Bowling Ball

Two identical bowling balls are rolling on a horizontal floor without slipping. The initial speed of both balls is V = 9.9 m/s. Ball A encounters a frictionless ramp, reaching a maximum vertical height H_A above the floor. Ball B on the other hand rolls up a regular ramp (i.e. without slipping), reaching a maximum vertical height H_B above the floor. Which ball goes higher, and by how much?

Both balls start with the same total kinetic energy:

$$K_i = K_{trans} + K_{rot} = \frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2$$

- note that both balls roll without slipping, thus $\omega_0 = \frac{v_0}{R}$
- You can use your equation sheet to find $I_{sphere} = \frac{2}{5}MR^2$

Ball A encounters a frictionless ramp. There is no torque applied, thus we know $v_f = 0$ and $\omega_f = \omega_0$

$$E_i = K_i + \cancel{U_i}^{\,0}$$

$$E_f = K_f + U_f$$

where $K_f = \frac{1}{2}I\omega_0^2$ and $U_f = Mgh_A$

Energy is conserved,

$$K_i = K_f + U_f \rightarrow \frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}I\omega_0^2 + Mgh_A$$

Solve for h_A to find

$$\boxed{h_A = \frac{v_0^2}{2g} = 5 \text{ meters}}$$

Ball B encounters a ramp with friction. The ball rolls without slipping, so it must be that $v_f = 0$ and $\omega_f = 0$

$$E_i = K_i + \cancel{U_i}^{\,0} = \frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_0}{R}\right)^2 = \frac{7}{10}Mv_0^2$$

$$E_f = \cancel{K_f}^{\,0} + U_f = Mgh_B$$

Once again, energy is conserved (note: static friction does not do work)

$$E_i = E_f \rightarrow \frac{7}{10}Mv_0^2 = Mgh_B$$

Solve for h_B to find

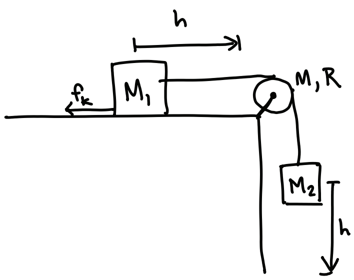
$$\boxed{h_B = \frac{7v^2}{10g} = 7 \text{ meters}}$$

$\boxed{\text{Ball B goes 2 meters higher than Ball A.}}$

Two Blocks and a Pulley

Block 1 (mass M_1) rests on a horizontal surface. A horizontal string is attached to the block, passing over a pulley to block 2 (mass M_2) which hangs vertically a distance h from the floor. The pulley is a uniform cylinder of mass M and radius R. The string has negligible mass and the pulley has no friction. The string does not slip on the pulley. The coefficient of sliding friction between block 1 and the horizontal surface is μ. The whole system is held in place, then released from rest. What is the speed of block 2 just before it hits the floor?

This problem can be approached using dynamics ($\Sigma F = ma$ and $\Sigma \tau = I\alpha$) followed by kinematics, but it will be slightly simpler to use energy, as shown here.



We'll use the Work-Energy Theorem for this problem:

$$W_{Net} = \Delta K$$

We only need to look at work done by external forces (gravity, friction) on the system:

$$W_{Net} = W_G + W_f \qquad \text{(tension is an internal force)}$$

$$W_G = -\Delta U_G = M_2 g h$$

$$W_f = \int \vec{f}_k \cdot d\vec{S} = (-1)\mu M_1 g h$$

Now we find ΔK:

