

Your Comments

This material is a little abstract. Can you explain these phase diagrams and how to draw the right triangles to find ϕ in more detail?

Is there ever going to be more than one loop/ more than one of each piece of the circuit?
If so, can we please go through an example?

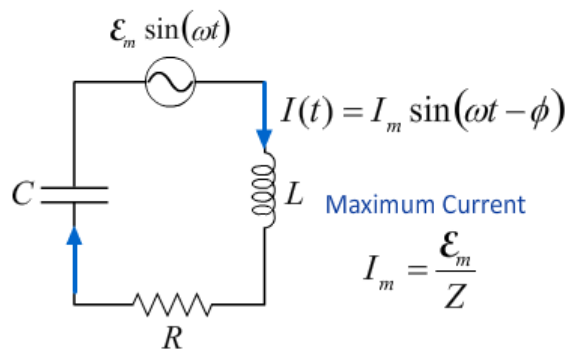
So we just get done with a decently difficult exam and this is what you throw at us??!?!?!?

Between this and the exam, I kinda sorta really, really hate E&M now. And did we really set up a differential equation, examine three separate complex instances, introduce phasers (which confuse the $\int (\mathbf{B} \cdot d\mathbf{A})$ out of me), and graph phasers all to arrive at ... a problem solved by simple geometry and algebra? Not only do I hate E&M, E&M constantly confuses me with how it can be so complex and yet so simple.

I really like the pretty diagrams... I just wish I understood them haha.

Sorry, i didn't really do this prelecture/checkpoint. but i'll come back to it in a few days!
studying for a CS exam :0

This pre-lecture went way over my head. Going over everything in class will definitely help because I need some general explanation of what things are in order to understand the details of the example.



Physics 212

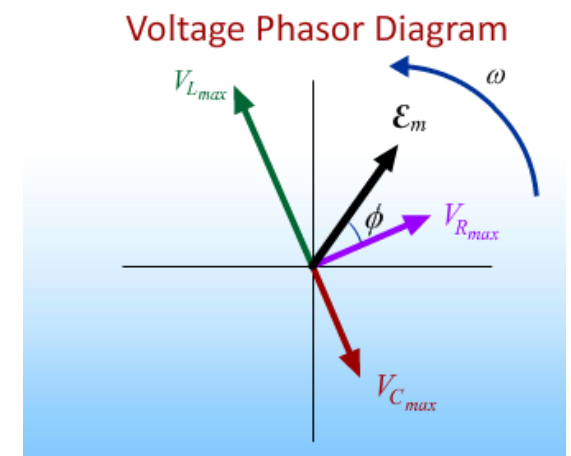
Lecture 20

Today's Concept:

AC Circuits

Maximum currents & voltages

Phasors: A Simple Tool



Big Idea

KVR

$$L \frac{d^2 Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} - \mathcal{E}_m \sin(\omega t) = 0$$

Maximum Values (easy $V=IR$)

$$I_{max} = \mathcal{E}_{max} / Z$$

$$V_{Rmax} = I_{max} R$$

$$V_{Lmax} = I_{max} X_L$$

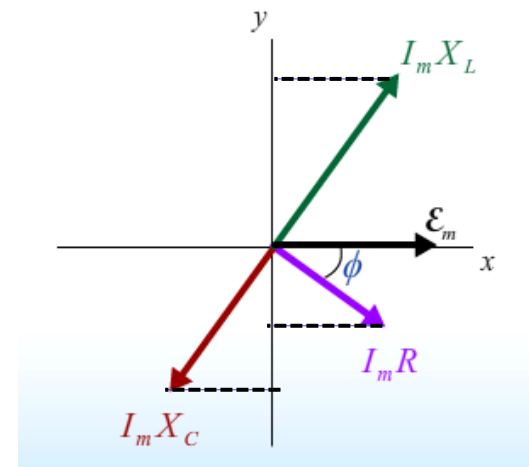
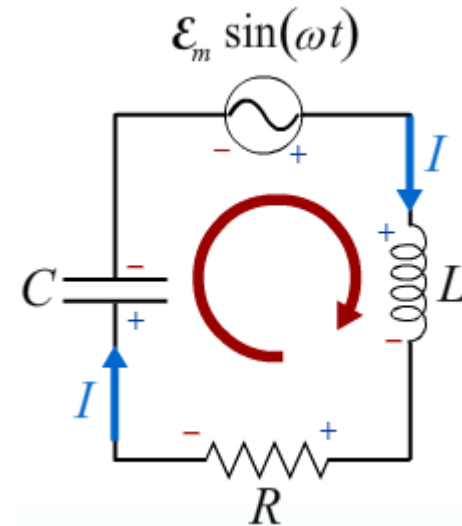
$$V_{Cmax} = I_{max} X_C$$

Value at specific time (phasors)

