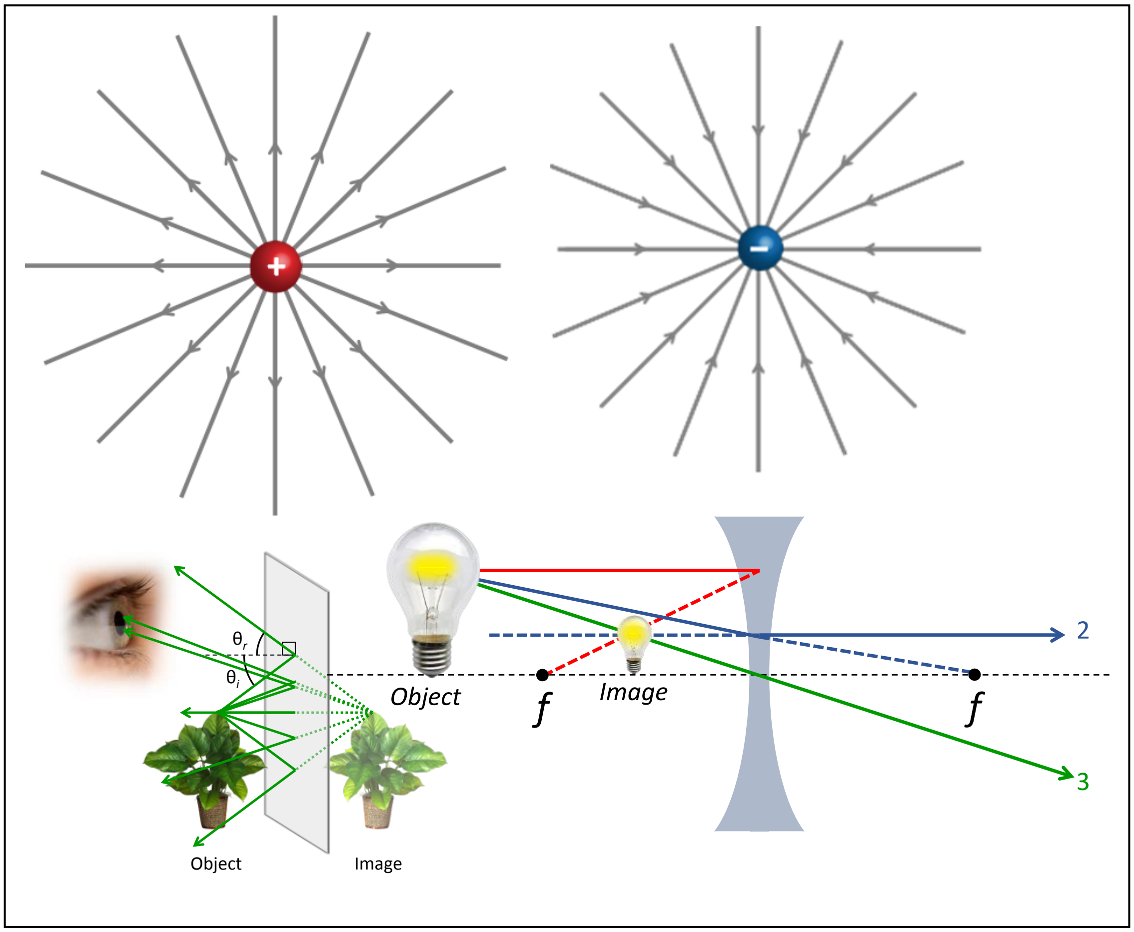
PHYSICS 102

University of Illinois at Urbana-Champaign



Discussion Problems

Fall 2014

Physics Department
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My Study Plan

In the table below fill out your study plan for Physics 102 this semester.

For each day of the week include time to:

1) Complete your homework.
2) Study for the week's quiz.
3) Study for the upcoming exam.

Remember a good study schedule includes setting aside ***30 to 60 minutes each day*** to work on your physics homework and studying. Don't forget to mark your **discussion section** time and the **office hour you plan to attend each week**.

Time	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8:00 AM							
9:00 AM							
10:00 AM							
11:00 AM							
12:00 Noon							
1:00 PM							
2:00 PM							
3:00 PM							
4:00 PM							
5:00 PM							
6:00 PM							
7:00 PM							
8:00 PM							
OTHER							

Welcome to Discussion!

Welcome to your Physics 102 Discussion section. Your discussion section will be led by a teaching assistant (TA) who has been trained in the techniques used in the Department of Physics at the University of Illinois at Urbana-Champaign.

My Discussion Section is: ____

My TA's name is: ____

My TA's office hour is: ____

Discussion "closes the loop". In discussion you will work through problems designed to help you not only work through the technical aspects of physics problems, but put these concepts into context.

The Goals of the Course:

By the end of this course you will be able to:

- describe the physics concepts in problems involving
  - electricity & magnetism
  - optics
  - modern physics.
- execute basic problem-solving strategy for problems in
  - electricity & magnetism
  - optics
  - modern physics.

The Discussion Manual

The Discussion Manual consists of one lesson for each week of the semester. Each lesson contains the following structure:

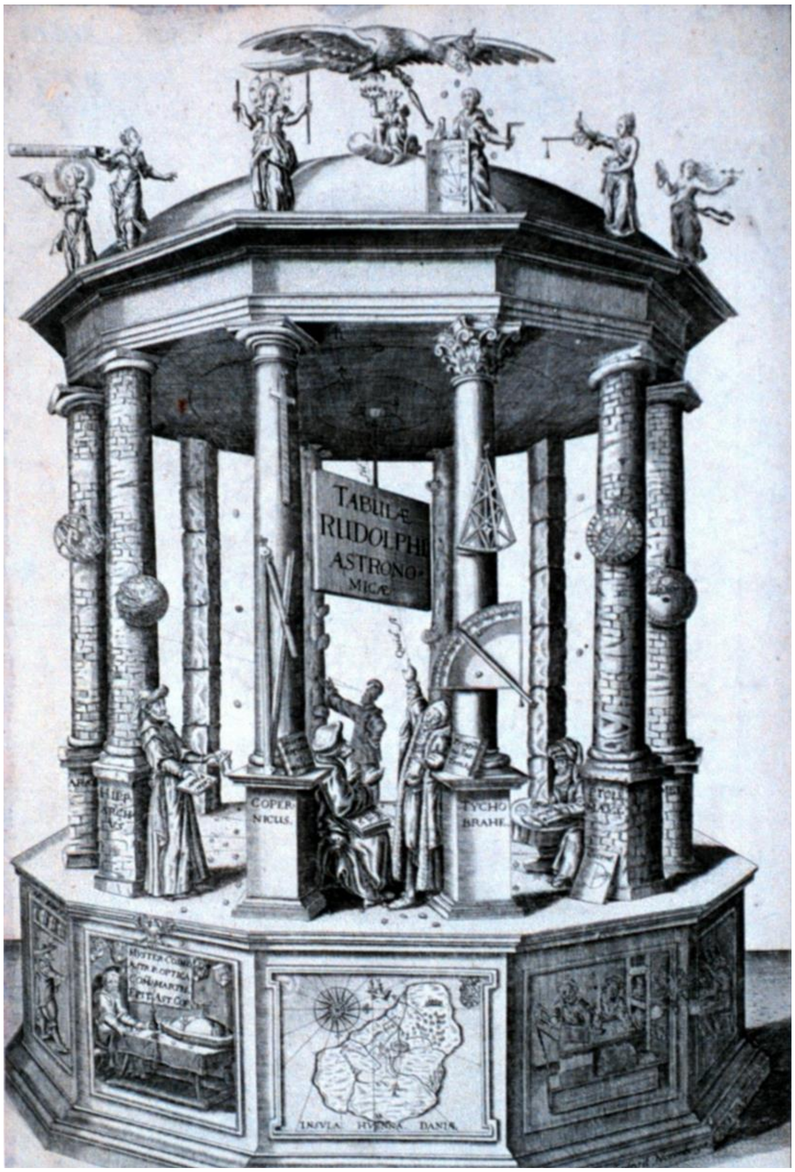
- A summary section containing:
  - A glossary of important terms for the lesson.
  - A summary of key concepts of the lesson.
- The situations to be solved during the discussion section.

