

Last Name: _____ First Name _____ ID _____

Discussion Section: _____ Discussion TA Name: _____

Instructions—

Please turn off your cell phone and put it away.

Calculators may not be shared. Please keep yours on your own desk.

This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the center of its circle.
2. Print your **NETWORK ID** in the designated spaces at the right side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.
3. Print **YOUR LAST NAME** in the designated spaces at the left side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.
4. **You may find the version of this Exam Booklet at the top of the next page.** Mark the version circle in the TEST FORM box at the bottom right on your answer sheet. **DO THIS NOW!**
5. Print your UIN# in the **STUDENT NUMBER** designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box
6. Sign your name (**DO NOT PRINT**) on the **STUDENT SIGNATURE** line.
7. On the **SECTION** line, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

Before starting work, check to make sure that your test booklet is complete. In addition to these instructions, *you should have 9 numbered pages plus one (1) Formula Sheet.*

Academic Integrity: Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including expulsion.

This Exam Booklet is Version A. Mark the **A** circle in the TEST FORM box at the bottom right on your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of **89** points, composed of three types of questions.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- A) If you mark only one answer and it is the correct answer, you earn **6** points.
- B) If you mark two answers, one of which is the correct answer, you earn **3** points.
- C) If you mark three answers, one of which is the correct answer, you earn **2** points.
- D) If you mark no answers, or more than three, you earn 0 points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- A) If you mark only one answer and it is the correct answer, you earn 3 points.
- B) If you mark a wrong answer or no answers, you earn 0 points.

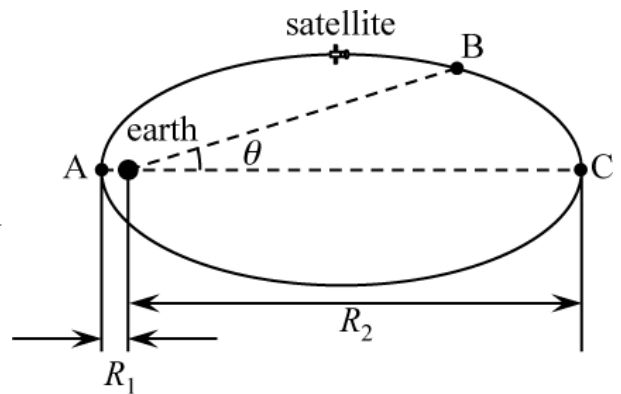
TF: *true-false questions, each worth 2 points.*

No partial credit.

- A) If you mark only one answer and it is the correct answer, you earn 2 points.
- B) If you mark the wrong answer or neither answer, you earn 0 points.

The next two questions pertain to the situation described below.

A satellite moves in an elliptical orbit around the earth, as shown in the figure. The orbit's distance from the center of the earth varies from a minimum of R_1 to a maximum of R_2 . Three points, labeled A, B, and C are shown on the orbit; A and C correspond to points in the orbit that are closest to, and farthest from, the earth. The intermediate point B lies on the orbit at an angle θ from the orbit's major axis.



Assume that the values of R_1 , R_2 and θ are 3×10^7 m, 4×10^8 m, and 25 degrees, respectively.

- 1) The magnitudes of the angular momenta of the satellite at the points A, B, and C are L_A , L_B and L_C . Which of the following statements is true?

- a. $L_A = L_B = L_C$
- b. $L_A > L_B > L_C$
- c. $L_A < L_B < L_C$

- 2) Radar observations from earth reveal that the satellite is moving with speed $v_A = 8$ km/s as it passes through point A. The satellite's speed at point C is

- a. 0.6 km/s
- b. 106.67 km/s
- c. 0.04 km/s

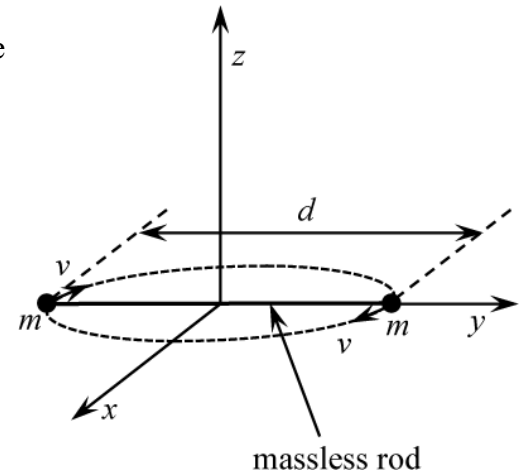
3) A bowling ball rolls without slipping down an inclined plane. The plane is fixed so that it remains stationary as gravity accelerates the ball down the incline. As it rolls, the ball's angular momentum remains constant.

