Key concepts this week:

- Kinematics of rotation
  - Use angular quantities instead of linear ones
  - Moment of inertia (resistance to angular acceleration) involves:
    - How much mass there is
    - **Where** the mass is located (this didn't matter in linear kinematics!)
- Dynamics of rotation
  - Parallel axis theorem: moving the axis of rotation from the center of mass changes the moment of inertia in a predictable way ($I_{\text{new}} = M_{\text{total}}D^2 + I_{\text{CM}}$)
  - Torque (the rotational analog of force) involves:
    - How much force is applied
    - The direction of the force
    - **Where** the force is applied (this didn't matter in linear kinematics!)
    - These pieces were all unified in a right hand rule for $\vec{\tau} = \vec{r} \times \vec{F}$
  - Newton's 2nd Law analog for rotation: $\vec{\tau}_{\text{net}} = I\vec{\alpha}$

Space Station

A space station is constructed in the shape of a wheel 22 m in diameter, with essentially all its mass (5.0×10^5 kg) at the rim. Once the space station is completed, it is set rotating at a rate such that an object at the rim experiences a radial acceleration equal to the Earth's gravitational acceleration g, thus simulating Earth's gravity. To accomplish this, two small rockets are attached on opposite sides of the rim, each able to provide a 100 N force. How long will it take to reach the desired rotation rate and how many revolutions will the space station make in this time?

Rotating Tip

In the Physics 211 Laboratory one group of students has decided to pursue their own experiments. They make a simple pendulum from a weight attached to a string of length L. They attach the other end of the string to a fixed support. They hold the weight with the string taut and horizontal and then released it. With their motion sensor they measure the speed of the weight as the string passes through the vertical. Remembering that all objects fall with the same acceleration, the students do a second experiment. They attach one end of a uniform stick of length L to the support, which acts as a pivot. They hold the stick horizontal and release it. They then measure the speed of the tip of the stick with their motion sensor. The mass of the pendulum weight and the mass of the stick are the same. Do they measure the same speed?

Review: Skate-Board Exhibition

You are helping your friend prepare for his skate-board clowning stunt. For his program, he plans to take a running start and then jump onto a gigantic 7 kg stationary skateboard. He and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. He has measured his maximum running speed to jump safely on the skateboard at 6 m/s, and he wants to know how high above ground level he will go as he rolls up the slope. He tells you his weight is 70 kg.

Review: Pothole

A car is traveling along a horizontal road when it suddenly encounters a pothole, in which the level of the road abruptly changes by a height h. The suspension of the car can be considered as a single spring having a spring constant of 110,000 N/m that can compress a maximum distance of 0.4 m. The mass of the car is 1200 kg. What is the maximum value of h that the car can tolerate before bottoming out (i.e., when the springs reach their maximum compression)? For simplicity, you may assume that the spring is not compressed initially.

Review: elastic collisions: Train Cars

A 1000 kg train car is moving rightwards at a speed 3 m/s towards another car, of 2000 kg, also moving rightwards, but at 1 m/s. They collide elastically through a spring of stiffness 10,000 N/m.

What is the final speed of the second car?

How much does the spring get compressed?

Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67\times10^{-11}\ Nm^2/kg^2$)
$\mathbf{F}_{\mathbf{spring}} = -k\,\mathbf{\Delta x}$

Friction
$f = \mu_k N$ (kinetic)
$f \le \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot d\mathbf{l}$
$W = \mathbf{F}\cdot\mathbf{\Delta r} = F\,\Delta r\,\cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i\mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i\mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i\mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i\mathbf{v}_i$
$\Sigma\mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse
$\mathbf{I} = \int \mathbf{F}\, dt$
$\Delta\mathbf{P} = \mathbf{F}_{av}\Delta t$

Collisions:
If $\Sigma\mathbf{F}_{EXT} = 0$ in some direction, then
$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:
$\Sigma m_i\mathbf{v}_i$ (before) $= \Sigma m_i\mathbf{v}_i$ (after)

<u>In addition, if the collision is elastic:</u>
* $E_{before} = E_{after}$
* *Rate of approach = Rate of recession*
* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics
$s = R\theta,\ v = R\omega,\ a = R\alpha$
$$\left.\begin{array}{l}\theta = \theta_0 + \omega_0 t + {}^1/_2\alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta\end{array}\right\}$$

Rotational Dynamics
$I = \Sigma m_i r_i^2$
$I_{parallel} = I_{CM} + MD^2$
$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$
$I_{hoop} = MR^2$
$I_{solid\text{-}sphere} = {}^2/_5 MR^2$
$I_{spherical\ shell} = {}^2/_3 MR^2$
$I_{rod\text{-}cm} = {}^1/_{12} ML^2$
$I_{rod\text{-}end} = {}^1/_3 ML^2$
$\tau = I\alpha$ (rotation about a fixed axis)
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$