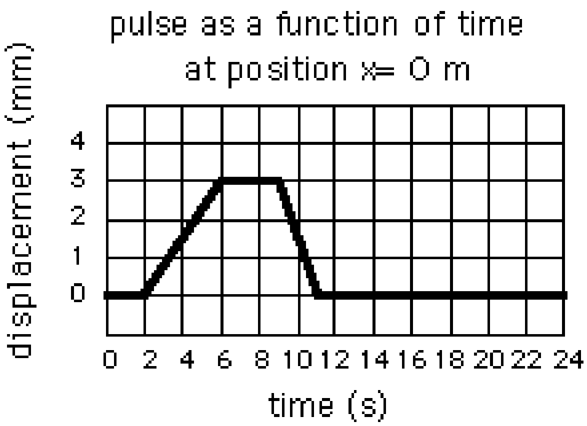
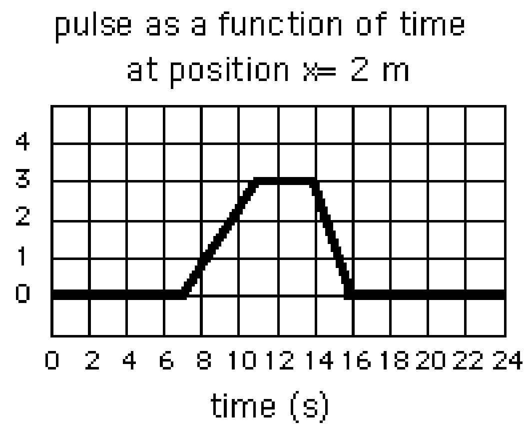
Physics 211 Week 13

Transverse Waves: Pulse on String Solutions

A pulse travels on a string. Its shape does not change. The displacement of the string as a function of time is shown in the figures below at two positions, x = 0 m and x = 2 m. These figures show that the string raises slowly to a height of 3 mm, remains at that height for some time, and lowers quickly back to the equilibrium position.

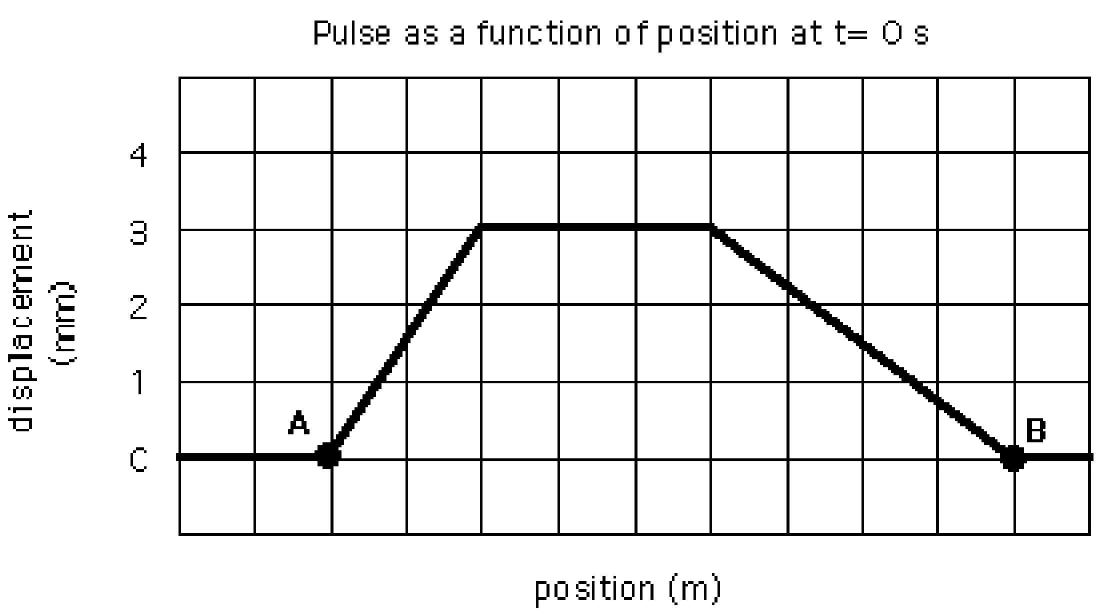




How fast is the pulse traveling along the string?

A strategy: Look at a spot on the wave such as the leading edge at 2 seconds in the plot for x = 0. This spot arrives at x= 2m in t = 7 seconds in the plot at the right. The velocity of this "spot" is thus $V = \Delta x / \Delta t = (2 - 0)/(7-2)= 0.4$ m/s

Another view of the pulse is shown in the figure below, the displacement of the string at t = 0 s. What is the distance between points A and B?



We note that when plotted as a displacement versus position plot the wave shape is appears reflected from its appearance when plotted as displacement versus time when the wave is traveling to the right as is the case here. This is easy to understand since if one were at a fixed position to the right of point B, one would first experience a gentle rise from 0 to 3 mm displacement followed by a more abrupt fall from 3 mm to 0 mm. This behavior is consistent with the two displacement versus time plots above.

If we concentrate on the displacement versus time plot at x=0 we see that point B passes x = 0 at 2 seconds while point A passes x=0 at 11 seconds. Hence the wave takes 11 - 2 = 9 seconds to pass through its entire length. Since we established that the wave is traveling with a propagation velocity of 0.4 m/s, points A and B must be separated by **9 s × 0.4 m/s = 3.6 m**.

