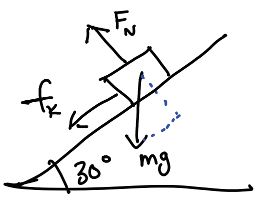
Block on Ramp

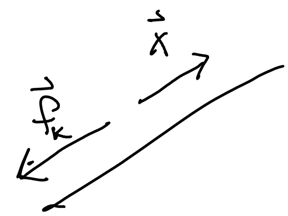
A block starts with a speed of 15 m/s at the bottom of a ramp that is inclined at an angle of 30° with the horizontal. The coefficient of kinetic friction between the block and the plane is μ=0.25. The block goes up the ramp, momentarily comes to rest, then slides back down the ramp. What is the speed of the block when it reaches the bottom of the ramp?

Friction is a nonconservative force, so we'll need to look at the block's whole path:

Part 1: Block goes up the ramp and stops

$\vec{f}_k$ and $\vec{x}$ are antiparallel





We know $W_{Total} = \Delta KE$

$$W_{f_1} = \int \vec{f}_k \cdot d\vec{S} = (\mu F_N)(x)(-1)$$

$$W_{G_1} = \int \vec{F}_G \cdot d\vec{S} = (F_{Gx})(x)(-1)$$

Using the FBD and Newton's 2nd law,

ΣF_y: $F_N - mg\cos\theta = 0$ to find F_N

and $F_{Gx} = mg\sin\theta$

finally, find $\Delta K_1 = K_f - K_o = \frac{1}{2}m\cancel{v_f^2}^{\,0} - \frac{1}{2}mv_o^2$

Use $W_{f_1} + W_{G_1} = \Delta K_1$ to find $\boxed{x = \frac{v_o^2}{2g(\mu\cos\theta + \sin\theta)} = 16.00\,m}$

Part 2: Block slides down the ramp

Now that we know the length of the path, we can find the work done on the block as it slides down the ramp:

$W_2 = W_{f_2} + W_{G2} \longrightarrow$ $W_{f2} = (\mu F_N)(x)(-1)$ ← friction points opposite to motion

$W_{G2} = (F_{Gx})(x)(+1)$ ← gravity is parallel to motion

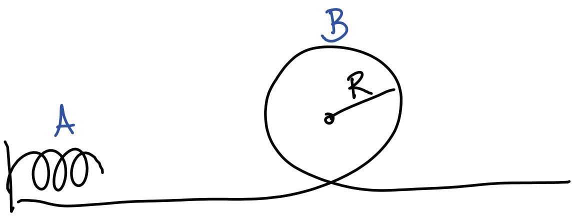
this will cause a change in kinetic energy

$$\Delta K_2 = \frac{1}{2}mv_f^2 - \frac{1}{2}m\cancel{v_{top}^2}^{\,0} = \frac{1}{2}mv_f^2$$

Solve $W_2 = \Delta K_2$ to find $\boxed{v_f = 9.44\,m/s}$

Roller Coaster

A roller coaster car has a mass of 840 kg. It is launched horizontally from a giant spring, with spring constant 31,000 N/m into a frictionless vertical loop-the-loop track of radius 6.2m. What is the minimum amount that the spring must be compressed if the car is to stay on the track?



We begin by looking at the energy when the car is at points A and B:

$E_A = K_A + U_A$ Where $K_A = 0$ and $U_A = U_S = \frac{1}{2}kx^2$

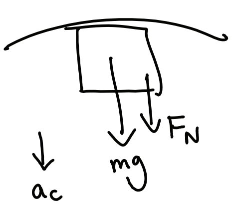
$\rightarrow E_A = \frac{1}{2}kx^2$

$E_B = K_B + U_B$ Where $K_B = \frac{1}{2}mv^2$ and $U_B = U_G = mg(2R)$

$\rightarrow E_B = \frac{1}{2}mv^2 + 2mgR$

Energy is conserved, so use $\boxed{E_A = E_B}$ to write an expression with the unknowns x_{min} and v_{min}

Now use Newton's 2nd Law to find v_{min}



When the car goes at its slowest possible speed, $F_N \rightarrow 0$

Thus, $F_N + mg = ma_c$ becomes $\cancel{m}g = \cancel{m}a_c$

So $g = \frac{v^2}{R} \rightarrow \boxed{v = \sqrt{gR}}$

Plug this in to the $E_A = E_B$ expression and solve to find

$$\boxed{x = 2.87\text{ m}}$$

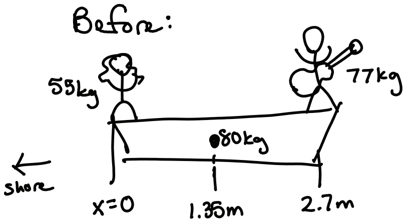
Romeo and Juliet

Romeo, who is sitting in the rear of their boat in still water, entertains Juliet by playing his guitar. After the serenade, Juliet, who was sitting in the front of the boat (closest to shore), carefully moves to the rear to plant a kiss on Romeo's cheek. The 80-kg boat is facing shore and the 55-kg Juliet moves 2.7 m (relative to the boat) towards the 77-kg Romeo. How far does the boat move? Does it move toward or away from the shore?

