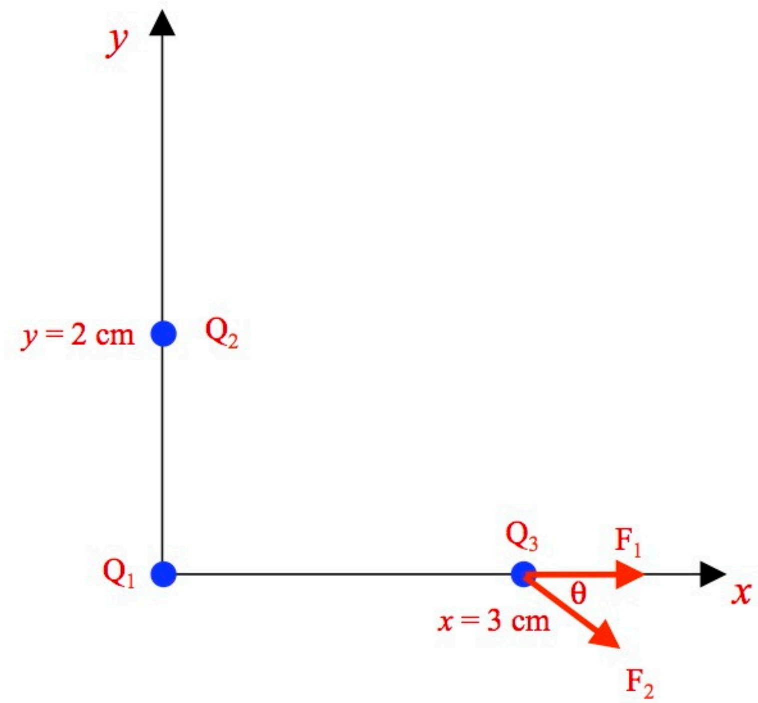
Week 1 Solutions

1. Force on one charge in a set of charges

Three charges, all with magnitude +4 μC are placed at the following locations on an (x,y) grid. In doing this problem you will find the net force on Q_3 due to Q_1 and Q_2.

Q_1: (0 cm, 0 cm)
Q_2: (0 cm, 2 cm) **Watch out for cm's (centimeters) in your calculations!**
Q_3: (3 cm, 0 cm)

a) Draw a picture of the charges.



b) On your picture, draw the two arrows which represent the force that Q_3 feels from each of the other two charges.

c) Make horizontal and vertical component arrows if necessary.

d) Apply Coulomb's law in turn and express your result in component form .

$F_x = F_1 + F_2 \cos\theta = 252 \text{ N}$

$F_y = -F_2 \sin\theta = -61.6 \text{ N}$

$\tan\theta = 2/3$
$\theta = 33.7°$

$$F_1 = k\frac{Q_1 Q_3}{r_{13}^2} = 9\times10^9 \frac{(4\times10^{-6})^2}{(0.03)^2} = 160 \text{ N}$$

$$F_2 = k\frac{Q_1 Q_2}{r_{23}^2} = 9\times10^9 \frac{(4\times10^{-6})^2}{(0.02)^2 + (0.03)^2} = 111 \text{ N}$$

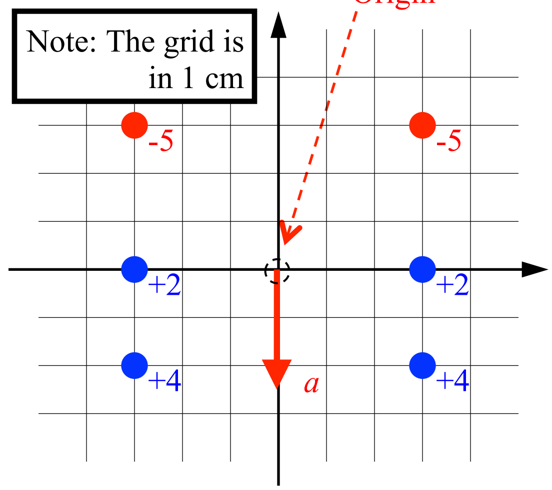
Week 1 Solutions

2. Electric field from a grid of charges in a plane

a) Find the electric field at the origin for the collection of charges shown in the figure below. Give your result in components. The charges are fixed at the locations shown and the grid is marked off in 1 cm increments.

