Electricity & Magnetism Lecture 2

Today's Concepts:

A) The Electric Field

B) Continuous Charge Distributions

Lab manuals will be available in bookstore this weekend!

Comments: Yours and Ours

"Is it possible to put in exponents for smart physics? so instead of typing .0001 can we type 1x10^-4 ?." YES 1e-4

"I'd like to go over examples that have more than two charges acting on a test charge " OK!

"ways to remember equations? I recommend the formula sheet see home page

"When answering the checkpoints, I was very careful to avoid saying "the force of the field," or anything along those lines, because I'm not sure what exactly that implies. Does it make sense to talk about the force exerted by an electric field? If so, what exactly does it mean?." Yes, It means the electric field is the key player!

"Is it necessary for us to know the calculus that goes into making the integral from -infinity to +infinity of k(dQ/s^2)*cos(θ) become 2kλ/r? If so, can you go over it quickly? If not, never mind:) You must set up the integral, but wolfram alpha can then integrate it for you

"Will exams be more like homework or discussion questions? This week's discussion was pretty tough but I am really understanding the homework." Yes! Most like old exams

The integration is a little weird. I hope we talk about continuous charge distribution. Also the checkpoints are super hard. I feel like i don't learn enough from the prelectures.

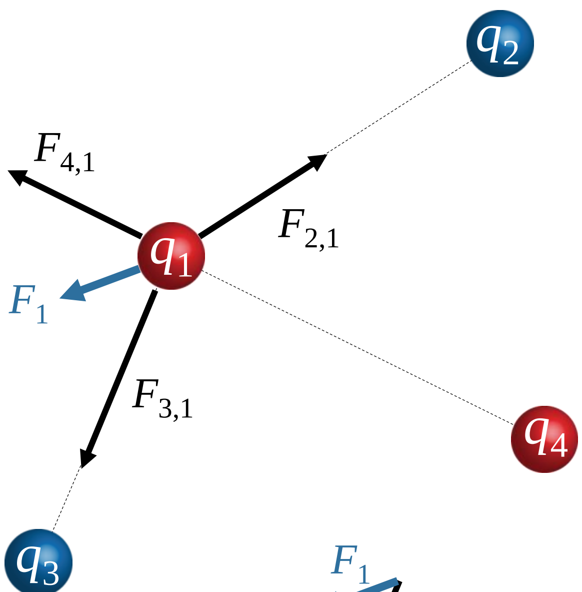
Yes! Yes! Yes! No…your brain is doing a lot you don't know about

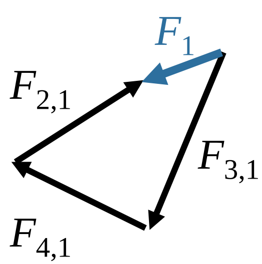
Please respect your fellow students – close laptops, cell phones on "airplane" mode

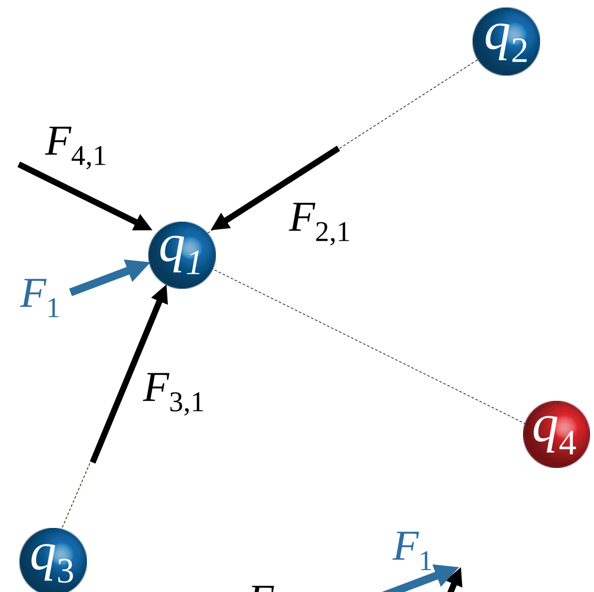
Don't panic!......Don't be intimidated by integrals!

Coulomb's Law (from last time)

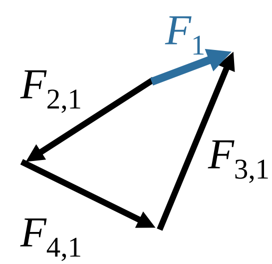
If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:







$+q_1 \rightarrow -q_1 \Rightarrow$ direction reversed



MATH:

$$\vec{F}_1 = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{k q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{k q_1 q_4}{r_{14}^2}\hat{r}_{14} \longrightarrow \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{k q_2}{r_{12}^2}\hat{r}_{12} + \frac{k q_3}{r_{13}^2}\hat{r}_{13} + \frac{k q_4}{r_{14}^2}\hat{r}_{14}$$

Electric Field

"Does the point we use to find the field always assumed to be a positive point charge? In other words, when it refers to a point, can we assume the charge is positive?"

The electric field E at a point in space is simply the force per unit charge at that point.

