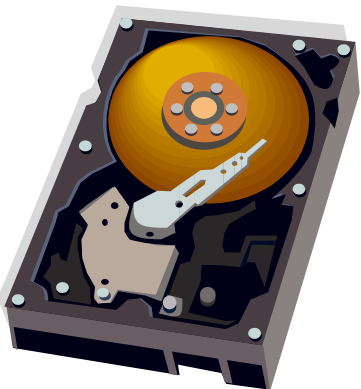


Physics 212

Lecture 17

Today's Concept:

Faraday's Law



$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$



Your Comments

Where did the $\text{emf}(\text{induced}) = \oint \mathbf{E} \cdot d\mathbf{l}$ come from? How does that match up with the $\text{emf} = BLv$ we learned for the bar, for instance?

When are things flipped i.e. when do you use your left hand instead of the right hand rule?

If nature is constantly opposing the change in flux (inducing currents to fight that change), how does flux ever change?

Since the practice exams contain questions that won't be on our exam, is the exam going to be shorter? Or will it be the same length as other exams, just with extra questions on our covered topics?

WHY WOULD YOU DO THIS TO US? We have a midterm this thursday and I am scared for it as it is and you give us Faraday's law right before? WHY????

I'm unsure if I used the RHR correctly for checkpoint 2, and checkpoint 3 I just flat out don't understand. Plz help, I just want to enjoy my spring break in peace :(

Why my words never go to the screen?

I have 5 mid-terms this week. If I don't make it, please tell my parents that I loved them!

Some Exam Stuff

Exam 2: Thursday March 19 at 7:00

- Covers material in Lectures 9 – 16 (NOT Today's material)
- Sign up in Gradebook for Conflict Exam at 5:15pm
- Link in Gradebook if you have a double-conflict

Exam Preparation:

- Old Exams are a good way to assess what you need to know
- Prelecture of Fall 2010 solutions available (“prelecture”)
- Video Solutions of Spring 2014 Hour Exam 2 (“optional HW”)
- Extra office hours posted on web

No Lab or Quiz this week

Faraday's Law

Faraday's Law:

$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

Looks scary but it's not – its amazing and beautiful!



A changing magnetic flux produces an electric field.

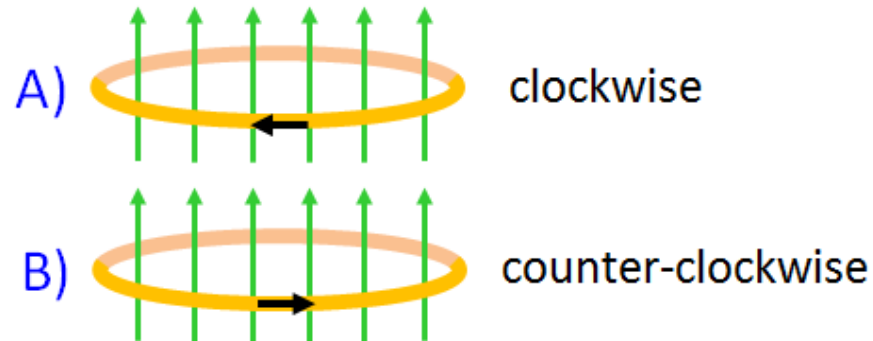


Electricity and magnetism are deeply connected.

Checkpoint 1



Suppose a current flows in a horizontal conducting loop in such a way that the magnetic flux produced by this current points upward.



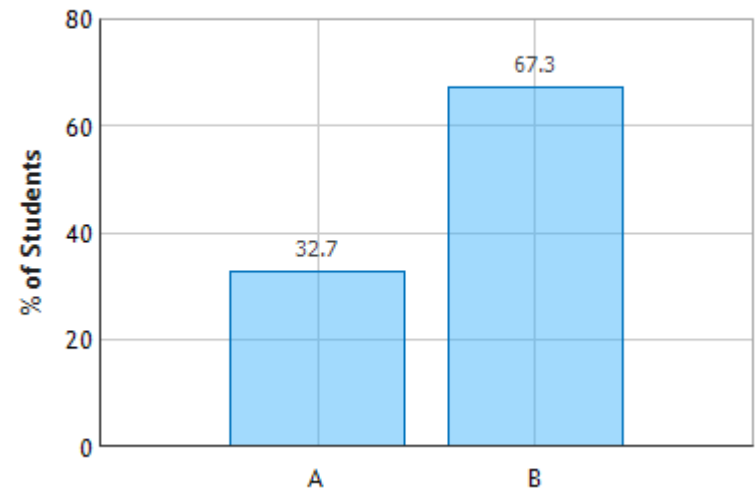
As viewed from above, in which direction is this current flowing?

A. clockwise

B. counterclockwise

Right hand rule. B field created by current points upwards.

Loop of Current: Question 1 (N = 790)



Faraday's Law:

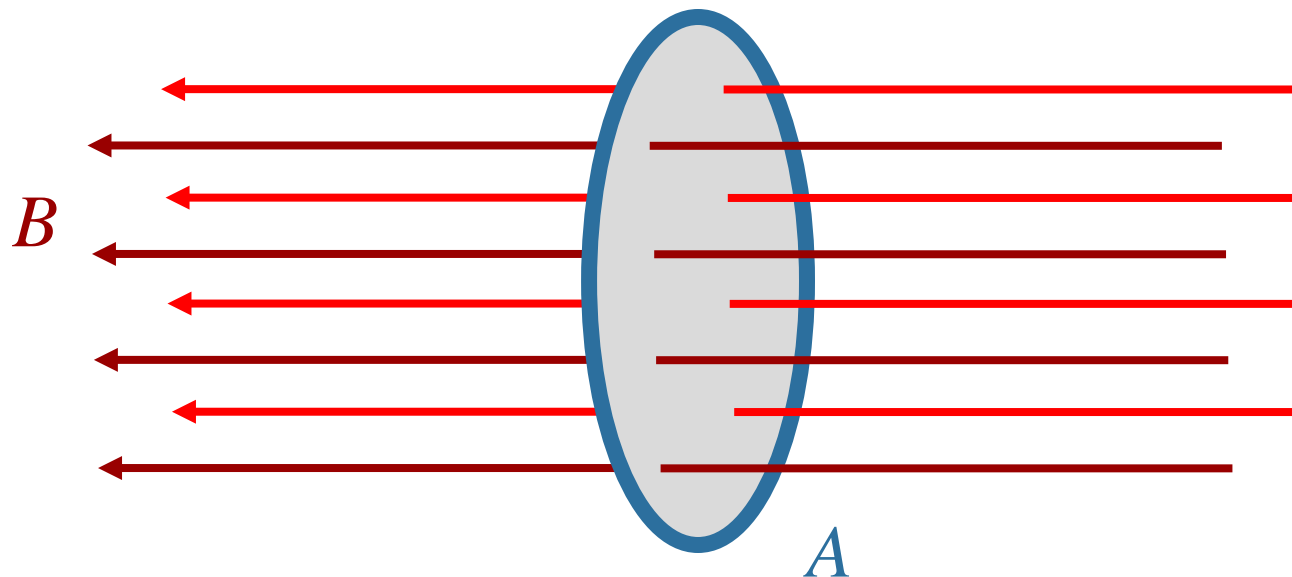
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.



