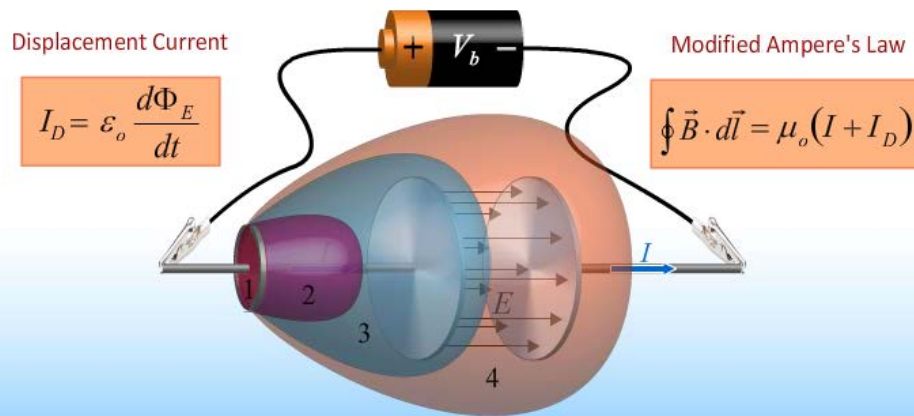


# Physics 212

## Lecture 22

### DISPLACEMENT CURRENT and EM WAVES



# Your Comments

Could you explain why  $E_0/B_0=c$ (the speed of light)?

Okay, this is getting ridiculous; you can't just throw a bunch of equations at me with a lightning-fast explanation and expect me to understand what is going on beyond a very superficial level.

It hurt... it all hurt... man down

I have not learn anything about differential equations. It is really hard for me to understand all the things in pre lecture.

Frankly, the prelecture is just becoming annoying. I cannot focus on the material since I don't know what the heck is going on. Your lecture clarifies everything though.

A photon checks into a hotel. The bell boy asks if he needs any help with his bags. The photon says, "No Thanks, I'm traveling light"

The answer to my question about the title "Maxwell's Equations"!! I realize now why you didn't answer at the time, because it was coming up. What a relief to finally know. Also, this prelecture was difficult for me conceptually. But i think the lecture should straighten most of it out. Im just not sure what the take away is right now.

Hardest pre-lecture so far. The formulas and graphs don't mean anything to me. Please make it mean something!

# What We Knew Before Prelecture 22

## Pre-MAXWELL'S EQUATIONS

Gauss' Law for E Fields

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_o}$$

Gauss' Law for B Fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

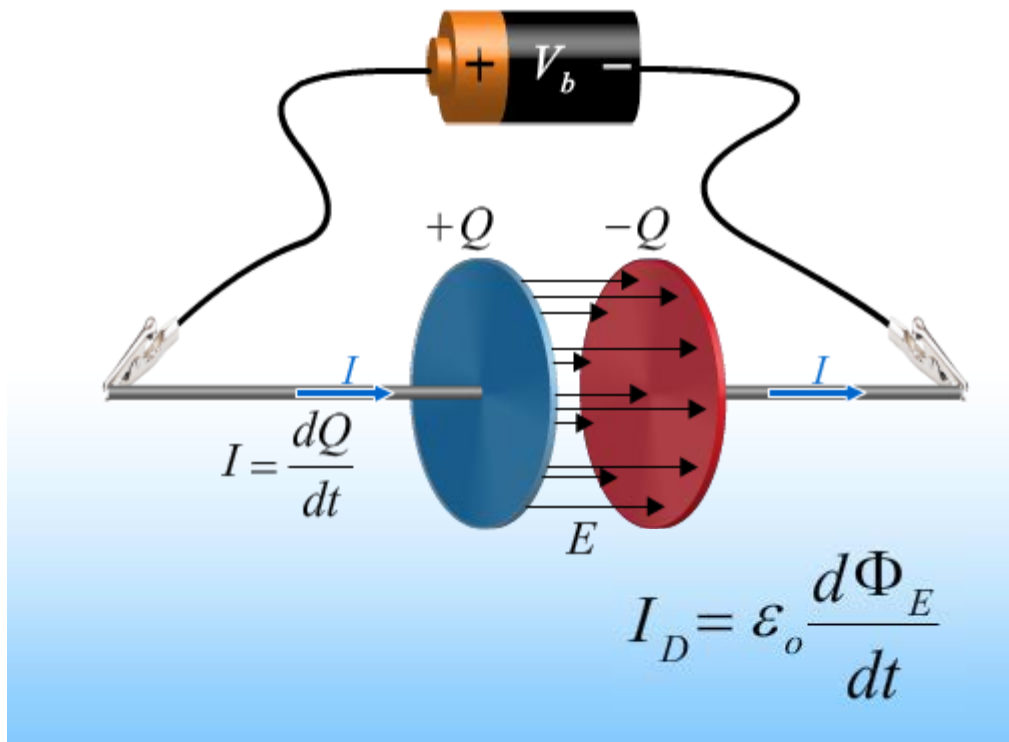
Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

# After Prelecture 21: Modify Ampere's Law

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{\text{enclosed}} = \mu_o (I + I_D)$$



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$



$$\Phi = EA = \frac{Q}{\epsilon_0}$$



$$Q = \epsilon_0 \Phi$$



$$\frac{dQ}{dt} = \epsilon_0 \frac{d\Phi}{dt} \equiv I_D$$

# Displacement Current

Real Current:

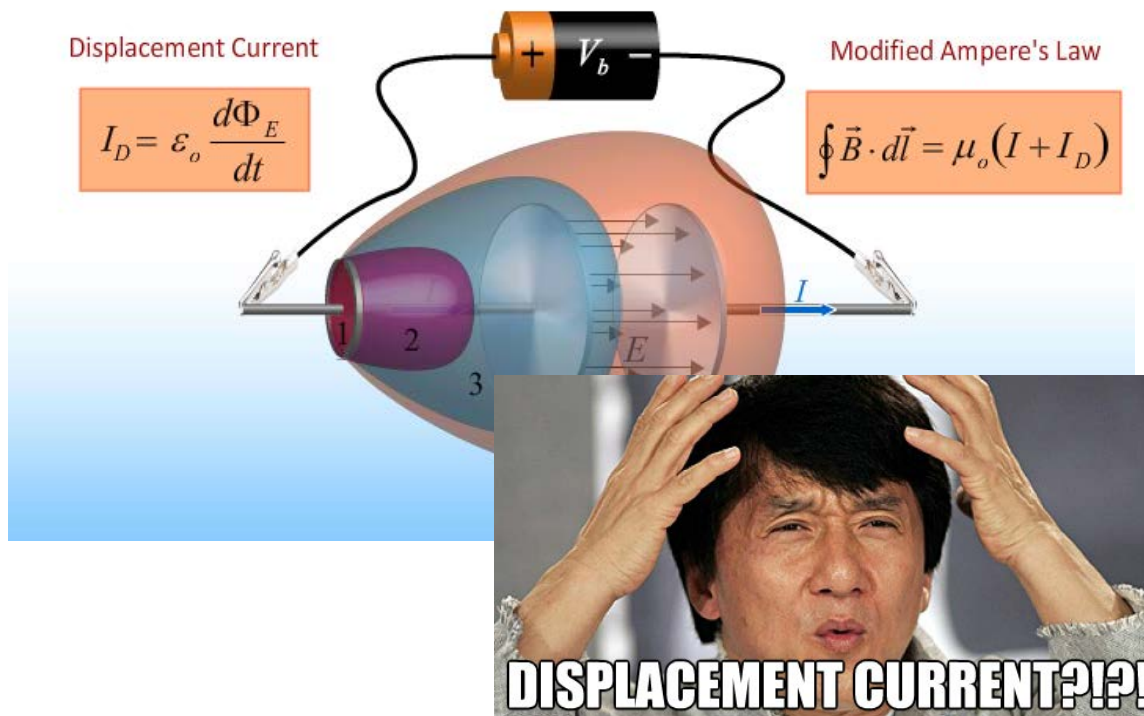
Charge  $Q$  passes through area  $A$  in time  $t$ :

