Physics 212
Lecture 6

Today's Concept:

Electric Potential

(Defined in terms of Path Integral of Electric Field)

Your Comments

When I see a sign that says, "DANGER! HIGH VOLTAGE!", I think,
"Wow, this place has a lot of potential."

Can you go over the difference between electric potential energy and electric potential?

What do dr and dl represent? I know dr's a scalar and dl's a vector, but why did we change variable names?

Just out of curiosity, what situation would have an electric field of 0 N./C.? Obviously in an empty space void of charges, there would be no electric field, and thus no potential difference generated by any charge, but what about in the middle of two charges with the same sign? Electric field lines cannot cross, so there are no electric field lines in the center between two charges of the same sign. But isn't the potential still positive or negative depending on the charge, or no?

Electric potential energy and electric potential?? what?! For the most part I understood everything, some topics were a little difficult to grasp at first.

Dude, what. okay. so... wait. 1. Will we need to use the spherical/cylindrical forms of the Gradient? Or naw. 2. Can you go over Work and how it relates to U and V again?

I don't get the example of a solid spherical insulator, and the first and third checkpoints. The nightmare I just went through when we were learning Gauss Law is repeating.

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

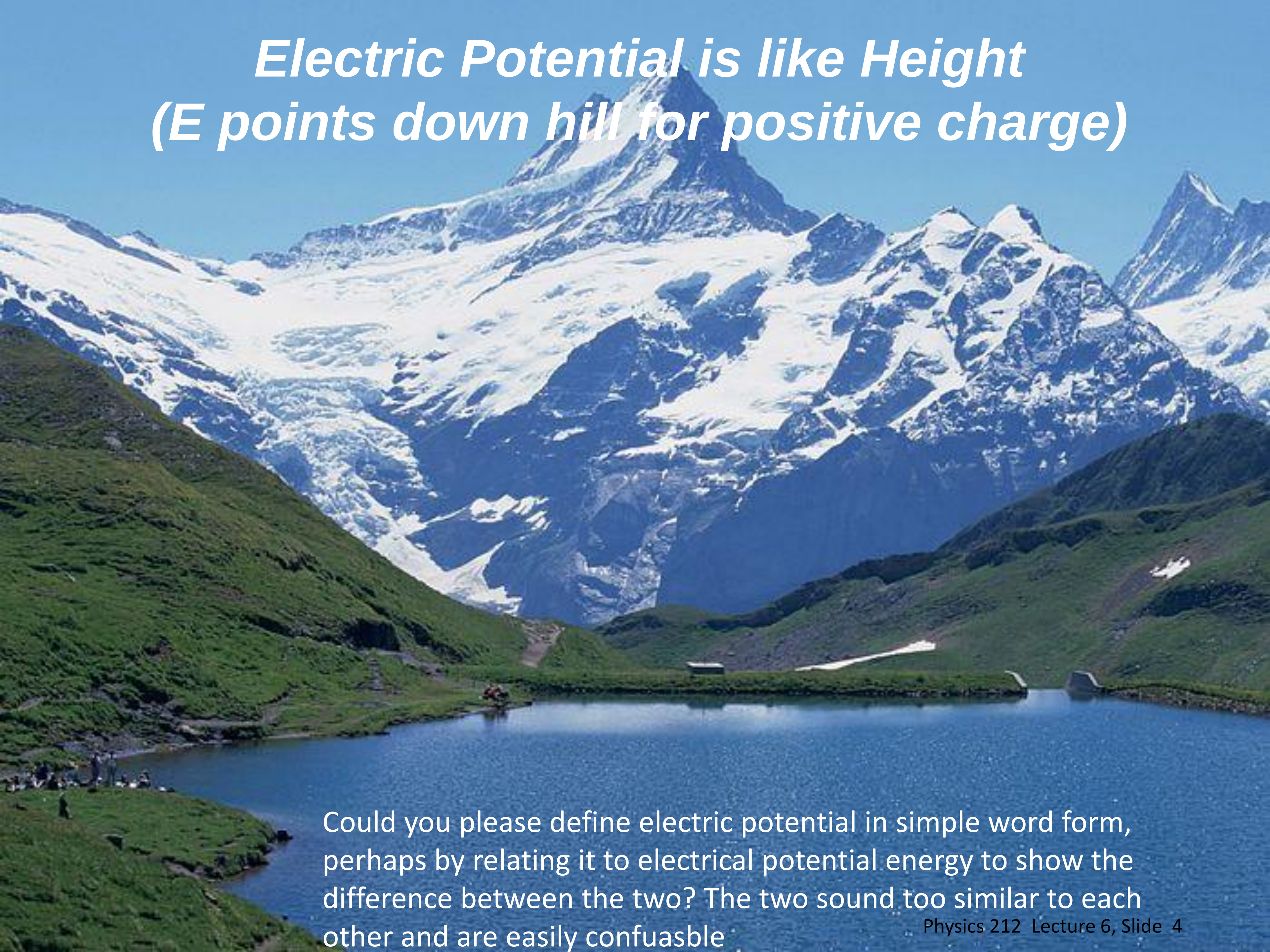
We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

