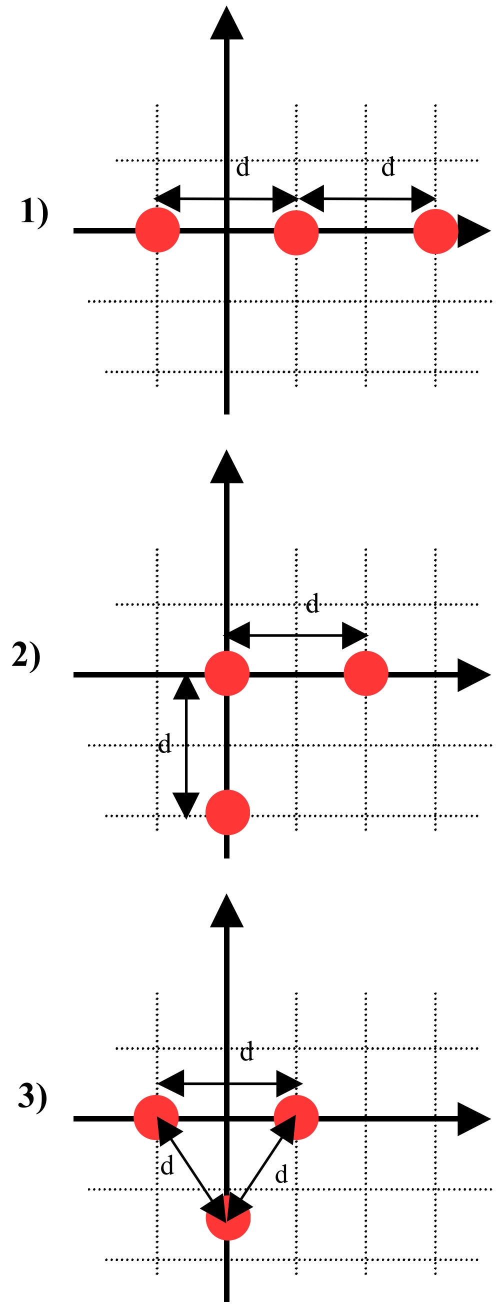
Week 2 Solutions

1. Potential energy associated with some simple charge distributions

All charges = -4nC



a) Which configuration requires the *least* work to assemble? Demonstrate your work with a complete calculation.

In these calculations, I use Q for the charge. Don't use numbers unless necessary.

1) The left charge requires no energy (always true for
 the first charge)
 The middle charge requires kQ^2/d.
 The right charge requires $kQ^2/2d + kQ^2/d. = (3/2)kQ^2/d$.
 The total is: $(5/2)\ kQ^2/d$.

2) The charge at (0,0) requires no energy.
 The charge at $(0,-d)$ requires kQ^2/d.
 The charge at $(d,0)$ requires
 $kQ^2/d + kQ^2/(\sqrt{2}d). = (1+1/\sqrt{2})\ kQ^2/d$.
 The total is $(2+1/\sqrt{2})\ kQ^2/d$.

3) The charge at $(-d,0)$ requires no energy.
 The charge at $(+d,0)$ requires kQ^2/d.
 The charge on the $-y$ axis requires $2(kQ^2/d)$.
 The total is $3(kQ^2/d)$.

$2.5 < 2.7 < 3$, so configuration 1) requires the least energy.

b) If the charges were positive instead, would that change
your answer? Discuss your answer.

No, because the energy depends on Q^2.

Week 2 Solutions

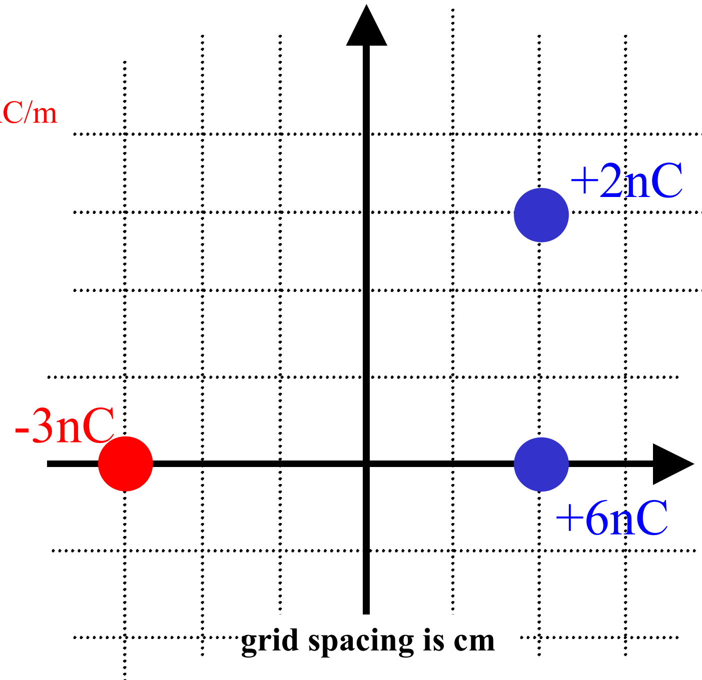
2: Electric potential from a collection of charges

a) Calculate the electric potential at the point (0,0). Use V = 0 at infinity.

Each charge contributes kQ/r, so:

$V(0,0) = k(-3/0.03 + 6/0.02 + 2/0.0361)$ nC/m
$= 2299$ V

Remember: 1 nC $= 10^{-9}$ C.



b) How large would the negative charge have to be in order to bring the potential at the origin to 0 V?