Think of Φ_B as the number of field lines passing through the surface

There are many ways to change this...

Faraday's Law:

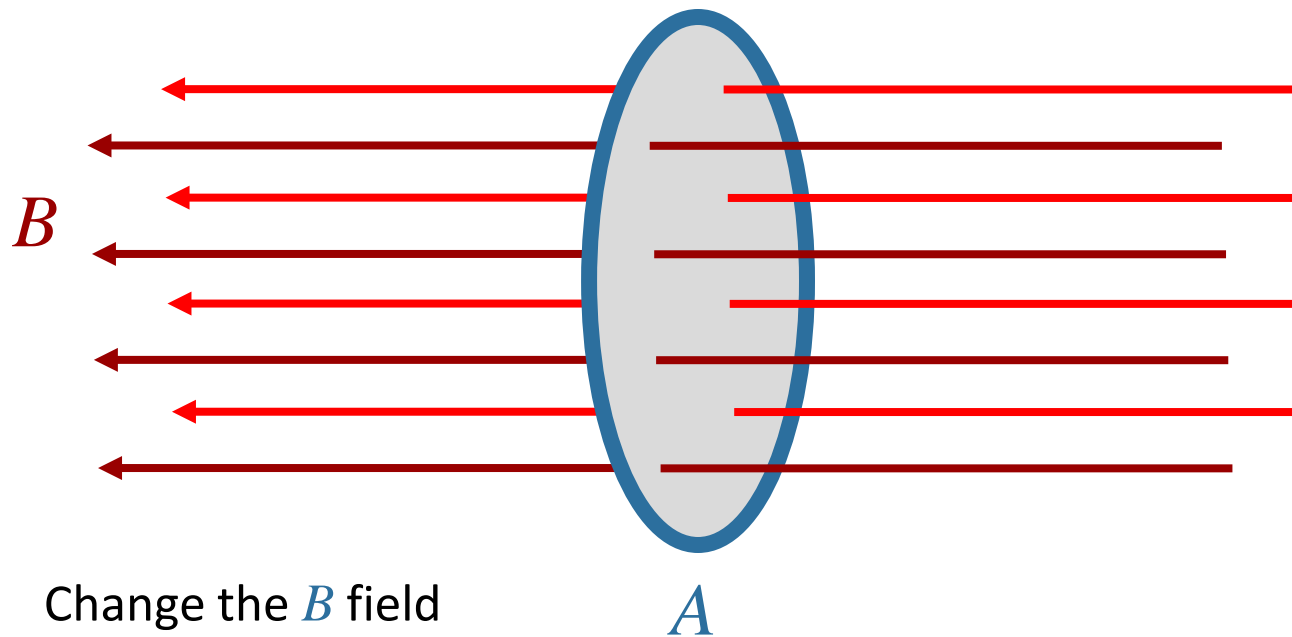
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where

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Faraday's Law:

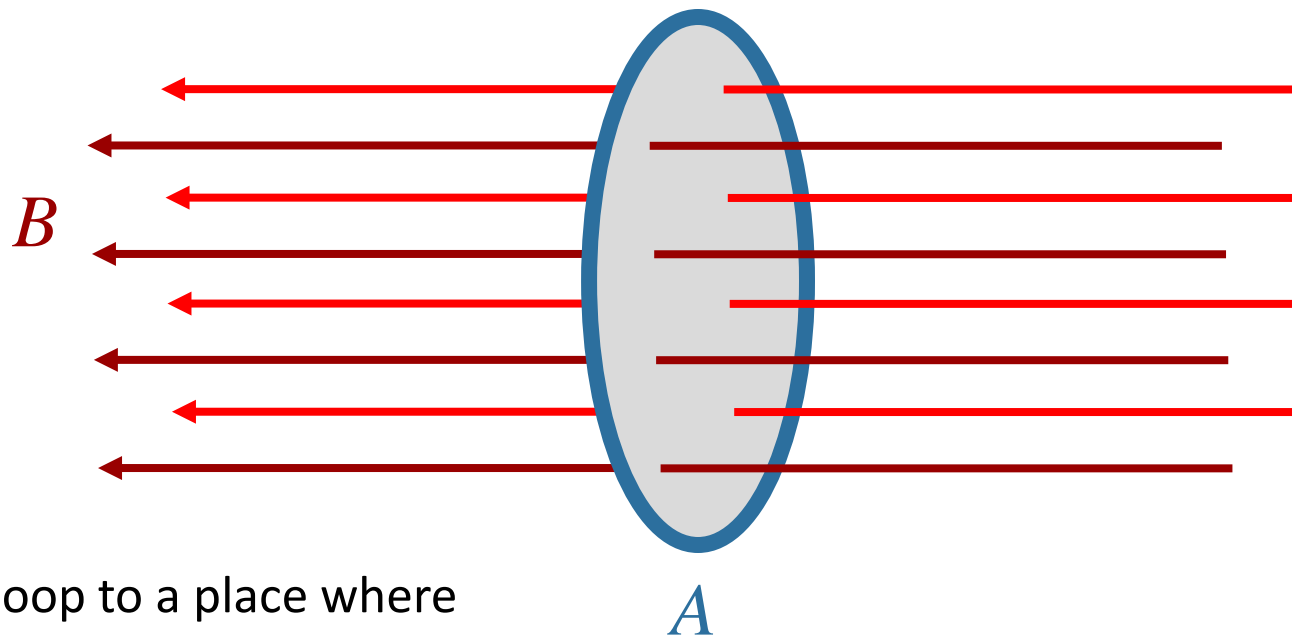
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.



Move loop to a place where the B field is different

Faraday's Law:

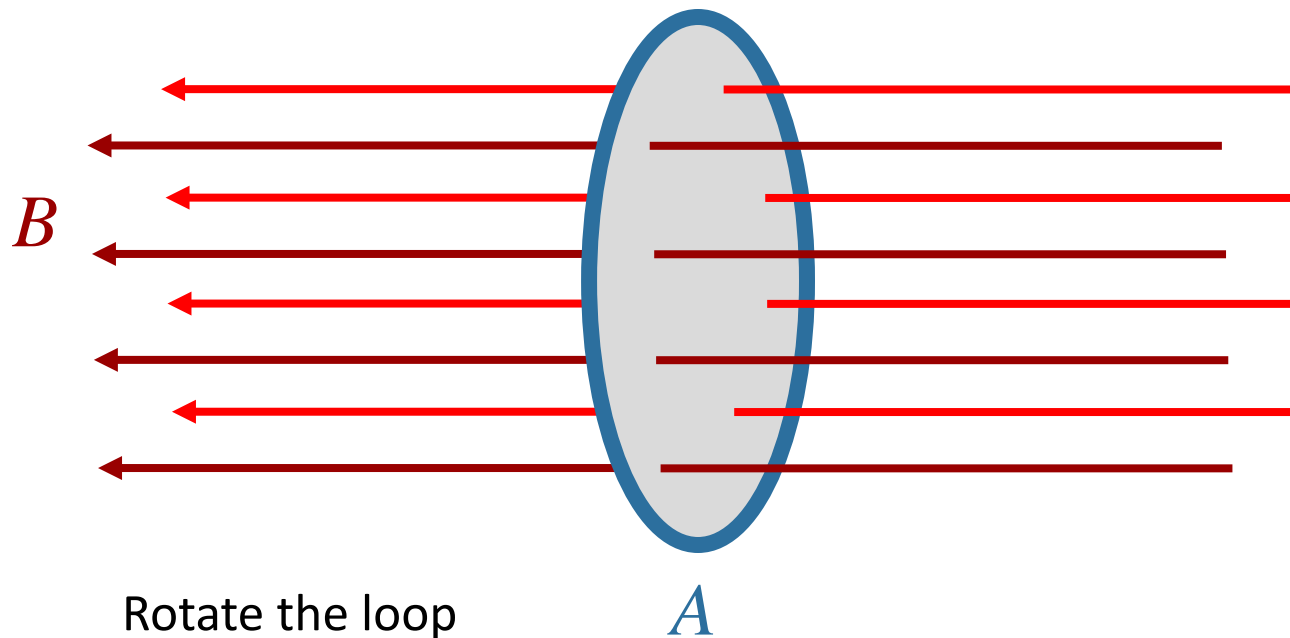
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Faraday's Law:

