Two-Dimensional Kinematics: Heading North (Solutions)

You are the navigator of a TWA flight scheduled to fly from New Orleans due north to St. Louis, a distance of 673 miles. Your instruments tell you that there is a steady wind from the northwest with a speed of 155 mph. The pilot sets the air speed at 510 mph and asks you to find the estimated flying time. What do you tell her?

Conceptual Analysis:

- The velocity of the plane with respect to the air is not the same as the velocity of the plane with respect to the ground.
- The wind from the northwest can be assumed to have equal easterly and southerly components.
- To avoid being blown off course, the pilot will need to head the plane in the westerly direction so that her resultant path with respect to the ground will be due north as opposed to being pushed off course to the east by the wind.

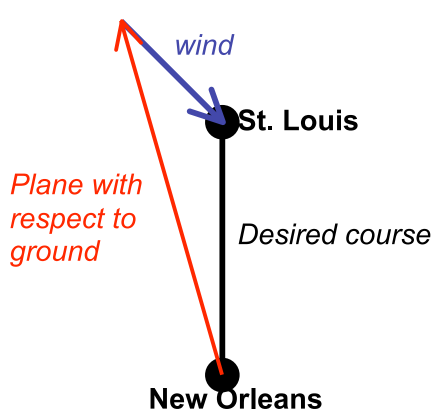
Strategic Analysis:

- Find the easterly component of the wind to determine the westerly component of the pilot's path.
- Use the given air speed and westerly component to determine the northerly component of the plane's air speed.
- Find the southerly component of the wind to determine the northerly component of the pilot's speed with respect to the ground.
- Use the ground speed and distance traveled to estimate the flying time.

Quantitative Analysis:

- Begin by labeling the given quantities:
  - d distance from New Orleans to St. Louis
  - v_w speed of the wind
  - v_a air speed of the plane

  We are looking for
  - t the estimated flying time
- Let's draw a picture to visualize the problem.



- The wind can be broken down into x and y components.

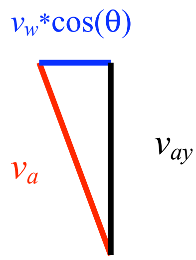
  $v_{wx} = v_w*\cos(\theta)$ (westerly component of the wind)

  $v_{wy} = v_w*\sin(\theta)$ (southerly component of the wind)

- The air speed of the plane is made up of a westerly component and northerly component. Since the westerly component of the plane's speed must equal the easterly component of the wind, we can set the two equal to each other.

$$v_{wx} = v_{ax} = v_w * \cos(\theta)$$

- Next, we can use vector addition to determine the northerly component of the speed of the plane in the air.



$$v_a^2 - v_w^2 * \cos^2(\theta) = v_{ay}^2$$

$$v_{ay} = \text{sqrt}[v_a^2 - v_w^2 * \cos^2(\theta)]$$

- Knowing v_{ay} and using the southerly component of the wind, we can find the ground speed of the plane.

$$v_{ay} - v_w * \sin(\theta) = v_g$$

- Now we can use the ground speed and the total distance to find the time.

$$d / v_g = t$$

- Inserting the values and solving:

$$t = d / (\text{sqrt}[v_a^2 - v_w^2 * \cos^2(\theta)] - v_w \sin\theta)$$

$$t = 673 \; miles / (\text{sqrt}[(510 \; mph)^2 - (155 \; mph)^2 * \cos^2(45^o)] - (155 \; mph * \sin 45^o))$$

$$t = 1.73 \; hr$$

- The estimated flying time is 1.73 hours.

Phys 211 Discussion Problem from session 2
Two-Dimensional Kinematics: Against the Grain

You are on the west bank of a river which flows due south and want to swim to the east bank. You have told your friends to meet you on the east bank directly opposite your starting point. Before starting out, you realize that, since the river is flowing swiftly at a speed of 12 ft/s and since your fastest swimming speed in still water is only 5 ft/s, you will inevitably be carried downstream. Nevertheless, you want to minimize the effort expended by your friends in walking downstream to meet you. Your guide book to the region tells you that the width of the river is 300 ft. After a quick calculation, you call your friends on your cellular phone and tell them to start walking to a new meeting point. How far downstream of the original meeting point should you tell them to walk?