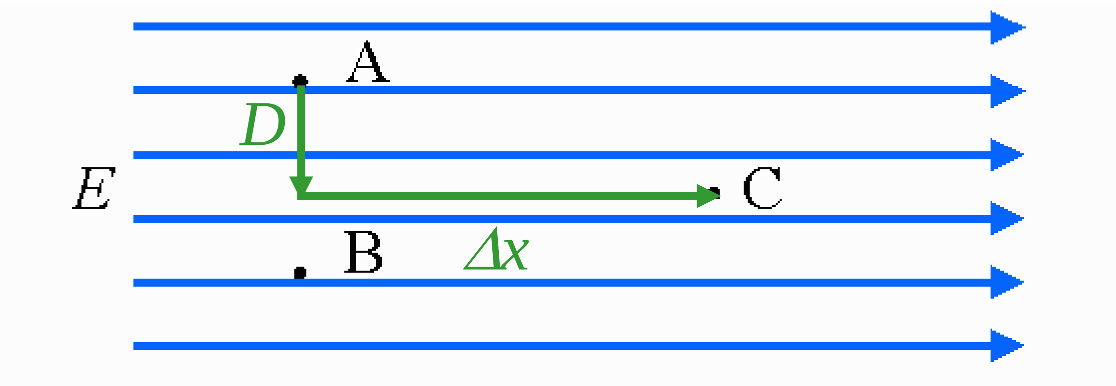
Electric Potential is like Height
(E points down hill for positive charge)



Could you please define electric potential in simple word form, perhaps by relating it to electrical potential energy to show the difference between the two? The two sound too similar to each other and are easily confuasble

Electric Potential from E field

Consider the three points A, B, and C located in a region of constant electric field as shown.



What is the sign of $\Delta V_{AC} = V_C - V_A$?

A) $\boxed{\Delta V_{AC} < 0}$ B) $\Delta V_{AC} = 0$ C) $\Delta V_{AC} > 0$

E points down hill

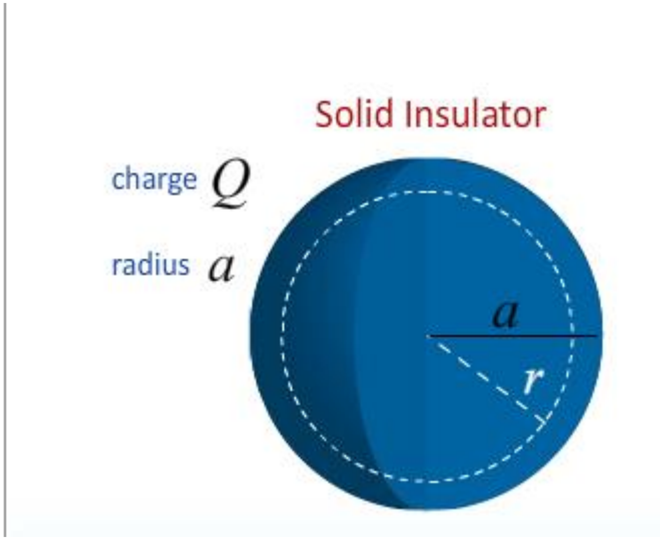
Remember the definition: $\Delta V_{A \to C} = -\int_A^C \vec{E} \cdot d\vec{l}$

Choose a path (any will do!)

$$\Delta V_{A \to C} = -\int_A^D \vec{E} \cdot d\vec{l} - \int_D^C \vec{E} \cdot d\vec{l} \longrightarrow \Delta V_{A \to C} = 0 - \int_D^C \vec{E} \cdot d\vec{l} = -E\Delta x < 0$$

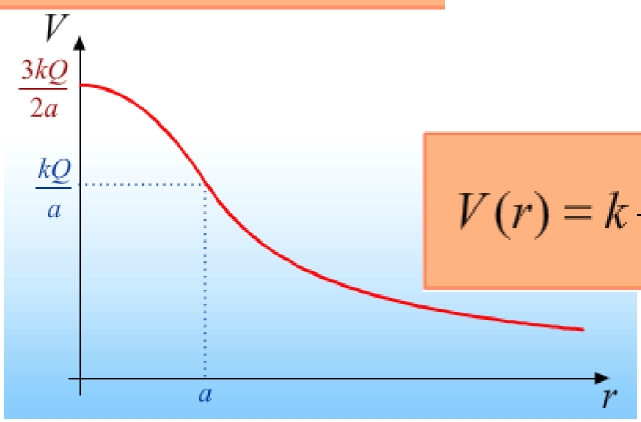
Charged Spherical Insulator

Please talk about the potential on the inside of an insulating sphere.



$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$ For $r < a$

$$\boxed{V(r) = k\frac{Q}{2a^3}(3a^2 - r^2)}$$ For $r < a$



$$\boxed{V(r) = k\frac{Q}{r}}$$ For $r > a$

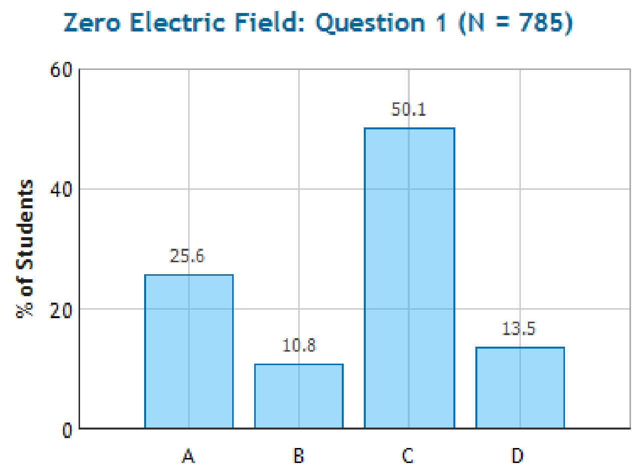
CheckPoint 2

If the electric field is zero in a region of space, what does that tell you about the electric potential in that region?

