Your Comments

Please explain what I just watched in terms we can understand, please! The concepts aren't hard, but all that math....

Nothing made sense- just a bunch of algebra that I don't know how to use in the correct physics ways.

I don't understand what peak voltage means and how to find it.

I keep hearing the word resonance. What does it actually mean? Also, the derivation of the Q equation was pretty confusing. I don't think the prelecture went into it at all.

I have seen in the prelectures that the equation for Q(t) often uses sin(wt+φ) and cos(wt+φ) interchangeably. Can we do this at our convenience by just assuming that φ has changed automatically? Would this affect the fact that in the phasor diagram the value of a phasor is its projection at the y axis and hence only a sine must be taken of the value? Please clarify!

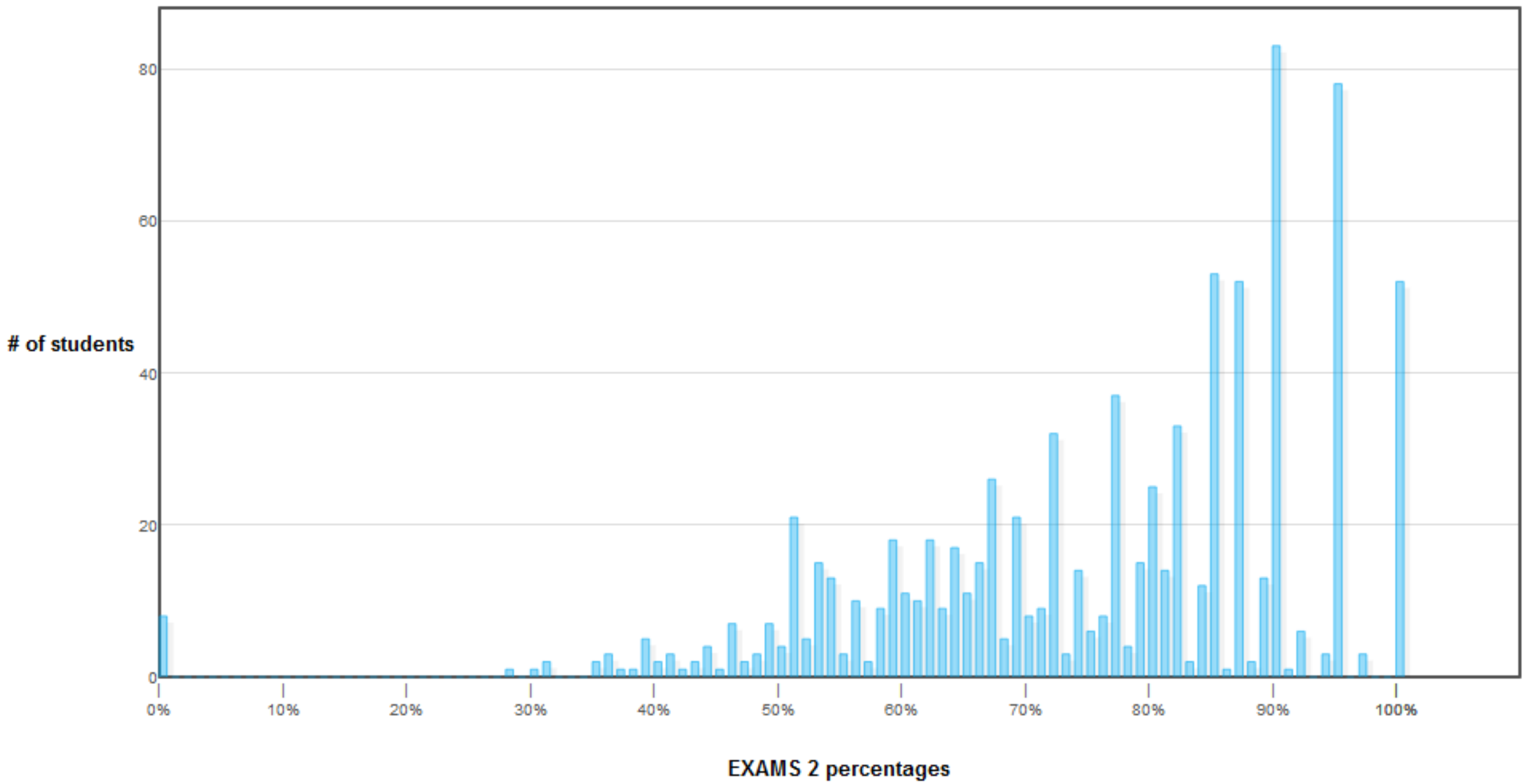
This φ depends on initial conditions, related to initial rotation of phasors. Magnitude of voltages oscillates in time.

There are 26 characters in the English alphabet, and Physicists chose Q twice!

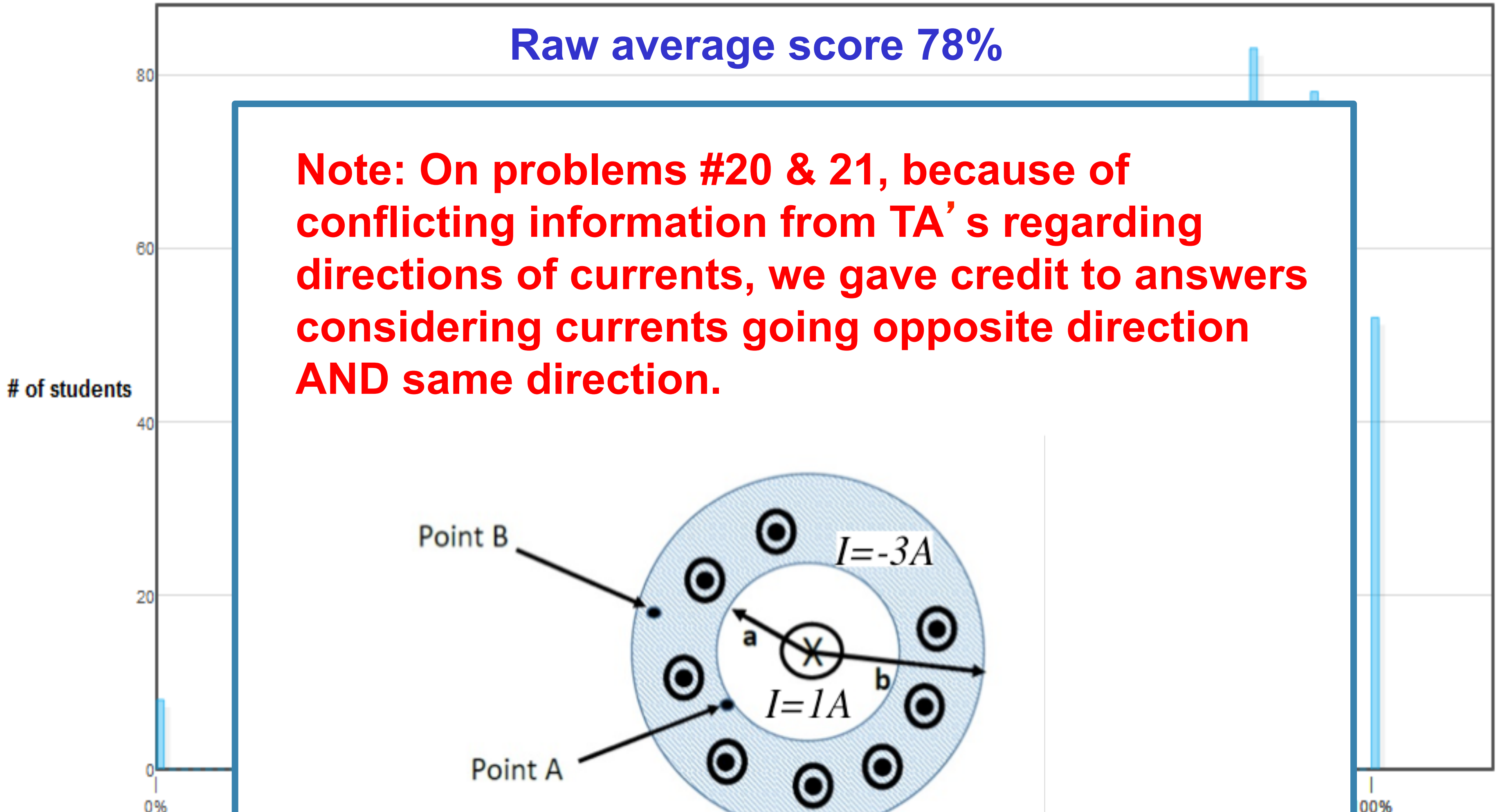
Transformers, more than meets the eye!