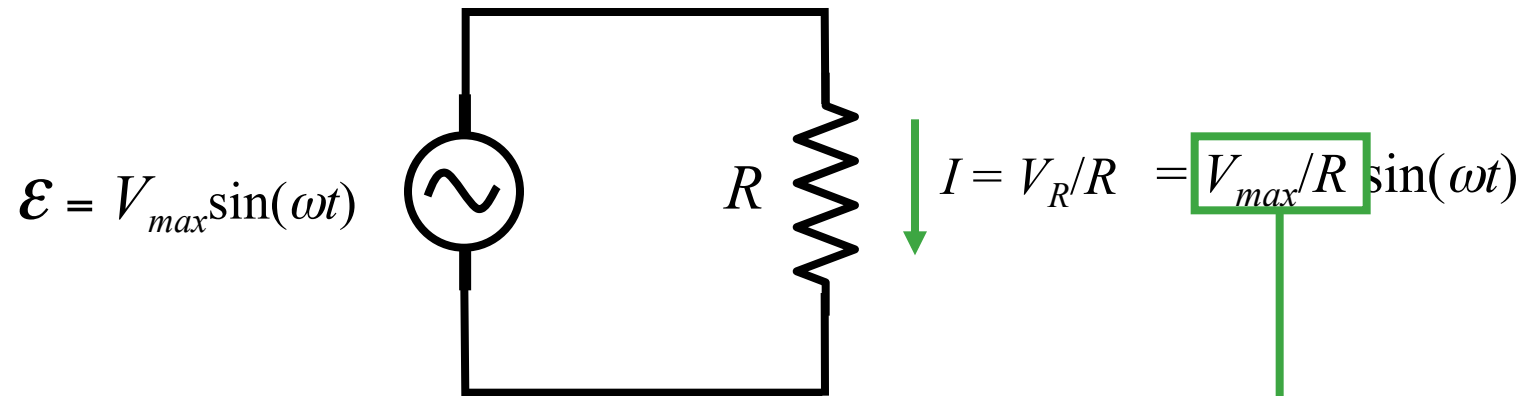
y component gives voltage

V-Inductor Leads current

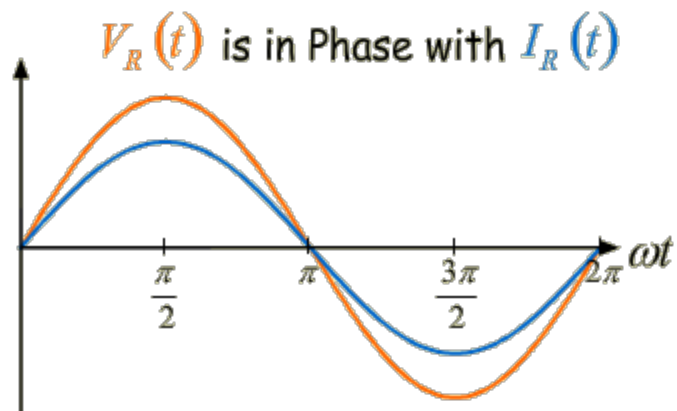
V-Capacitor Lags current



Resistors

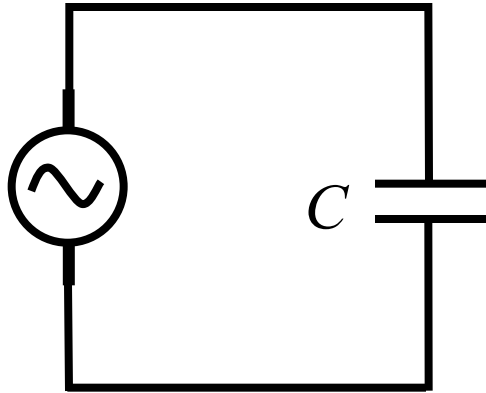


Amplitude = V_{max}/R



Capacitors

$$\mathcal{E} = V_{max}\sin(\omega t)$$



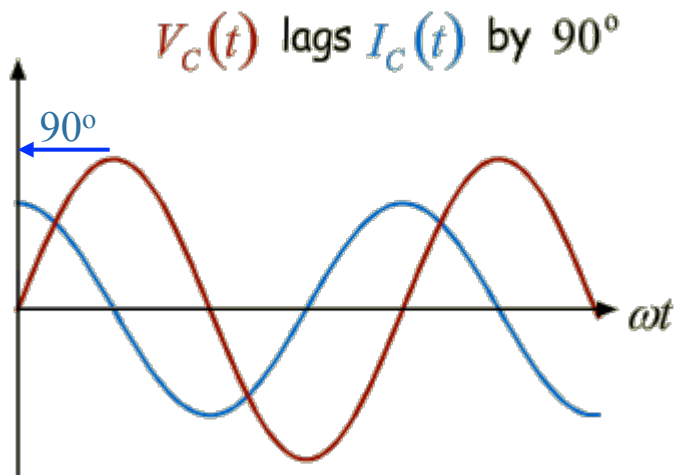
$$Q = CV = CV_{max}\sin(\omega t)$$

$$I = dQ/dt$$

$$I = V_{max}\omega C \cos(\omega t)$$

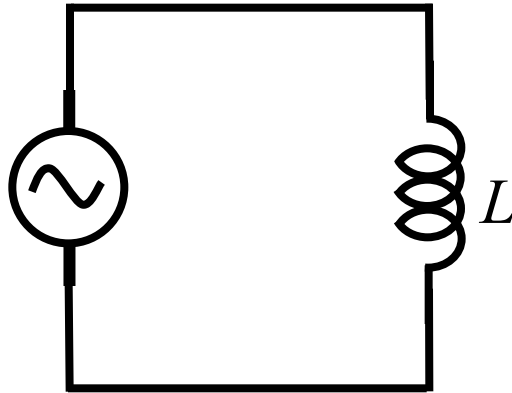
$$\text{Amplitude} = V_{max}/X_C$$

where $X_C = 1/\omega C$
is like the “resistance”
of the capacitor
 X_C depends on ω



Inductors

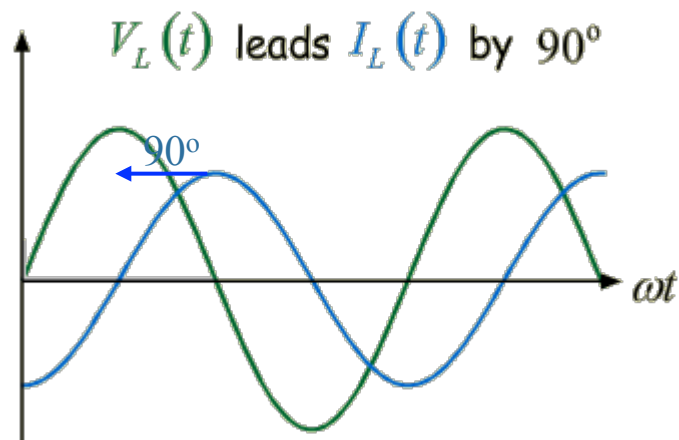
$$\mathcal{E} = V_{max} \sin(\omega t)$$



$$L \, dI/dt = V_L = V_{max} \sin(\omega t)$$

$$I = -V_{max}/\omega L \cos(\omega t)$$

$$\text{Amplitude} = V_{max}/X_L$$

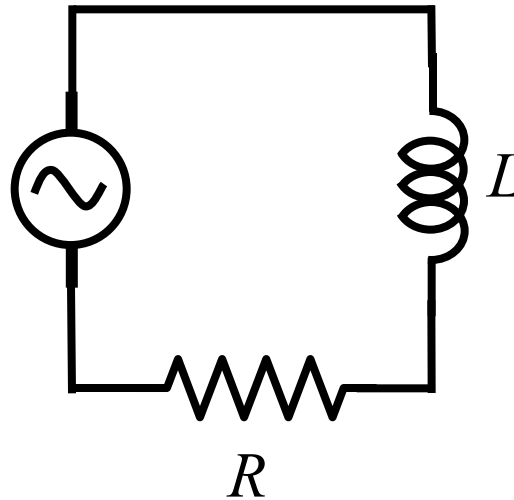


where $X_L = \omega L$
is like the “resistance”
of the inductor
 X_L depends on ω

RL Clicker Question



An RL circuit is driven by an *AC* generator as shown in the figure.



$$X_L = \omega L$$

As $\omega \rightarrow 0$, so does X_L

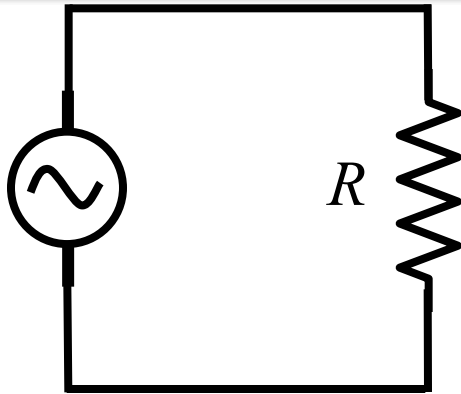


As $\omega \rightarrow 0$,
resistance of circuit R
current gets bigger

For what driving frequency ω of the generator will the current through the resistor be largest

- A) ω large
- B) Current through R doesn't depend on ω
- C) ω small

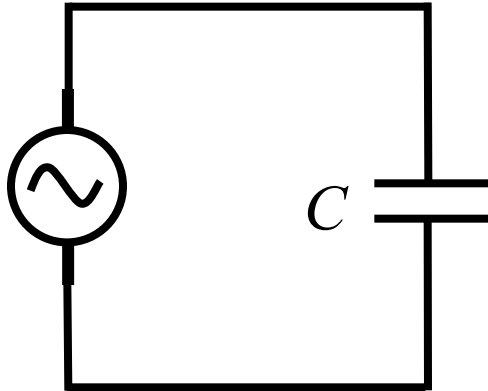
Summary



$$I_{max} = V_{max}/R$$

V_R in phase with I

Because resistors are simple



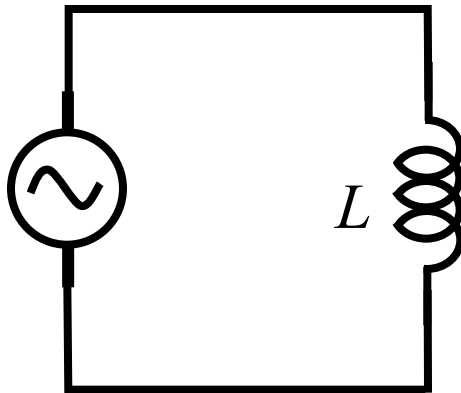
$$I_{max} = V_{max}/X_C$$

$$X_C = 1/\omega C$$

V_C 90° behind I

Current comes first since it charges capacitor

Like a wire at high ω



$$I_{max} = V_{max}/X_L$$

$$X_L = \omega L$$

V_L 90° ahead of I

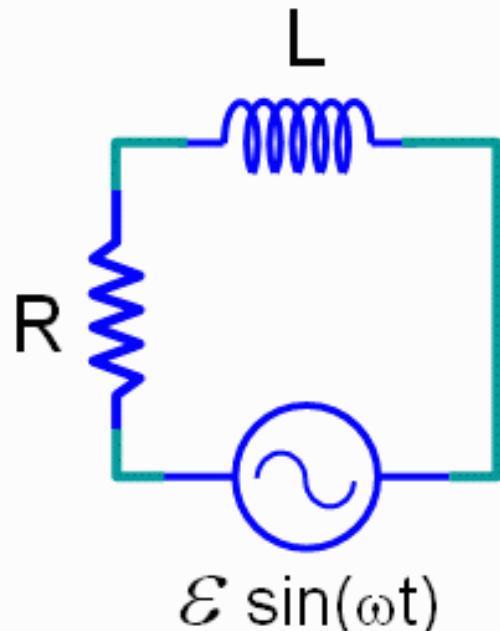
Opposite of capacitor

Like a wire at low ω

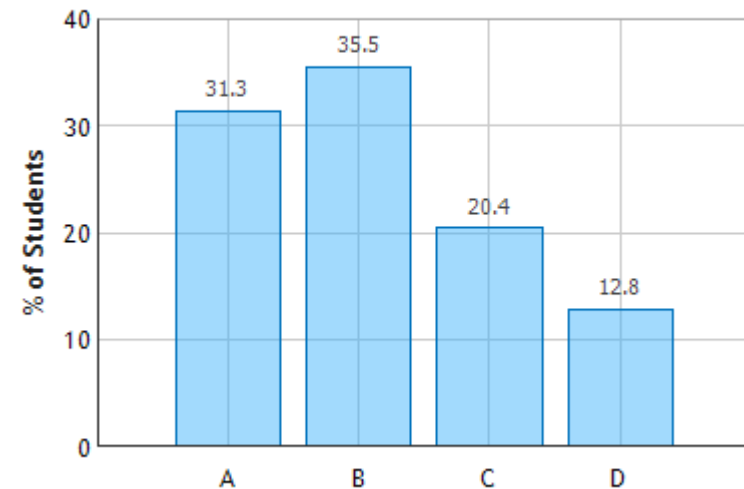
CheckPoint 1c



A RL circuit is driven by an AC generator as shown in the figure.