- a. True
- b. False

The next two questions pertain to the situation described below.

Two point masses m are connected by a massless rod of length d . The two-mass object rotates in the $x - y$ plane, with its center of mass at the origin, as shown in the diagram. The speed of each mass is v .

Assume that $m = 7 \text{ kg}$, $d = 0.4 \text{ m}$, and $v = 3 \text{ m/s}$.



4) The direction of the angular momentum of the object is

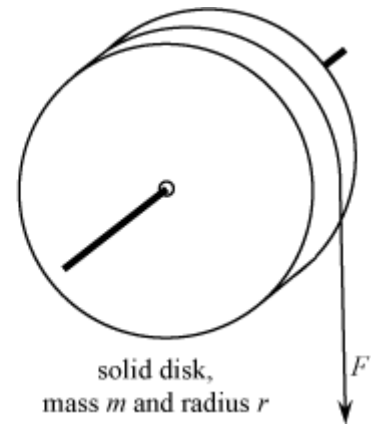
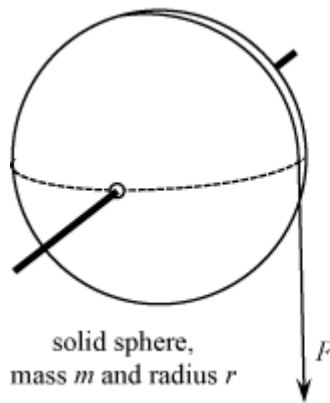
- a. In the negative x direction
- b. In the positive y direction
- c. In the negative z direction

5) The magnitude of the object's angular momentum is

- a. $8.4 \text{ kg m}^2/\text{s}$
- b. $630 \text{ kg m}^2/\text{s}$
- c. $63 \text{ kg m}^2/\text{s}$
- d. $1.12 \text{ kg m}^2/\text{s}$
- e. $42 \text{ kg m}^2/\text{s}$

The next three questions pertain to the situation described below.

A solid sphere of mass m and radius r can rotate without friction around a horizontal axis which passes through its center. A string wrapped around the sphere applies a constant force F , which generates a torque acting on the sphere. A solid cylinder, also with mass m and radius r , can rotate without friction around a horizontal axis which passes through its center. It too suffers the effects of an identical force F applied by

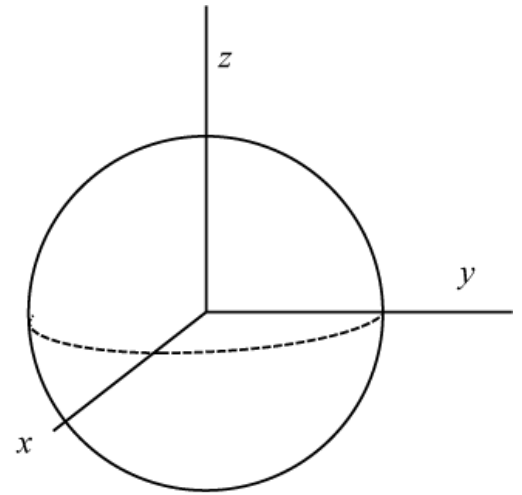


a string wrapped around the cylinder. For time $t < 0$, the sphere and cylinder are held fixed, but at time $t = 0$ they are allowed to rotate freely. The strings do not slip as they exert torques on the two objects.

Please assume that $m = 45 \text{ kg}$, $F = 60 \text{ N}$, and $r = 0.45 \text{ m}$. Recall that the moments of inertia of spheres and cylinders are $\frac{2mr^2}{5}$ and $\frac{mr^2}{2}$, respectively.

