Your Comments

I have seen the light.

Can you explain the EM wave picture? I'm really not sure how to interpret that graph/wave/thing. Also, elaborate on the differences between electric flux and magnetic flux? Do they affect each other directly, can you have one without the other, etc?

When do we learn the differential forms of Maxwell's Equations?

The waves equations and more and how they apply to em waves

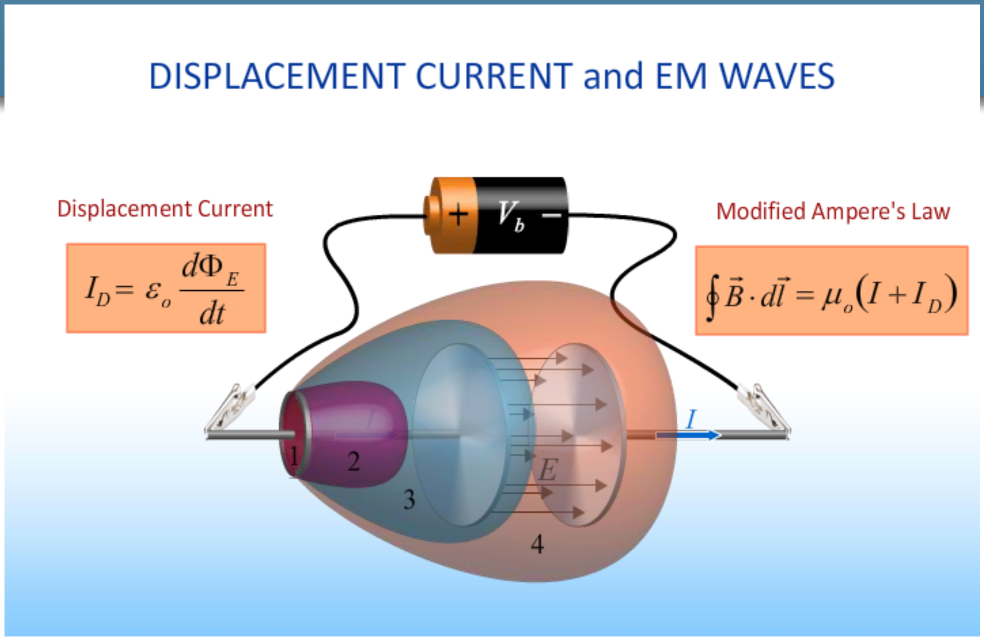
Please go over the displacement current more and how it compares in magnitude to the current through the circuit. Also, the magnitude of the electric and magnetic fields at different locations at different points on the wave. For the self propagating wave do the electric and magnetic fields cause the other to oscillate continuously so that it doesn't need a medium besides space?

You been lyin' to us about Ampere's Law the whole time?!

Isn't it technically impossible to define a reference frame at all for EM waves if they always travel at c? It seems like you could look from say a photon going one direction and another in the opposite direction which should then be going 2c with respect to that photon which then doesn't work because it is faster than c. I find that pretty weird.

I'm sorry but the way they got the speed of light out of the two constants we know blew my freaking mind. It was right under our noses all along. *mind explodes*

Physics 212
Lecture 22



What We Knew Before Prelecture 22

Pre-Maxwell's Equations

Gauss' Law for E Fields

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

Gauss' Law for B Fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

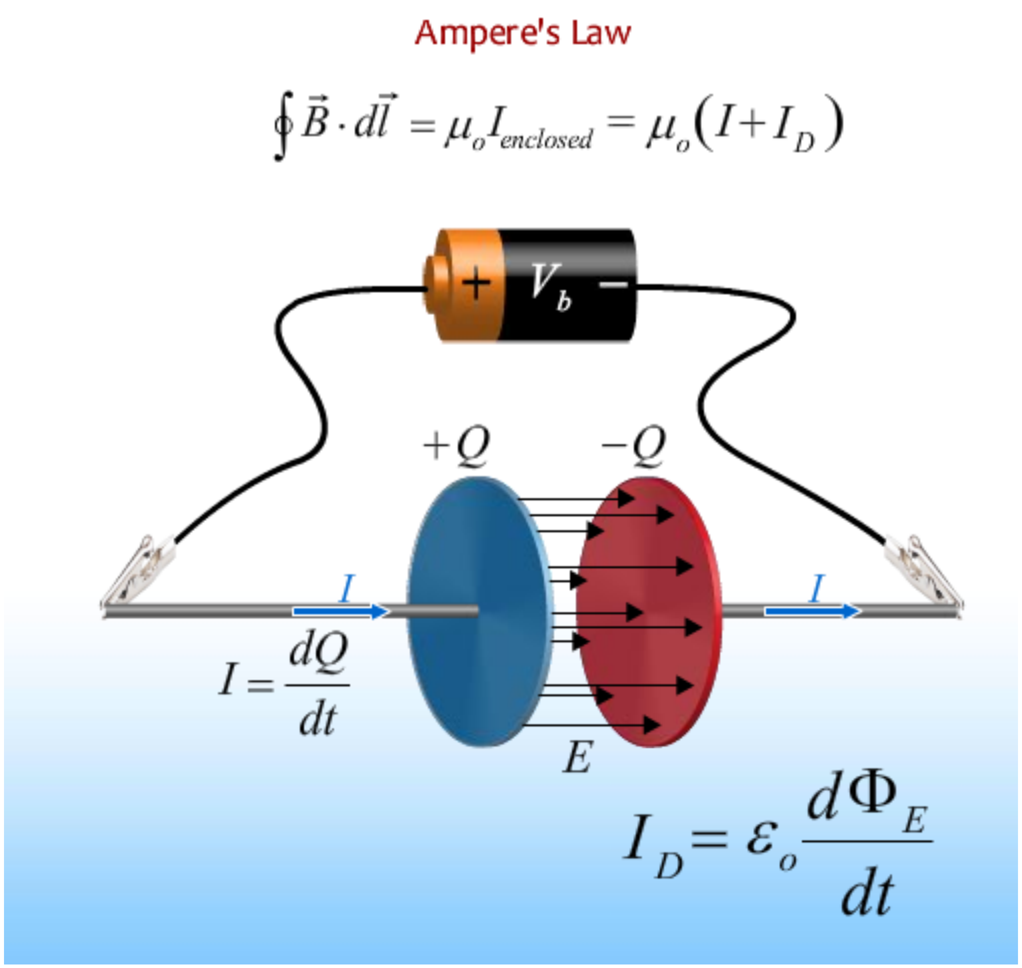
Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