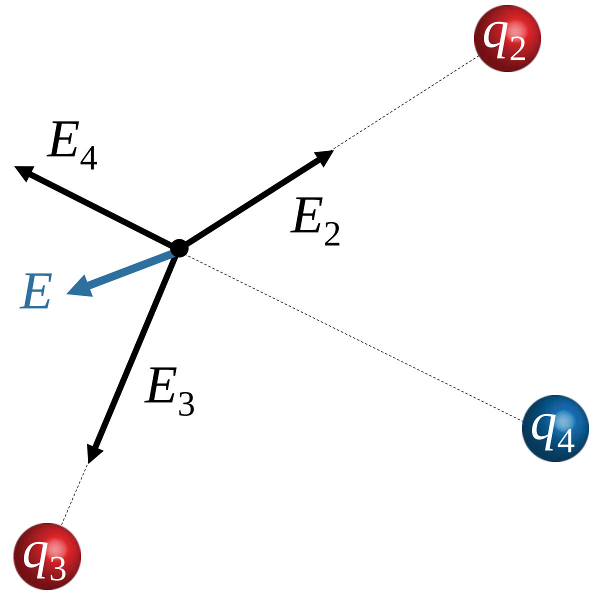
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

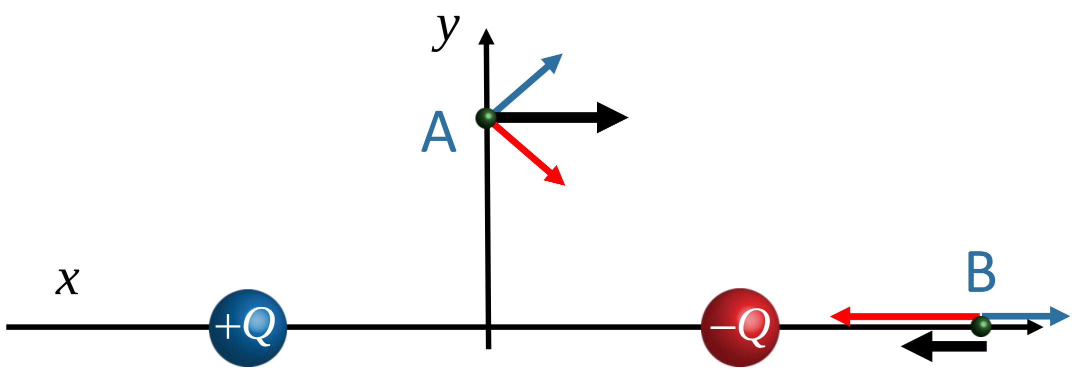
Superposition

$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$$



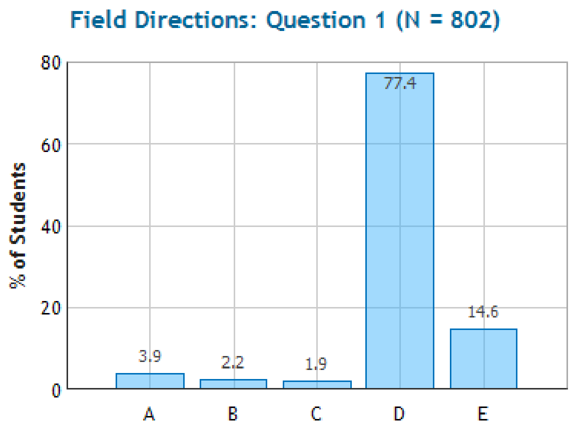
Field points toward negative and Away from positive charges.

CheckPoint



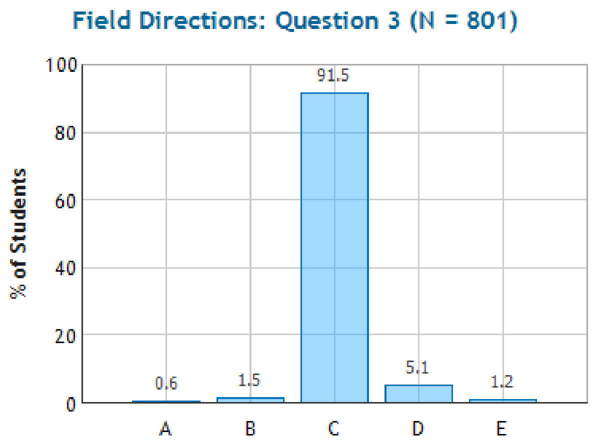
Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is direction at point A



