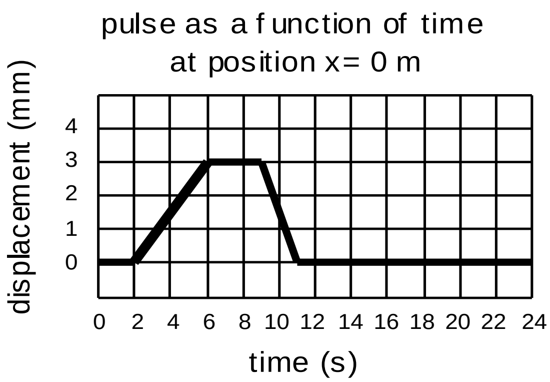
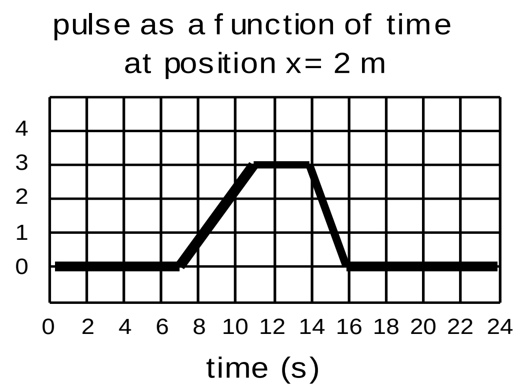
Key concepts this week:

- Transverse waves on a string take the general form $y(x,t) = f(x-vt)$ (or $f(x+vt)$, depending on the direction the wave is going) for *any* continuous function *f*.
- Do not confuse the transverse displacement of the material y (usually in mm) with the coordinate x (typically in meters) at which that bit of material lies.
- Do not confuse the material velocity $\partial y/\partial t$ of the matter moving the direction perpendicular to the string, with the speed v with which the wave moves.
- Wave speed is determined by the properties of the medium. For transverse waves on a string, this is given by the expression $v = \sqrt{T/\mu}$, where T is the tension in the string and μ is the mass *per unit length* of the string
  - The wave speed depends **only** on the medium (not on the amplitude, wavelength, frequency, etc.)
- *Harmonic* waves are sinusoidal in both space and time (i.e. $y(x,t) = A\cos(kx - \omega t)$)
- The energy carried by a wave is proportional to the square of the displacement amplitude and frequency of the wave: $K_{\max} : \omega^2 A^2$ (in particular, for a string with $y = A \sin(kx-\omega t)$ the average power is $(1/2)\ \mu\ (A\omega)^2\ v$)
- Free vibration of a standing wave in a string of length L fixed at both ends must have a half-wavelength that fits into the length, i.e., L must be a multiple of $\lambda/2$

Pulse on String

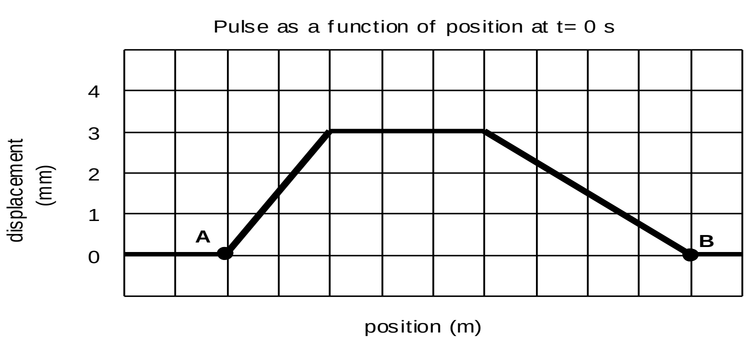
A pulse travels on a string. Its shape does not change. The displacement of the string as a function of time is shown in the figures below at two positions, x = 0 m and x = 2 m. These figures show that the string rises slowly to a height of 3 mm, remains at that height for some time, and lowers quickly back to the equilibrium position.





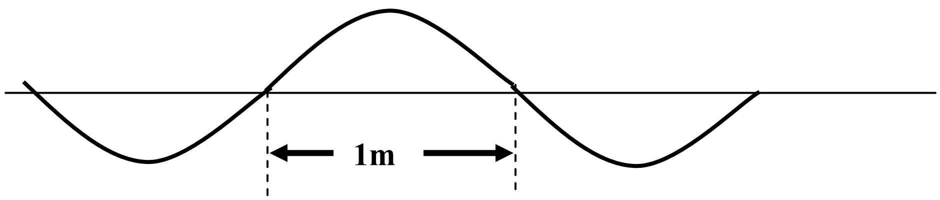
How fast is the pulse traveling along the string?

What is the distance between points A and B?



Note: the graph above shows another view of the pulse, the displacement of the string at t=0. It may be useful to realize that this latter picture is a snapshot of the string and the first frame of a movie in which that shape is moving steadily to the right.

Power Train



A long wire under tension is carrying the traveling wave illustrated above. A one meter section of the wire has a mass of 1 gram. The oscillation frequency is f = 256 Hz and the average power passing a point is 10 Watts. Find the tension on the wire and the wave amplitude.