There is no outside force acting on the boat, so its CM will stay in the same place relative to the water.

To make this problem easier, we'll find how the CM moves relative to the boat and then shift the boat relative to the water to keep the first statement true.

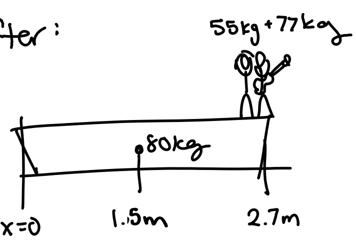
Before:



Find the CM of the system:

$$R_{cm,1} = \frac{\sum mr}{\sum m} = \frac{(55)(0) + (80)(1.35) + (77)(2.7)}{55 + 80 + 77} = 1.49\,m$$

After:



$$R_{CM_2} = \frac{(55)(2.7) + (80)(1.35) + (77)(2.7)}{55 + 80 + 77} = 2.19\,m$$

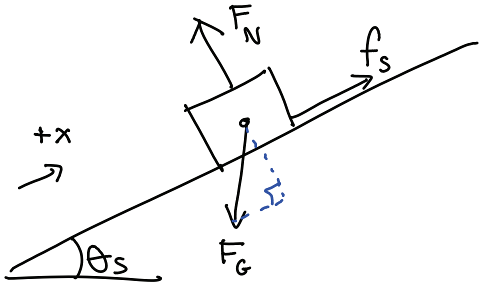
In order for the CM of the system to stay the same relative to the water, the boat's position must change by $-\Delta R_{CM}$

$$-\Delta R_{CM} = -(R_{CM2} - R_{CM1}) = -0.7\,m$$

The boat moves 0.7m towards the shore

Review: Determine Friction by Angle

A block of mass M rests on an incline of length L which makes an angle θ with the horizontal. The angle is slowly increased until the block starts to move. Let the angle at which the block starts to move be θ_s. Show how the coefficient of static friction can be determined from the measurement of θ_s.



When the block just starts to move, we know

$$f_s = f_{s\,max} = \mu F_N$$

Using Newton's 2nd Law, we find F_N

$$F_N - Mg\cos\theta_s = 0 \rightarrow Mg\cos\theta_s = F_N$$

Now write Newton's 2nd law in the x direction:

$$f_s - Mg\sin\theta_s = 0$$

plug in $f_s = \mu Mg\cos\theta_s$ to get

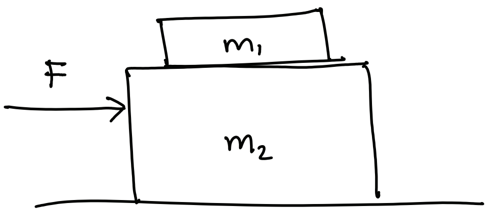
$$\mu Mg\cos\theta_s - Mg\sin\theta_s = 0$$

Solving for μ, you should find

$$\boxed{\mu = \tan\theta_s}$$

Review: Box, Book, and Friction

A box of mass 12 kg rests on top of a horizontal surface. A physics book of mass 3 kg rests on top of the box. A force is applied to box, and the box and book accelerate together from rest to 1.2 m/s in 0.5 s. The box is then brought to a stop in 0.33 s, during which time the book slides off. What is the range of possible values for the coefficient of static friction between the two blocks?



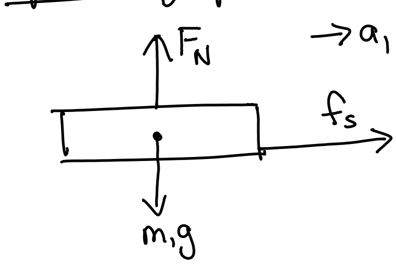
In this ituation, we're looking at two accelerations:

$$a_1 = \frac{\Delta v_1}{\Delta t_1} = \frac{1.2\,m/s}{0.5\,s} = 2.4\,m/s^2$$

$$a_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{-1.2\,m/s}{0.33\,s} = -3.64\,m/s^2$$

We are concerned with μ_s between m_1 and m_2, so we'll only need FBDs for m_1:

speeding up:



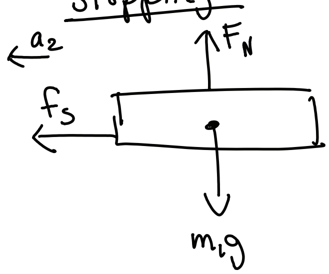
$$\Sigma F_y: \; F_N - m_1 g = 0 \rightarrow F_N = m_1 g$$

$$\Sigma F_x: \; f_s = m_1 a_1$$

By definition, $f_s \leq f_{s,max}$, so

$$f_{s,max} \geq m_1 a_1 \rightarrow \mu \cancel{m_1} g \geq \cancel{m_1} a_1 \rightarrow \mu \geq \frac{a_1}{g}$$

