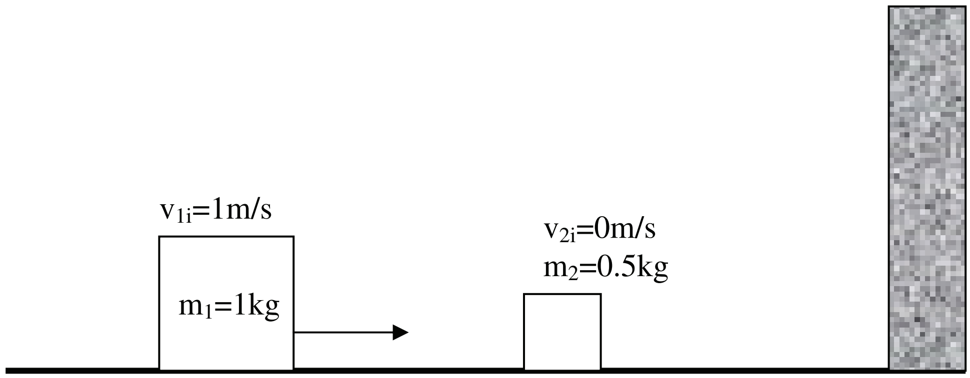
Elastic Blocks

Initially, Block 1 with mass m_1=1kg is moving on a frictionless table with velocity v_1=1 m/s and block 2 with mass m_2=0.5 kg is at rest. Block 1 collides **elastically** with block 2.



Find the velocities of block 1 and block 2 after the collision.

Afterwards, block 2 collides elastically with a wall. What impulse does the wall give to block 2?

Both collisions are elastic, thus Kinetic energy is conserved...

Collision 1: m_1 hits m_2

Let's use the CM reference frame approach. (Note: this only works for elastic collisions!)

1. Find $\vec{v}_{CM}$: $\vec{v}_{CM} = \frac{\vec{P}_{total}}{m_{total}} = \frac{(1kg)(1m/s)+(0.5kg)(0m/s)}{1kg+0.5kg} = \frac{2}{3}$ m/s

2. Convert v_{1i} to v_{1i}^* and v_{2i} to v_{2i}^* (the * tells us we're in the CM frame)

   By relative motion, $v_{Block,CM} = v_{Block,Lab} + v_{Lab,CM} = v_{Block,Lab} - v_{CM,Lab}$

   $v_{1i}^* = +1 m/s - 2/3 m/s = 1/3 m/s$ $\quad$ $v_{2i}^* = 0 m/s - 2/3 m/s = -2/3 m/s$

3. Because the collision is elastic, $v_{1i}^* = -v_{1f}^*$ and $v_{2i}^* = -v_{2f}^*$

   $v_{1f}^* = -1/3 m/s$ $\quad$ $v_{2f}^* = +2/3 m/s$

4. Now convert back to the lab reference frame:

   $v_{block,lab} = v_{block,CM} + v_{CM,Lab}$

   $v_{1f} = v_{1f}^* + 2/3 = \boxed{+1/3 m/s}$ $\quad$ $v_{2f} = v_{2f}^* + 2/3 = \boxed{+4/3 m/s}$

Collision 2: m_2 bounces off the wall.

This collision is elastic, so $K_i = K_f$, thus $v_f = -v_i = -4/3$ m/s

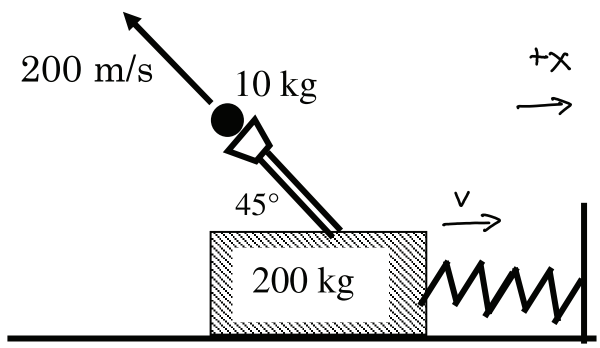
Impulse is simply change of momentum, so

$$\text{Impulse} = \Delta p = \underbrace{(0.5kg)(-4/3)}_{p_f} - \underbrace{(0.5kg)(+4/3)}_{p_i} = \boxed{-4/3 \ kg\,m/s}$$

Note: you can also solve collision 1 in the lab frame using $p_{i,total} = p_{f,total}$ and $K_{i,total} = K_{f,total}$ but the math is a little harder.