$E_x = 0$ (The charges cancel in pairs.)

$$E_y = k\frac{2\times 5\ \mu C}{(0.03^2+0.03^2)\ cm^2}\cos 45^\circ$$
$$+k\frac{2\times 4\ \mu C}{(0.02^2+0.03^2)\ cm^2}\cos 56.3^\circ$$
$$= 6.61\times 10^7\ N/C$$

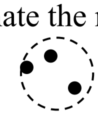


b) A 15 mg particle has a charge of –0.3μC. What is the magnitude of the initial acceleration of such a particle if it is released from the origin? In what direction would it initially travel?

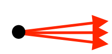
a = (Indicate direction on drawing with an arrow)
Only the *y*-component is non-zero, because only E_y is non-zero.
Acceleration is along $-y$, because q is negative.

Magnitude of a: $a = F/m = qE/m = 1.32\times 10^6\ m/s^2$.

c) Consider this same charge placed 100 meters away from the origin along any direction you choose. Describe the motion of the charge if it was released from such a point. How would you estimate the magnitude of the force on this charge?



Group of charges



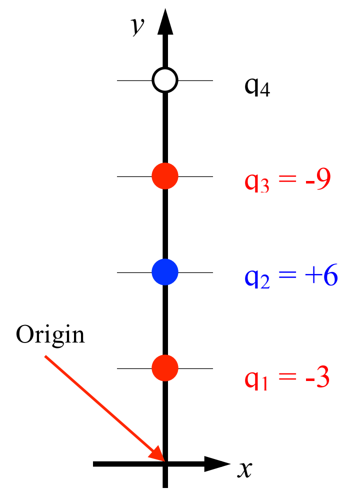
E-fields from charges

When a group of charges is very far away (compared to the size of the group), then all the E-field vectors point in approximately the same direction. The total electric field can be calculated assuming that all the charges are at the same place, and only the total charge is important. In this case, $Q_{tot} = +2\ \mu C$. Our particle is negatively charged, so it will feel an attractive force, pointing toward the group.

Week 1 Solutions

3. Charge stack…

a) Four charges are fixed to the y axis as shown at positions 1, 2, 3 and 4 m, respectively. What must the charge q_4 be if the electric field at the origin is 0 N/C ?



We want:

$$0 = -\left(k\frac{q_1}{y_1^2} + k\frac{q_2}{y_2^2} + k\frac{q_3}{y_3^2} + k\frac{q_4}{y_4^2} \right)$$

The minus sign is because all the charges are at positive y. Positive charges at positive y create an E-field at the origin that points in the $-y$ direction.

Solve for q_4:

$$q_4 = -y_4^2\left(\frac{q_1}{y_1^2} + \frac{q_2}{y_2^2} + \frac{q_3}{y_3^2} \right)$$

$$= -4^2\left(\frac{-3}{1^2} + \frac{6}{2^2} + \frac{-9}{3^2} \right)$$

$$= 40$$

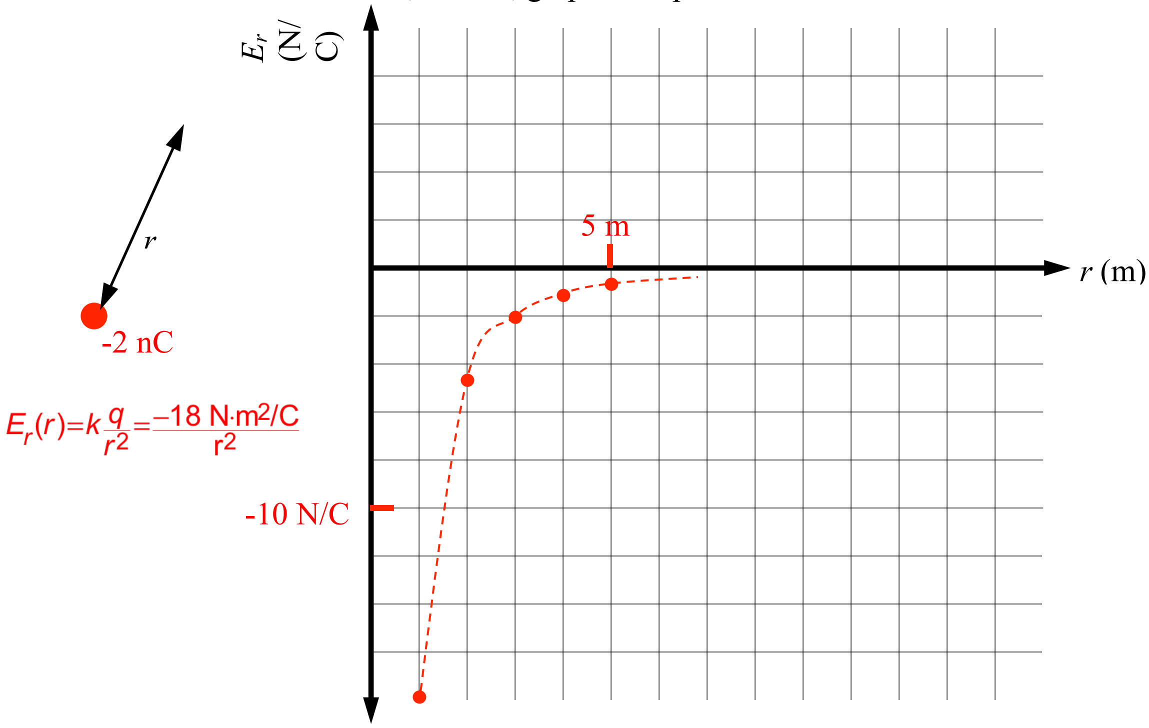
b) What is the force on a 12 μC charge placed at the origin? Find the magnitude and direction of the force. Assume that the charge, q_4, you calculated in part a) is in place.

