AC Problems

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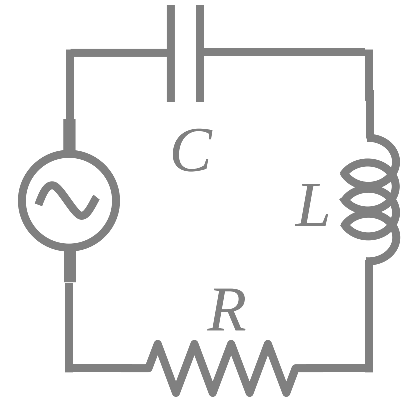


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

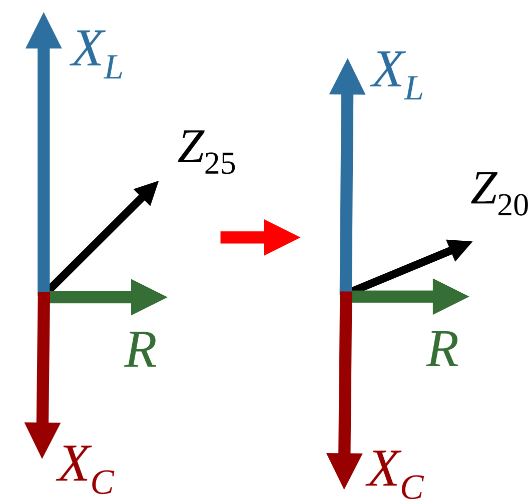
What happens to the impedance if we decrease the angular frequency to 20 rad/sec?

A) Z increases

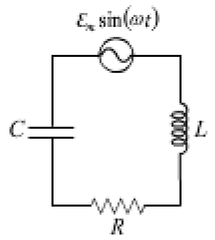
B) Z remains the same

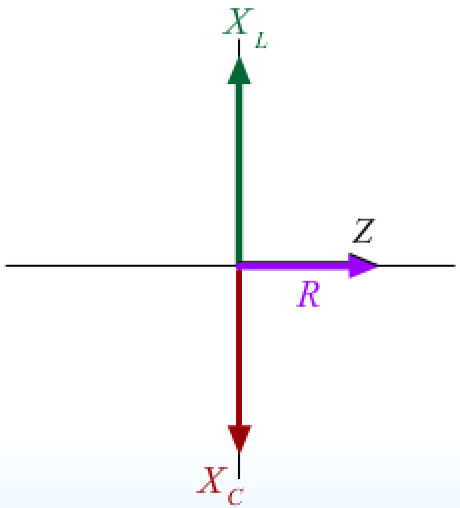
C) Z decreases

$(X_L - X_C)$: (200 – 100) -> (160 – 125)



Resonance





Resonance

I_m is a maximum $\longrightarrow$ $I_m = \dfrac{\mathcal{E}_m}{R}$

$\omega = \omega_o$

Z minimized $\longrightarrow$ $X_L = X_C$

$\phi = 0^{o}$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

Light-bulb Demo

Resonance

Frequency at which voltage across inductor and capacitor cancel

R is independent of ω

X_L increases with ω

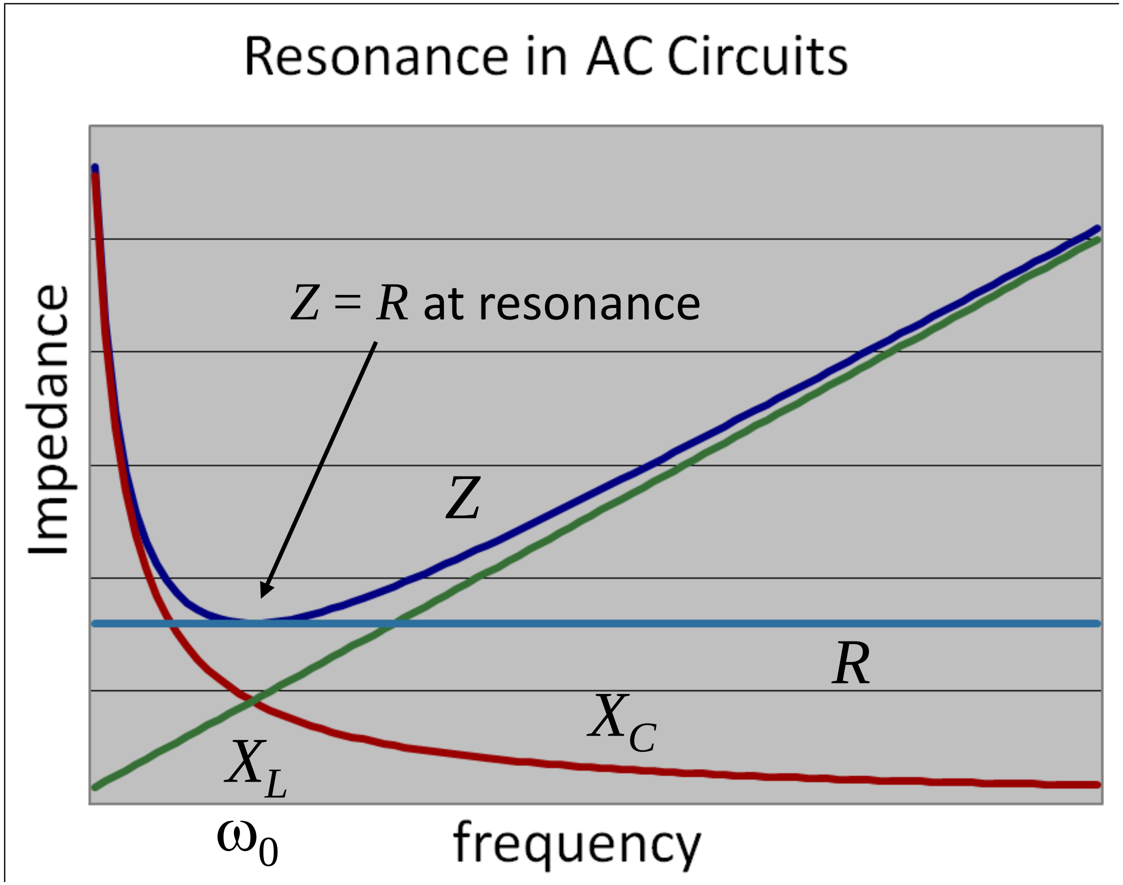
$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