After Prelecture 21: Modify Ampere's Law



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\downarrow$$

$$\Phi = EA = \frac{Q}{\varepsilon_0}$$

$$\downarrow$$

$$Q = \varepsilon_0 \Phi$$

$$\downarrow$$

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi}{dt} \equiv I_D$$

Displacement Current

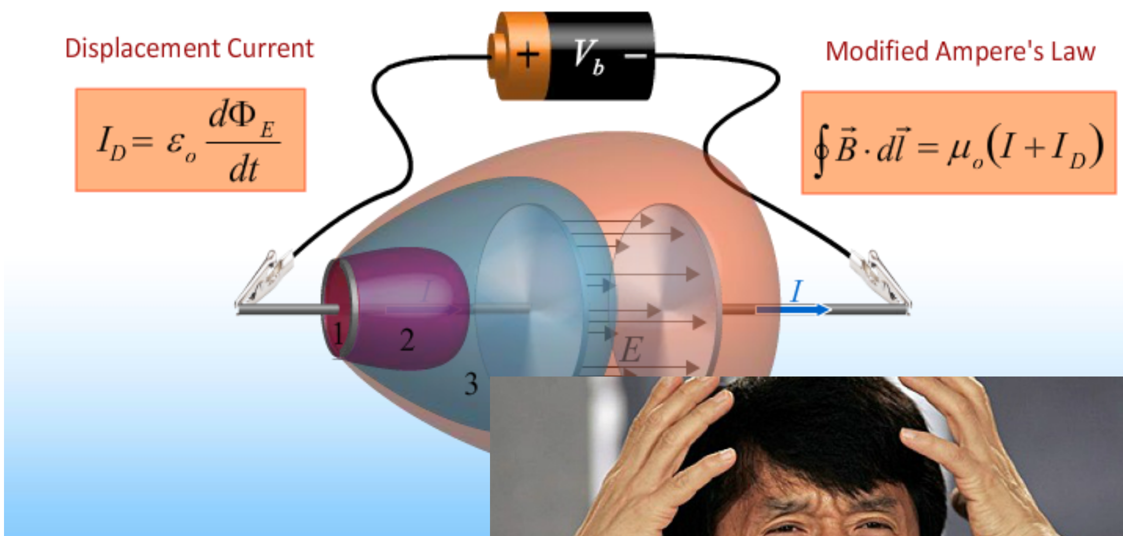
Real Current: Charge Q passes through area A in time t:

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area A' changes in time

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

$\updownarrow$

Modified Ampere's Law

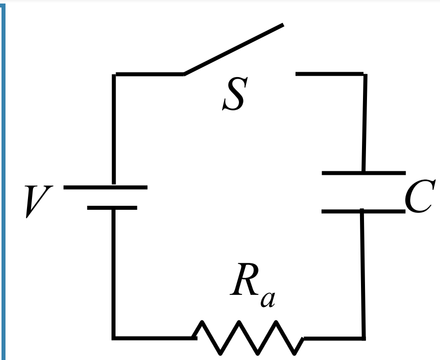
$$\oint \vec{B}\cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

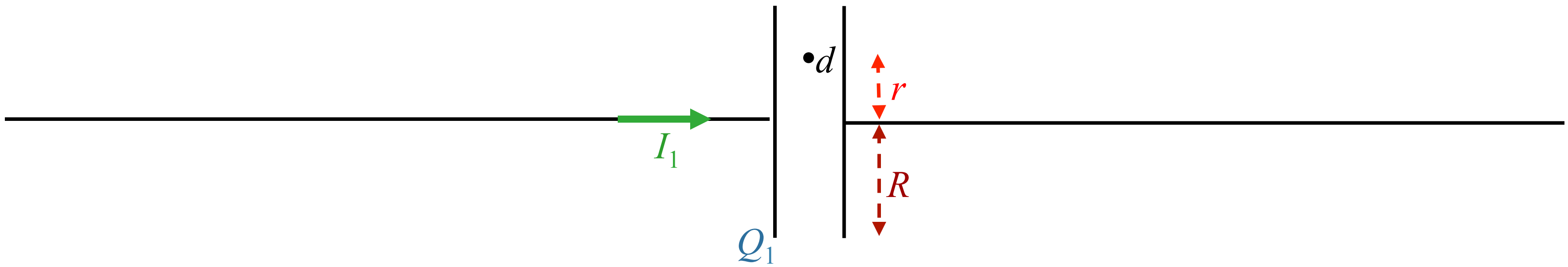
Free space

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.

At time t_1, what is the magnetic field B_1 at a radius r (point d) in between the plates of the capacitor?



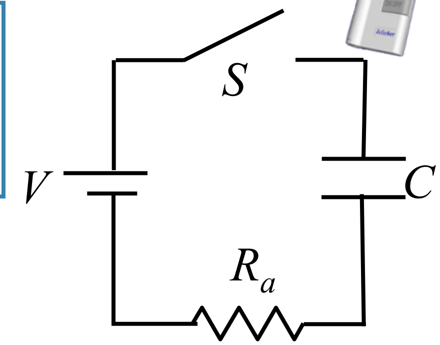


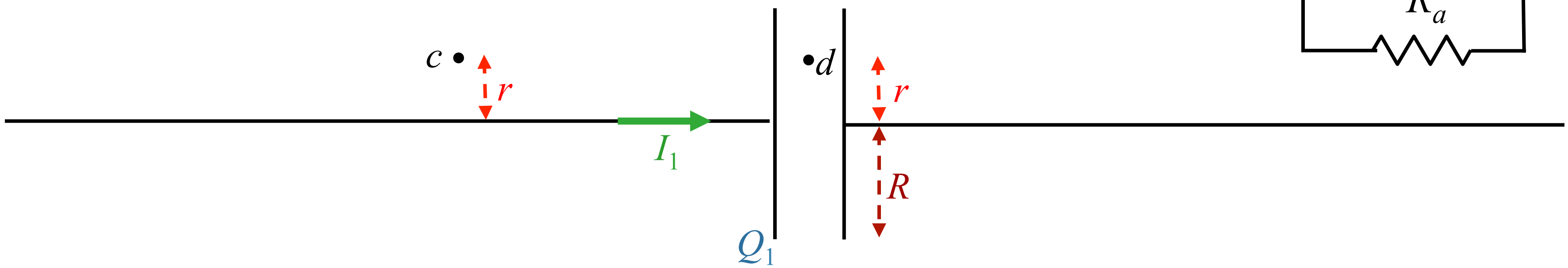
Conceptual and Strategic Analysis

Charge Q_1 creates electric field between the plates of C
Charge Q_1 changing in time gives rise to a changing electric flux between the plates
Changing electric flux gives rise to a displacement current I_D in between the plates
Apply (modified) Ampere's law using I_D to determine B

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.
Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.



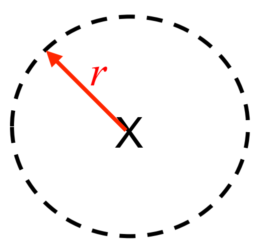


Compare the magnitudes of the B fields at points c and d.

