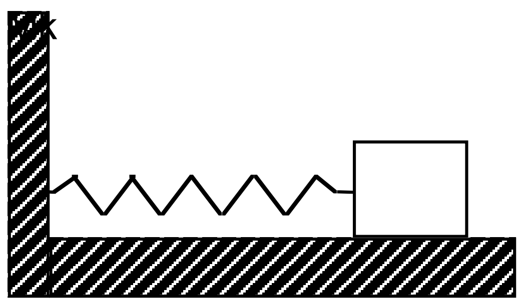
Physics 211 Week 12

Simple Harmonic Motion: Equation of Motion

A mass M rests on a frictionless table and is connected to a spring of spring constant k. The other end of the spring is fixed to a vertical wall as shown in the figure. At time t = 0 s the mass is at x = 2.6cm and moving to the right at a velocity of 47 cm/s. It is at this position with this **speed** next at t = 0.2s. Find an expression for the position as a function of time and in so doing find the frequency and the amplitude of oscillation.

We can begin this problem by writing the equations of motion for the position and the velocity, keeping in mind that the velocity is simply a derivative of the position with respect to time. Next, we can write the equations at time t = 0. We can obtain the angular frequency from the given period in the problem. Then solve the two coupled equations for the amplitude and the initial phase. You should obtain

$$x(t) = 3.0\cos\left(31.4t - \frac{\pi}{6}\right)$$

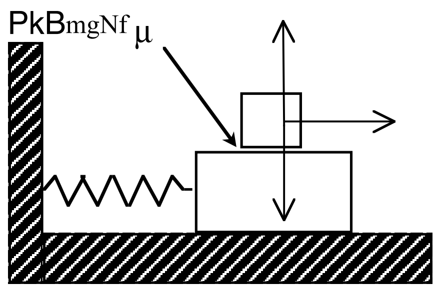


Physics 211 Week 12

Simple Harmonic Motion: Plate, Block, and Spring

A flat plate P of mass 5.0 kg is attached to a spring of spring constant k = 60 N/m and executes horizontal simple harmonic motion by sliding across a frictionless surface. A block B of mass 2.0 kg rests on the plate and the coefficient of static friction between the block and the plate is μ= 0.60. What is the maximum amplitude of oscillation that the plate-block system can have in order that the block not slip on the plate?

The block will slip when the maximum acceleration exceeds the force of static friction on the block. We can relate the maximum acceleration to the amplitude by using the angular frequency. The angular frequency can be found from the mass and spring constant given. It is important to remember to use the combined mass when finding the frequency. You should obtain a maximum amplitude of *0.69 m*.



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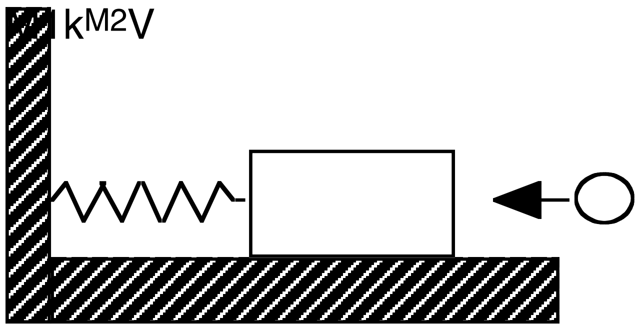
Simple Harmonic Motion: Block, Clay, and Spring

A block of mass M_1 = 5 kg is attached to a spring of spring constant k = 20 N/m and rests on a frictionless horizontal surface. A wad of clay of mass M_2=2 kg and traveling horizontally with speed v = 14 m/s hits and sticks to the block. Find the frequency and amplitude for the subsequent simple harmonic oscillations.



Conceptual Analysis:

- The collision of the clay and block is inelastic.
- Momentum is conserved in the collision of the clay and block.
- The maximum velocity after the collision occurs is related to the amplitude of the oscillations by the angular frequency.
- The angular frequency depends only on the mass and spring constant, not on the initial amplitude or velocity.

