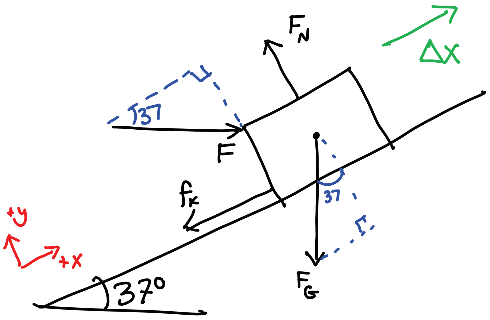
# Block on Incline

A block of mass 3 kg is moved up an incline that makes an angle of 37° with the horizontal under the action of a constant *horizontal* force of 40 N. The coefficient of kinetic friction between the block and the incline is 0.1. The block is initially at rest. What is the kinetic energy of the block after it has been displaced 2 m along the incline?

There are several ways to solve this problem, but we'll only show two:




As always, start with a diagram with all the forces and other relevant information (e.g., axes, direction of displacement)

Both methods take advantage of the Work-Energy theorem, $W_{NET} = \Delta K$ to find $K_f$ ($K_f = \Delta K$ since $K_0 = 0$)

**Method 1**: We are dealing with constant forces, so we'll use $W = \vec{F} \cdot \vec{S}$ to calculate work. The dot product selects out only the parallel components of the vectors, and since $\vec{S}$ points in the +x direction,

$$W = \vec{F} \cdot \vec{S} = (\Sigma F_x)(2)$$

Using the diagram above,

$$\Sigma F_x = F\cos 37° - mg\sin 37° - f_k$$

$f_k = \mu_k F_N$

Using Newton's 2nd law in the y direction,

$$\Sigma F_y = -F\sin 37° - mg\cos 37° + F_N = 0$$

so $F_N = F\sin 37° + mg\cos 37°$

and $f_k = \mu_k (F\sin 37° + mg\cos 37°)$

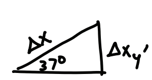
$$\Sigma F_x = F\cos 37° - mg\sin 37° - \mu(F\sin 37° + F_G\cos 37°)$$
$$= 9.49 \text{ N}$$

Finally, $W = (\Sigma F_x)(2) = \boxed{18.96 \text{ J}}$

**Method 2**: Find the work done on the box by each force and add them to find $W_{NET}$

$W_F = \vec{F} \cdot \vec{S} = (F_x)(2\text{m}) = (F\cos 37°)(2) = +63.89 \text{ J}$

$W_g = \vec{F}_G \cdot \vec{S} = -F_{Gx}(2\text{m})$ or $-F_G(\Delta x_{y'})$ ← $\Delta x_{y'}$ is height change




$= -(mg\sin 37°)(2)$ or $-mg(2\sin 37°)$

$= -35.42 \text{ J}$

$W_N = \vec{F}_N \cdot \vec{S} = 0$ ($\vec{F}_N$ is perpendicular to $\vec{S}$)

$W_f = \vec{f}_k \cdot \vec{S} = -f_k(2) = -\mu(F\sin 37° + mg\cos 37°)(2)$ ← found using the calculation above (in the box)

$= -9.52 \text{ J}$

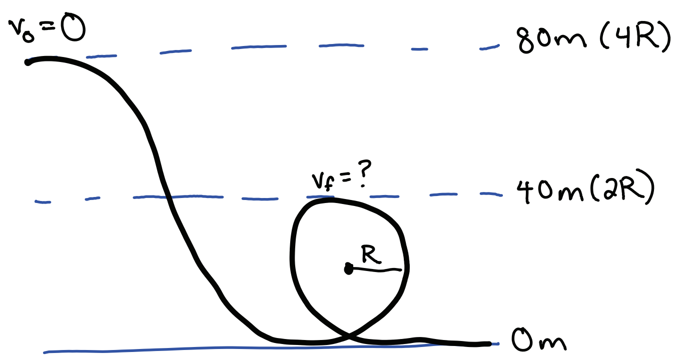
$W_{NET} = W_F + W_g + W_N + W_f = \boxed{18.96 \text{ J}}$

# Loop

You are designing a new roller-coaster. The main feature of this particular design is to be a vertical circular loop-the-loop where riders will feel like they are being squished into their seats even when they are in fact upside-down (at the top of the loop).

