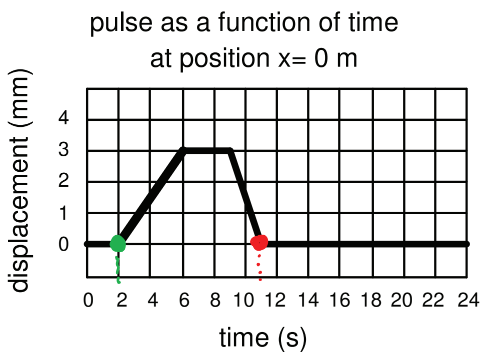
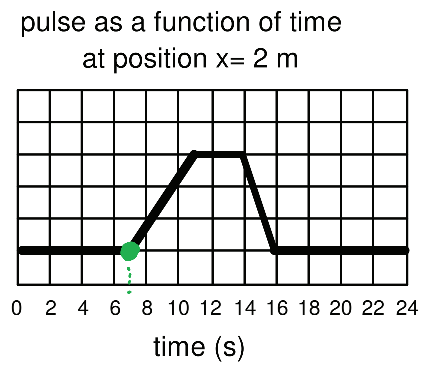
Pulse on String

A pulse travels on a string. Its shape does not change. The displacement of the string as a function of time is shown in the figures below at two positions, x = 0 m and x = 2 m. These figures show that the string rises slowly to a height of 3 mm, remains at that height for some time, and lowers quickly back to the equilibrium position.





How fast is the pulse traveling along the string?

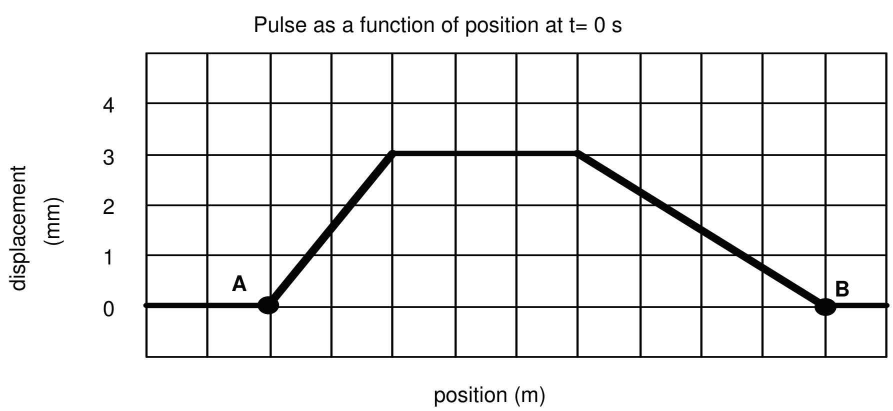
We can use the graphs above to find the amount of time it takes for one part of the pulse to travel $\Delta x = 2m$. The front of the pulse is marked in green above. From this, we see $t_{x=0} = 2s$ and $t_{x=2} = 7s$

Thus, $v = \frac{\Delta x}{\Delta t} = \frac{2m}{5s} = \boxed{0.4 \, m/s}$

Another view of the pulse is shown in the figure below, the displacement of the string at t = 0 s. What is the distance between points A and B?

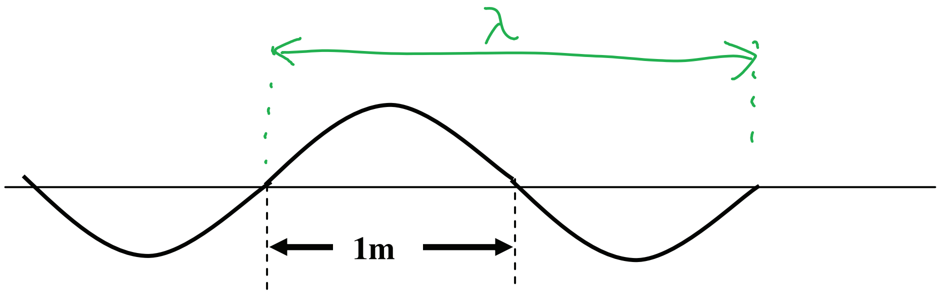
Once again using the plot for x = 0 above, we find the time it takes for the whole pulse (front to back, marked with green and red, respectively) to pass x = 0 is $\Delta t = 11s - 2s = 9\,sec$

Now use $\Delta x = v \Delta t$ to find $\Delta x = (0.4 \, m/s)(9s) = \boxed{3.6m}$



It may be useful to realize that this latter picture is a snapshot of the string, and the first frame of a movie in which that shape is moving steadily to the right.