$E = 0 \Rightarrow F = 0$

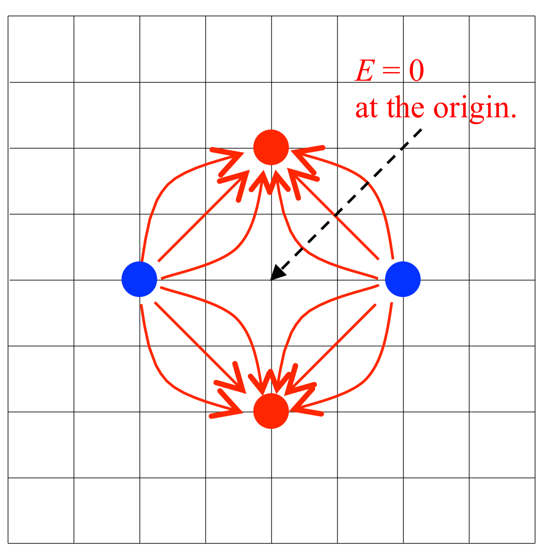
Week 1 Solutions

4. Pictures and plots of electric fields

a) Make a plot of the electric field strength versus radius from the center of a charge of **-**2 nC. Take the time to do this plot carefully. We will come back to this graph NEXT WEEK and add another, related, graph on top of it.



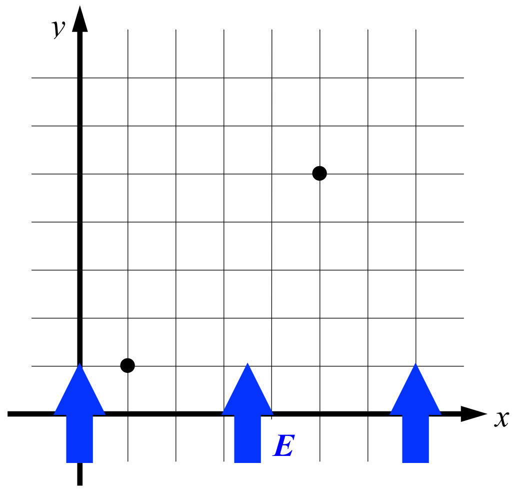
b) Sketch the electric field lines for the charge configuration shown. Note that all charges have the same magnitude but some are positive and some negative.



Week 1 Solutions

5. A charge in a uniform electric field

A uniform electric field with magnitude 3.5 x 10^5 N/C is oriented in the $+y$ direction in an (x,y) plane. You move a +4nC charge from the point (1 m, 1 m) to the point (5 m, 5 m) in this plane.



How much WORK is required BY YOU to move the charge from start to finish?
Assume that the charge is at rest at the beginning and end of its travel.

The assumption means that the charge has no kinetic energy at the beginning ant at the end. Therefore, the total work done must be zero (no change in energy). Calculate the work done by the electric field. Your work must cancel it.

Work is force × distance. Only motion along the force direction contributes!!
$F = qE$ = 0.014 N (in the $+y$ direction). So, W_{field} = 0.014 N × 4 m = 0.056 J.
Positive work means that the field adds 0.056 Joules to the particle's energy.
Your job is to prevent that. You must do -0.056 J of work.

Does the work depend on the path taken? Explain.

No. The work done only depends on the distance moved along y, which is completely determined by the locations of the end points.