You should come to discussion prepared to work the problems. It is your responsibility to:

1) Attend every lecture during the week.
2) Complete your homework
3) Attend at least one office hour prepared with good questions.
4) Read and make notes on the summary for each lesson before arriving at your discussion section.
5) Read and make notes on the questions in each lesson before arriving at your discussion section.
6) Come to your discussion section prepared with good questions.

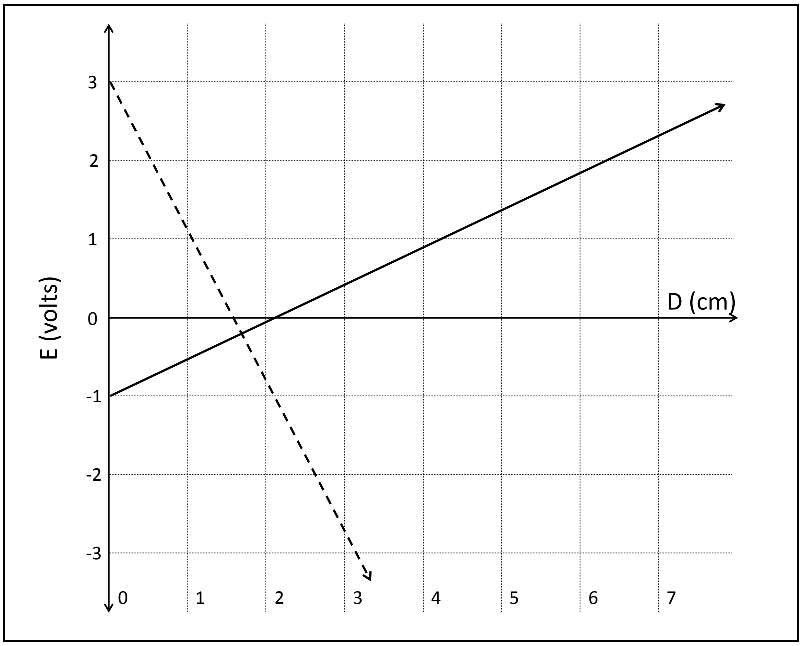
Your teaching assistant will help you formulate good questions.

Week 1 Math Review



Situation 1: Slopes and lines:

Using the graph and the *slope-intercept* form for a straight line [$y = mx + b$], fill in the table:



	Slope (m) with units	Intercept (b) with units	Equation in Slope-Intercept Form
Solid Line			
Dashed Line			

Situation2: Quadratic Formula:

Find all values for x satisfying: $2x^2 - 5x + 3 = 0$.

Situation 3: Logarithms and Exponents.

Rules and Notation for Logarithms and Exponents:

Exponents:

$$y^n z^n = (yz)^n$$
$$(y^n)^m = y^{nm}$$

Logarithms: $x = \log_B y$, x is the *logarithm* of y in base B.

$$y = B^x$$
$$B \equiv \text{Base};\ x\text{: exponent}$$

$\log y$	*Logarithm of y in base 10*
$\ln y$	*Natural logarithm of y. Natural logarithms have base e.*

1) Using the rules for exponents find the following values:

$$13^3 15^3 = (\quad)^3 =$$
$$\left(13^3\right)^2 = (\quad)^{(\ )} =$$

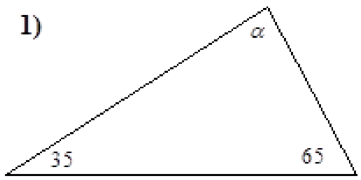
2) At what time has the charge on a capacitor has decayed to 10% of its original value. The charge on a discharging capacitor is: $Q(t) = Q_0 e^{-t/\tau}$. Let τ = 0.2 μs. The quantity Q_0 is the value $Q(t = 0)$.

3) The charge on a charging capacitor is $Q(t) = Q_0\left[1 - e^{-t/\tau}\right]$. If Q_0 is 3 μC (*micro-Coulombs*), how much charge is on the capacitor after τ seconds, i.e., after one time constant?

Situation 4: Geometry time!

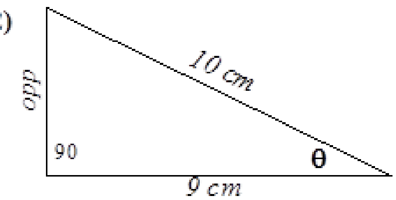
Fill in the blanks! You may look up any expressions you do not know.

1)



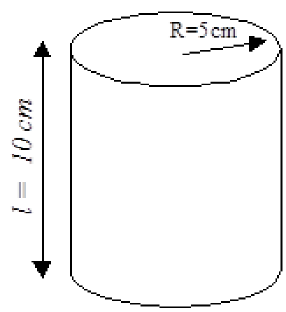
α = ______

2)



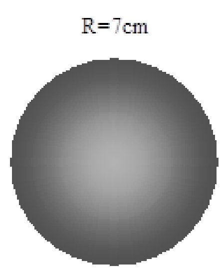
opp = ________
area = ________
θ = ________

3)



area of lid = ________
area of can = ________
volume of can = ________

4) area of sphere = ________
volume of sphere = ________

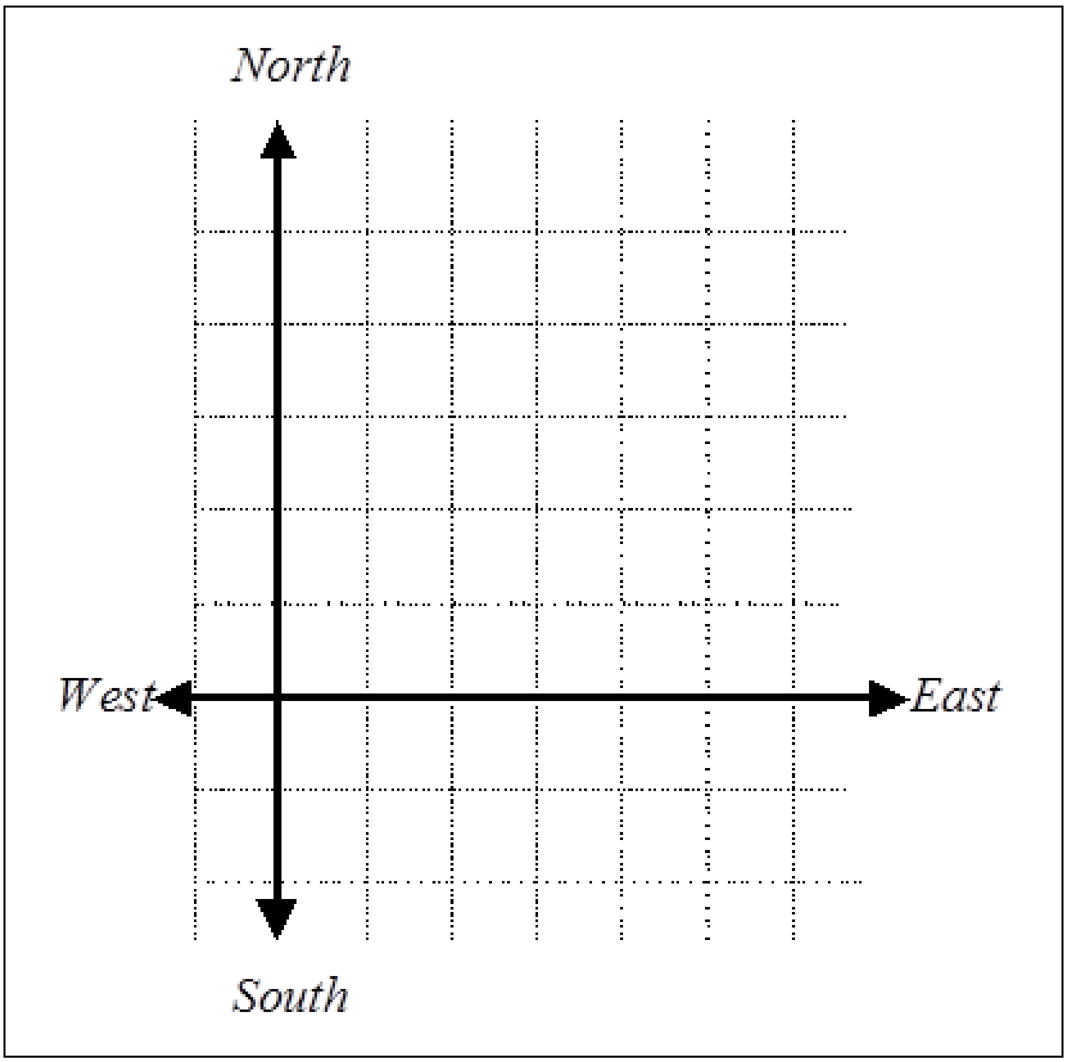


Situation 5: Vectors!