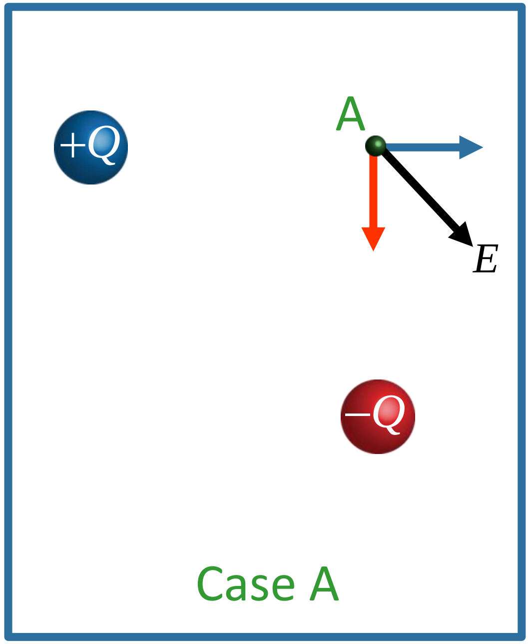
a) Up b) down c) Left d) Right e) zero

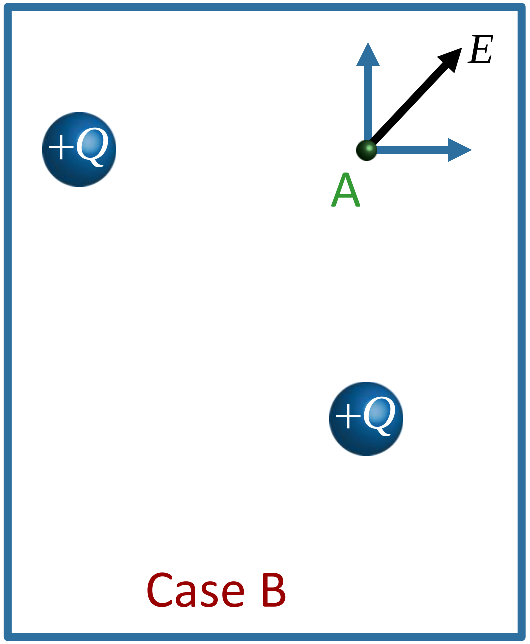
What is direction at point B



a) Up b) down c) Left d) Right e) zero

CheckPoint



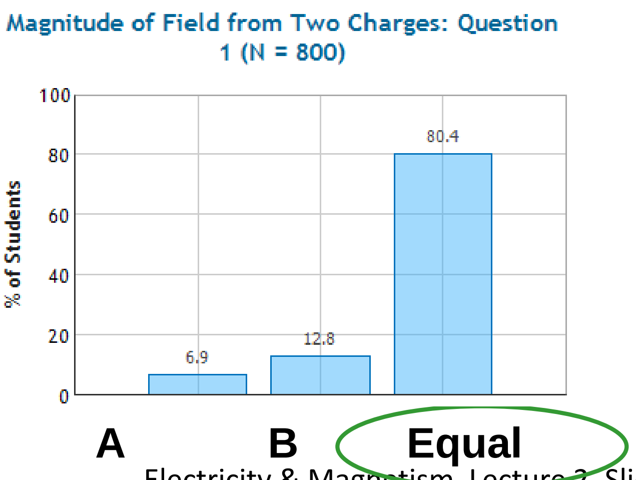


In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest? (Select C if you think they are equal)

"The electric field cancels out in case two because of the two repulsive positive charges, so it is larger in case 1."

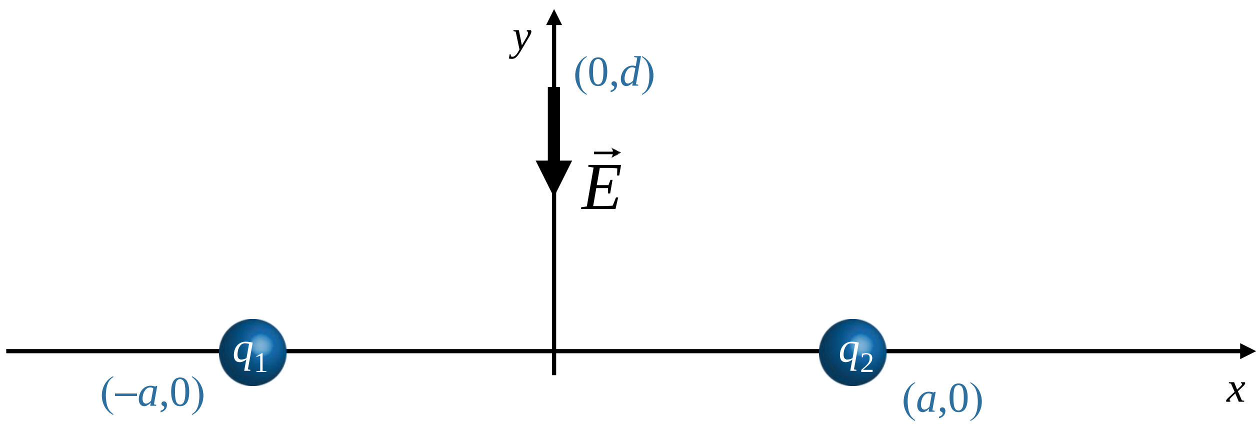
"the components of case 2 are additive while case 1 cancels itself out."

"The resulting forces from each charge will be perpendicular to each other, so the x and y components will not cancel in either case.