Strategic Analysis:

- Use conservation of momentum to find the speed of the clay-block combination immediately after the collision.
- Use the given masses and spring constant to find the angular frequency.
- Use the relationship of the maximum velocity and angular frequency to find the amplitude.

Quantitative Analysis:

- Begin by labeling the given quantities

M_1 mass of the block
M_2 mass of the clay
v initial speed of the clay
v' speed of the clay and block after the collision
k spring constant

We are looking for

f frequency of oscillation = $\omega/2\pi$
A the amplitude of the oscillation

- We will use momentum conservation to find the speed of the block and clay after the collision. The initial momentum is only the motion the clay. The final momentum after the inelastic collision must include the sum of the masses.

$M_2 v = (M_1 + M_2)v'$ $\quad\Rightarrow v' = \dfrac{M_2 v}{M_1 + M_2}$ inserting given values to obtain

$v' = 4\ m/s$

- We can find the frequency of oscillation from the given spring constant and masses.

$\omega = \sqrt{\dfrac{k}{M_1 + M_2}}$ inserting given values to obtain $\omega = 1.7\ s^{-1}$

- The maximum velocity of the oscillations occurs immediately following the collision. Recall that the maximum velocity is related to the amplitude by the angular frequency

$$v' = A\omega \quad \Rightarrow \quad A = \frac{v'}{\omega}$$

- We then put together the two parts to obtain

$$\Rightarrow A = \frac{v'}{\omega} = \frac{4\,m/s}{1.7s^{-1}} = 2.4m$$

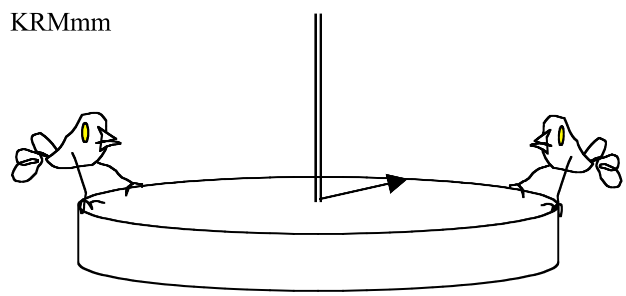
- In summary, the amplitude is 2.4 *m*, and the frequency ω is 1.7 s^{-1}.

Physics 211 Week 12

Oscillations: Bird Feeder Torsion Pendulum

A bird feeder consists of a solid circular disk of mass M=0.34 kg and radius R=0.25 m suspended by a wire attached at the center. Two birds, each of mass m=65 g land at opposite ends of a diameter, and the system goes into torsional oscillation with a frequency f=2.6 Hz. What is the torsional constant of the wire?

The frequency of a torsion pendulum depends on the torsional constant and the moment of inertia. $f = (1/2\pi)\ \mathrm{sqrt}(\kappa/I)$. We know the frequency and we are looking for the constant. We can find the moment of inertia by summing the moments of inertia of the three components that make up the torsion pendulum: the two birds and the bird feeder. The bird feeder is treated as a disk, and the two birds are treated as point masses. After finding the moment of inertia of the system, you should be able to use it with the given frequency to find the torsional constant. You should obtain *5* $kg*m^2/s^2$.



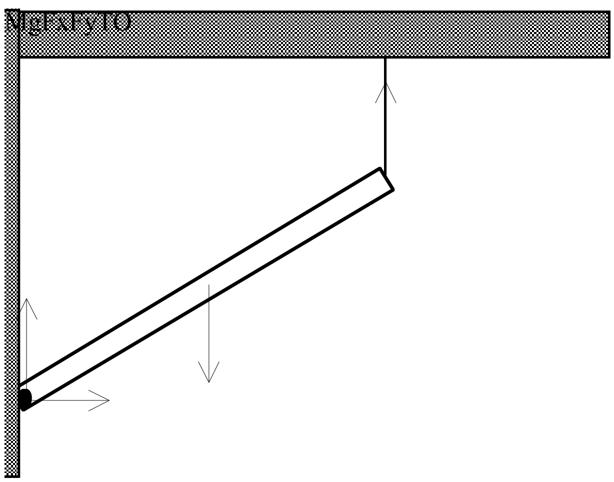
Physics 211 Week 12
Statics & Dynamics: Falling Meter Stick

A uniform meter stick of mass 1.5kg is attached to the wall by a frictionless hinge at one end. On the opposite end it is supported by a vertical massless string such that the stick makes an angle of 40° with the horizontal.