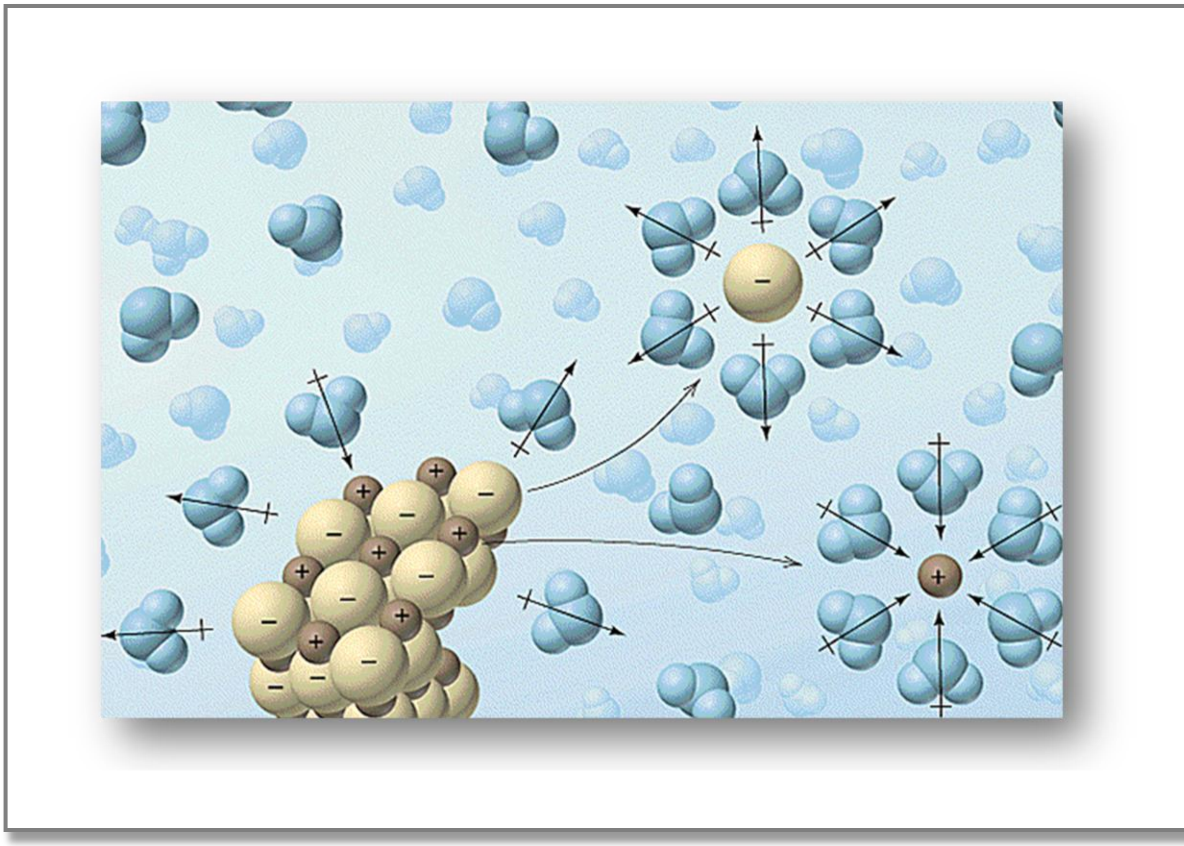
From your textbook:

> A jogger runs $145\ m$ in a direction $20°\ east\ of\ north$ and then $105\ m$ in a direction $35°\ south\ of\ east.$ Determine the magnitude and direction of the resultant vector for these two displacements.

1. Sketch each vector on the grid provided. Both vectors start at the origin.
2. Determine the components of each vector.
3. Add the components and record your result as the components of the final displacement vector.
4. Determine the magnitude of the final displacement vector and the angle with respect to one of the directions on the grid.
5. Sketch the final displacement vector



Week 2 Lectures 1 and 2—Electric Charges, Electric Forces, and Electric Dipoles



Summary

Glossary

***Attraction:** the result of a force that tends to draw two or more bodies together.*

***charging by Induction:** imparting charge on a neutral object by virtue of its proximity to a charged object.*

***Conductors:** materials with freely moving electrons.*

***Coulomb (C):** the unit of electrical charge.*

***Electric dipole:** a positive and negative charge pair of the equal magnitude q separated by a distance d.*

***Electric dipole moment ($\vec{p}$):** the measure of the separated positive and negative charges in an electric dipole (a vector quantity)*

***Electric quadrupole:** an anti-parallel pair of electric dipoles, separated by the dipole separation distance.*

***Electrical charge (q):** an intrinsic property of matter responsible for the electro-static force.*

***Insulators:** materials with tightly bound electrons.*

***Principle of Superposition:** the method of calculating forces (or fields) experienced by a charged object by a collection of charges. The force (or field) on that object is calculated from each of the charges individually, and taking the resulting vector sum.*

***Repulsion:** the result of a force that pushes two or more bodies away from each other.*

Key Ideas

The Electro-static Force: Coulomb's Law

Coulomb's Law describes the force between two charged objects. This force is the electro-static force:

- **Electro**: electrically charged objects.
- **Static**: the objects are not moving.
- **Force**: the interaction between two objects

Coulomb's Law:

$$F_{12} = k\frac{q_1 q_2}{r^2};\, k = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

- q_1: the electrical charge on object 1
- q_2: the electrical charge on object 2
- r^2: the distance between the two objects squared

Units Check!

- The unit of charge (q_1, q_2) is the Coulomb (C).
- The unit of distance (r) is the meter (m).
- The unit of force is the Newton $(N = kg\ m/s^2)$ Do the units work out?

Direction

Force is a vector. Coulomb's Law gives us the magnitude. What is the direction of the electro-static force?

Repulsion: If q_1 and q_2 have the same sign (positive or negative) the force ***repels the charged objects*** from each other with equal and opposite force.

Attraction: If q_1 and q_2 have opposite signs (one positive, the other negative) ***charges are attracted*** to one another.

Important Fact: If a charge has a mass m, the Coulomb force $\vec{\boldsymbol{F}}$ gives the charge an *instantaneous acceleration* with magnitude:

$$|\vec{\boldsymbol{a}}| = \frac{|\vec{\boldsymbol{F}}|}{m}$$

Do you know why?

Situations for Lecture 1: Electric Charges and Coulomb's Law

Situation 1: Charging it Up

Consider the following situation:

Three metal spheres, labeled 1, 2, and 3, rest on glass stands.

Spheres 1 and 2 carry a net zero charge. Sphere 3 carries an unknown *positive* charge.

The following steps occur:

1) You place spheres 1 and 2 on a table separated by $30\ cm$. Sphere 1 is on the *left* of sphere 2
2) You place sphere 3 near sphere 2 on the *right* of sphere 2. Sphere 2 and sphere 3 do not touch.
3) You attach sphere 1 and sphere 2 with a conducting wire.
4) You remove the wire.
5) You remove sphere 3 from the table.

You would like to know the charge on sphere 3 from the resulting charges of spheres 1 and 2 if you can figure it out.

1) List the important information in this problem setup. Discuss with your teammates why this information is important. Take notes on the discussion.