Call the unknown charge q. We want: $0 = q/0.03 + 6/0.02 + 2/.0361$. So, $q = -10.7$ nC.

c) Assume this change has been made: The potential at the origin is zero and the value of the negative charge is what you have found in part b). Now draw a path from infinity (start at the top, right corner of the page) to the origin along which you would have to do the least work to move a big positive test charge. Discuss this problem with your group. What am I asking?!

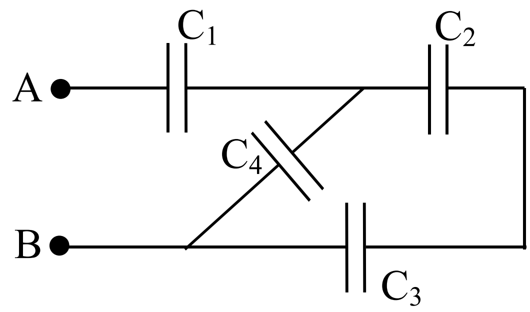
Because $V(0,0) = 0$, no work is required to bring a charge in from infinity, where $V = 0$ also.

This answer is independent of the path used. If this weren't the case, energy would not be conserved. You could go one direction along ont path and return along the other, and not end up with the original energy.

Week 2 Solutions

3. Capacitor Network #1

a) What is the equivalent capacitance of the following network, where all capacitors have a capacitance of 4 μF?



Every capacitor has the same capacitance. Call it C. Work this problem in stages:

C_2 and C_3 are in series: $\frac{1}{C_{23}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_{23} = \frac{C}{2}$

C_{23} and C_4 are in parallel: $C_{234} = C + \frac{C}{2} = \frac{3}{2}C$.

C_1 and C_{234} are in series, so $C_{eq} = C_{1234} = \frac{3}{5}C = 2.4\ \mu\text{F}$.

b) If a battery were placed between A and B and supplied a potential difference of 15 V, then how much energy is stored in the entire network?

The energy stored is $\frac{1}{2} C_{eq}V^2 = 2.7\times10^{-4}$ J (= 0.27 mJ) .

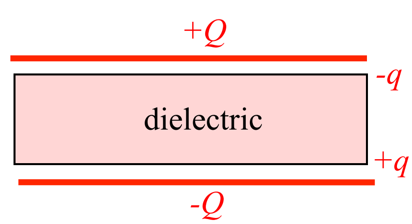
c) What charge is stored on C_1 (*i.e.*, on one of its plates)?

As C_{eq} is charged up, all the current flows onto the left-hand plate of C_1. (Look at the figure above.) So, $Q_1 = C_{eq}V = 36\ \mu$C.

d) Now suppose a dielectric material with $\kappa = 5.4$ is placed between the plates of all capacitors in this problem. What happens to your answer to b) and to c)? Where does the extra energy come from?

The dielectric increases every capacitance (and, therefore, C_{eq}) by a factor of κ. So, the answers to b) and c) both increases by a factor of κ.

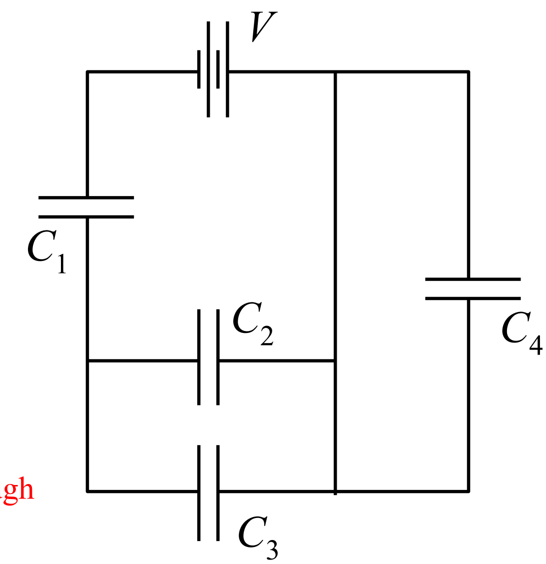
The extra energy is in the dielectric. It takes energy to separate the $\pm q$ charges inside the dielectric. This charge separation is the reason the capacitance increases. It decreases the electric field between the plates, which means that for a given Q, V is smaller. $C = Q/V$.



Week 2 Solutions

4. Capacitor Network #2

In the network of capacitors shown, the following quantities are known:
$C_1 = 6\ \mu F$, $C_3 = 0.5\ \mu F$, $q_1 = 6\ \mu C$, $U_3 = 25\ \mu J$. Label the figure with them.



a) What is the voltage across C_1?

$V_1 = q_1/C_1 = 1$ V

b) What is the charge on C_3?

$U_3 = \frac{1}{2} q_3^2/C_3$, so $q_3 = \sqrt{(2U_3C_3)} = 5\ \mu C$.

c) What is the charge on C_2?

The current that flows through C_1 must flow through C_2 or C_3. Therefore, $q_2 + q_3 = q_1$. $\Rightarrow$ $q_2 = 1\ \mu C$.

d) What is the capacitance of C_2?

The voltage across C_2 must equal that across C_3. Therefore $C_2 = C_3*(q_2/q_3) = 0.1\ \mu F$.

e) What is the equivalent capacitance of the system?

C_4 is irrelevant to this problem, because it's shorted out by a wire. So, C_{eq} is C_1 in series with (C_2 & C_3 in parallel). $C_{eq} = 0.55\ \mu F$.