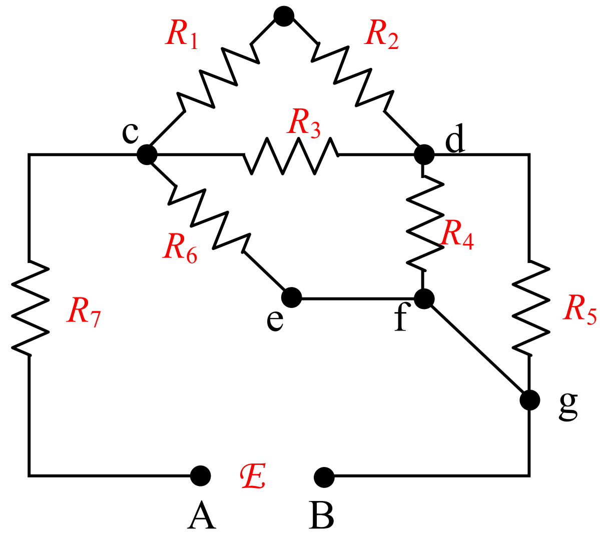
Week 3 Solutions

1. Equivalent Resistance ... and fun with topology

All resistors have the same resistance R = 3 Ω.



a) Simplify this resistor network and then compute the equivalent resistance between points A and B.

See my labels for the resistors.

- R_1 and R_2 are in series, so $R_{12} = 2R$.
- R_3 and R_{12} are in parallel, so $R_{123} = 2R/3$.
- R_4 and R_5, are in parallel, so $R_{45} = R/2$.
- R_{45} and R_6 are in series, so $R_{456} = 3R/2$.
- R_{123} and R_{45} are in series, so $R_{12345} = 7R/6$.
- R_{12345} and R_6 are in parallel, so $R_{123456} = 7R/13$.
- R_{123456} is in series with R_7, so $R_{eq} = 20R/13 = 4.615\ \Omega$.

b) Suppose a 15 V power supply was connected between A and B. What power would be dissipated in this network of resistors?

$\mathcal{E} = IR_{eq}$, so $I = \mathcal{E}/R_{eq.} = (15\text{ V})/(4.615\ \Omega) = 3.25$ A.

The power is voltage times current: $P = \mathcal{E}I = \mathcal{E}^2/R_{eq} = 48.8$ W.

c) What is the potential difference between points c and g, i.e., $V_c - V_g$?

You don't need to worry about R_1 through R_6.

$V_{cg} = \mathcal{E} - IR_7$ (IR_7 is the voltage drop across R_7) = 15 V - (3.25 A)*(3 Ω) = 5.25 V

d) If these resistors were all identical light bulbs, which one would be brightest? Which one would be the least bright?

All the current flows through R_7. All of the other resistors have other resistors in parallel, so none of them carry all the current. Therefore bulb #7 will be the brightest.

The dimmest is more complicated: (B is brightness)

- $B_1 = B_2 < B_3$ (R_1 and R_2 in series, and $R_{12} > R_3$.)
- $B_4 = B_5 < B_3$ and $> B_1$. (R_{45} in series with R_{123}. R_4 and R_5 each get half the current, R_3 gets more than half, and R_1 and R_2 each get less than half.
- $B_6 >$ any of B_1 - B_5, because V_6 is the entire voltage across that part of the network.

So, bulbs #1 and #2 are the dimmest.

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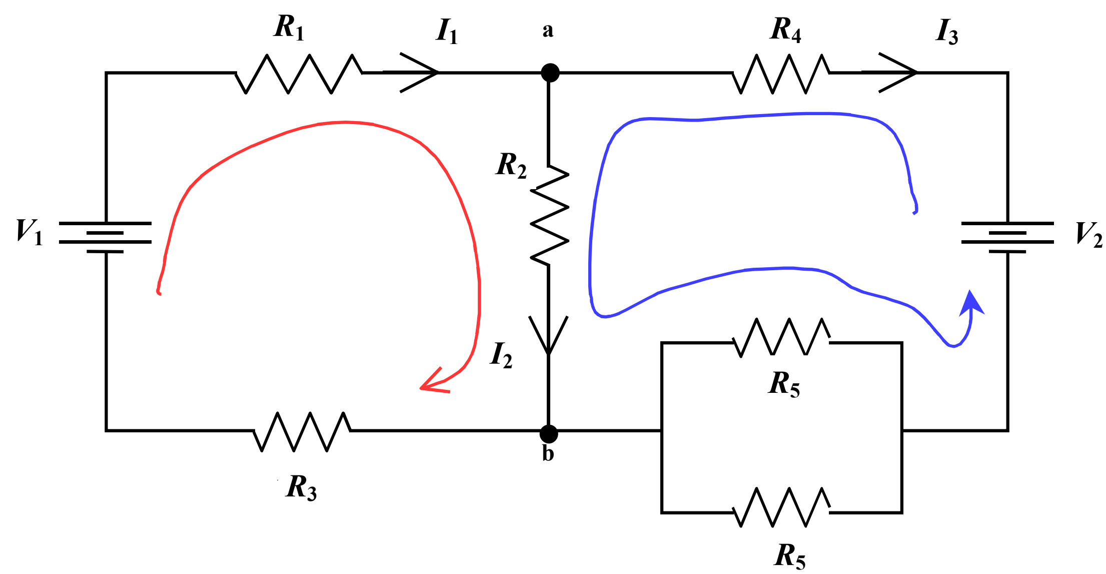
2: Kirchhoff Double Loop Problem



a) Write down the junction rule for the currents that meet at **b**.

