Session 2 Solution

Wednesday, January 29, 2014 12:48 PM

Heading North

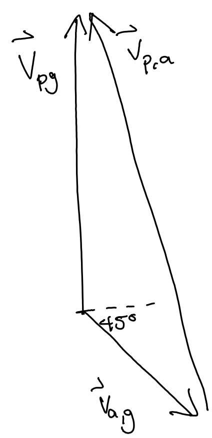
You are the navigator of an AA flight scheduled to fly from New Orleans due north to St. Louis, a distance of 673 miles. Your instruments tell you that there is a steady wind from the northwest with a speed of ~~155~~ 105 mph. The pilot sets the air speed at ~~510~~ 575 mph and asks you to find the estimated flying time. What do you tell her?

This problem asks for time to travel 673 miles relative to the ground. That's easy:

$$\boxed{\vec{x} = \vec{V}_{plane,ground}\, t}$$

Well, maybe not so easy. We know $|\vec{V}_{plane,air}| = 575$ mph and $\vec{V}_{air,ground} = 105$ mph from NW.

We also know that $\vec{V}_{plane,ground}$ will point due north (or else they won't land in St. Louis)



Using vector addition,

$$\boxed{\vec{V}_{p,g} = \vec{V}_{p,a} + \vec{V}_{a,g}}$$

Looking at components,

① $V_{p,ax} + V_{a,gx} = 0$

② $V_{p,ay} + V_{a,gy} = |\vec{V}_{p,g}|$

The airplane flies at an angle to cancel the effect of the wind. Let's define θ as degrees North of West and solve:

$$\overset{575}{V_{p,a}} \cos\theta - \overset{105}{V_{a,g}} \cos 45° = 0 \qquad \boxed{\theta = 82.6°}$$

From here, use y components to find $|\vec{V}_{p,g}|$: $V_{p,a} \sin\theta - V_{a,g} \sin 45° = |V_{p,g}|$

$|\vec{V}_{p,g}| = 495.9$ mph

With the original $\vec{x} = \vec{V}_{p,g}\, t$ equation, you'll find $\boxed{t = 1.357 \text{ hours}}$

Against the Grain

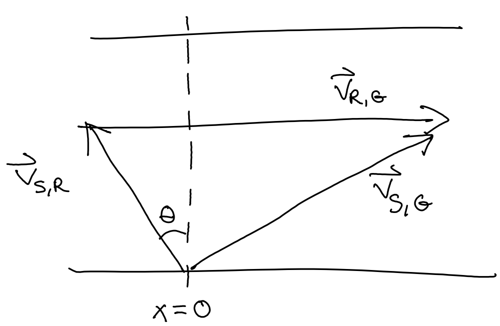
You are on the west bank of a river which flows due south and want to swim to the east bank. You have told your friends to meet you on the east bank directly opposite your starting point. Before starting out, you realize that, since the river is flowing swiftly at a speed of 12 ft/s and since your fastest swimming speed in still water is only 5 ft/s, you will inevitably be carried downstream. Nevertheless, you want to minimize the effort expended by your friends in walking downstream to meet you. Your guide book to the region tells you that the width of the river is 300 ft. After a quick calculation, you call your friends on your cellular phone and tell them to start walking to a new meeting point. How far downstream of the original meeting point should you tell them to walk?

This is the only discussion question where you'll actually use calculus.

Recall: you can find the minimum of a function by finding where its derivative is equal to zero

The situation described in the problem looks like this

S - swimmer ; R - river ; G - ground



$$\boxed{\vec{V}_{S,G} = \vec{V}_{SR} + \vec{V}_{RG}}$$

Time spent in the river will be governed by the y-component of your path:

$300\text{ft} = V_{SG,y}\, t$ where $\underbrace{V_{SG,y} = V_{SR}\cos\theta}_{\text{From the diagram}}$

During this time, you will drift a distance $x = (V_{SG,x})t$

We also know $\underbrace{V_{SG,x} = 12 - 5\sin\theta}_{\text{from the diagram}}$

Combining these four equations, you'll get $\boxed{x = -300\tan\theta + 720\sec\theta}$

Setting $\frac{dx}{d\theta} = 0$, you will find your minimum occurs at $\theta = 24.6°$

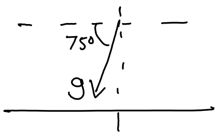
and $\boxed{x_{min} = 654.5\text{ft}}$

Review: Which Way Is Up?

Tilted Land is flat, much like central Illinois, but in Tilted Land gravity does not point straight down! It points down at an angle of 75 degrees with respect to the ground. (In central Illinois gravity points straight down at an angle of 90 degrees with respect to the ground.) This circumstance causes major changes in everyday life, but people manage. Suppose that a person in Tilted Land throws a ball straight up (that is, perpendicular to the surface of the land) at 20 m/s. Where does the ball hit the ground?

This is 2D kinematics with a twist!

Instead of our usual $\downarrow g$ we have



First, find a_x and a_y:

$$a_x = -9.81\cos 75^\circ \qquad a_y = -9.81\sin 75^\circ$$

We follow our usual process from here.

The y-component dictates the amount of time the ball stays in the air:

$$\boxed{y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2}$$

here $y = y_0 = 0$ and $v_{0y} = v_0 = 20\,m/s$

Solve for t and plug it into the x-component equation to find out where the ball lands

$$\boxed{x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2}$$

where $x_0 = 0$ and $v_{0x} = 0$.

Solving the two equations, you should find

$$\boxed{t = 4.22s}$$

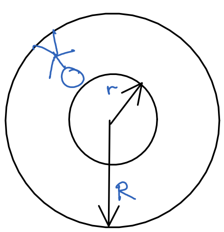
$$\boxed{x = -22.61\,m}$$

HAL - 2001

A Hollywood producer has decided to film a remake of the movie **2001: A Space Odyssey**. You have been hired as a consultant for the movie to make sure the science is correct. The producer wanted to have an Illinois physics student for the job since HAL was invented in Urbana (January 12, 1997!). Part of the movie takes place on a space station very far from any gravitating body. This station is a large wheel-like structure where people live and work on the rim. In order to create "artificial gravity", the space station rotates about its axis.

One space station design is a large wheel-like structure where people live and work on the rim. In order to create "artificial gravity", the space station rotates about its axis. It is desired that the gravity be equal to 0.85 times that of the Earth. Since centripetal acceleration depends on the distance to the axis of rotation, it is not possible for the "artificial gravity" to be the same at the head and the feet of a standing person (~1.8 m). In order to minimize any possible discomfort, suppose that the difference in the "artificial gravity can only be 1%. The special effects department wants to know the diameter and the rate of rotation of a space station that meets these specifications.

When something rotates, its rotational speed is ω (omega). ω has the same value at any distance from the rotation axis, unlike v, so we'll use it to make this problem easier.



By the problem description,

$r = R - 1.8\,m$

at R, ① $a_{c,R} = 0.85g$

at r, ② $a_{c,r} = 0.99\, a_{c,R}$

We also know $a_{c,R} = \omega^2 R$ and $a_{c,r} = \omega^2 r$

Putting these into ②, $\cancel{\omega^2}(R-1.8) = 0.99\,\cancel{\omega^2}R$

From this, $R = 180\,m$

Then use ①: $\omega^2 R = 0.85g$ to find $\omega = 0.214\,rad/s$

Converting ($D = 2R$ and $f = \frac{\omega}{2\pi}$)

$\boxed{D = 360\,m,\ f = 0.034\,Hz}$