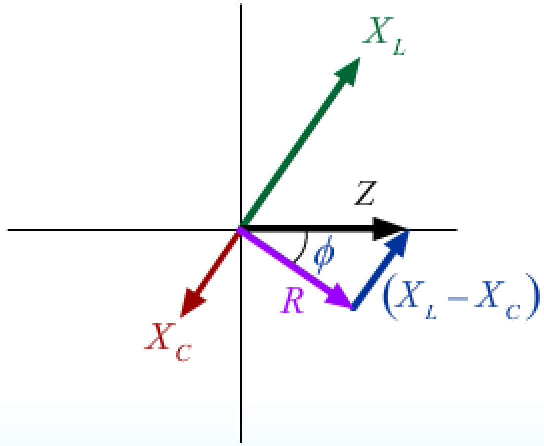
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance



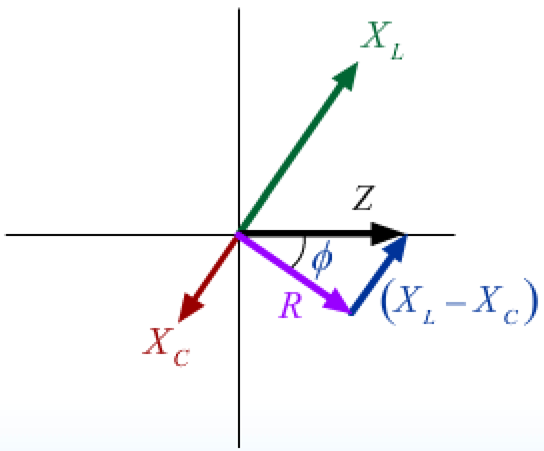
Resonance: $X_L = X_C$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

Off Resonance



$$I_m = \frac{\mathcal{E}_m}{Z}$$

$$I_m = \frac{\mathcal{E}_m}{R}\frac{R}{\underbrace{\boxed{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}}_{\mathbf{Z}}}$$



$$x \equiv \frac{\omega}{\omega_o}$$

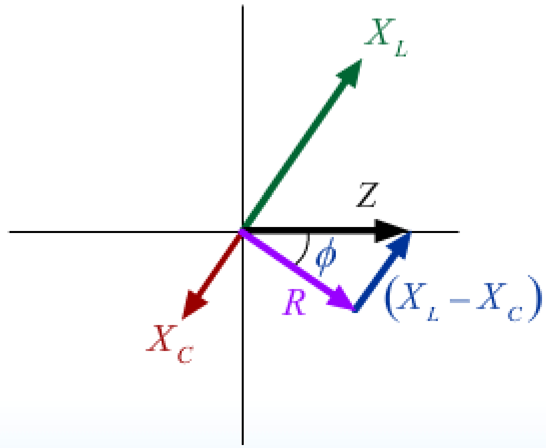
$$Q^2 \equiv \frac{L}{R^2 C}$$

$$Q \equiv 2\pi \frac{U_{\max}}{\Delta U}$$

$$\boxed{I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1 + Q^2 \frac{\left(x^2 - 1\right)^2}{x^2}}}}$$

U_{max} = max energy stored
ΔU = energy dissipated
in one cycle at resonance

Off Resonance



$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

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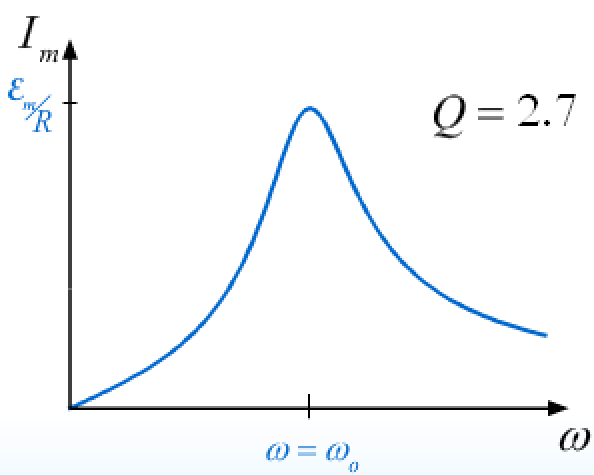
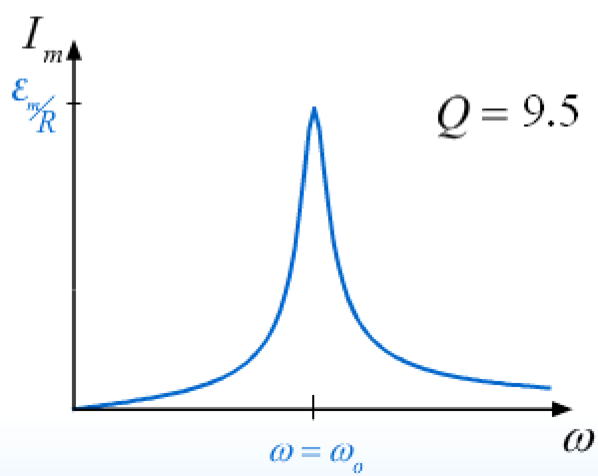
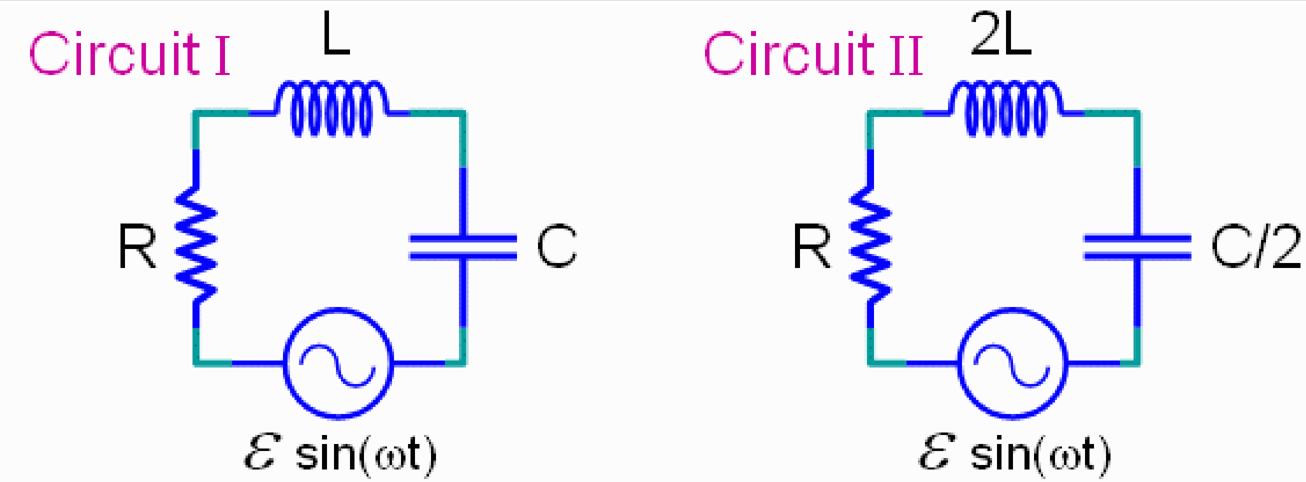




