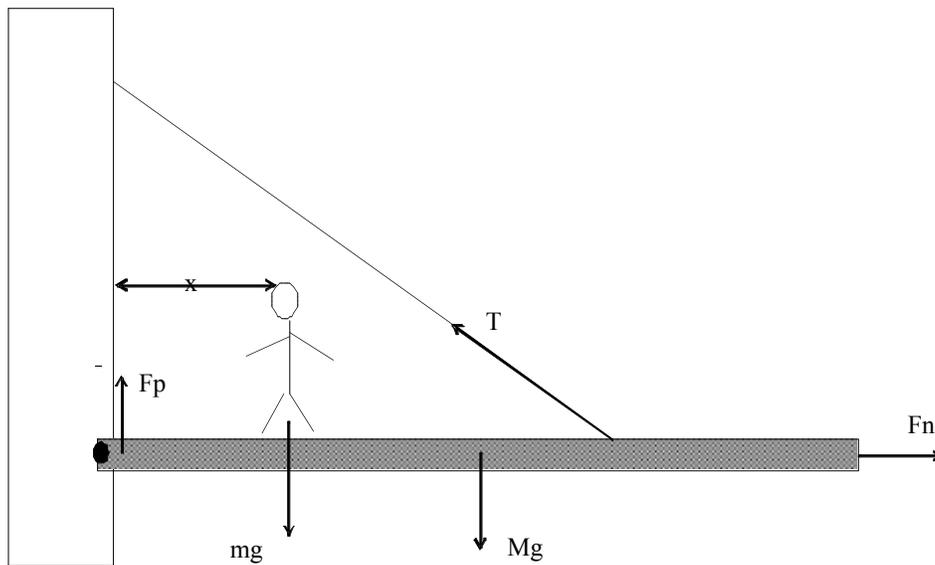


## Statics: Walking the Plank (Solution)

A uniform horizontal beam 8 m long is attached by a frictionless pivot to a wall. A cable making an angle of  $37^\circ$ , attached to the beam 5 m from the pivot point, supports the beam, which has a mass of 600 kg. The breaking point of the cable is 8000 N. A man of mass 95 kg walks out along the beam. How far can he walk before the cable breaks?

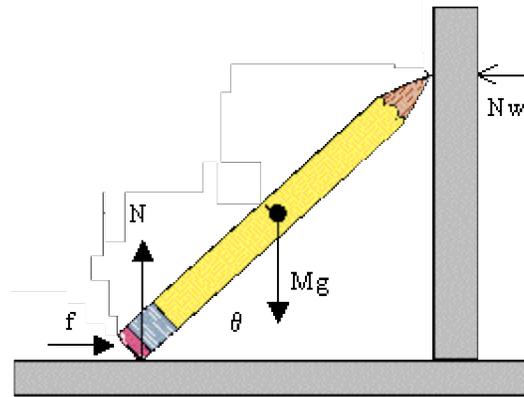
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Before the cable breaks, all of the torques and forces are in equilibrium. Draw a free body diagram and use it to write net force and net torque equations. Measuring torques about the given pivot will eliminate the unknown force from the pivot. You'll be left with torques from the weight of the plank, weight of the man, and the tension. Using the maximum value of the tension as the measured value of the tensional force, you can find the maximum distance that the man can walk and still have his torque balanced so that the cable does not break. You will find that he can walk  $0.57\text{ m}$ .

## Rotational Statics and Dynamics: Leaning Pencil

A picture below shows a pencil with its sharpened end resting against a smooth vertical surface and its eraser end resting on the floor. The center of mass of the pencil is 9 cm from the end of the eraser and 11 cm from the tip of the lead. The coefficient of static friction between the eraser and floor is  $\mu = 0.80$ . What is the minimum angle  $\theta$  the pencil can make with the floor such that it does not slip?