1. Find the tension in the string and the magnitude and direction of the force exerted on the stick by the hinge.
2. Suppose the string is cut. Find the angular acceleration of the stick immediately thereafter.

A strategy: We will apply Newton's 2nd Law for translation and rotation, where we know that the linear and angular accelerations are both zero. Take the hinge as the pivot point for the torque to keep the unknown hinge force from appearing in the rotational equation.

To find the angular acceleration after the string is cut, we'll use Newton's 2nd Law for rotation, inserting the torques and moment of inertia about the hinge.



$$\sum F_x = F_x = 0$$
$$\sum F_y = F_y + T - Mg = 0$$
$$\sum \tau_o = TL\cos\theta - Mg\frac{L}{2}\cos\theta = 0$$

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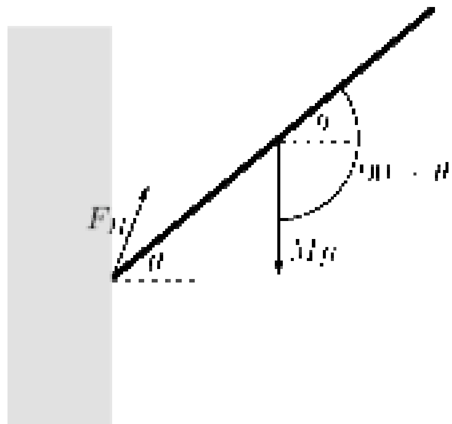
1. Find the tension and force from the hinge.
From the third equation:

$$T = \frac{1}{2}Mg = 7.4N$$

From the first equation, we know that the hinge force has no x-component. We can use the second equation to find its y-component:

$$F_y + T - Mg = 0$$

$$\begin{aligned} F_y &= Mg - T \\ &= Mg - \frac{Mg}{2} \\ &= \frac{Mg}{2} = 7.36 \text{ N upward} \end{aligned}$$



2. Find the angular acceleration after the string is cut.
We use Newton's 2nd Law for rotation about the hinge. We need to calculate the torque each force generates about the hinge. We also know that a rod has moment of inertia 1/3 ML^2 about an end.

$$\tau_{\text{net}} = I\alpha$$

$$\tau_{\text{hinge force}} + \tau_{\text{weight}} = I_{\text{about hinge}}\alpha$$

$$r_F F \sin\theta_{rF} + r_W W \sin\theta_{rW} = \left(\frac{1}{3}ML^2\right)\alpha$$

The hinge force acts at the pivot point, so it has zero radius and therefore generates zero torque. The gravitational force acts at the center of mass, which is half the meterstick length from the hinge. From the geometry in the figure, the angle between the radius vector and the weight is $90^\circ+\theta$. Remember that $\sin(90^\circ+\theta)=\cos(\theta)$

$$(0)F\sin\theta_{rF} + \left(\frac{L}{2}\right)(Mg)\sin\left(90^\circ+\theta\right) = \left(\frac{1}{3}ML^2\right)\alpha$$

$$\cancel{M}g\frac{\cancel{L}}{2}\cos\theta = \left(\frac{1}{3}\cancel{M}L^{\cancel{2}}\right)\alpha$$

$$\frac{3}{2}\frac{g}{L}\cos\theta = \alpha$$

$$\alpha = \frac{3}{2}\frac{9.81\frac{\text{m}}{\text{s}^2}}{1 \text{ m}}\cos 40^\circ = 11.27\,\tfrac{\text{rad}}{\text{s}^2}$$