CheckPoint 1a



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

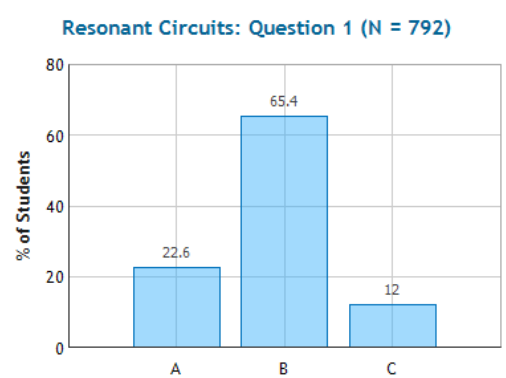
Compare the peak voltage across the resistor in the two circuits

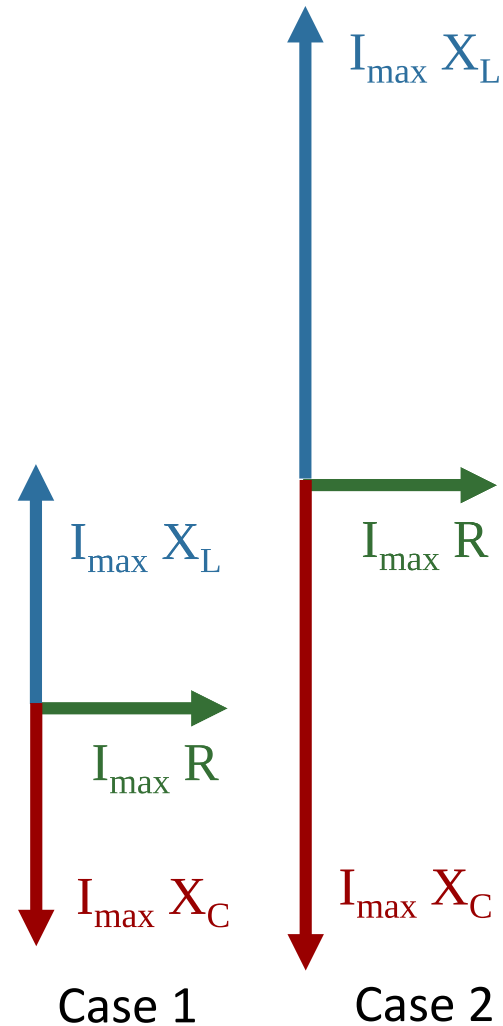
A. $V_I > V_{II}$ $\boxed{\textbf{B. } V_I = V_{II}}$ **C.** $V_I < V_{II}$