======

Conceptual Analysis:

- If you swim straight across you will get to the opposite bank the most quickly (as discussed in Lecture) but not necessarily the least distance downsteam.
- Your velocity with respect to the water (at speed 5 f/sec) is not the same as your velocity with respect to the ground.
- You need to choose the optimal direction to swim in relative to the water, to minimize the distance D down stream

Strategic Analysis:

- Posit an angle θ to head in. And seek D as a function of θ.
- Find the component of your net velocity perpendicular to the bank, and thus the time to cross, in terms of θ.
- Use the time to cross, and the component of your net velocity dow-stream to find where you land.
- Minimize this as a function of θ.

Quantitative Analysis:

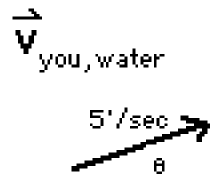
- Begin by labeling a few key quantities:

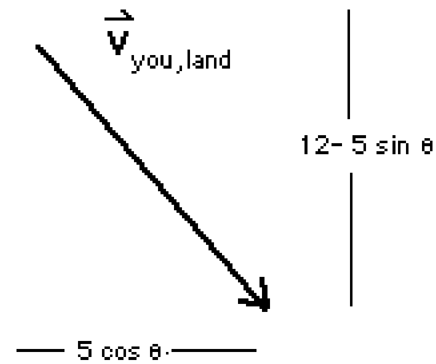
  D distance downstream that you land
  d distance across (-= 300 feet)
  θ your heading relative to the water (0 degrees being straight across)
  v_x your speed perpendicular to the banks
  v_y your speed downstream relative to the banks
  t the time to cross

Fundamental relation for relative motion

$$\vec{V}_{you,land} = \vec{V}_{you,water} + \vec{V}_{water,land}$$

The first term has two components: 5feet/sec times cosθ across, and 5 ft/sec times sin θ in the upstream direction. The last term is just 12 feet/sec downsteam. The net velocity is $v_x = 5\cos\theta$ across and $v_y = 12 - 5\sin\theta$ downstream





We conclude that the time to get across is 300 feet/ v_x, or

$t = d / v_x = 300 \text{ ft} / 5(\text{ft/sec}) \cos\theta$.

We also conclude that the distance D we go downstream in that time is

$D = t\, v_y = (300 \text{ ft} / 5(\text{ft/sec}) \cos\theta)\,(12\text{ft/sec}-5(\text{ft/sec})\sin\theta)$

which must be minimized as a function of the heading θ.

That can be done by taking the derivative analytically and setting it to zero. The answer is $\theta = \arcsin(5/12) = 24.6°$. This may be substituted into the above $D(\theta)$ to find the minimum D.

Alternatively, D can be minimized by plotting $D(\theta)$ and picking out the minimum that way.

The answer is 655 feet. The time to cross was 66 seconds.

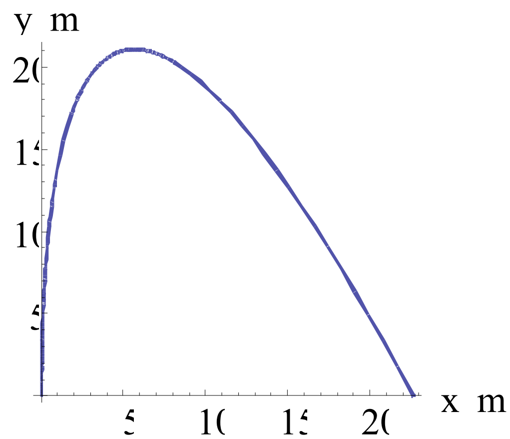
(This can be compared to where we'd land if we went straight across using $\theta = 0$) This minimizes the time to cross, giving t = 60 seconds. But we land 720 = 12*60 feet downstream.)

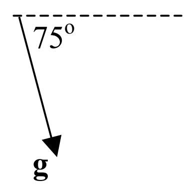
Physics 211 - Week 2
Two-Dimensional Kinematics: Which Way Is Up? (Solutions)

Tilted Land is flat, much like central Illinois, but in Tilted Land gravity does not point straight down! It points down at an angle of 75 degrees with respect to the ground. (In central Illinois gravity points straight down at an angle of 90 degrees with respect to the ground.) This circumstance causes major changes in everyday life, but people manage. Suppose that a person in Tilted Land throws a ball straight up at 20 m/s. Where does the ball hit the ground?