Nice work on hour exam 2

Raw average score 78%



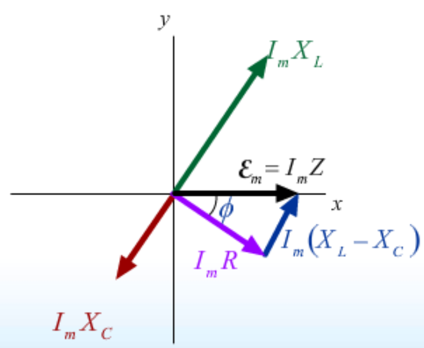
Nice work on hour exam 2

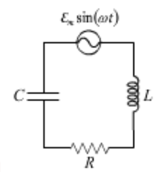


Note: On problems #20 & 21, because of conflicting information from TA's regarding directions of currents, we gave credit to answers considering currents going opposite direction AND same direction.

Physics 212
Lecture 21

Voltage Phasor Diagram





Phase Relation

$$\tan\phi = \frac{X_L - X_C}{R}$$

Impedance

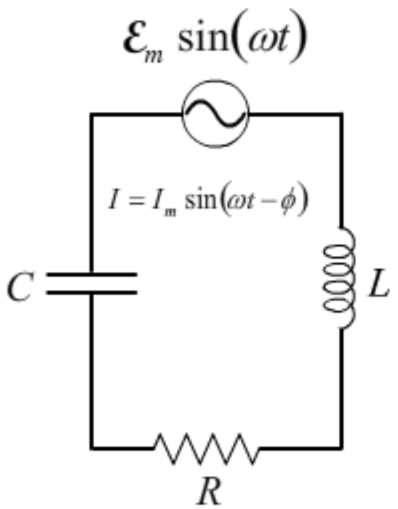
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Maximum Current

$$I_m = \frac{\mathcal{E}_m}{Z}$$

Looks intimidating, but isn't bad!

The Driven LCR Circuit



Frequency Dependence of Maximum Current

$$I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1+Q^2\frac{(x^2-1)^2}{x^2}}}$$

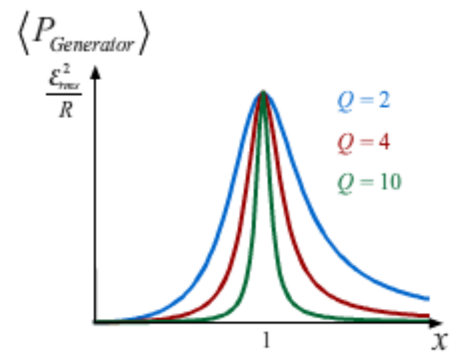
Average Power per Cycle

$$\langle P_{Generator}\rangle = \frac{\mathcal{E}_{rms}^2}{R}\frac{x^2}{x^2+Q^2(x^2-1)^2}$$

where $x \equiv \frac{\omega}{\omega_o}$ & $Q^2 = \frac{L}{R^2C}$

Quality Factor

$$Q \equiv 2\pi\left[\frac{U_{max}}{\Delta U}\right]_{cycle} \xrightarrow{\text{evaluate at}} \omega = \omega_o$$



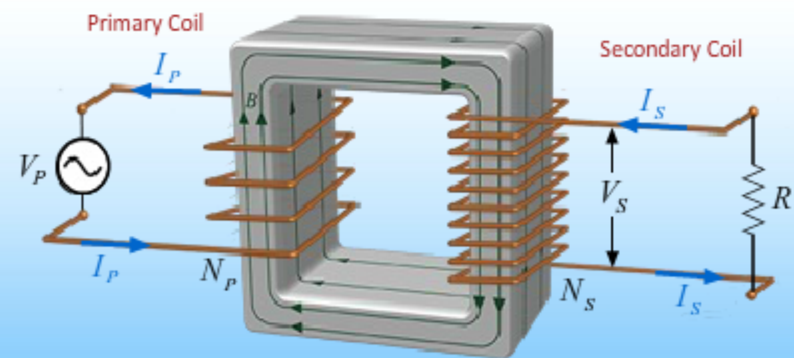
Transformers

Voltage Relation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Current Relation

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

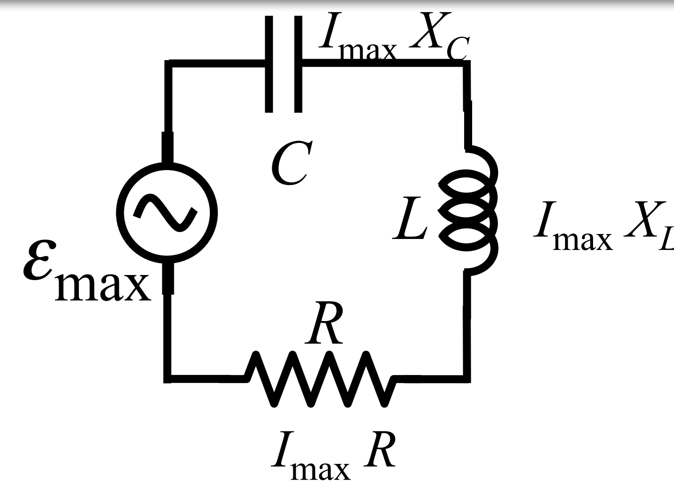


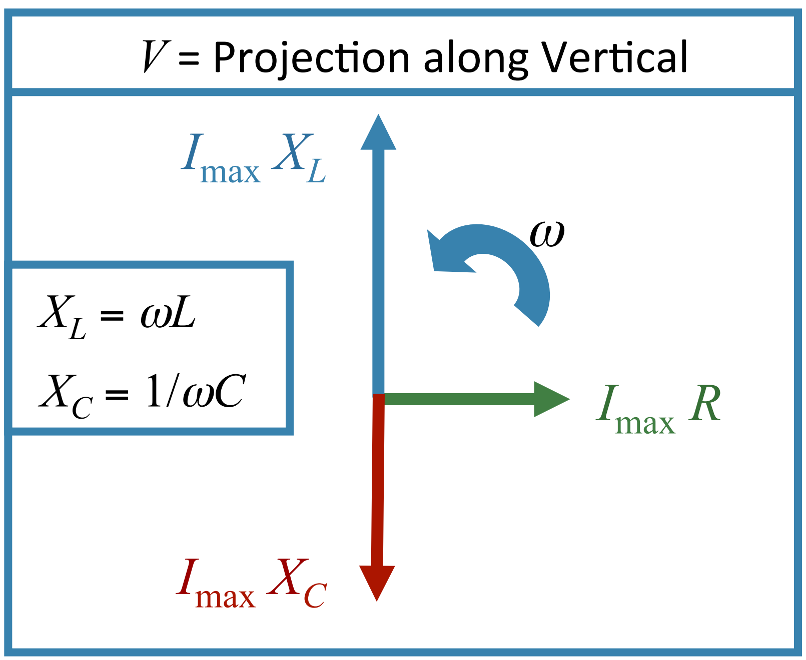
AC Circuits & Phasors

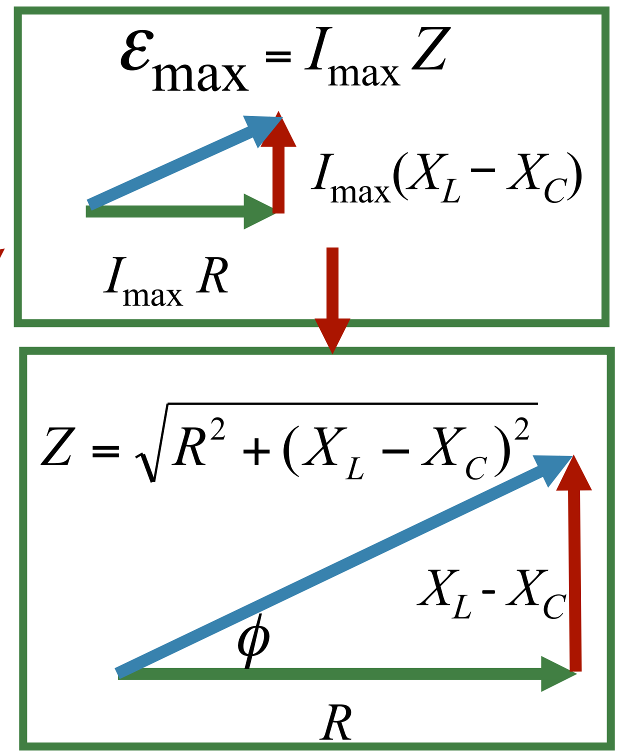
PHASORS ARE THE KEY !
FORMULAS ARE NOT !