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area  $A'$  changes in time

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

## DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



Modified Ampere's Law

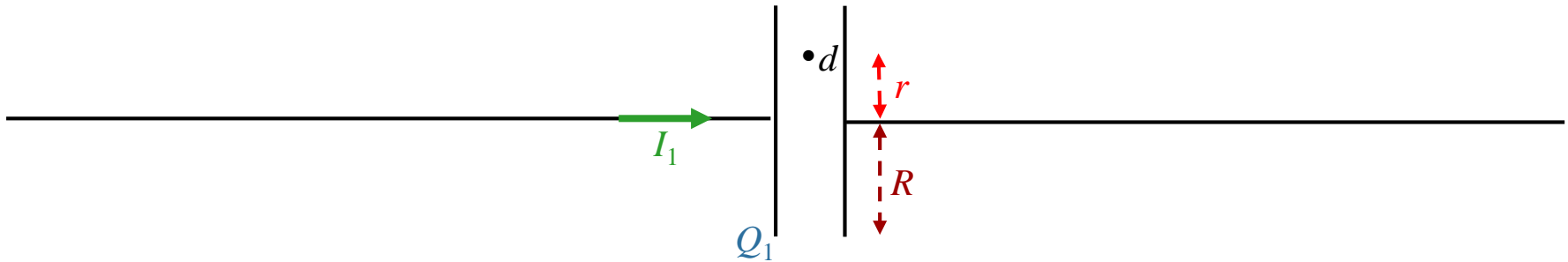
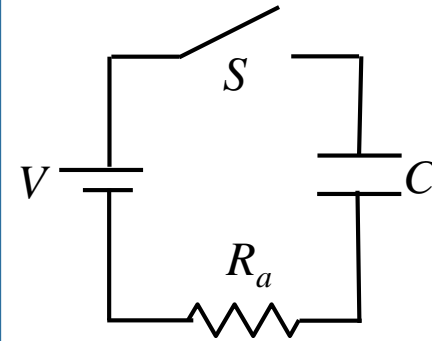
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Free space

# Calculation

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed. Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .

At time  $t_1$ , what is the magnetic field  $B_1$  at a radius  $r$  (point  $d$ ) in between the plates of the capacitor?



## Conceptual and Strategic Analysis

Charge  $Q_1$  creates electric field between the plates of  $C$

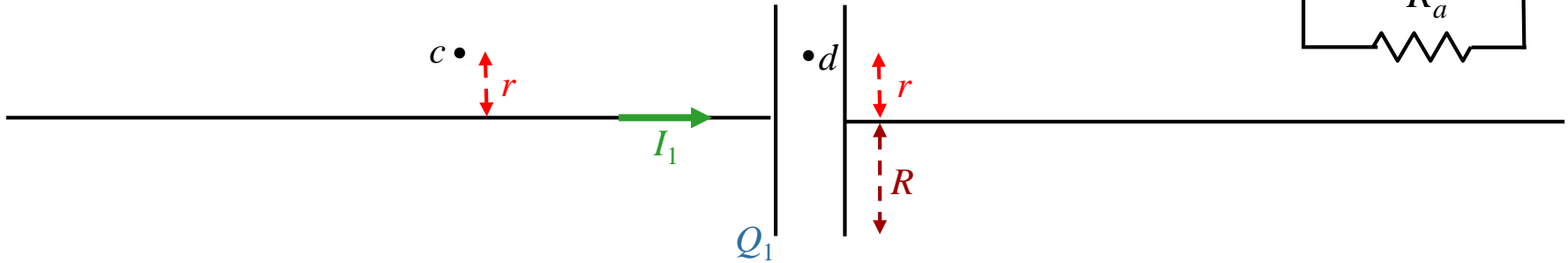
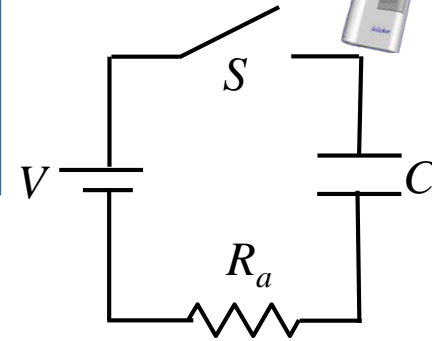
Charge  $Q_1$  changing in time gives rise to a changing electric flux between the plates

Changing electric flux gives rise to a displacement current  $I_D$  in between the plates

Apply (modified) Ampere's law using  $I_D$  to determine  $B$

# Calculation

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed. Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .



Compare the magnitudes of the  $B$  fields at points  $c$  and  $d$ .

A)  $B_c < B_d$

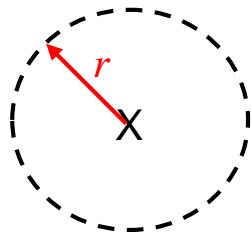
B)  $B_c = B_d$

C)  $B_c > B_d$

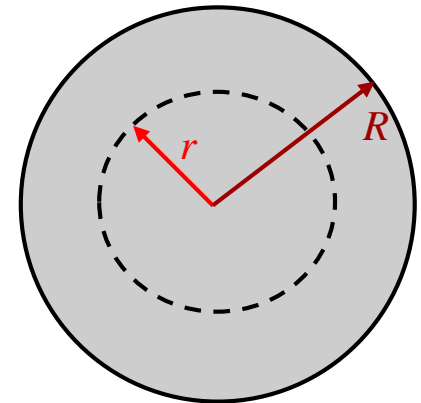
What is the difference?