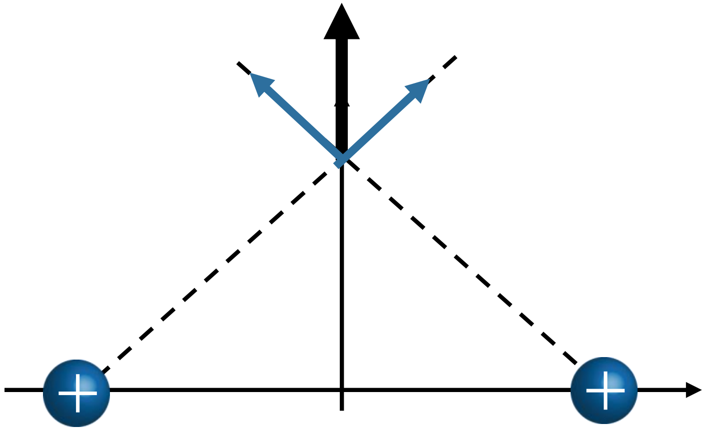
Two Charges

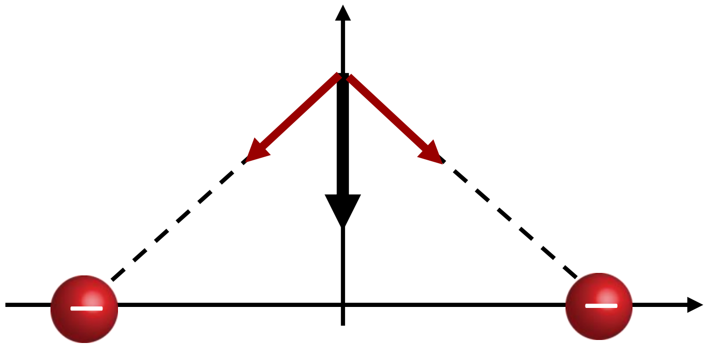
Two charges q_1 and q_2 are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.

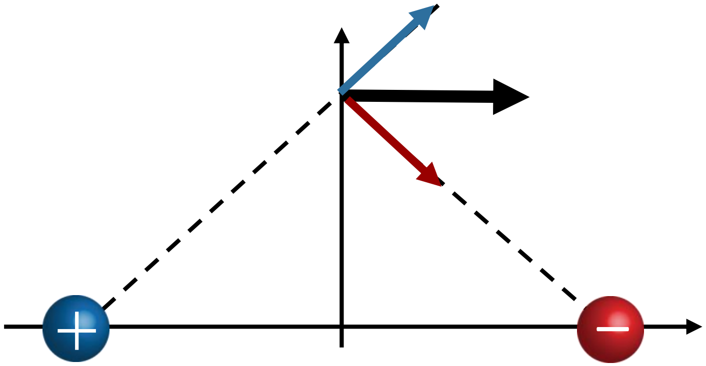


Which of the following statements is true:

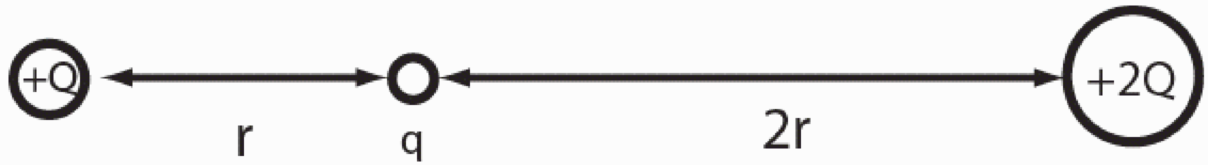
A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related







CheckPoint

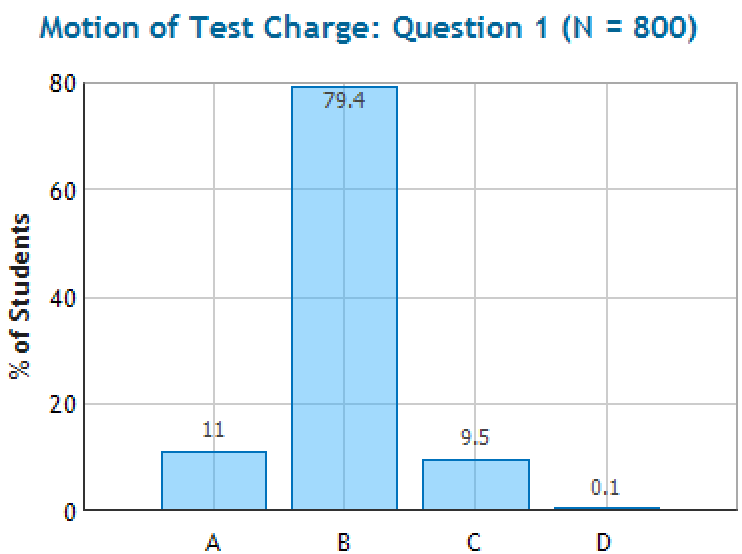


A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q. How will the test charge move immediately after being released?

8) How will the test charge move immediately after being released?