START WITH PHASOR DIAGRAM

DEVELOP FORMULAS FROM THE DIAGRAM !!







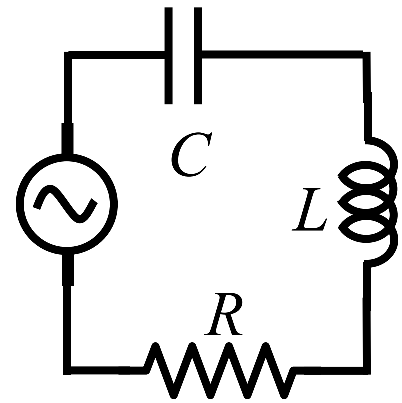
Peak AC Problems

"Ohms" Law for each element

NOTE: Good for PEAK values only)

$$V_{gen} = I_{\max} Z$$
$$V_{Resistor} = I_{\max} R$$
$$V_{inductor} = I_{\max} X_L$$
$$V_{Capacitor} = I_{\max} X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$

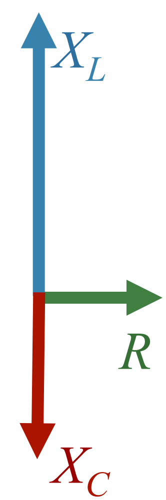


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

$$X_L = \omega L = 200\Omega \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} = 122\Omega$$

$$X_C = \frac{1}{\omega C} = 100\Omega \qquad I_{\max} = \frac{V_{gen}}{Z} = 0.13A$$



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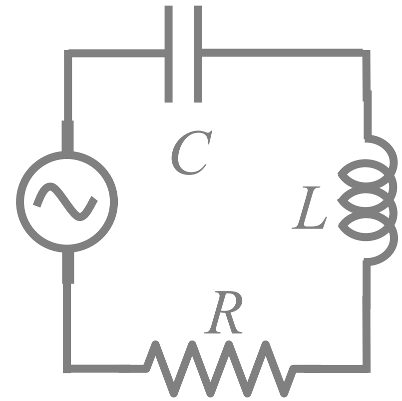


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

Which element has the largest peak voltage across it?

A) Generator

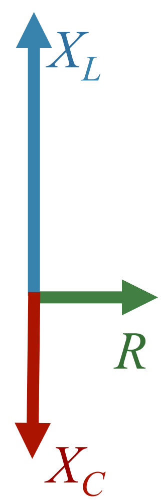
B) Inductor

C) Resistor

D) Capacitor

E) All the same.

$$V_{\max} = I_{\max} X$$



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Peak AC Problems

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$$V_{Resistor} = I_{\max} R$$
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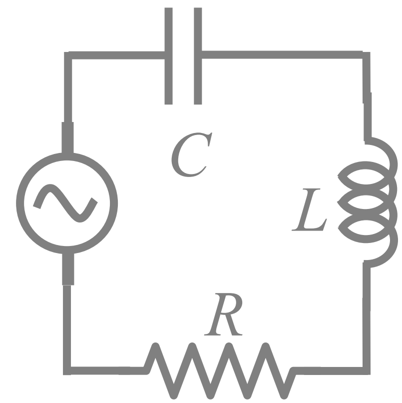
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$



Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

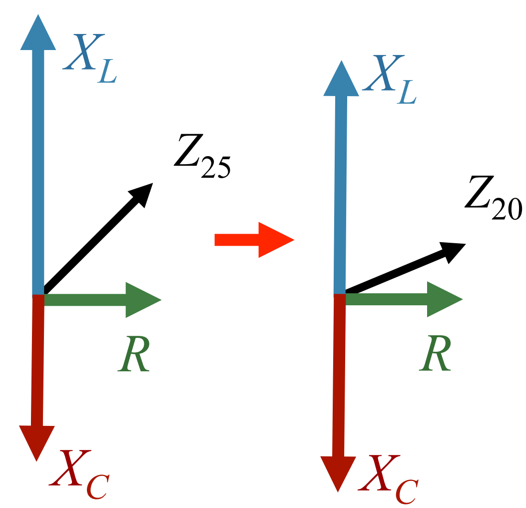
What happens to the impedance if we decrease the angular frequency to 20 rad/sec?

A) Z increases

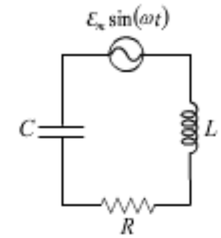
B) Z remains the same

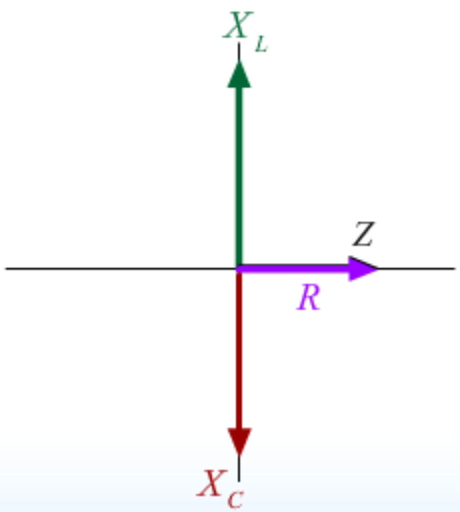
C) Z decreases

$(X_L - X_C)$: (200 − 100) -> (160 − 125)