Resonance: $X_L = X_C$

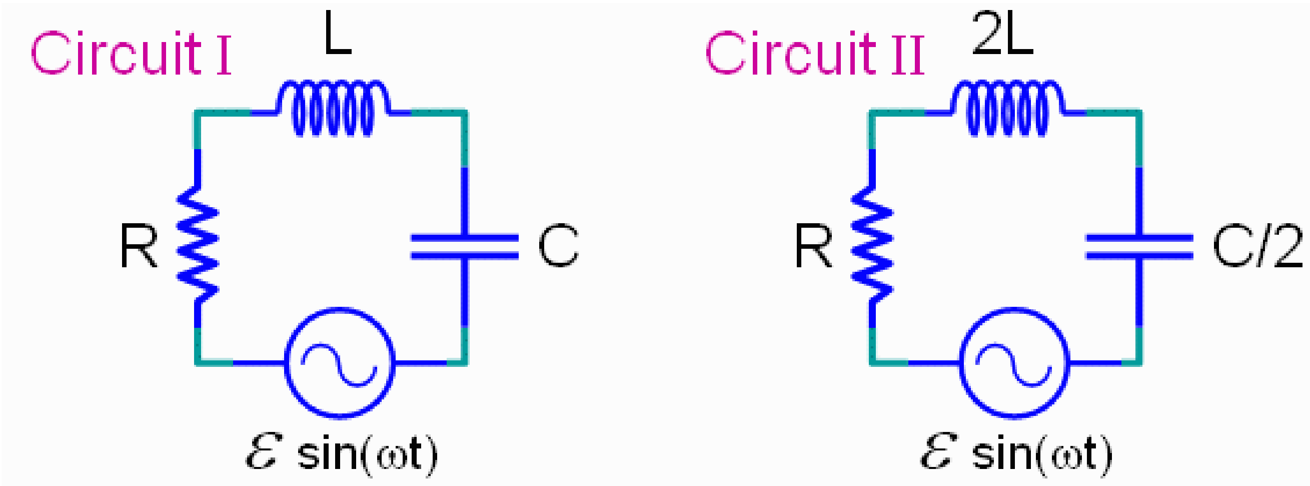
$Z = R$

Same since R doesn't change





CheckPoint 1b

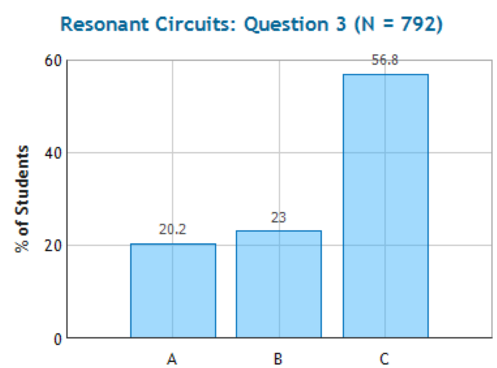


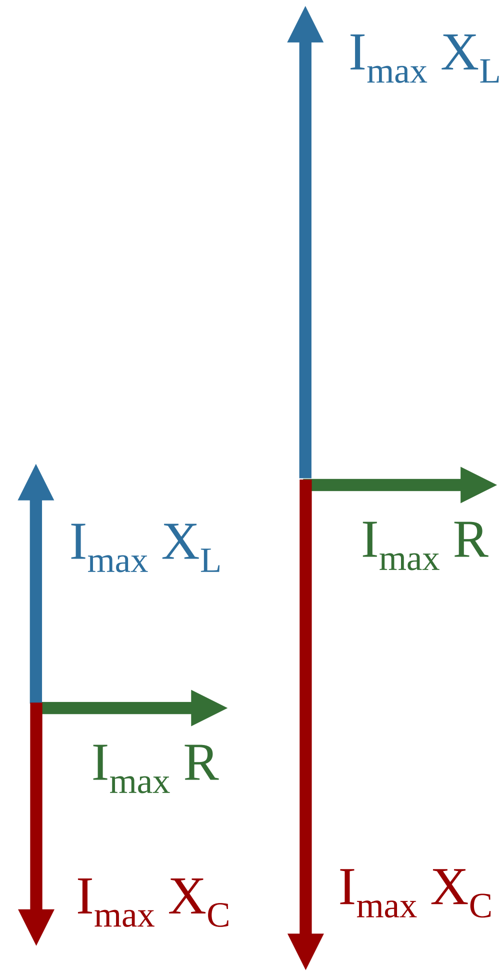
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

Compare the peak voltage <u>across the inductor</u> in the two circuits

A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

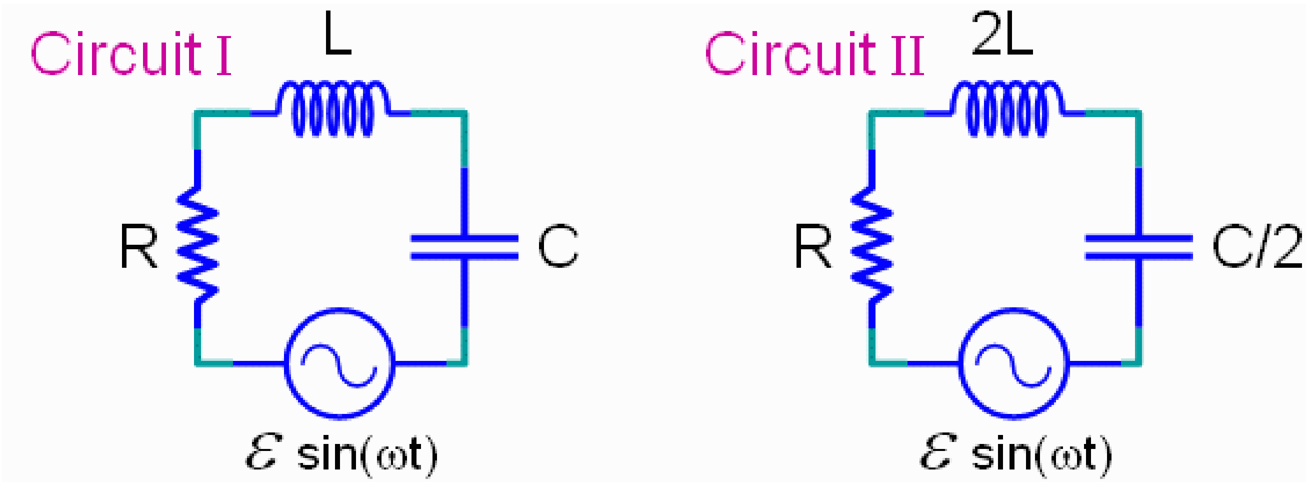
Voltage in second circuit will be twice that of the first because of the $2L$ compared to L.