AC Circuit 1: Question 5 (N = 729)



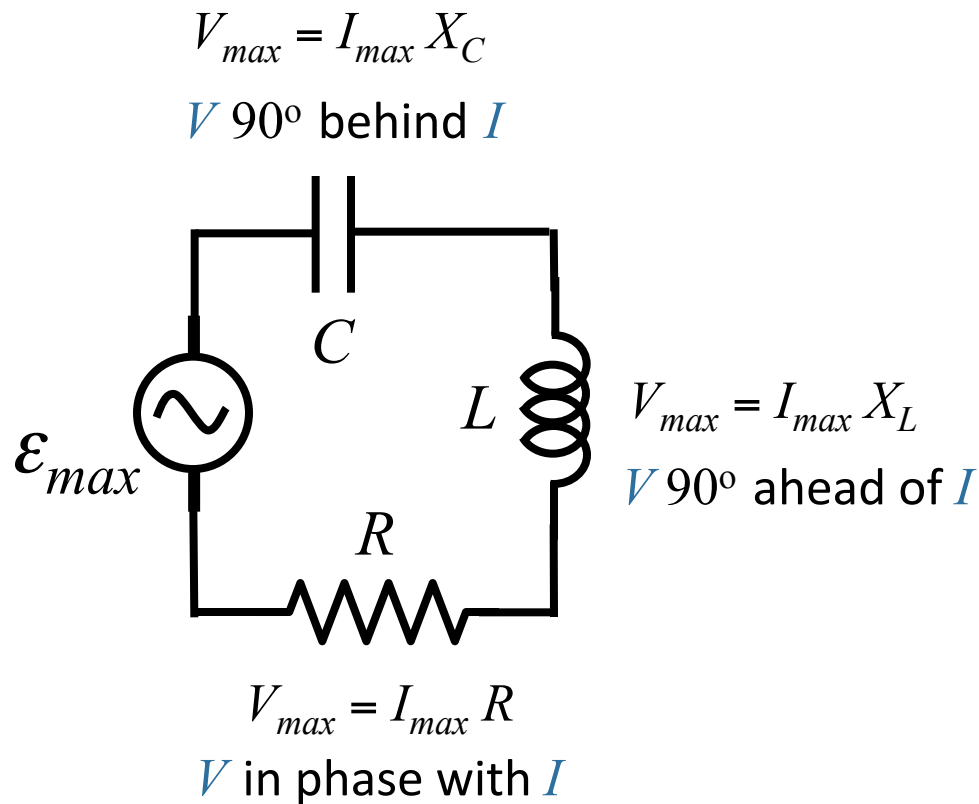
The phase difference between the CURRENT through the resistor and the CURRENT through the inductor is

- A)** Is always zero
- B)** Is always 90°
- C)** Depends on the value of L and R
- D)** Depends on L, R and the generator voltage

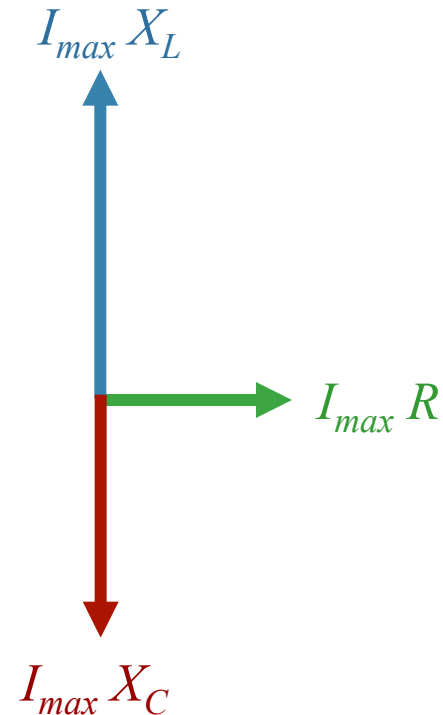
The CURRENT is THE CURRENT
There is only 1 current in this circuit
Same everywhere in circuit

Driven RLC Circuit

Makes sense to write everything in terms of I since this is the same everywhere in a one-loop circuit:



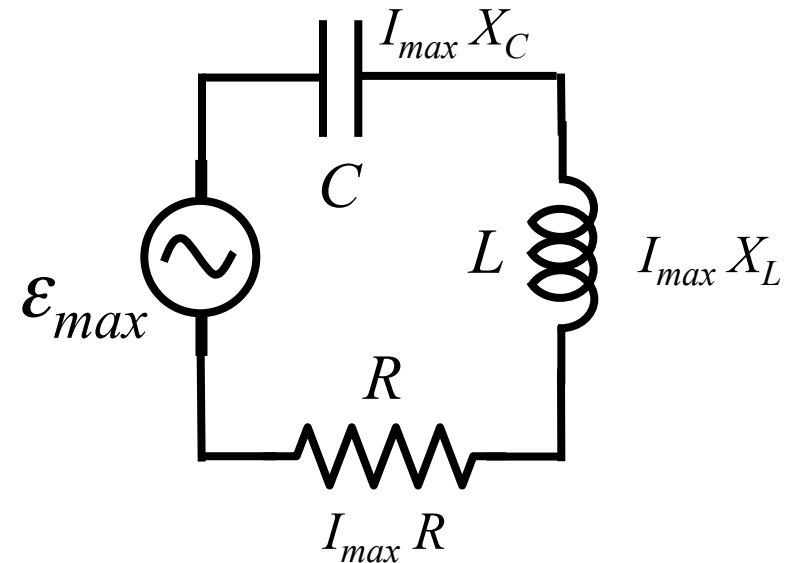
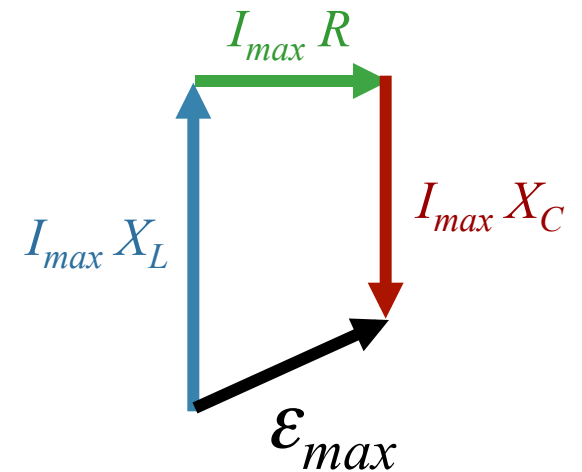
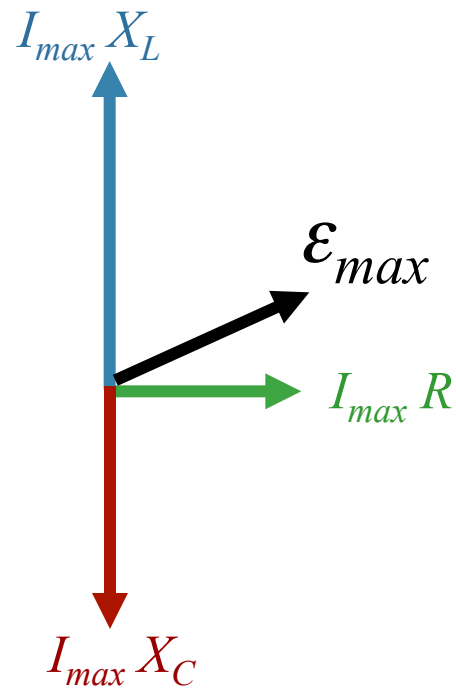
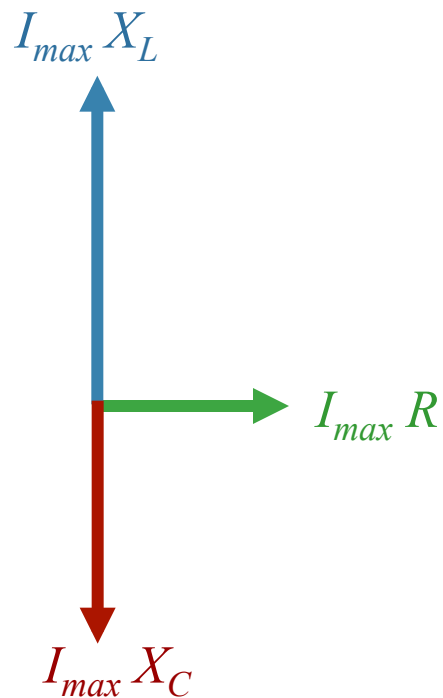
Phasors make this simple to see



