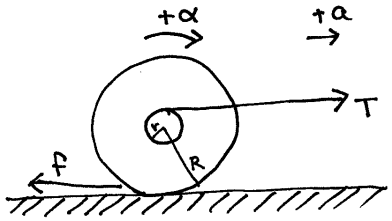
Review: Rolling Spool

A spool with outer radius R and inner radius r rolls without slipping on a horizontal surface. The inner part may be approximated as a uniform cylinder (radius r) of mass m. The two rims may be thought of as disks (of radius R) and mass M each. Total mass is m + 2M.

The spool is pulled by a rope of tension T wrapped around the inner radius as pictured. What is the acceleration of the center of mass of the spool?



A secret force is missing on the original diagram: Static friction makes the spool "roll without slipping." I've drawn it above. We'll ignore the y-direction since it's clear that $F_N = mg$. Also note the acceleration above.

The acceleration of the spool is governed by

$$\sum \tau = I\alpha \quad \text{and} \quad \sum F_x = ma$$

Let's write these:

$$\sum \tau = I\alpha$$

$$\tau_T = +Tr$$

$$\tau_f = +fR$$

$$I = \sum_n I_n = \frac{1}{2}mr^2 + 2\left(\frac{1}{2}MR^2\right)$$

$$\alpha = \frac{a}{R}$$

Combine these: $\boxed{Tr + fR = \frac{1}{2}\left(mr^2 + 2MR^2\right)\left(\frac{a}{R}\right)}$

$$\sum F_x = ma$$

$$\sum F_x = T - f$$

$$m_{Total} = m + 2M$$

$$\boxed{T - f = (m + 2M)a}$$

Working from the two boxed equations, you will find

$$\boxed{a = \frac{T(R+r)}{m_{Total}R + \frac{I_{Total}}{R}}}$$