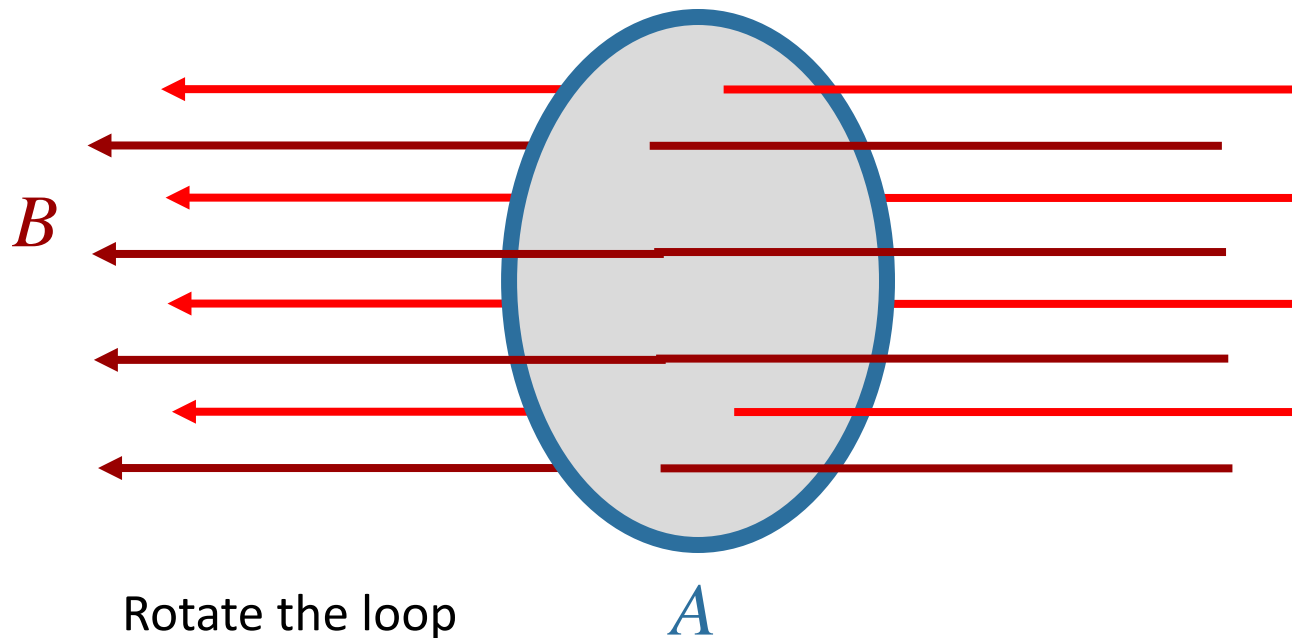
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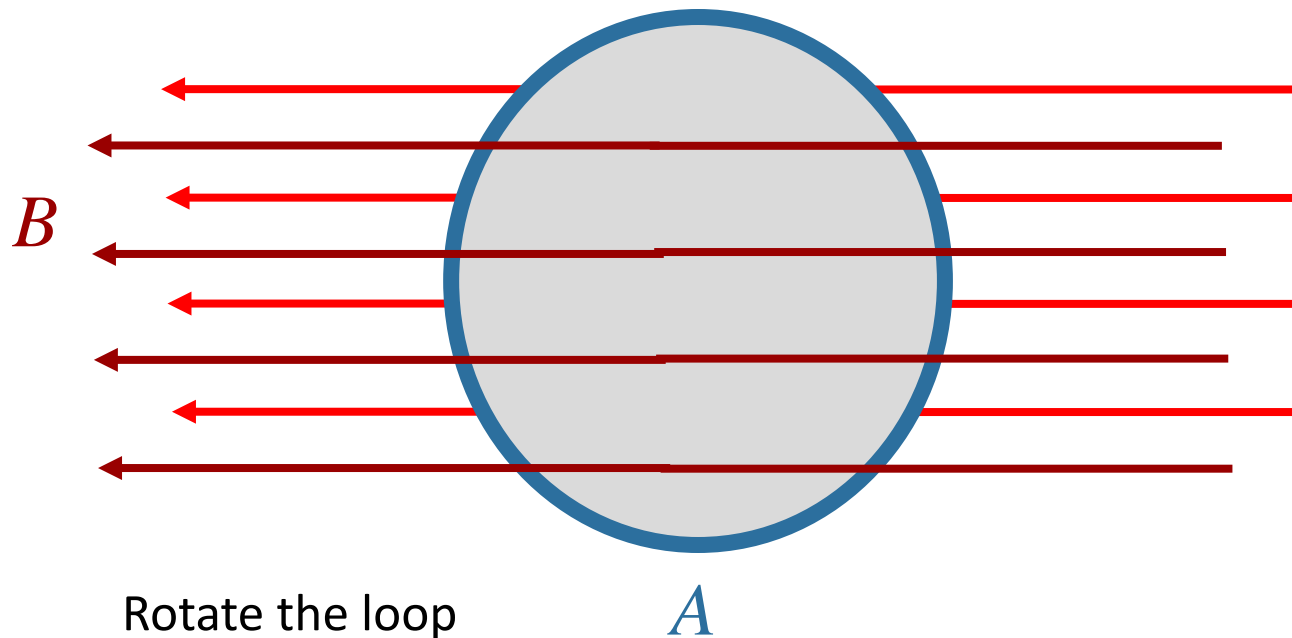
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Faraday's Law:

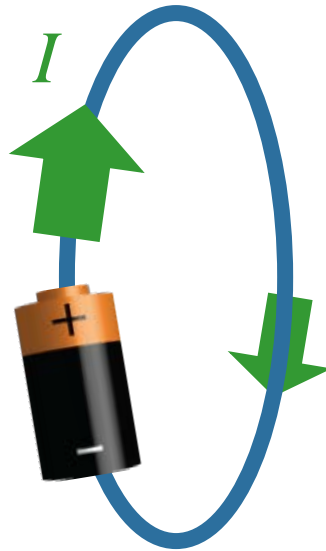
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

In Practical Words:

- 1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.
- 2) The *emf* will make a current flow if it can (like a battery).