Apply (modified) Ampere's Law

point  $c$ :  
 $I(\text{enclosed}) = I_1$



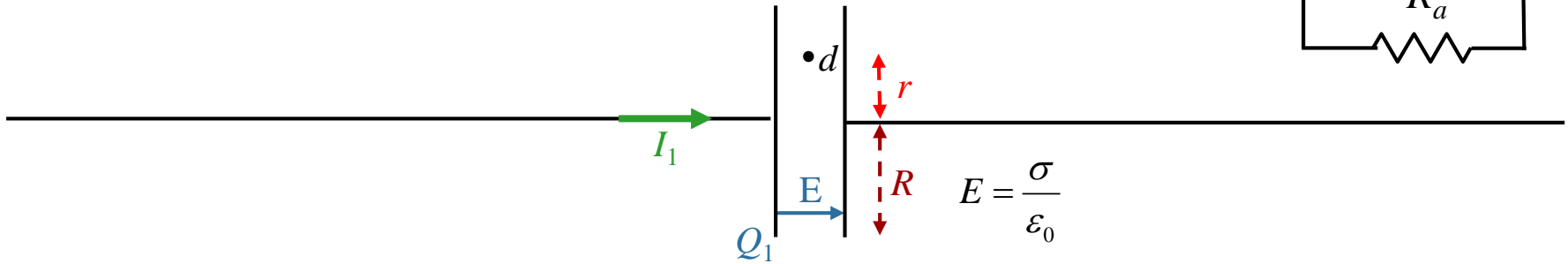
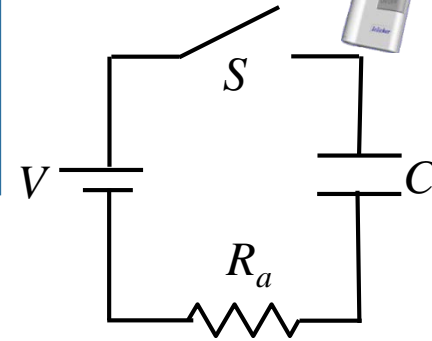
point  $d$ :  
 $I_D(\text{enclosed}) < I_1$



# Calculation

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed.

Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .



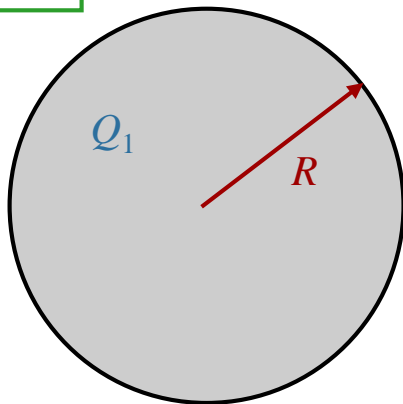
What is the magnitude of the electric field between the plates?

A)  $E = \frac{Q_1}{\pi R^2 \epsilon_0}$

B)  $E = \frac{Q_1 \pi R^2}{\epsilon_0}$

C)  $E = \frac{Q_1}{\epsilon_0}$

D)  $E = \frac{Q_1}{r}$



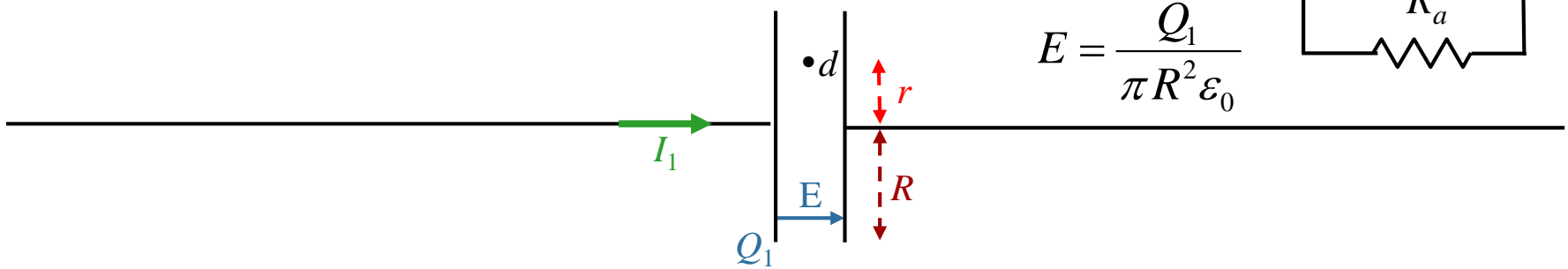
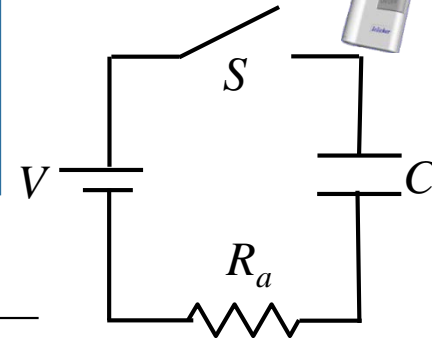
$$E = \frac{\sigma}{\epsilon_0} \rightarrow \sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi R^2} \rightarrow E = \frac{Q_1}{\epsilon_0 \pi R^2}$$



# Calculation

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed.

Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .



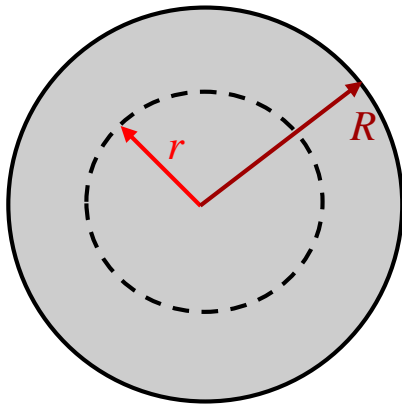
What is the electric flux through a circle of radius  $r$  in between the plates?

A)  $\Phi_E = \frac{Q_1}{\epsilon_0} \pi r^2$

B)  $\Phi_E = \frac{Q_1}{\epsilon_0} \pi R^2$

C)  $\Phi_E = \frac{Q_1 r^2}{\epsilon_0 R^2}$

D)  $\Phi_E = \frac{Q_1 \pi r^2}{\epsilon_0 R^2}$

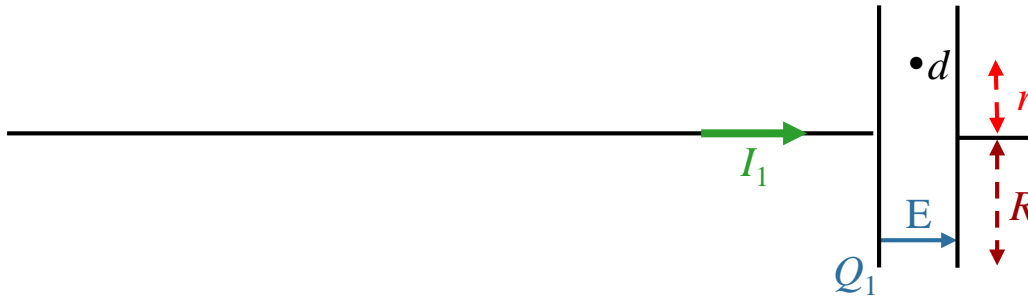
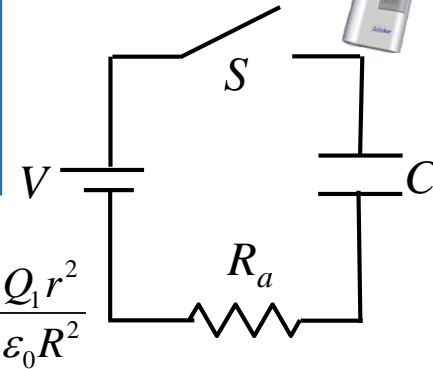


$$\Phi_E = \vec{E} \cdot \vec{A} \longrightarrow \Phi_E = \frac{Q_1}{\epsilon_0 \pi R^2} \pi r^2 \longrightarrow \Phi_E = \frac{Q_1}{\epsilon_0} \frac{r^2}{R^2}$$

# Calculation

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed.

Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .



$$\Phi_E = \frac{Q_1 r^2}{\epsilon_0 R^2}$$

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

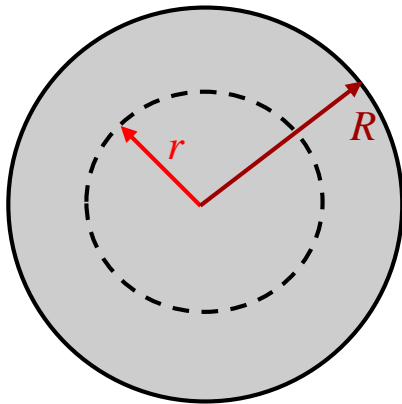
What is the displacement current enclosed by circle of radius  $r$ ?

A)  $I_D = I_1 \frac{R^2}{r^2}$

B)  $I_D = I_1 \frac{r}{R}$

C)  $I_D = I_1 \frac{r^2}{R^2}$

D)  $I_D = I_1 \frac{R}{r}$



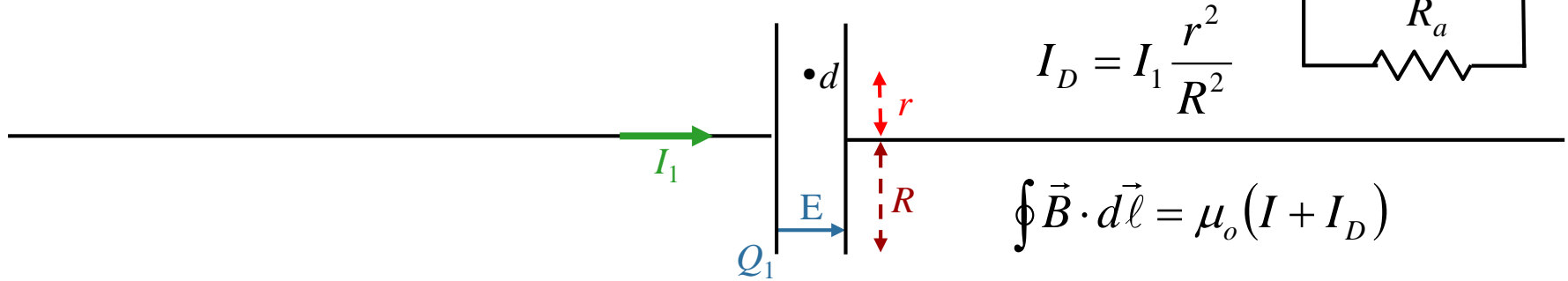
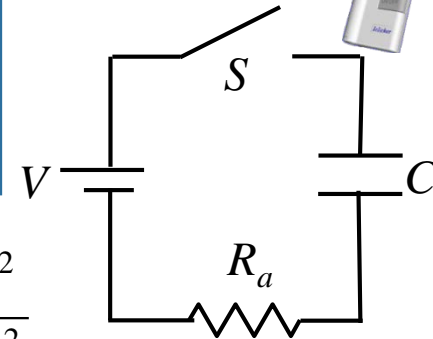
$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

→  $I_D = I_1 \frac{r^2}{R^2}$

# Calculation

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed.

Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .



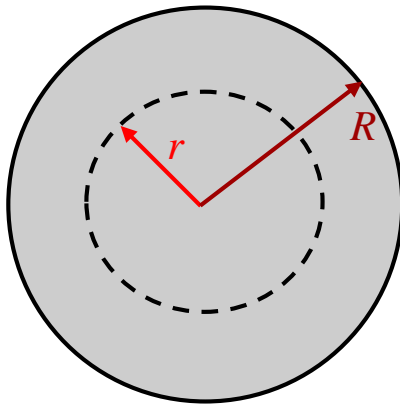
What is the magnitude of the  $B$  field at radius  $r$  ?

A)  $B = \frac{\mu_0 I_1}{2\pi R}$

B)  $B = \frac{\mu_0 I_1}{2\pi r}$

C)  $B = \frac{\mu_0 I_1}{2\pi} \frac{R}{r^2}$

D)  $B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$



Ampere's Law:  $\oint \vec{B} \cdot d\vec{\ell} = \mu_o (I + I_D)$

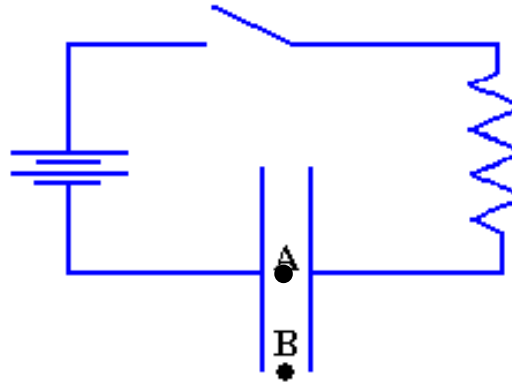
$$\rightarrow B \cdot 2\pi r = \mu_0 \left( 0 + I_1 \frac{r^2}{R^2} \right)$$

$$\rightarrow B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

# CheckPoint 1B



At time  $t=0$  the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed

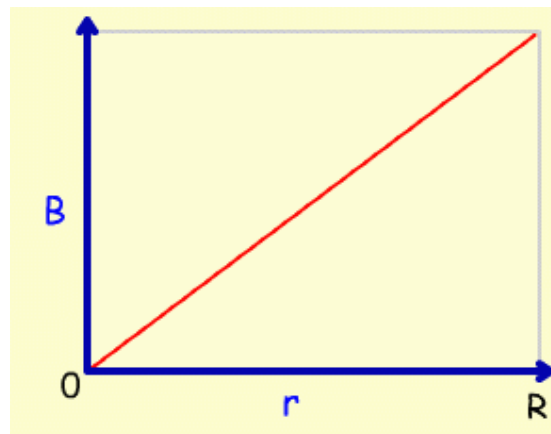
**A.**  $B_A < B_B$

**B.**  $B_A = B_B$

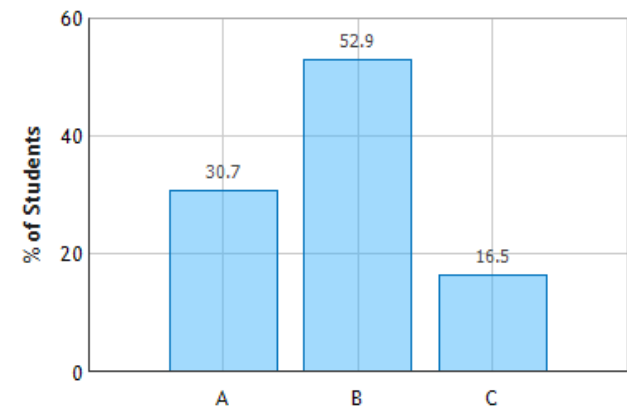
**C.**  $B_A > B_B$

From the calculation we just did:

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$



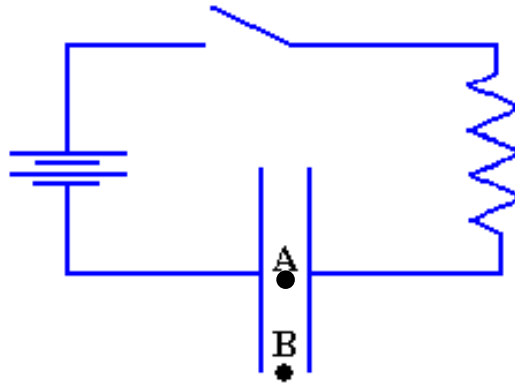
Displacement Current: Question 3 (N = 753)



# CheckPoint 1a



At time  $t=0$  the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



After the switch is closed, there will be a magnetic field at point A that increases as the current in the circuit increases:

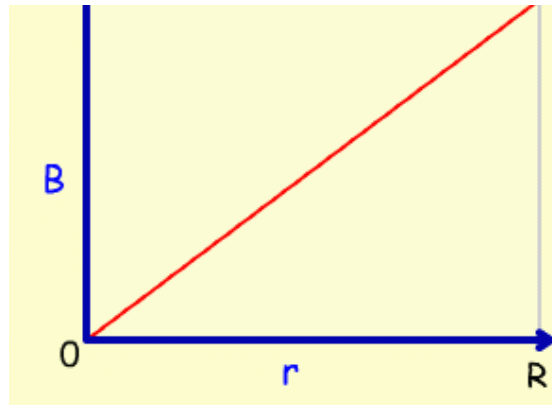
**A.** True

**B.** False

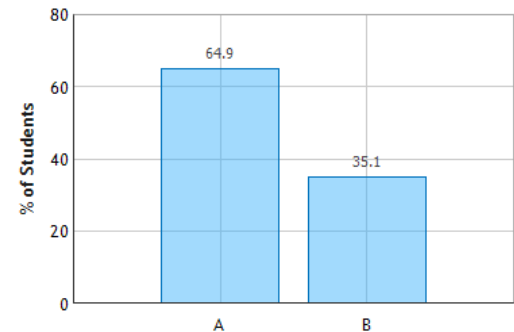
$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

$B$  is proportional to  $I$   
but

At center,  $B = 0$  !!



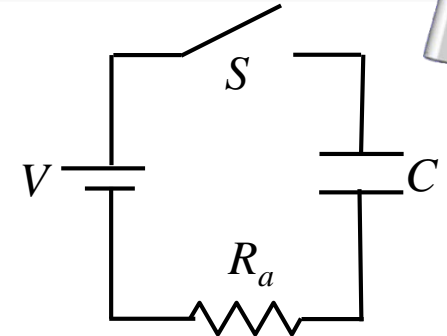
Displacement Current: Question 1 (N = 755)



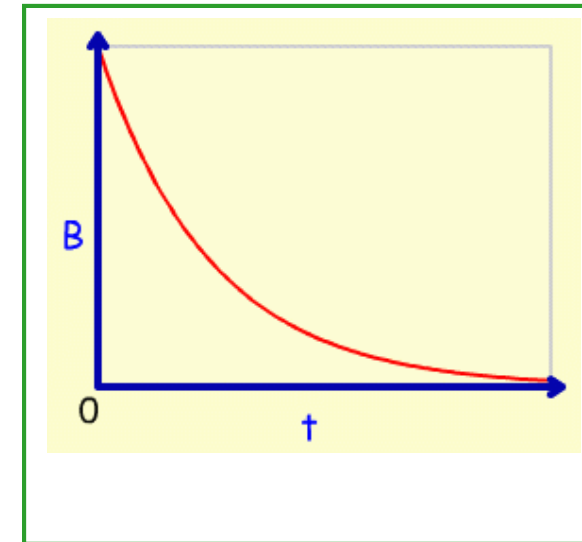
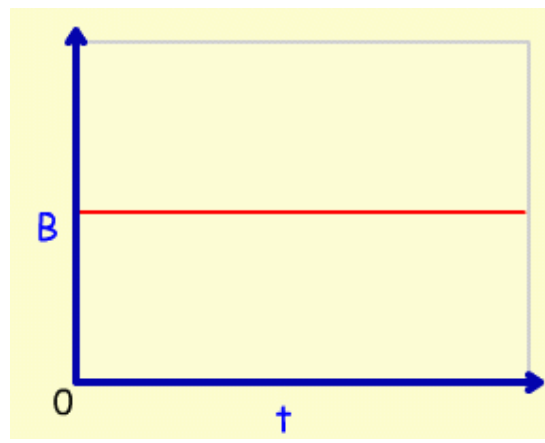
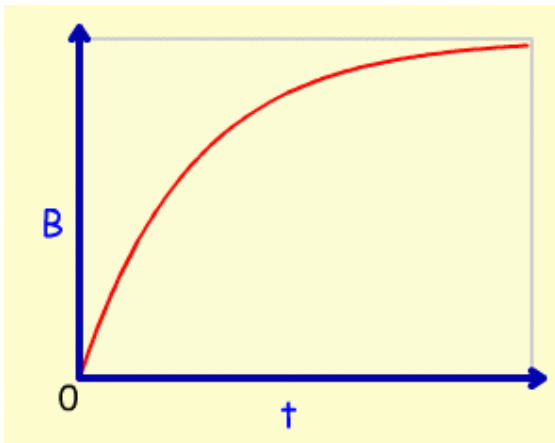
# Follow-Up

Switch  $S$  has been open a long time when at  $t = 0$ , it is closed. Capacitor  $C$  has circular plates of radius  $R$ . At time  $t = t_1$ , a current  $I_1$  flows in the circuit and the capacitor carries charge  $Q_1$ .

What is the time dependence of the magnetic field  $B$  at a radius  $r$  between the plates of the capacitor?



$$B_1 = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$



$B$  at fixed  $r$  is proportional to the current  $I$

Close switch:  $V_C = 0 \Rightarrow I = V/R_a$  (maximum)

$I$  exponentially decays with time constant  $\tau = R_a C$

# Follow-Up 2

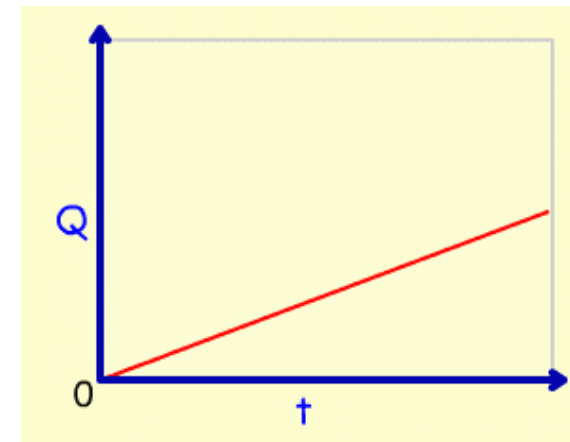
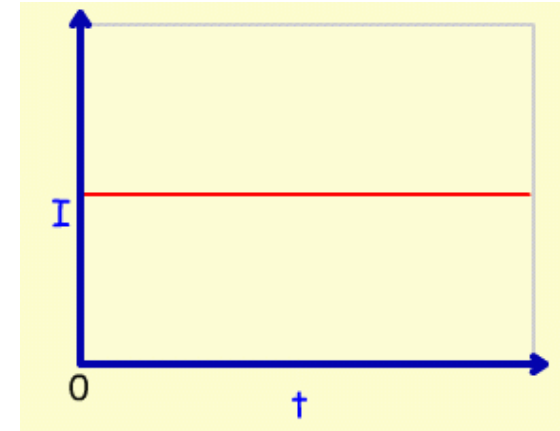


Suppose you were able to charge a capacitor with constant current (does not change in time).

