Key concepts this week:

- Definition of Angular Momentum
  - Point particle ($\vec{L} = \vec{r} \times \vec{p}$)
  - Rigid, extended object ($\vec{L} = I\vec{\omega}$)
- Handy form for rotational kinetic energy ($K = \dfrac{L^2}{2I}$)
- Using Newton's 2nd Law for rotation
  - A net torque produces a change in angular momentum ($\tau_{\text{net}} = \dfrac{d\vec{L}}{dt}$)
  - No net torque $\Rightarrow$ angular momentum doesn't change (i.e. is conserved)
  - If there IS a net torque, then its magnitude and direction determines the change in angular momentum (e.g. precession)

Rotating Astronauts

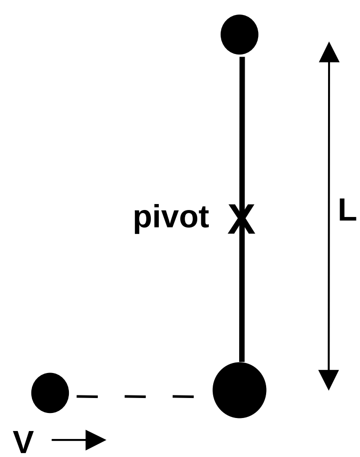
Two astronauts, each having a mass 75 kg, are connected by a light rope 10 meters long. They are isolated in space, and are initially revolving around their common center of mass once every 6 seconds. They now pull in on the rope, halving the distance between them. How much work is done by the astronauts as they shorten the rope?

Spinning Disks

Two disks are mounted on frictionless bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, which has a mass of 1 kg and radius of 2 m, is set spinning at 45 rad/s. The second disk, which has a mass of 2 kg and a radius of
3 m, is set spinning at 25 rad/s in the opposite direction. They then couple together. What is their angular speed after coupling?

Dumbbell Collision

A dumbbell consists of two balls, one of mass M and the other of mass 2M, connected by a light rod of length L. The dumbbell is mounted vertically on a pivot with the heavier ball at the bottom. The pivot is located at the midpoint of the rod. The system, which is initially at rest, is free to rotate about the pivot. A wad of putty of mass M and initial velocity V collides with and sticks to the lower mass, as shown in the diagram. In terms of the quantities given, what is the minimum value of V for which the dumbbell will make it all the way around?



Faster than a Speeding Bullet

You are assigned the job of designing a simple system to measure the speed of a bullet. The system you come up with works in the following way: You shoot the bullet into a hardwood rod of mass 5 kg and length 1.2 m, mounted on a frictionless axle which passes through its center and is perpendicular to its length. The axle is vertical, so the rod rotates in the horizontal plane after the bullet hits it. The rod is initially at rest and oriented perpendicular to the path of the bullet. You shoot a bullet of mass 2.0 grams at 400 m/s into the rod a distance of 0.45 m to one side of the axle, where it embeds itself and stops.

1. How much time will it take the rod to make one full revolution after the bullet hits it?
2. What is the ratio of the initial to the final kinetic energy of the system?

Review Rotational Statics: Tipping Crates

A uniform rectangular crate of height H and width W weighs Mg. It is desired to tip (i.e., roll) the crate by applying a horizontal force F to one of the upper edges.

(a) Assuming that the crate does not slip, what is the minimum force necessary to tip the crate?

(b) What minimum coefficient of static friction between the crate and the ground is required such that the crate tips but does not slide?

Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67\text{x}10^{-11}\ Nm^2/kg^2$)
$\mathbf{F}_{spring} = -k\,\mathbf{\Delta x}$

Friction
$f = \mu_k N$ (kinetic)
$f \leq \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot d\mathbf{l}$
$W = \mathbf{F}\cdot\mathbf{\Delta r} = F\,\Delta r\,cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i \mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i \mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i \mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i \mathbf{v}_i$
$\Sigma \mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse
$\mathbf{I} = \int \mathbf{F}\,dt$
$\Delta \mathbf{P} = \mathbf{F}_{av}\Delta t$

Collisions:
If $\Sigma \mathbf{F}_{EXT} = 0$ in some direction, then
$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:
$\Sigma m_i \mathbf{v}_i$ (before) $= \Sigma m_i \mathbf{v}_i$ (after)

<u>In addition, if the collision is elastic:</u>
* $E_{before} = E_{after}$
* *Rate of approach = Rate of recession*
* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics
$s = R\theta,\ v = R\omega,\ a = R\alpha$
$$\left.\begin{aligned}\theta &= \theta_0 + \omega_0 t + {}^1/_2\alpha t^2\\ \omega &= \omega_0 + \alpha t\\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta\end{aligned}\right\}$$

Rotational Dynamics
$I = \Sigma m_i r_i^2$
$I_{parallel} = I_{CM} + MD^2$
$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$
$I_{hoop} = MR^2$
$I_{solid\text{-}sphere} = {}^2/_5 MR^2$
$I_{spherical\ shell} = {}^2/_3 MR^2$
$I_{rod\text{-}cm} = {}^1/_{12} ML^2$
$I_{rod\text{-}end} = {}^1/_3 ML^2$
$\tau = I\alpha$ (rotation about a fixed axis)
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$

Work & Energy
$K_{rotation} = {}^1/_2 I\omega^2$,
$K_{translation} = {}^1/_2 MV_{cm}^2$
$K_{total} = K_{rotation} + K_{translation}$
$W = \tau\theta$

Statics
$\Sigma \mathbf{F} = 0$, $\Sigma\tau = 0$ (about any axis)

Angular Momentum:
$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
$L_z = I\omega_z$
$\mathbf{L}_{tot} = \mathbf{L}_{CM} + \mathbf{L}^*$
$\boldsymbol{\tau}_{ext} = d\mathbf{L}/dt$
$\boldsymbol{\tau}_{cm} = d\mathbf{L}^*/dt$
$\Omega_{precession} = \tau / L$