Case 1 Case 2

CheckPoint 1c

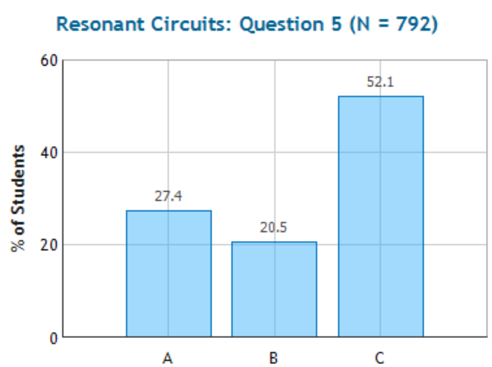


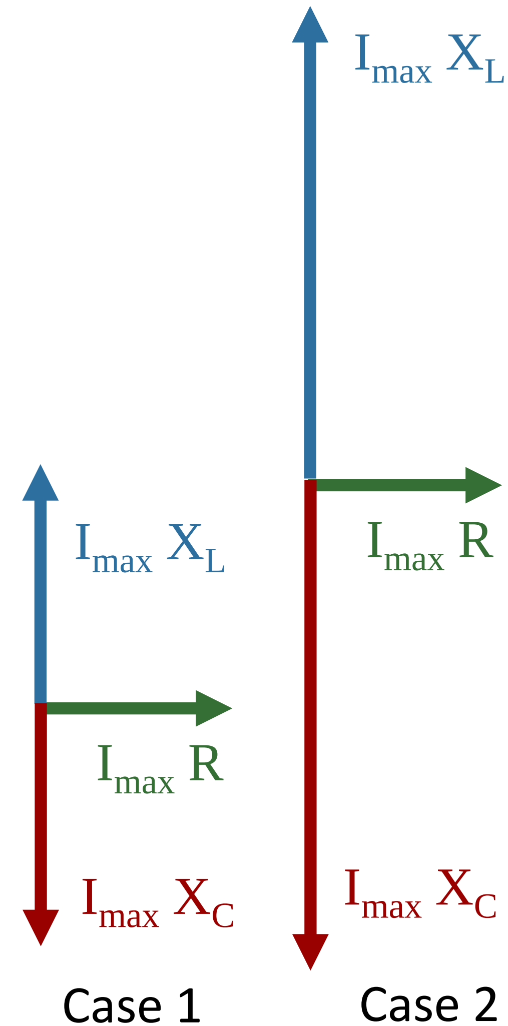
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

Compare the peak voltage across the capacitor in the two circuits

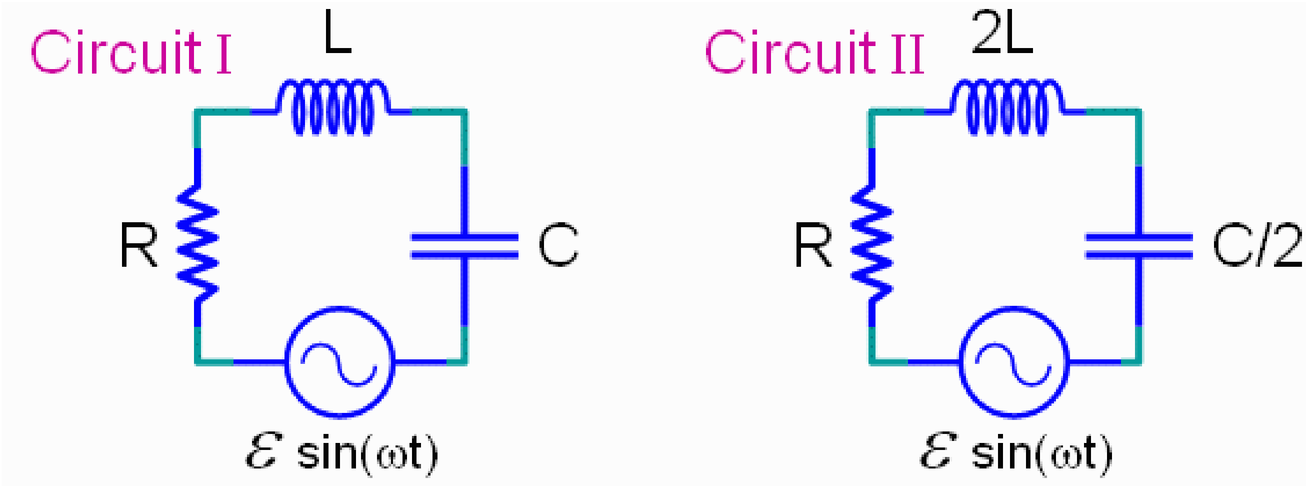
A. $V_I > V_{II}$ **B.** $V_I = V_{II}$ $\boxed{\textbf{C. } V_I < V_{II}}$

The peak voltage will be greater in circuit 2 because the value of X_C doubles.





CheckPoint 1D



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

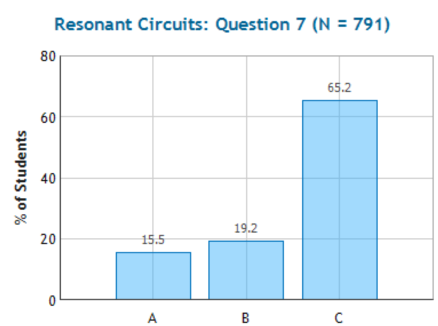
At the resonant frequency, which of the following is true?

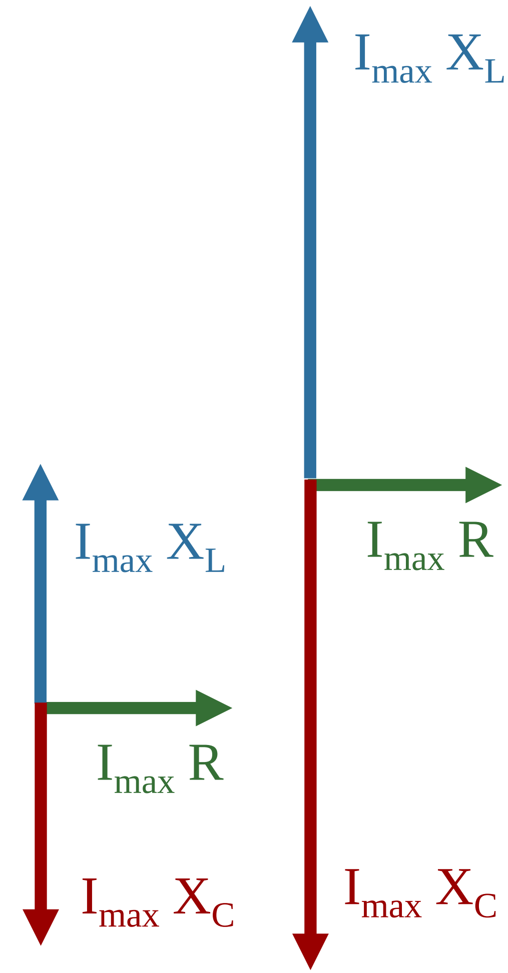
A. Current leads voltage across the generator

B. Current lags voltage across the generator

C. Current is in phase with voltage across the generator

The voltage across the inductor and the capacitor are equal when at resonant frequency, so there is no lag or lead.