Power Train



A long wire under tension is carrying the traveling wave illustrated above. A one meter section of the wire has a mass of 1 gram. The oscillation frequency is f = 256 Hz and the average power passing a point is 10 Watts. Find the tension on the wire and the wave amplitude.

First, let's find the tension.

On a string,

$$v = \sqrt{\frac{T}{\mu}} \qquad \text{So } T = \mu v^2$$

From the problem description, we know $\mu = 1\,g/m = 0.001\,kg/m$

And we find velocity using $v = \lambda f = (2)(256) = 512\,m/s$

Plug these in to find $\boxed{T = 262\,N}$

Next, we use average power to find the amplitude, A.

Average power is given by the expression $\tilde{P} = \frac{1}{2}\mu(\underbrace{A\omega}_{\text{speed of wire displacement}})^2 v$

We know μ, v, and $\tilde{P}$, and $= 2\pi f$.

Plug these in to find

$\boxed{A = 3.89\,mm}$

For more details on the expression for average power, visit hyperphysics.phy-astr.gsu.edu/hbase/waves/powstr.html

[redacted]

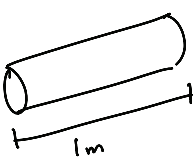
Building a guitar

You are designing a new lightweight electric guitar using hi-tech composite materials, and one of your first jobs is to calculate how much material needs to be in the neck and head-stock. This will depend on how strong you want it to be, which in turn is determined by the total tension in the strings since the neck has to support this without bending too much. You make the simplifying assumption that the tension in all six strings is about the same, and decide to calculate what the tension is in the high-E string since you happen to have a brand new one handy.

You read on the E string package that its diameter is 0.009" and that it is made of steel, which your physics book tells you has a density of about 8 g/cm³. The distance between the bridge and the nut on your guitar is supposed to be 26", and the vibration frequency of an open high-E string should be 330 Hz. How much tension must the neck support?

Let's start by converting all these mismatched units into something we can use.

• Finding μ:



μ is mass per unit length, so $\mu = \rho A L$

$d = 0.009'' = (0.009)(2.54)\,\text{cm} = 0.2286\,\text{cm}$

$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = 4.104 \times 10^{-4}\,\text{cm}^2$

$L = 1\,\text{m} = 100\,\text{cm}$

$\mu = (8\,\text{g/cm}^3)(4.104\times10^{-4}\,\text{cm}^2)(100\,\text{cm}) = 0.328\,\text{g/m}$

We need this in kg/m, so we divide by 1000:

$\mu = 3.28 \times 10^{-4}\,\text{kg/m}$

Now, we use $v = \sqrt{\frac{T}{\mu}}$ to find T.

Find v using $v = \lambda f$, where $f = 330\,\text{Hz}$ and $\lambda = 2L = 52'' = 1.32\,\text{m}$

$v = 435.9\,\text{m/s}$

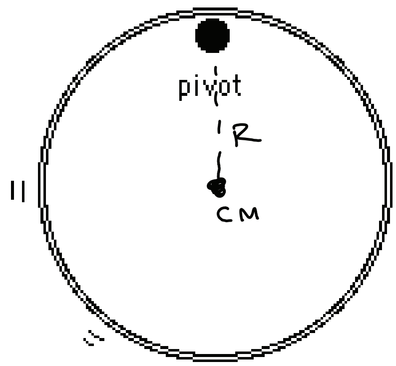
Then $T = \mu v^2 = 62.38\,\text{N}$

Since the guitar has six strings, its neck must be able to support

$6T = \boxed{374.3\,\text{N}}$

Review: Not So Simple Pendulum: Hoop

A thin uniform hoop of mass M and radius R is hung on a thin horizontal rod that acts like a pivot. It is set oscillating with small amplitude. Find the period of oscillation of the hoop, in terms of R and M and g. The hoop does not slip on the rod.



For a physical pendulum, we know

$$\omega = \sqrt{\frac{mgR_{CM}}{I}}$$

distance from pivot to cm

We also know $\omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega}$

this is period, not tension

First, we use the parallel axis theorem to find the hoop's moment of inertia about the pivot:

$$I = I_{CM} + MR^2$$
$$= MR^2 + MR^2 = 2MR^2$$

Now we can find the frequency:

$$\omega = \sqrt{\frac{MgR}{2MR^2}} = \sqrt{\frac{g}{2R}}$$

Finally, we find the period

$$T = \frac{2\pi}{\omega} = \boxed{2\pi \frac{2R}{g}}$$