○ to the left ○ to the right ○ stay still ○ other

A B C D



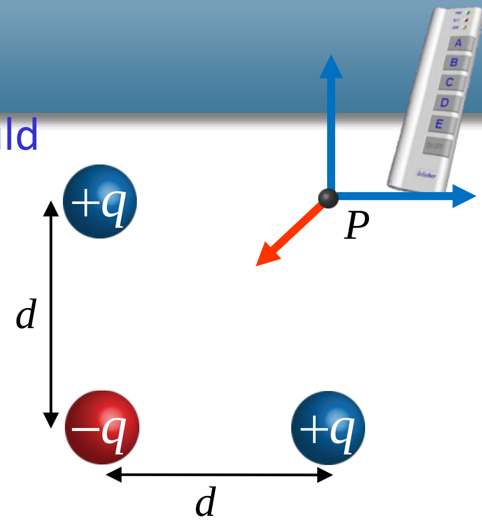
(A left)The value of r has a greater weight on the direction of the resultant vector because E is proportional to 1/(r^2). Therefore, the resultant force on q points to the left

"(B right) Q applies a greater electromagnetic force because it is closer to q. Since the force due to Q points to the right, q will initially be pushed to the right."

"(C Still) The charges and their distances are proportional, so q should not move after they are released."

Example

"More examples of electric fields of different charge distributions would be nice (for example, charges arranged in a circle or square).!"



What is the direction of the electric field at point P, the unoccupied corner of the square?

A) ↗

B) ↙

C) $E = 0$

D) Need to know d

E) Need to know d & q

Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2} \cos\frac{\pi}{4} \right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2} \sin\frac{\pi}{4} \right)$$

Continuous Charge Distributions

"Can we go over electric fields for uniform distributions in more depth? Especially how to find and set up the integral to calculate the magnitude of the electric field."

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

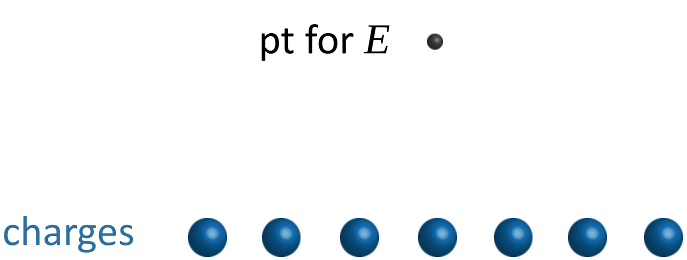
WHAT DOES THIS MEAN ?

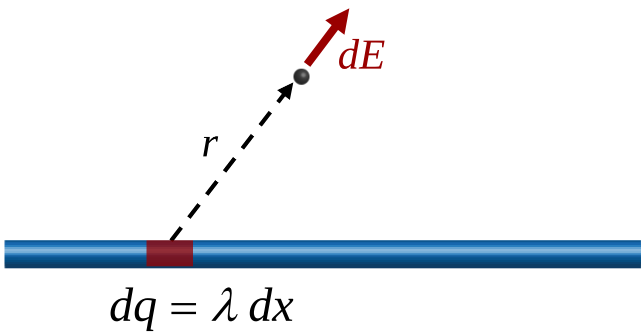
Integrate over all charges (dq)

r is vector from dq to the point at which E is defined

Linear Example:

$$\lambda = Q/L$$





Charge Density

"Can we further discuss linear charge density?"

Linear $(\lambda = Q/L)$ Coulombs/meter

Surface $(\sigma = Q/A)$ Coulombs/meter^2

Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?.

A) A sphere w/ radius 4 meters and volume charge density ρ = 2 C/m^3

B) A sphere w/ radius 4 meters and surface charge density σ = 2 C/m^2

C) Both A) and B) have the same net charge.

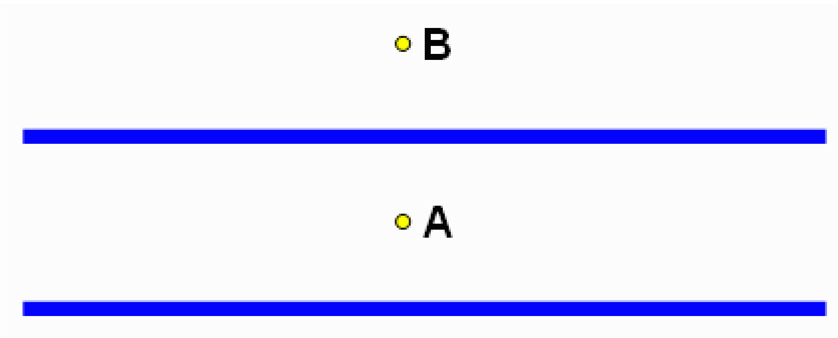
$$Q_A = \rho V = \rho \tfrac{4}{3}\pi R^3$$

$$Q_B = \sigma A = \sigma 4\pi R^2$$

$$\frac{Q_A}{Q_B} = \frac{\rho \frac{4}{3}\pi R^3}{\sigma 4\pi R^2} = \frac{1}{3}\frac{\rho}{\sigma}R = \frac{1}{3}\frac{2}{2}4 = \frac{4}{3}$$