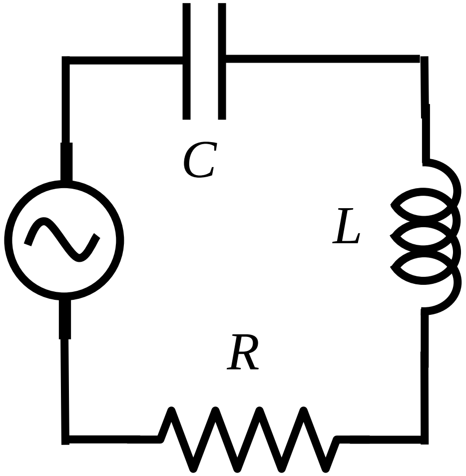


Case 1 Case 2

Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I,V are always in phase!



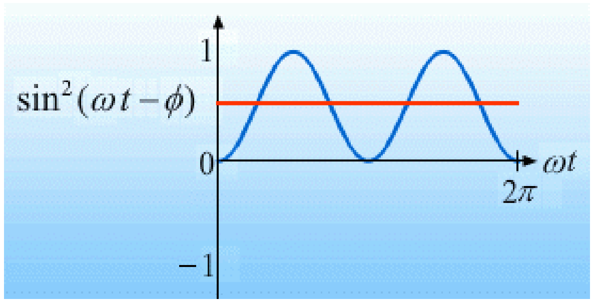
$$P = IV$$
$$= I^2 R$$

Average Power

Inductor and Capacitor $= 0 \quad (< \sin(\omega t) \cos(\omega t) > = 0)$

Resistor

$$<I^2R> = <I^2> R = ½\, I^2_{peak}\, R$$



RMS = Root Mean Square

$I_{peak} = I_{rms}$ sqrt(2)

$\rightarrow$ $\boxed{< I^2 R > = I_{rms}{}^2 R}$

Power Line Calculation

If you want to deliver 1,500 Watts at 100 Volts over transmission lines w/ resistance of 5 Ohms. How much power is lost in the lines?

- Current Delivered: $I = P/V = 15$ Amps
- Loss $= IV$ (on line) $= I^2 R = 15*15 * 5 = 1{,}125$ Watts!

If you deliver 1,500 Watts at 10,000 Volts over the same transmission lines. How much power is lost?

- Current Delivered: $I = P/V = .15$ Amps
- Loss $= IV$ (on line) $= I^2 R = 0.125$ Watts

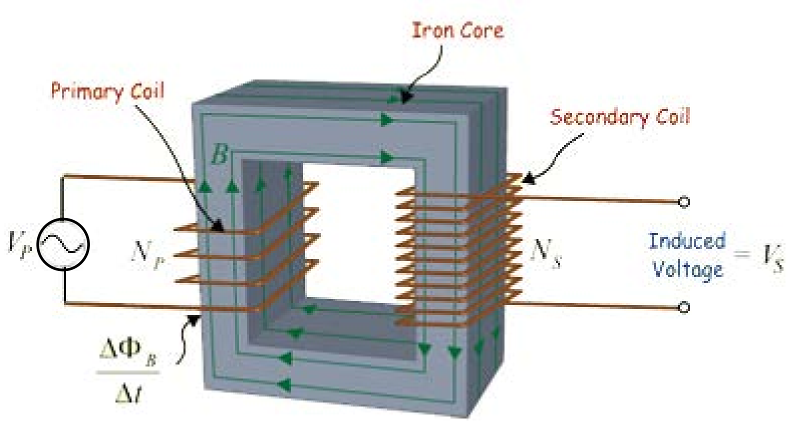
DEMO

Transformers

Application of Faraday's Law

- Changing EMF in Primary creates changing flux
- Changing flux, creates EMF in secondary

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$



Efficient method to change voltage for AC.

Power Transmission Loss $= I^2R$

Power electronics

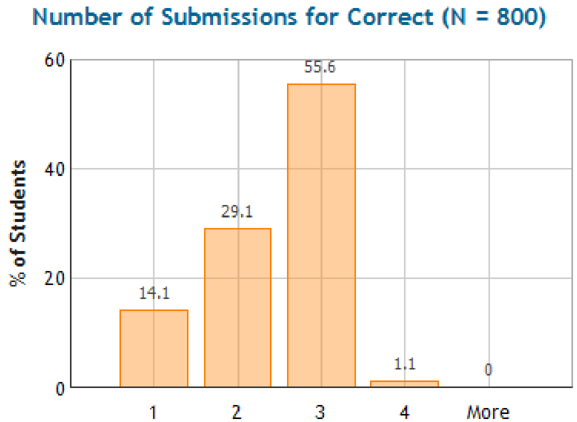
Demo

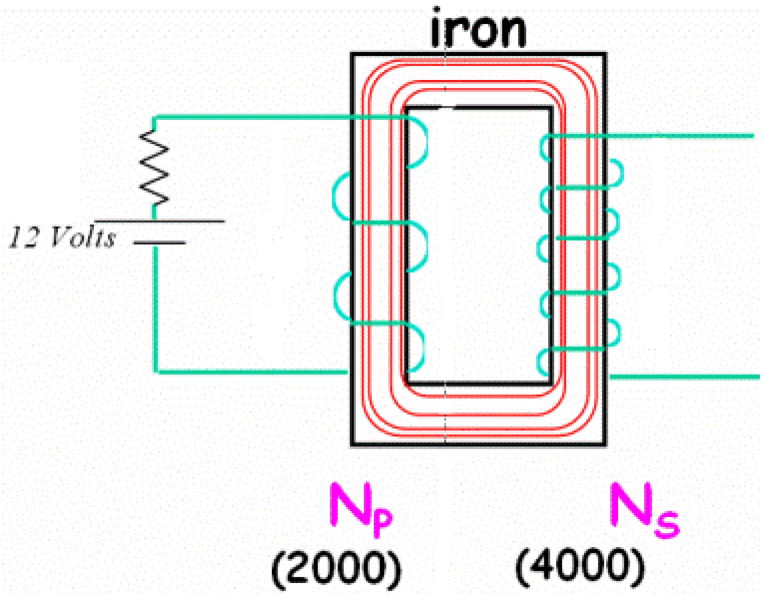
Tranformers

I don't understand the second question of the prelecture. It said something about the changing currents...like changing flux?