2) Sketch each step listed in the problem setup. Make sure your entire team understands each sketch and agrees it's correct. Take notes on the discussion

3) Answer the following questions:
   a. The metal spheres are (choose one) conductors/insulators.
   b. The glass stands are (choose one) conductors/insulators.
   c. The charge on sphere 1 is (choose one) positive/negative/zero.
   d. The charge on sphere 2 is (choose one) positive/negative/zero.
4) You measure the force between sphere 1 and sphere 2. You find $F_{12} = 3\ N$ and the spheres *attract each other*. What is the magnitude of the charge on sphere 1 and sphere 2?

5) You now remove sphere 1 and replace it with sphere 3. You measure the force between sphere 2 and sphere 3 and find $F_{13} = 8\ N$ and *attractive.* What is the charge on sphere 3?

6) This problem is an example of ***charging by induction***. Discuss with your teammates how you might use charging by induction in your field of study. Try to think of at least 2 examples per person.

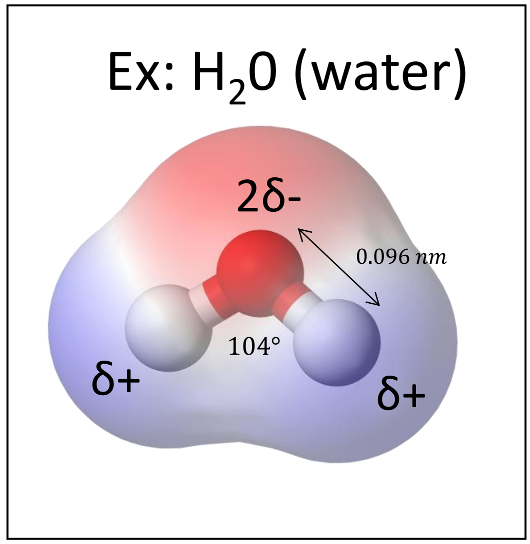
Situations for Lecture 2: The Electric Dipole

Situation 1: The Salt in the Water

Consider the following situation:

In a liter of distilled, de-ionized water you dissolve 1 gram of sodium chloride (NaCl). Using the information in the table and the figures discuss with your teammates the questions in this situation:

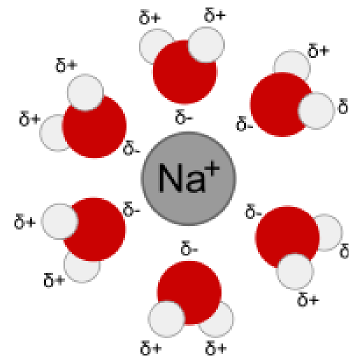
Radius of water molecule	$2.8\ \text{Å}$
Radius of sodium ion Na^+	$1.1\ \text{Å}$
Electric Dipole charge, $\lvert\delta\rvert$, (water)	$0.35e$
Dipole moment	$\vec{p} = q\vec{d}$
Average intermolecular distance for water molecules	0.27 nm



1) How will the water molecules interact with the sodium ions in the water. Take notes on your discussion.

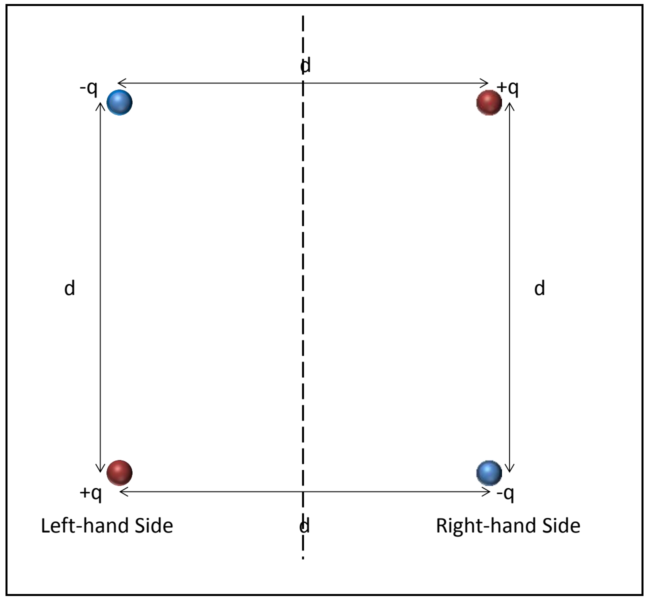
2) Calculate the magnitude and direction of the electric dipole moment of a water molecule.

3) Assuming the diagram accurately describes the hydration shell of a sodium ion in water, how will the water molecules organize around a sodium ion. Be specific about the physical processes involved. Take notes on your discussion.



"Hydration shell"

Situation 2: The Electric Quadrupole



Consider the following collection of charges, known as an electric quadrupole. We will use the electric quadrupole to explore the *Principle of Superposition.*

1) With your teammates discuss the similarities and differences between the electric dipole and the electric quadrupole. Take notes on your discussion.

2) Consider the left-hand pair of charges. These two charges form an electric dipole. Calculate the force on the negative charge of the left-hand dipole due to the positive charge in the left-hand dipole.

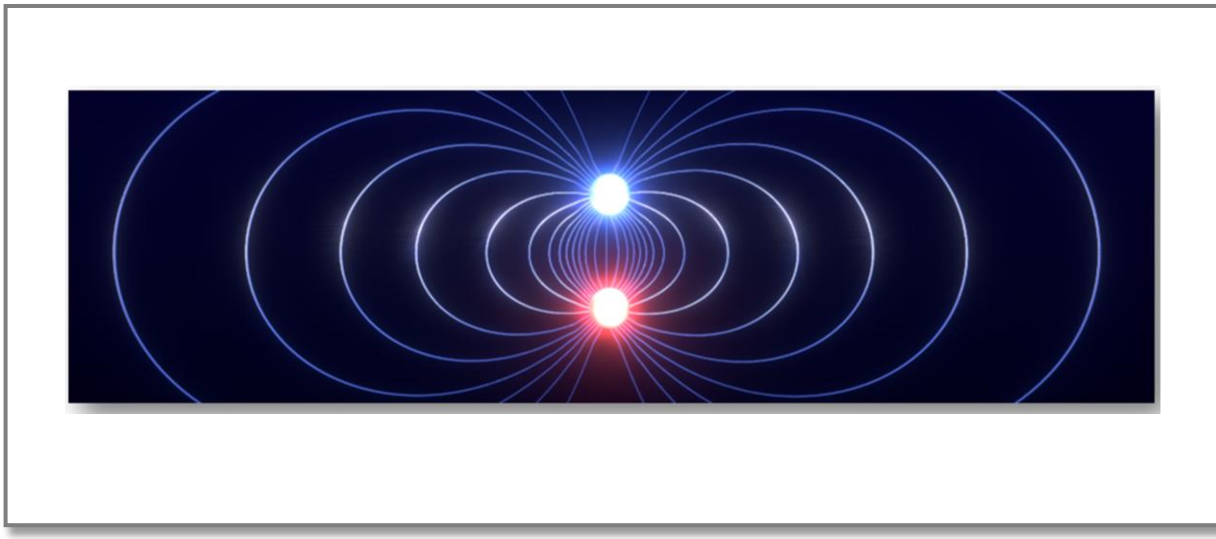
3) Now use the Principle of Superposition to calculate the force on the left-hand negative charge from all of the other charges. Remember: **Force is a vector!**
   a. Calculate the force on the left-hand negative charge from the right-hand positive charge.

   b. Calculate the force on the left-hand negative charge from the right-hand negative charge.

   c. Together with your result from 2) take the *vector sum* of the forces you calculated.

4) Where might electric dipoles, quadrupoles, or the Principle of Superposition be important in your field of study, or a field in which you are interested? Discuss this with your teammates, and come up with 2 examples per person. Take notes on your discussion.

Week 3 Lecture 3—The Electric Field



Summary

Glossary

Electric field ($\vec{E}$): ***the force per unit charge at a given point. Units:*** N/C

Field lines: ***a technique for visualizing the electric field generated by a charge or collection of charges. Field lines leave positive charges and arrive at negative charges.***

Test charge(q): ***an assumed (sometimes called fictitious) electrical charge used to calculate electric fields. Always assumed to be positive.***

Key Ideas

The Electric Field : Also called the *electro-static field*.