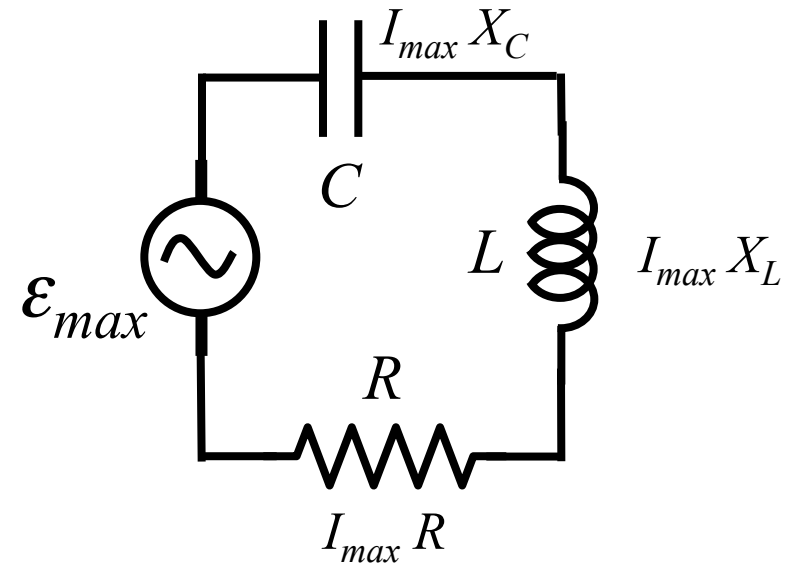
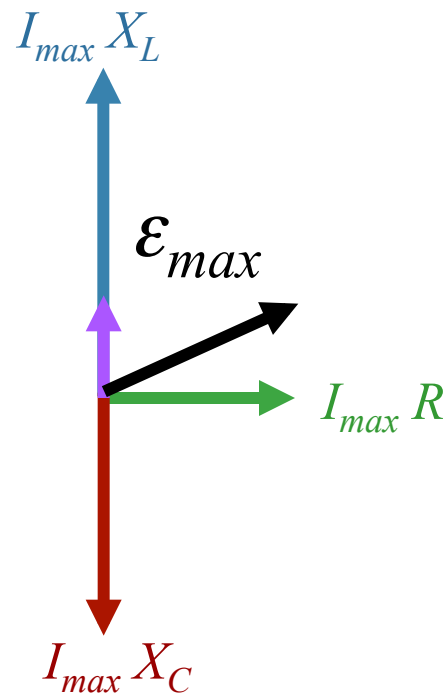
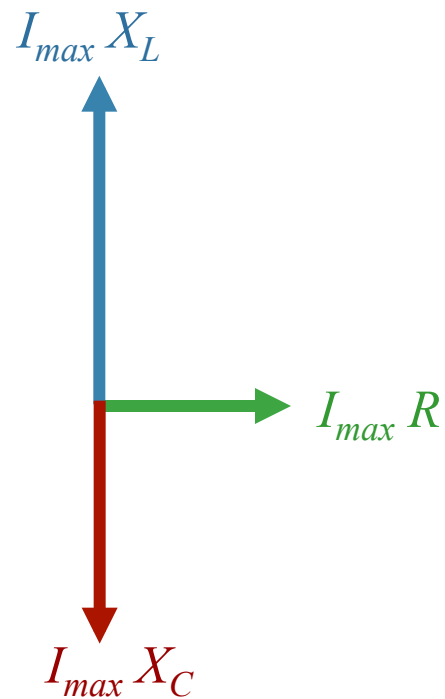
Always looks the same.
Only the lengths will change

The Voltages still Add Up

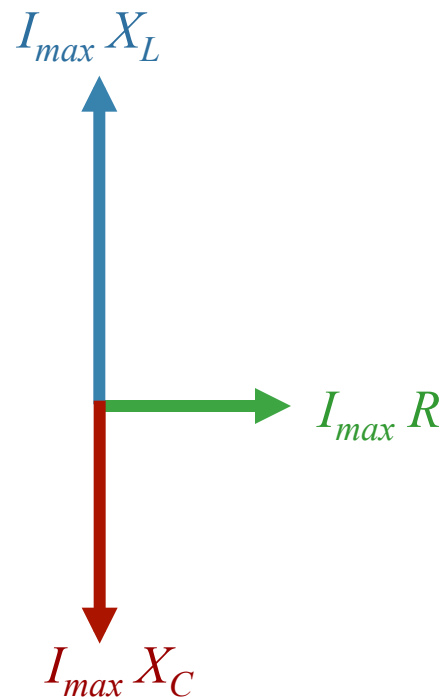
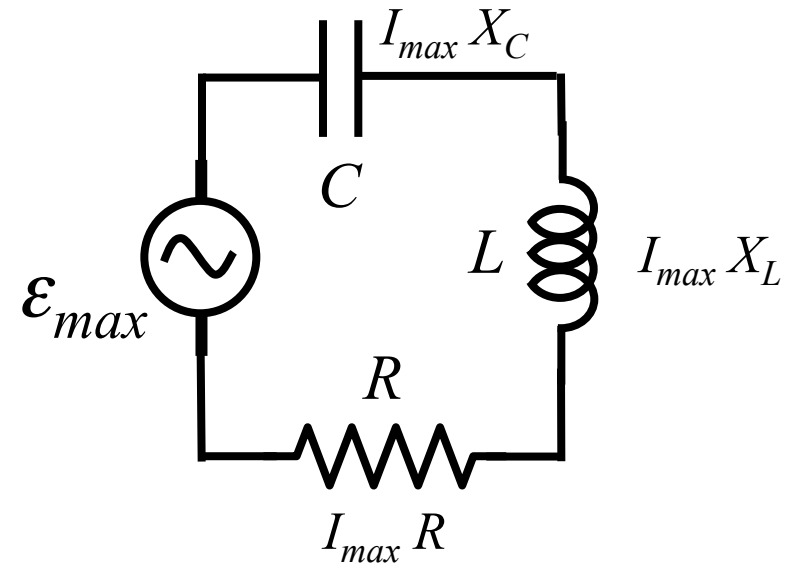
But now we are adding vectors:



Make this Simpler



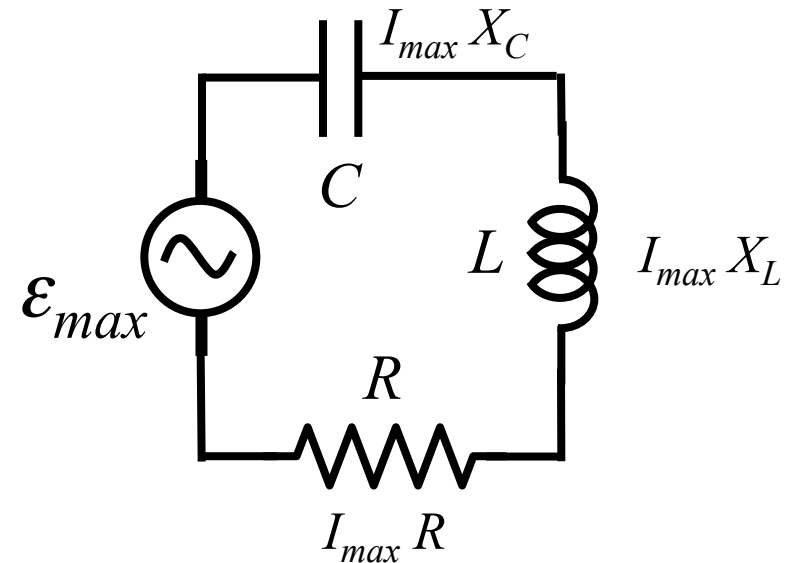
Make this Simpler



$$\mathcal{E}_{max} = I_{max} Z$$

A phasor diagram showing the total voltage \mathcal{E}_{max} as the hypotenuse of a right triangle. The horizontal leg is labeled $I_{max} R$ (green arrow). The vertical leg is labeled $I_{max}(X_L - X_C)$ (purple arrow). The hypotenuse is labeled $\mathcal{E}_{max} = I_{max} Z$ (black arrow).