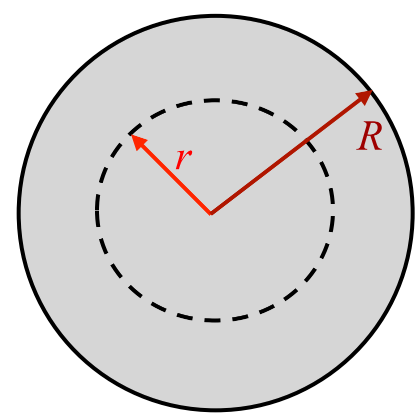
A) $B_c < B_d$ B) $B_c = B_d$ C) $\boxed{B_c > B_d}$

What is the difference?
Apply (modified) Ampere's Law

point c:
I(enclosed) $= I_1$

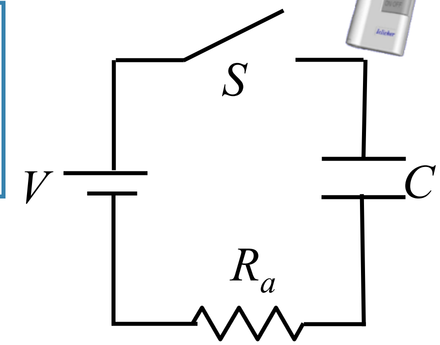


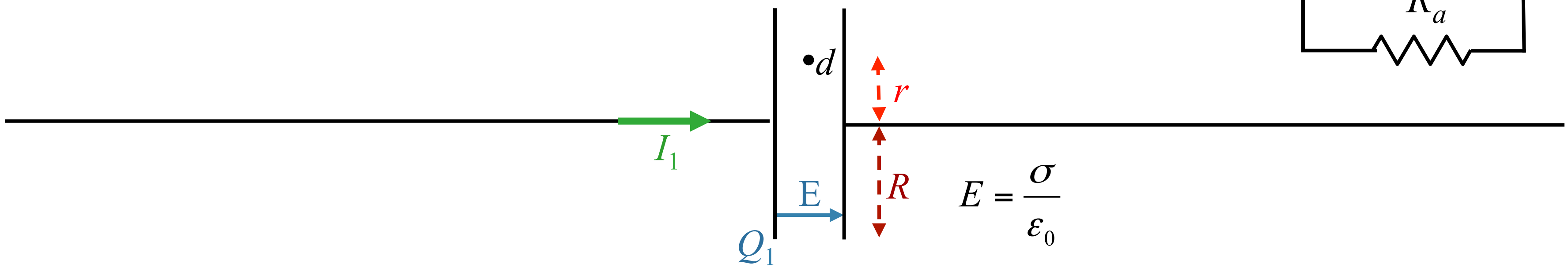
point d:
I_D(enclosed) $< I_1$



Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$E = \frac{\sigma}{\varepsilon_0}$$

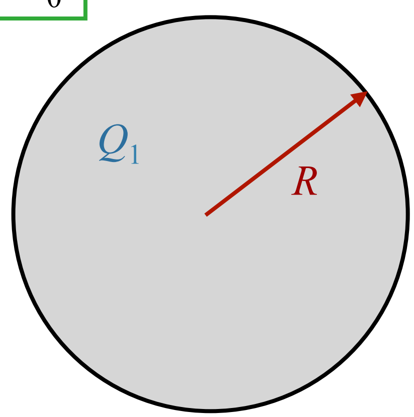
What is the magnitude of the electric field between the plates?

A) $E = \dfrac{Q_1}{\pi R^2 \varepsilon_0}$

B) $E = \dfrac{Q_1 \pi R^2}{\varepsilon_0}$

C) $E = \dfrac{Q_1}{\varepsilon_0}$

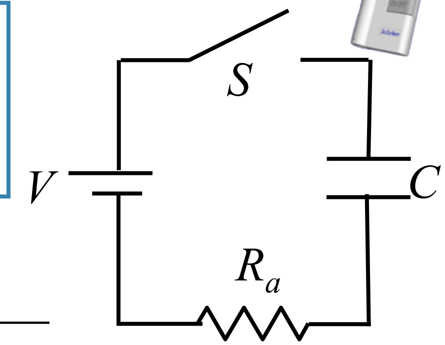
D) $E = \dfrac{Q_1}{r}$

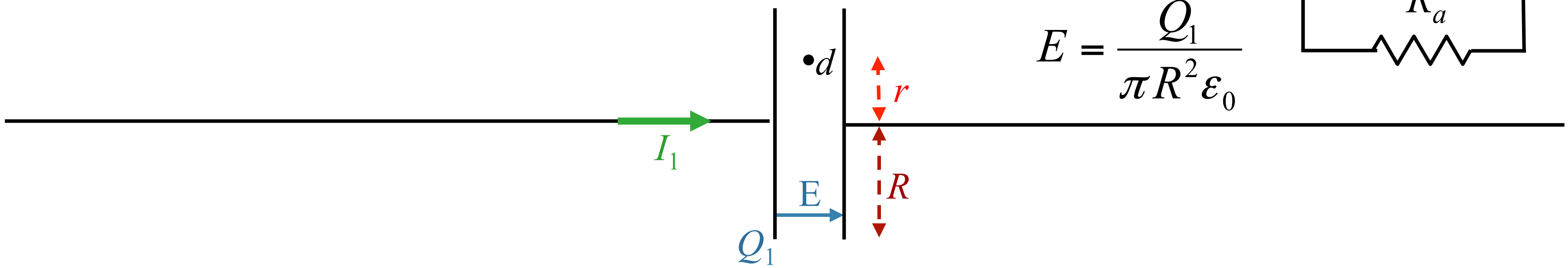


$$E = \frac{\sigma}{\varepsilon_0} \rightarrow \sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi R^2} \rightarrow E = \frac{Q_1}{\varepsilon_0 \pi R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$E = \frac{Q_1}{\pi R^2 \varepsilon_0}$$

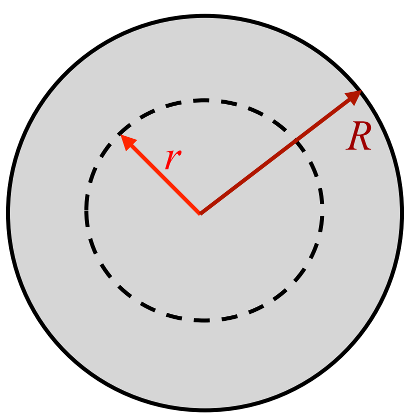
What is the electric flux through a circle of radius r in between the plates?

A) $\Phi_E = \dfrac{Q_1}{\varepsilon_0}\pi r^2$

B) $\Phi_E = \dfrac{Q_1}{\varepsilon_0}\pi R^2$

C) $\boxed{\Phi_E = \dfrac{Q_1 r^2}{\varepsilon_0 R^2}}$

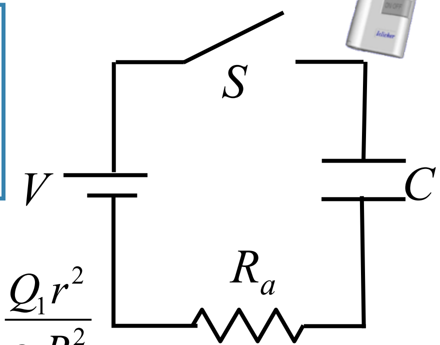
D) $\Phi_E = \dfrac{Q_1 \pi r^2}{\varepsilon_0 R^2}$



