Physics 211 Week 11

Angular Momentum: Rotating Astronauts

Two astronauts, each having a mass 75 kg, are connected by a light rope 10 meters long. They are isolated in space, and are initially revolving around their common center of mass once every 6 seconds. They now pull in on the rope, halving the distance between them. How much work is done by the astronauts as they shorten the rope?

The work done by the astronauts as they shorten the rope will be equal to the change in kinetic energy of the system. The kinetic energy will be changed because of the change in the moment of inertia of the system as the radius of the astronauts about the center of rotation is shortened. The angular speed is also changed as they shorten the rope. However, as the astronauts pull in on the rope, the system maintains the same angular momentum. You can use the constant value of angular momentum to determine the relationship between the original and final angular velocities. A numerical value for the initial angular velocity can be found from the given period. After finding the new angular velocity and moment of inertia, you can find the change in kinetic energy and therefore find the work done by the astronauts in shortening the rope. You should obtain a value of 6168 J.

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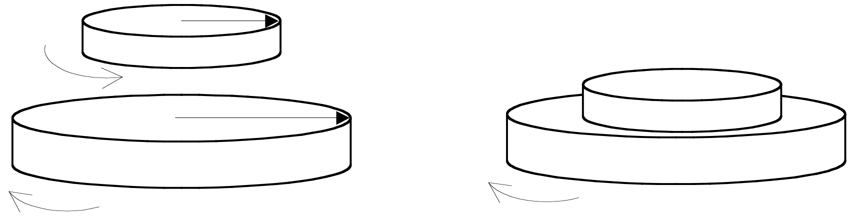
Angular Momentum: Spinning Disks

Two disks are mounted on frictionless bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, which has a mass of 1 kg and radius of 2 m, is set spinning at 45 rad/s. The second disk, which has a mass of 2 kg and a radius of 3 m, is set spinning at 25 rad/s in the opposite direction. They then couple together. What is their angular speed after coupling?

R^{1}R^{2}W^{1}W^{2}W^{f}InitialFinal

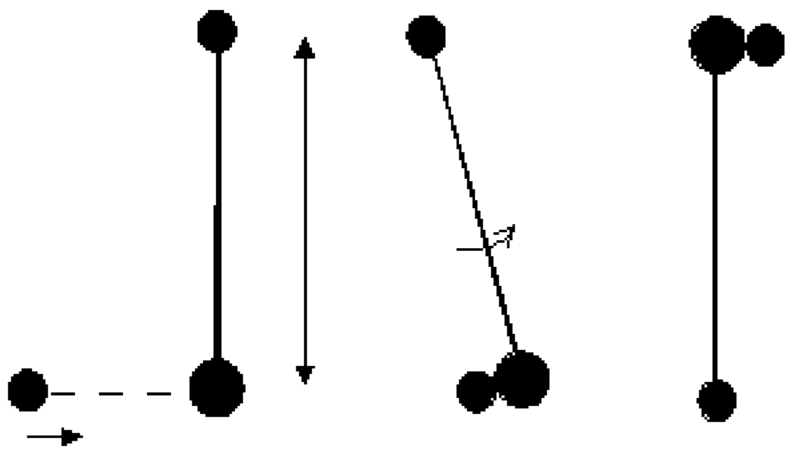


In the collision of the two spinning disks, angular momentum is conserved. Angular momentum is the product of the moment of inertia and angular velocity. The angular momentum of the pair before they collide is the sum of their individual angular momentums; however, remember that they are spinning in opposite directions and so their angular momentums' have opposite signs. After they collide the angular momentum is the product of their combined moment of inertia and the new angular speed. Solving this equation you should find the new angular speed is 12.3 rad/s.

Physics 211 Week 11

Angular Momentum: Dumbbell Collision

A dumbbell consists of two balls, one of mass M and the other of mass 2M, connected by a light rod of length L. The dumbbell is mounted vertically on a pivot with the heavier ball at the bottom. The pivot is located at the midpoint of the rod. The system, which is initially at rest, is free to rotate about the pivot. A wad of putty of mass M and initial velocity V collides with and sticks to the lower mass, as shown in the diagram. In terms of the quantities given, what is the minimum value of V for which the dumbbell will make it all the way around?



During the collision, the angular momentum is conserved; therefore, after the collision, the dumbbell will have an angular velocity. It must have enough angular velocity to get all the way around. What will stop it from reaching the top? Work done by gravity.

Use the conservation of angular momentum to find the angular velocity after the collision. Then find the rotational kinetic energy of the dumbbell when it first begins to rotate. The rotational energy of the dumbbell plus the gravitational potential energy of the top of the dumbbell must equal the gravitational potential energy of the collided masses when they are in the top position for the dumbbell to be able to make it all the way around. Solving the conservation of angular momentum equation and the conservation of energy equation, you will find a needed initial velocity of

$$v = \sqrt{16gL}$$

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Angular Momentum: Faster than a Speeding Bullet

You are assigned the job of designing a simple system to measure the speed of a bullet. The system you come up with works in the following way: You shoot the bullet into a hardwood rod of mass of 5 kg and a length of 1.2 m, mounted on a frictionless axle which passes through it's center and is perpendicular to its length. The axle is vertical, so the rod rotates in the horizontal plane after the bullet hits it. The rod is initially at rest and oriented perpendicular to the path of the bullet. You shoot a bullet of mass 2.0 grams at 400 m/s into the rod a distance of 0.45 m to one side of the axle, where it embeds itself and stops.

1. How much time will it take the rod to make one full revolution after the bullet slams into it?
2. What is the ratio of the initial to the final kinetic energy of the system?

For the first part of the problem, you can use the conservation of angular momentum to find the final angular speed and use the speed to find the period. You can find the initial angular momentum by taking the cross product of the linear momentum and the distance of closest approach. When determining the final angular momentum, you will need to determine the momentum of inertia of the rod-bullet system; however, the mass of the bullet is negligible compared to the massive rod, so the bullet contribution to the moment of inertia can be ignored. You should obtain a period of 10.47 second.

For the second part of the problem you need to make a ratio of the kinetic energies. The kinetic energy can be found by squaring the angular momentum and dividing by the moment of inertia. The angular momentum is the same both before and after the collision, so the kinetic energies are simply inversely proportional to the moment of inertia. The ratio of the initial to final kinetic energy will be the same as the ratio of the final to initial moment of inertia. You can find the moments of inertia in the same way you did for part one. You should obtain a ratio of 1481:1.