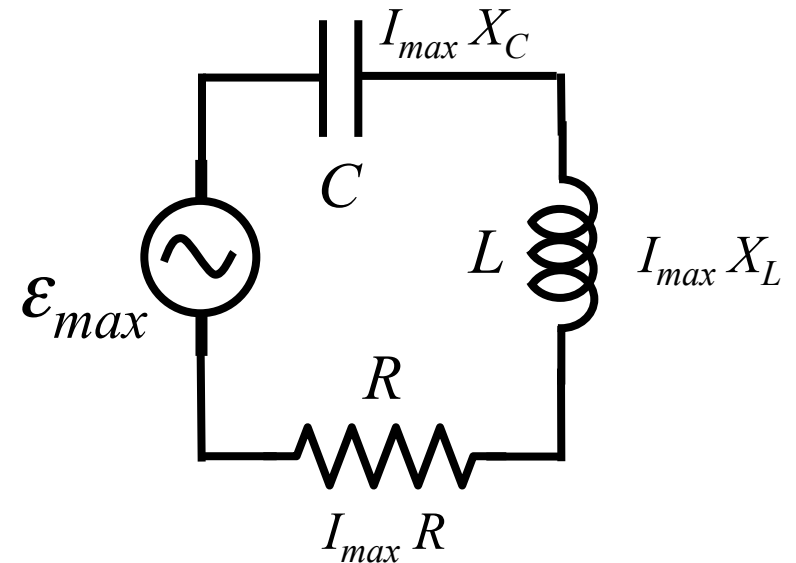
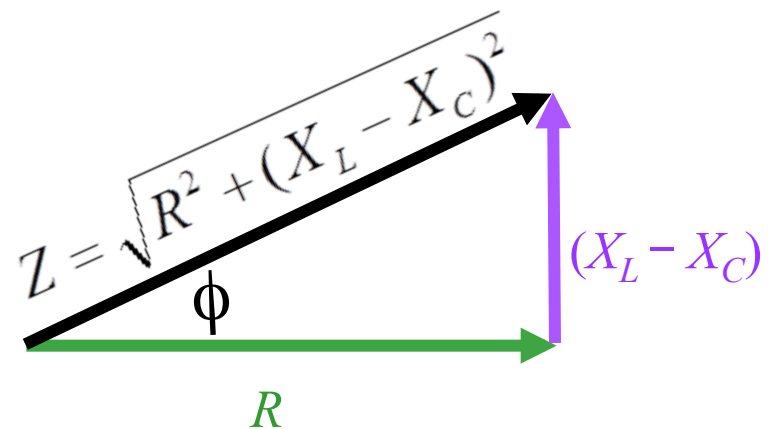
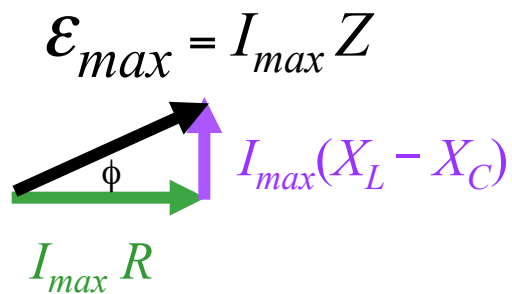
Make this Simpler



$$\mathcal{E}_{max} = I_{max} Z$$

A phasor diagram showing the relationship between the maximum EMF \mathcal{E}_{max} and the maximum current I_{max} . The total EMF \mathcal{E}_{max} is represented by a black vector. The voltage drop across the resistor $I_{max} R$ is represented by a green vector along the horizontal axis. The net voltage drop across the inductor and capacitor $I_{max}(X_L - X_C)$ is represented by a purple vector perpendicular to the green vector.

Make this Simpler



Impedance Triangle

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

Summary

$$V_{Cmax} = I_{max} X_C$$

$$V_{Lmax} = I_{max} X_L$$

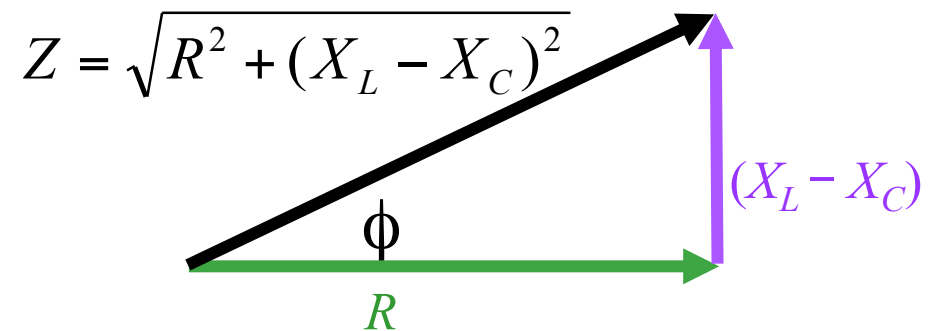
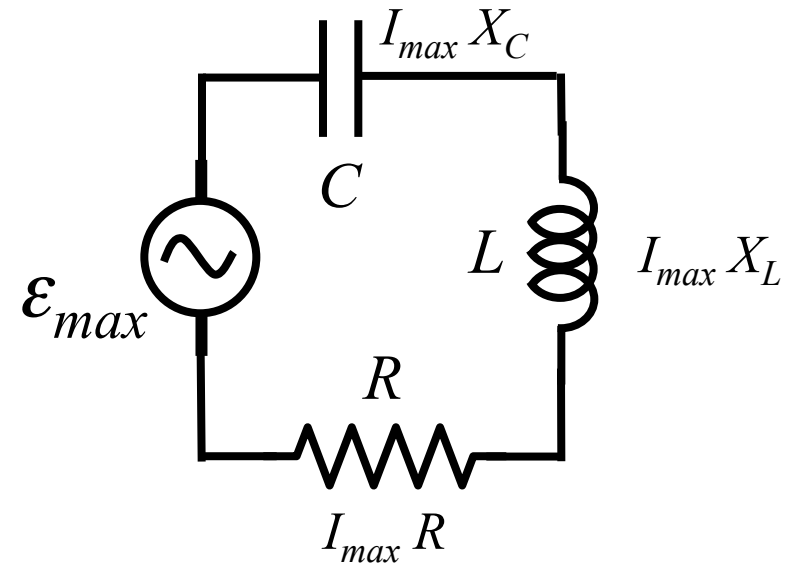
$$V_{Rmax} = I_{max} R$$

$$\mathcal{E}_{max} = I_{max} Z$$

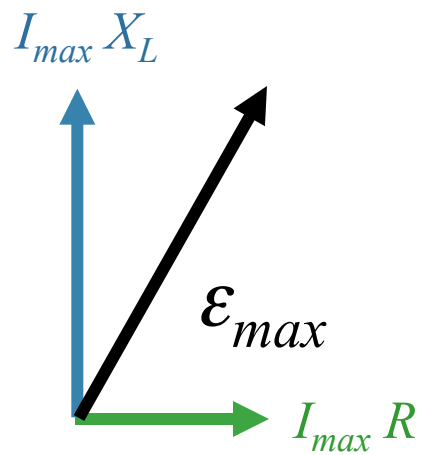
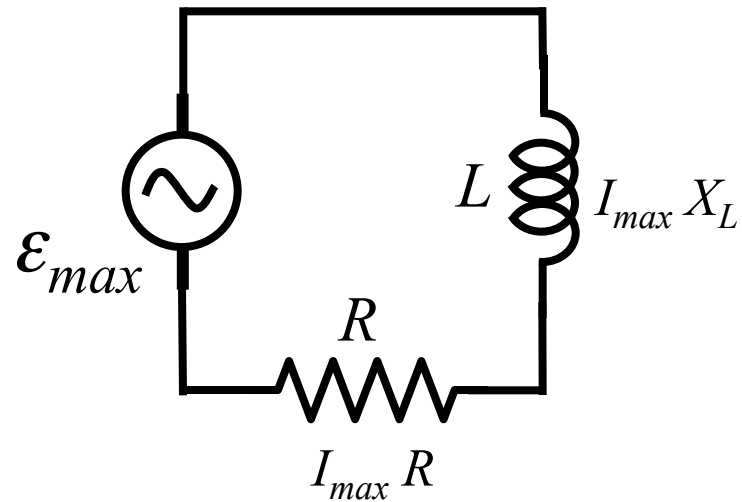
$$I_{max} = \mathcal{E}_{max} / Z$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan(\phi) = \frac{X_L - X_C}{R}$$