- 6) After the **sphere** has been experiencing the applied torque for 1 seconds, what is the angular momentum of the sphere?
 - a. $60 \text{ kg m}^2/\text{s}$
 - b. $27 \text{ kg m}^2/\text{s}$
 - c. $3.65 \text{ kg m}^2/\text{s}$
 - d. $441.45 \text{ kg m}^2/\text{s}$
 - e. $1350 \text{ kg m}^2/\text{s}$
- 7) After the string wound around the **cylinder** has been pulled through a distance of $d = 2 \text{ m}$, what is the kinetic energy of the cylinder?
 - a. 54 J
 - b. 9.11 J
 - c. 120 J
- 8) After the torques applied by the strings to the sphere and the cylinder have been acting for 3 seconds, a technician measures the angular velocities ω_{sphere} and ω_{cylinder} of the rotating objects. The technician finds
 - a. $\omega_{\text{sphere}} < \omega_{\text{cylinder}}$
 - b. $\omega_{\text{sphere}} = \omega_{\text{cylinder}}$
 - c. $\omega_{\text{sphere}} > \omega_{\text{cylinder}}$

A spherical deep space probe is stabilized by a pair of internal gyroscopes. The rotation axis of the first gyro, which is spinning with angular speed ω_1 , points along the positive x axis. The second gyro, turning with angular speed ω_2 , points along the y axis. Both gyroscopes are solid disks with mass $m_{\text{gyro}} = 18 \text{ kg}$ and radius $r_{\text{gyro}} = 0.4 \text{ m}$. The orientation of the probe is well maintained by the gyroscopes, so that its antennas are properly aimed at the earth.



solid sphere,
mass m_1 and radius r

After years of flawless operation, the lubricant in both gyroscopes freezes, and the gyros grind to a stop. The probe begins to spin about an axis whose orientation is fixed in space.

Please assume that the gyroscopes' angular speeds were $\omega_1 = 3 \times 10^3$ radians per second and $\omega_2 = 2 \times 10^3$ radians per second before the malfunction took place.

9) The axis about which the probe spins can be parallel to which of the following vectors?

- a. $-0.71\hat{x} - 0.71\hat{y}$
- b. $-0.83\hat{y} + 0.55\hat{z}$
- c. $0.83\hat{x} + 0.55\hat{y}$
- d. \hat{z}
- e. $0.17\hat{y} - 0.99\hat{z}$

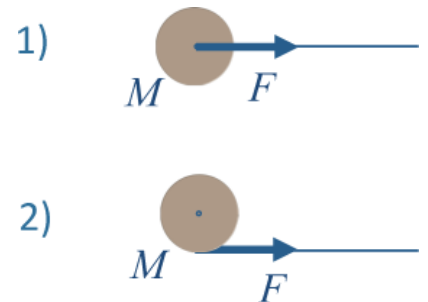
A pizza chef places a fully loaded, perfectly circular pizza onto a frictionless table in a restaurant kitchen. He then tosses an unloaded ball of pizza dough into the air to shape it into an equally perfect circle of the same diameter as the finished pizza on the table. He does this by spinning the ball with impressively high angular velocity around the dough ball's vertical axis. By the time the dough ball comes down, it has become a perfectly uniform, rapidly rotating disk with exactly the same radius as the pizza on the table. Unfortunately, the chef forgets to catch the airborne pizza, which lands on, and sticks to, the pizza on the table. The two disks are perfectly aligned and rotate madly on the table.

Assume the pizza already on the table has mass $m_1 = 2$ kg and radius $r = 0.5$ m, while the disk of unbaked dough has mass $m_2 = 0.3$ kg, radius $r = 0.5$ m, and angular velocity of magnitude $\omega = 50$ radians per second.

- 10) What is the magnitude of the angular velocity of the combined mess on the table after the flying disk lands on the pizza?
- a. 6.52 radians per second
 - b. 43.48 radians per second
 - c. 7.5 radians per second

The next two questions pertain to the situation described below.