Faraday's Law:

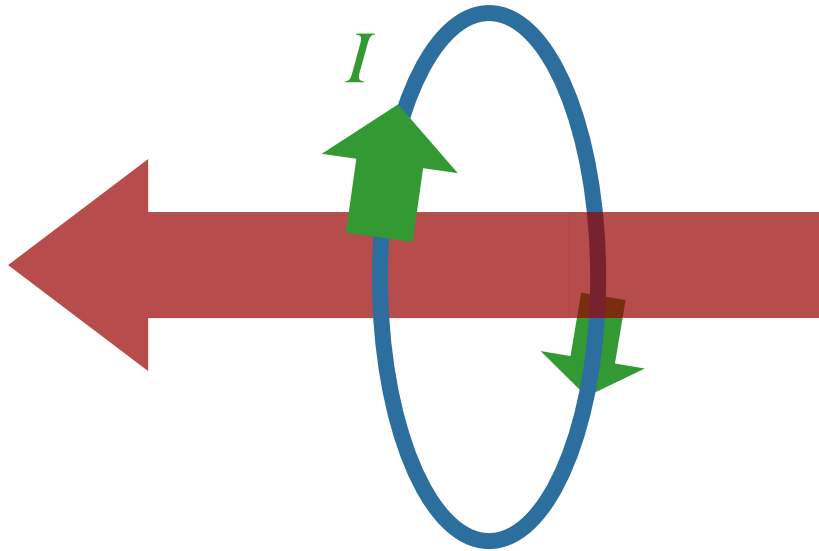
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

In Practical Words:

- 1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.
- 2) The *emf* will make a current flow if it can (like a battery).
- 3) The current that flows induces a new magnetic field.



Faraday's Law:

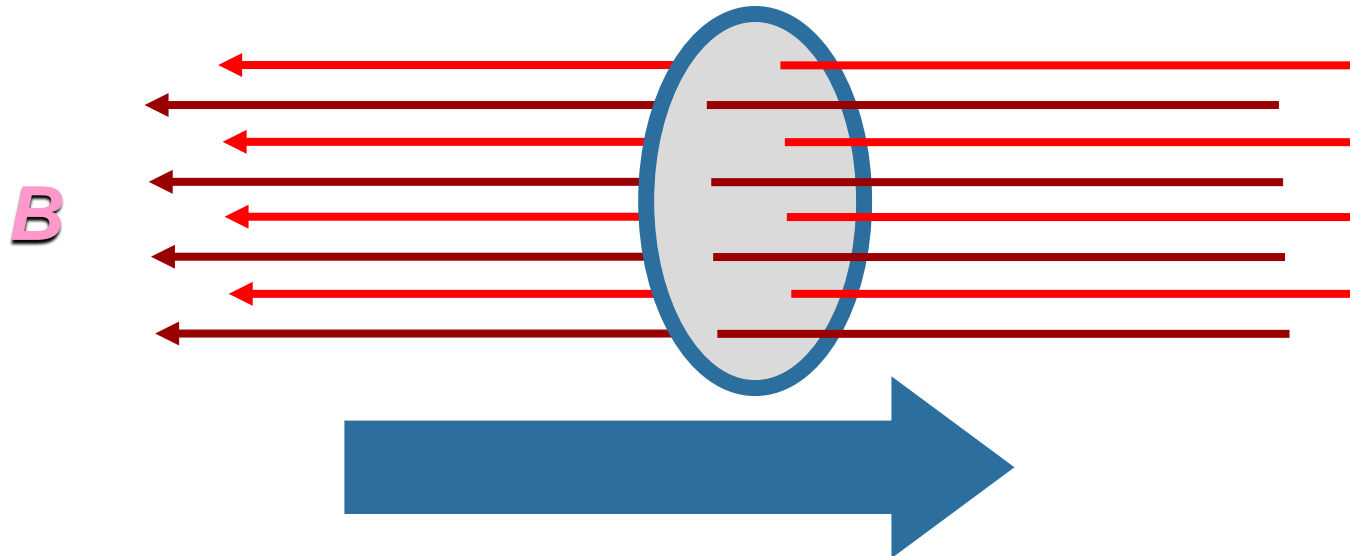
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

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In Practical Words:

- 1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.
- 2) The *emf* will make a current flow if it can (like a battery).
- 3) The current that flows induces a new magnetic field.
- 4) The new magnetic field opposes the change in the original magnetic field that created it. (Lenz' Law)



Demo
Coil and magnet

Faraday's Law:

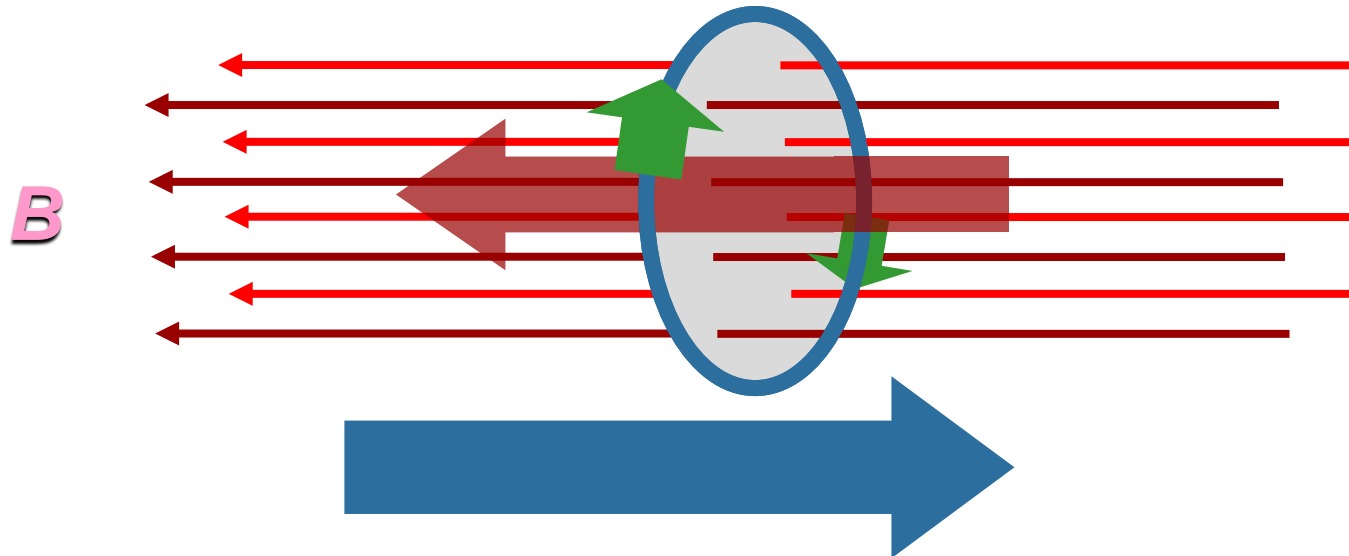
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

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In Practical Words:

- 1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.
- 2) The *emf* will make a current flow if it can (like a battery).
- 3) The current that flows induces a new magnetic field.
- 4) The new magnetic field opposes the change in the original magnetic field that created it. (Lenz' Law)



Demo
Back emf & braking

Faraday's Law:

$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

Executive Summary:

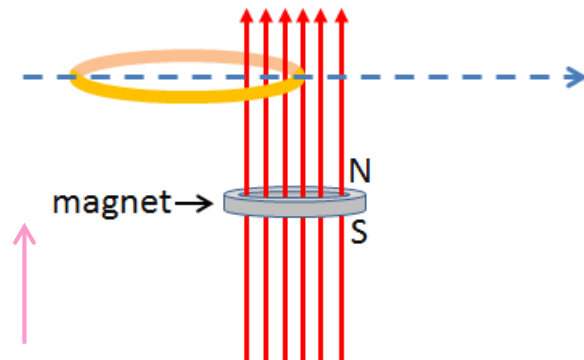


$emf \rightarrow$ current \rightarrow field a) induced **only** when **flux is changing**
b) **opposes the change**

Checkpoint 2

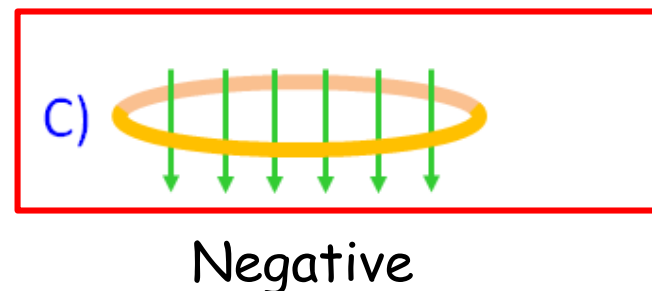
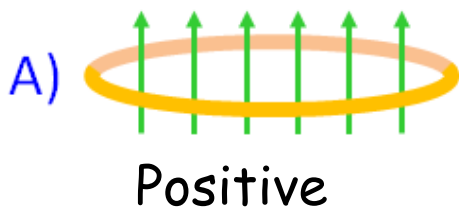


A magnet makes the vertical magnetic field shown by the red arrows. A horizontal conducting loop is entering the field as shown.

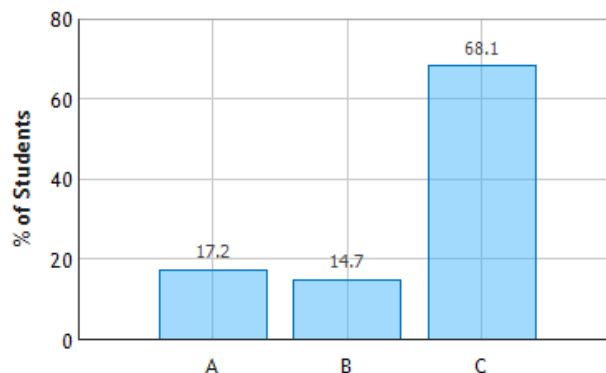


Direction of positive flux

At the instant shown above, what is the sign of the additional flux produced by the current induced in the loop?



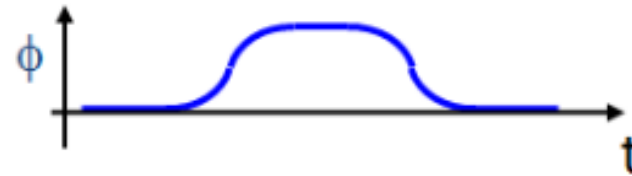
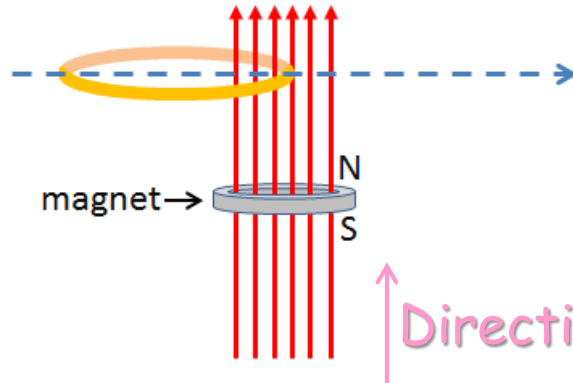
Direction of Induced Flux: Question 1 (N = 790)



Checkpoint 3

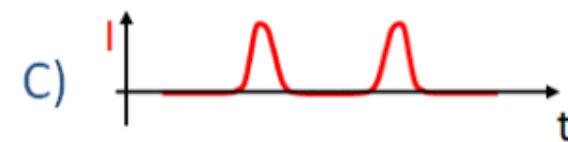
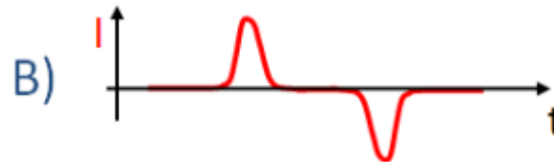


A magnet makes the vertical magnetic field shown by the red arrows. A horizontal conducting loop is entering the field as shown.



Direction of positive flux

The upward flux through the loop as a function of time is shown by the blue trace. Which of the red traces below it best represents the current induced in the loop as a function of time as it passes over the magnet? (Positive means counter-clockwise as viewed from above):



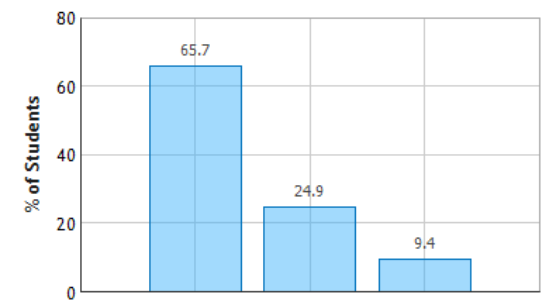
Flux is changing!

Induced flux is initially negative (opposing increasing positive flux – last checkpoint)

THEREFORE, initial *induced current* must be CW as viewed from above

- Current direction from right-hand rule ☺

Direction of Induced Current: Question 1 (N = 788)

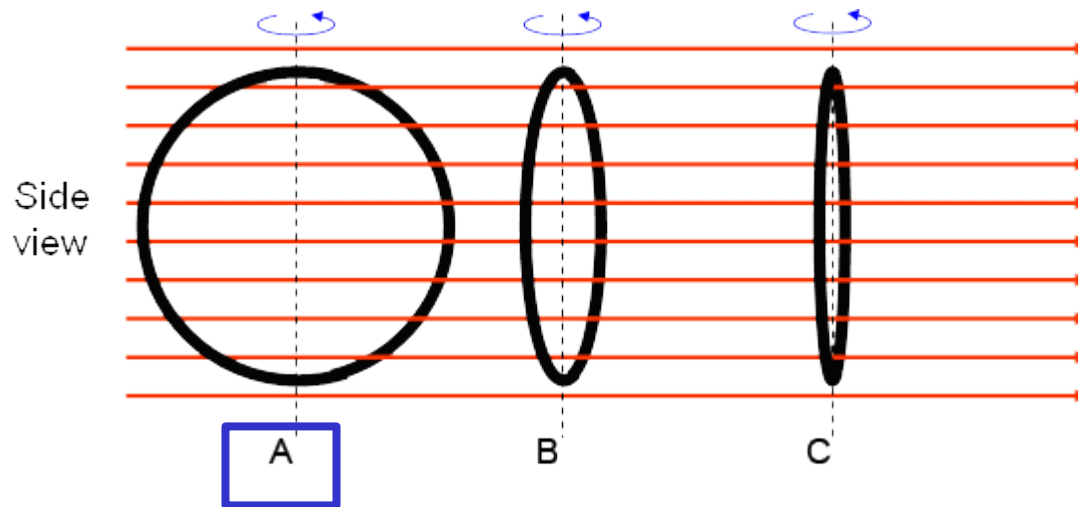


Prelecture: Faraday's Law



A circular wire loop is placed in a uniform magnetic field pointing to the right. The loop is rotated with *constant angular velocity* around a vertical axis (dashed line).