$K_i = 0$ (system begins at rest)

$$K_f = K_{1f} + K_{2f} + K_{pf}$$

$$= \frac{1}{2}M_1 v_f^2 + \frac{1}{2}M_2 v_f^2 + \frac{1}{2}I\omega_f^2$$

$I_{cylinder} = \frac{1}{2}MR^2$

$\omega_{no\ slipping} = \frac{V}{R}$

$$= \frac{1}{2}M_1 v_f^2 + \frac{1}{2}M_2 v_f^2 + \frac{1}{4}M v_f^2$$

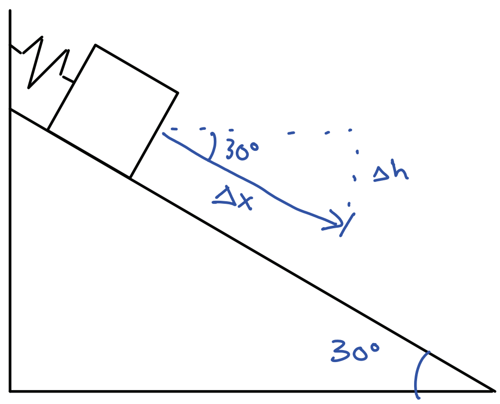
$$\Delta K = K_f - K_i = K_f$$

Set $W_G + W_f = K_f$ to find

$$\boxed{v_f = \sqrt{\frac{2gh(M_2 - \mu M_1)}{M_1 + M_2 + \frac{1}{2}M}}}$$

Review: Block Slide

In an effort to combine several aspects of her recent physics lectures, an enterprising student poses for herself the following question. An unstretched spring is attached to a 1.5 kg block on a ramp which makes an angle of 30° with respect to the horizontal. The other end of the spring is fixed. The mass is released and it slides down the ramp and stretches the spring. There is friction between the block and the ramp with a coefficient of 0.3. The spring has a constant of 30 N/m. Undaunted by the complexity of her problem, she computes the maximum distance that the block slides down the ramp. What is her answer?



This problem is exam 2 material. We'll use the Work-Energy Theorem to solve it:

$$W_{Net} = \Delta K \quad \text{where } \Delta K = 0$$

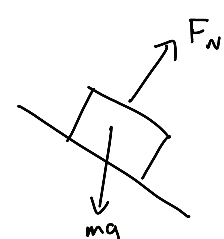
$$W_{Net} = W_G + W_S + W_f$$

$$W_G = -\Delta U_G = -(0 - mg\Delta h) = mg\Delta x \sin 30°$$

$$W_S = -\Delta U_S = -\left(\tfrac{1}{2}k\Delta x^2 - 0\right) = -\tfrac{1}{2}k\Delta x^2$$

$$W_f = \int \vec{F}_k \cdot d\vec{S} = (-1)(\mu F_N)(\Delta x)$$

$$= -\mu mg\Delta x \cos 30°$$



$F_N - mg\cos\theta = 0$

Solve $W_G + W_S + W_f = 0$

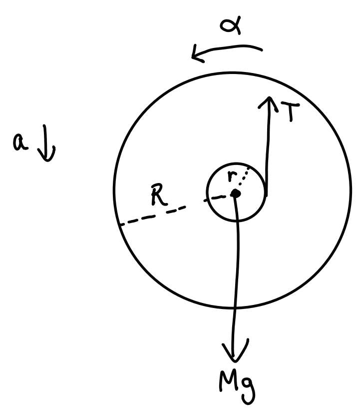
to find

$$\Delta x = \frac{2mg}{k}(\sin 30° - \mu\cos 30°) = \boxed{0.24\,m}$$

Yo^2

A yo-yo has mass M and outer radius R. The central stem has negligible mass and radius r and string of negligible mass is wrapped around it. The string is taut and held vertically and the yo-yo is released from rest. How long does it take for the yo-yo to hit the ground, which is a distance h below the initial position?