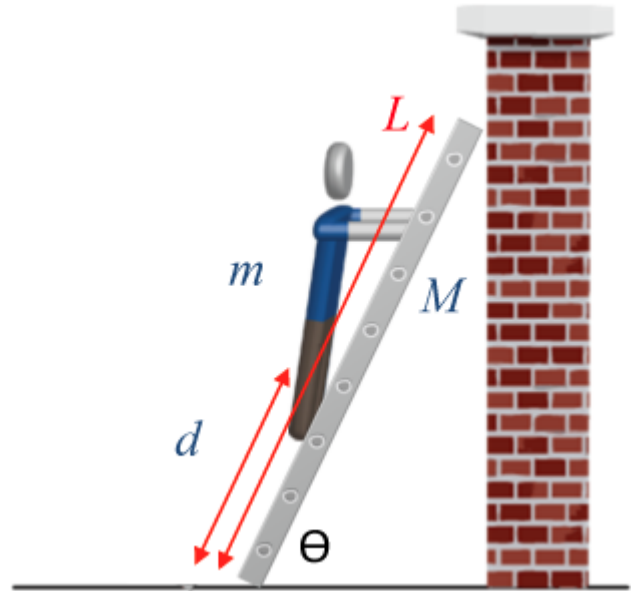
Two pucks (uniform, solid disks with moment of inertia $I = 1/2MR^2$) are pulled by strings which exert identical forces along a frictionless surface. One of the pucks has the string attached to its center, while the other has the string wrapped around it so that the string unwinds from the puck as it is pulled. The pucks are initially at rest.



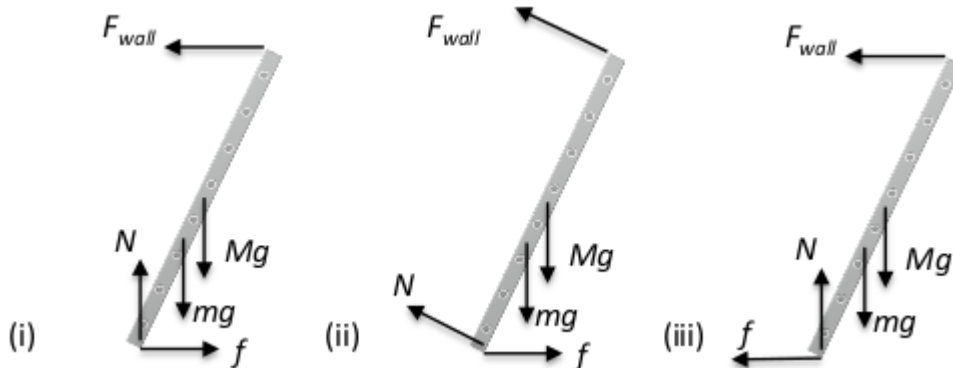
- 11) Which puck requires less time to travel a distance d ?
- a. Puck 1
 - b. Both pucks require the same amount of time to travel the distance d
 - c. Puck 2
- 12) What is the ratio of the total kinetic energy of puck 1 K_1 to the total kinetic energy of puck 2 K_2 while the strings are being pulled?
- a. $K_1/K_2 = 1$
 - b. $K_1/K_2 = 3$
 - c. $K_1/K_2 = 2$
 - d. $K_1/K_2 = 1/3$
 - e. $K_1/K_2 = 1/2$

The next two questions pertain to the situation described below.

A ladder of length $L = 8$ meters and mass $M = 15$ kg leans on a wall at an angle of $\theta = 47$ degrees. The coefficient of friction between the floor and the ladder is μ_s . There is no friction between the ladder and the wall. A man of mass $m = 75$ kg climbs a distance d up the ladder.



13) Which diagram best represents the free body diagram of the ladder?



