Space Station

A space station is constructed in the shape of a wheel 22 m in diameter, with essentially all its weight (5.0 x 10^5 kg) at the rim. Once the space station is completed, it is set rotating at a rate such that an object at the rim experiences a radial acceleration equal to the Earth's gravitational acceleration g, thus simulating Earth's gravity. To accomplish this, two small rockets are attached on opposite sides of the rim, each able to provide a 100 N force. How long will it take to reach the desired rotation rate and how many revolutions will the space station make in this time?

In this problem,

1. Two rockets cause a (hoop) space station to have an <u>angular</u> acceleration.
2. The space station's final rotation speed gives a radial (centripetal) acceleration equal to g.
3. The angular acceleration occurs over some time and distance.

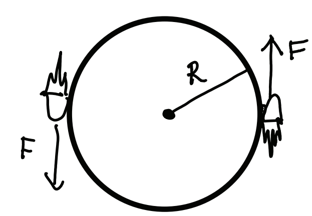
① Each rocket exerts a force F=100N at R=11m

$$\tau_{rocket} = \vec{F}_{rocket} \times \vec{r} = FR$$

This torque causes a hoop ($I = MR^2$) to accelerate:

$$\sum \tau = 2\tau_{rocket} = I\alpha$$

solve to find $\boxed{\alpha = \frac{2F}{MR}}$



② We are told $a_c = g$ and we know $a_c = \omega^2 R$

$$\omega_f^2 R = g \rightarrow \boxed{\omega_f = \sqrt{\frac{g}{R}}}$$

③ Now we know ω_0, ω_f, and α. Use kinematics to find t and θ:

Finding t:

$$\omega_f = \cancel{\omega_0}^{\,0} + \alpha t$$

$$t = \frac{\omega_f}{\alpha} = \frac{M}{2F}\sqrt{gR}$$

$$\boxed{t = 2.6 \times 10^4 \text{ s}}$$

Finding θ:

$$\omega_f^2 = \cancel{\omega_0}^{2\,0} + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{\omega_f^2}{2\alpha} = \frac{Mg}{4F}$$

We are looking for revolutions, where 1 rev = 2π rad

$$\frac{\Delta\theta}{2\pi} = \frac{Mg}{8\pi F} = \boxed{1952 \text{ revs}}$$

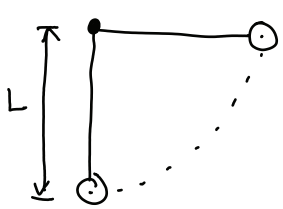
Rotating Tip

In the Physics 211 Laboratory one group of students has decided to pursue their own experiments. They make a simple pendulum from a weight attached to a string of length L. They attach the other end of the string to a fixed support. They hold the weight with the string taut and horizontal and then released it. With their motion sensor they measure the speed of the weight as the string passes through the vertical. Remembering that all objects fall with the same acceleration, the students do a second experiment. They attach one end of a uniform stick of length L to the support, which acts as a pivot. They hold the stick horizontal and release it. They then measure the speed of the tip of the stick with their motion sensor. The mass of the pendulum weight and the mass of the stick are the same. Do they measure the same speed?

When we look at pendula that aren't point masses, we have to look at rotational quantities:

- $K_{rot.} = \frac{1}{2} I \omega^2$ where I is the moment of inertia about the axis of rotation
- $U_G = mgy_{CM}$ Gravitational potential is determined by the location of the object's CM

Weight on String



$I_{\text{point mass}} = ML^2$

$E_i = U_i + \cancel{K_i}^{\,0} = MgL$

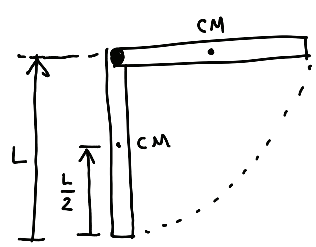
$E_f = \cancel{U_f}^{\,0} + K_f = \frac{1}{2} I \omega^2 = \frac{1}{2} M v^2$

