Physics 211 Discussion Session 8

Material covered this week:

Rotation – Equations change a little, but problem-solving methods stay the same!

Rotational Kinematics

$x \to \theta$; $v \to \omega$; $a_T \to \alpha$

Basic Rotational Dynamics

In addition to $\Sigma \vec{F} = m\vec{a}$, we now have $\Sigma \vec{\tau} = I\vec{\alpha}$

$\vec{F} \to \vec{\tau}$ $(\vec{\tau} \equiv \vec{r} \times \vec{F})$; $m \to I$; $a_T \to \alpha$

* If there is <u>no</u> slipping $\omega = \frac{v}{R}$, $\alpha = \frac{a_T}{R}$, $\theta = \frac{x}{R}$

Conservation of Energy with Rotation

$KE_{Total} = \underset{\text{rot. axis}}{KE_{Trans,}} + \underset{\text{about axis}}{KE_{rot}}$ where $KE_{rot} = \frac{1}{2} I \omega^2$

When calculating U, consider CM motion.

Review

Collisions (linear)

Conservation of Energy (linear)

Space Station

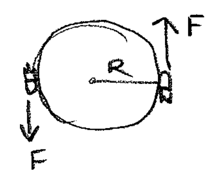
A space station is constructed in the shape of a wheel 22 m in diameter, with essentially all its weight (5.0×10^5 kg) at the rim. Once the space station is completed, it is set rotating at a rate such that an object at the rim experiences a radial acceleration equal to the Earth's gravitational acceleration g, thus simulating Earth's gravity. To accomplish this, two small rockets are attached on opposite sides of the rim, each able to provide a 100 N force. How long will it take to reach the desired rotation rate and how many revolutions will the space station make in this time?

We dive right in to rotational problems with this multi-conceptual situation. Let's rewrite the problem:

1. 2 rockets cause a space station to have an angular acceleration.
2. The space station's final rotation speed gives a radial (centripetal) acceleration equal to g
3. The angular acceleration occurs over some time and distance.

Taking this one part at a time:

1. The rockets exert a total torque $\tau = 200R$
   The space station is a wheel with $I = MR^2$
   Find α using $\tau = I\alpha$



2. We are told $a_c = g$. Use $a_c = \omega^2 R$ to find ω_f
3. Kinematics time! By now you know $\omega_0 = 0$, ω_f, and α. You should have no problem finding t and θ. Note: 1 revolution = 2π radians

$$\boxed{t = 2.6 \times 10^4\ \text{s} \qquad 1952 \text{ revolutions}}$$

Rotating Tip

In the Physics 211 Laboratory one group of students has decided to pursue their own experiments. They make a simple pendulum from a weight attached to a string of length L. They attach the other end of the string to a fixed support. They hold the weight with the string taut and horizontal and then released it. With their motion sensor they measure the speed of the weight as the string passes through the vertical. Remembering that all objects fall with the same acceleration, the students do a second experiment. They attach one end of a uniform stick of length L to the support, which acts as a pivot. They hold the stick horizontal and release it. They then measure the speed of the tip of the stick with their motion sensor. The mass of the pendulum weight and the mass of the stick are the same. Do they measure the same speed?

The weight at the end of the string looks like our familiar conservation of energy problem, but the stick is different...

Let's calculate the speed for both:

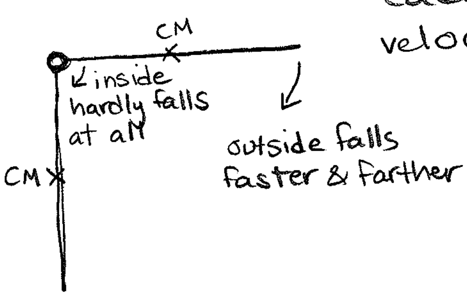
1 Pendulum weight

$E_i = E_f$

$mgL = \frac{1}{2}mv^2$ all of U converts to KE

$\boxed{v = \sqrt{2gL}}$

2 Stick : We can't do the exact same method because each part of the stick moves at a different velocity and falls a different height



We'll use rotational energy instead!