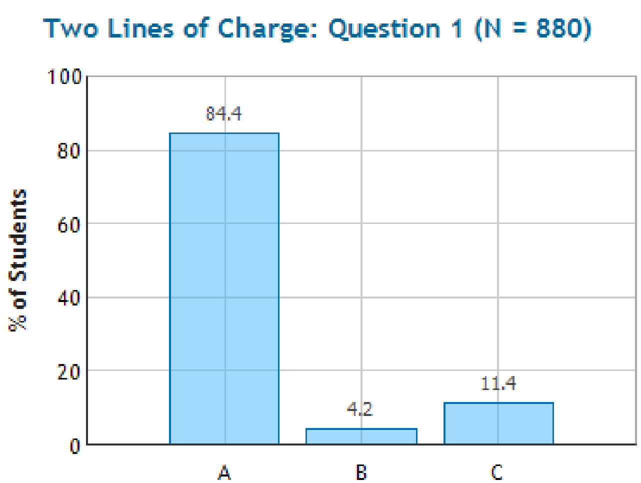
CheckPoint

Two infinite lines of charge are shown below.



Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown. How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- $E_A < E_B$
- $E_A = E_B$
- $E_A > E_B$

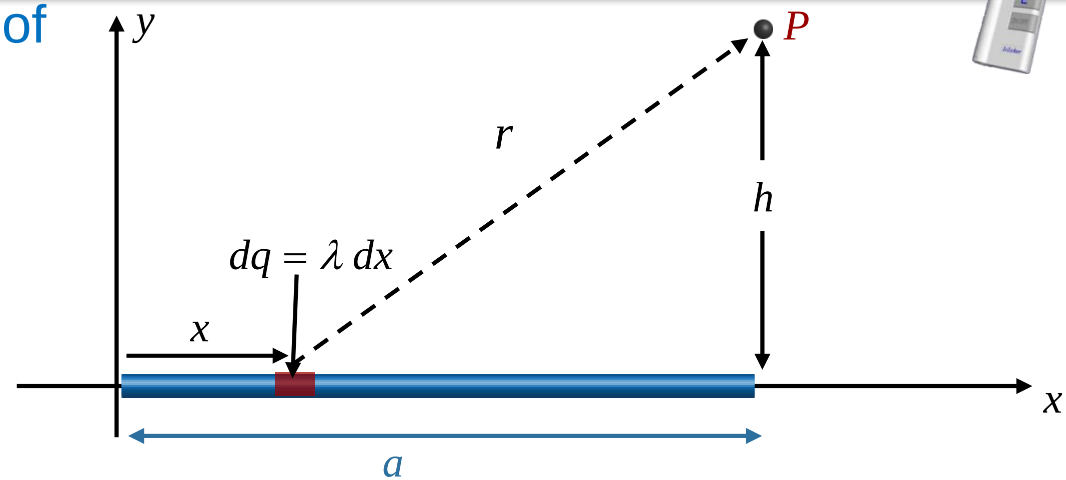


"I was wondering if we could go over the 3rd question from the pre-lecture? I was mainly confused on how they got the denominator for the answer."

"All y-components of e-field acting on A due to both infinite lines of charge cancel because A is between the two charge distributions. This is not true for B since B is above both charge distributions. Therefore, Ea < Eb."

Calculation

"PLEASE, go over the integration of the electric field for the line."



Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?

We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is $\dfrac{dq}{r^2}$?

A) $\dfrac{dx}{x^2}$

B) $\dfrac{dx}{a^2 + h^2}$

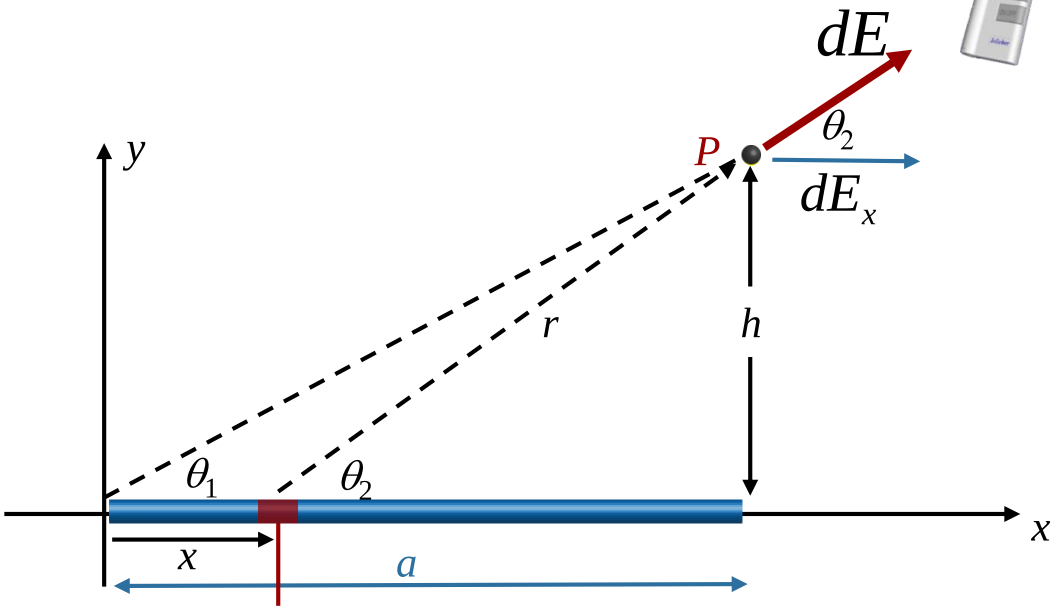
C) $\dfrac{\lambda dx}{a^2 + h^2}$

D) $\dfrac{\lambda dx}{(a-x)^2 + h^2}$

E) $\dfrac{\lambda dx}{x^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

We want:

$$E_x = \int dE_x$$

What is dE_x?