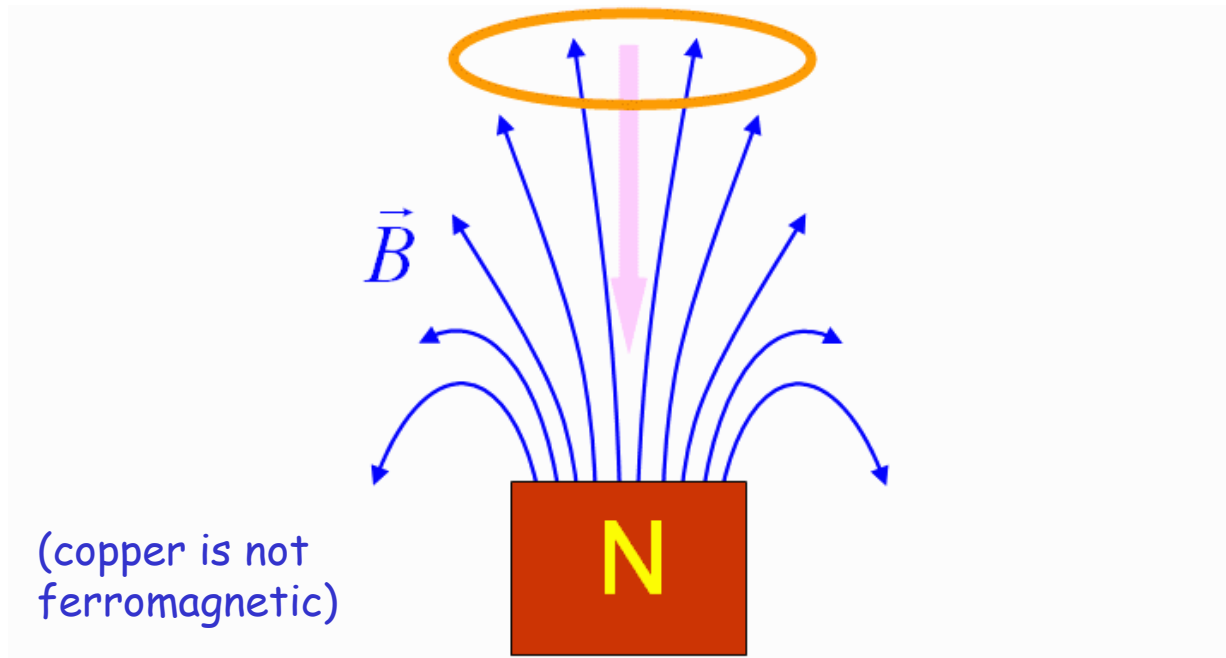
At which of the three times shown is the induced *emf* greatest?



$emf = -d\Phi/dt \rightarrow$ largest where slope of Φ vs t largest

Cool Example

A horizontal copper ring is dropped from rest directly above the north pole of a permanent magnet



As the ring falls, in which direction will the induced field point?

A. up

B. down

C. No induced field

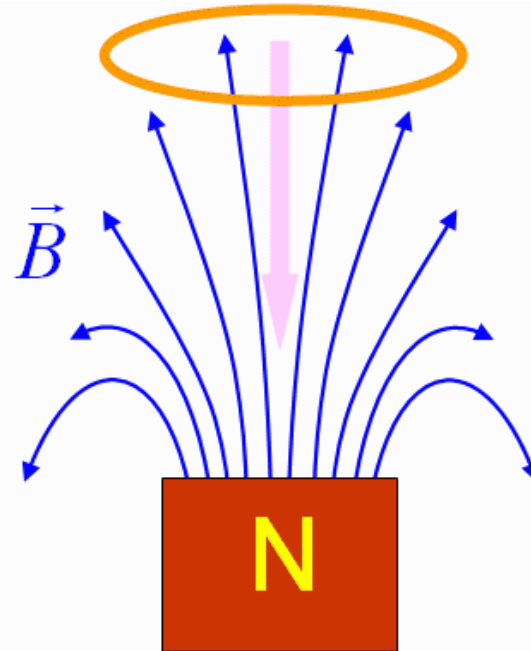
Choose direction for positive flux to be in same direction as external field (easiest), then

Increase in flux from external field is opposed by induced flux with opposite sign

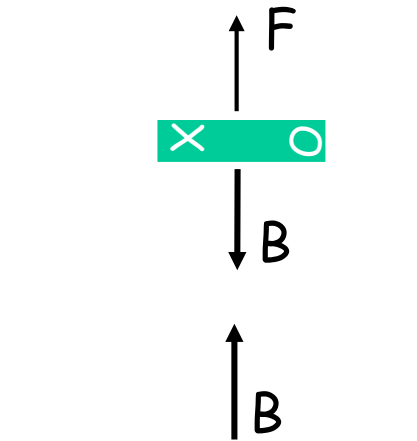
Cool Example



A horizontal copper ring is dropped from rest directly above the north pole of a permanent magnet



(copper is not
ferromagnetic)



Like poles repel

→ $F_{\text{total}} < mg$

→ $a < g$

Will the acceleration a of the falling ring in the presence of the magnet be any different than it would have been under the influence of just gravity (i.e. g)?

A. $a > g$

B. $a = g$

C. $a < g$

This one is hard !