Gun Cushion

A cannon fires a 10 kg projectile with a velocity of 200 m/s at an angle of 45° with respect to vertical. The cannon, which has a mass of 200 kg, is cushioned by a relaxed, horizontal spring which is designed to absorb the recoil. Assume that the gun stands on a frictionless surface. Compute the minimum spring constant required so that the spring compresses by less than 20 centimeters.



Two processes occur in this problem: ① the cannon fires (momentum is conserved) and ② the spring absorbs the recoil (energy is conserved)

① The cannon fires the cannonball. Momentum is conserved only in the x direction.

$p_{ix} = (200\text{kg})(0\,\text{m/s}) + (10\text{kg})(0\,\text{m/s}) = 0$

$p_{fx} = (200\text{kg})(v) + (10\text{kg})(-200\cos 45°)$

setting $p_{ix} = p_{fx}$, find $v = 10\cos 45° = 7.07\,\text{m/s}$

② The spring absorbs all of the cannon's kinetic energy

$K_i + \cancel{U_i}^{\,0} = \cancel{K_f}^{\,0} + U_f$

$\frac{1}{2}mv^2 = \frac{1}{2}k\underbrace{x}_{0.2\,\text{m}}^2$

Solve to find $k = \frac{mv^2}{x^2} = \boxed{250{,}000\ \text{N/m}}$

Gone Fission

When a Uranium-235 nucleus decays, two lighter nuclei are ejected with high kinetic energy. It is this energy that is harnessed in reactors (and bombs). A pair of neutrons is typically also ejected, each of which can cause another ^{235}U nucleus to fission, leading to a chain reaction.

A fun way to model a ^{235}U nucleus is to say that it is really composed of two lighter nuclei, call them A and B. Say nucleus A has a mass of 90 amu (atomic mass units), and nucleus B has a mass of 145 amu, and that initially A and B are at rest and have a massless spring squeezed between them. We will also need to pretend that there is something holding the pair together, preventing them from being blown apart by the spring (a little latch, say). When a neutron happens to bump into this setup it releases the latch, allowing the spring to expand, sending A and B off in opposite directions at high speed. The total kinetic energy of A and B (the uranium fission products) after this happens is known to be about 200 MeV.

Use this simple model to figure out the following:

1. What are the speeds of both A and B after the ^{235}U nucleus fissions?
2. Assuming the spring was initially compressed 10 femtometers (the size of a nucleus), what is the spring constant?

Useful conversions:
1 amu = $1.7\text{x}10^{-27}$ kg
1 MeV = $1.6\text{x}10^{-13}$ J
1 femtometer = 10^{-15} m

Convert right away:

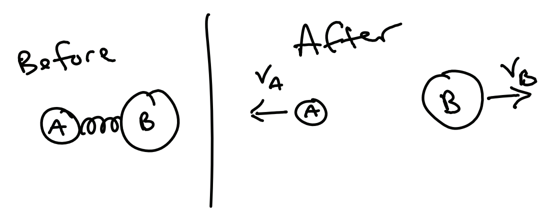
$m_A = 90\,\text{amu} = 1.53 \times 10^{-25}$ kg

$m_B = 145\,\text{amu} = 2.465 \times 10^{-25}$ kg

$K_f = 200\,\text{MeV} = 3.2 \times 10^{-11}$ J

$\Delta x = 10\,\text{fm} = 10^{-14}$ m

1. Momentum is conserved in the collision



$p_i = m_A(0) + _B(0) = 0$

$p_f = m_A v_A + m_B v_B$

Thus $m_A v_A + m_B v_B = 0$

We also know the final kinetic energy

$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = K_f$

Combine these two equations and solve (using SI units) to find

$\boxed{|v_A| = 1.61 \times 10^7 \text{ m/s}}$ and $\boxed{|v_B| = 9.97 \times 10^6 \text{ m/s}}$

2. The energy of the nuclei comes from the "spring"

$U_i + \cancelto{0}{K_i} = \cancelto{0}{U_f} + K_f$

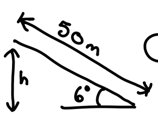
$\frac{1}{2} k x^2 = K_f$

$k = \frac{2K_f}{x^2} = \boxed{6.4 \times 10^{17} \text{ N/m}}$

Review: Lone Ranger

A wagon with two boxes of gold and having a total mass of 300 kg is cut loose from the horses by an outlaw when the wagon is at rest 50 m up a 6° slope. The outlaw plans to have the wagon roll down the slope and then across 50m of level ground before finally falling over a cliff into a canyon where his confederate waits. But, unknown to the outlaws, the Lone Ranger (mass of 80 kg) and Tonto (mass of 60 kg) are waiting in a tree 40 m from the cliff. They time their fall so that they drop vertically into the wagon just as the wagon passes beneath them. They require 5.0 s to grab the gold and jump out of the wagon. Will they make it before the wagon goes over the cliff?