The electric field, $\vec{\boldsymbol{E}}$, is a vector quantity. It has a ***magnitude*** and a ***direction.*** The electric field is calculated from the electro-static force using Coulomb's Law:

$$|\vec{\boldsymbol{E}}| \equiv \frac{|\vec{\boldsymbol{F}}|}{q} = \frac{kQ}{r^2}; \text{ (Units: } N/C)$$

Let's define the charges in this expression:

- q is the ***test charge.*** The test charge is typically a positive charge.
- Q is the field-producing charge. The field producing charge can be either positive or negative.

Magnitude: the strength of the force an *arbitrary charge* q would experience from the field producing charge Q if q is placed at a given point in space.

- Units: *force per charge* (N/C)
- $|\vec{\boldsymbol{F}}| = q|\vec{\boldsymbol{E}}|$

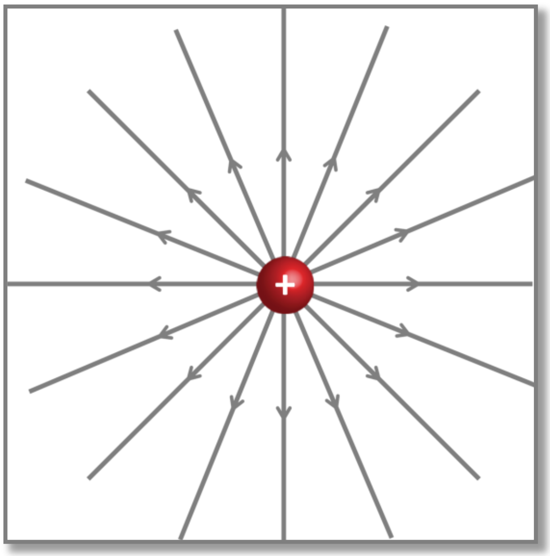
Direction: the acceleration of a positive charge released from a point in space. (Note: The direction of acceleration is opposite for negative charges.)

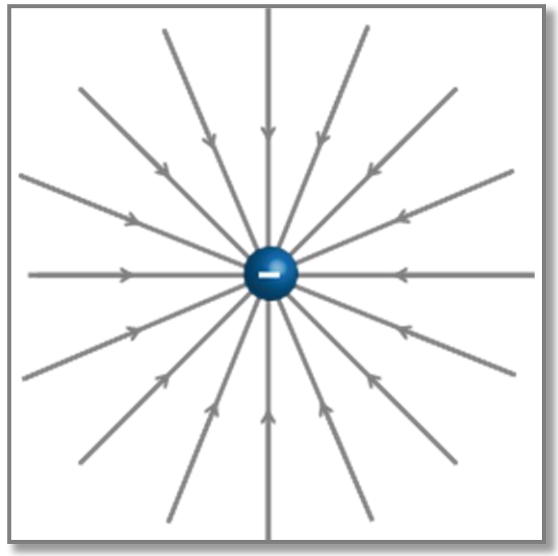
Advantages of the electric field:

- It is a *property of a charge or collection of charges*
  - the electric field exists for a single point charge
  - the electric force does not exist for a single point charge.
- Electro-static forces can be calculated for different test charges.
- For a collection of charges, apply the ***Principle of Superposition***.

Field lines

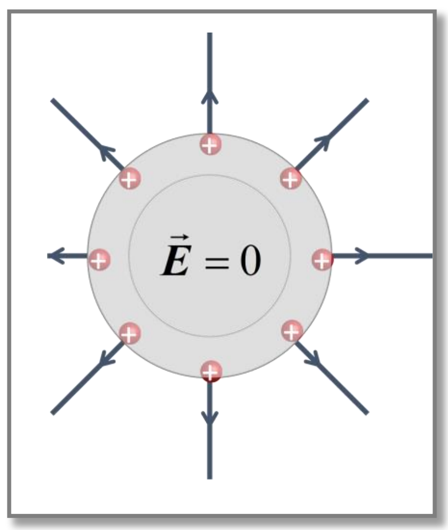
Field lines are used to visualize the electric field. They are indicated by arrows which *leave* positive charges and *arrive* at negative charges.





Rules for field lines:

- The density of lines indicates the *strength of the electric field*.
- *Field lines never cross*.
- Field lines are always perpendicular to a conducting surface.
- The electric field inside conductors is *always zero* in electro-static conditions.
- The electric field inside an insulator is non-zero.

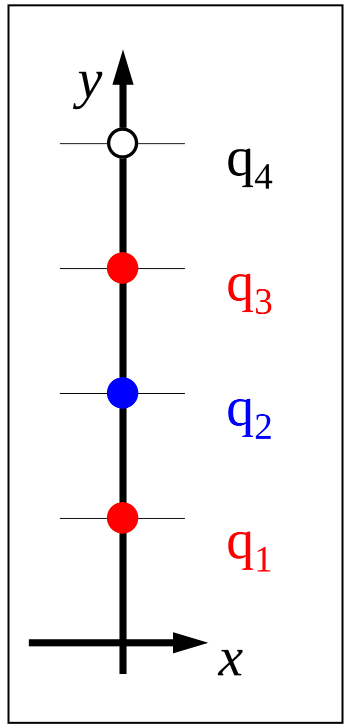


Situations for Lecture 3: The Electric Field

Situation 1: The Charge Stack

A charge stack of 4 different charges is shown in the figure. They are fixed in position along the y-axis. The values and positions are given in the table below:

Charge	Value	Position
q_1	$3\ \mu C$	$1\ m$
q_2	$-6\ \mu C$	$2\ m$
q_3	$+9\ \mu C$	$3\ m$
q_4	$unknown$	$4\ m$



1) Discuss with your teammates your expectations for the strength of the electric field at the origin ($y = 0\ m$) of the charge stack. Make sure you discuss the effect of each charge separately, and the result of them together. Take notes on your discussion.

2) To produce an electric field $|\vec{E}| = 0\ N/C$ at the origin:
   - q_4 must be positive or negative (circle one)
   - q_4 must have magnitude ______________

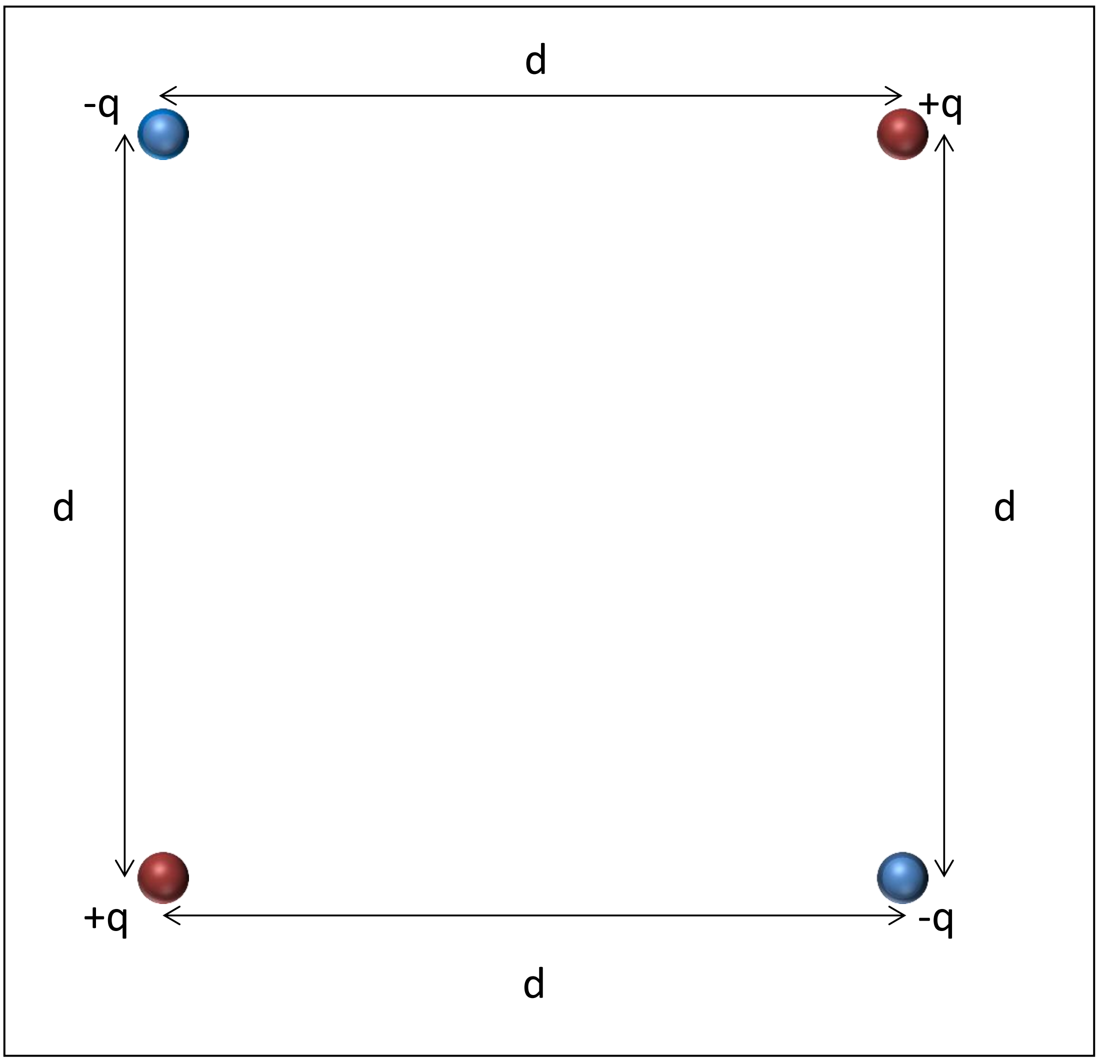
3) What is the force on a $12\ \mu C$ charge placed at the origin? Find the magnitude and direction of the force. Assume that the charge, q_4, you calculated in part 2) is in place. Remember to discuss your problem solving strategy first

