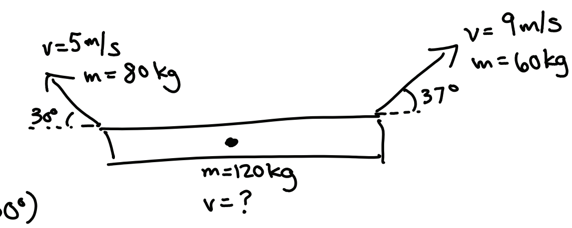
Along Came a Spider...

A man and a woman are sitting in a sleigh that is at rest on frictionless ice. The mass of the man is 80 kg, that of the woman is 60 kg, and that of the sleigh is 120 kg. The people suddenly see a poisonous spider on the floor of the sleigh and jump out. The man jumps to the left with a velocity of 5.0 m/s relative to the ground at 30° above the horizontal. The woman jumps to the right at 9.0 m/s relative to the ground at 37° above the horizontal. What is the velocity (magnitude and direction) of the sleigh after they have both jumped out?

There is no external force on the sleigh in the horizontal direction, so momentum will be conserved in x (but not in y):

Find the initial momentum:

Nothing is moving. $p_0 = 0$



Find the final momentum in x:

man: $p_x = -mv\cos\theta = -(80)(5\cos 30°)$
$= -346.4 \text{ kg m/s}$

woman: $p_x = +mv\cos\theta = (60)(9\cos 37°) = 431.26 \text{ kg m/s}$

sleigh: $p_x = mv = 120v$

$p_f = p_{man} + p_{woman} + p_{sleigh} = 84.86 + 120v$

Momentum is conserved, so $p_i = p_f$

$0 = 84.26 + 120v$

Solve to find

$$\boxed{v = -0.707 \text{ m/s}}$$

Run the Plank

In frozen Minnesota the Winter Sports Carnival includes some unusual events. Since it is dangerous to run on ice, each runner runs on a heavy (240 kg) and long (40 m) wooden plank, which itself rests on the smooth and horizontal ice. One of the competitors is a 60 kg woman who runs the length of the plank in 4.4 seconds, quite an impressive time. Her performance is viewed by a crowd huddled on the ice. The performance that they see is less impressive. With what speed do they see her moving?

IMPORTANT NOTE: you can only apply conservation of momentum if you have velocities in the same reference frame!

Relative to the plank, the woman's velocity is

$$v_{WP} = \frac{40\text{m}}{4.4\text{s}} = 9.09\,\text{m/s}$$

We'll have to use relative motion to relate this to the woman's velocity relative to the ice:

$$\boxed{\vec{v}_{WI} = \vec{v}_{WP} + \vec{v}_{PI}} = 9.09 + \vec{v}_{PI}$$

$\vec{v}_{WI}$ and $\vec{v}_{PI}$ are both relative to the ice, so we can now use conservation of momentum:

$$p_o = 0 \quad \text{(nothing moving)}$$

$$p_f = m_W\vec{v}_{WI} + m_P\vec{v}_{PI} = 60\vec{v}_{WI} + 240\vec{v}_{PI}$$

set $p_o = p_f$ to get $\boxed{60\vec{v}_{WI} + 240\vec{v}_{PI} = 0}$

With the relative motion equation, we now have two equations with two unknowns. Solve them to find

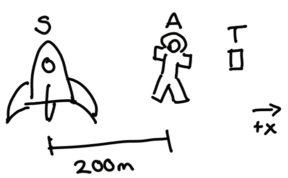
$$\boxed{v_{WI} = 7.27\,\text{m/s}}$$

Space Shuttle Emergency

You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has also been damaged and no longer works. She is 200 m from the shuttle and moving with it (i.e., she is not moving with respect to the shuttle). She is drifting in space with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10-kg tool kit and throw it away with all her strength, so that it has a speed of 8 m/s *relative to her*. In the script, she survives, but is this correct? Her mass, including her space suit but not her tool kit, is 80 kg.

We'll solve this problem using exactly the same steps we used for the previous problem.

