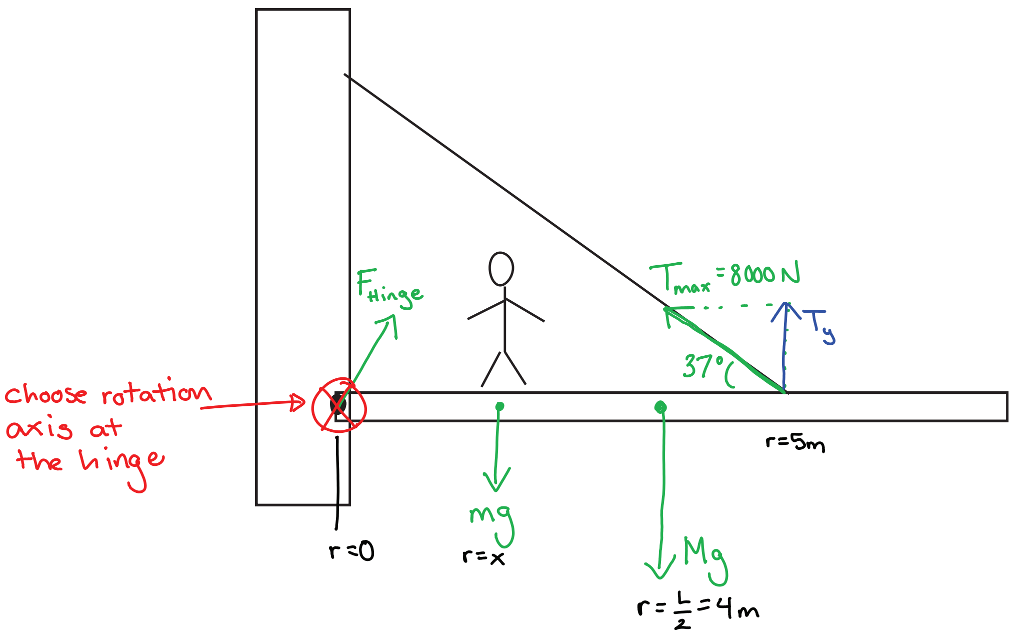
Walking the Plank

A uniform horizontal beam 8 m long is attached by a frictionless pivot to a wall. A cable making an angle of 37°, attached to the beam 5 m from the pivot point, supports the beam, which has a mass of 600 kg. The breaking point of the cable is 8000 N. A man of mass 95 kg walks out along the beam. How far can he walk before the cable breaks?



For <u>any</u> rotation axis we choose, it <u>must</u> be true that

$$\sum \vec{\tau} = \vec{\tau}_{man} + \vec{\tau}_{beam} + \vec{\tau}_{wire} + \vec{\tau}_{hinge} = 0$$

We don't know F_{hinge}, so let's make life easier by choosing a rotation axis at the hinge, as shown above.

Now, calculate torques from each force:

CW torque is negative

$$\vec{\tau}_{man} = \vec{r}_{man} \times \vec{F}_{man} = (x)(95g)(-1) = -931.95x$$

$$\vec{\tau}_{beam} = \vec{r}_{beam} \times \vec{F}_{beam} = (4)(600g)(-1) = -23544$$

$$\vec{\tau}_{wire} = \vec{r}_{wire} \times \vec{F}_{wire} = (5)(T_y)(+1) = (5)(T_{max}\sin 37°) = 24072.6$$

$$\vec{\tau}_{hinge} = \vec{r}_{hinge} \times \vec{F}_{hinge} = (0)(F_{hinge}) = 0$$

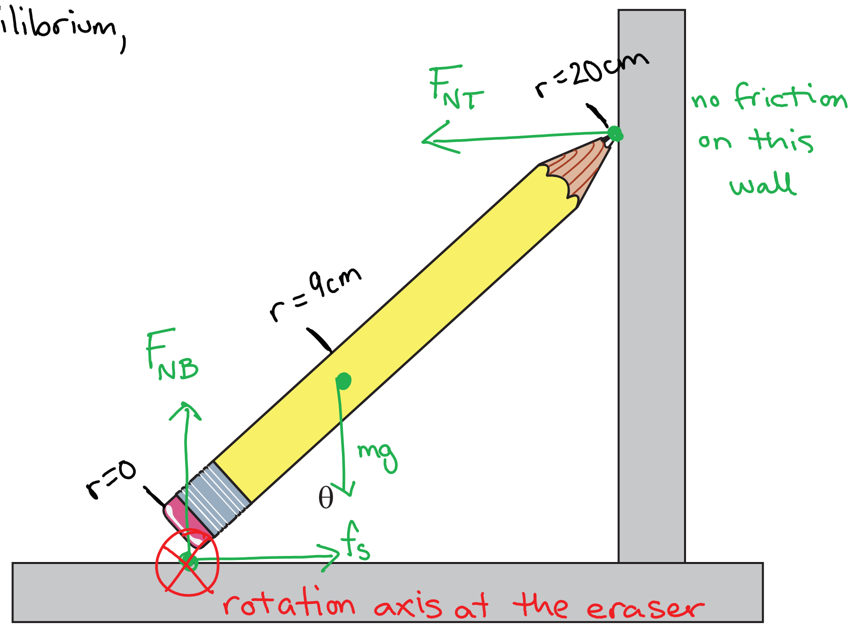
Set the sum of these torques to be equal to zero and Solve to find

$$\boxed{x = 0.567 \text{ m}}$$

Leaning Pencil

A picture below shows a pencil with its sharpened end resting against a smooth vertical surface and its erase end resting on the floor. The center of mass of the pencil is 9 cm from the end of the eraser and 11 cm from the tip of the lead. The coefficient of static friction between the eraser and floor is $\mu = 0.80$. What is the minimum angle θ the pencil can make with the floor such that it does not slip?