You can look up $I_{Rod,end} = \frac{1}{3}ML^2$

then $KE_f = \frac{1}{2} I_{r,end}\,\omega^2$

and $U_i = \frac{1}{2}Mg\left(\frac{L}{2}\right)$ note: calculate ΔU using the object's CM

Let $U_i = KE_f$ and convert ω to v to get

$\boxed{V_{rod} = \sqrt{3gL}}$ is faster.

Review: Skate-Board Exhibition

You are helping your friend prepare for his skate-board clowning stunt. For his program, he plans to take a running start, and then jump onto a gigantic 7 kg stationary skateboard. He and the skateboard will glide in a straight line along a short, level section of track, then up a sloped concrete wall. He has measured his maximum running speed to jump safely on the skateboard at 6 m/s, and he wants to know how high above ground level he will make as he rolls up the slope. He tells you his weight is 70 kg.

Aha! A collision followed by a height change! By now you should be a pro at this, but for fun let's walk through it.

When your friend jumps on the skateboard, momentum is conserved:

$$\vec{p}_i = m_f v_1$$

$$\vec{p}_f = (m_f + m_B) v_2$$

Let $\vec{p}_i = \vec{p}_f$ to find v_2.

Now he goes up the wall. His KE is converted to U_g:

$$KE_i = \frac{1}{2}(m_f + m_B) v_2^2$$

$$U_f = (m_f + m_B) g H$$

Note that both he and the skateboard go up the wall.

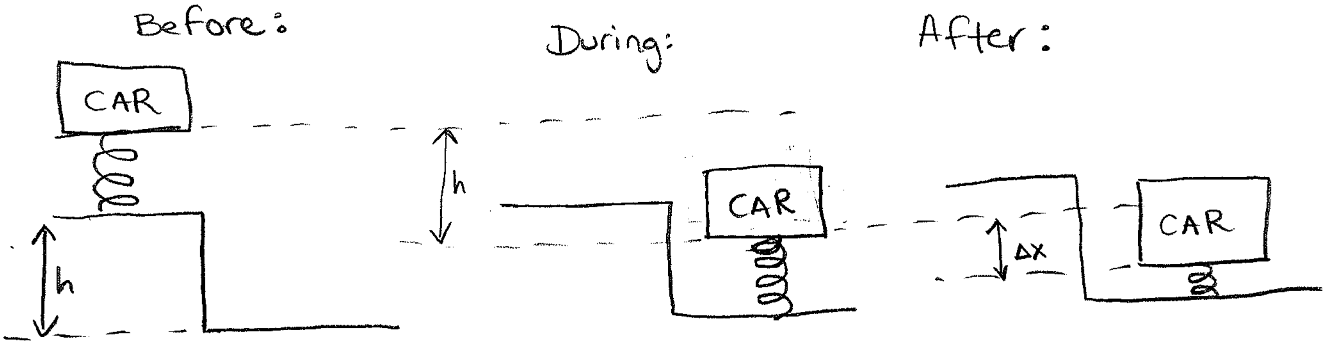
Let $KE_i = U_f$ to find H

$$\boxed{H = 1.52 \text{ m}}$$

Review: Pothole

A car is traveling along a horizontal road when it suddenly encounters a pothole, in which the level of the road abruptly changes by a height *h*. The suspension of the car can be considered as a single spring having a spring constant of 110,000 N/m that can compress a maximum distance of 0.4 m. The mass of the car is 1200 kg. What is the maximum value of *h* that the car can tolerate before bottoming out (i.e. when the springs reach their maximum compression)? For simplicity, you may assume that the spring is not compressed initially.

