Key concepts this week:

- Force is negative slope of potential energy ($F = -\dfrac{\mathrm{d}U}{\mathrm{d}x}$ or $\vec{F} = -\vec{\nabla}U$)
- Center of Mass
  - Definition ($\vec{R}_{CM} = \dfrac{1}{M_{\text{Total}}}\sum_i m_i \vec{r}_i$)
  - Dynamics
    - Newton's 2nd Law for the system ($\vec{A}_{CM} = \dfrac{\vec{F}_{\text{net, external}}}{M_{\text{Total}}}$)
    - Center of Mass Equation ($\Delta K_{\text{CM}} = \int \vec{F}_{\text{net, external}} \bullet d\vec{l}_{\text{CM}}$)

Block on Ramp

A block starts with a speed of 15 m/s at the bottom of a ramp that is inclined at an angle of 30^o with the horizontal. The coefficient of kinetic friction between the block and the plane is $\mu=0.25$. The block goes up the ramp, momentarily comes to rest, then slides back down the ramp. What is the speed of the block when it reaches the bottom of the ramp?

Roller Coaster

A roller coaster car has a mass of 840 kg. It is launched horizontally from a giant spring, with spring constant 31,000 N/m into a frictionless vertical loop-the-loop track of radius 6.2m. What is the minimum amount that the spring must be compressed if the car is to stay on the track?

Romeo and Juliet

Romeo, who is sitting in the rear of their boat in still water, entertains Juliet by playing his guitar. After the serenade, Juliet, who was sitting in the front of the boat (closest to shore), carefully moves to the rear to plant a kiss on Romeo's cheek. The 80-kg boat is facing shore and the 55-kg Juliet moves 2.7 m (relative to the boat) towards the 77-kg Romeo. How far does the boat move? Does it move toward or away from the shore?

Review: Determine Friction by Angle

A block of mass M rests on an incline of length L which makes an angle θ with the horizontal. The angle is slowly increased until the block starts to move. Let the angle at which the block starts to move be θ_s. Show how the coefficient of static friction can be determined from the measurement of θ_s.

Review: Box, Book, and Friction

A box of mass 12 kg rests on top of a horizontal surface. A physics book of mass 3 kg rests on top of the box. A force is applied to the box, and the box and book accelerate together from rest to 1.2 m/s in 0.5 s. The box is then brought to a stop in 0.33 s, during which time the book slides off. What is the range of possible values for the coefficient of static friction between the book and the box?

Review: Blocks with Friction

A block of mass M_1 rests on top of a block or mass M_2 that rests on a horizontal surface. A light rope attached to the bottom block is used to pull on it with a force F. The coefficient of sliding friction between M_2 and the horizontal surface is μ_2. When the M_2 is pulled (and therefore accelerates), the frictional force between the blocks is not big enough to keep M_1 stuck to it, hence M_1 slides on M_2. The coefficient of kinetic friction between the two blocks is μ_1. Find the acceleration of each block in terms of F, the masses M_1 and M_2, and the coefficients μ_1 and μ_2.

Review: Car on Icy Curve

Race-track turns are often "banked" (tilted inward) so that cars can take them at high speed without skidding. Consider a circular track 2 km in length (i.e. circumference) banked at an angle of 20^o, and just for fun suppose the track is covered in ice (after a bad storm, let's say). With what speed does a car have to drive in order to make it around the track?

Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67\text{x}10^{-11} \text{ Nm}^2/\text{kg}^2$)
$\mathbf{F}_{\mathbf{spring}} = -k\,\boldsymbol{\Delta}\mathbf{x}$

Friction
$f = \mu_k N$ (kinetic)
$f \le \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot\mathbf{dl}$
$W = \mathbf{F}\cdot\boldsymbol{\Delta}\mathbf{r} = F\,\Delta r\,\cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i\mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i\mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i\mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i\mathbf{v}_i$
$\Sigma\mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$