Setting $E_i = E_f$,

$MgL = \frac{1}{2} M v^2$

$\boxed{v = \sqrt{2gL}}$

Stick



$I_{\text{rod,end}} = \frac{1}{3} ML^2$

$E_i = \cancel{K_i}^{\,0} + U_i = MgL$

$E_f = K_f + U_f = \frac{1}{2} I \omega^2 + Mg\left(\frac{L}{2}\right)$

Setting $E_i = E_f$,

$MgL = \frac{1}{2} I \omega^2 + Mg\left(\frac{L}{2}\right)$

$\omega = \frac{v}{r}$

$Mg\left(\frac{L}{2}\right) = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{1}{3} ML^2\right)\left(\frac{v}{L}\right)^2$

Solve the above equation to find

$\boxed{v = \sqrt{3gL}}$

The stick goes faster.

Review: Skate-Board Exhibition

You are helping your friend prepare for his skate-board clowning stunt. For his program, he plans to take a running start, and then jump onto a gigantic 7 kg stationary skateboard. He and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. He has measured his maximum running speed to jump safely on the skateboard at 6 m/s, and he wants to know how high above ground level he will make as he rolls up the slope. He tells you his weight is 70 kg.

When your friend jumps on the skateboard, momentum is conserved:

$$\vec{p}_i = m_f v_1$$

$$\vec{p}_f = (m_f + m_b) v_2$$

Set $\vec{p}_i = \vec{p}_f$ to find $v_2 = \frac{m_f v_1}{m_f + m_b} = 5.45\,\text{m/s}$

Then they go up a ramp. Energy is conserved:

$$E_i = K_i + \cancel{U_i}^{\,0} = \frac{1}{2}(m_f + m_b) v_2^2$$

$$E_f = \cancel{K_f}^{\,0} + U_f = (m_f + m_b) g h$$

Set $E_i = E_f$

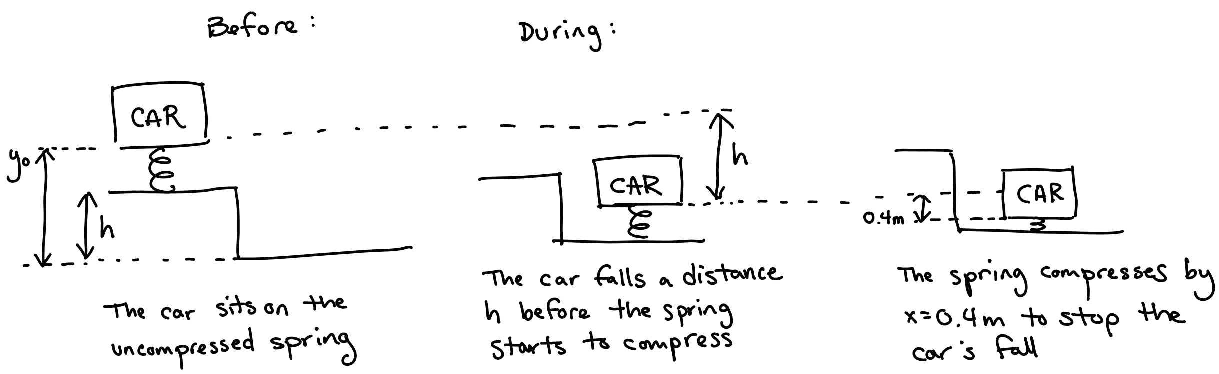
$$\frac{1}{2}\cancel{(m_f + m_b)} v_2^2 = \cancel{(m_f + m_b)} g h$$

$$h = \frac{v^2}{2g} = \boxed{1.52\,\text{m}}$$

Review: Construction Zone

A car is traveling along a horizontal road in a construction zone when it suddenly encounters an area where the road is being resurfaced, in which the level of the road is abruptly lowered by a height h. The suspension of the car can be considered as a single spring having a spring constant of 110,000 N/m that can compress a maximum distance of 0.4m. The mass of the car is 1200 kg. What is the maximum value of h that the car can tolerate before bottoming out (i.e. when the springs reach their maximum compression)? For simplicity, you may assume that the spring is not compressed initially.