A) $dE \cos\theta_1$

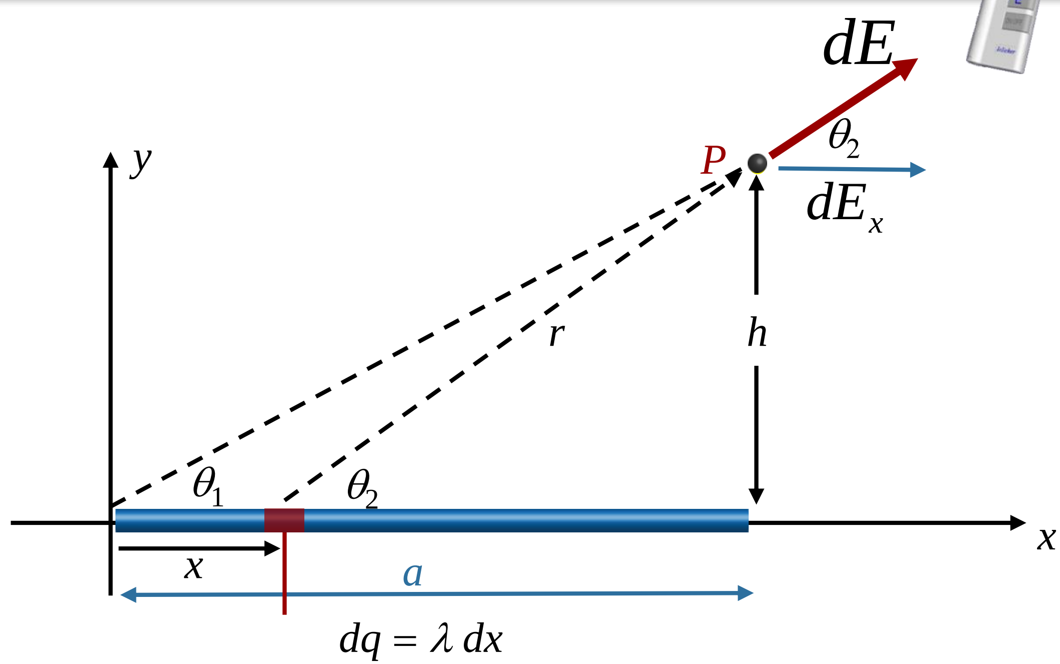
B) $dE \cos\theta_2$

C) $dE \sin\theta_1$

D) $dE \sin\theta_2$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

What is E_x ?

A) $\int_0^a \frac{k\lambda \cos\theta_2 dx}{(a-x)^2 + h^2}$

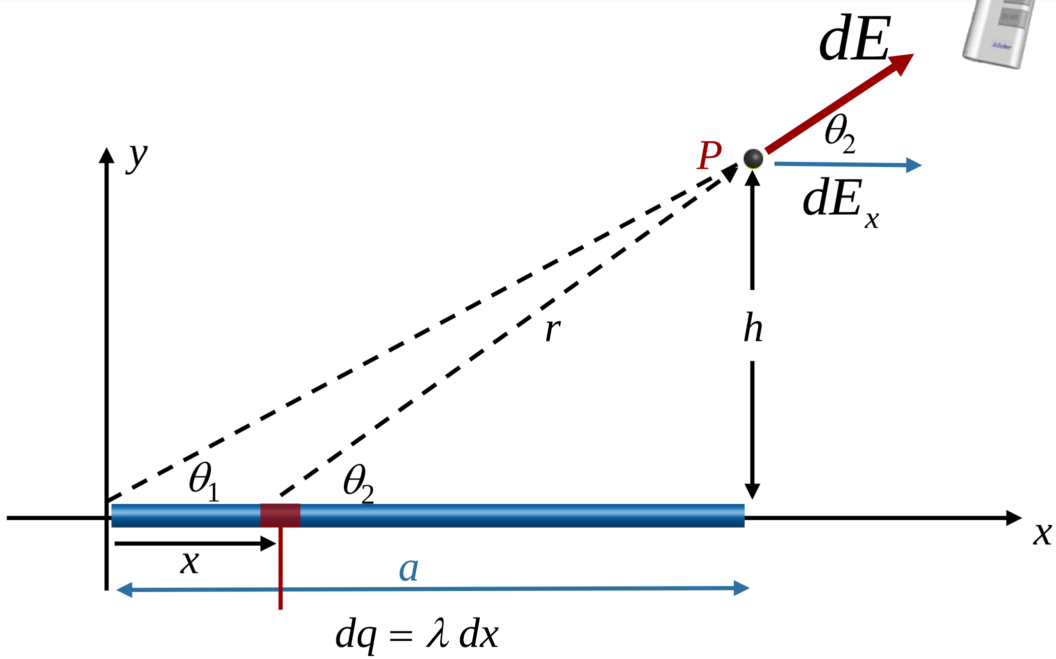
B) $\lambda k \cos\theta_2 \int_0^a \frac{dx}{h^2 + (x-a)^2}$

C) A and B are both OK

$\cos\theta_2$ DEPENDS ON x!

