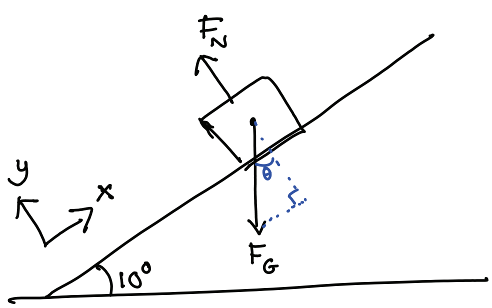
Exit Ramp

On a trip to the Colorado Rockies, you notice that when the freeway goes steeply down a hill, there are emergency exits every few miles. These emergency exits are straight ramps which leave the freeway and are sloped uphill. They are designed to stop runaway trucks and cars that lose their brakes on downhill stretches of the freeway even if the road is covered with ice. You are curious, so you stop at the next emergency exit to take some measurements. You determine that the exit rises at an angle of 10° from the horizontal and is 100m long. What is the maximum speed of a truck that you are sure will be stopped by this road, even if the frictional force of the road surface is negligible?

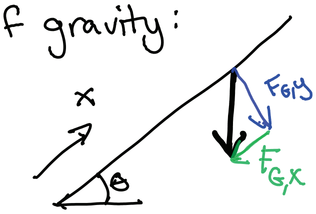
We begin this problem with a FBD:



Since the acceleration of the vehicle is only in the x direction, we won't worry about anything in y...

Find the x component of gravity:

$F_{G,x} = F_G \sin\theta$



Using Newton's 2nd Law in the x direction

$$\sum F_x = -F_G \sin\theta = ma$$

$$-\cancel{m}g\sin\theta = \cancel{m}a$$

$$\boxed{a = -g\sin\theta}$$

Now we use the other information from the problem: given $x = 100$, $v = 0$ and $a = -g\sin\theta$, what is v_0?

Well,

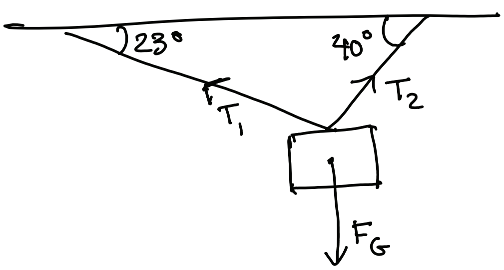
$$\cancel{v}^{\,0\,2} = v_0^2 + 2a\Delta x$$

$$v_0 = \sqrt{-2a\Delta x}$$ plug in the numbers to get

$$\boxed{v_0 = 18.46 \, m/s}$$

Hanging Sculpture

You are part of a team to help design the atrium of a new building. Your boss, the manager of the project, wants to suspend a 10-kg sculpture high over the room by hanging it from the ceiling using thin, clear fishing line (string) so that it will be difficult to see how the sculpture is held up. The only place to fasten the fishing line is to a wooden beam which runs around the edge of the room at the ceiling. The fishing line that she wants to use is known to be able to support a vertically hanging mass of 10 kg, so she suggests attaching two lines to the sculpture to be safe. Each line would come from the opposite side of the ceiling to attach to the hanging sculpture. Her initial design has one line making an angle of 23° with the ceiling and the other line making an angle of 40° with the ceiling. She knows you are taking physics, so she asks you if her design can work.



In order for your friend's design to work:

(1) The sculpture must be in static equilibrium (i.e., $\Sigma F_x = 0$ and $\Sigma F_y = 0$)

(2) $T_1 < 98N$ <u>and</u> $T_2 < 98N$

Let's satisfy condition (1) first:

$$\Sigma F_x = -T_{1x} + T_{2x} = 0$$

$$-T_1 \cos 23° + T_2 \cos 40° = 0$$

$$\boxed{T_1 \cos 23° = T_2 \cos 40°}$$

$$\Sigma F_y = T_{1y} + T_{2y} - F_G = 0$$

$$\boxed{T_1 \sin 23° + T_2 \sin 40° - mg = 0}$$

Use the boxed equations above to solve for T_1 and T_2.

You will find

$$\boxed{\begin{aligned} T_1 &= 84.34N \\ T_2 &= 101.35N \end{aligned}}$$

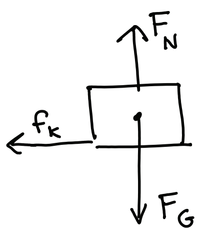
Since $T_2 > 98N$, <u>the sculpture will not work</u>.

Friction Intro

(a) A block of mass M = 5 kg slides on a horizontal table. The kinetic coefficient of friction between the block and the table is μ = 0.38. If the initial speed of the block is 8 m/s, how many seconds does is slide before stopping?

(b) A block of mass M = 15 kg is pulled across a horizontal table by a string. The kinetic coefficient of friction between the block and the table is μ = 0.69. If the speed of the block is to be constant at 2 m/s, what must the tension in the string be?

(a) Start with a FBD:



In the y direction we have

$$\sum_i F_y = F_N - F_G = 0 \rightarrow F_N = F_G = mg$$

In the x direction,

$$\sum_i F_x = -f_k = ma$$

where $f_k = \mu F_N = \mu mg$

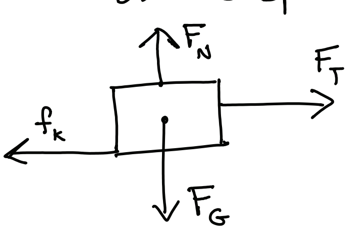
thus $-\mu \cancel{m} g = \cancel{m} a \rightarrow \boxed{a = -\mu g}$

From here, use kinematics.