Upward B field increases as loop falls

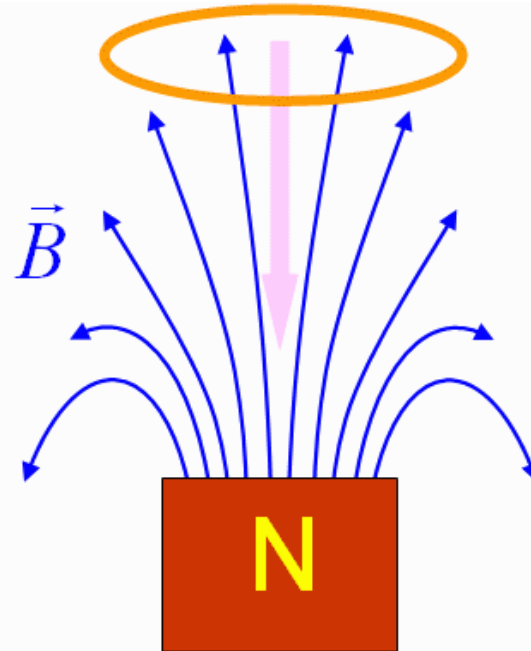
Clockwise current (viewed from top) is induced

Demo !
dropping magnets

Cool Example



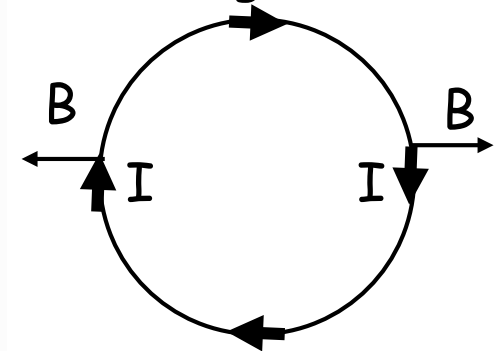
A horizontal copper ring is dropped from rest directly above the north pole of a permanent magnet



(copper is not ferromagnetic)

HOW
IT
WORKS

Looking down



$\vec{I} \times \vec{B}$ points UP

→ $F_{\text{total}} < mg$

→ $a < g$

Demo !
e-m cannon

Will the acceleration a of the falling ring in the presence of the magnet be any different than it would have been under the influence of just gravity (i.e. g)?

A. $a > g$

B. $a = g$

C. $a < g$

B field increases upward as loop falls

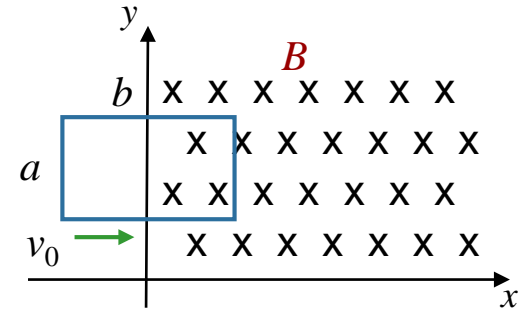
Clockwise current (viewed from top) is induced

Main Field produces horizontal forces

"Fringe" Field produces vertical force

Calculation

A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the direction and the magnitude of the force on the loop when half of it is in the field?

Conceptual Analysis

Once loop enters B field region, flux will be changing in time
Faraday's Law then says emf will be induced

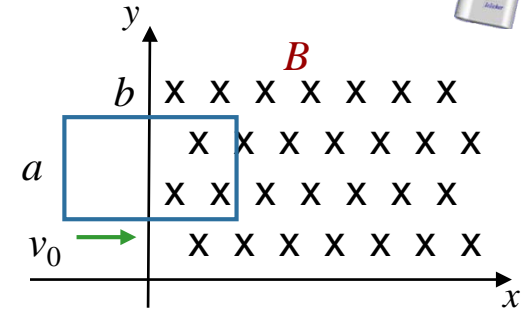
Strategic Analysis

- Find the emf
- Find the current in the loop
- Find the force on the current

Calculation



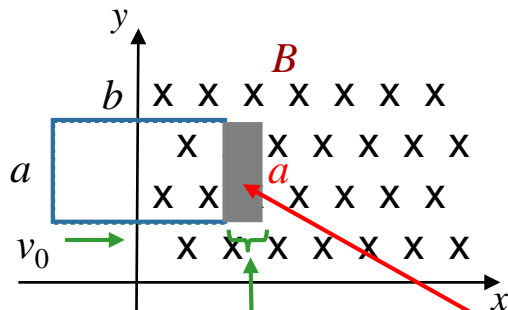
A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



$$emf = -\frac{d\Phi_B}{dt}$$

What is the magnitude of the emf induced in the loop just after it enters the field?

- A) $\varepsilon = Babv_0^2$ B) $\varepsilon = \frac{1}{2} Bav_0$ C) $\varepsilon = \frac{1}{2} Bbv_0$ **D) $\varepsilon = Bav_0$** E) $\varepsilon = Bbv_0$



In a time dt
it moves by $v_0 dt$

The area in field
changes by $dA = v_0 dt a$

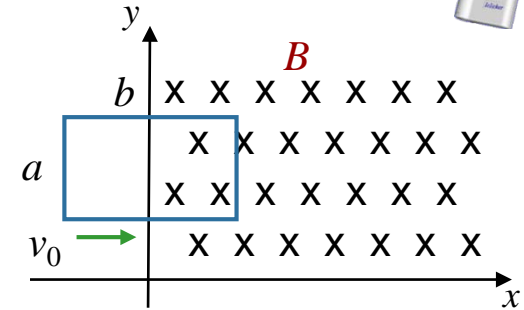
→ Change in Flux = $d\Phi_B = BdA = Bav_0 dt$

→ $\frac{d\Phi_B}{dt} = Bav_0$

Calculation



A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



$$\text{emf} = -\frac{d\Phi_B}{dt}$$

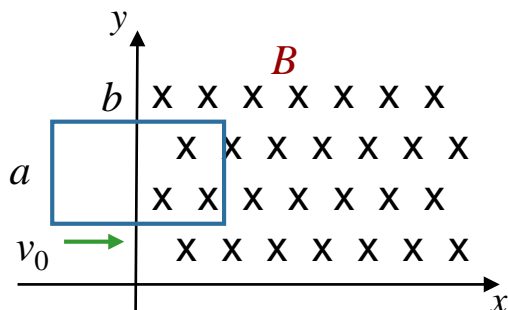
What is the direction of the current induced in the loop just after it enters the field?

A) clockwise

B) counterclockwise

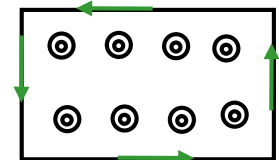
C) no current is induced

emf is induced in direction to oppose the change in flux that produced it



Flux is increasing into the screen

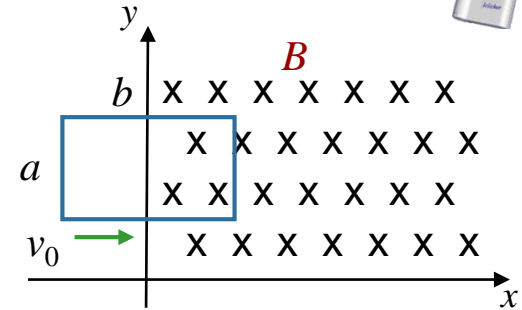
Induced emf produces flux out of screen



Calculation



A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



$$emf = -\frac{d\Phi_B}{dt}$$

What is the direction of the net force on the loop just after it enters the field?