We'll start with a drawing, as this is tough to visualize:



The car will fall a total distance of $H = h + 0.4$

To keep things simple, we'll set $y=0$ where the car bottoms out

$E_i = mgH \qquad E_f = \frac{1}{2}kx^2$

$$mgH = \frac{1}{2}kx^2$$

$$mg(h+x) = \frac{1}{2}kx^2$$

Solve for h to find

$$\boxed{h = 0.35\,m}$$

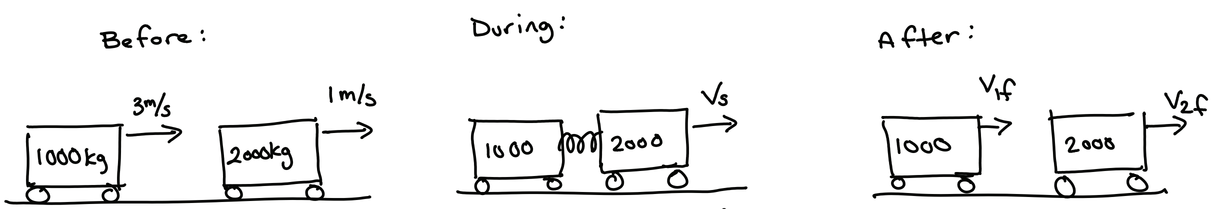
Review: elastic collisions: Train Cars

A 1000 kg car is moving rightwards at a speed 3 m/s towards another car, of mass 2000 kg, also moving rightwards, but at 1 m/s. They collide elastically through a spring of stiffness 10,000 N/m.

1. What is the final speed of the second car?

2. How much does the spring get compressed?

Let's begin with three snapshots of the collision:



Spring is compressed. At this moment, both cars move together at the shared speed V_s

1. The overall collision is elastic. Thus, $\vec{P}_{total}$ and K_{total} are conserved.

CM method:

$$v_{cm} = \frac{\vec{P}_{1i} + \vec{P}_{2i}}{m_1 + m_2} = \frac{5}{3}\ m/s$$

$$V_{2i}^{*} = V_{2i} - V_{CM} = -\frac{2}{3}\ m/s$$

$$V_{2f}^{*} = -V_{2i}^{*} = +\frac{2}{3}\ m/s$$

$$V_{2f} = V_{2f}^{*} + V_{CM} = \boxed{\frac{7}{3}\ m/s}$$

Staying in the lab frame:

$$\vec{P}_i = m_1 V_{1i} + m_2 V_{2i} = 5000\ kg\,m/s \qquad \vec{P}_f = m_1 V_{1f} + m_2 V_{2f}$$

$$\vec{P}_i = \vec{P}_f \rightarrow \boxed{m_1 V_{1f} + m_2 V_{2f} = 5000\ kg\,m/s}$$

$$K_i = \tfrac{1}{2} m_1 V_{1i}^2 + \tfrac{1}{2} m_2 V_{2i}^2 = 5500\,J \qquad K_f = \tfrac{1}{2} m_1 V_{1f}^2 + \tfrac{1}{2} m_2 V_{2f}^2$$

$$K_i = K_f \rightarrow \boxed{\tfrac{1}{2} m_1 V_{1f}^2 + \tfrac{1}{2} m_2 V_{2f}^2 = 5500\,J}$$

Solve the system of equations to find $V_{2f} = \frac{7}{3}\ m/s$

2. With the spring compressed, both cars move with speed

$$V_s = V_{CM} = \frac{5}{3}\ m/s$$

At this instant, the spring is storing $|\Delta K|$ energy:

$$|\Delta K| = \tfrac{1}{2} k x^2$$

Find ΔK: $\Delta K = K_f - K_i = \frac{1}{2}(m_1 + m_2) V_{CM}^2 - 5500 = -1333.33\ J$

Solve $\frac{1}{2} k x^2 = 1333.33$

to find $\boxed{x = 0.52\,m}$