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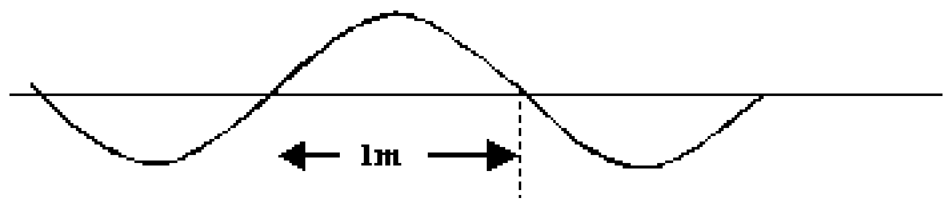
Transverse Waves: Power Train Solutions



A long wire under tension is carrying the traveling wave illustrated above. A one meter section of the wire has a mass of 1 gram. The oscillation frequency is f = 256 Hz and the average power passing a point is 10 Watts. Find the tension on the wire and the wave amplitude.

A strategy: You can determine the wavelength from the sketch. From the oscillation frequency and wavelength you can find the propogation velocity. The expression for the average power can then be used to solve for the amplitude and the expression for the velocity of propagation can be used to solve for the tension.

The 1 meter section corresponds to one half wavelength which means λ = 2m.

$V = \lambda\, f = (2m)\times(256Hz) = 512m/s$

The mass density is given by:

$\mu = 1gm/m = 0.001\,kg/m$

We can now solve for the amplitude using:

$$\overline{P} = \tfrac{1}{2}\mu V(a\omega)^2 \Rightarrow a = \sqrt{\frac{2\overline{P}}{\mu V\omega^2}} = \sqrt{\frac{2\overline{P}}{\mu V(2\pi f)^2}} = \sqrt{\frac{2(10W)}{(0.001kg/m)(512m/s)(2\pi\times 256s^{-1})^2}} = 0.00389m$$

And the tension using:

$$V = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu V^2 = (0.001kg/m)\times(512m/s)^2 = 262\ N$$

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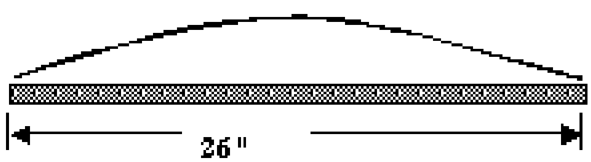
Transverse Waves: Building a guitar

You are designing a new lightweight electric guitar using hi-tech composite materials, and one of your first jobs is to calculate how much material needs to be in the neck and head-stock. This will depend on how strong you want it to be, which in turn is determined by the total tension in the strings since the neck has to support this without bending too much. You make the simplifying assumption that the tension in all six strings is about the same, and decide to calculate what the tension is in the high-E string since you happen to have a brand new one handy.

You read on the E string package that it's diameter is 0.009" and that it is made of steel, which your physics book tells you has a density of about 8 gr/cm^3. The distance between the bridge and the nut on your guitar is supposed to be 26", and the vibration frequency of an open high-E string should be 330 Hz. How much tension must the neck support?

A strategy: Since the string is vibrating in its fundamental, the distance of 26" between the bridge and adjustment nut represents one half wavelength. Knowing the wavelength and the frequency of high-E allows us to calculate the velocity of wave propagation that is related to the string tension and mass density. We multiply the string tension by six to get the total tension that the neck must support. The volume mass density times the cross sectional area of a string gives the linear mass density

Let's begin by calculating the velocity of propagation.



Looking at the standing wave in its fundamental vibration its easy to see that $\lambda/2 = 26"$ or $\lambda = 2 \times 26" \times (0.0254)$ m/inch = 1.32 m

The velocity is then $V = \lambda f = (1.32\text{ m})\ 330\text{ Hz} = 435.9$ m/s

We next calculate the linear mass density μ. An easy way to do this is to compute the mass of 1 cm of the wire and divide by the 1 cm to compute the mass/cm.

The volume is $\pi r^2 \times$ length $= \pi\,(0.5 * 0.009 * 2.54)^2 \times 1 = 4.104 \times 10^{-4}$ cm^3 . The mass is then mass= 4.104×10^{-4} cm$^3 \times (8$ gm/ cm$^3) = 0.003$ gm $\Rightarrow \mu = 0.003283$ gm/cm =
0.003 gm/cm $\times$ (100 cm/m) $\times$ (0.001 kg/gm) = 3.283×10^{-4} kg/m

We next compute the tension in one string :

$$\sqrt{\frac{F}{\mu}} \Rightarrow F = \mu V^2 = (3.283\times10^{-4} kg/m)\times(435.9 m/s)^2 = 62.38\,N$$

Taking into all 6 strings we have $F_{TOTAL} = 6 \times 62.38$ N = 374.3 N

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Oscillations: Not So Simple Pendulum: Hoop

A thin uniform hoop of mass M and radius R is hung on a thin horizontal rod and set oscillating with small amplitude. Find the period of oscillation of the hoop.

To picture this set-up, imagine taking a ring and putting your pencil through. It oscillates around the edge of the hoop, which requires use of the parallel axis theorem to find the moment of inertia of the hoop. Then use the equation for the period of a physical pendulum with the expression you found for the moment of inertia of the hoop about the new axis. You should obtain a period of

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2R}{g}}$$