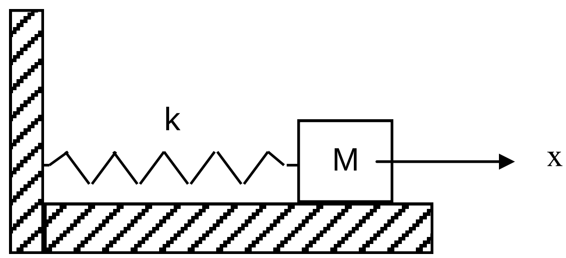
Key concepts this week:

- Simple Harmonic Motion
  - A restoring force leads to the signature differential equation of simple harmonic motion ($\frac{d^2x}{dt^2} = -\omega^2 x$ where ω is a constant)
  - The angular frequency is constant and does not depend on the initial conditions (e.g. $\omega = \sqrt{\frac{k}{m}}$)
  - The functional form of the displacement, velocity, and acceleration is sinusoidal
- Pendula
  - For small oscillations, pendula undergo simple harmonic motion
  - The angular frequency is determined by the size and shape (but not the mass) of the pendulum

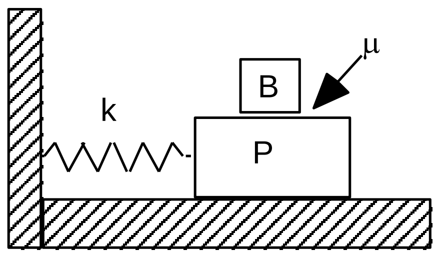
Equation of Motion

A mass M rests on a frictionless table and is connected to a spring of spring constant k. The other end of the spring is fixed to a vertical wall as shown in the figure. At time t = 0 s the mass is at x = 2.6 cm and moving to the right at a speed of 47 cm/s (the equilibrium position of the mass is at x = 0). It is at this position with this velocity next at t = 0.2 s. Find an expression for the position as a function of time and in so doing find the frequency and the amplitude of oscillation.



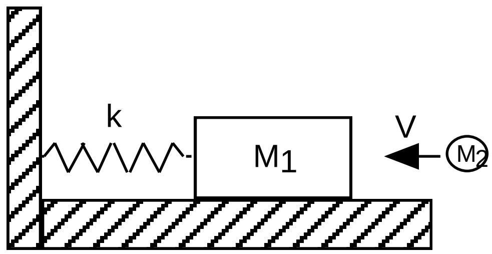
Plate, Block, and Spring

A flat plate P of mass 5.0 kg is attached to a spring of spring constant k = 60 N/m and executes horizontal simple harmonic motion by sliding across a frictionless surface. A block B of mass 2.0 kg rests on the plate and the coefficient of static friction between the block and the plate is μ= 0.60. What is the maximum amplitude of oscillation that the plate-block system can have in order that the block not slip on the plate?



Block, Clay, and Spring

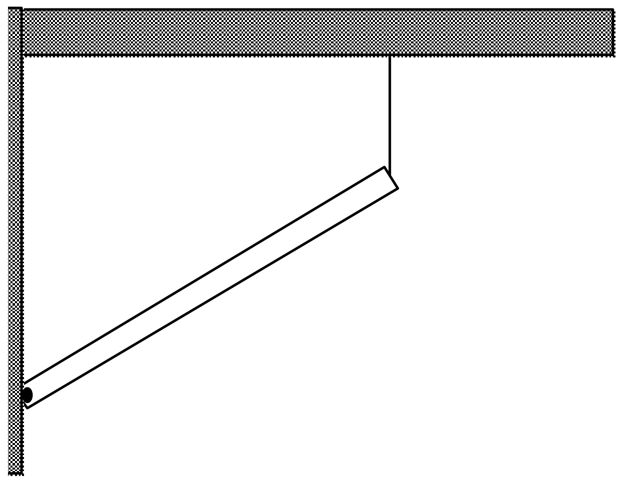
A block of mass M_1 = 5 kg is attached to a spring of spring constant k = 20 N/m and rests on a frictionless horizontal surface. A wad of clay of mass M_2=2 kg and traveling horizontally with speed V = 14 m/s hits and sticks to the block. Find the frequency and amplitude for the subsequent simple harmonic oscillations.



Bird Feeder Torsion Pendulum

A bird feeder consists of a solid circular disk of mass M = 0.34 kg and radius R = 0.25 m suspended by a wire attached at the center. Two birds, each of mass m = 65 g land at opposite ends of a diameter, and the system goes into torsional oscillation with a frequency f = 2.6 Hz. What is the torsional constant of the wire?

Review: Falling Meter Stick



A uniform meter stick of mass 1.5kg is attached to the wall by a frictionless hinge at one end. On the opposite end it is supported by a vertical massless string such that the stick makes an angle of 40^{o} with the horizontal.

1. Find the tension in the string and the magnitude and direction of the force exerted on the stick by the hinge.

Suppose the string is cut. Find the angular acceleration of the stick immediately thereafter.