Resonance





Resonance

I_m is a maximum $\longrightarrow$ $I_m = \dfrac{\mathcal{E}_m}{R}$

$\omega = \omega_o$

Z minimized $\longrightarrow$ $X_L = X_C$

$\phi = 0^o$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

Light-bulb Demo

Resonance

Frequency at which voltage across inductor and capacitor cancel

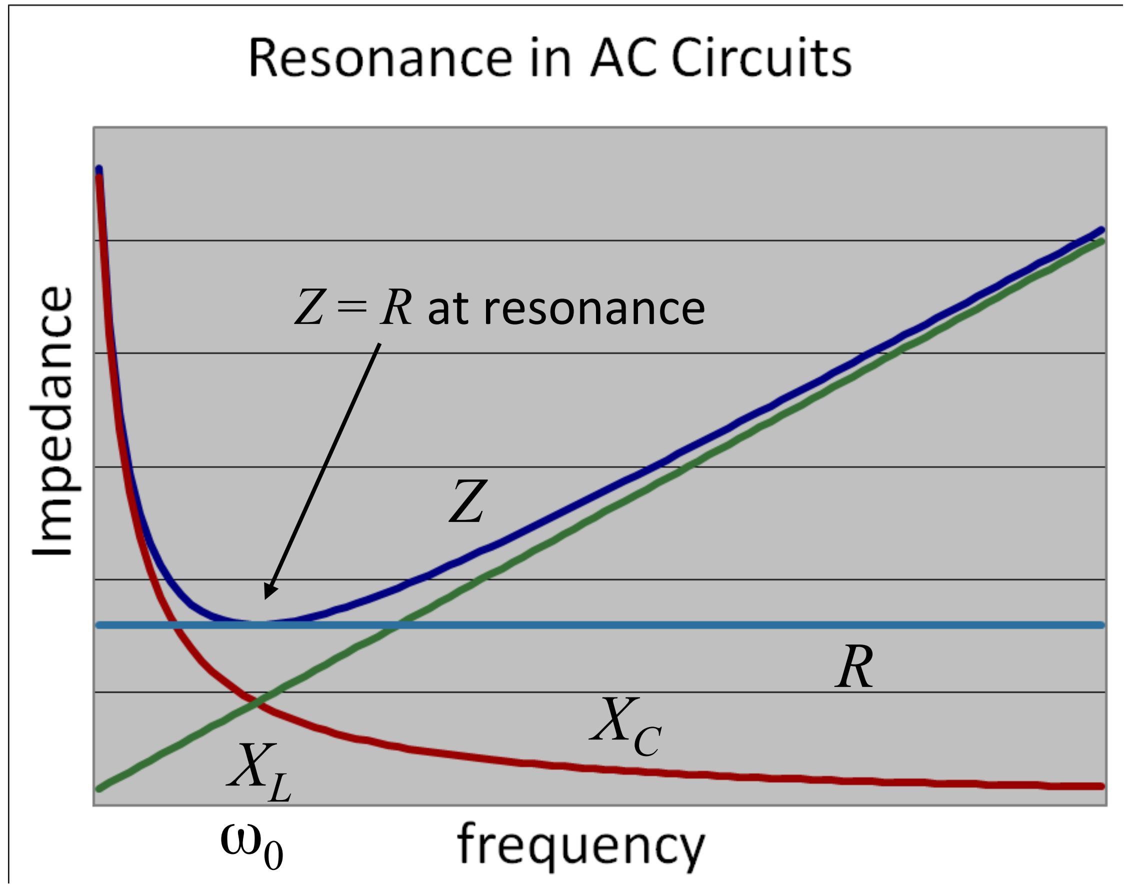
R is independent of ω

X_L increases with ω

$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

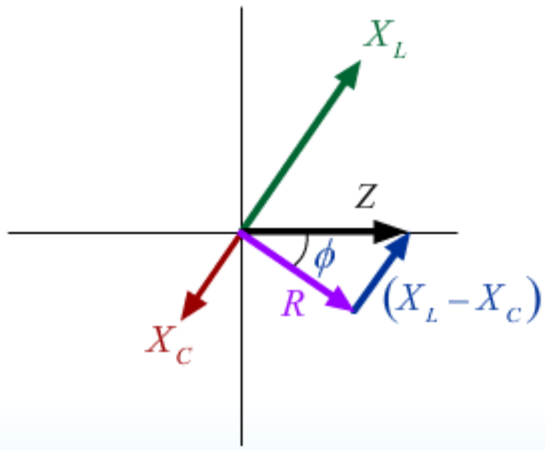


$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance

Resonance: $X_L = X_C$ $\quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

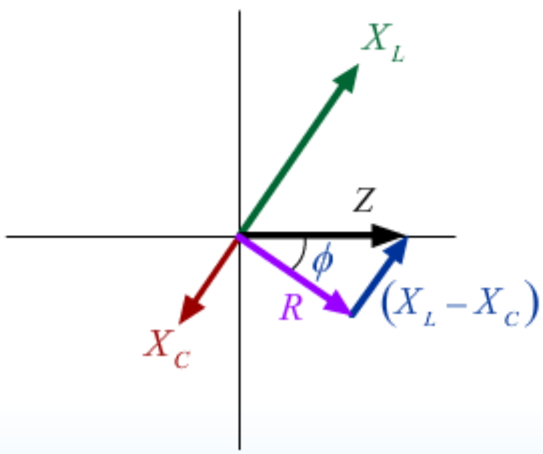
Off Resonance



$$I_m = \frac{\mathcal{E}_m}{Z}$$

$$I_m = \frac{\mathcal{E}_m}{R}\frac{R}{\boxed{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}}$$

Z

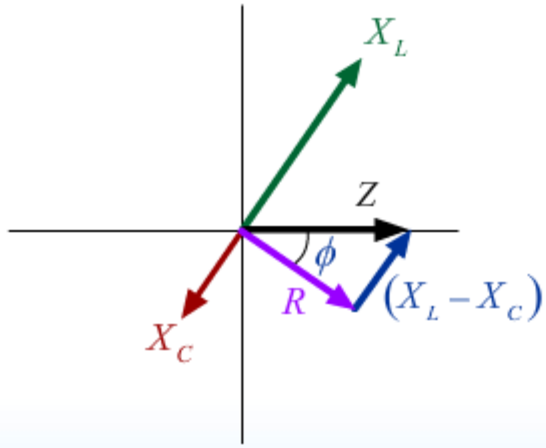


$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C} \longrightarrow Q \equiv 2\pi\frac{U_{\max}}{\Delta U}$$

$$\boxed{I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1 + Q^2\frac{(x^2-1)^2}{x^2}}}}$$

U_{max} = max energy stored
ΔU = energy dissipated
in one cycle at resonance

Off Resonance



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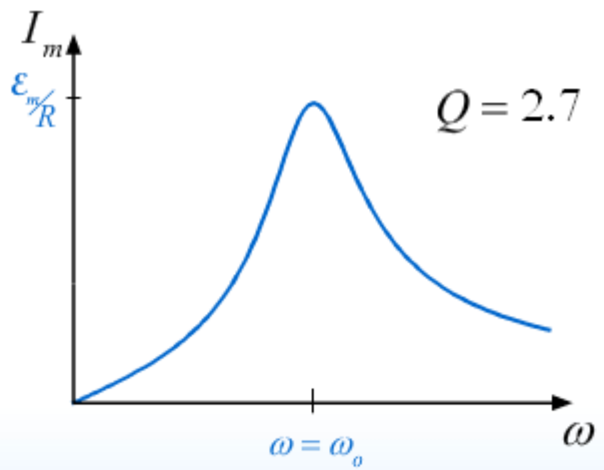
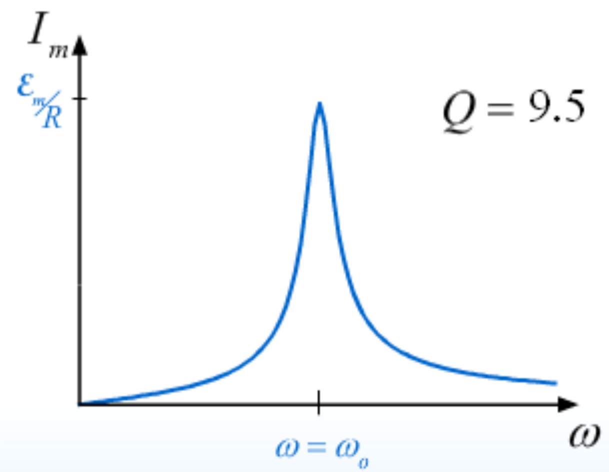
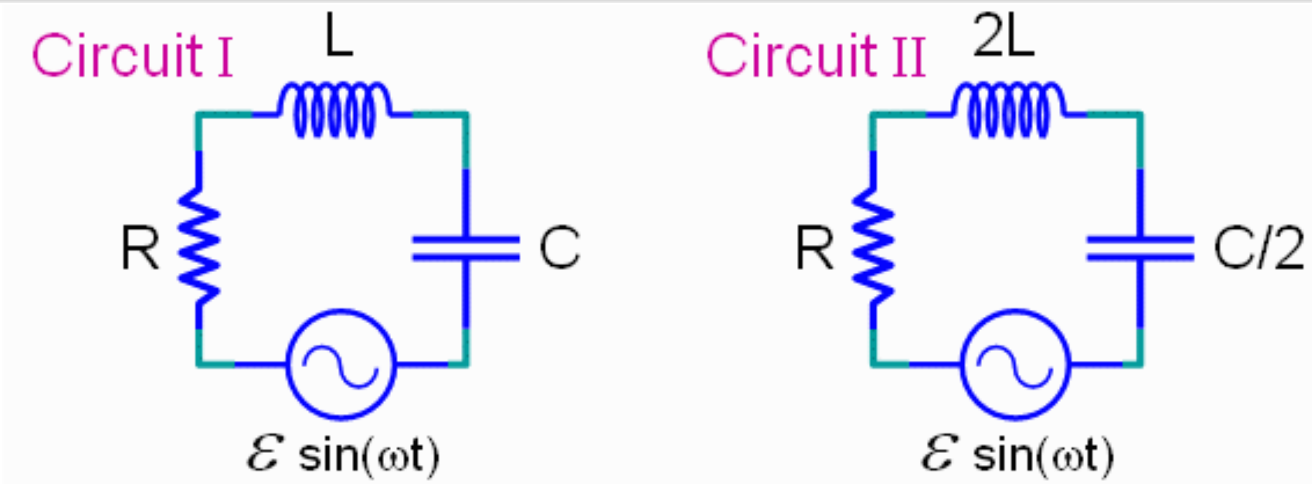




