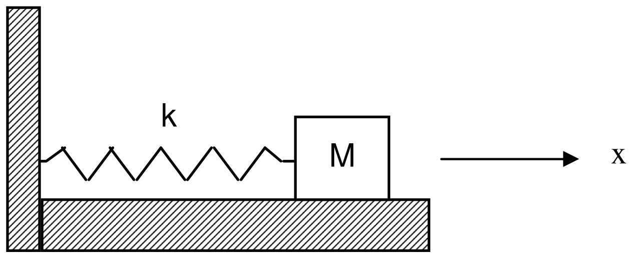
Equation of Motion

A mass M rests on a frictionless table and is connected to a spring of spring constant k. The other end of the spring is fixed to a vertical wall as shown in the figure. At time t = 0 s the mass is at x = 2.6 cm and moving to the right at a speed of 47 cm/s (the equilibrium position of the mass is at x = 0). It is at this position with this velocity next at t = 0.2 s. Find an expression for the position as a function of time and in so doing find the frequency and the amplitude of oscillation.



In this problem, we seek to use the given information to fill in the unknowns A, ω, and ϕ in the equation $x(t) = A\cos(\omega t + \phi)$

Finding ω: This is the easiest task. After 0.2 s, we know the mass has gone through one full cycle, thus $T = 0.2\,s$ ← Period, not tension

and $\omega = \frac{2\pi}{T} = \boxed{10\pi \text{ rad/s}}$

Finding A and ϕ: Now we set up a system of equations using the initial conditions

$$x(t) = A\cos(\omega t + \phi) \qquad v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$x(t=0) = A\cos\phi \qquad v(t=0) = -A\omega\sin\phi$$

$$\Downarrow \qquad \Downarrow$$

$$\boxed{2.6 = A\cos\phi} \qquad \boxed{+47 = -A\omega\sin\phi}$$

Plug in $\omega = 10\pi$ rad/s and solve to find $A = 3.0$ cm and $\phi = -0.52$ rad

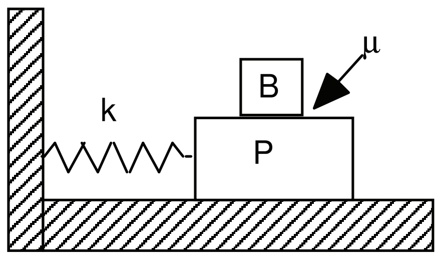
If you're not getting this answer, check that your calculator is in radian mode

So the equation for the position is

$$\boxed{x(t) = 3.0\cos(10\pi t - 0.52)}$$

Plate, Block, and Spring

A flat plate P of mass 5.0 kg is attached to a spring of spring constant k = 60 N/m and executes horizontal simple harmonic motion by sliding across a frictionless surface. A block B of mass 2.0 kg rests on the plate and the coefficient of static friction between the block and the plate is μ= 0.60. What is the maximum amplitude of oscillation that the plate-block system can have in order that the block not slip on the plate?



In order for the block not to slip, it must be true that

$f_s \geq m_b a$

Of course, the largest value for f_s is $f_{s,max} = \mu F_N = \mu m_B g$

and

$f_{s,max} = m_b a_{max} \rightarrow \mu \cancel{m_B} g = \cancel{m_B} a_{max} \rightarrow \boxed{a_{max} = \mu g}$

Now we use our equations of motion to find a_{max}:

$a(t) = -A\omega^2 \cos(\omega t + \phi)$

The maximum of $\cos(\omega t + \phi)$ is 1, so

$\boxed{a_{max} = A\omega^2}$

where $\omega = \sqrt{\frac{k}{M_{total}}} = \sqrt{\frac{k}{m_p + m_b}} = \sqrt{\frac{60}{7}}$

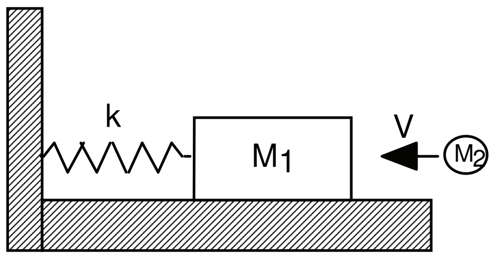
Plugging this in above, we have

$\mu g = A\omega^2 \rightarrow \mu g = A\left(\frac{k}{m_p + m_b}\right)$

Solve to find $\boxed{A = 0.686 \text{ m}}$

Block, Clay, and Spring

A block of mass $M_1 = 5$ kg is attached to a spring of spring constant k = 20 N/m and rests on a frictionless horizontal surface. A wad of clay of mass M_2=2 kg and traveling horizontally with speed V = 14 m/s hits and sticks to the block. Find the frequency and amplitude for the subsequent simple harmonic oscillations.