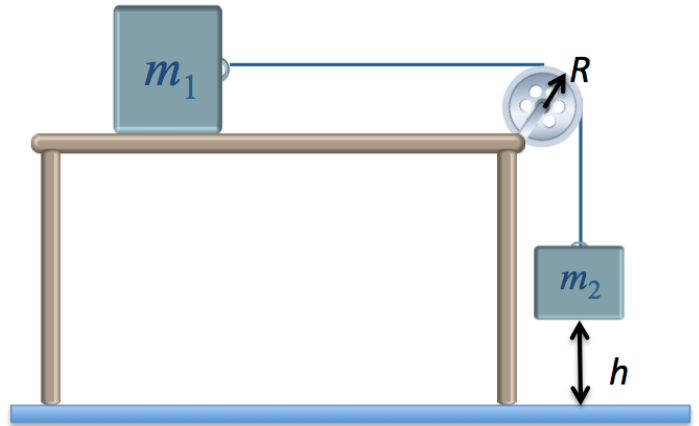
- a. Diagram (ii)
- b. Diagram (i)
- c. Diagram (iii)

14) What is the minimum value of static friction that would allow the man to climb to the top of the ladder?

- a. 0.63
- b. 0.85
- c. 0.39
- d. 0.78
- e. 0.34

The next two questions pertain to the situation described below.

A block of mass $m_1 = 3$ kg rests on a frictionless surface. A massless string, attached to the block and running over a pulley of mass $m_p = 0.3$ kg and radius $R = 0.25$ meters is also attached to a second block of mass $m_2 = 3$ kg. The system is initially at rest, with the second block a height $h = 1$ meter above the ground. At time $t = 0$ the system is allowed to move, and the second block begins to descend. Assume that the string does not stretch and does not slip on the pulley, and treat the pulley as a solid, uniform disk.



15) What is the angular speed of the pulley ω_1 immediately before the mass hits the ground?

- a. 39.62 s^{-1}
- b. 12.37 s^{-1}
- c. 17.29 s^{-1}
- d. 79.24 s^{-1}
- e. 19.81 s^{-1}

16) How would the angular velocity, ω_2 , change relative to ω_1 in the previous question if the pulley were instead a hoop with massless spokes? Assume that the hoop-like and disk-like pulleys have the same mass.

- a. $\omega_2 > \omega_1$
- b. $\omega_2 < \omega_1$
- c. $\omega_2 = \omega_1$

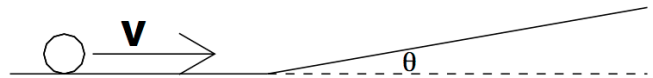
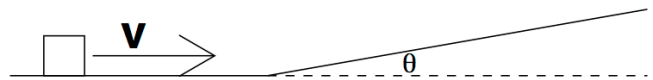
Two solid disks are made from the same material and have the same thickness. The radius R_L of the larger disk is twice that of the radius R_S of the smaller disk. The disks' moments of inertia about axes through their centers are I_L and I_S for the larger and smaller disks, respectively.

17) What is the ratio I_L / I_S ? (Hint: think about how the mass of the disks depends on their radius.)

- a. 8
- b. 16
- c. 4

The next two questions pertain to the situation described below.

A box and a solid cylinder of equal mass M are moving across horizontal surfaces which are joined to ramps, as shown in the figure. The surface on which the box moves is frictionless, but the surface on which the ball rolls is not, so that the ball rolls without slipping. Both ramps have the same angle of inclination θ .



18) If both objects have the same velocities v just as they reach the ramps, what is the ratio of the changes in elevations of the centers of mass of the two objects, $h_{\text{Box}}/h_{\text{Cyl}}$?

- a. $2/3$
- b. 1
- c. $1/2$
- d. $4/3$
- e. $3/4$

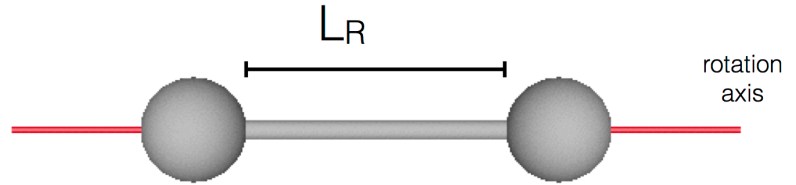
19) Now imagine that both ramps are frictionless, and that the box and the cylinder arrive at the start of their ramps with the same initial velocities v . How does the new height, $H_{\text{Cyl,noFriction}}$ compare to h_{Box} ?