CheckPoint 1a



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

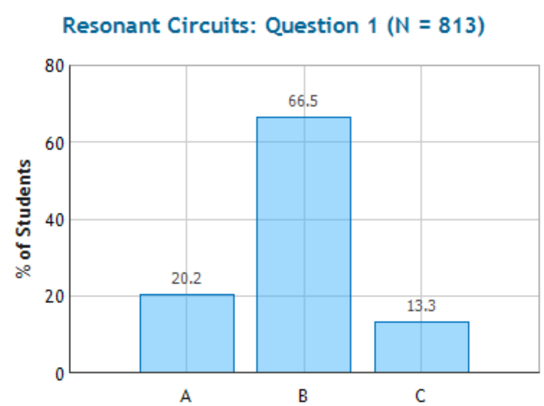
Compare the peak voltage across the resistor in the two circuits

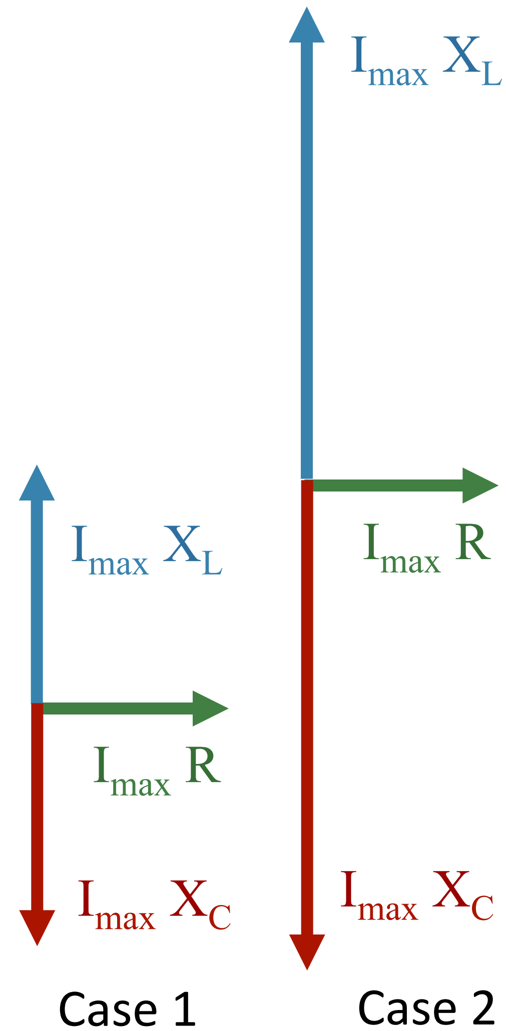
A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

Resonance: $X_L = X_C$

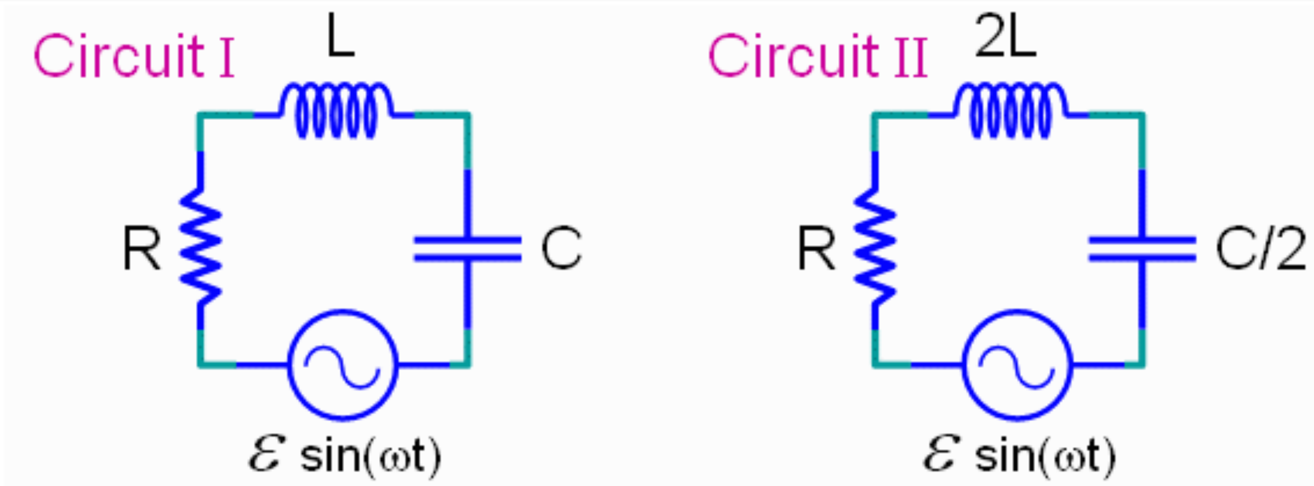
$Z = R$

Same since R doesn't change





CheckPoint 1b

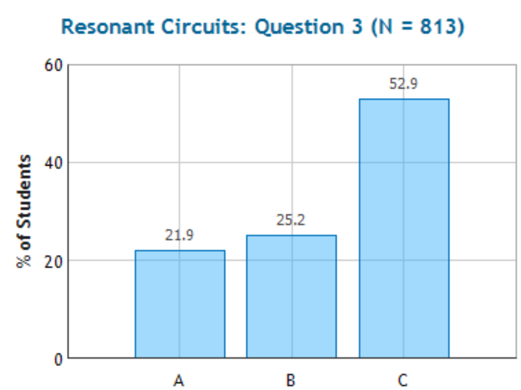


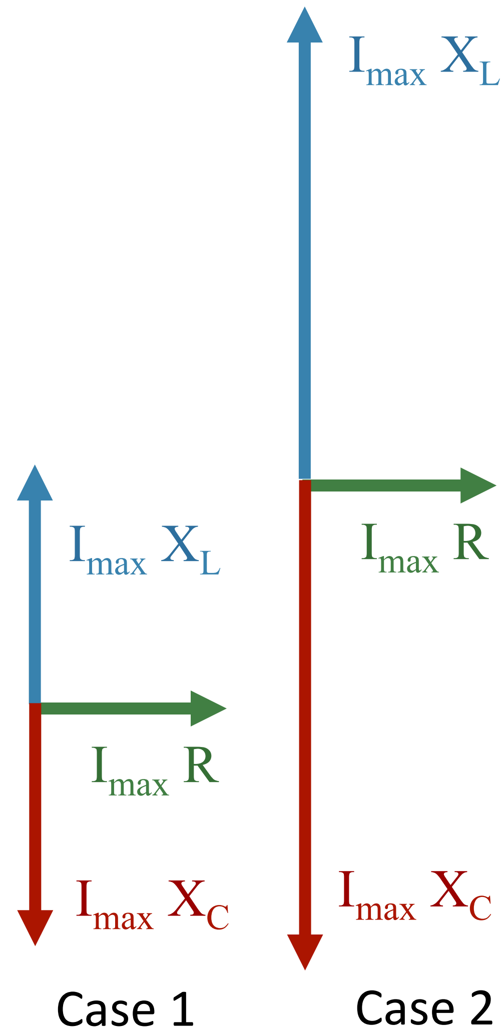
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