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

What is $\cos\theta_2$?

A) $\dfrac{x}{\sqrt{a^2+h^2}}$

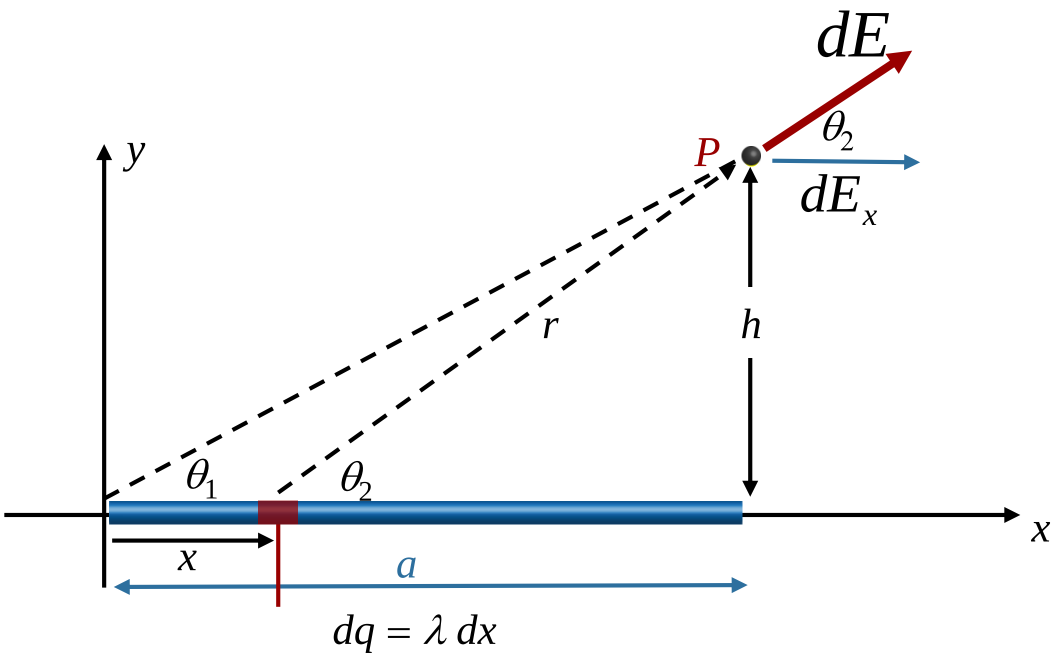
B) $\dfrac{a-x}{\sqrt{(a-x)^2+h^2}}$

C) $\dfrac{a}{\sqrt{a^2+h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2+h^2}}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know: $$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

$$\cos\theta_2 = \frac{a-x}{\sqrt{(a-x)^2 + h^2}}$$

Putting it all together

$$E_x(P) = \lambda k \int_0^a dx \frac{a-x}{\left((a-x)^2 + h^2\right)^{3/2}}$$

$$E_x(P) = \frac{\lambda k}{h}\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right)$$

Homework Problem

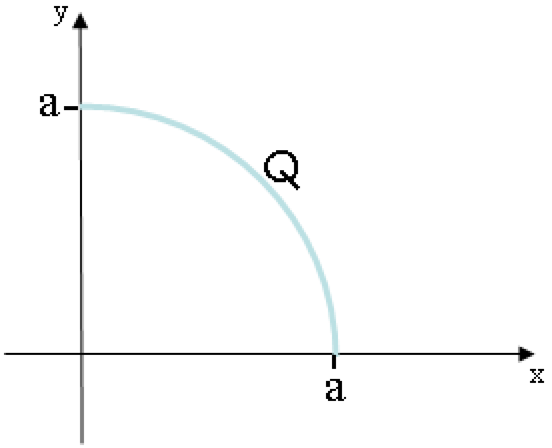
Homework: Coulomb's Law

Deadline: 100% until Tuesday, January 22 at 8:00 AM

Electric Field from Arc of Charge

1 2 3 4 5

A total charge Q = -4.2 μC is distributed uniformly over a quarter circle arc of radius a = 7.7 cm as shown.



Please try to solve this problem on your own. Only use help if you get stuck, and have thought hard about it for at least 10 minutes!

1) What is λ the linear charge density along the arc?

C/m Submit

2) What is E_x, the value of the x-component of the electric field at the origin (x,y) = (0,0) ?

N/C Submit Help