- a. $H_{\text{Cyl,noFriction}} < h_{\text{Box}}$
- b. $H_{\text{Cyl,noFriction}} > h_{\text{Box}}$
- c. $H_{\text{Cyl,noFriction}} = h_{\text{Box}}$

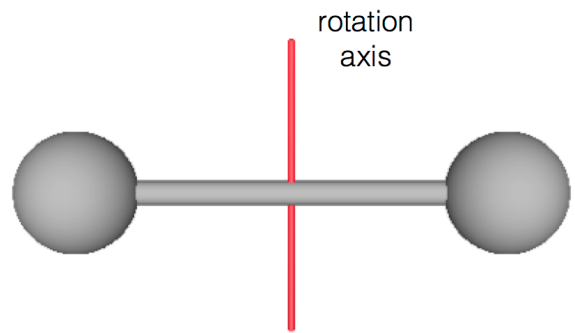
The next two questions pertain to the situation described below.

The object shown in the diagram is built from two identical spheres of radius $R_S = 0.14$ m and mass $M_S = 2.1$ kg, which are joined by a rod of length $L_R = 0.25$ m, mass $M_R = 1.6$ kg, and radius $R_R = 0.03$ m.

- 20) What is the moment of inertia of the object pictured about the axis through the center of and parallel to the rod connecting the two spheres?



- a. 0.0569 kg m^2
b. 0.0336 kg m^2
c. 0.0387 kg m^2
d. 0.0358 kg m^2
e. 0.0172 kg m^2
- 21) A different object is built from a massless rod and a pair of solid spheres of mass $M = 7$ kg and radius $R = 0.1$ m. The rod's length is chosen to keep the center of each sphere a distance $D = 0.3$ m from the midpoint of the rod. What is the object's moment of inertia for rotations around an axis which is perpendicular to the rod, and which runs through the rod's center, as shown in the figure?



- a. 0.66 kg m^2
b. 1.32 kg m^2
c. 2.58 kg m^2
d. 0.06 kg m^2
e. 0.42 kg m^2

Phys 211 Formula Sheet

Kinematics

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$$

$$\mathbf{v}_{A,B} = \mathbf{v}_{A,C} + \mathbf{v}_{C,B}$$

Uniform Circular Motion

$$a = v^2/r = \omega^2 r$$

$$v = \omega r$$

$$\omega = 2\pi/T = 2\pi f$$

Dynamics

$$\mathbf{F}_{\text{net}} = m\mathbf{a} = d\mathbf{p}/dt$$

$$\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$$

$$F = mg \text{ (near earth's surface)}$$

$$F_{12} = -Gm_1 m_2 / r^2 \text{ (in general)}$$

$$\text{(where } G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2\text{)}$$

$$\mathbf{F}_{\text{spring}} = -k \Delta \mathbf{x}$$

Friction

$$f = \mu_k N \text{ (kinetic)}$$

$$f \leq \mu_s N \text{ (static)}$$

Work & Kinetic energy

$$W = \int \mathbf{F} \cdot d\mathbf{l}$$

$$W = \mathbf{F} \cdot \Delta \mathbf{r} = F \Delta r \cos \theta$$

$$\text{(constant force)}$$

$$W_{\text{grav}} = -mg\Delta y$$

$$W_{\text{spring}} = -k(x_2^2 - x_1^2)/2$$

$$K = mv^2/2 = p^2/2m$$

$$W_{\text{NET}} = \Delta K$$

Potential Energy

$$U_{\text{grav}} = mgy \text{ (near earth surface)}$$

$$U_{\text{grav}} = -GMm/r \text{ (in general)}$$

$$U_{\text{spring}} = kx^2/2$$

$$\Delta E = \Delta K + \Delta U = W_{\text{nc}}$$

Power

$$P = dW/dt$$

$$P = \mathbf{F} \cdot \mathbf{v} \text{ (for constant force)}$$

System of Particles

$$\mathbf{R}_{\text{CM}} = \sum m_i \mathbf{r}_i / \sum m_i$$

$$\mathbf{V}_{\text{CM}} = \sum m_i \mathbf{v}_i / \sum m_i$$

$$\mathbf{A}_{\text{CM}} = \sum m_i \mathbf{a}_i / \sum m_i$$

$$\mathbf{P} = \sum m_i \mathbf{v}_i$$

$$\Sigma \mathbf{F}_{\text{EXT}} = M \mathbf{A}_{\text{CM}} = d\mathbf{P}/dt$$

Impulse

$$\mathbf{I} = \int \mathbf{F} dt$$

$$\Delta \mathbf{P} = \mathbf{F}_{\text{av}} \Delta t$$

Collisions:

If $\Sigma \mathbf{F}_{\text{EXT}} = 0$ in some direction, then

$\mathbf{P}_{\text{before}} = \mathbf{P}_{\text{after}}$ in this direction:

$$\Sigma m_i \mathbf{v}_i \text{ (before)} = \Sigma m_i \mathbf{v}_i \text{ (after)}$$

In addition, if the collision is elastic:

* $E_{\text{before}} = E_{\text{after}}$

* *Rate of approach = Rate of recession*

* *The speed of an object in the Center-of-Mass reference frame is unchanged by an elastic collision.*

Rotational kinematics

$$s = R\theta, v = R\omega, a = R\alpha$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha \Delta \theta$$

Rotational Dynamics

$$I = \Sigma m_i r_i^2$$

$$I_{\text{parallel}} = I_{\text{CM}} + MD^2$$

$$I_{\text{disk}} = I_{\text{cylinder}} = \frac{1}{2} MR^2$$

$$I_{\text{hoop}} = MR^2$$

$$I_{\text{solid-sphere}} = \frac{2}{5} MR^2$$

$$I_{\text{spherical shell}} = \frac{2}{3} MR^2$$

$$I_{\text{rod-cm}} = \frac{1}{12} ML^2$$

$$I_{\text{rod-end}} = \frac{1}{3} ML^2$$

$$\tau = I\alpha \text{ (rotation about a fixed axis)}$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, |\tau| = rF \sin \phi$$

Work & Energy

$$K_{\text{rotation}} = \frac{1}{2} I \omega^2$$

$$K_{\text{translation}} = \frac{1}{2} M V_{\text{cm}}^2$$

$$K_{\text{total}} = K_{\text{rotation}} + K_{\text{translation}}$$

$$W = \tau \theta$$

Statics

$$\Sigma \mathbf{F} = 0, \Sigma \tau = 0 \text{ (about any axis)}$$

Angular Momentum:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$L_z = I\omega_z$$

$$\mathbf{L}_{\text{tot}} = \mathbf{L}_{\text{CM}} + \mathbf{L}^*$$

$$\boldsymbol{\tau}_{\text{ext}} = d\mathbf{L}/dt$$

$$\boldsymbol{\tau}_{\text{cm}} = d\mathbf{L}^*/dt$$

$$\Omega_{\text{precession}} = \tau / L$$

Simple Harmonic Motion:

$$d^2x/dt^2 = -\omega^2 x$$

$$\text{(differential equation for SHM)}$$

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -A \omega \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

$$\omega^2 = k/m \text{ (mass on spring)}$$

$$\omega^2 = g/L \text{ (simple pendulum)}$$

$$\omega^2 = mgR_{\text{CM}}/I \text{ (physical pendulum)}$$

$$\omega^2 = \kappa/I \text{ (torsion pendulum)}$$

General harmonic transverse waves:

$$y(x,t) = A \cos(kx - \omega t)$$

$$k = 2\pi/\lambda, \omega = 2\pi f = 2\pi/T$$

$$v = \lambda f = \omega/k$$

Waves on a string:

$$v^2 = \frac{F}{\mu} = \frac{\text{(tension)}}{\text{(mass per unit length)}}$$

$$\bar{P} = \frac{1}{2} \mu v \omega^2 A^2$$

$$\frac{d\bar{E}}{dx} = \frac{1}{2} \mu \omega^2 A^2$$

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2} \text{ Wave Equation}$$

Fluids:

$$\rho = \frac{m}{V} \quad p = \frac{F}{A}$$

$$A_1 v_1 = A_2 v_2$$

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

$$F_B = \rho_{\text{liquid}} g V_{\text{liquid}}$$

$$F_2 = F_1 \frac{A_2}{A_1}$$