Compare the peak voltage across the inductor in the two circuits

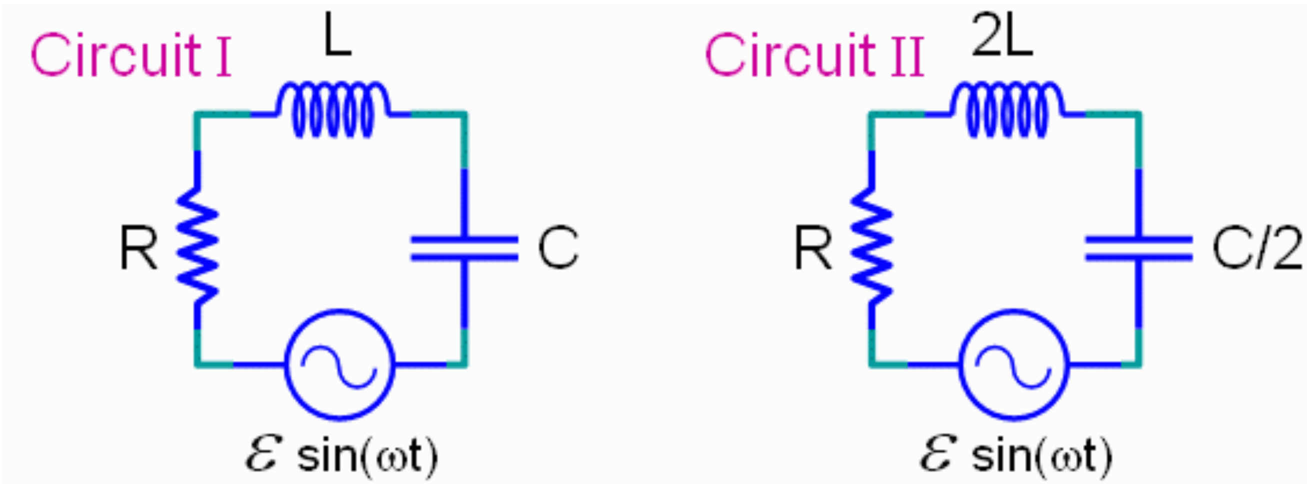
A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

Voltage in second circuit will be twice that of the first because of the $2L$ compared to L.





CheckPoint 1c

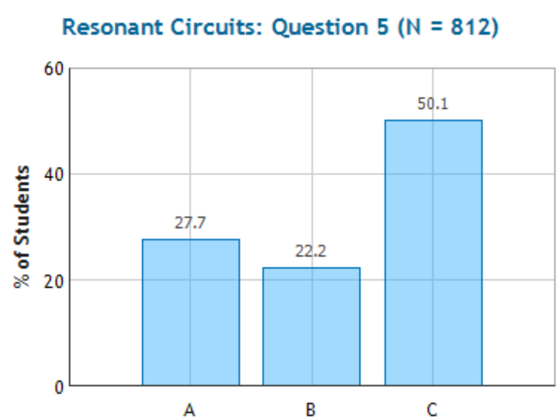


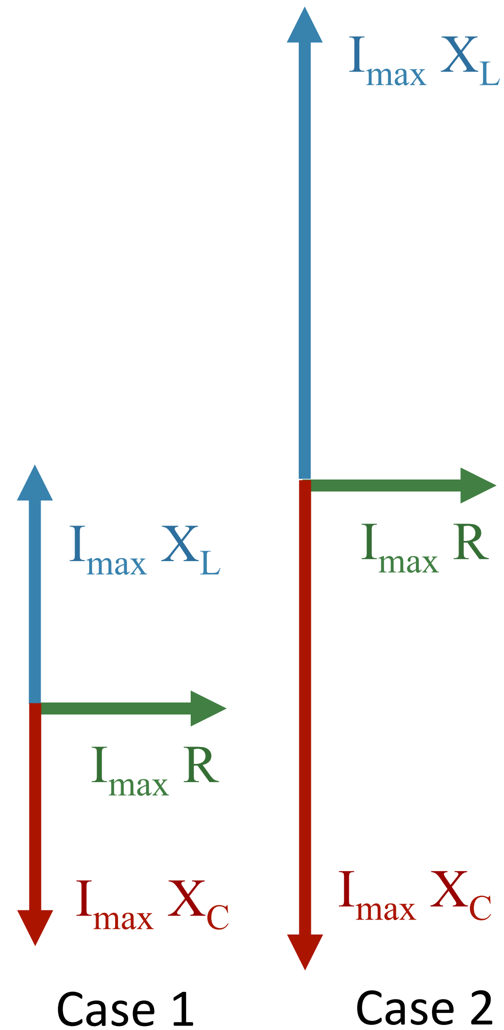
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

Compare the peak voltage across the capacitor in the two circuits

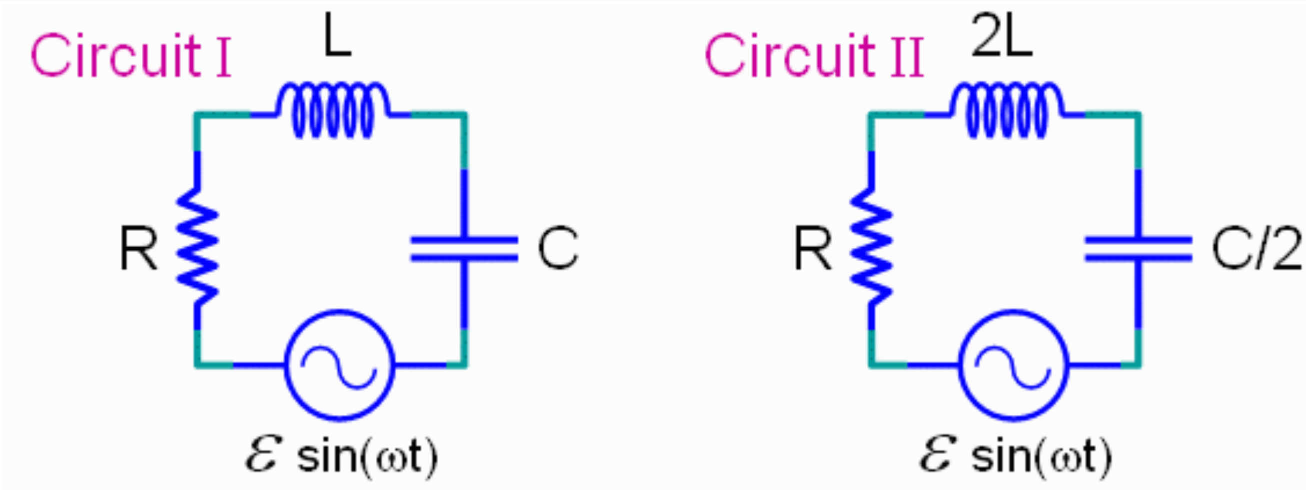
A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

The peak voltage will be greater in circuit 2 because the value of X_C doubles.





CheckPoint 1D



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

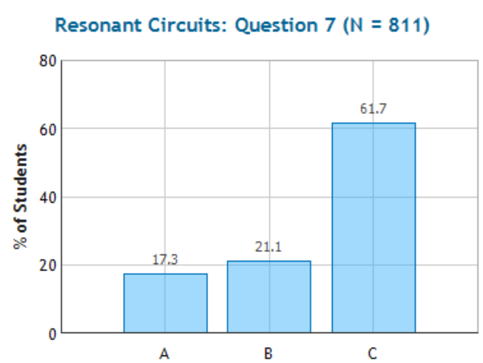
At the resonant frequency, which of the following is true?