The coaster starts at rest a height of 80m above the ground, speeds up as it descends to ground level, and then enters the loop which has a radius of 20m. Suppose a rider is sitting on a bathroom scale that initially reads W (when the coaster is horizontal and at rest). What will the scale read when the coaster is moving past the top of the loop?
(You can assume that the coaster rolls on the track without friction).

To solve this problem, we'll have to conserve energy and use dynamics.




We begin by conserving energy:

$E_0 = E_f$

Find $E_0 = U_0 + K_0$

$U_0 = mgh_0 \qquad K_0 = \frac{1}{2}m\cancel{v_0^2}^{\,0}$

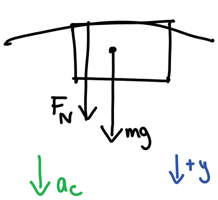
Find $E_f = U_f + K_f$

$U_f = mgh_f \qquad K_f = \frac{1}{2}mv_f^2$

setting $E_0 = E_f$, $\quad mgh_0 = mgh_f + \frac{1}{2}mv_f^2$

$$v_f = \sqrt{2g(h_0 - h_f)} = \sqrt{2g(2R)}$$

From here, we use dynamics to relate $v_f$ to normal force ("weight" on the scale) at the top of the loop.




$\sum F_y = F_N + mg = ma_c \qquad$ where $a_c = \frac{v^2}{R}$

$F_N + mg = m\frac{v^2}{R}$

$F_N = m\left(\frac{(2g*2R)}{R} - g\right) = mg(4-1) = 3mg$

Since a scale will usually measure $W = mg$, the scale reads $\boxed{3W}$ at the top of the loop.

# Colliding Binary

Two identical stars, each having mass and radius $M = 2x10^{29}$ kg and $R = 7x10^{8}$ m, are initially at rest in outer space. Their initial separation is the same as the distance between our sun and the earth, $D = 1.5x10^{11}$ m. Their gravitational interaction causes the stars to be pulled toward one another. Find the speed of the stars just before they collide, i.e., when their centers are a distance 2R apart.

To solve this problem, we'll need

$$G = 6.67\times10^{-11}\ m^3/kg\,s^2 \quad \text{and} \quad U_G = -G\frac{mM}{r}$$

which are both in the exam formula sheet but are not in the discussion booklet.

As the stars move, energy is conserved:

Far apart: $E_0$

$$U_0 = -G\frac{M^2}{D} \qquad K_0 = 0$$

Colliding: $E_f$

$$U_f = -G\frac{M^2}{2R} \qquad K_f = \underbrace{\tfrac{1}{2}Mv_f^2 + \tfrac{1}{2}Mv_f^2}_{\text{Note that both stars are moving!}} = Mv_f^2$$

Set $E_0 = E_f$:

$$U_0 + K_0 = U_f + K_f$$

$$-G\frac{M^2}{D} = -G\frac{M^2}{2R} + Mv_f^2$$

$$v_f^2 = GM\left(\frac{1}{2R} - \frac{1}{D}\right)$$

$$v_f = \sqrt{GM\left(\frac{1}{2R} - \frac{1}{D}\right)}$$

Plugging in the numbers, find

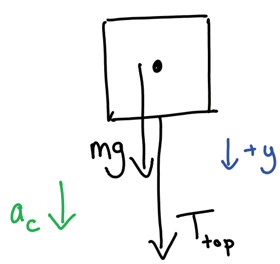
$$\boxed{v_f = 97158\ m/s}$$

# Review: Noise-Maker

After watching the movie "Crocodile Dundee," you and some friends decide to make a communications device invented by the Australian Aborigines. It consists of a noise-maker swung in a vertical circle on the end of a string. You are worried about whether the string you have will be strong enough, so you decide to calculate the tension in the string when the device is swung with constant speed. You and your friends can't agree whether the maximum tension will occur when the noise-maker is at the highest point in the circle, at the lowest point in the circle, or will always be the same. To settle the argument, you decide to calculate the tension at the highest point and at the lowest point and then compare them.

