Physics 211 **Week 13**

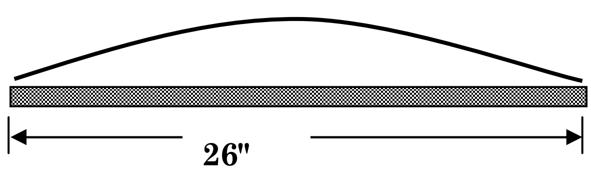
Transverse Waves: Building a guitar

You are designing a new lightweight electric guitar using hi-tech composite materials, and one of your first jobs is to calculate how much material needs to be in the neck and head-stock. This will depend on how strong you want it to be, which in turn is determined by the total tension in the strings since the neck has to support this without bending too much. You make the simplifying assumption that the tension in all six strings is about the same, and decide to calculate what the tension is in the high-E string since you happen to have a brand new one handy.

You read on the E string package that it's diameter is 0.009" and that it is made of steel, which your physics book tells you has a density of about 8 gr/cm^3. The distance between the bridge and the nut on your guitar is supposed to be 26", and the vibration frequency of an open high-E string should be 330 Hz. How much tension must the neck support?

A strategy: Since the string is vibrating in its fundamental, the distance of 26" between the bridge and adjustment nut represents one half wavelength. Knowing the wavelength and the frequency of high-E allows us to calculate the velocity of wave propagation that is related to the string tension and mass density. We multiply the string tension by six to get the total tension that the neck must support. The volume mass density times the cross sectional area of a string gives the linear mass density

Let's begin by calculating the velocity of propagation.
Looking at the standing wave in its fundamental vibration its easy to see that $\lambda/2$ = 26" or $\lambda = 2 \times 26" \times (0.0254)$ m/inch = 1.32 m



The velocity is then $V = \lambda f = (1.32 \text{ m})\ 330 \text{ Hz} = 435.9$ m/s

We next calculate the linear mass density μ. An easy way to do this is to compute the mass of 1 cm of the wire and divide by the 1 cm to compute the mass/cm.

The volume is $\pi r^2 \times \text{length} = \pi\,(0.5 * 0.009 * 2.54)^2 \times 1 = 4.104 \times 10^{-4}$ cm^3 . The mass is then mass= 4.104×10^{-4} cm$^3 \times (8$ gm/ cm$^3) = 0.003$ gm $\Rightarrow \mu = 0.003283$ gm/cm = 0.003 gm/cm $\times$ (100 cm/m) $\times$ (0.001 kg/gm) = 3.283×10^{-4} kg/m

We next compute the tension in one string :

$$\sqrt{\frac{F}{\mu}} \Rightarrow F = \mu V^2 = (3.283 \times 10^{-4} kg/m) \times (435.9 m/s)^2 = 62.38\,N$$

Taking into all 6 strings we have $F_{TOTAL} = 6 \times 62.38$ N = 374.3 N