Does a  $B$  field exist in between the plates of the capacitor?

A) YES

B) NO



Constant current  $\Rightarrow Q$  increases linearly with time

Therefore  $E$  increases linearly with time ( $E = Q/(A\epsilon_0)$ )

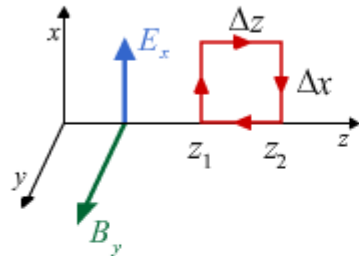
$dE/dt$  is not zero  $\Rightarrow$  Displacement current is not zero

$\Rightarrow B$  is not zero !

# Let there be light!

## Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



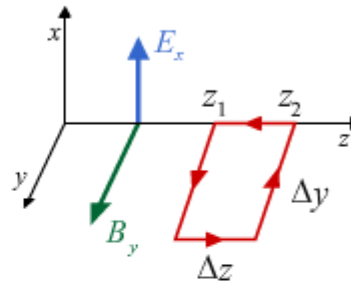
Plane Wave Solution

$$\vec{E} \rightarrow \vec{E}(z,t)$$

$$\vec{B} \rightarrow \vec{B}(z,t)$$

## Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \epsilon_o \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$



$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z} \frac{\partial B_y}{\partial t}$$

$$\frac{\partial B_y}{\partial z} = -\mu_o \epsilon_o \frac{\partial E_x}{\partial t}$$

$$\frac{\partial}{\partial t} \frac{\partial B_y}{\partial z} = -\mu_o \epsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \epsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

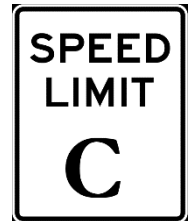
## Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \epsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

## Speed of Electromagnetic Wave

$$v = \frac{1}{\sqrt{\mu_o \epsilon_o}} = c = 3.00 \times 10^8 \text{ m/s}$$

Speed of Light !



## Special Relativity (1905)

Speed of Light is Constant

Albert Einstein



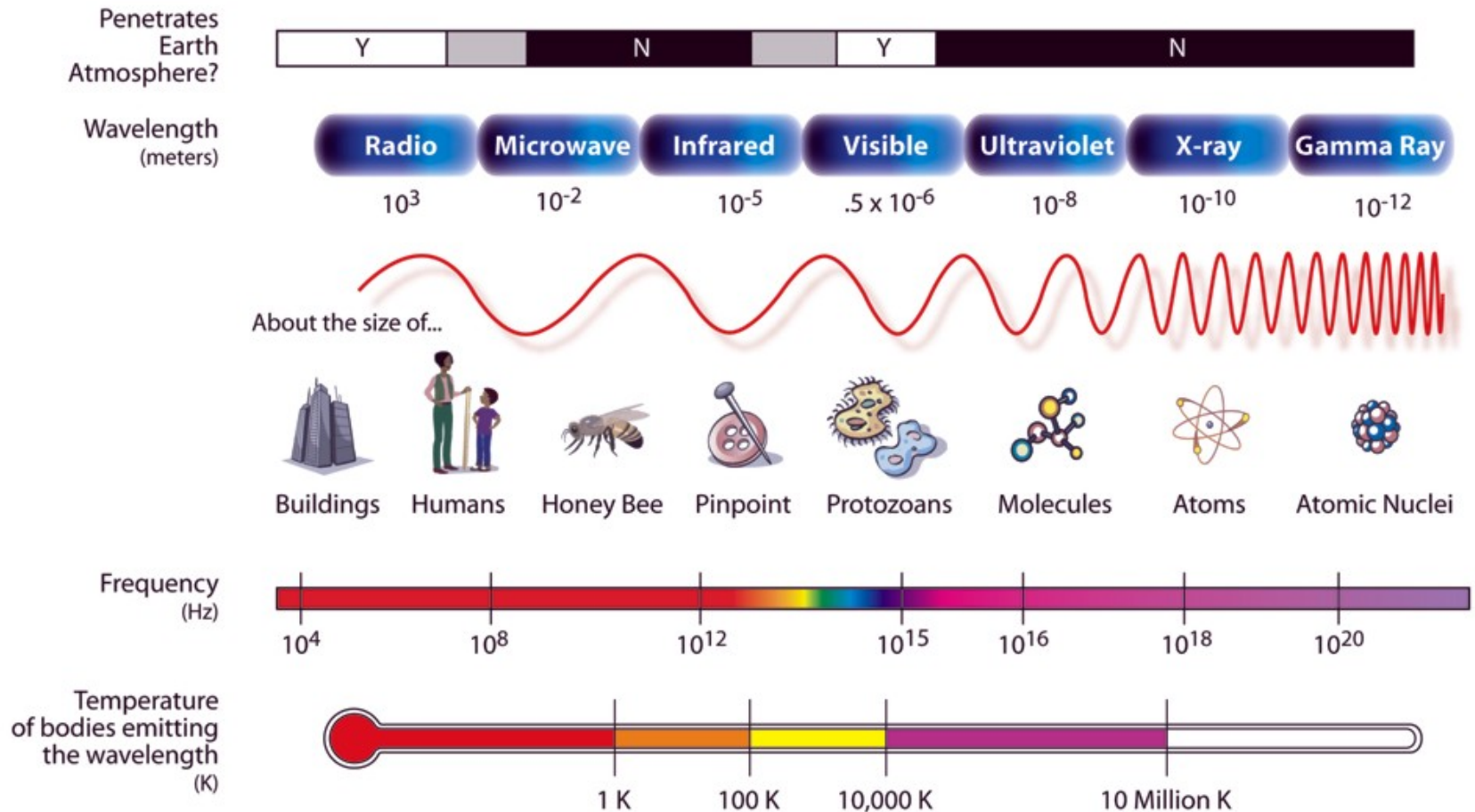
“How can light move at the same velocity in any inertial frame of reference? That's really trippy.”

see PHYS 225



# Electromagnetic Spectrum

## THE ELECTROMAGNETIC SPECTRUM



# We learned about waves in Physics 211

1-D Wave Equation

$$\frac{d^2 h}{dx^2} = \frac{1}{v^2} \frac{d^2 h}{dt^2}$$

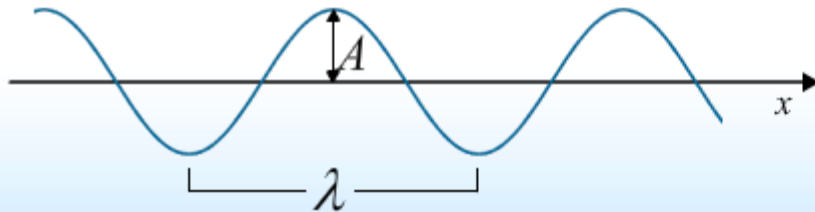


Solution

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

Common Example: Harmonic Plane Wave

$$h(x,t) = A \cos(kx - \omega t)$$



## Variable Definitions

Amplitude:  $A$

Wave Number:  $k = \frac{2\pi}{\lambda}$

Wavelength:  $\lambda$

Angular Frequency:  $\omega = \frac{2\pi}{T}$

Period:  $T$

Frequency:  $f = \frac{1}{T}$

Velocity:  $v = \lambda f = \frac{\omega}{k}$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \epsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