Kinematics
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + at^2/2$
$v^2 = v_0^2 + 2a(x-x_0)$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$

$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$

Uniform Circular Motion
$a = v^2/r = \omega^2 r$
$v = \omega r$
$\omega = 2\pi/T = 2\pi f$

Dynamics
$\mathbf{F}_{net} = m\mathbf{a} = d\mathbf{p}/dt$
$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

$F = mg$ (near earth's surface)
$F_{12} = -Gm_1m_2/r^2$ (in general)
(where $G = 6.67\times10^{-11}\ Nm^2/kg^2$)
$\mathbf{F}_{\mathbf{spring}} = -k\,\mathbf{\Delta x}$

Friction
$f = \mu_k N$ (kinetic)
$f \le \mu_S N$ (static)

Work & Kinetic energy
$W = \int \mathbf{F}\cdot d\mathbf{l}$
$W = \mathbf{F}\cdot\mathbf{\Delta r} = F\,\Delta r\,\cos\theta$
(constant force)

$W_{grav} = -mg\Delta y$
$W_{spring} = -k(x_2^2 - x_1^2)/2$

$K = mv^2/2 = p^2/2m$
$W_{NET} = \Delta K$

Potential Energy
$U_{grav} = mgy$ (near earth surface)
$U_{grav} = -GMm/r$ (in general)
$U_{spring} = kx^2/2$
$\Delta E = \Delta K + \Delta U = W_{nc}$

Power
$P = dW/dt$
$P = \mathbf{F}\cdot\mathbf{v}$ (for constant force)

System of Particles
$\mathbf{R}_{CM} = \Sigma m_i\mathbf{r}_i / \Sigma m_i$
$\mathbf{V}_{CM} = \Sigma m_i\mathbf{v}_i / \Sigma m_i$
$\mathbf{A}_{CM} = \Sigma m_i\mathbf{a}_i / \Sigma m_i$
$\mathbf{P} = \Sigma m_i\mathbf{v}_i$
$\Sigma\mathbf{F}_{EXT} = M\mathbf{A}_{CM} = d\mathbf{P}/dt$

Impulse
$\mathbf{I} = \int \mathbf{F}\,dt$
$\Delta\mathbf{P} = \mathbf{F}_{av}\Delta t$

Collisions:
If $\Sigma\mathbf{F}_{EXT} = 0$ in some direction, then
$\mathbf{P}_{before} = \mathbf{P}_{after}$ in this direction:
$\Sigma m_i\mathbf{v}_i$ (before) $= \Sigma m_i\mathbf{v}_i$ (after)

In addition, if the collision is elastic:
* $E_{before} = E_{after}$
* *Rate of approach = Rate of recession*
* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics
$s = R\theta,\ v = R\omega,\ a = R\alpha$
$$\left.\begin{aligned}\theta &= \theta_0 + \omega_0 t + {}^1/_2\alpha t^2\\ \omega &= \omega_0 + \alpha t\\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta\end{aligned}\right\}$$

Rotational Dynamics
$I = \Sigma m_i r_i^2$
$I_{parallel} = I_{CM} + MD^2$
$I_{disk} = I_{cylinder} = {}^1/_2MR^2$
$I_{hoop} = MR^2$
$I_{solid\text{-}sphere} = {}^2/_5MR^2$
$I_{spherical\ shell} = {}^2/_3MR^2$
$I_{rod\text{-}cm} = {}^1/_{12}ML^2$
$I_{rod\text{-}end} = {}^1/_3\,ML^2$
$\tau = I\alpha$ (rotation about a fixed axis)
$\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$, $|\tau| = rF\sin\phi$

Work & Energy
$K_{rotation} = {}^1/_2 I\omega^2$,
$K_{translation} = {}^1/_2 MV_{cm}^2$
$K_{total} = K_{rotation} + K_{translation}$
$W = \tau\theta$

Statics
$\Sigma\mathbf{F} = 0$, $\Sigma\tau = 0$ (about any axis)

Angular Momentum:
$\mathbf{L} = \mathbf{r}\times\mathbf{p}$
$L_z = I\omega_z$
$\mathbf{L}_{tot} = \mathbf{L}_{CM} + \mathbf{L}^*$
$\boldsymbol{\tau}_{ext} = d\mathbf{L}/dt$
$\boldsymbol{\tau}_{cm} = d\mathbf{L}^*/dt$
$\Omega_{precession} = \tau / L$

Simple Harmonic Motion:
$d^2x/dt^2 = -\omega^2 x$
(differential equation for SHM)

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A\sin(\omega t + \phi)$
$a(t) = -\omega^2 A\cos(\omega t + \phi)$

$\omega^2 = k/m$ (mass on spring)
$\omega^2 = g/L$ (simple pendulum)
$\omega^2 = mgR_{CM}/I$ (physical pendulum)
$\omega^2 = \kappa/I$ (torsion pendulum)