4) Discuss with your teammates how you might find a charge stack in your field of study or a field of interest to you. Try to find 2 examples per person. Take notes on your discussion.

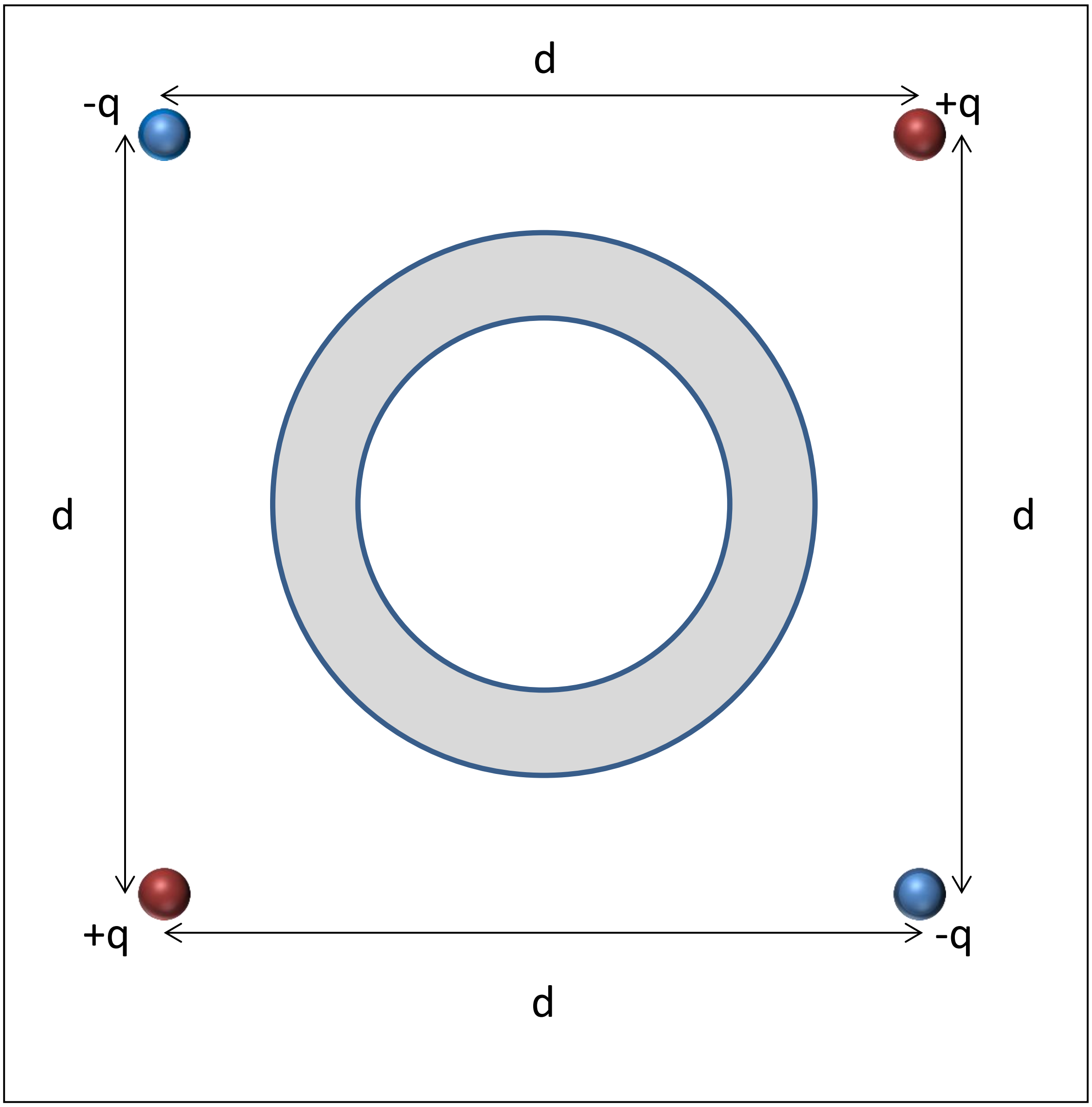
Situation 2: The Electric Quadrupole Revisited

Consider the electric quadrupole we worked in the problems for Lecture 2.

1) Draw the field lines which represent the electric field of the electric quadrupole.



2) Consider the same electric quadrupole with a conducting shell centered within the charges as shown in the figure. Draw the field lines now with the conductor in place. Discuss the results with your teammates and take notes on the discussion.



Situation 3: The Salt in the Water Revisited

Consider the hydration shell problem from Lecture 2. Below is the table of useful information from that problem.

Radius of water molecule	2.8 Å
Radius of sodium ion Na^+	1.1 Å
Electric Dipole charge, $\lvert\delta\rvert$, (water)	$0.35e$
Dipole moment	$\vec{p} = q\vec{d}$
Average intermolecular distance for water	0.27 m

1) Calculate the electric field produced by the sodium ion. Remember the charge of an Na^+ sodium ion is $1e$. You may leave your results in terms of the charge on the electron, e. Sketch your result.

2) The torque on an electric dipole in an electric field is: $\tau = pE\sin\theta$. Calculate the magnitude of the torque on the electric dipole moment of the water molecules for the following orientations with respect to a field line from the sodium ion:

Angle	Torque
$90°$	
$45°$	
$30°$	
$60°$	
$180°$	

Make a plot of your results and discuss with your teammates the consequences of this result.

Week 4 Lectures 4 and 5—Electric Potential Energy and Electric Potential Difference

Summary

Glossary

***Electric potential difference (V):** a property of a charge configuration related to the energy which can be stored by adding a charge into the configuration. It is analogous to the relationship between the electric field and the electro-static force. Units: J/C also called the Volt.*

***Electrical potential energy:** the energy stored in an electric field.*

***Gravitational potential energy:** the energy stored in the gravitational field.*

***Potential energy (U):** stored energy—that is due only to the configuration of objects. Measured in Joules (J) as a difference from a known reference value.*

Key Ideas

Electrical Potential Energy

Potential energy (U) is energy stored by a configuration of objects and can be positive or negative. It is measured in Joules and always implies a difference *with respect to* a reference point. This reference is defined as *zero potential energy*. For example the height of an object in a gravitational field (gravitational potential energy) is the *configuration*; the zero potential energy is often ground level.