But first, let's draw a picture to establish our signs:



Relative to the astronaut, we know

$$\vec{V}_{TA} = +8\,m/s$$

By relative motion,

$$\boxed{\vec{V}_{TS} = \vec{V}_{TA} + \vec{V}_{AS}} = 8 + \vec{V}_{AS}$$

$\vec{V}_{TS}$ and $\vec{V}_{AS}$ share the same reference frame, so we conserve momentum:

$$\vec{p}_o = 0 \quad \text{(system at rest)}$$

$$\vec{p}_f = m_A\vec{V}_{AS} + m_T\vec{V}_{TS}$$

Setting $\vec{p}_o = \vec{p}_f$ and plugging in the masses, we have

$$\boxed{80\vec{V}_{AS} + 10\vec{V}_{TS} = 0}$$

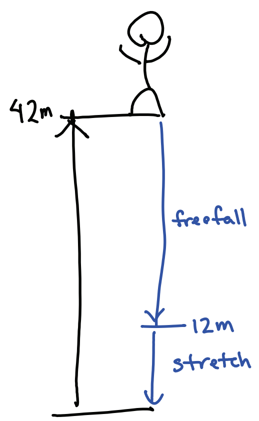
Combine this with the relative motion equation above to find $\boxed{V_{AS} = -0.89\,m/s}$

It will take $t = \frac{200\,m}{0.89\,m/s} = 225\,s$ to get back to the shuttle.

This is less than four minutes (240 s) so the script is correct.

Review: Bungee Jumpin'

In order to raise money for a scholarship fund, you convince the Physics 211 lecturer to bungee jump from a crane. To add some interest, the jump will be made from 42 m above a deep pool of Jello. A 30 m long bungee cord would be attached to his ankle. First you must convince him that your plan is safe for a person of his mass, roughly 70 kg. He knows that as the bungee cord begins to stretch, it will exert a force that has the same properties as the force exerted by a spring. Your plan has your lecturer stepping off a platform and being in free fall for the 30 m before the cord begins to stretch. You must determine the elastic constant of the bungee cord so that it stretches exactly 12 m, which will just keep his head out of the Jello.



We'll apply conservation of energy to find k:

$$E_o = U_o + K_o$$

$$U_o = mgh_o \qquad K_o = 0$$

$$E_f = U_f + K_f$$

$$U_f = mg\cancelto{0}{h_f} + \frac{1}{2}kx^2 \qquad K_f = 0$$

Setting $E_o = E_f$,

$$mgh_o = \frac{1}{2}kx^2$$

$$k = \frac{2mgh_o}{x^2} = \frac{2(70)(9.81)(42)}{(12)^2}$$

$$\boxed{k = 400.575\ N/m^2}$$

Review: Speeding Ticket?

You are driving your car uphill along a straight road. Suddenly you see a car run a red light and enter the intersection just ahead of you. You slam on your brakes and skid in a straight line to a stop, leaving skid marks 100 feet long. A policeman observes the whole incident and, much to your shock, gives *you* a ticket for exceeding the speed limit of 30 mph. When you get home, you consult your Physics 211 notes and estimate that the coefficient of kinetic friction between your tires and the road was 0.60 and the coefficient of static friction was 0.80. You estimate that the hill made an angle of 10^o with the horizontal. You look up in your owner's manual and find that your car weighs 2050 lbs. Will you fight the traffic ticket in court?

There are a few ways to approach this, but we'll just show one:

Since we're working in unusual units, let's go over the things we know.

$\Delta x = 100\,ft$ $\quad F_G = mg = 2050\,lbs$

$g = 32.2\,ft/s^2$ $\quad 1\text{ mile} = 5280\,ft$

We can find the car's initial speed using the work energy theorem:

$W_{NET} = \Delta K$

where $\Delta K = \cancel{K_f}^{\,0} - K_o = -\frac{1}{2}mv_o^2$

$W_{NET} = W_G + W_f$

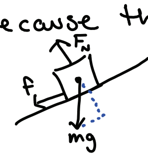
$W_G = \int \vec{F}_G \cdot d\vec{S} = (mg\sin 10^\circ)(\Delta x)(-1)$

$W_f = \int \vec{F}_f \cdot d\vec{S} = f_k(\Delta x)(-1)$

to find f_k (kinetic because the car is skidding)

$f_k = \mu_k F_N$

$= \mu_k mg\cos 10^\circ$



$F_N = mg\cos 10^\circ$

$W_f = (\mu_k mg\cos 10^\circ)\,\Delta x\,(-1)$

Putting these together, we get the expression

$$-mg\Delta x\sin 10^\circ - \mu_k mg\Delta x\cos 10^\circ = -\frac{1}{2}mv^2$$

Solve this to find

$$v_o = \sqrt{2g\Delta x(\sin 10^\circ + \mu_k\cos 10^\circ)} = \boxed{70.17\,ft/s}$$

Converting to mph, $\boxed{v_o = 48\,mph}$ so you were speeding.