In order to be in static equilibrium, both $\Sigma F = 0$ and $\Sigma \tau = 0$ must be satisfied. We will use both of these to help us solve this problem.



① $\Sigma F_y = 0$:

$$F_{NB} - mg = 0 \rightarrow F_{NB} = mg$$

② $\Sigma F_x = 0$:

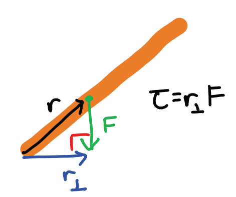
$$f_s - F_{NT} = 0 \rightarrow f_s = F_{NT} = \underbrace{\mu F_{NB}}_{f_s \text{ is maximized}} = \mu mg$$

③ $\Sigma \tau = 0$: We'll get our easiest equation by putting the rotation axis at the eraser

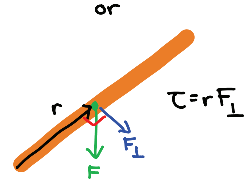
$$\Sigma \tau = \tau_{floor} + \tau_{pencil} + \tau_{wall} = 0$$

$F_{NT} = f_s$

$$0 + (0.09\cos\theta)(mg)(-1) + (0.2\sin\theta)(F_{NT})(+1) = 0$$



or

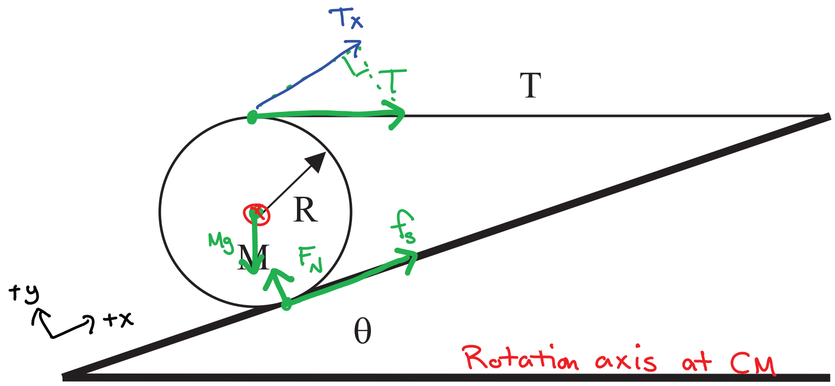


Solve the above equation to find

$$\boxed{\theta = 29.35°}$$

Cylinder Held on Inclined Plane

A cylinder of mass M and radius R is in static equilibrium as shown in the diagram. The cylinder rests on an inclined plane making an angle θ with the horizontal and is held by a horizontal string attached to the top of the cylinder and to the inclined plane. There is friction between the cylinder and the plane. What is the tension in the string T?



Once again, we'll take advantage of $\Sigma\tau = 0$ and $\Sigma F = 0$:

$\Sigma\tau = 0$:

$$\Sigma\tau = \tau_{string} + \tau_{friction} + \cancel{\tau_{normal}}^{\,0} + \cancel{\tau_{gravity}}^{\,0}$$

$$= (R)(T)(-1) + (R)(f_s)(+1)$$

$$-RT + Rf_s = 0 \rightarrow f_s = T$$

$\tau_{normal} = 0$ because $\vec{F}_N$ is parallel to $\vec{r}$.