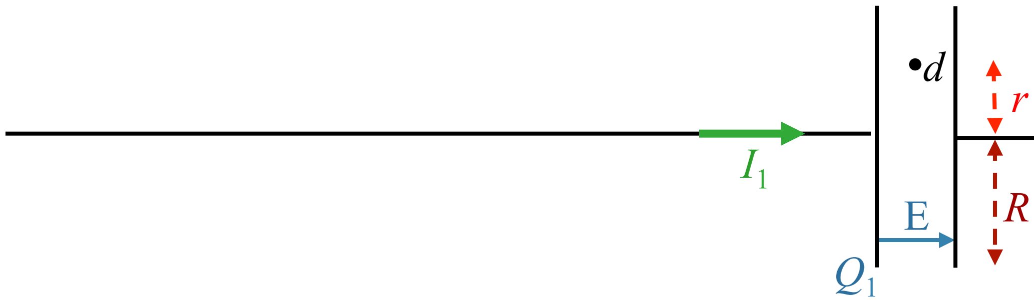
$$\Phi_E = \vec{E} \cdot \vec{A} \;\rightarrow\; \Phi_E = \frac{Q_1}{\varepsilon_0 \pi R^2}\pi r^2 \;\rightarrow\; \Phi_E = \frac{Q_1}{\varepsilon_0}\frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.



$$\Phi_E = \frac{Q_1 r^2}{\varepsilon_0 R^2}$$



$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

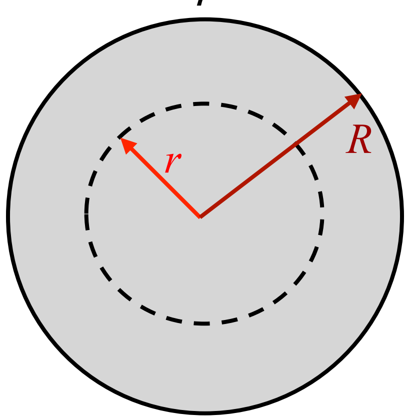
What is the displacement current enclosed by circle of radius r ?

A) $I_D = I_1 \dfrac{R^2}{r^2}$

B) $I_D = I_1 \dfrac{r}{R}$

C) $\boxed{I_D = I_1 \dfrac{r^2}{R^2}}$

D) $I_D = I_1 \dfrac{R}{r}$

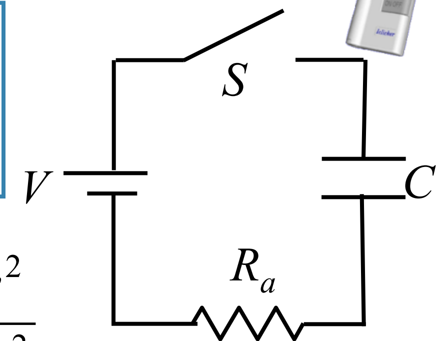


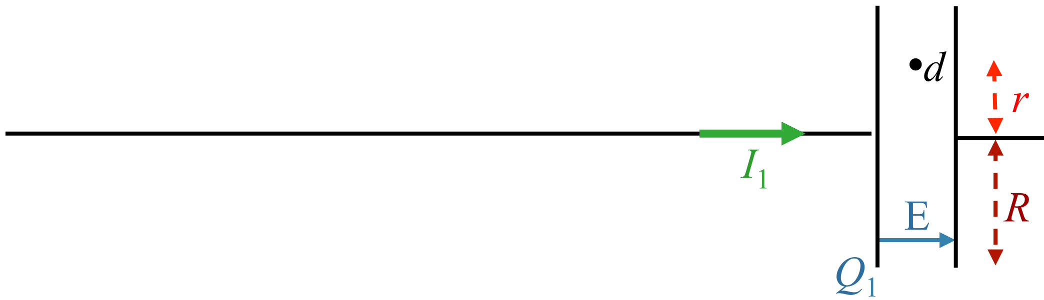
$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ_1}{dt}\frac{r^2}{R^2} = I_1 \frac{r^2}{R^2}$$

$$\rightarrow \quad I_D = I_1 \frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$I_D = I_1 \frac{r^2}{R^2}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o (I + I_D)$$

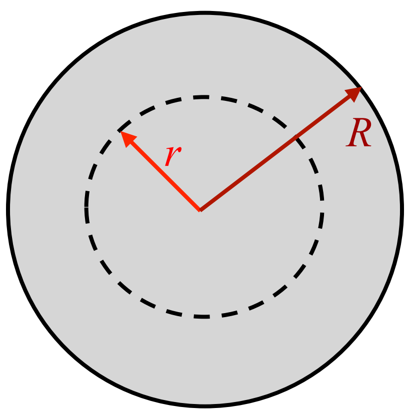
What is the magnitude of the B field at radius r ?

A) $B = \dfrac{\mu_0 I_1}{2\pi R}$

B) $B = \dfrac{\mu_0 I_1}{2\pi r}$

C) $B = \dfrac{\mu_0 I_1}{2\pi} \dfrac{R}{r^2}$

D) $B = \dfrac{\mu_0 I_1}{2\pi} \dfrac{r}{R^2}$



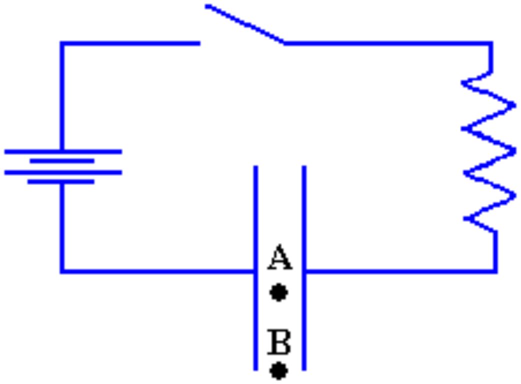
Ampere’s Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_o (I + I_D)$

$$\rightarrow \quad B \cdot 2\pi r = \mu_0 \left(0 + I_1 \frac{r^2}{R^2} \right)$$

$$\rightarrow \quad B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

CheckPoint 1B

At time t=0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed

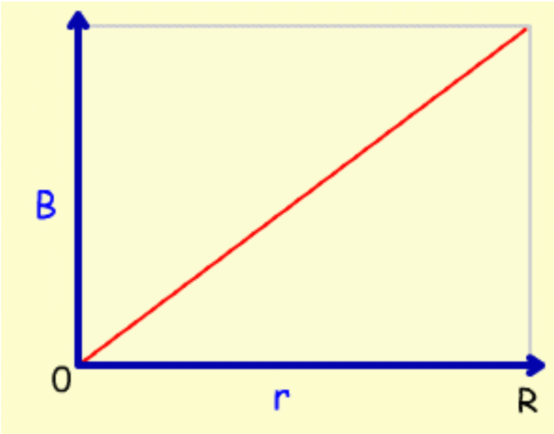
A. $B_A < B_B$

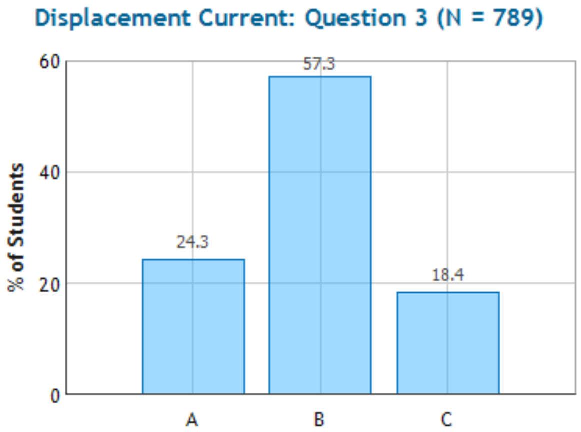
B. $B_A = B_B$

C. $B_A > B_B$

From the calculation we just did:

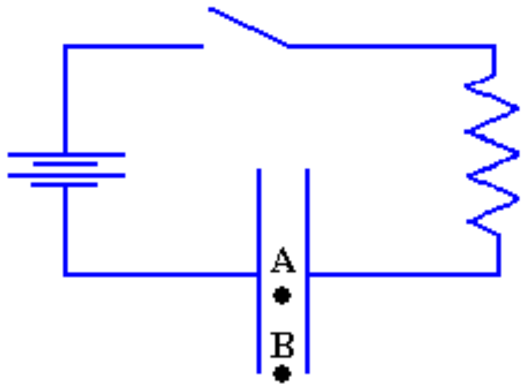
$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$





CheckPoint 1a

At time t=0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



After the switch is closed, there will be a magnetic field at point A that increases as the current in the circuit increases:

A. True

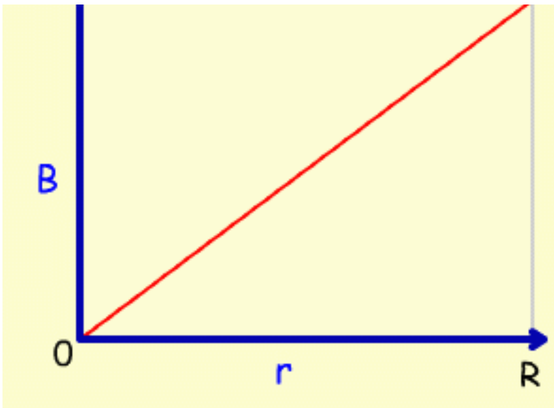
B. False

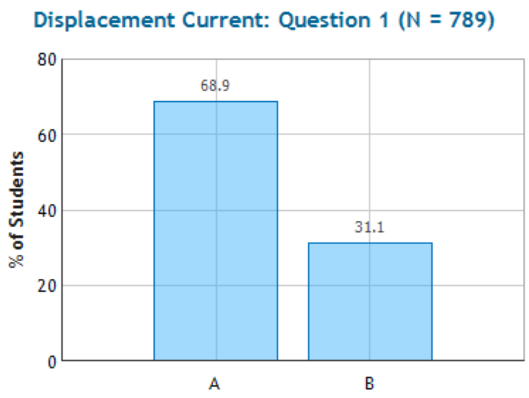
$$B = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$

B is proportional to I

but

At center, $B = 0$!!

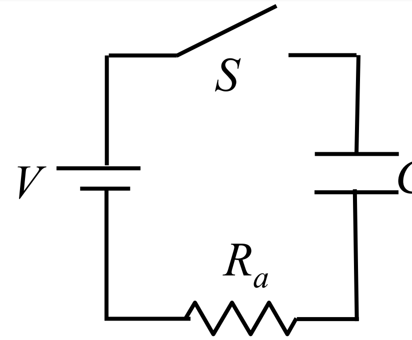




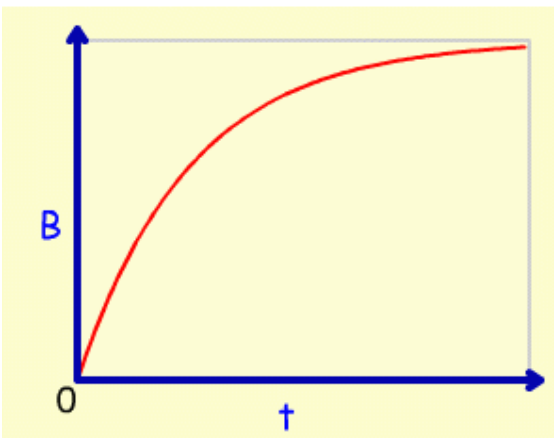
Follow-Up

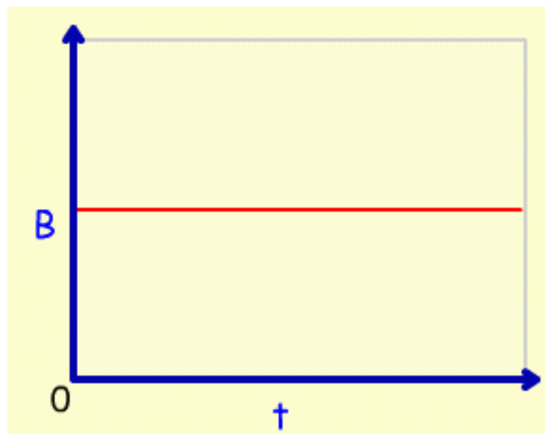
Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.

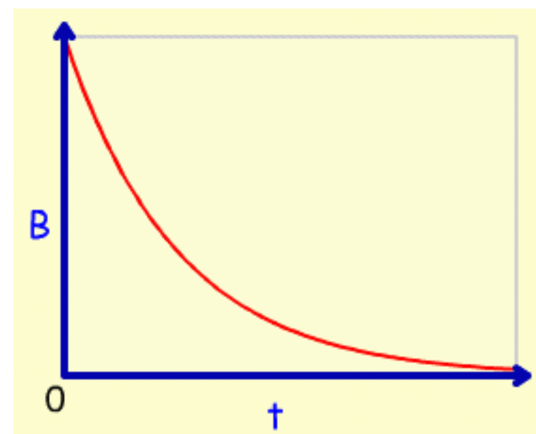
What is the time dependence of the magnetic field B at a radius r between the plates of the capacitor?



$$B_1 = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$







B at fixed r is proportional to the current I

Close switch: $V_C = 0$ [glyph] $I = V/R_a$ (maximum)

I exponentially decays with time constant $\tau = R_a C$

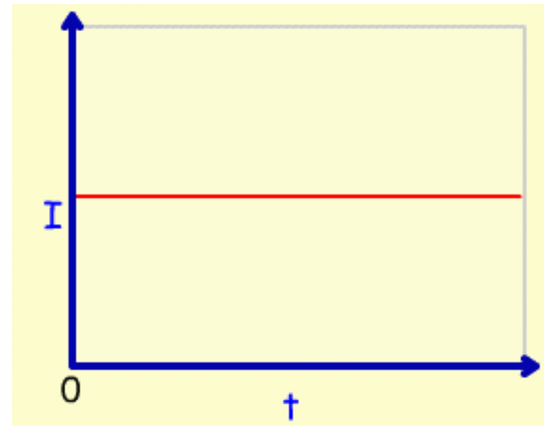
Follow-Up 2

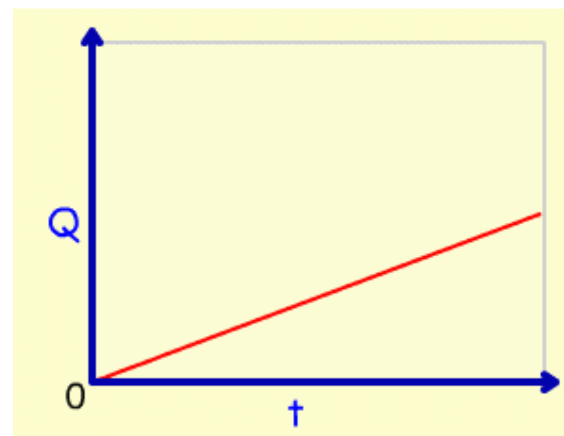
Suppose you were able to charge a capacitor with constant current (does not change in time).