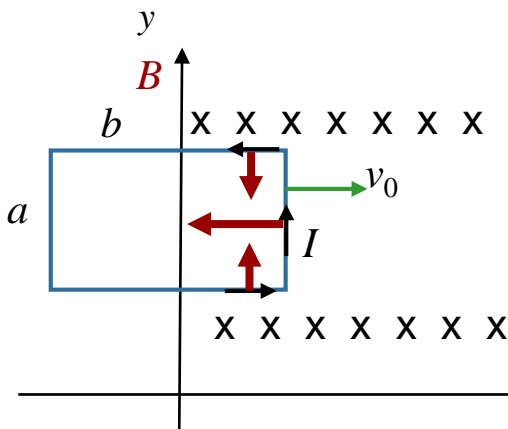
A) $+y$

B) $-y$

C) $+x$

D) $-x$

Force on a current in a magnetic field: $\vec{F} = I\vec{L} \times \vec{B}$



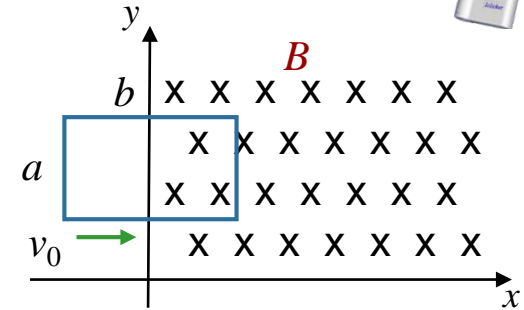
Force on top and bottom segments cancel (red arrows)

Force on right segment is directed in $-x$ direction.

Calculation



A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the magnitude of the net force on the loop just after it enters the field?

$$\vec{F} = I\vec{L} \times \vec{B} \quad \varepsilon = Bav_0 \quad emf = -\frac{d\Phi_B}{dt}$$

A) $F = 4aBv_0R$

B) $F = a^2Bv_0R$

C) $F = a^2B^2v_0^2 / R$

D) $F = a^2B^2v_0 / R$

$\vec{F} = I\vec{L} \times \vec{B} \quad \rightarrow \quad F = ILB \quad \text{since } \vec{L} \perp \vec{B}$

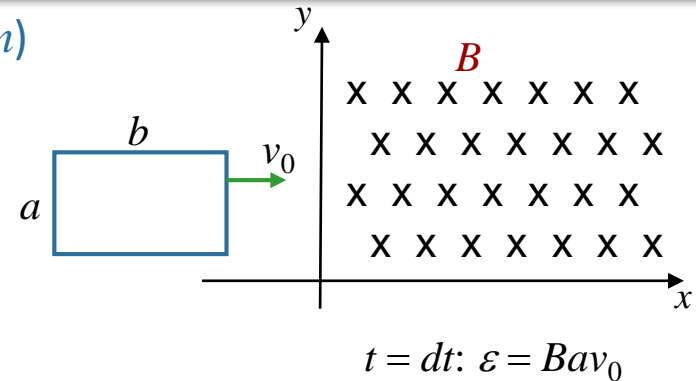
$I = \frac{\varepsilon}{R} = \frac{Bav_0}{R} \quad \rightarrow \quad F = \left(\frac{Bav_0}{R} \right) aB = \frac{B^2a^2v_0}{R}$

ILB

Follow Up

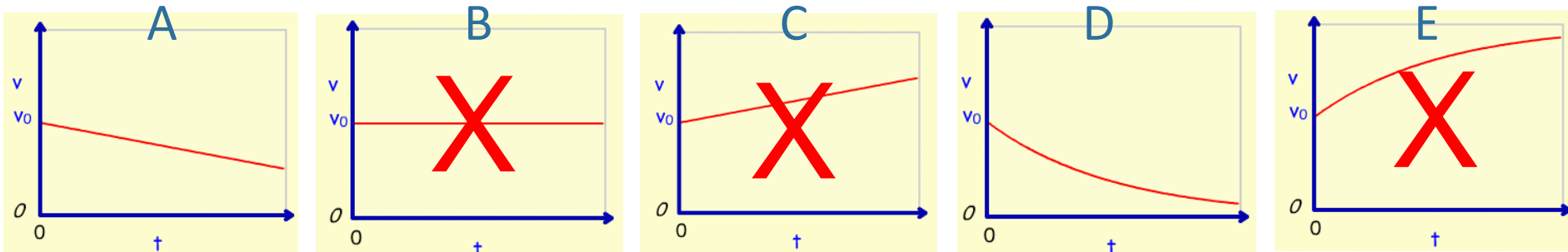


A rectangular loop (sides = a, b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



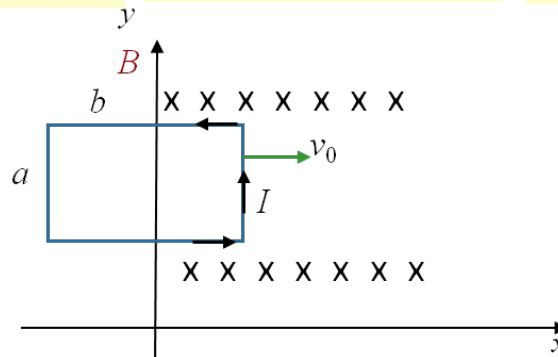
What is the velocity of the loop when half of it is in the field?

Which of these plots best represents the velocity as a function of time as the loop moves from entering the field to halfway through?



This is not obvious, but we know v must decrease

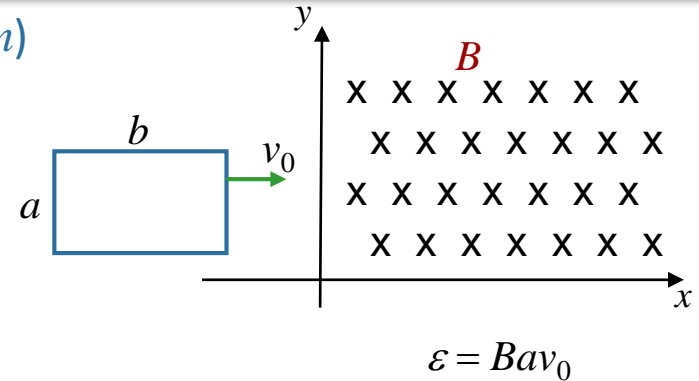
Why?



F_{right} points to left
Acceleration negative
Speed must decrease

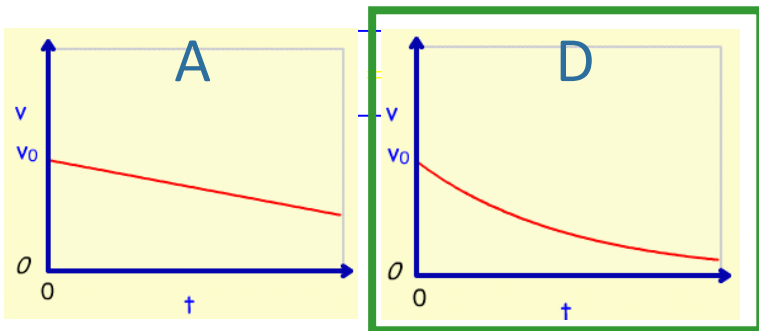
Follow Up

A rectangular loop (sides = a, b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the velocity of the loop when half of it is in the field?

Which of these plots best represents the velocity as a function of time as the loop moves from entering the field to halfway through?



Why D, not A?

F is not constant, depends on v

$$F = -\frac{a^2 B^2 v}{R} = m \frac{dv}{dt} \quad \longrightarrow \quad v = v_0 e^{-\alpha t}$$

where $\alpha = \frac{a^2 B^2}{mR}$

Challenge: Look at energy



Claim: The decrease in kinetic energy of loop is equal to the energy dissipated as heat in the resistor. Can you verify?