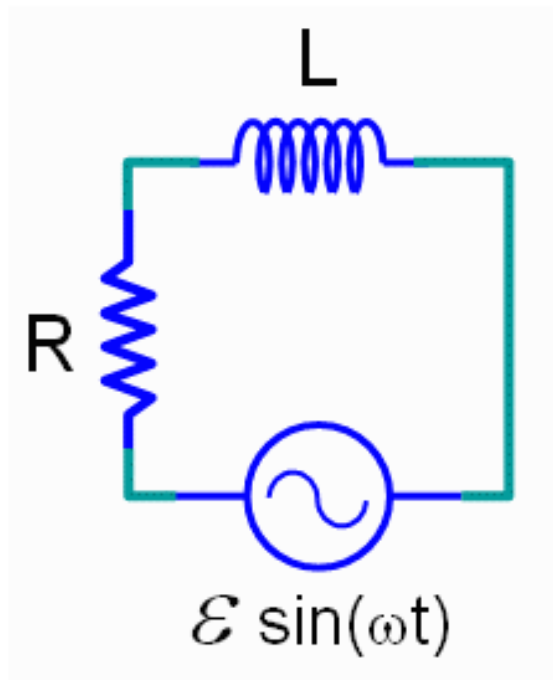
Example: RL Circuit $X_c = 0$



Checkpoint 1a



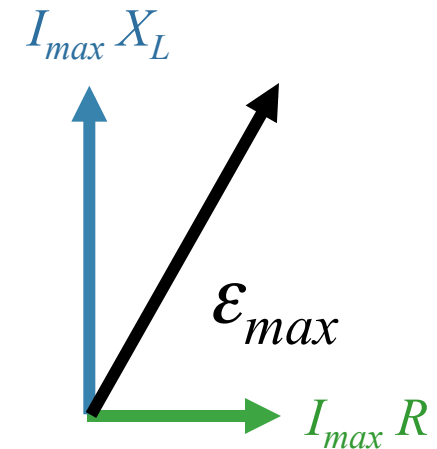
2) A RL circuit is driven by an AC generator as shown in the figure.



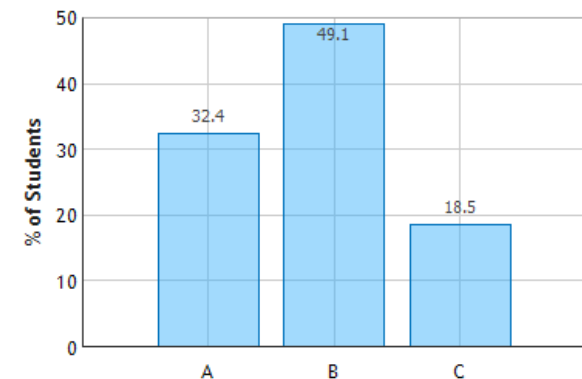
The voltages across the resistor and generator are

- A)** Always out of phase
- B)** Always in phase
- C)** Sometimes in and sometimes out of phase

Draw Voltage Phasors



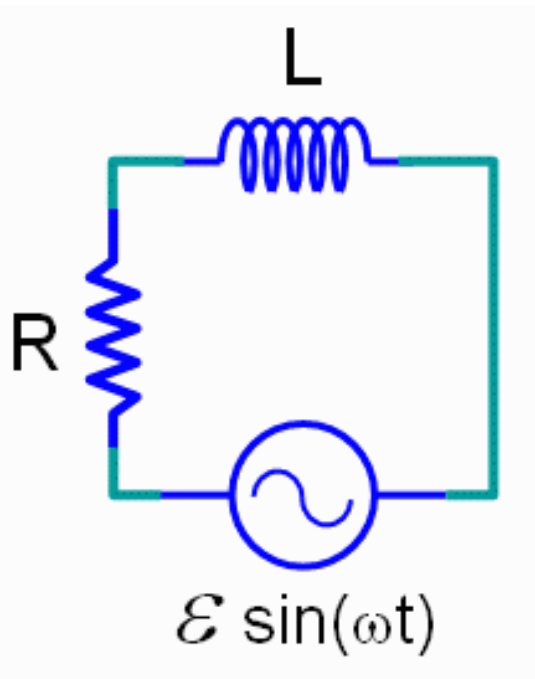
AC Circuit 1: Question 1 (N = 729)



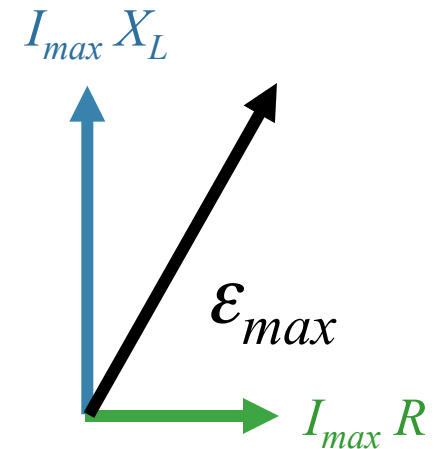
CheckPoint 1b



A RL circuit is driven by an AC generator as shown in the figure.



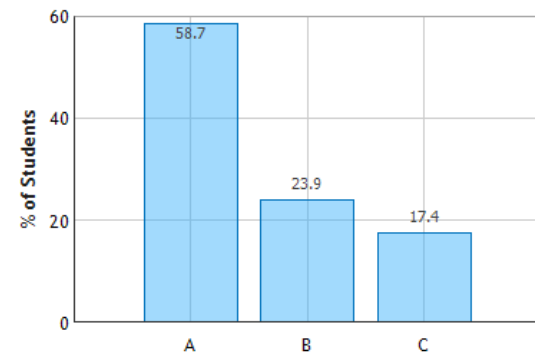
Draw Voltage Phasors



The voltages across the resistor and inductor are

- ☒ **A) Always out of phase** **B) Always in phase**
C) Sometimes in and sometimes out of phase

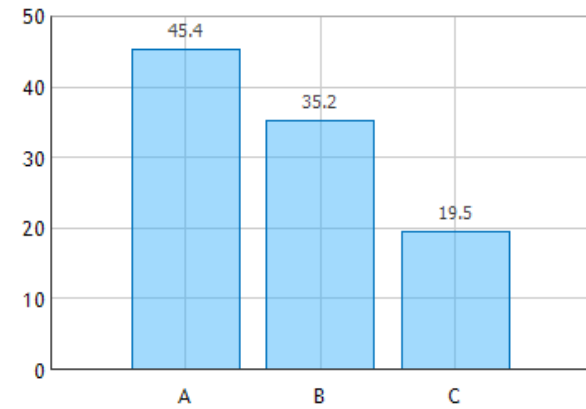
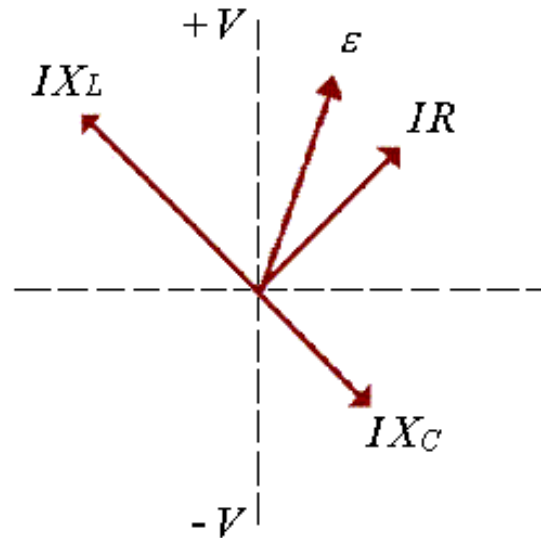
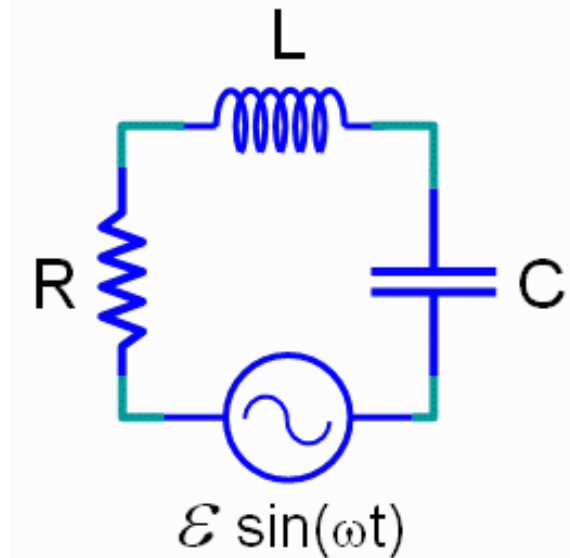
AC Circuit 1: Question 3 (N = 729)



CheckPoint 2a



A driven RLC circuit is represented by the phasor diagram below.



