Key concepts this week:

- No-slip condition
  - angular displacement of rotation is related to the comparable linear displacement if there's no slipping ($v = \omega R$, etc)
- Rotation and Energy
  - Work done by torque (rotational center of mass equation): $\int_{\theta_i}^{\theta_f} \tau_{\text{net}} d\theta = \Delta\left(\frac{1}{2} I_{\text{CM}} \omega^2\right)$
  - Total kinetic energy of a translating and spinning object has two pieces:
    $K_{\text{total}} = K_{\text{CM}} + K_{\text{about CM}} = \left(\frac{1}{2} M_{\text{total}} v_{\text{CM}}^2\right) + \frac{1}{2} I_{\text{CM}} \omega^2$

Bowling Ball

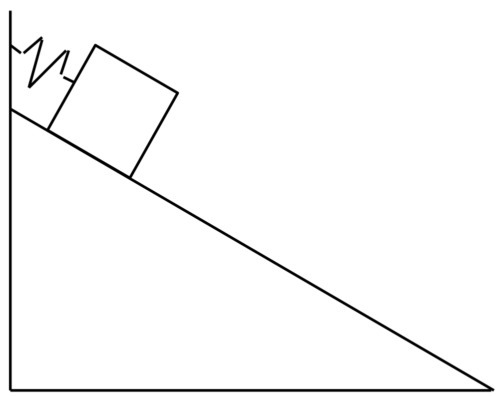
Two identical bowling balls are rolling on a horizontal floor without slipping. The initial speed of both balls is $V = 9.9$ m/s. Ball A encounters a frictionless ramp, reaching a maximum vertical height H_A above the floor. Ball B on the other hand rolls up a regular ramp (i.e. without slipping), reaching a maximum vertical height H_B above the floor. Which ball goes higher, and by how much?

Two Blocks and a Pulley

Block 1 (mass M_1) rests on a horizontal surface. A horizontal string is attached to the block, passing over a pulley to Block 2 (mass M_2) which hangs vertically a distance h from the floor. The pulley is a uniform cylinder of mass M and radius R. The string has negligible mass and the axle of the pulley has no friction. The string does not slip on the pulley. The coefficient of sliding friction between block 1 and the horizontal surface is μ. The whole system is held in place, then released from rest. What is the speed of block 2 just before it hits the floor?

Review: Block Slide

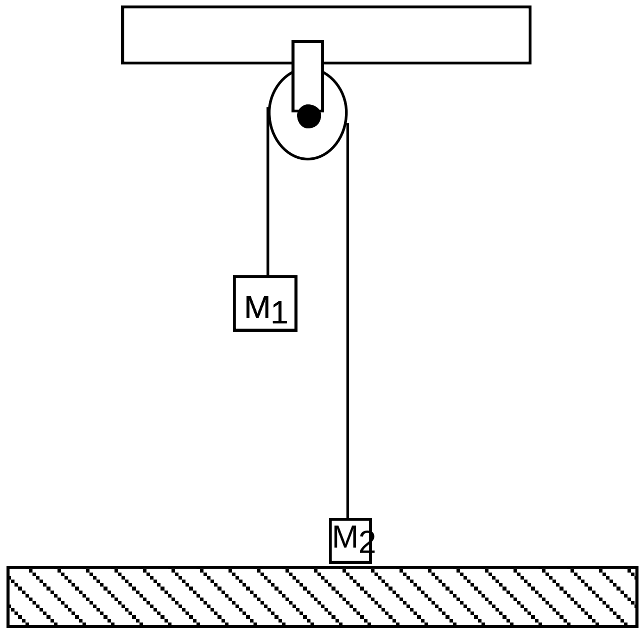
In an effort to combine several aspects of her recent physics lectures, an enterprising student poses for herself the following question. An unstretched spring is attached to a 1.5 kg block on a ramp which makes an angle of 30° with respect to the horizontal. The other end of the spring is fixed. The mass is released and it slides down the ramp and stretches the spring. There is friction between the block and the ramp with a coefficient of 0.3. The spring has a constant of 30 N/m. Undaunted by the complexity of her problem, she computes the maximum distance that the block slides down the ramp. What is her answer?



Yo^2

A yo-yo has mass M and outer radius R. The central stem has negligible mass and radius r and string of negligible mass is wrapped around it. The string is taut and held vertically and the yo-yo is released from rest. How long does it take for the yo-yo to hit the ground, which is a distance h below the initial position?

Atwood's Machine Revisited



Consider a realistic Atwood's machine where the pulley is not massless. Instead it is a disk of radius 0.1 m and mass 3 kg. The heavier weight has mass M_1=5 kg and the lighter weight has mass M_2= 2 kg. The system is released from rest when the lighter mass is on the floor and the heavier mass is 1.8 m above the floor. How long does it take the heavier mass to hit the floor?

Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$)
$\mathbf{F}_{spring} = -k\,\boldsymbol{\Delta}\mathbf{x}$

Friction
$f = \mu_k N$ (kinetic)
$f \le \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot d\mathbf{l}$
$W = \mathbf{F}\cdot\boldsymbol{\Delta}\mathbf{r} = F\,\Delta r\,\cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i \mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i \mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i \mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i \mathbf{v}_i$
$\Sigma \mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse
$\mathbf{I} = \int \mathbf{F}\,dt$
$\Delta\mathbf{P} = \mathbf{F}_{av}\Delta t$

Collisions:
If $\Sigma\mathbf{F}_{EXT} = 0$ in some direction, then
$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:
$\Sigma m_i \mathbf{v}_i$ (before) $= \Sigma m_i \mathbf{v}_i$ (after)

<u>In addition, if the collision is elastic:</u>
* $E_{before} = E_{after}$
* *Rate of approach = Rate of recession*
* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics
$s = R\theta,\ v = R\omega,\ a = R\alpha$

$$\left.\begin{aligned}\theta &= \theta_0 + \omega_0 t + {}^1/_2\alpha t^2\\ \omega &= \omega_0 + \alpha t\\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta\end{aligned}\right\}$$

Rotational Dynamics
$I = \Sigma m_i r_i^2$
$I_{parallel} = I_{CM} + MD^2$
$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$
$I_{hoop} = MR^2$
$I_{solid\text{-}sphere} = {}^2/_5 MR^2$
$I_{spherical\ shell} = {}^2/_3 MR^2$
$I_{rod\text{-}cm} = {}^1/_{12} ML^2$
$I_{rod\text{-}end} = {}^1/_3 ML^2$
$\tau = I\alpha$ (rotation about a fixed axis)
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$

Work & Energy
$K_{rotation} = {}^1/_2 I\omega^2$,
$K_{translation} = {}^1/_2 MV_{cm}^2$
$K_{total} = K_{rotation} + K_{translation}$
$W = \tau\theta$

Statics
$\Sigma\mathbf{F} = 0$, $\Sigma\tau = 0$ (about any axis)