A) The electric potential is zero everywhere in this region.
B) The electric potential is zero at at least one point in this region.
C) The electric potential is constant everywhere in this region.
D) There is not enough information given to distingush which of the above answers is correct.

Remember the definition

$$\Delta V_{A\to B} = -\int_A^B \vec{E}\cdot d\vec{l}$$



$$\vec{E} = 0 \quad \longrightarrow \quad \Delta V_{A\to B} = 0 \quad \longrightarrow \quad V \text{ is constant!}$$

E from V

If we can get the potential by integrating the electric field:

$$\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

We should be able to get the electric field by differentiating the potential?

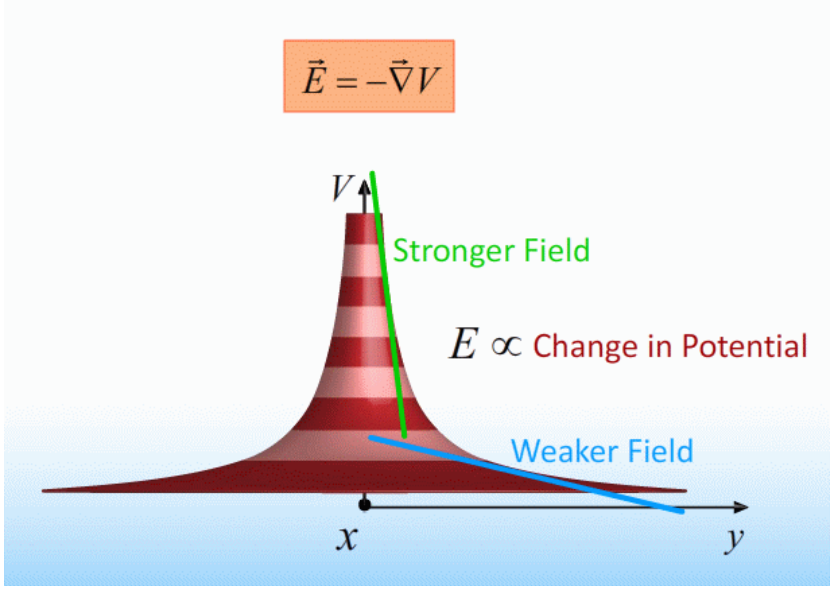
$$\vec{E} = -\vec{\nabla} V$$

In Cartesian coordinates:

$$E_x = -\frac{\partial V}{dx}$$

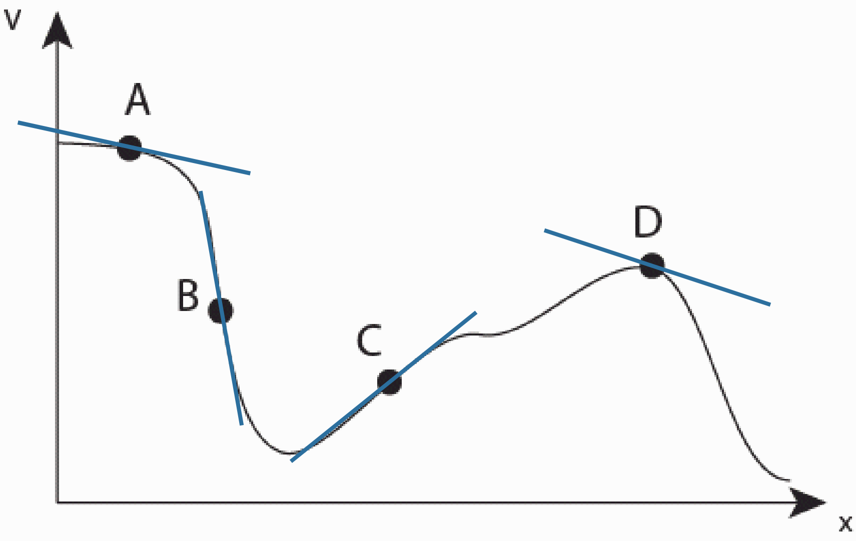
$$E_y = -\frac{\partial V}{dy}$$

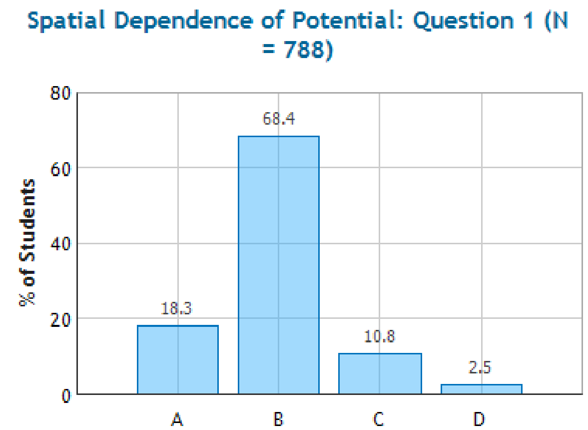
$$E_z = -\frac{\partial V}{dz}$$



CheckPoint 1a

2) The electric potential in a certain region is plotted in the following graph





At which point is the magnitude of the electric field greatest?

"A) The E field is the greatest where the voltage is the greatest, they are directly related."

"B) The electric field is the derivative of the electric potential. B has the steepest slope which indicates the greatest electric field."