This one definitely needs a picture:



The car is on the uncompressed spring

The car falls a distance h before the spring begins to compress

The spring compresses by Δx to stop the car's fall

The total height change experienced by the car will be $h + \Delta x$
All its gravitational potential energy converts to spring potential

$$U_g = U_s$$
$$mg(h + \Delta x) = \frac{1}{2} k \Delta x^2$$

Solving the above equation, find

$$\boxed{h = 0.35 \text{ m}}$$

Review: elastic collisions: Train Cars

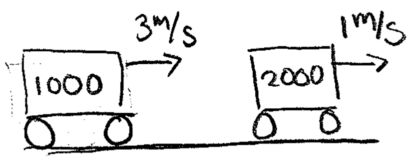
A 1000 kg train car is moving rightwards at a speed 3 m/s towards another car, of 2000 kg, also moving rightwards, but at 1 m/s. They collide elastically through a spring of stiffness 10,000 N/m.

1. What is the final speed of the second car?

2. How much does the spring get compressed?

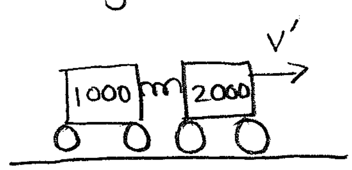
There are 3 relevant times for this problem:

Before collision



$\vec{p}_{before} = 5000 \text{ kg m/s}$

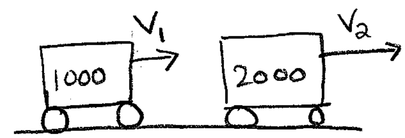
During collision



$\vec{p}_{during} = 5000 \text{ kg m/s}$

both cars move together at the same speed (collision temporarily inelastic)

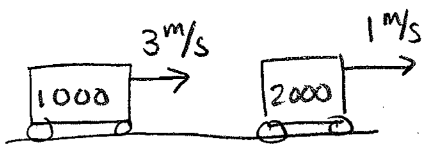
After collision



$\vec{p}_{after} = 5000 \text{ kg m/s}$

Collision was elastic

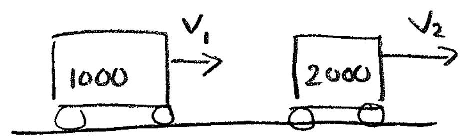
Let's answer (1) first. To find the final speed of the second car, we consider the times before and after the collision.



Elastic collision

Momentum conserved

KE conserved



Using the ground reference frame or the CM reference frame, set

$$\vec{p}_i = \vec{p}_f \quad \text{and} \quad KE_i = KE_f$$

to find $\boxed{v_2 = 7/3 \text{ m/s}}$

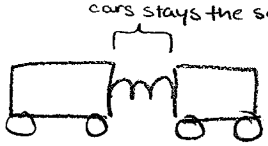
momentum of each object

Note: at any time, $\vec{p} = m_A v_A + m_B v_B = \sum_i \vec{p}_i$

Train cars (cont'd)

Ok, cool. Now we're going to answer (2).
The spring is compressed during the collision. At some moment, it reaches its maximum compression.

At this moment, the spring is neither expanding nor being compressed. Therefore, both cars **must** be moving at the same speed



By now you know that their speed is $V_{cm} = \frac{\sum_i m_i v_i}{\sum_i m_i}$

The spring is compressed, let's say by Δx, so it is storing energy

$$U_s = \frac{1}{2} k \Delta x^2$$

Where could this energy have come from? It must have started out as KE, since that's all we had before the collision.

By conservation of energy,

$$E_i = KE_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$

$$E_f = U_s + KE_f = \frac{1}{2} k \Delta x^2 + \frac{1}{2} m_1 V_{cm}^2 + \frac{1}{2} m_2 V_{cm}^2$$

Set $E_i = E_f$ to find Δx.

$$\boxed{\Delta x = 0.52\,m}$$

Note: v_2, the velocity of the 2000kg car, has no effect on the answer to part (2). Here we are looking at before and during, not after.