For coils connected to battery, what is voltage in secondary?

A) 0 Volts
B) 6 Volts
C) 12 Volts





Wrong Answer: 2 ✗

Feedback: Actually if this was connected to an AC source, the secondary would have twice the primary voltage. However, the battery voltage does not change in time, so after the battery has been connected for a while, there will not be a changing current to create a voltage across the secondary.

Calculation from last lecture

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

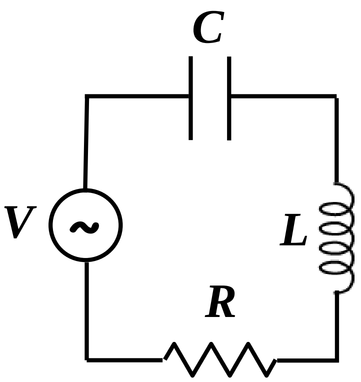
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



Conceptual Analysis

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components

Strategic Analysis

Use V_{max} and I_{max} to determine Z

Use impedance triangle to determine R

Use V_{Cmax} and impedance triangle to determine X_L

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

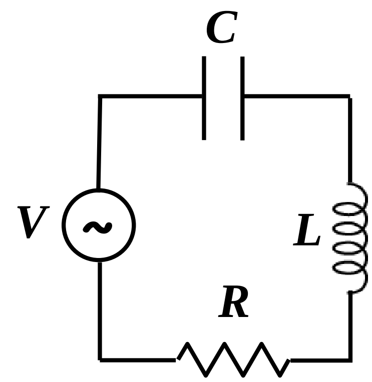
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



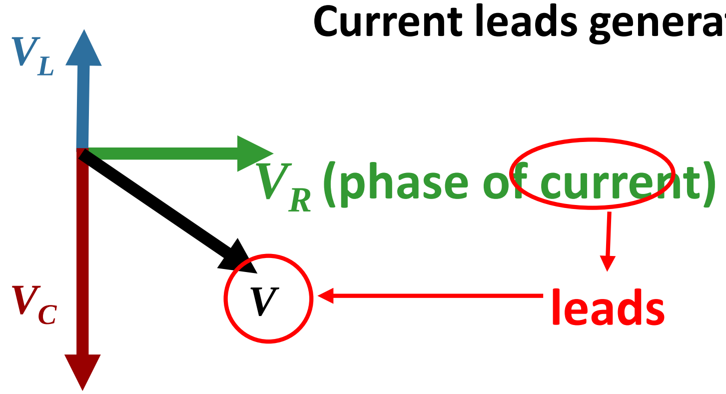
Compare X_L and X_C at this frequency:

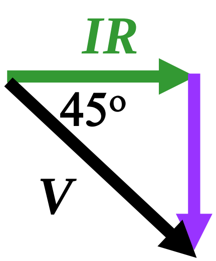
A) $X_L < X_C$ B) $X_L = X_C$ C) $X_L > X_C$ D) Not enough information

This information is determined from the phase

Current leads generator voltage

$$V_L = I_{\max} X_L$$
$$V_C = I_{\max} X_C$$





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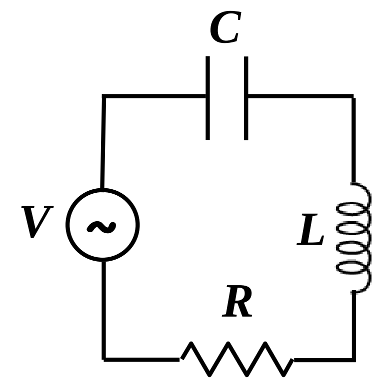

What is Z, the total impedance of the circuit?

A) $70.7\ k\Omega$ B) $\boxed{50\ k\Omega}$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{100V}{2mA} = 50k\Omega$$

Calculation

Consider the harmonically driven series LCR circuit shown.

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$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

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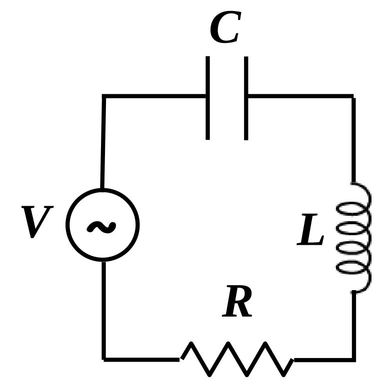


$Z = 50k\Omega$

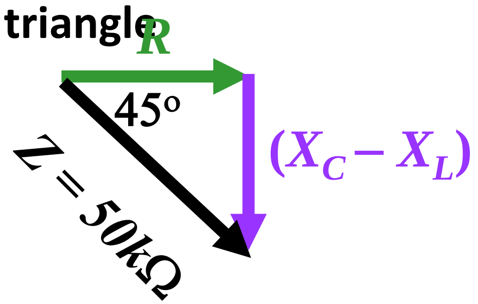
$\sin(45) = .707$

$\cos(45) = .707$

What is R?

A) $70.7\ k\Omega$ B) $50\ k\Omega$ $\boxed{\text{C) } 35.4\ k\Omega}$ D) $21.1\ k\Omega$

Determined from impedance triangle



$$\cos(45) = \frac{R}{Z} \rightarrow R = Z\cos(45^o)$$

$$= 50\ k\Omega \times 0.707$$

$$= 35.4\ k\Omega$$

Calculation

Consider the harmonically driven series LCR circuit shown.