"C) greatest positive derivative c"

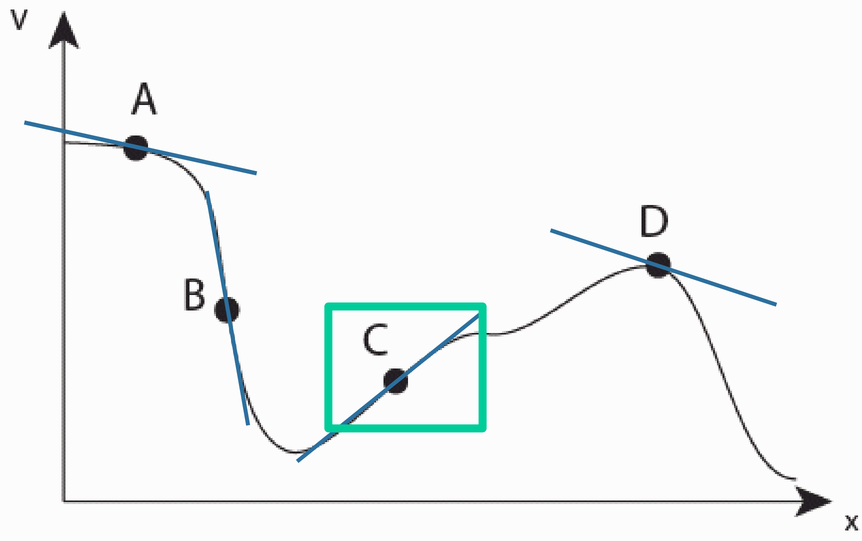
How do we get E from V?

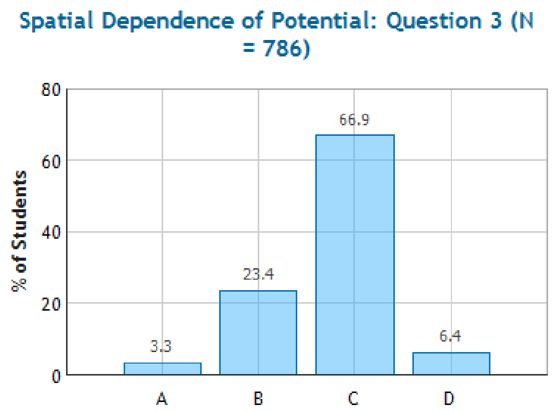
$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow$$

Look at slopes!

CheckPoint 1b

2) The electric potential in a certain region is plotted in the following graph





At which point is the electric field pointing in the negative x direction?

"B) The slope is negative at B"

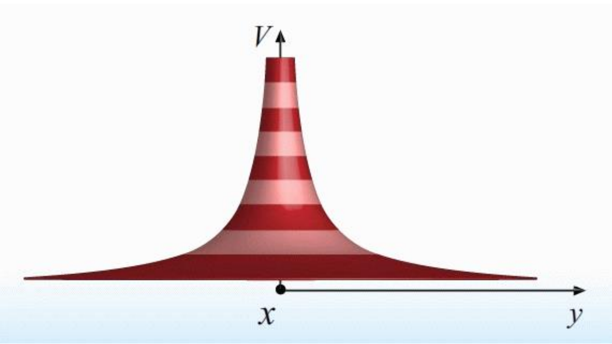
"C) Slope is positive, so the E-field is negative."

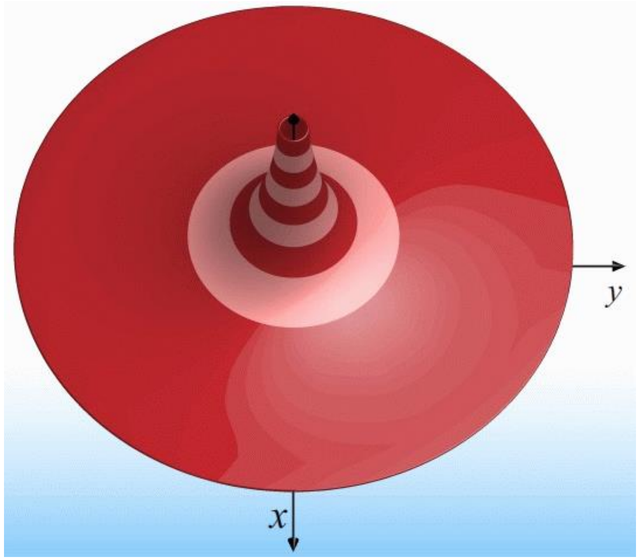
How do we get E from V?

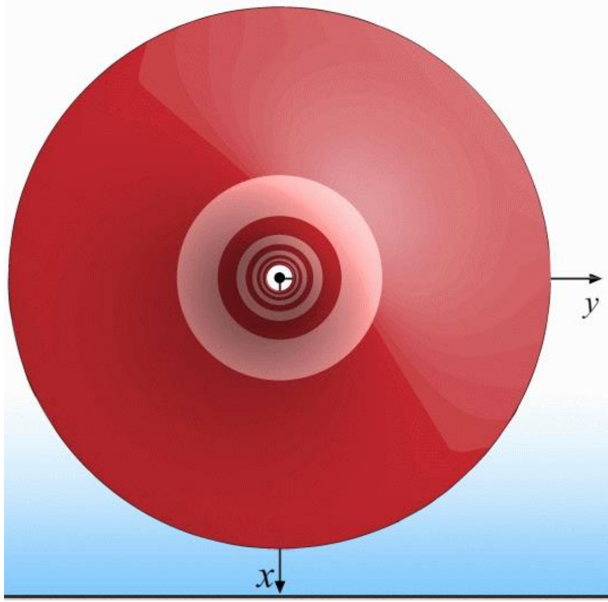
$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow \quad \text{Look at slopes!}$$