I_1 flows out of **b**; I_2 and I_3 flow in. The current out must equal the current in, so $I_1 = I_2 + I_3$.

b) Write an algebraic expression for the Loop Rule for the red (left) loop.

The sum of the voltage changes around a loop must be zero. That's conservation of energy. So (starting at **b** and going clockwise): $-I_1R_3 + V_1 - I_1R_1 - I_2R_2 = 0$

c) Now do the same for the blue (right) loop. First simplify the resistance of the two R_5 resistors into an equivalent resistance. Be careful of which way the "positive" current goes here through the resistors and the battery.

As suggested by the blue arow, start at **b** and go counterclockwise. It actually doesn't matter which way you go. Reversing direction just puts a minus sign on every term, and it cancels out. $+I_3(R_5/2) + V_2 + I_3R_4 - I_2R_2 = 0$ The I_3R terms are positive, because we are "swimming upstream" (against the current). V_2 doesn't depend on the direction of current flow.

If you are now given the values of R_1, R_2, R_3, R_4, R_5, V_1, and V_2, you are left with three equations for three unknown currents. You could now solve for each current I_1 , I_2 and I_3. (You do not need to solve for the currents in this problem.)

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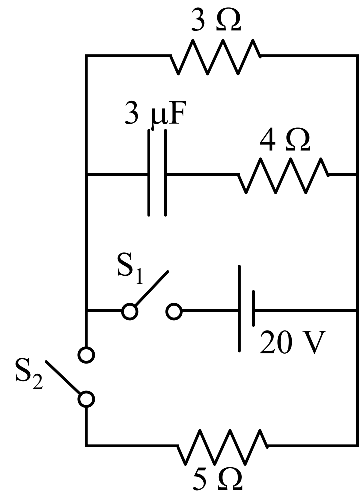
3. Circuit with resistors and capacitance



Initially, S_1 and S are open, and the capacitor is discharged.

a) At time t_0, switch S_1 is closed (while S_2 remains open). What is the current drawn from the battery at the instant t_0?
The capacitor is discharged, so there is no voltage across it. Therefore, the 3 Ω and 4 Ω resistors each have 20 V across them, because each is connected between the battery's terminals. The 5 Ω resistor does not participate.
Therefore, $I_{batt} = I_3 + I_4 = (20\text{ V})/(3\ \Omega) + (20\text{ V})/(4\ \Omega)$
$= 6.67\text{ A} + 5\text{ A} = 11.67\text{ A}$.

b) What is the current through the battery a long time later?
After the capacitor charges up, no current flows through it. Therefore, $I_{batt} = I_3 = 6.67$ A.

c) How much energy is stored on the capacitor a long time after the switch is closed?
No current means no voltage change across the resistor. Therefore, $V_{cap} = 20$ V.
The energy stored in a capacitor is $U = ½CV^2 = 0.5(3\times10^{-6}\text{ F})(20\text{ V})^2 = 6\times10^{-4}$ J.

d) Would your answer to c) change if the 3 Ω resistor at the top of the circuit were removed? Explain.
No. The calculation in c) does not depend on anything having to do with the 3 Ω resistor.

e) Now S_1 is opened and S_2 is closed at exactly the same instant. How much current goes through the 4 Ω resistor immediately after this event?
The battery is disconnected, but the capacitor is charged up and, for a short time, acts like a 20 V battery. To find the total current calculate the equivalent resistance:
R_3 and R_5 are in parallel: $R_{35} = 15/8\ \Omega$. R_{35} is in series with R_4, so $R_{eq} = 5.875\ \Omega$.
$I_{tot} = (20\text{ V})/(5.875\text{ W}) = 3.40$ A. That's I_4, because all the current flows through R_4.

f) Which direction does the current go through the 4 Ω resistor, to the left or to the right?
The left side of the capacitor is the positive side, because it was charged up by the battery. Therefore current flows from left to right through the resistor.

g) How long does it take for the current to fall to 1/e of the value you obtained in e)?
The time constant of the circuit is $\tau = R_{eq}C = (5.875\text{ W})(3\times10^{-6}\text{ F}) = 1.76\times10^{-5}$ s.
It takes one time constant to fall to 1/e, so $t = \tau$.