Does a B field exist in between the plates of the capacitor?

A) YES

B) NO





Constant current $\Rightarrow$ Q increases linearly with time

Therefore E increases linearly with time $(E = Q/(A\varepsilon_0))$

dE/dt is not zero $\Rightarrow$ Displacement current is not zero

$\Rightarrow$ B is not zero !

Faraday's Law

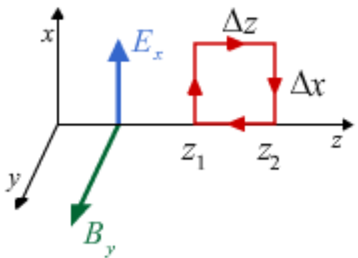
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

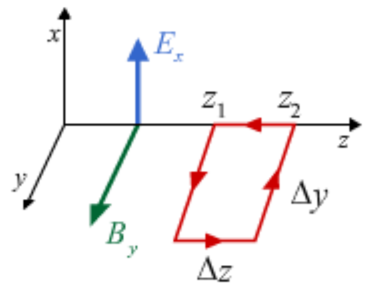
Plane Wave Solution

$$\vec{E}\rightarrow \vec{E}(z,t)$$
$$\vec{B}\rightarrow \vec{B}(z,t)$$

Modified Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o\varepsilon_o \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$





$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z}\frac{\partial B_y}{\partial t}$$

$$\frac{\partial B_y}{\partial z} = -\mu_o\varepsilon_o \frac{\partial E_x}{\partial t}$$

$$\frac{\partial}{\partial t}\frac{\partial B_y}{\partial z} = -\mu_o\varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_o\varepsilon_o \frac{\partial^2 E_x}{\partial t^2}}$$

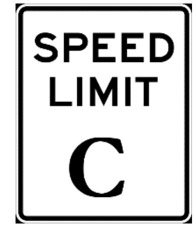
Wave Equation

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_o\varepsilon_o \frac{\partial^2 E_x}{\partial t^2}}$$

Speed of Electromagnetic Wave

$$\boxed{v = \frac{1}{\sqrt{\mu_o\varepsilon_o}}} = c = 3.00\times 10^8 \text{ m/s}$$

Speed of Light !



Special Relativity (1905)

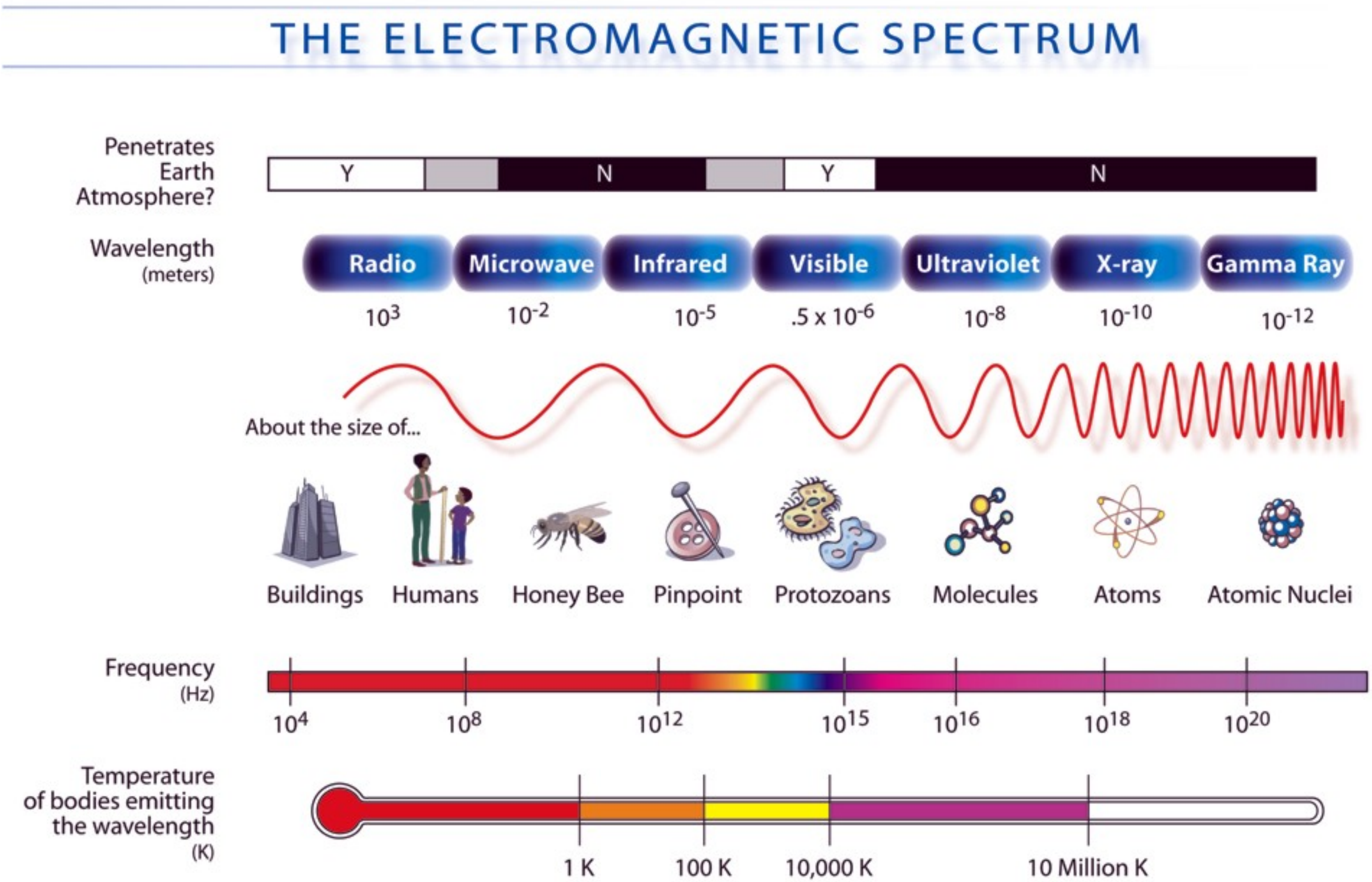
Speed of Light is Constant

Albert Einstein

"How can light move at the same velocity in any inertial frame of reference? That's really trippy. "

see PHYS 225

Electromagnetic Spectrum



We learned about waves in Physics 211

1-D Wave Equation

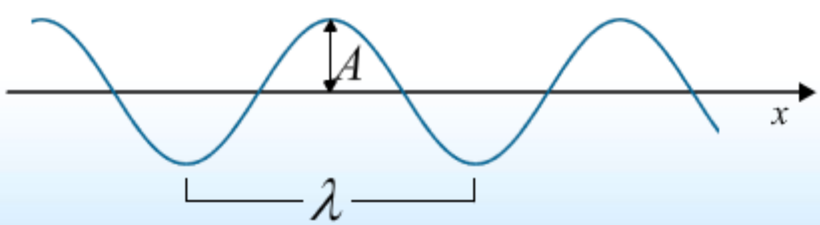
$$\frac{d^2h}{dx^2} = \frac{1}{v^2}\frac{d^2h}{dt^2}$$