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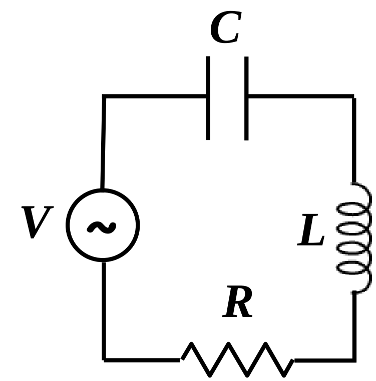
$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?

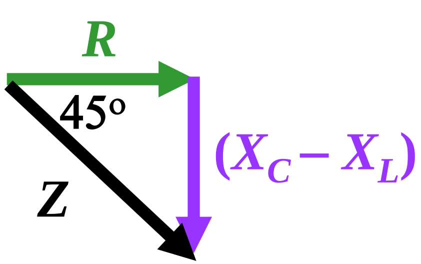


$Z = 50k\Omega$

$R = 35.4k\Omega$

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $\boxed{21.1\ k\Omega}$

We start with the impedance triangle:



$$\frac{X_C - X_L}{R} = \tan 45° = 1 \rightarrow X_L = X_C - R$$

What is X_C?

$$V_{Cmax} = I_{max} X_C$$

$$X_C = \frac{113}{2} = 56.5k\Omega$$

$$X_L = 56.5\ k\Omega - 35.4\ k\Omega$$

Follow-Up from last lecture

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

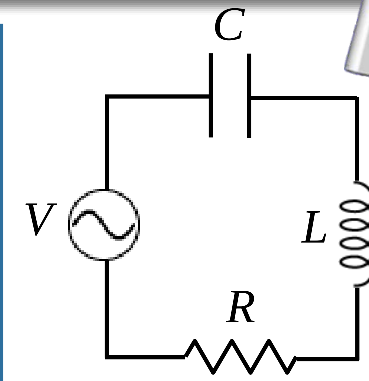
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

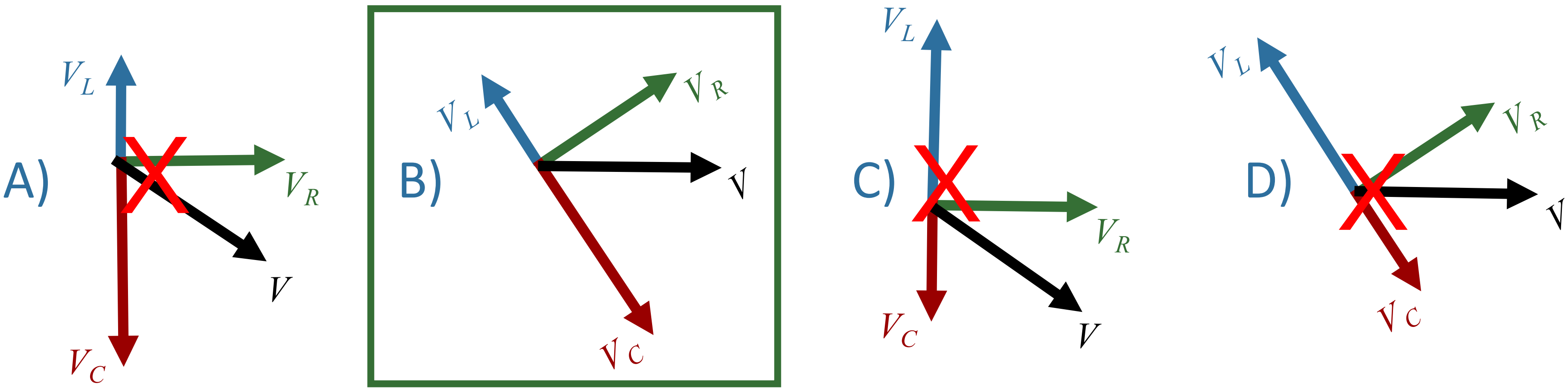
L and R are unknown.

What does the phasor diagram look like at t = 0? (assume $V = V_{max} \sin\omega t$)



$$R = 25\sqrt{2}\ k\Omega$$

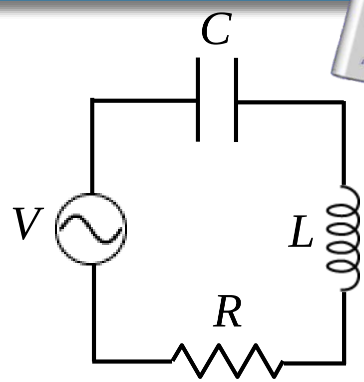
$$X_L = 15\sqrt{2}\ k\Omega$$



$V = V_{max} \sin\omega t$ -> V is horizontal at $t = 0$ ($V = 0$)

$\vec{V} = \vec{V}_L + \vec{V}_C + \vec{V}_R$ → $V_L < V_C$ if current leads generator voltage

Follow-Up from Last Lecture



Consider the harmonically driven series LCR circuit shown.

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$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

L and R are unknown.

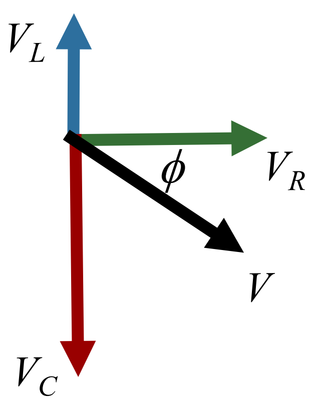
How should we change ω to bring circuit to resonance?

A) decrease ω

B) increase ω

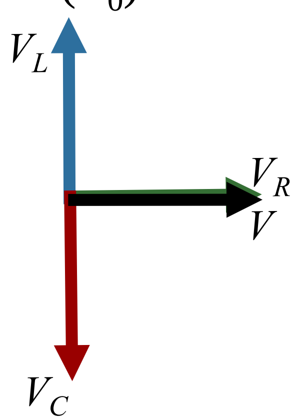
C) Not enough info

Original ω



At resonance

(ω_0)



At resonance

$X_L = X_C$

$\downarrow$

X_L increases

X_C decreases

$\rightarrow$ ω increases