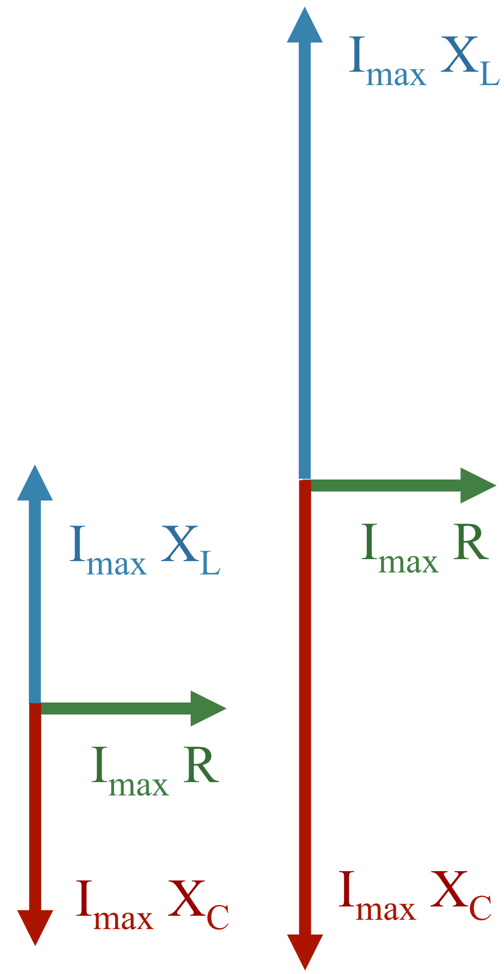
A. Current leads voltage across the generator

B. Current lags voltage across the generator

$\boxed{\textit{C.}\ \text{Current is in phase with voltage across the generator}}$

The voltage across the inductor and the capacitor are equal when at resonant frequency, so there is no lag or lead.



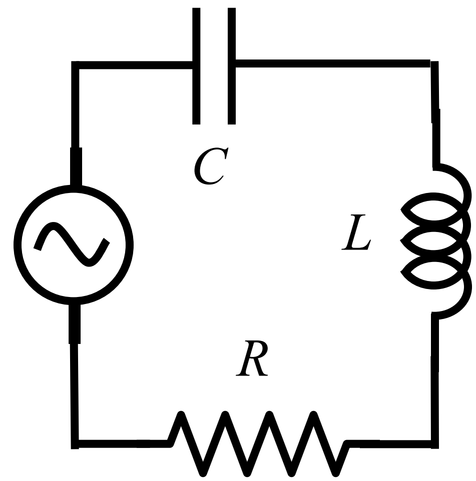


Case 1 Case 2

Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I, V are always in phase!



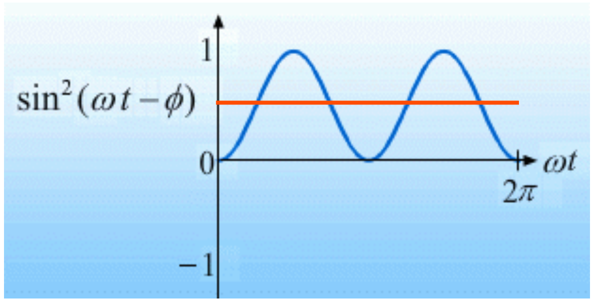
$$P = IV$$
$$= I^2 R$$

Average Power

Inductor and Capacitor = 0 ($\langle \sin(\omega t) \cos(\omega t) \rangle = 0$)

Resistor

$\langle I^2 R \rangle = \langle I^2 \rangle R = \frac{1}{2} I^2_{peak} R$



RMS = Root Mean Square

$I_{peak} = I_{rms}$ sqrt(2)

$\langle I^2 R \rangle = I_{rms}^2 R$

Power Line Calculation

If you want to deliver 1,500 Watts at 100 Volts over transmission lines w/ resistance of 5 Ohms. How much power is lost in the lines?

- Current Delivered: $I = P/V = 15$ Amps
- Loss = IV (on line) = $I^2 R = 15*15 * 5 = 1{,}125$ Watts!

If you deliver 1,500 Watts at 10,000 Volts over the same transmission lines. How much power is lost?

- Current Delivered: $I = P/V = .15$ Amps
- Loss = IV (on line) = $I^2R = 0.125$ Watts

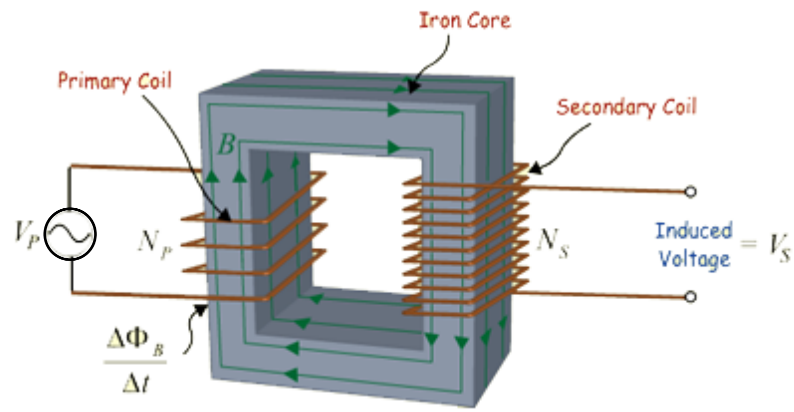
DEMO

Transformers

Application of Faraday's Law

- Changing EMF in Primary creates changing flux
- Changing flux, creates EMF in secondary

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$



Efficient method to change voltage for AC.

Power Transmission   Loss = I^2R

Power electronics

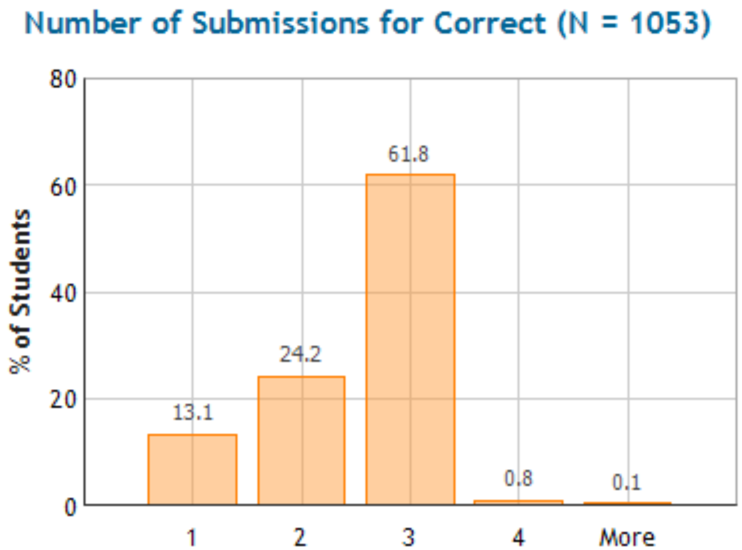
Demo

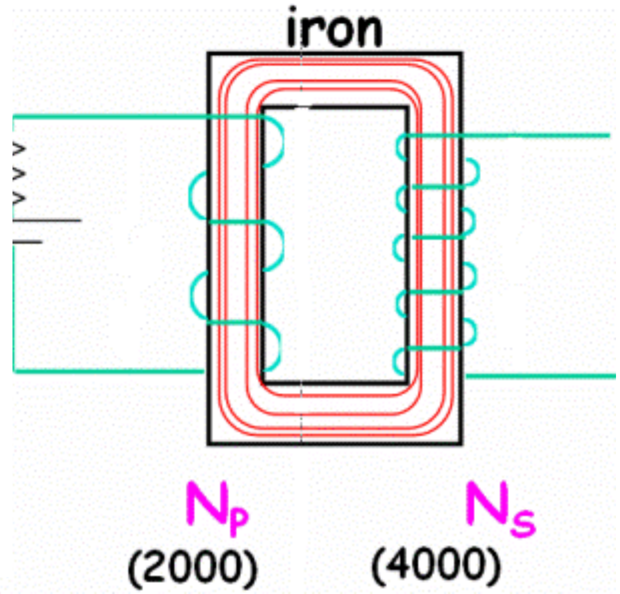
Tranformers

I don't understand the second question of the prelecture. It said something about the changing currents...like changing flux?

For coils connected to battery, what is voltage in secondary?