$\tau_{gravity} = 0$ because $\vec{r} = 0$

$\Sigma F_x = 0$: Using tilted axes,

$$\Sigma F_x = T\cos\theta + f_s - Mg\sin\theta$$

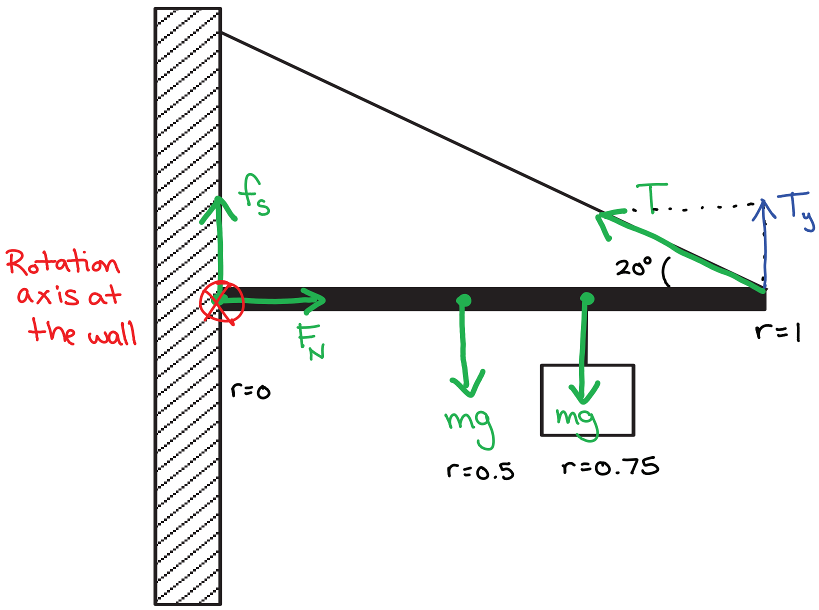
$$T\cos\theta + T - Mg\sin\theta = 0$$

Solve for T to find

$$\boxed{T = \frac{Mg\sin\theta}{1+\cos\theta}}$$

Weight on Stick

One end of a uniform meter stick of mass 0.25 kg is placed against a vertical wall. The other end is held by a lightweight cord making an angle $\theta = 20^\circ$ with the stick. A block with the same mass as the meter stick is suspended from the stick a distance 0.75 m from the wall. Find the tension in the cord and the frictional force between the stick and the wall.



This situation must satisfy $\Sigma\tau = 0$ and $\Sigma F = 0$

$\Sigma\tau = 0$: It's easiest to set r=0 (the rotation axis) at the wall so that $\tau_{friction} = \tau_{Normal} = 0$

$$\Sigma\tau = \cancel{\tau_{friction}}^{0} + \cancel{\tau_{Normal}}^{0} + \tau_{stick} + \tau_{block} + \tau_{cord}$$

$$= (0.5)(mg)(-1) + (0.75)(mg)(-1) + (1)(T_y)(+1) = 0$$

$T\sin 20^\circ$

Solve to find $\boxed{T = 8.963\text{N}}$

$\Sigma F_y = 0$:

$$\Sigma F_y = f_s - mg - mg + T_y = 0$$

$$f_s = 2mg - T\sin 20^\circ$$

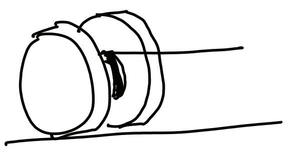
Use T from above to find $\boxed{f_s = 1.839\text{N}}$

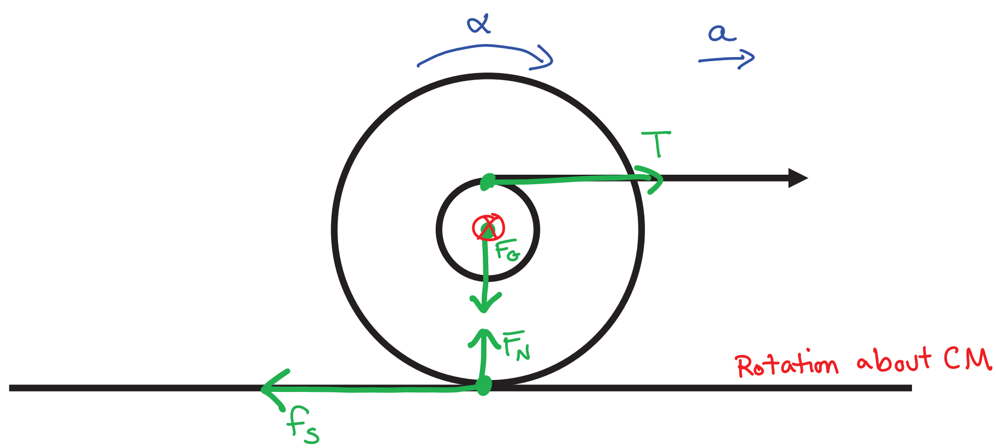
Review: Rolling Spool

A spool with outer radius R and inner radius r rolls without slipping on a horizontal surface. The inner part may be approximated as a uniform cylinder (radius r) of mass m. The two rims may be thought of as disks (of radius R) and mass M each. Total mass is m + 2M.

The spool is pulled by a rope of tension T wrapped around the inner radius as pictured. What is the acceleration of the center of mass of the spool?

side view:





The spool is not slipping on the floor, thus we know

① there is static friction at the floor (drawn above)

② $a_{cM} = \alpha R$ (not αr)

Using Newton's 2nd Law:

$\Sigma \tau = I\alpha$

$\Sigma \tau = \tau_T + \tau_f = rT + Rf$ $\quad\quad I = I_{small} + 2I_{big} = \frac{1}{2}mr^2 + MR^2$

So we have

$$\boxed{rT + Rf = \left(\tfrac{1}{2}mr^2 + MR^2\right)\left(\frac{a_{cM}}{R}\right)}$$

$\Sigma F_x = (m+2M)a_{cM}$

$\Sigma F_x = T - f \rightarrow \boxed{T - f = (m+2M)a_{cM}}$

Combine the two equations to find

$$\boxed{a_{cM} = \frac{T(R+r)}{3MR + mR + \frac{mr^2}{2R}}}$$