Stopping:



ΣF_y: same as before

$$\Sigma F_x: \; f_s = m_1 a_2$$

The book slides, so $f_s > f_{s,max}$

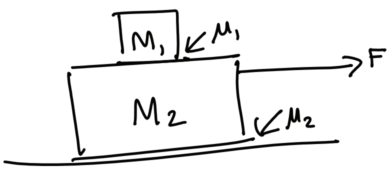
$$f_{s,max} < m_1 a_2 \rightarrow \mu \cancel{m_1} g < \cancel{m_1} a_2 \rightarrow \mu < \frac{|a_2|}{g}$$

Plugging in a_1 and a_2 as found above,

$$\boxed{0.24 \leq \mu < 0.37}$$

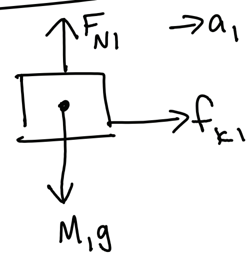
Review: Blocks with Friction

A block of mass M_1 rests on top of a block of mass M_2 that rests on a horizontal surface. A light rope attached to M_2 is used to pull on it with a force F. The coefficient of sliding friction between M_2 and the horizontal surface is μ_2. When M_2 is pulled (and therefore accelerates), the frictional force between the blocks is not big enough to keep M_1 stuck to it, hence M_1 slides on M_2. The coefficient of kinetic friction between the two blocks is μ_1. Find the acceleration of each block in terms of F, the masses M and the coefficients μ_1 and μ_2.



Solve this problem by considering each block separately:

Top block:



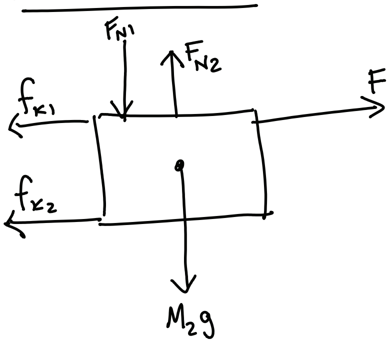
$\sum F_y: F_{N1} - M_1 g = 0$

$\sum F_x: f_{k1} = M_1 a_1$

$\hookrightarrow f_{k1} = \mu_1 F_{N1} = \mu_1 (M_1 g)$

from these, $\boxed{a_1 = \mu_1 g}$

Bottom block:



For this block, note that f_{k1} and F_{N1} are Newton's 3rd Law pairs of the forces we considered for the top block and thus we already know their magnitude.

$\sum F_y: F_{N2} - F_{N1} - M_2 g = 0$

$\hookrightarrow F_{N2} = F_{N1} + M_2 g = (M_1 + M_2) g$

$\sum F_x: F - f_{k2} - f_{k1} = M_2 a_2$

$\hookrightarrow f_{k1} = \mu_1 M_1 g$

$\hookrightarrow f_{k2} = \mu_2 F_{N2} = \mu_2 (M_1 + M_2) g$

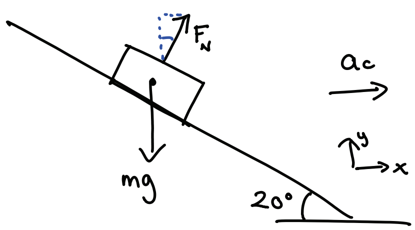
Solving for a_2, find

$$\boxed{a_2 = \frac{F - \mu_2 (M_1 + M_2) g - \mu_1 M_1 g}{M_2}}$$

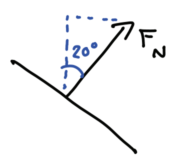
Review: Car on Icy Curve

Race-track turns are often "banked" (tilted inward) so that cars can take them at high speed without skidding. Consider a circular track 2 km in length (i.e. circumference) banked at an angle of 20°, and just for fun suppose the track is covered in ice (after a bad storm, let's say). With what speed does a car have to drive in order to make it around the track?

Let's draw a FBD to see what's going on:



Since a_c is horizontal, it will be easiest not to tilt our axes and break F_N into components:



$F_{Nx} = F_N \sin 20°$

$F_{Ny} = F_N \cos 20°$

Using Newton's 2nd Law:

$\Sigma F_y: \; F_{Ny} - mg = 0$

$F_N \cos 20° = mg$

$F_N = \dfrac{mg}{\cos 20°}$

$\Sigma F_x: \; F_{Nx} = ma_c$

$F_N \sin 20° = m\dfrac{v^2}{R}$

Combine these to find

$$v = \sqrt{gR\tan 20°}$$

The circumference of the track is 2km, so find R:

$2\text{km} = 2\pi R \rightarrow R = \frac{1}{\pi}\text{km} = \frac{1000}{\pi}\text{m}$

Plug the value of R in meters into the expression for v to get

$$\boxed{v = 33.7\ \text{m/s}}$$