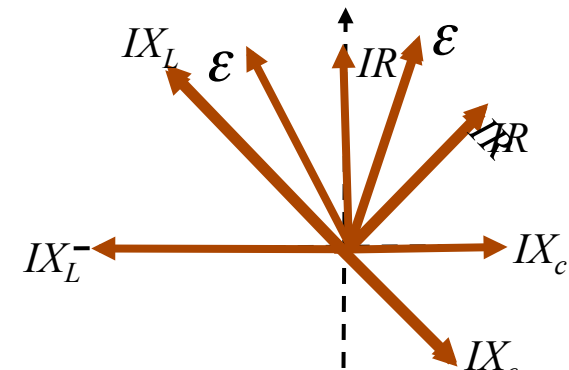
The vertical axis of the phasor diagram represents voltage. When the current through the circuit is maximum, what is the potential difference across the inductor?

A) $V_L = 0$

B) $V_L = V_{L,\max}/2$

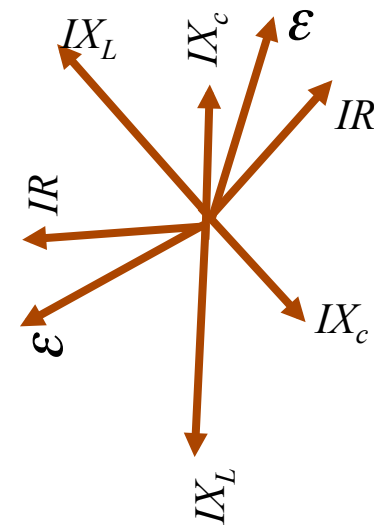
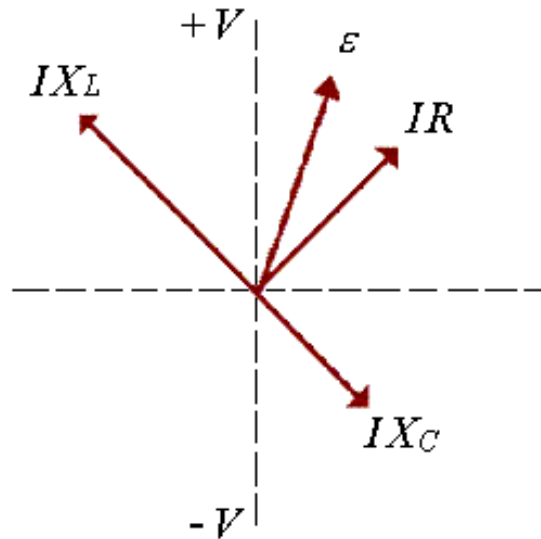
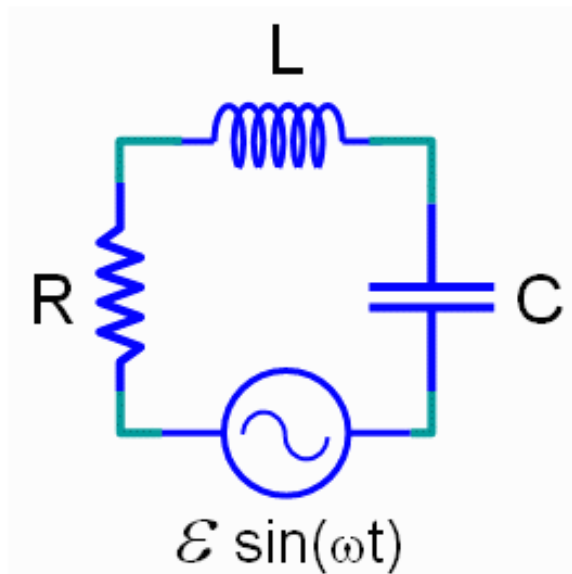
C) $V_L = V_{L,\max}$

What does the voltage phasor diagram look like when the current is a maximum?



Checkpoint 2b

A driven RLC circuit is represented by the phasor diagram below.



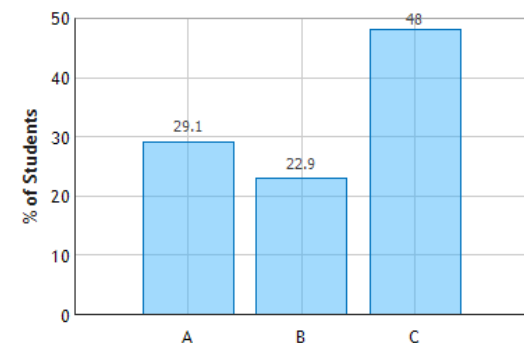
When the capacitor is fully charged, what is the magnitude of the voltage across the inductor?

A) $V_L = 0$

B) $V_L = V_{L,\max}/2$

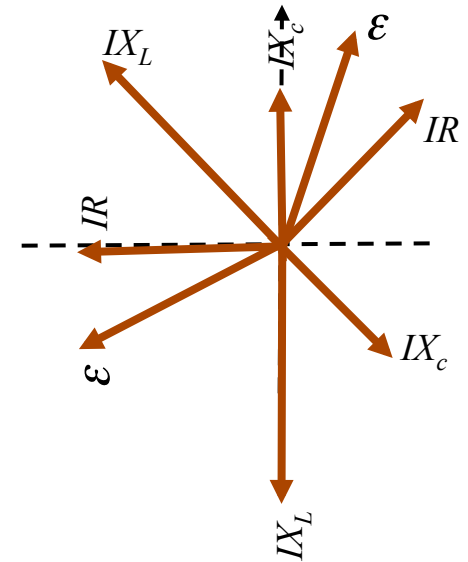
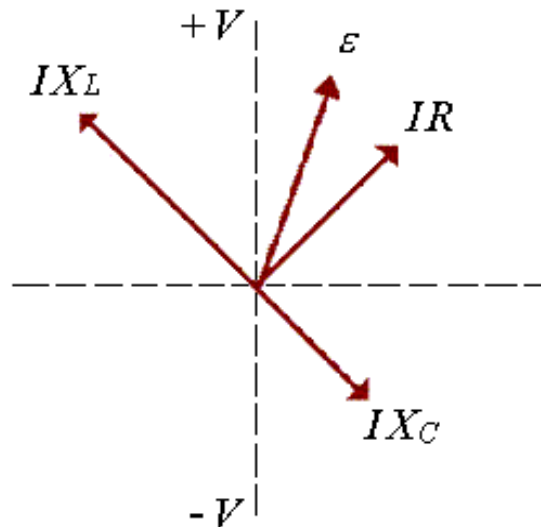
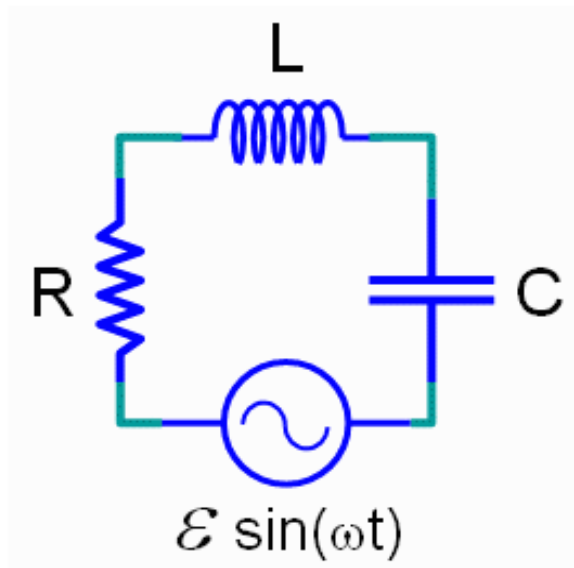
C) $V_L = V_{L,\max}$

What does the voltage phasor diagram look like when the capacitor is fully charged?



Checkpoint 2C

A driven RLC circuit is represented by the phasor diagram below.



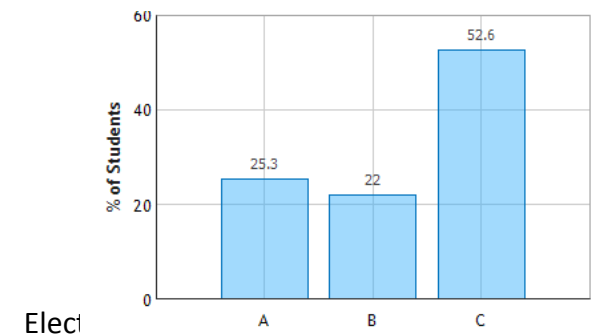
When the voltage across the capacitor is at its positive maximum, $V_C = +V_{C,\max}$, what is the voltage across the inductor?

