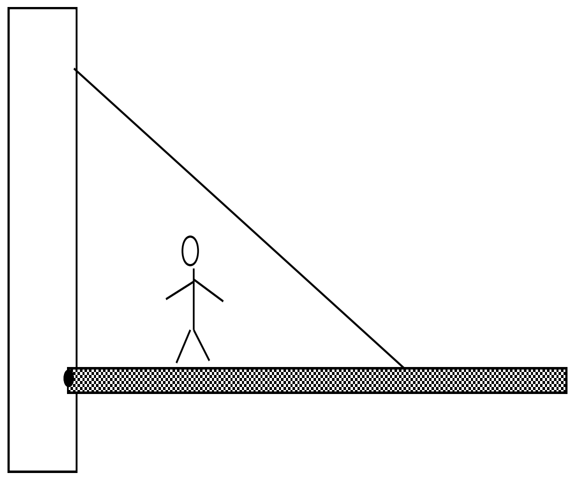
Key concepts this week:

- No-slip condition
  - angular displacement of rotation is related to the comparable linear displacement if there's no slipping ($v = \omega R$, etc)
- Rotational Statics
  1. zero linear (CM) acceleration ⇒ the net force on the system is zero
  2. zero angular acceleration ⇒ the net torque about *any* axis is zero
  3. we can use these two conclusions to determine magnitude, direction, and location of forces on the system

  - Linear and angular accelerations are both zero ⇒ $\begin{cases} F_{\text{net},x} = Ma_x = 0 \\ F_{\text{net,y}} = Ma_y = 0 \\ \tau_{\text{net}} = I\alpha = 0 \end{cases}$
  - A stationary object won't tip if its center of mass is within the object's footprint on the surface below
  - The gravitational potential energy of an extended object uses the height of the object's center of mass ($U_{\text{gravity}} = MgY_{\text{CM}}$)

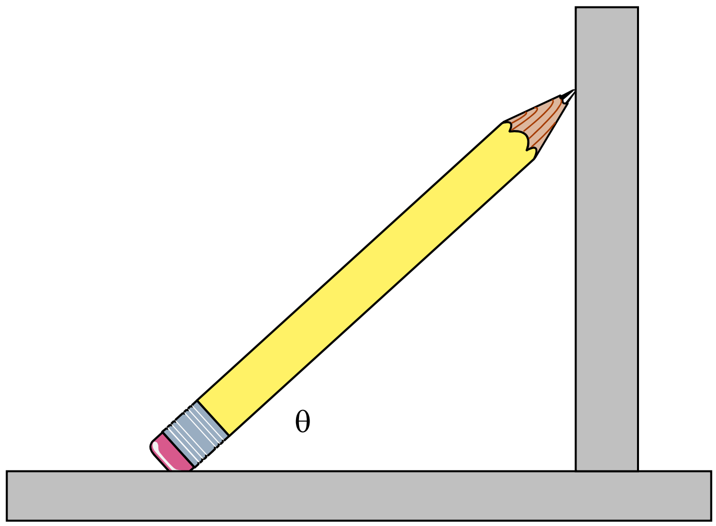
Walking the Plank

A uniform horizontal beam 8 m long is attached by a frictionless pivot to a wall. A cable making an angle of 37^{o}, attached to the beam 5 m from the pivot point, supports the beam, which has a mass of 600 kg. The breaking point of the cable is 8000 N. A man of mass 95 kg walks out along the beam. How far can he walk before the cable breaks?



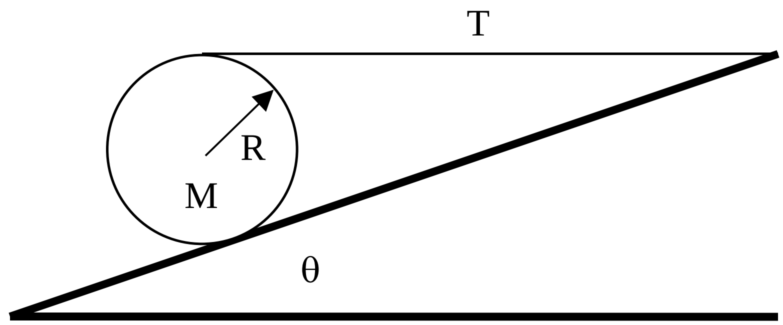
Leaning Pencil

A picture below shows a pencil with its sharpened end resting against a smooth vertical surface and its eraser end resting on the floor. The center of mass of the pencil is 9 cm from the end of the eraser and 11 cm from the tip of the lead. The coefficient of static friction between the eraser and floor is $\mu = 0.80$. What is the minimum angle θ the pencil can make with the floor such that it does not slip?



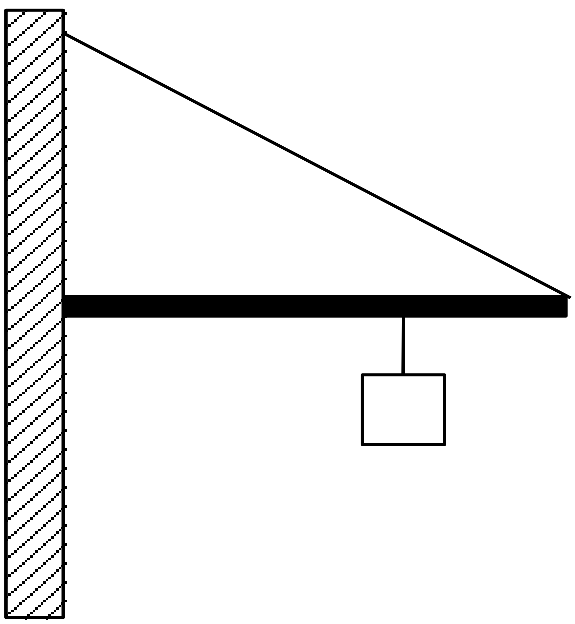
Cylinder Held on Inclined Plane

A cylinder of mass M and radius R is in static equilibrium as shown in the diagram. The cylinder rests on an inclined plane making an angle θ with the horizontal and is held by a horizontal string attached to the top of the cylinder and to the inclined plane. There is friction between the cylinder and the plane. What is the tension in the string T?