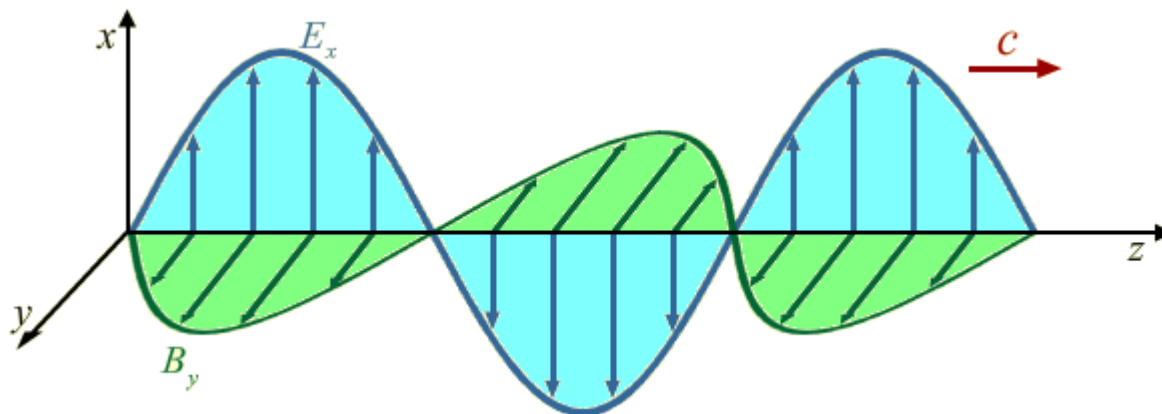
$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \epsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

Two Important Features

1.  $B_y$  is in phase with  $E_x$
2.  $B_o = \frac{E_o}{c}$

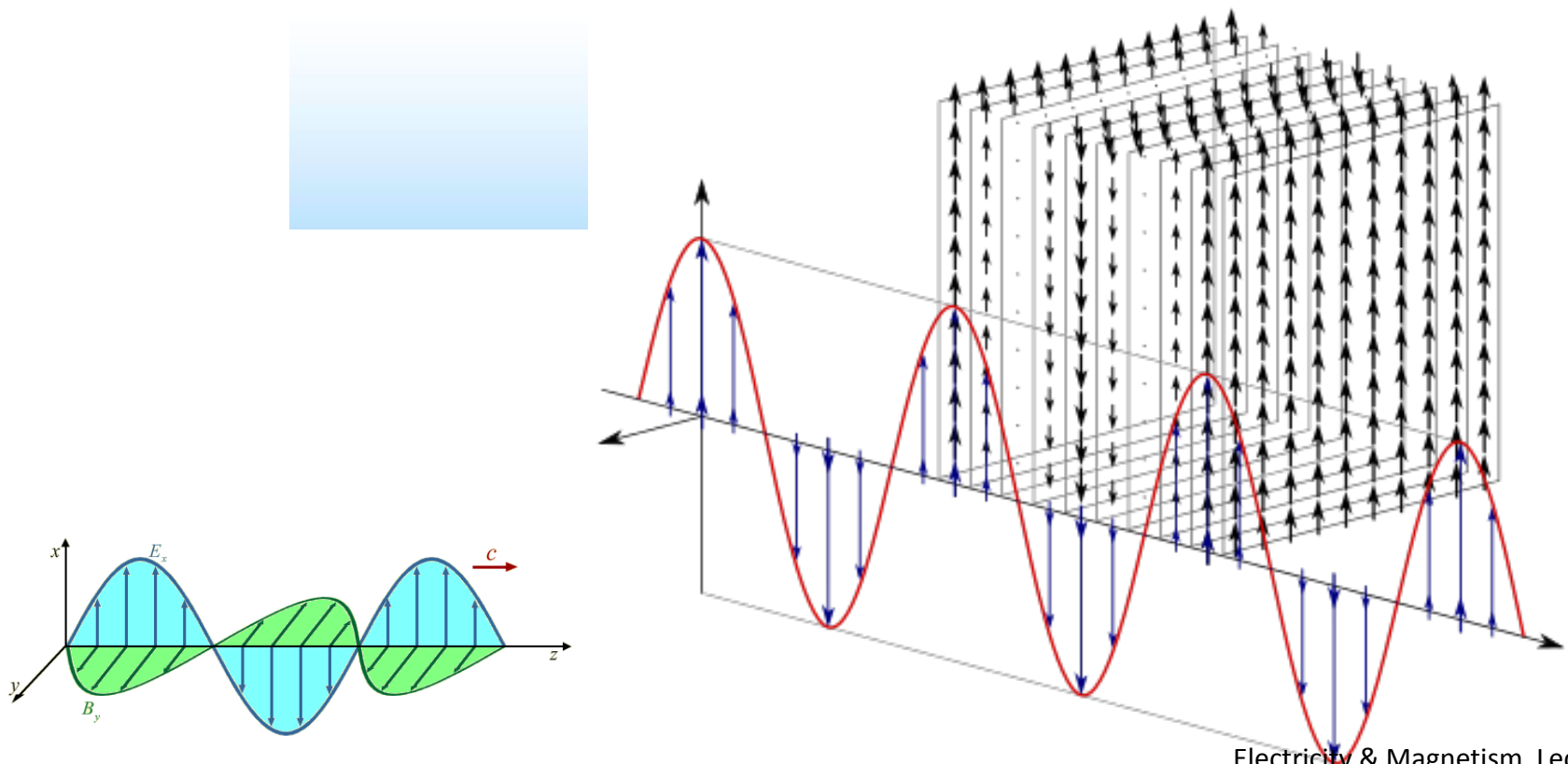


$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \epsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \epsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

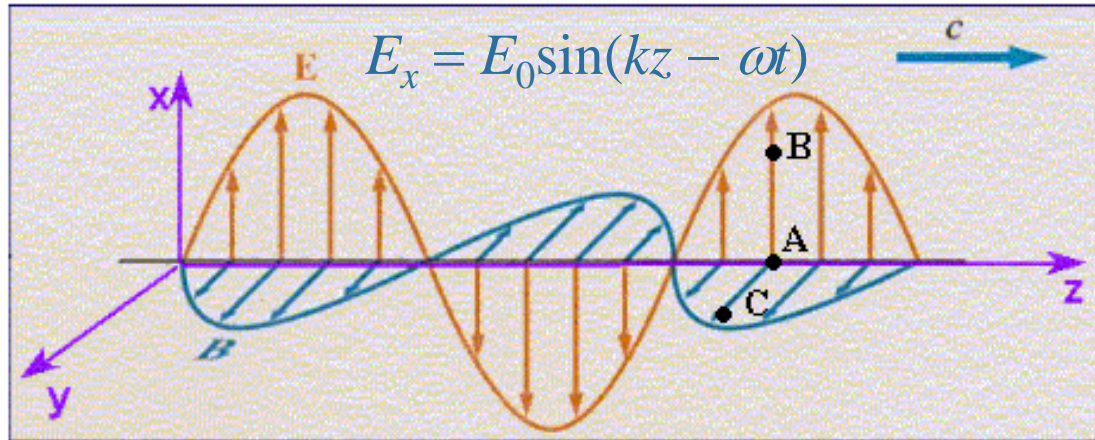
$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$



# CheckPoint 2a



An electromagnetic plane-wave is traveling in the  $+z$  direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same  $z$  coordinate.