Given $a = -\mu g$, $v_0 = 8\,m/s$ and $v_f = 0\,m/s$, we find t:

$$v = v_0 + at \rightarrow 0 = 8 - \mu g t \rightarrow \boxed{t = 2.15\,s}$$

(b) The block's speed is constant, thus $a = 0$.



$$\sum_i F_y = F_N - F_G = 0 \rightarrow F_N = F_G = mg$$

$$\sum_i F_x = F_T - f_k = 0$$

where $f_k = \mu F_N = \mu mg$

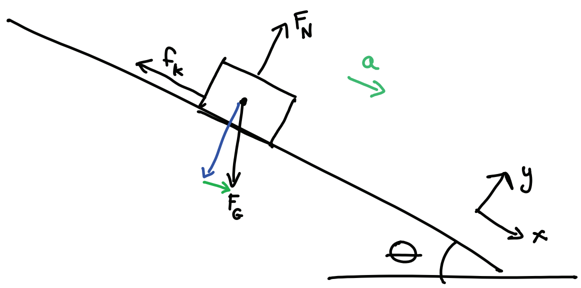
so $F_T - \mu mg = 0$

$$\boxed{F_T = \mu mg = 101.53\,N}$$

Friction by Time

A block of mass M is released from rest on an incline of length L which makes an angle θ with the horizontal. The block slides down the incline and reaches the end of the incline in time T. Show how the coefficient of kinetic friction can be determined from the measurement of time T. That is, find a formula for μ_k in terms of the givens: M, L, T, g and θ.

Begin this problem, as always, with a FBD.



We'll use Newton's 2nd Law in the x and y directions to find a:

$$\Sigma F_y = F_N - F_{Gy} = 0$$

where $F_{Gy} = Mg\cos\theta$

$$F_N = Mg\cos\theta$$

$$\Sigma F_x = -f_k + F_{Gx} = Ma$$

where $f_k = \mu F_N = \mu Mg\cos\theta$

and $F_{Gx} = Mg\sin\theta$

$$-\mu \cancel{M} g\cos\theta + \cancel{M} g\sin\theta = \cancel{M} a$$

$$\boxed{a = g\sin\theta - \mu g\cos\theta}$$

Now that we have a, we can get into the kinematics part of the problem. The block starts at rest ($v_o = 0$) and slides a length L in time T:

$$x = \cancel{x_o}^{0} + \cancel{v_o}^{0} t + \frac{1}{2}at^2$$

$$L = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)T^2$$

Using algebra to solve for μ, you will find

$$\boxed{\mu = \tan\theta - \frac{2L}{gT^2\cos\theta}}$$

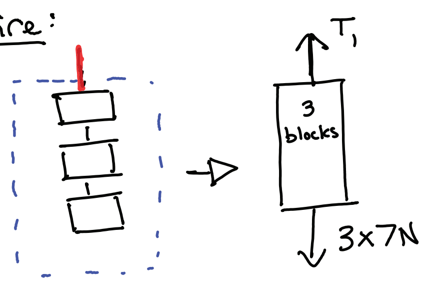
Emergency Stop

Your friend has been hired to design the interior of a special executive express elevator for a new office building. This elevator has all the latest safety features and will stop with an acceleration of g/3 in the case of an emergency. The management would like a decorative lamp hanging from the unusually high ceiling of the elevator. He designs a lamp which has three sections which hang one directly below the other. Each section is attached to the previous one by a single thin wire, which also carries the electric current. The lamp is also attached to the ceiling by a single wire. Each section of the lamp weighs 7.0 N. Because the idea is to make each section appear that it is floating on air without support, he wants to use the thinnest wire possible. Unfortunately the thinner the wire, the weaker it is. Since he knows that you have taken a course in physics, he asks you to calculate the force on each wire in case of an emergency stop.

There are a few different ways to approach this problem, but here we'll do it top to bottom.

Each wire causes all the lamp sections below it to accelerate at $a = +g/3$

Top wire:

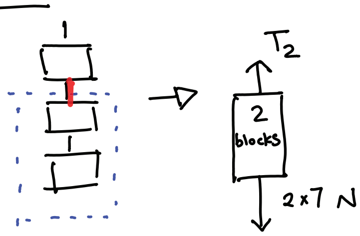


$$\Sigma F_y = T_1 - (3\times7) = \frac{(3\times7)}{g}a$$

$$T_1 - 21 = \frac{\cancel{3\times7}}{\cancel{g}}\left(\frac{\cancel{g}}{\cancel{3}}\right)$$

$$T_1 - 21 = 7 \rightarrow \boxed{T_1 = 28\,N}$$

Middle wire:

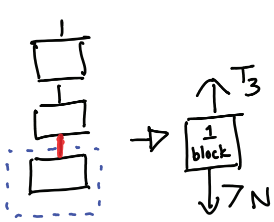


$$\Sigma F_y = T_2 - (2\times7) = \frac{(2\times7)}{\cancel{g}}\left(\frac{\cancel{g}}{3}\right)$$

$$T_2 - 14 = \frac{14}{3}$$

$$\boxed{T_2 = 18.67N}$$

Bottom wire:



$$\Sigma F_y = T_3 - 7 = \frac{7}{\cancel{g}}\left(\frac{\cancel{g}}{3}\right)$$

$$T_3 - 7 = \frac{7}{3}$$

$$\boxed{T_3 = 9.33N}$$