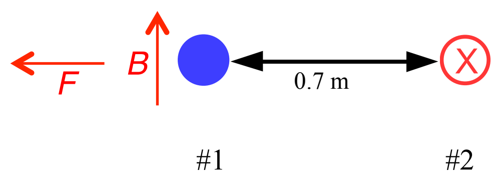
Week 5 Solutions

1. Magnetic field from a wire and force on a second wire

Two long straight wires (15 m each) are positioned perpendicular to the page as indicated in the figure. In wire #1, a current of 4 A is coming out of the page. In wire #2, a current of 2 A is going into the page.



a) What is the magnetic field at the location of wire #1 due to wire #2?
Calculate the magnitude and put an arrow on the drawing to indicate the direction.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 2}{2\pi \times 0.7} = 5.71\times10^{-7}\ \text{T}$$, pointing up (see the drawing).

b) What is the magnitude and direction of the force on wire #1 caused by this field?
Again, put an arrow on the drawing and label it.

$$F = ILB = 4\times15\times\left(5.71\times10^{-7}\right) = 3.43\times10^{-6}\ \text{N}$$, pointing left (see the drawing).

c) Can you make any general conclusions about the force between two parallel wires when the currents in the wires run:

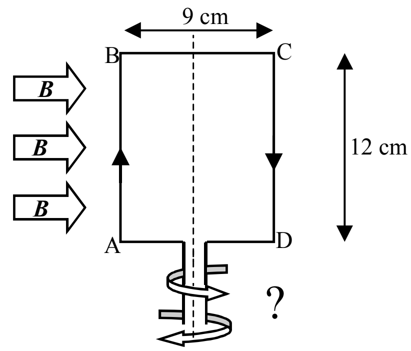
1. in opposite directions. They repel.

2. the same direction. They attract.

Week 5 Solutions

2: Torque on a Loop

A wire loop with a current $I = 8$ A flowing through in the direction shown sits in a uniform magnetic field of strength $B = 3.3$ mT. The initial orientation of the loop is flat on the page and the magnetic field runs left to right as indicated.



a) What is the magnitude and direction of the force on wire segment CD?

$ILB = 8 \times 0.12 \times 3.3{\times}10^{-3} = 3.17{\times}10^{-3}$ N, out of the page.

b) What is the magnitude of the torque on the loop in this position?

The force on AB has the same magnitude as that on CD, but it points into the page. Each force exerts a torque about the axis: $\tau = Fd = 3.17{\times}10^{-3}$ N $\times$ 0.045 m = $1.43{\times}10^{-4}$ N·m, so the total torque is $2.86{\times}10^{-4}$ N·m.

c) Which way does it twist? See the arrows and pick one.

The left wire is being pushed in of the paper, so the lower twisty arrow is correct.

d) If the magnetic field were instead oriented into the page, would the loop feel a torque? If so, describe the sense of the torque in this case.

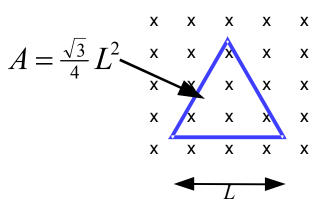
If the magnetic field is perpendicular to the page, then all forces lie in the plane of the page (perpendicular to B), and therefore point through the rotation axis. Therefore there is no lever arm and no torque.

Week 5 Solutions

3. Determining Flux through various Surfaces

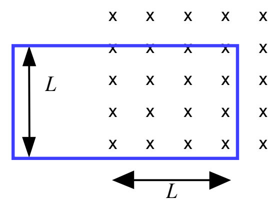
a) Determine the flux through the following loops. The magnitude of the magnetic field is just B, and L is a characteristic length of the objects below as shown.

i) Equilateral triangle



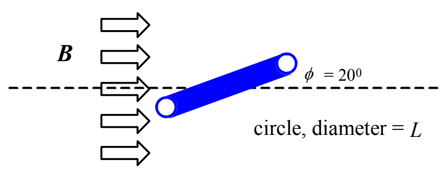
Flux = BA

ii) Rectangular loop partially in field



Flux = BL^2

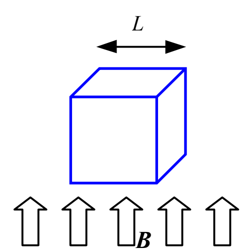
iii) Circular loop at angle f



Flux = $B(\pi L^2/4)\sin\phi$

iv) Cubic box: Flux through **Top**

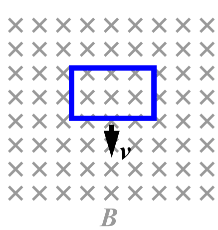
v) Flux through any **Side**.



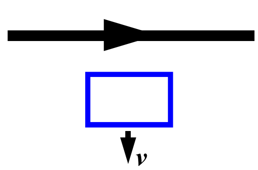
Flux(top) = BL^2

Flux(side) = 0

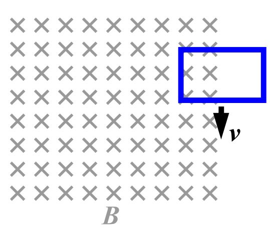
b) Each loop is pulled at a constant speed v as shown in the presence of a magnetic field. Which way will the induced current in the loops flow? (Clockwise, Counterclockwise, or No current will flow...)