A) 0 Volts
B) 6 Volts
C) 12 Volts





Wrong Answer: 2 ✗

Feedback: Actually if this was connected to an AC source, the secondary would have twice the primary voltage. However, the battery voltage does not change in time, so after the battery has been connected for a while, there will not be a changing current to create a voltage across the secondary.

Follow-Up from last lecture

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

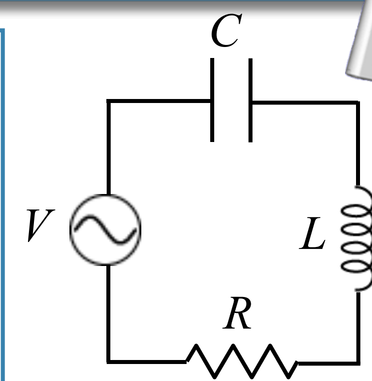
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

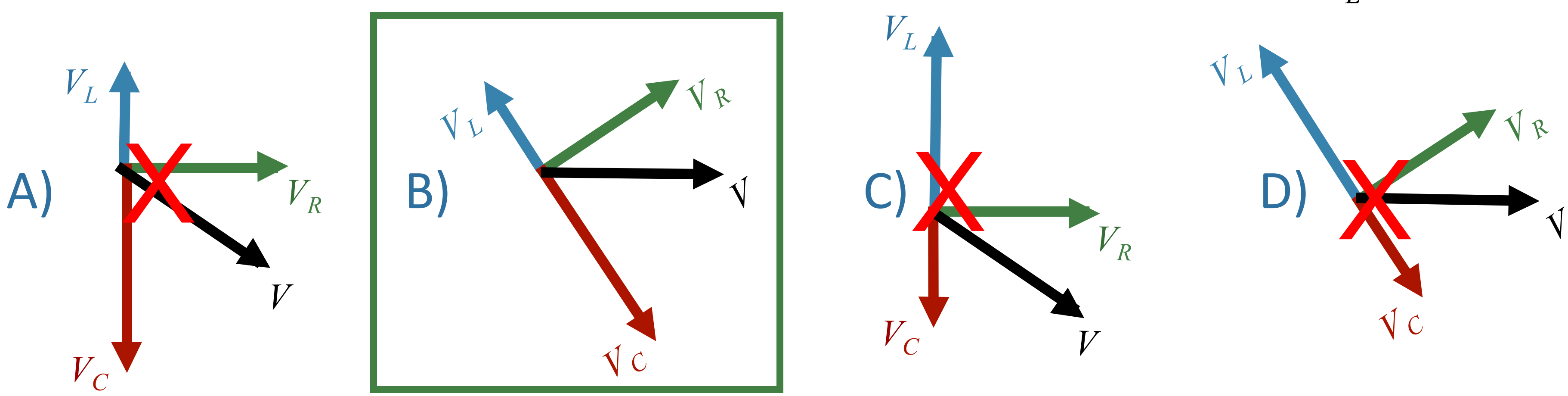
L and R are unknown.

What does the phasor diagram look like at t = 0? (assume $V = V_{max}\sin\omega t$)



$$R = 25\sqrt{2}\ k\Omega$$

$$X_L = 15\sqrt{2}\ k\Omega$$



$V = V_{max}\sin\omega t$ -> V is horizontal at $t = 0$ ($V = 0$)

$$\vec{V} = \vec{V}_L + \vec{V}_C + \vec{V}_R \quad \longrightarrow \quad V_L < V_C \text{ if current leads generator voltage}$$

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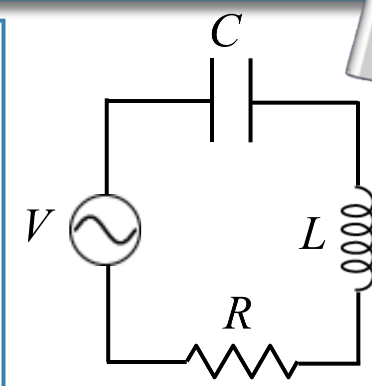


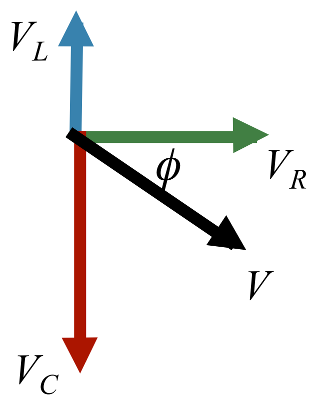
How should we change ω to bring circuit to resonance?

A) decrease ω

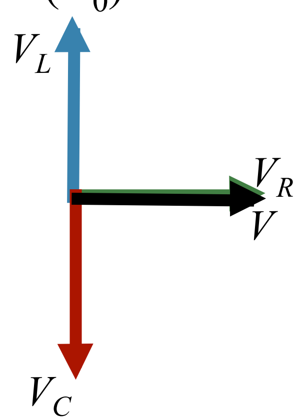
B) increase ω

C) Not enough info

Original ω



At resonance (ω_0)



At resonance

$X_L = X_C$

$\downarrow$

X_L increases

X_C decreases $\rightarrow$ ω increases

More Follow-Up

Consider the harmonically driven series LCR circuit shown.

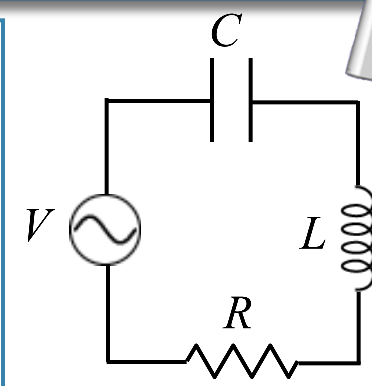
$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2)) $\rightarrow$ $X_C = 40\sqrt{2}\ k\Omega$

The current leads generator voltage by 45^{o} (cos = sin = 1/sqrt(2))

L and R are unknown.



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$

By what factor should we increase ω to bring circuit to resonance?
i.e. if $\omega_0 = f\omega$, what is f?

A) $f = \sqrt{2}$

B) $f = 2\sqrt{2}$

C) $\boxed{f = \sqrt{\frac{8}{3}}}$

D) $f = \sqrt{\frac{8}{5}}$

If ω is increased by a factor of f:

X_L increases by factor of f

X_C decreases by factor of f

$\rightarrow$

$$X_L \rightarrow f \cdot 15\sqrt{2}$$

$$X_C \rightarrow (1/f) \cdot 40\sqrt{2}$$

At resonance $X_L = X_C$

$$\rightarrow 15f = \frac{40}{f} \rightarrow f^2 = \frac{40}{15} \rightarrow f = \sqrt{\frac{8}{3}}$$

Current Follow-Up

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

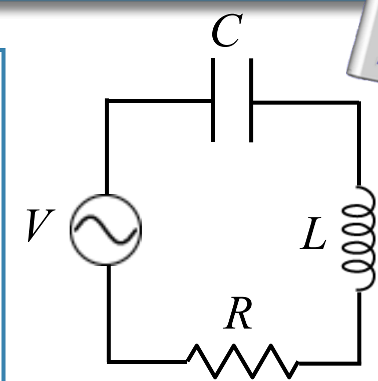
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2)) $\longrightarrow$ $X_C = 40\sqrt{2}\ k\Omega$

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

L and R are unknown.

What is the maximum current at resonance



$$R = 25\sqrt{2}\ k\Omega$$

$$X_L = 15\sqrt{2}\ k\Omega$$

$$\omega_0 = \sqrt{\frac{8}{3}}\,\omega$$

A) $I_{\max}(\omega_0) = \sqrt{2}\,mA$

B) $\boxed{I_{\max}(\omega_0) = 2\sqrt{2}\,mA}$

C) $I_{\max}(\omega_0) = \sqrt{\frac{8}{3}}\,mA$

At resonance
$X_L = X_C$ $\longrightarrow$ $Z = R$ $\longrightarrow$ $I_{\max}(\omega_0) = \dfrac{V_{\max}}{R} = \dfrac{100}{25\sqrt{2}} = 2\sqrt{2}\,mA$