Electrical potential energy is related to the work done on an object in an electric field:

$$W_{AB} = U_A - U_B$$

This is the *work done by the electric field* when moving a charge from point A to point B.

Potential energy is always associated with the work required to *configure* the system. To build a brick wall, it takes work to assemble the bricks. That energy is stored as potential energy (gravitational). To collect a number of positive electric charges and place them near each other takes work. They will push apart! A group of closely spaced charges can store a lot of potential energy.

Work can be negative! If it is you that is moving the charge, the work you do is the negative of the work done by the electric field!

Electric Potential Difference (Potential)

The *electric potential difference* (V) provides a way to calculate potential energy (or work) done by an electric field. To figure out the work required to move a charge, q, from point A to point B in an electric field, define *electric potential difference* as *potential energy per unit of charge*. It is expressed formally as follows:

$$V_{AB} = V_B - V_A = -\frac{W_{AB}}{q_0} = \frac{U_B}{q_0} - \frac{U_A}{q_0}$$

Let's define some terms:

- $V_B - V_A$: The potential at the stopping point (B) minus the potential at the starting point (A)
- V_{AB}: The potential difference (Units J/C)—Just like potential energy it is always calculated (or measured) with respect to a known reference value.

To find the potential energy of the charge in an electric field:

$$U = qV_{AB}$$

We can calculate the electric potential difference from a point charge:

$$V = \frac{kq}{r}$$

A few notes:

- This is the potential difference between a particle at a point r and infinity (what is $1/\infty$?)
- You'll notice that this looks very similar to Coulomb's Law (how is it different?) That's not an accident. While it is beyond the scope of this cours, if you are interested, talk with your TA who will be happy to provide you with more information.

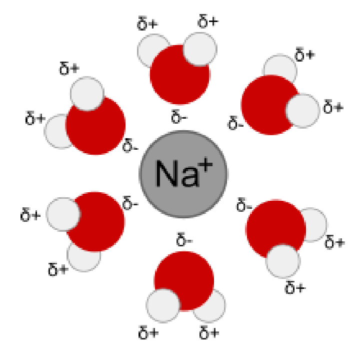
Situations for Lecture 4: Electrical Potential Energy and Work

Situation 1: The Salt in the Water—Potential Energy Edition

Let's return to our hydrated sodium ion.

In the problems for Lecture 3 we calculated the torque a water molecule would experience as it aligned itself with the electric field of the sodium ion. Let's add to the table:

Angle	Force Experienced by δ_+	Torque	Potential energy of dipole
$90°$			
$45°$			
$30°$			
$60°$			
$180°$			



"Hydration shell"

Fill in the above table with the requested quantities for water molecules oriented at the given angles with respect to the sodium ion's electric field. Study your results from Lecture 3 to help you.
Recall: $U_{dip} = -pEcos\theta$.

Discuss your results with your teammates. Take notes on your discussion.

Situations for Lecture 5: Electrical Potential Difference

Situation 1: Making the Electric Quadrupole

Let's return to our electric quadrupole. This time we are going to analyze the fields, potentials, and energy stored in the electric quadrupole. Using the information in the table answer the following questions:

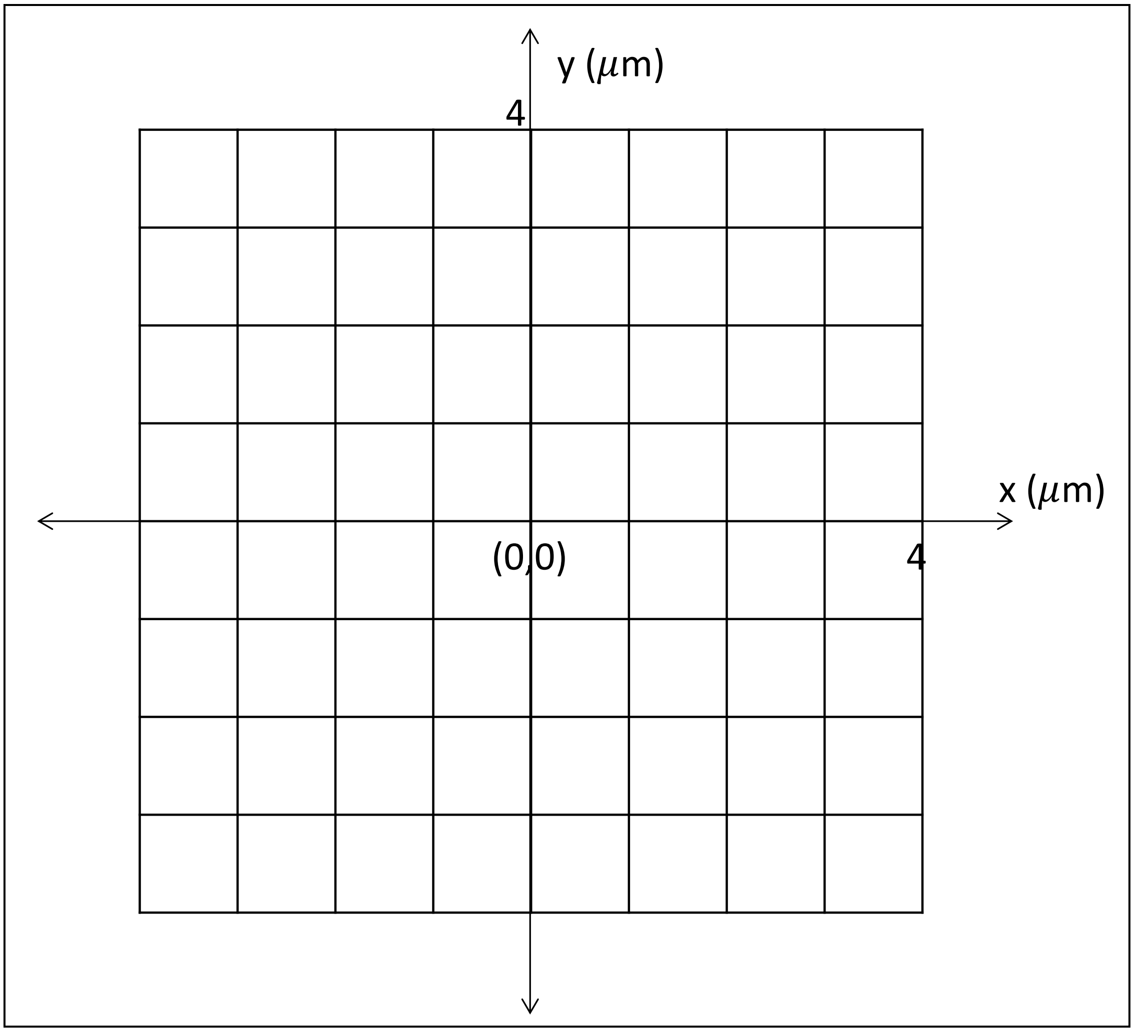
Charge	Location (x,y)
$q_1 = 100\ \mu C$	$(-3\mu m, -3\ \mu m)$
$q_2 = -100\ \mu C$	$(-3\ \mu m, 3\ \mu m)$
$q_3 = +100\ \mu C$	$(3\ \mu m, 3\ \mu m)$
$q_2 = -100\ \mu C$	$(-3\ \mu m, -3\ \mu m)$

1) Assembling the quadrupole:
   a. Discuss with your teammates how you would assemble the electric quadrupole described above. What will you experience as you put the quadrupole together? Be specific. Take notes on your discussion.

b. Start with q_1. On the grid on the next page assemble the electric quadrupole. Fill in the table below with the values for the electro-static force, electric field, electric potential difference, and electric potential energy experienced by each charge as it is moved into its final position.