Solution

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

Common Example: Harmonic Plane Wave

$$\boxed{h(x,t) = A\cos(kx - \omega t)}$$



Variable Definitions

Amplitude:	A
Wave Number:	$k = \frac{2\pi}{\lambda}$
Wavelength:	λ
Angular Frequency:	$\omega = \frac{2\pi}{T}$
Period:	T
Frequency:	$f = \frac{1}{T}$
Velocity:	$v = \lambda f = \frac{\omega}{k}$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

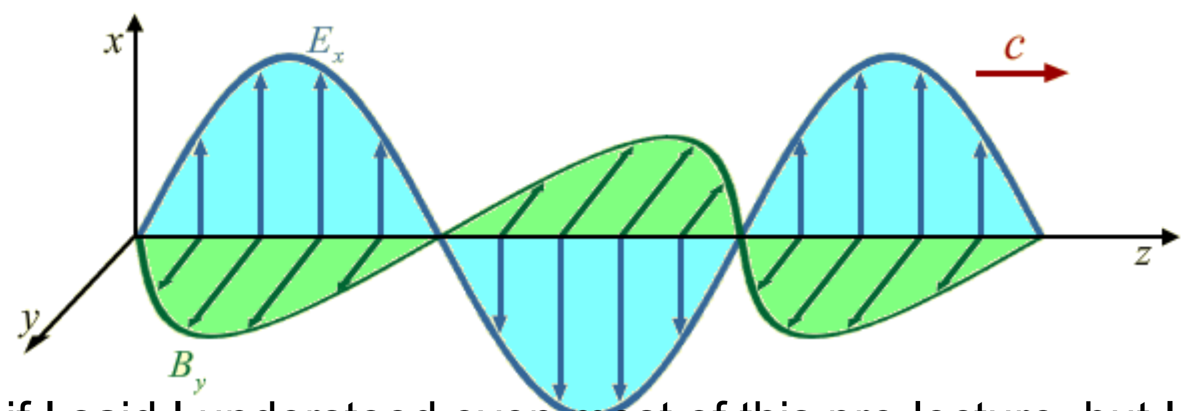
$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

Two Important Features

1. B_y is in phase with E_x
2. $B_o = \frac{E_o}{c}$



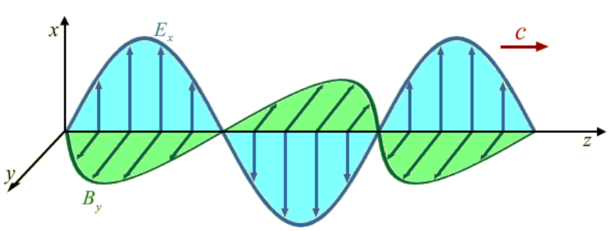
I'd be lying if I said I understood even most of this pre-lecture, but I think that part that was the least clear to me was the slide with the two 3D-Graphs, each with a loop in a different plane

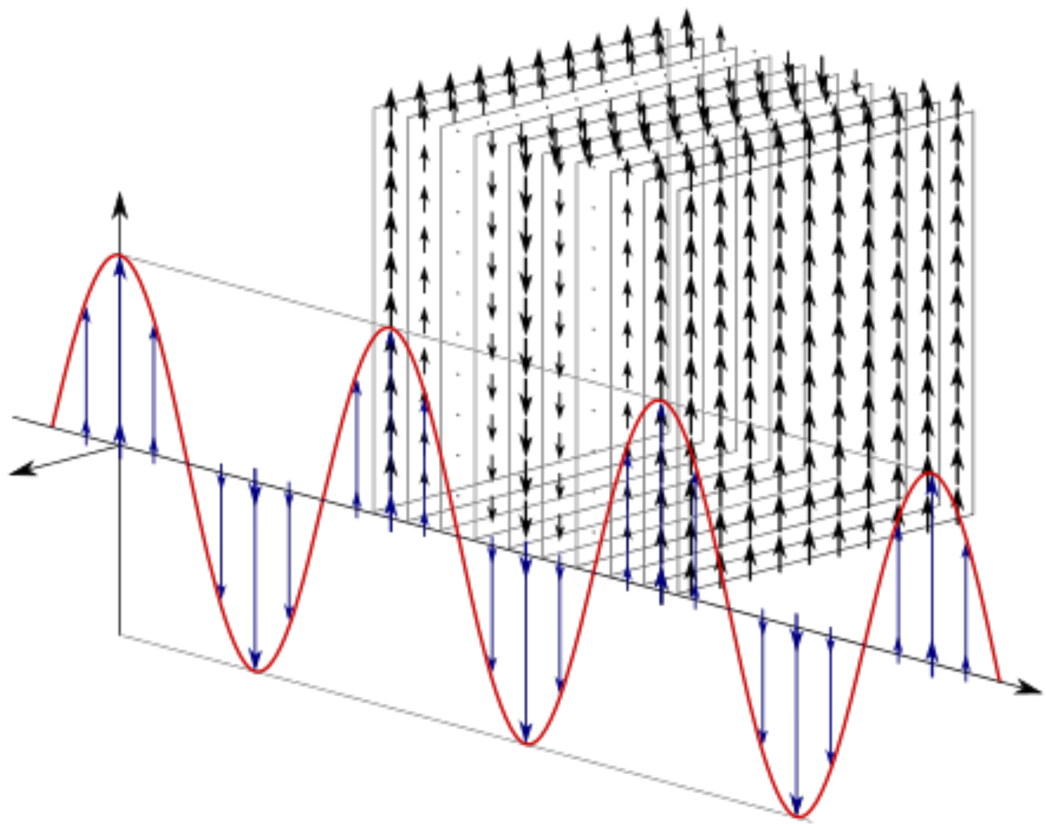
$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