Conceptual Analysis:

- This problem is different from our usual projectile problems.
- There are components of acceleration due to gravity in both the x and y directions.
- The motion of the projectile can be described by a tilted parabola as shown below.





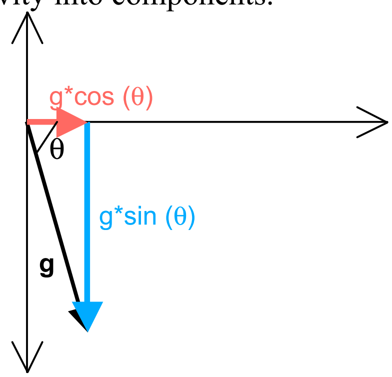
Strategic Analysis:

- This problem can be solved in the same manner as a normal projectile problem you just need to account for the acceleration in the x-direction.
- Find the components of acceleration in the x and y directions.
- Find the time in the air from the information about the y-direction.
- Use the total time in the air to determine the distance traveled in the x-direction.

Quantitative Analysis:

- We can begin by assigning labels to the given quantities:
  - θ angle gravity points down at with respect to the ground
  - v_o initial velocity of the ball
  - g acceleration due to gravity

  We are looking for
  - x the horizontal distance traveled by the ball
- Next we can find the acceleration in the x and y directions by breaking the tilted gravity into components.



y: $a_y = g*\sin(\theta)$

x: $a_x = g*\cos(\theta)$

- Find the time in the air using

$$y = y_o + v_o*t + \_*a*t^2$$

the given values, and your knowledge that the ball will begin and end on the ground (position y = 0):

$$0 = 0 + v_o*t + \_*g*\sin(\theta)*t^2$$

$$t = 2v_o / [g*\sin(\theta)]$$

(You could also find the amount of time it takes for the velocity to reach zero in the y-direction, the peak of the parabola. Even for a tilted parabola, the time to reach the peak is half the total time in the air.)

- Next, use the time in the air to find the distance the ball travels in the x-direction using

$$x = x_o + v_o*t + \_*a*t^2$$

the given values, and your knowledge that the initial velocity in the x direction is zero:

$$x = \_*g*\cos(\theta)*(t)^2$$

inserting your value for the time:

$$x = \_*g*\cos(\theta)*(2v_o / [g*\sin(\theta)])^2$$

$$x = \_*(9.81\ m/s^2)*\cos(75^o)*(2*(20\ m/s) / [(9.81\ m/s^2)*\sin(75^o)])^2$$

$$x = 22.6\ m$$

The ball will hit the ground 22.6m from where it was thrown in the direction that the x-component of gravity in Tilted Land points.

Physics 211 - Week 2
Two-Dimensional Kinematics: 2001

A Hollywood producer has decided to film a remake of the movie 2001: A Space Odyssey. You have been hired as a consultant for the movie to make sure the science is correct. The producer wanted to have an Illinois physics student for the job since HAL was invented in Urbana (January 12, 1997!). Part of the movie takes place on a space station very far from any gravitating body. This station is a large wheel-like structure where people live and work on the rim. In order to create "artificial gravity", the space station rotates about its axis.

One space station design is a large wheel-like structure where people live and work on the rim. In order to create "artificial gravity", the space station rotates about its axis. It is desired that the gravity be equal to 0.85 times that of the Earth. Since centripetal acceleration depends on the distance to the axis of rotation, it is not possible for the “artificial gravity” to be the same at the head and the feet of a standing person (say ~1.8 m). In order to minimize any possible discomfort, suppose that the difference in the “artificial gravity can only be 1%. The special effects department wants to know the diameter and the rate of rotation of a space station that meets these specifications.

The artificial gravity will be provided by the centripetal acceleration; we need to find both the diameter and the rate of rotation for the station. Since we have two unknown quantities, we should try to create two equations to describe the motion of the station. We know that the values of the artificial gravity at the head and the feet of a person should be within 1% of each other, so it is a good idea to set up one equation for each position. Using the given height of the standing person, you should be able to develop a set of equations comparing the value of the artificial gravity to the centripetal acceleration for two positions- the head and the feet. Solving them, you should obtain a diameter of 360m and a rotation rate of 0.034 revolutions per second.