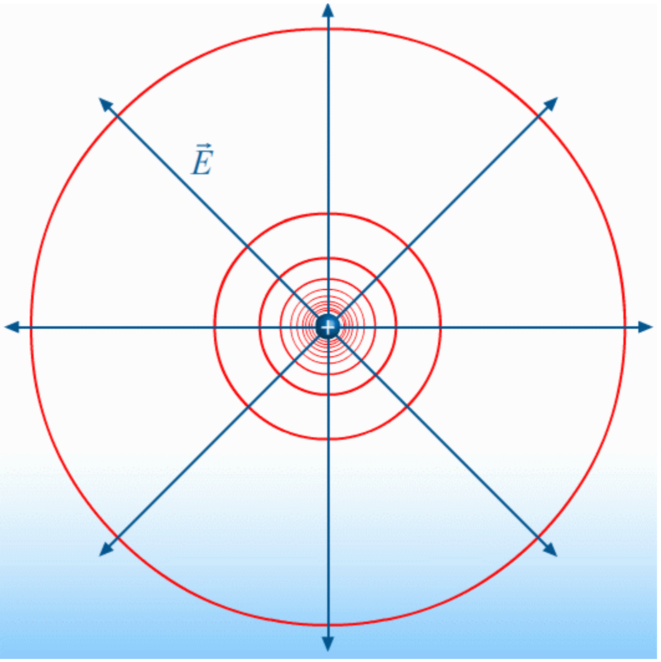
Equipotentials

Equipotentials are the locus of points having the same potential.







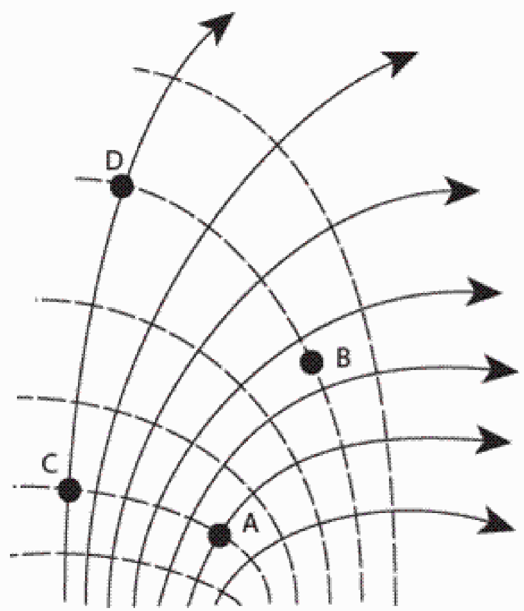


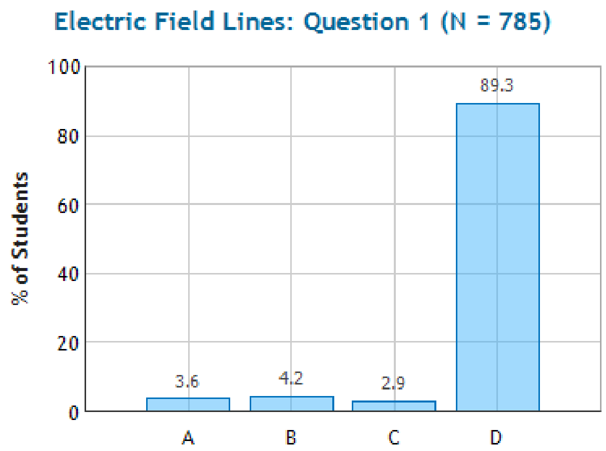
Equipotentials are
ALWAYS
perpendicular to the electric field lines.

The SPACING of the equipotentials indicates
The STRENGTH of the electric field.

CheckPoint 3a

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



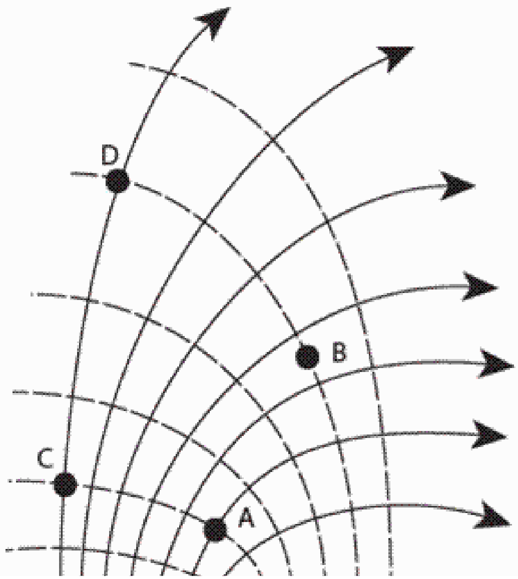


At which point is the magnitude of the electric field the smallest?

"The E field lines are spaced farther apart at D and the equipotential lines are less steep here."