Weight on Stick

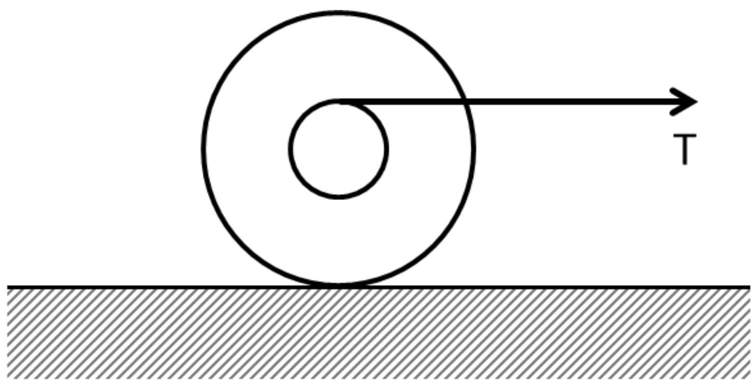
One end of a uniform meter stick of mass 0.25 kg is placed against a vertical wall. The other end is held by a lightweight cord making an angle $\theta = 20^{o}$ with the stick. A block with the same mass as the meter stick is suspended from the stick a distance 0.75 m from the wall. Find the tension in the cord and the frictional force between the stick and the wall.



Review: Rolling Spool

A spool with outer radius R and inner radius r rolls without slipping on a horizontal surface. The inner part may be approximated as a uniform cylinder (radius r) of mass m. The two rims may be thought of as disks (of radius R) and mass M each. Total mass is m + 2M.

The spool is pulled by a rope of tension T wrapped around the inner radius as pictured. What is the acceleration of the center of mass of the spool?



Kinematics

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$

$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion

$a = v^2/r = \omega^2 r$

$v = \omega r$

$\omega = 2\pi/T = 2\pi f$

Dynamics

$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$

$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)

$F_{12} = -Gm_1m_2/r^2$ (in general)

(where $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$)

$\mathbf{F}_{spring} = -k\,\mathbf{\Delta x}$

Friction

$f = \mu_k N$ (kinetic)

$f \leq \mu_S N$ (static)

Work & Kinetic energy

$W = \int \mathbf{F} \cdot \mathbf{dl}$

$W = \mathbf{F} \cdot \mathbf{\Delta r} = F\,\Delta r \cos\theta$

(constant force)

$W_{grav} = -mg\Delta y$

$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$

$W_{NET} = \Delta K$

Potential Energy

$U_{grav} = mgy$ (near earth surface)

$U_{grav} = -GMm/r$ (in general)

$U_{spring} = kx^2/2$

$\Delta E = \Delta K + \Delta U = W_{nc}$

Power

$P = dW/dt$

$P = \mathbf{F} \cdot \mathbf{v}$ (for constant force)

System of Particles

$\mathbf{R}_{CM} = \Sigma m_i \mathbf{r}_i / \Sigma m_i$

$\mathbf{V}_{CM} = \Sigma m_i \mathbf{v}_i / \Sigma m_i$

$\mathbf{A}_{CM} = \Sigma m_i \mathbf{a}_i / \Sigma m_i$

$\mathbf{P} = \Sigma m_i \mathbf{v}_i$

$\Sigma \mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse

$\mathbf{I} = \int \mathbf{F}\, dt$

$\Delta \mathbf{P} = \mathbf{F}_{av} \Delta t$

Collisions:

If $\Sigma \mathbf{F}_{EXT} = 0$ in some direction, then

$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:

$\Sigma m_i \mathbf{v}_i$ (before) $= \Sigma m_i \mathbf{v}_i$ (after)

<u>In addition, if the collision is elastic:</u>

* $E_{before} = E_{after}$

* *Rate of approach = Rate of recession*

* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics

$s = R\theta,\ v = R\omega,\ a = R\alpha$

$$\left.\begin{array}{l} \theta = \theta_0 + \omega_0 t + {}^1\!/_2 \alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta \end{array}\right\}$$

Rotational Dynamics

$I = \Sigma m_i r_i^2$

$I_{parallel} = I_{CM} + MD^2$

$I_{disk} = I_{cylinder} = {}^1\!/_2 MR^2$

$I_{hoop} = MR^2$

$I_{solid\text{-}sphere} = {}^2\!/_5 MR^2$

$I_{spherical\ shell} = {}^2\!/_3 MR^2$

$I_{rod\text{-}cm} = {}^1\!/_{12} ML^2$

$I_{rod\text{-}end} = {}^1\!/_3 ML^2$

$\tau = I\alpha$ (rotation about a fixed axis)

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$

Work & Energy

$K_{rotation} = {}^1\!/_2 I\omega^2$,

$K_{translation} = {}^1\!/_2 MV_{cm}^2$

$K_{total} = K_{rotation} + K_{translation}$

$W = \tau\theta$

Statics

$\Sigma \mathbf{F} = 0$, $\Sigma\tau = 0$ (about any axis)