When the clay hits the block, momentum is conserved:

$$p_i = M_2 V = 28 \text{ kg m/s}$$

$$p_f = (M_1 + M_2) v_f$$

Set $p_i = p_f$ to find $\boxed{v_f = 4 \text{ m/s}}$

Now, the block and clay start to oscillate. Since the collision took place at x=0, we know that v_f from above is v_{max}!

$$V_{max} = A\omega = 4 \text{ m/s}$$

We find ω with our usual equation

$$\omega = \sqrt{\frac{k}{M_{total}}} = \sqrt{\frac{k}{m_1 + m_2}} = \boxed{1.69 \text{ rad/s}}$$

And plug it in to the V_{max} equation to find A

$$A = \frac{V_{max}}{\omega} = \boxed{2.35 \text{ m}}$$

Bird Feeder Torsion Pendulum

A bird feeder consists of a solid circular disk of mass M = 0.34 kg and radius R = 0.25 m suspended by a wire attached at the center. Two birds, each of mass m = 65 g land at opposite ends of a diameter, and the system goes into torsional oscillation with a frequency f = 2.6 Hz. What is the torsional constant of the wire?

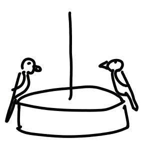
The frequency of a torsion pendulum is given by

$$\omega = \sqrt{\frac{\kappa}{I}}$$

We know $\omega = 2\pi f = 5.2\pi$

"Kappa"

So all we need is I_{total} to find κ



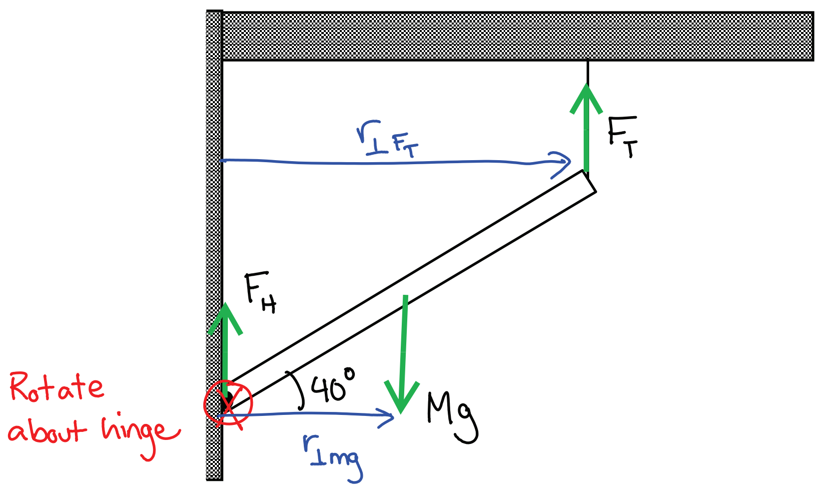
The bird feeder is a disk $(I_{disk} = \frac{1}{2}MR^2)$
With two point-mass birds sitting at its edge $(I_{birds} = 2mR^2)$
Thus,

$$I_{total} = I_{disk} + I_{birds} = 0.01875\ \text{kg m}^2$$

Finally, we find κ:

$$\kappa = \omega^2 I_{total} = \boxed{5.0\ \text{kg}\ \text{m}^2/\text{s}^2}$$

Falling Meter Stick



A uniform meter stick of mass 1.5kg is attached to the wall by a frictionless hinge at one end. On the opposite end it is supported by a vertical massless string such that the stick makes an angle of 40° with the horizontal.

1. Find the tension in the string and the magnitude and direction of the force exerted on the stick by the hinge.
2. Suppose the string is cut. Find the angular acceleration of the stick immediately thereafter.

1. The meter stick is in equilibrium, so

$\Sigma\tau = 0$:

$$\Sigma\tau = \tau_{Mg} + \tau_{F_T} = F_T(L\cos 40^\circ) - Mg\left(\tfrac{L}{2}\cos 40^\circ\right) = 0$$

From this equation, you can find $\boxed{F_T = 7.36\,N}$

$\Sigma F_y = 0$:

$$\Sigma F_y = F_T + F_H - Mg = 0$$

Plug in F_T from above to find $\boxed{F_H = +7.36\,N}$

2. When the rope is cut, the stick will rotate about the hinge

$\Sigma\tau = I\alpha$ where $\Sigma\tau = \tau_G = Mg\left(\tfrac{L}{2}\cos 40^\circ\right)$ and $I = I_{rod,end} = \tfrac{1}{3}ML^2$

Plug these in to find $\alpha = \dfrac{3g\cos 40^\circ}{2L} = \boxed{11.27\ \text{rad/s}^2}$

Not getting this answer? Make sure your calculator is back in degree mode