CheckPoint 3b

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



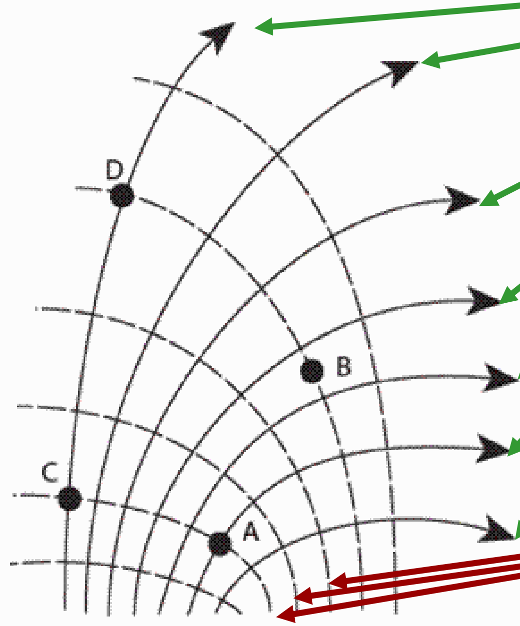
Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

Less than ½ got this correct!

Hint

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



What are these?

ELECTRIC FIELD LINES!

What are these?

EQUIPOTENTIALS!

What is the sign of $W_{AC} =$ work done by E field to move negative charge from A to C ?

A) $W_{AC} < 0$

B) $\boxed{W_{AC} = 0}$

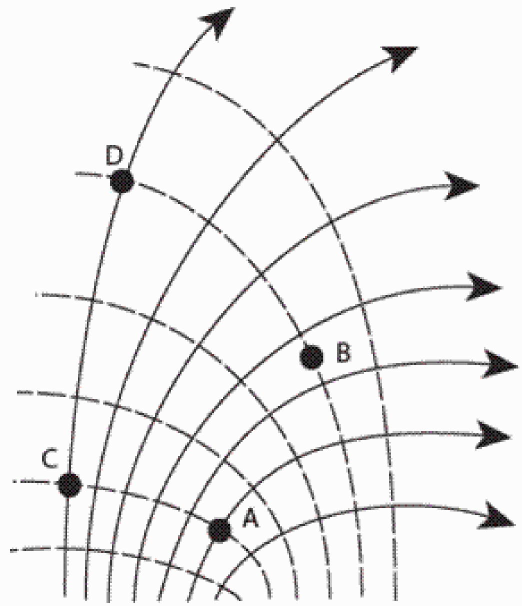
C) $W_{AC} > 0$

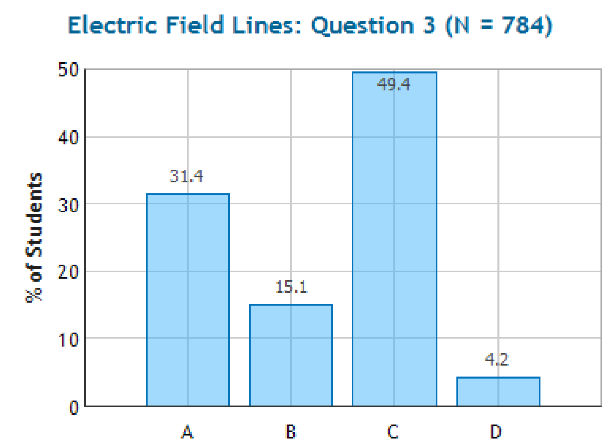
A and C are on the same equipotential → $W_{AC} = 0$

Equipotentials are perpendicular to the E field: No work is done along an equipotential

CheckPoint 3b Again?

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.





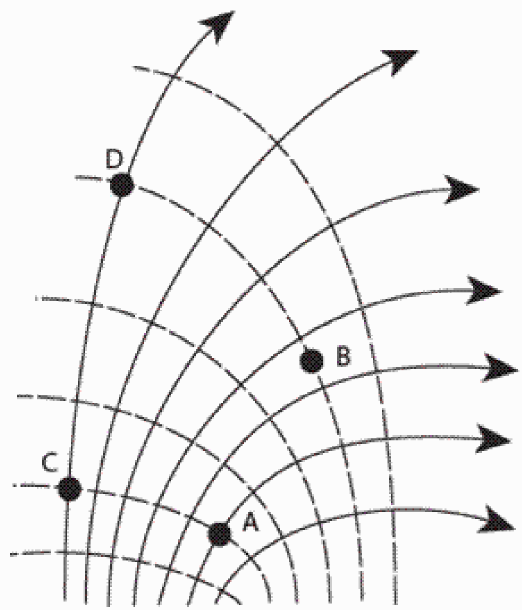
Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

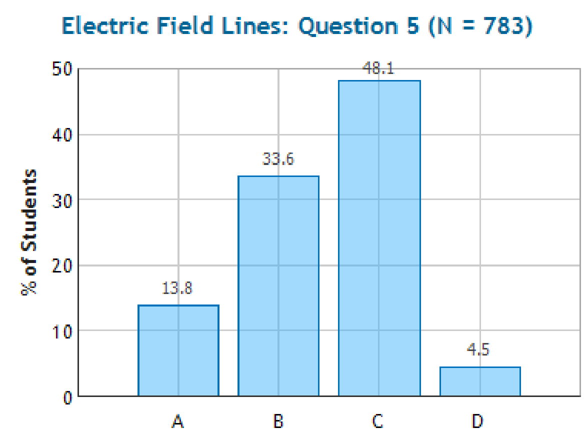
A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

- A and C are on the same equipotential
- B and D are on the same equipotential
- Therefore the potential difference between A and B is the SAME as the potential between C and D

CheckPoint 3c

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.





Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from A to D

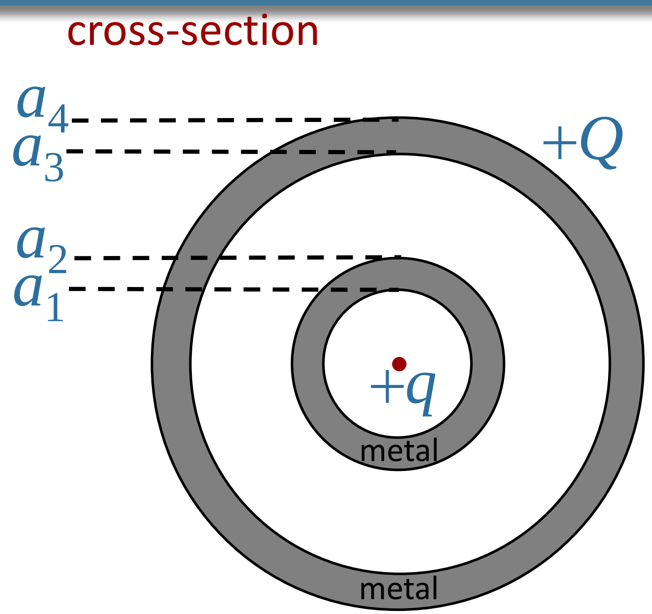
A) More work from A to B

B) More work from A to D

C) Same

D) Can not determine w/o performing calculation

Calculation for Potential



Point charge q at center of concentric conducting spherical shells of radii a_1, a_2, a_3, and a_4. The inner shell is uncharged, but the outer shell carries charge Q.

What is V as a function of r?

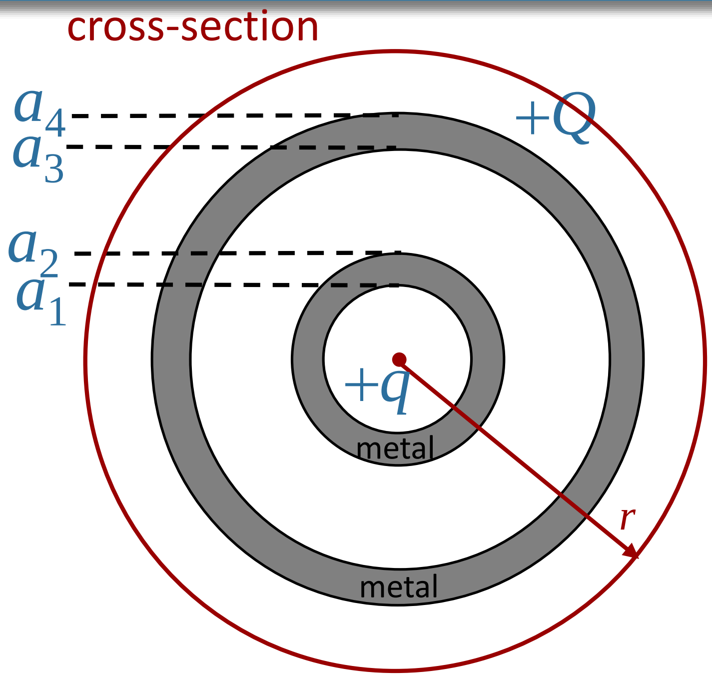
Conceptual Analysis:

- Charges q and Q will create an E field throughout space
- $V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$

Strategic Analysis:

- Spherical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V

Calculation: Quantitative Analysis



$r > a_4$: What is $E(r)$ outside spheres?

A) 0

B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_0}\frac{Q+q}{r}$

D) $\boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}}$

E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Why?

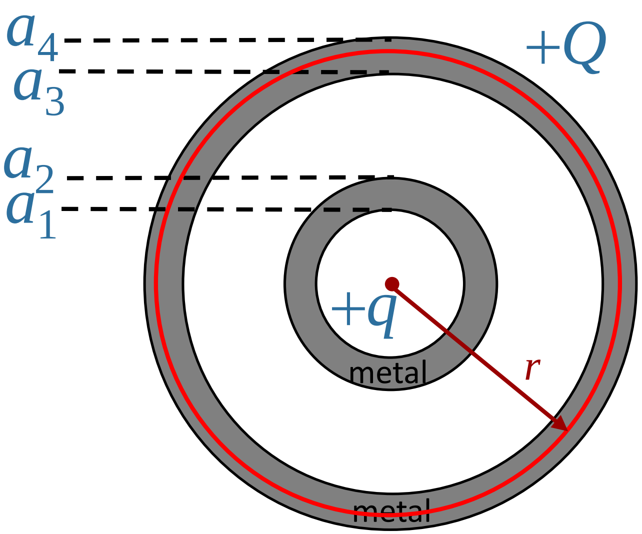
Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

$$E4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}$$

Calculation: Quantitative Analysis

cross-section



$a_3 < r < a_4$: What is $E(r)$ Inside outer metal sphere?

$\boxed{\text{A) } 0}$

B) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

C) $\dfrac{1}{2\pi\varepsilon_0}\dfrac{q}{r}$

D) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{-q}{r^2}$

E) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q-q}{r^2}$

Applying Gauss' law, what is $Q_{enclosed}$ for red sphere shown?