No current.
Flux is constant.



Clockwise
B points into paper.
Flux is decreasing.

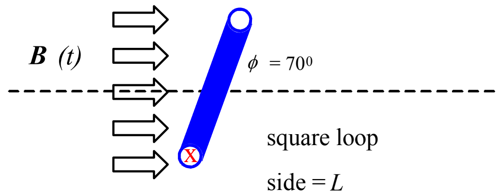


No current.
Flux is constant.

Week 5 Solutions

4. Time-Dependent Magnetic Field and Induced Current

A square loop with side $L = 0.25$ m is shown in a side-on view. A horizontal magnetic field has the time-dependent magnitude $B(t) = 3.5\ \text{T} - \left(0.02\ \dfrac{\text{T}}{\text{s}}\right)t$.



a) What is the flux through the loop at time t = 10 seconds?

Flux = $BL^2\sin\phi$. At t = 10 sec, B = 3.3 T, so Flux = 0.194 T·m^2.

b) At what time, is the flux through the loop zero? At this time, what is the magnitude of the EMF around the loop?

B = 0 when t = (3.5 T)/(0.02 T/s) = 175 sec.

The EMF depends on the rate at which the flux is changing, not on the flux itself.
EMF = (ΔFlux/Δt) = $(\Delta B/\Delta t)L^2\sin\phi$ = (0.02 T/s).(0.25 m)^2·(sin70°) = 1.17×10^{-3} V.

c) More generally, at any time after t = 0, what is the induced current in the loop if the net resistance of the loop is 15 Ω? Make a sketch of the induced current versus time for this problem.

$\Delta B/\Delta t$ is a constant -0.02 T/s. Therefore, the induced EMF is also constant. The induced current is I = EMF/R = 7.83×10^{-5} A.

d) Mark an (X) or a dot (·) on the figure to indicate the direction of the induced current.

See the x in the lower limb of the loop above. The current in the loop must oppose the decrease in the magnetic flux.

Week 5 Solutions

5. A Loop Enters a Uniform Magnetic Field

A rectangular loop with width w = 3 cm and length l = 5 cm is pulled at a uniform speed v = 1.5 m/s into, through, and then out of, a uniform magnetic field of strength B = 3.3 mT (into the page). The picture below shows this situation and the axes below the figure correspond to the position of the front edge of the loop. Thus, the first dashed line is at a time when the loop just begins to enter the magnetic field. The second dashed line is when it just begins to exit.

a) Plot the total flux versus time. Pay attention to the scale of your plot. You can set this accurately based on the information given above.

The maximum flux is $BA = (3.3\times10^{-3}\text{ T})(0.03\text{ m})(0.05\text{ m}) = 4.95\times10^{-6}\text{ T}\cdot\text{m}^2$.
It reaches that value a time $t = l/v = (0.05\text{ m})/(1.5\text{ m/s}) = 0.033$ sec after beginning to enter the field. As the loop leaves the field, the flux returns (linearly) to zero at the same rate.

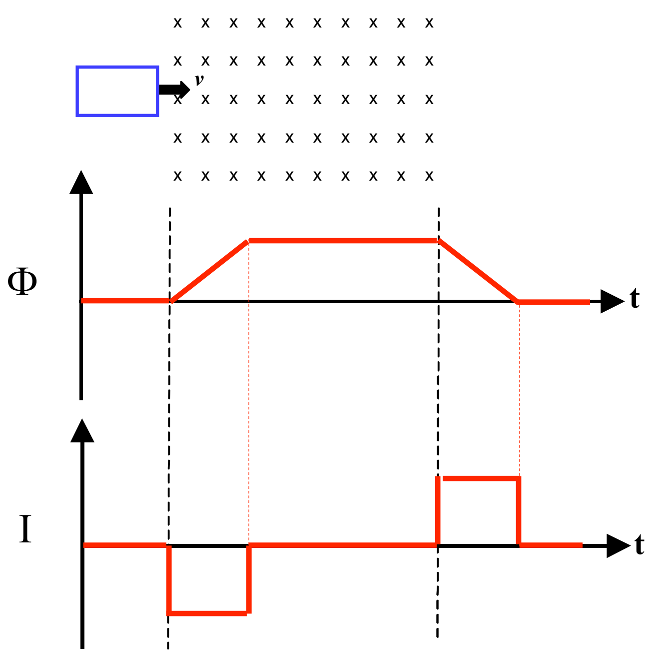
b) If the loop has 15 turns of wire, with a resistance per unit length of 8 Ω/m, what is the total resistance of the loop? Now plot the induced current in the loop on the graph below, again paying attention to the vertical scale (in Amperes).

The total resistance of the wire is R = (8 Ω/m)(15)(2*0.03 m + 2*0.05 m) = 19.2 Ω.

There is current only when the flux is changing. Because the flux is changing at a constant rate during those intervals, the current is constant:

$$I = (\Delta\text{Flux}/\Delta t)/R = (4.95\times10^{-6}\text{ T}\cdot\text{m}^2)/(0.033\text{ sec})/(19.2\ \Omega) = 3.14\times10^{-6}\ A.$$

The direction of current flow is opposite when the loop is leaving the field.



When the leading edge of the loop enters the field.

When the leading edge of the loop leaves the field.