Conceptual Analysis:

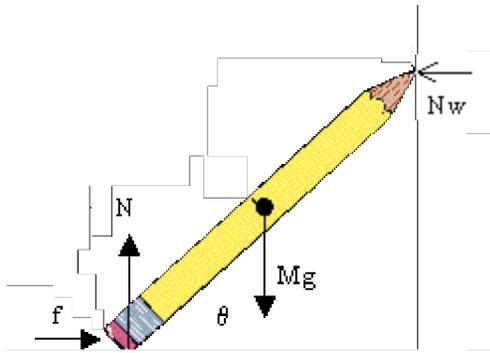
- The net torque and net force on the pencil are both zero.
- There are four forces acting on the pencil: the normal force from the floor, the normal force from the wall, the force of gravity, and the force of friction.
- The pencil wants to slide down the wall and along the floor; therefore, the force of friction along the floor must point towards the wall to balance the normal force from the wall.

Strategic Analysis:

- Create net force and torque equations.
- Solve the equations for the angle.

Quantitative Analysis:

- Begin by labeling the given quantities:
  - $\mu$  coefficient of static friction between the floor and the eraser
  - $d_1$  distance from the eraser to the center of mass
  - $d_2$  distance from the center of mass to the tip of the lead
- We are looking for
  - $\theta$  minimum angle between the pencil and the floor



- In the x-direction, there is zero net force. The two forces in the x-direction are the force of friction and the normal force from the wall, so we know that:

$$F_x: f = N_w$$

- In the y-direction, there is also zero net force. The two forces in the y-direction are the force of gravity and the normal force from the floor, so we know that:

$$F_y: N = Mg$$

- Putting the pivot at the bottom of the pencil eliminates 2 of the 4 forces. The only two forces creating torque about this pivot are the force of gravity and normal force from the floor. The two torques must be equal for the pencil to stay in place so we know that

$$N_w * (d_1 + d_2) * \sin\theta = Mg * d_1 * \cos\theta$$

- Using substitution with the net force equations to eliminate the normal force in the torque equation, we have

$$N_w = f = \mu N = \mu Mg$$

- So, solving:

$$\mu Mg * (d_1 + d_2) * \sin\theta = Mg * d_1 * \cos\theta$$

$$\mu * (d_1 + d_2) * \sin\theta = d_1 * \cos\theta$$

$$\theta = \tan^{-1}\left(\frac{d_1}{\mu(d_1 + d_2)}\right)$$

$$= \tan^{-1}\left(\frac{9}{0.8(9 + 11)}\right) = 29.35^\circ$$

The minimum angle that the pencil can make with the floor without slipping is  $29.35^\circ$ .

**Rotational Statics and Dynamics: Cylinder Held on Inclined Plane**

A cylinder of mass  $M$  and radius  $R$  is in static equilibrium as shown in the diagram. The cylinder rests on an inclined plane making an angle  $\theta$  with the horizontal and is held by a horizontal string attached to the top of the cylinder and to the inclined plane. There is friction between the cylinder and the plane. What is the tension in the string  $T$ ?

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The cylinder is not moving so the forces and torques are all balanced. To balance the torques, pivot the cylinder at the center. The torques about this pivot are due to the forces of tension and friction. Since both forces act at radius  $R$ , the two forces must be equal in magnitude. To balance the forces, choose axes that create ease in breaking the forces into components. With the  $x$ -axis parallel to the incline, you can balance the forces in the  $x$ -direction to obtain another equation relating the forces of friction, gravity, and tension. Knowing that the frictional force is equal to the tension, you can substitute into your net force equation and solve for the value of the tension. You should obtain

$$T = \frac{Mg \sin \theta}{1 + \cos \theta}$$

**Rotational Statics and Dynamics: Weight on Stick**

One end of a uniform meter stick of mass 0.25 kg is placed against a vertical wall. The other end is held by a lightweight cord making an angle  $\theta=20^\circ$  with the stick. A block with the same mass as the meter stick is suspended from the stick a distance 0.75 m from the wall. Find the tension in the cord and the frictional force between the stick and the wall.

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By balancing both the forces and torques, you can find both the tension and the frictional force. The net force in both the x and y direction is zero. The net force equation for the y direction will give you the relationship between the force of friction and tension. Next you can create a net torque equation. In picking your pivot, it is good to choose one end of the stick so at least one of the unknown force values is eliminated by having a zero radius. Using substitution, you should be able to obtain a tension of 8.963 N and a frictional force of 1.839 N.