A) q B) $-q$ $\boxed{\text{C) } 0}$

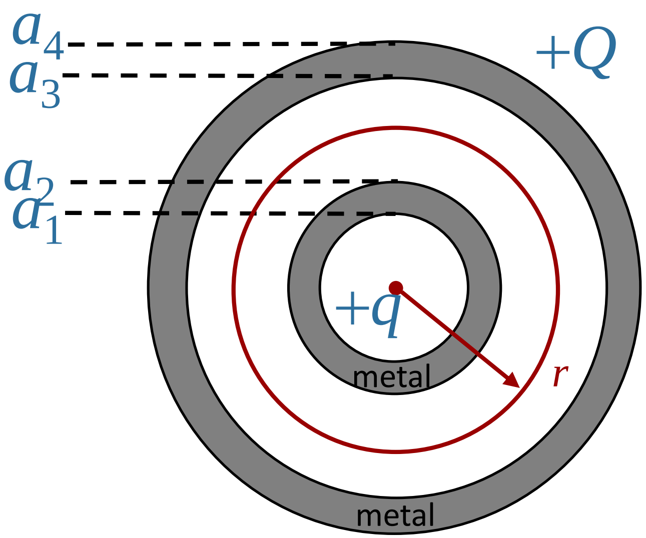
How is this possible?

$-q$ must be induced at $r = a_3$ surface $\longrightarrow$ charge at $r = a_4$ surface $= Q + q$

$$\sigma_3 = \frac{-q}{4\pi a_3^2} \qquad \sigma_4 = \frac{Q+q}{4\pi a_4^2}$$

Calculation: Quantitative Analysis

cross-section

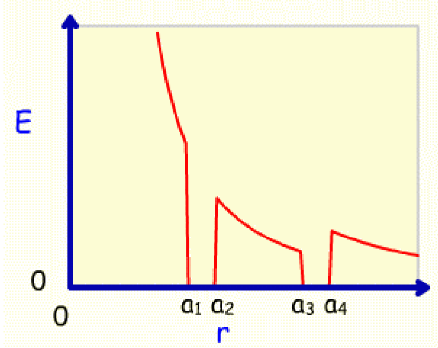


Continue on in…

$a_2 < r < a_3:$ $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$

$a_1 < r < a_2:$ $\boxed{E = 0}$

$r < a_1:$ $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$



$$V(r) = -\int_{r_0}^{r} \vec{E}\cdot d\vec{\ell}$$

To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: $r_0 =$ infinity)
2) Integrate!

$r > a_4:$ $V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r}$

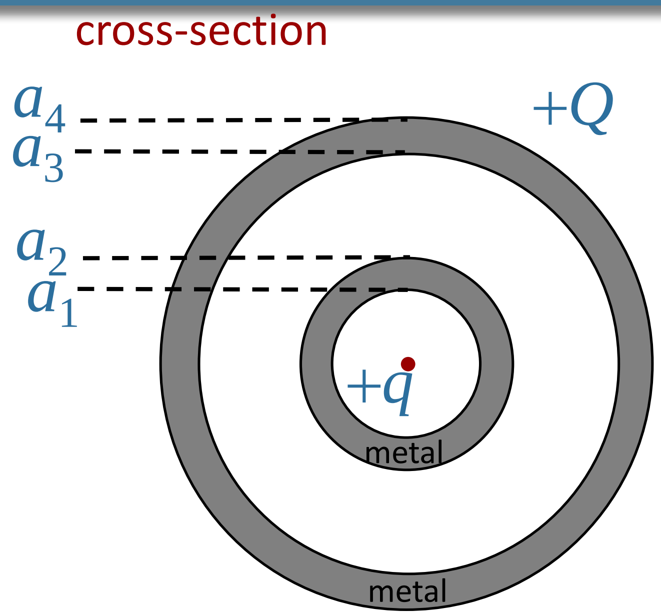
$a_3 < r < a_4:$

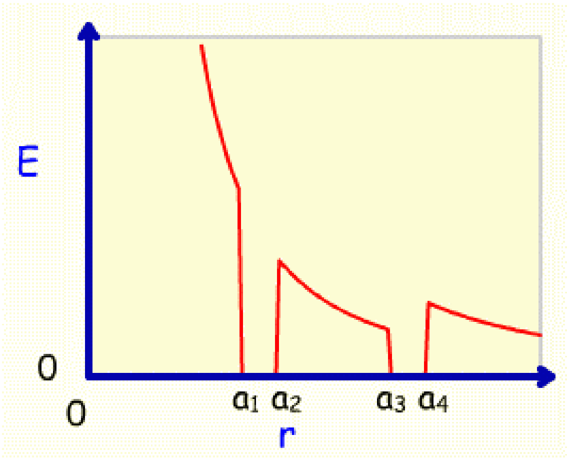
A) $V = 0$

B) $\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_4}}$

C) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_3}$

Calculation: Quantitative Analysis





$r > a_4:$ $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r}$$

$a_3 < r < a_4:$ $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_4}$$

$a_2 < r < a_3:$ $$V(r) = \Delta V(\infty \to a_4) + 0 + \Delta V(a_3 \to r)$$

$$V(r) = \frac{Q+q}{4\pi\varepsilon_0 a_4} + 0 + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a_3}\right)$$

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{r} - \frac{q}{a_3}\right)}$$

$a_1 < r < a_2:$ $$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{a_2} - \frac{q}{a_3}\right)$$

$0 < r < a_1:$ $$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{a_2} - \frac{q}{a_3} + \frac{q}{r} - \frac{q}{a_1}\right)$$