We'll use FBDs and Newton's 2nd law to look at both situations:

At the top:

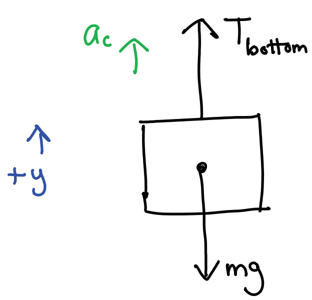



Letting +y be in the downward direction,

$$\sum F_y = mg + T_{top} = m a_c \quad \leftarrow v^2/R$$

$$\boxed{T_{top} = m\left(\frac{v^2}{R} - g\right)}$$

At the bottom:




Letting +y be in the upward direction,

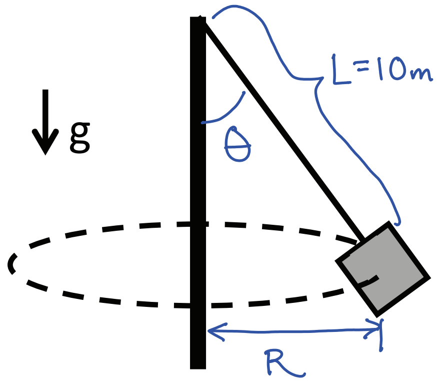
$$\sum F_y = T_{bottom} - mg = m a_c \quad \leftarrow v^2/R$$

$$\boxed{T_{bottom} = m\left(\frac{v^2}{R} + g\right)}$$

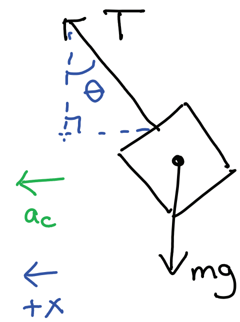
Since v is constant, the tension is the largest at the bottom of the circle.

# Carnival Ride

A neighbor's child wants to go to a carnival to experience the wild rides. The neighbor is worried about safety because one of the rides looks particularly dangerous. She knows that you have taken physics and so asks you for advice. The ride in question has a 4 kg chair which hangs freely from a 10 m long chain attached to a pivot on the top of a tall tower. When the child enters the ride, the chain is hanging straight down. The child is then attached to the chair with a seat belt and shoulder harness. When the ride starts up, the chain rotates about the tower. Soon the chain reaches its maximum speed and remains rotating at that speed, which corresponds to one rotation about the tower every 3 seconds. When you ask the operator, he says the ride is perfectly safe. He demonstrates this by sitting in the stationary chair. The chain creaks but holds, and he weighs 90kg. Has the operator shown that the ride is safe for a 25 kg child?




For clarity, geometry is labeled on the left. A FBD is shown on the right.




As shown above, the moving ride swings out to an angle $\theta$. The ride rotates such that $a_x = a_c$ and $a_y = 0$.

Using Newton's $2^{nd}$ Law,

$$\Sigma F_x = T_x = ma_c \quad \text{where } a_c = \omega^2 R$$
$$\text{and } T_x = T\sin\theta$$

$$T\sin\theta = m\omega^2 R$$

Find $\omega$: $\omega = 2\pi f = 2\pi\left(\frac{1}{3}\right) = \frac{2\pi}{3}$ rad/s

Find R: $R = L\sin\theta$

$$T\cancel{\sin\theta} = m\left(\frac{2\pi}{3}\right)^2 L\cancel{\sin\theta}$$

$$T = mL\left(\frac{2\pi}{3}\right)^2 = (29\text{kg})(10\text{m})\left(\frac{2\pi}{3}\right)^2 = \boxed{1272.1\text{ N}}$$

The ride operator has shown you that $T = (90\text{kg} + 4\text{kg}) * g = \boxed{922.1\text{N}}$ is safe. Clearly, this is not sufficient proof.