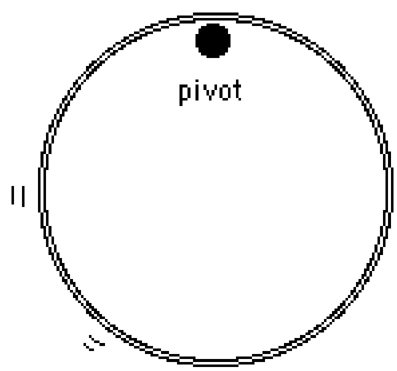
Building a guitar

You are designing a new lightweight electric guitar using hi-tech composite materials, and one of your first jobs is to calculate how much material needs to be in the neck and head-stock. This will depend on how strong you want it to be, which in turn is determined by the total tension in the strings since the neck has to support this without bending too much. You make the simplifying assumption that the tension in all six strings is about the same, and decide to calculate what the tension is in the high-E string since you happen to have a brand new one handy.

You read on the E string package that its diameter is 0.009" and that it is made of steel, which your physics book tells you has a density of about 8 g/cm^3. The distance between the bridge and the nut on your guitar is supposed to be 26", and the vibration frequency of an open high-E string should be 330 Hz. How much tension must the neck support?

Review: Not So Simple Pendulum: Hoop

A thin uniform hoop of mass M and radius R is hung on a thin horizontal rod that acts like a pivot. It is set oscillating with small amplitude. Find the period of oscillation of the hoop, in terms of R, M, and g. The hoop does not slip on the rod.



Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67\times10^{-11}\ Nm^2/kg^2$)
$\mathbf{F}_{spring} = -k\,\mathbf{\Delta x}$

Friction
$f = \mu_k N$ (kinetic)
$f \le \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot d\mathbf{l}$
$W = \mathbf{F}\cdot\mathbf{\Delta r} = F\,\Delta r\,\cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i\mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i\mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i\mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i\mathbf{v}_i$
$\Sigma\mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse
$\mathbf{I} = \int \mathbf{F}\,dt$
$\Delta\mathbf{P} = \mathbf{F}_{av}\Delta t$

Collisions:
If $\Sigma\mathbf{F}_{EXT} = 0$ in some direction, then
$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:
$\Sigma m_i\mathbf{v}_i$ (before) $= \Sigma m_i\mathbf{v}_i$ (after)

In addition, if the collision is elastic:
* $E_{before} = E_{after}$
* *Rate of approach = Rate of recession*
* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics
$s = R\theta,\ v = R\omega,\ a = R\alpha$

$$\left.\begin{array}{l}\theta = \theta_0 + \omega_0 t + {}^1/_2\alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta\end{array}\right\}$$

Rotational Dynamics
$I = \Sigma m_i r_i^2$
$I_{parallel} = I_{CM} + MD^2$
$I_{disk} = I_{cylinder} = {}^1/_2MR^2$
$I_{hoop} = MR^2$
$I_{solid\text{-}sphere} = {}^2/_5MR^2$
$I_{spherical\ shell} = {}^2/_3MR^2$
$I_{rod\text{-}cm} = {}^1/_{12}ML^2$
$I_{rod\text{-}end} = {}^1/_3\ ML^2$
$\tau = I\alpha$ (rotation about a fixed axis)
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$

Work & Energy
$K_{rotation} = {}^1/_2I\omega^2$,
$K_{translation} = {}^1/_2MV_{cm}^2$
$K_{total} = K_{rotation} + K_{translation}$
$W = \tau\theta$

Statics
$\Sigma\mathbf{F} = 0$, $\Sigma\tau = 0$ (about any axis)

Angular Momentum:
$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
$L_z = I\omega_z$
$\mathbf{L}_{tot} = \mathbf{L}_{CM} + \mathbf{L}^*$
$\boldsymbol{\tau}_{ext} = d\mathbf{L}/dt$
$\boldsymbol{\tau}_{cm} = d\mathbf{L}^*/dt$
$\Omega_{precession} = \tau / L$

Simple Harmonic Motion:
$d^2x/dt^2 = -\omega^2 x$
(differential equation for SHM)

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A\sin(\omega t + \phi)$
$a(t) = -\omega^2 A\cos(\omega t + \phi)$

$\omega^2 = k/m$ (mass on spring)
$\omega^2 = g/L$ (simple pendulum)
$\omega^2 = mgR_{CM}/I$ (physical pendulum)
$\omega^2 = \kappa/I$ (torsion pendulum)

General harmonic transverse waves:
$y(x,t) = A\cos(kx - \omega t)$
$k = 2\pi/\lambda$, $\omega = 2\pi f = 2\pi/T$
$v = \lambda f = \omega/k$

Waves on a string:

$$v^2 = \frac{F}{\mu} = \frac{(\text{tension})}{(\text{mass per unit length})}$$

$$\overline{P} = \frac{1}{2}\mu v\omega^2 A^2$$

$$\frac{d\overline{E}}{dx} = \frac{1}{2}\mu\omega^2 A^2$$

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad \text{Wave Equation}$$