Compare the magnitudes of the electric fields at points A and B

A.  $E_A < E_B$

**B.  $E_A = E_B$**

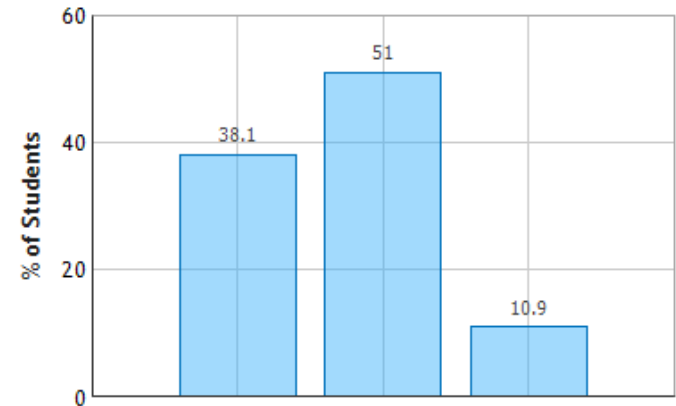
C.  $E_A > E_B$

$$E = E_0 \sin(kz - \omega t):$$

$E$  depends only on  $z$  coordinate for constant  $t$ .

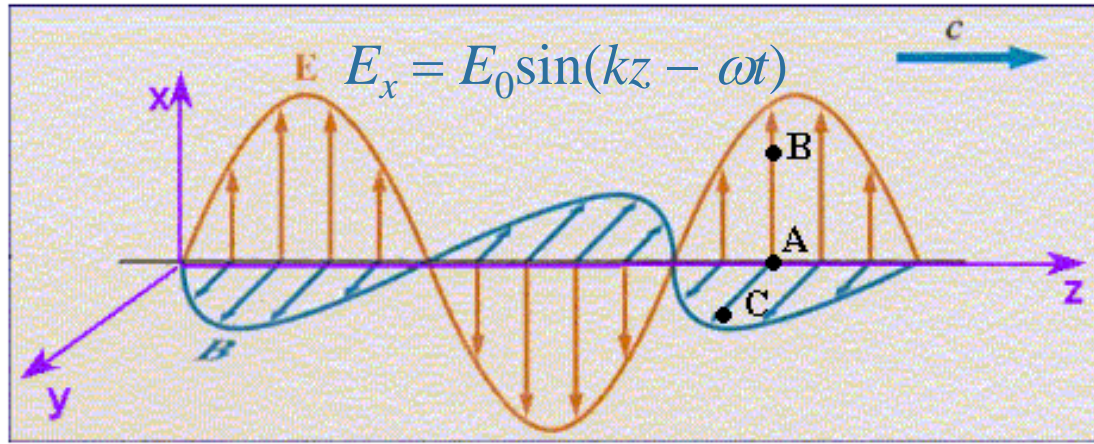
$z$  coordinate is same for A, B, C.

Electromagnetic Waves: Question 1 (N = 753)



# CheckPoint 2b

An electromagnetic plane-wave is traveling in the  $+z$  direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same  $z$  coordinate.



Compare the magnitudes of the electric fields at points A and C

A.  $E_A < E_C$

**B.  $E_A = E_C$**

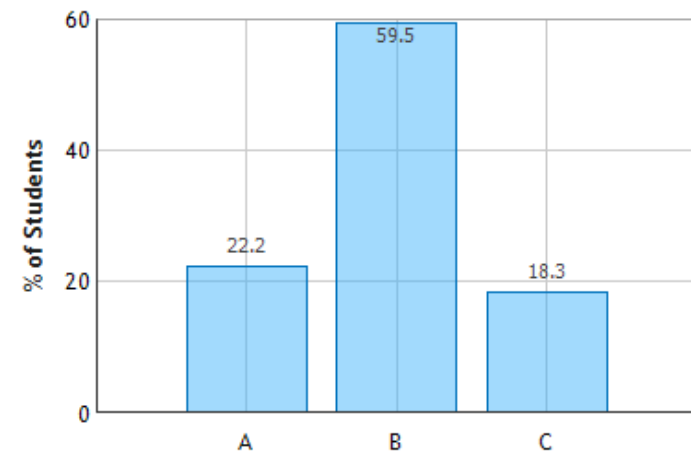
C.  $E_A > E_C$

$$E = E_0 \sin(kz - \omega t):$$

$E$  depends only on  $z$  coordinate for constant  $t$ .

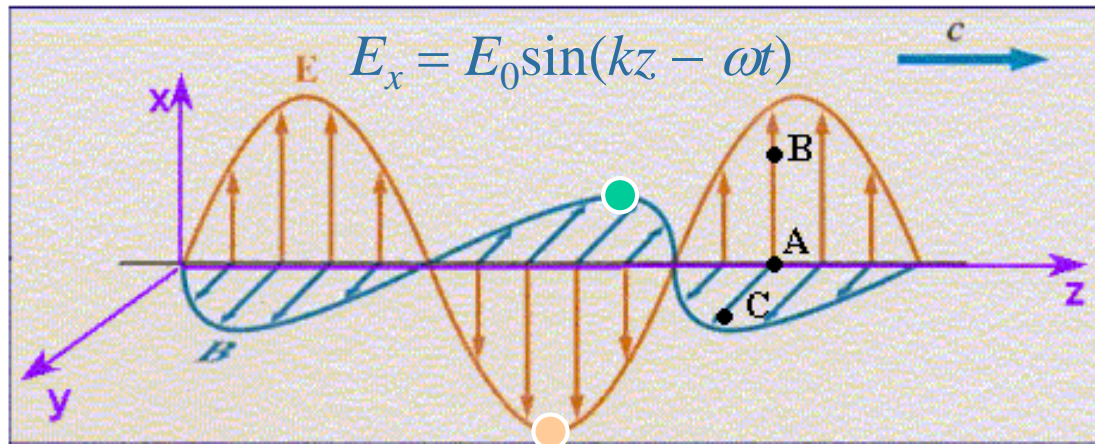
$z$  coordinate is same for A, B, C.

Electromagnetic Waves: Question 2 (N = 753)





# Clicker Question



Consider a point  $(x,y,z)$  at time  $t$  when  $E_x$  is negative and has its maximum value.

At  $(x,y,z)$  at time  $t$ , what is  $B_y$ ?

- A)  $B_y$  is positive and has its maximum value
- ☒ B)  $B_y$  is negative and has its maximum value
- C)  $B_y$  is zero
- D) We do not have enough information