A) $V_L = 0$

B) $V_L = V_{L,\max}$

C) $V_L = -V_{L,\max}$

What does the voltage phasor diagram look like when the voltage across capacitor is at its positive maximum?



Calculation

Consider the harmonically driven series LCR circuit shown.

$$V_{max} = 100 \text{ V}$$

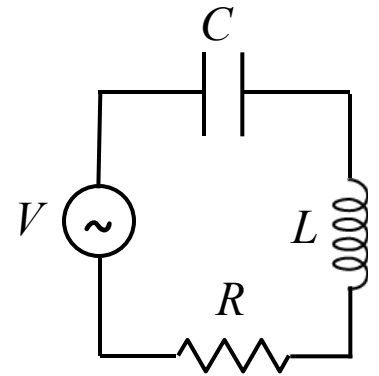
$$I_{max} = 2 \text{ mA}$$

$$V_{Cmax} = 113 \text{ V}$$

The current leads generator voltage by 45°

L and R are unknown.

What is X_L , the reactance of the inductor, at this frequency?



Conceptual Analysis

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components

Strategic Analysis

Use V_{max} and I_{max} to determine Z

Use impedance triangle to determine R

Use V_{Cmax} and impedance triangle to determine X_L

Calculation



Consider the harmonically driven series *LCR* circuit shown.

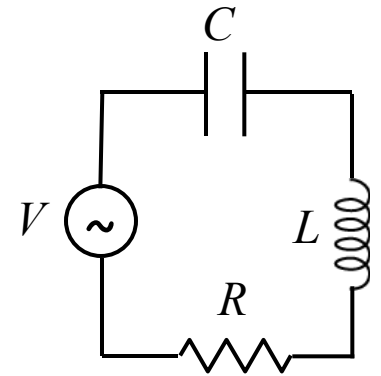
$$V_{max} = 100 \text{ V}$$

$$I_{max} = 2 \text{ mA}$$

$$V_{Cmax} = 113 \text{ V}$$

The current leads generator voltage by 45°

L and *R* are unknown.



What is X_L , the reactance of the inductor, at this frequency?

Compare X_L and X_C at this frequency:

A) $X_L < X_C$

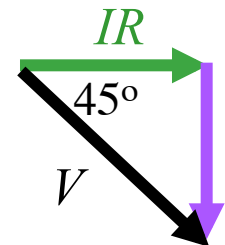
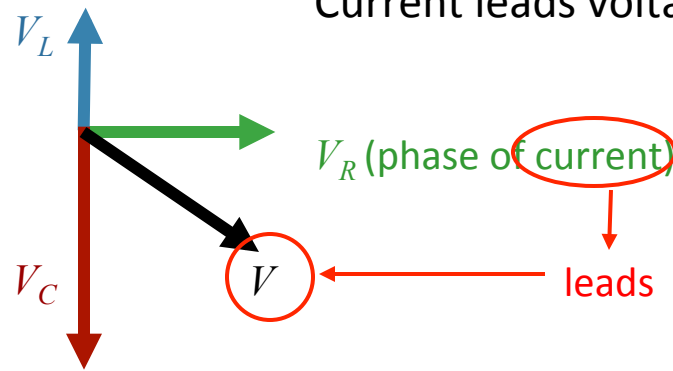
B) $X_L = X_C$

C) $X_L > X_C$

D) Not enough information

This information is determined from the phase
Current leads voltage

$$V_L = I_{max} X_L$$
$$V_C = I_{max} X_C$$



Calculation



Consider the harmonically driven series *LCR* circuit shown.

$$V_{\max} = 100 \text{ V}$$

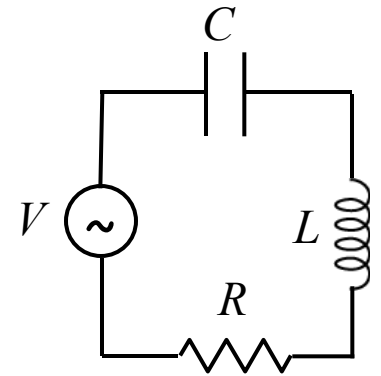
$$I_{\max} = 2 \text{ mA}$$

$$V_{C\max} = 113 \text{ V}$$

The current leads generator voltage by 45°

L and *R* are unknown.

What is X_L , the reactance of the inductor, at this frequency?



What is *Z*, the total impedance of the circuit?

A) $70.7 \text{ k}\Omega$

B) $50 \text{ k}\Omega$

C) $35.4 \text{ k}\Omega$

D) $21.1 \text{ k}\Omega$

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{100\text{V}}{2\text{mA}} = 50\text{k}\Omega$$

Calculation



Consider the harmonically driven series *LCR* circuit shown.

$$V_{max} = 100 \text{ V}$$

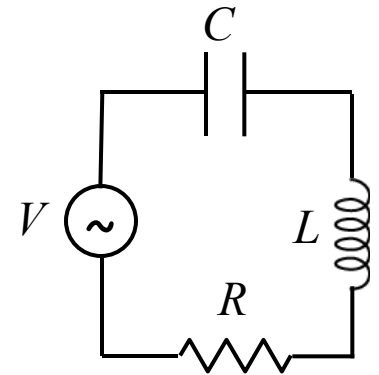
$$I_{max} = 2 \text{ mA}$$

$$V_{Cmax} = 113 \text{ V}$$

The current leads generator voltage by 45°

L and *R* are unknown.

What is X_L , the reactance of the inductor, at this frequency?



$$Z = 50 \text{ k}\Omega$$

$$\sin(45) = .707$$

$$\cos(45) = .707$$

What is *R*?

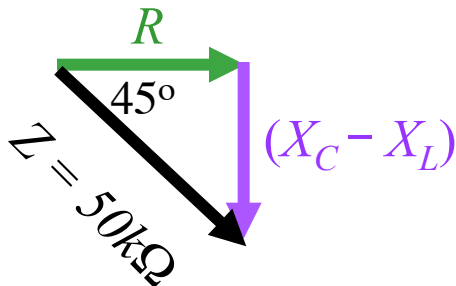
A) $70.7 \text{ k}\Omega$

B) $50 \text{ k}\Omega$

C) $35.4 \text{ k}\Omega$

D) $21.1 \text{ k}\Omega$

Determined from impedance triangle



$$\begin{aligned} \cos(45) &= \frac{R}{Z} \quad \longrightarrow \quad R = Z \cos(45^\circ) \\ &= 50 \text{ k}\Omega \times 0.707 \\ &= 35.4 \text{ k}\Omega \end{aligned}$$

Calculation



Consider the harmonically driven series *LCR* circuit shown.

$$V_{max} = 100 \text{ V}$$

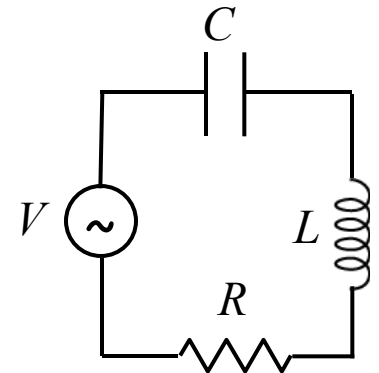
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The current leads generator voltage by 45°

L and *R* are unknown.

What is X_L , the reactance of the inductor, at this frequency?



$$Z = 50 \text{ k}\Omega$$

$$R = 35.4 \text{ k}\Omega$$

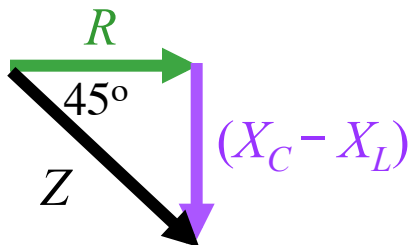
A) $70.7 \text{ k}\Omega$

B) $50 \text{ k}\Omega$

C) $35.4 \text{ k}\Omega$

D) $21.1 \text{ k}\Omega$

We start with the impedance triangle:



$$\frac{X_C - X_L}{R} = \tan 45^\circ = 1 \quad \longrightarrow \quad X_L = X_C - R$$

What is X_C ?

$$V_{Cmax} = I_{max} X_C$$

$$X_C = \frac{113}{2} = 56.5 \text{ k}\Omega$$

$$X_L = 56.5 \text{ k}\Omega - 35.4 \text{ k}\Omega$$