Once again, we have the option to use dynamics or energy to answer this question. This time, we'll use a dynamics approach:



Draw a FBD indicating the location of your forces and the directions of $\vec{a}$ and $\vec{\alpha}$

Note: the no-slip condition occurs at the inner radius r, so $\alpha = \frac{a}{r}$ (not $\frac{a}{R}$)

We write two Newton's 2nd Law equations:

Rotational: $\sum \tau = Tr = I\alpha$

$I = \frac{1}{2}MR^2$; $\alpha = \frac{a}{r}$

$$\boxed{Tr = \frac{1}{2}MR^2\left(\frac{a}{r}\right)}$$

Translational: $\sum F = M\vec{a}$

$$\boxed{T - Mg = -Ma}$$

Solve the system of equations to find $\boxed{a = \dfrac{g}{1 + R^2/2r^2}}$

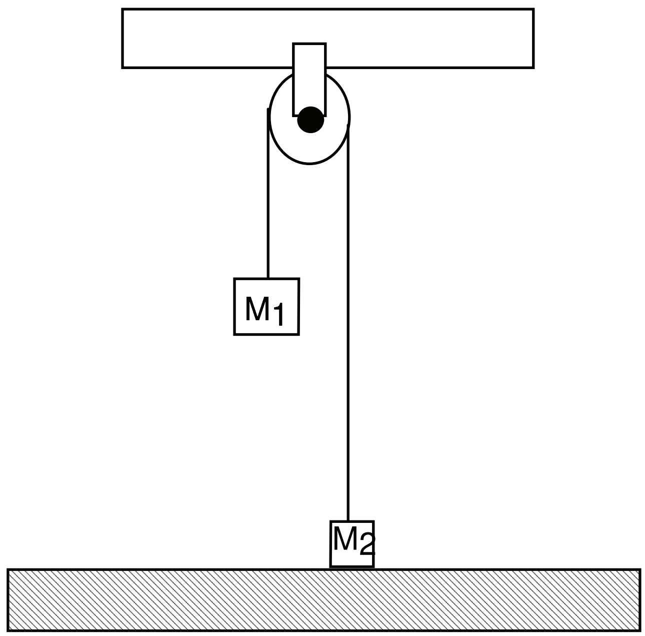
Now, use kinematics to find the time it takes to fall a height h:

$$0 = h - \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2h}{a}}$$

$$\boxed{t = \sqrt{\frac{2h\left(\frac{R^2}{2r^2} + 1\right)}{g}}}$$

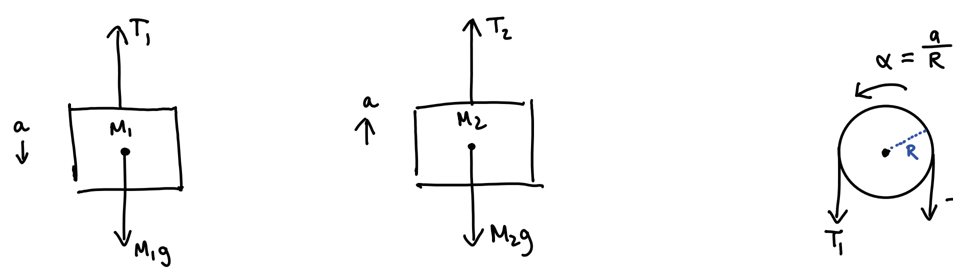
Atwood's Machine Revisited



Consider a realistic Atwood's machine where the pulley is not massless. Instead it is a disk of radius 0.1 m and mass 3 kg. The heavier weight has mass M_1=5 kg and the lighter weight has mass M_2= 2 kg. The system is released from rest when the lighter mass is on the floor and the heavier mass is 1.8 m above the floor. How long does it take the heavier mass to hit the floor?

Once again, we have the option of using energy or dynamics to approach this problem. Since we already did a pulley problem with energy, we'll do this one with dynamics

Start by drawing a FBD for each object, then write Newton's 2nd Law equations:



① $T_1 - M_1 g = -M_1 a$

② $T_2 - M_2 g = +M_2 a$

③ $\sum \tau = T_1 R - T_2 R = I\alpha$; $I_{cyl} = \frac{1}{2}MR^2$; $\alpha = \frac{a}{R}$

Combine ①, ②, and ③ to find $\boxed{a = 3.46\,m/s^2}$

The heavier mass falls a height of 1.8 m, so use kinematics to find time:

$0 = h - \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2h}{a}}$ $\qquad \boxed{t = 1.02\,s}$