Concepts this Week

Key Concepts:

1. A non-moving fluid has a pressure which increases with depth:

$$P = P_o + \rho g h$$

2. A submerged, or partially submerged, object feels an upward **buoyancy** force equal to the weight of the volume of water displaced.

3. In the approximation that the density of water is constant, **continuity** (essentially conservation of mass) requires that the average speed through a pipe increases as the pipe narrows, in inverse proportion to the cross sectional area.

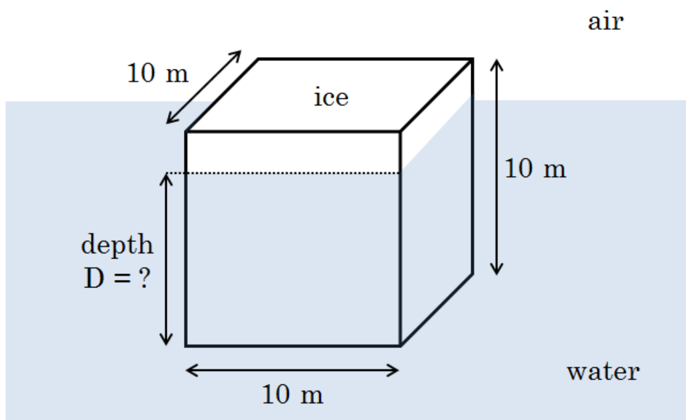
4. **Bernoulli's** principle (based on work-energy considerations and in the approximation that there is no pipe friction) indicates that the pressure in a pipe flow must diminish as the fluid speed increases (and of course also as the height increases, as in the above equation)

No Discussion Class this Week!

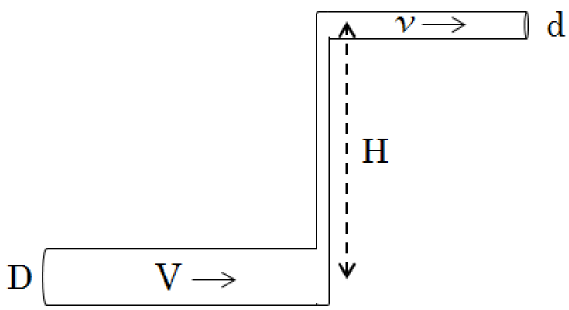
Solutions will be posted on SmartPhysics

Hydrostatics: Ice Cube

A large ice cube, 10m on each side, is floating in the Atlantic Ocean. The density of ice is $\rho_{ice} = 0.92 g/cm^3$ and the density of sea water is $\rho_w = 1.03\ g/cm^3$. A diver is going to dive to inspect the underside of the ice cube. How deep below the surface of the ocean is the bottom of the ice cube? At the bottom of the ice cube, what is the pressure? (Assume the air pressure at the surface of the ocean is 1 atm = 101,000 Pa.)



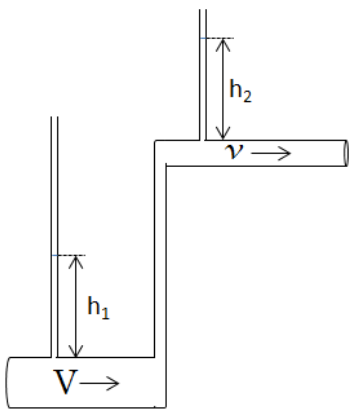
Steady Hydrodynamics: pipe flow



Water enters from the left through a pipe of diameter D = 0.1 m at a speed of V = 1 m/s and an absolute pressure of 2 atm. It then rises a height H = 3 m through the narrow pipe and then exits to the right through a horizontal pipe of diameter d = 0.05 m.

With what speed v does it move through the upper horizontal section?
What is the pressure in the upper horizontal section?

Two Pitot tubes (open to the atmosphere at their tops) are now attached to the above set of pipes. To what heights h_1 and h_2 does the water rise in each tube?



Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67\times10^{-11}\ Nm^2/kg^2$)
$\mathbf{F}_{spring} = -k\,\mathbf{\Delta x}$

Friction
$f = \mu_k N$ (kinetic)
$f \le \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot d\mathbf{l}$
$W = \mathbf{F}\cdot\mathbf{\Delta r} = F\,\Delta r\,\cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i \mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i \mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i \mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i \mathbf{v}_i$
$\Sigma \mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse
$\mathbf{I} = \int \mathbf{F}\,dt$
$\Delta\mathbf{P} = \mathbf{F}_{av}\Delta t$

Collisions:
If $\Sigma\mathbf{F}_{EXT} = 0$ in some direction, then
$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:
$\Sigma m_i \mathbf{v}_i$ (before) $= \Sigma m_i \mathbf{v}_i$ (after)

In addition, if the collision is elastic:
* $E_{before} = E_{after}$
* *Rate of approach = Rate of recession*
* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics
$s = R\theta,\ v = R\omega,\ a = R\alpha$

$$\left.\begin{array}{l}\theta = \theta_0 + \omega_0 t + {}^1/_2\alpha t^2 \\ \omega = \omega_0 + \alpha t \\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta\end{array}\right\}$$

Rotational Dynamics
$I = \Sigma m_i r_i^2$
$I_{parallel} = I_{CM} + MD^2$
$I_{disk} = I_{cylinder} = {}^1/_2 MR^2$
$I_{hoop} = MR^2$
$I_{solid\text{-}sphere} = {}^2/_5 MR^2$
$I_{spherical\ shell} = {}^2/_3 MR^2$
$I_{rod\text{-}cm} = {}^1/_{12} ML^2$
$I_{rod\text{-}end} = {}^1/_3 ML^2$
$\tau = I\alpha$ (rotation about a fixed axis)
$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, $|\tau| = rF\sin\phi$

Work & Energy
$K_{rotation} = {}^1/_2 I\omega^2$,
$K_{translation} = {}^1/_2 MV_{cm}^2$
$K_{total} = K_{rotation} + K_{translation}$
$W = \tau\theta$

Statics
$\Sigma\mathbf{F} = 0$, $\Sigma\tau = 0$ (about any axis)

Angular Momentum:
$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
$L_z = I\omega_z$
$\mathbf{L}_{tot} = \mathbf{L}_{CM} + \mathbf{L}^*$
$\boldsymbol{\tau}_{ext} = d\mathbf{L}/dt$
$\boldsymbol{\tau}_{cm} = d\mathbf{L}^*/dt$
$\Omega_{precession} = \tau / L$

Simple Harmonic Motion:
$d^2x/dt^2 = -\omega^2 x$
(differential equation for SHM)

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A\sin(\omega t + \phi)$
$a(t) = -\omega^2 A\cos(\omega t + \phi)$

$\omega^2 = k/m$ (mass on spring)
$\omega^2 = g/L$ (simple pendulum)
$\omega^2 = mgR_{CM}/I$ (physical pendulum)
$\omega^2 = \kappa/I$ (torsion pendulum)

General harmonic transverse waves:
$y(x,t) = A\cos(kx - \omega t)$
$k = 2\pi/\lambda$, $\omega = 2\pi f = 2\pi/T$
$v = \lambda f = \omega/k$

Waves on a string:

$$v^2 = \frac{F}{\mu} = \frac{(\text{tension})}{(\text{mass per unit length})}$$

$$\overline{P} = \frac{1}{2}\mu v\omega^2 A^2$$

$$\frac{d\overline{E}}{dx} = \frac{1}{2}\mu\omega^2 A^2$$

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad \text{Wave Equation}$$

Fluids:

$$\rho = \frac{m}{V} \qquad p = \frac{F}{A}$$

$$A_1 v_1 = A_2 v_2$$

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$F_B = \rho_{liquid}\, g V_{liquid}$$

$$F_2 = F_1 \frac{A_2}{A_1}$$