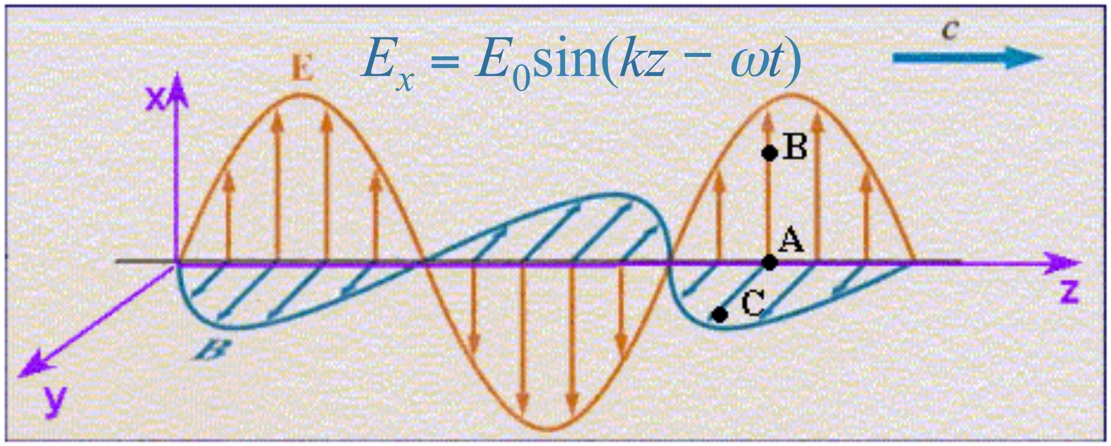
$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$





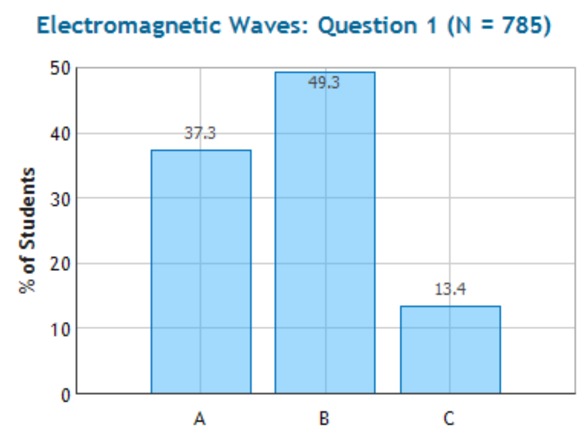
CheckPoint 2a

An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.



Compare the magnitudes of the electric fields at points A and B

A. $E_A < E_B$ **B.** $E_A = E_B$ **C.** $E_A > E_B$



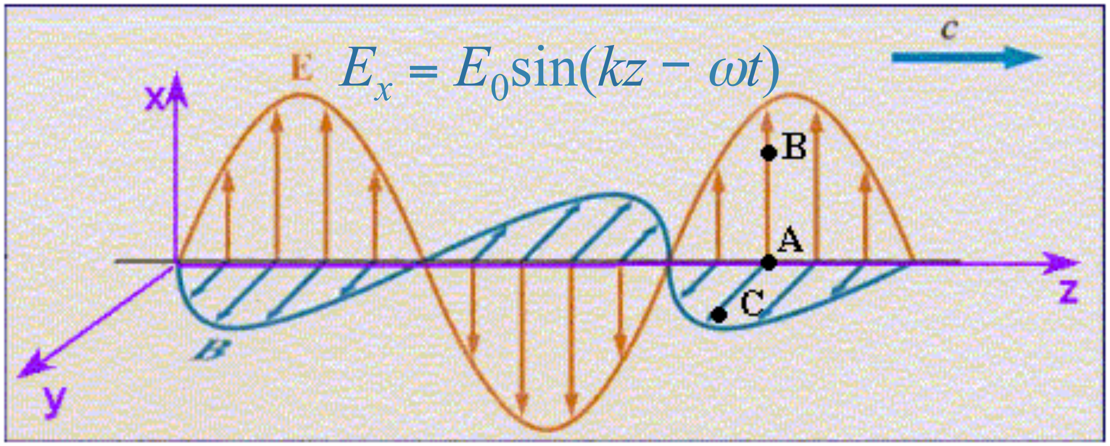
$E = E_0 \sin(kz - \omega t)$:

E depends only on z coordinate for constant t.

z coordinate is same for A, B, C.

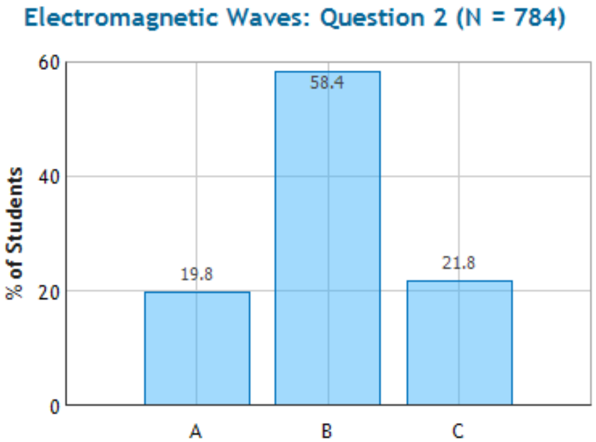
CheckPoint 2b

An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.



Compare the magnitudes of the electric fields at points A and C

A. $E_A < E_C$ **B.** $E_A = E_C$ **C.** $E_A > E_C$

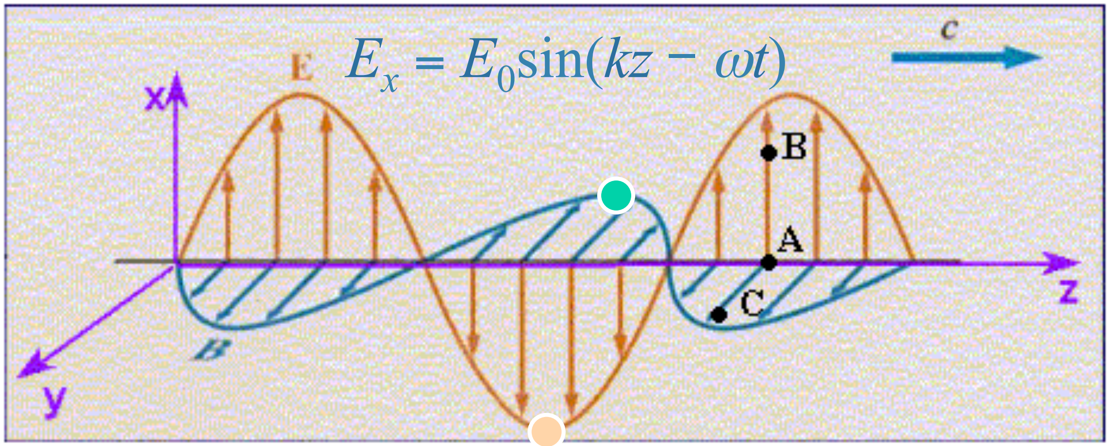


$E = E_0 \sin(kz - \omega t)$:

E depends only on z coordinate for constant t.

z coordinate is same for A, B, C.

Clicker Question



Consider a point (x,y,z) at time t when E_x is negative and has its maximum value.

At (x,y,z) at time t, what is B_y?

A) B_y is positive and has its maximum value

B) B_y is negative and has its maximum value

C) B_y is zero

D) We do not have enough information