In this problem, (1) the wagon rolls down the slope (U_G converted to K), (2) the two men jump on the wagon, causing it to slow down ($\vec{p}$ conserved), (3) then we want to know if 5s is enough time to steal the gold before the wagon goes off the cliff.



(1) Rolling down the slope

$$E_i = U_i + K_i = mgh + 0 = mg(50\sin 6^\circ)$$

$$E_f = U_f + K_f = 0 + \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_1^2$$

$$E_i = E_f \rightarrow mg(50\sin 6^\circ) = \tfrac{1}{2}mv_1^2$$

$$v_1 = \sqrt{100 g \sin 6^\circ} = \boxed{10.13\,\text{m/s}}$$

(2) The Lone Ranger and Tonto drop in (i.e., collide and stick to the wagon)

$$p_1 = m_{wagon} v_1 \qquad p_2 = (m_{wagon} + m_{LR} + m_T) v_2$$

$$p_1 = p_2 \rightarrow m_w v_1 = (m_w + m_{LR} + m_T) v_2$$

$$v_2 = \frac{m_w v_1}{m_w + m_{LR} + m_T} = \boxed{6.90\,\text{m/s}}$$

(3) Let's see how far they go in 5.0 s

$$x = v_2 t = (6.90)(5.0) = \boxed{34.5\text{m}}$$

The cliff is 40m away, so they get off the wagon in time.

Automobile Collision

Your friend has just been in a traffic accident and is trying to negotiate with the insurance company of the other driver to pay for fixing her car. She believes that the other car was speeding and therefore that the accident was the other driver's fault. She knows that you are taking Physics 211, so she asks you for help in proving her conjecture. She takes you out to the scene and describes what happened.

She was traveling North when she entered the fateful intersection. There was no stop sign, so she looked in both directions and did not see another car approaching. It was a bright, sunny, clear day. When she reached the center of the intersection, her car was struck by the other car which was traveling East. The two cars remained joined together after the collision and skidded to a stop. The speed limit on both roads entering the intersection was 80 km/hr. From the skid marks still visible on the street, you determine that after the collision the cars skidded 17m at an angle of 30° north of east before stopping. She has a copy of the police report which gives the year and make of each car. At the library you determine that the weight of her car was 1200 kg and that of the other car was 1000 kg, including the weight of the driver in each case. The coefficient of kinetic friction for a rubber tire skidding on dry pavement is 0.80. It is not enough to prove that the other driver was speeding to convince the insurance company. She must also show that she was under the speed limit. How do you advise her?

In this problem, 1 two cars collide and stick together and then 2 skid to a stop.
We know more about 2, so let's run this backwards:

2 Friction causes the cars to stop:

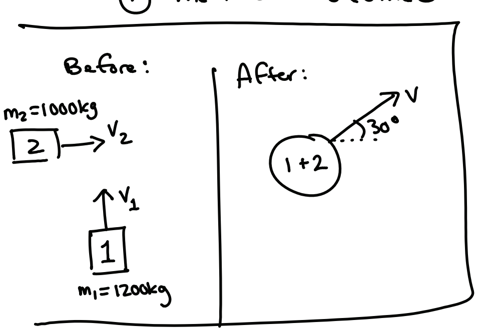
$$W_f = \int \vec{F}_k \cdot d\vec{S} = -\mu_k F_N x = -\mu_k (m_1+m_2) g x$$

By the Work Energy Theorem,

$$W_f = \Delta K = \cancel{K_f}^{\,0} - K_i$$

So $$-\mu_k (m_1+m_2) g x = -\frac{1}{2}(m_1+m_2)v^2 \rightarrow v = 16.33\,m/s$$

1 The two cars collide



This is a 2D collision, so we'll ok at x and y separately.

$p_{xi} = m_2 v_2 \qquad p_{xf} = (m_1+m_2) v \cos 30°$

$p_{yi} = m_1 v_1 \qquad p_{yf} = (m_1+m_2) v \sin 30°$

Set $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$ to find

$v_1 = 14.97\,m/s$ and $v_2 = 31.12\,m/s$

Convert these to km/h to get

$\boxed{v_1 = 53.91\,km/hr}$ and $\boxed{v_2 = 112\,km/hr}$

Your friend was not speeding
(but might be a ghost)