Charge	Electro-static Force	Electric Field	Electric Potential Difference	Electric Potential Energy
q_1				
q_2				
q_3				
q_4				

Assemble the electric quadrupole on the grid below:



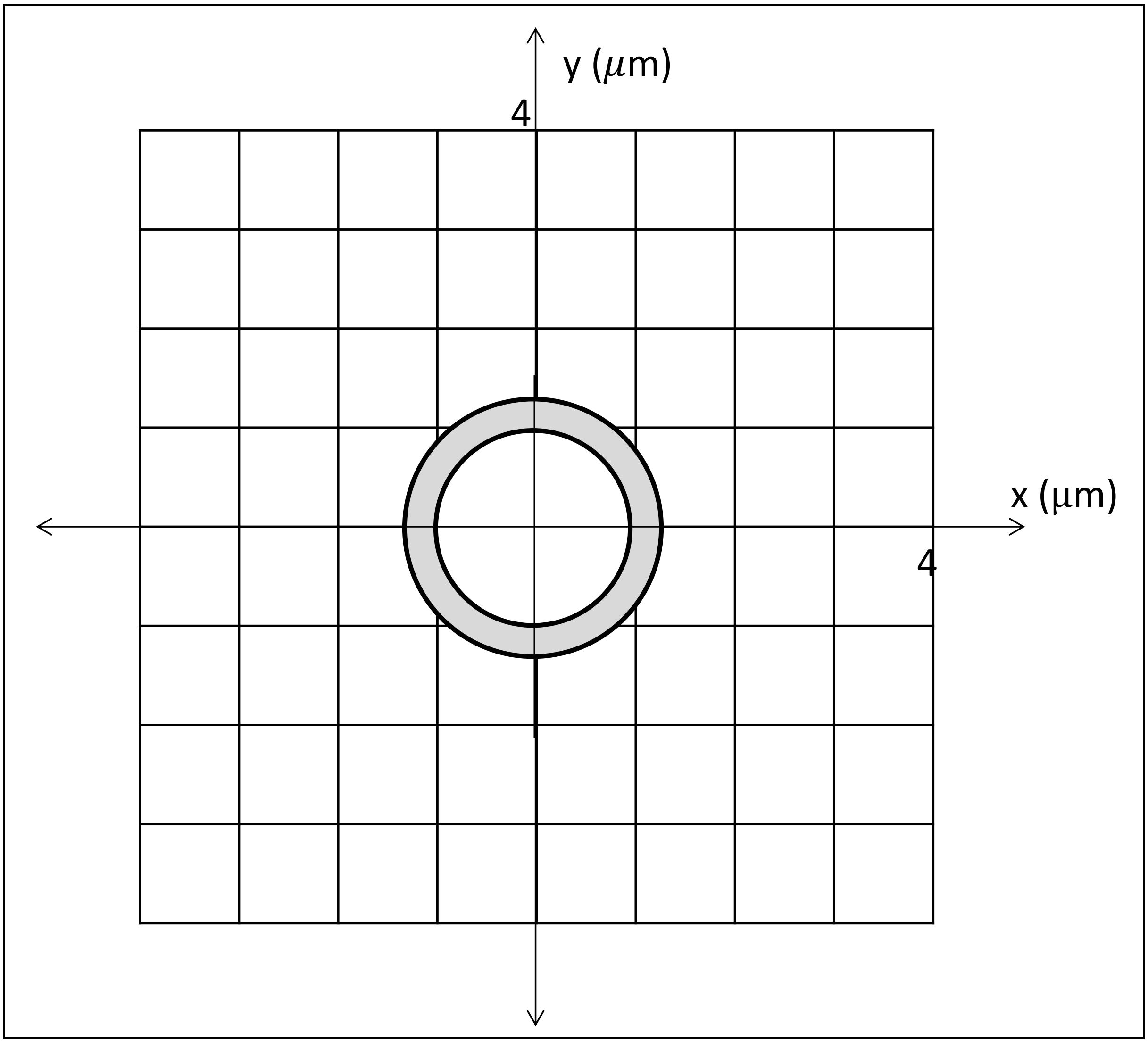
2) On your diagram of the electric quadrupole, draw the electric field lines representing the electric field for this configuration of charges.
3) Lines of equal potential (*equipotential* lines) run perpendicular to the lines of electric field. On your diagram, draw the equipotential lines representing the lines of equal potential difference from this configuration of charges.
4) Discuss the result with your teammates. Take notes on your discussion

Situation 2: The Quadrupole and the Conducting Shell

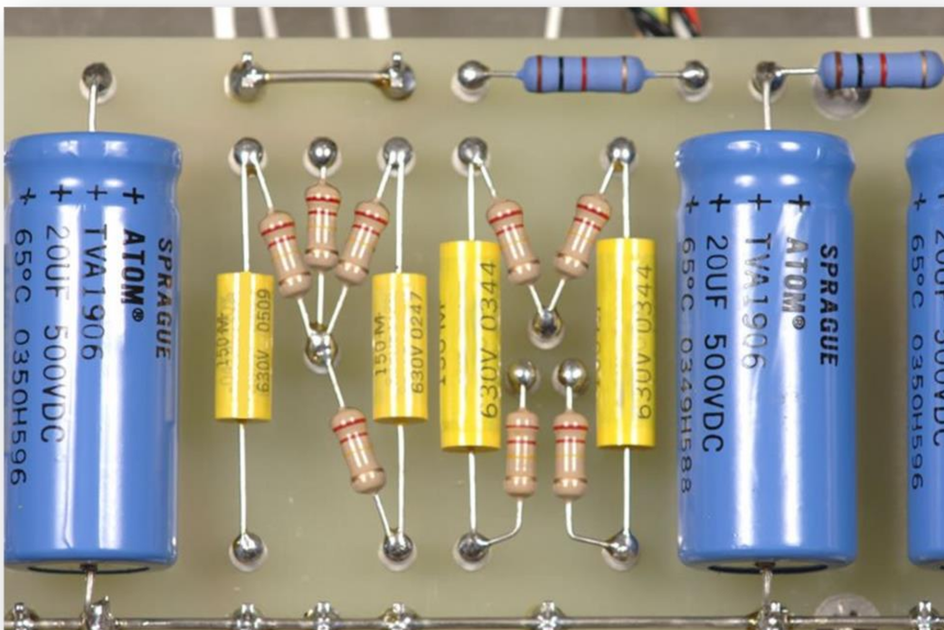
Now construct the electric quadrupole on the grid on the next page, including a conducting shell centered at the origin. Draw the electric field and equipotential lines for this configuration. Discuss with your teammates the similarities and differences between this configuration and the configuration without the conducting shell. Take notes on your discussion.

The table below contains the important information about the quadrupole.

Charge	Location (x,y)
$q_1 = 100\ \mu C$	$(-3\mu m, -3\ \mu m)$
$q_2 = -100\ \mu C$	$(-3\ \mu m, 3\ \mu m)$
$q_3 = +100\ \mu C$	$(3\ \mu m, 3\ \mu m)$
$q_2 = -100\ \mu C$	$(-3\ \mu m, -3\ \mu m)$


y (μm)
4
x (μm)
4

Week 5 Lectures 6 & 7—Circuit Elements



Summary

Glossary

***Battery:** a device that produces a constant potential difference.*

***Capacitance** (C)**:** a measure a capacitor's ability to store charge. It is the ratio of the charge on the conductor plates to the potential difference between the conductors. (Units: Farad F).*

***Capacitor:** a device which stores electrical charge.*

***Current** (I)**:** the flow of charge. Units: Amperes (A) (Coulombs/second)*

***Dielectric** (κ)**:** an insulating material.*

***Dielectric constant:** a measure of the insulating strength of a non-conducting material.*

***Electromotive force** (ε)**:** the stable potential difference produced by a battery.*

***Ohm's Law:** the ratio of the potential difference across the terminals of a resistor to the current which passes through it.*

***Parallel circuits:** circuits in which all of the circuit elements share a common voltage.*

***Power:** the rate of change of energy (Units W)*

***Resistance** (R)**:** a property of a device to impede the flow of charge. (Units: Ohms Ω)*

***Resistivity** (ρ)**:** a property of a material that measures the material's ability to impede the flow of charge. (units: $\Omega \cdot \mathrm{m}$)*

***Series circuits:** circuits in which all of the circuit elements share a common current.*

Key Ideas

Current

Current is the flow of charge. It is given the symbol I and unit Ampere ($A = C/s$). Current flows in response to an electric potential difference. Current is calculated:

$$I = \frac{\Delta Q}{\Delta t}$$

Let's define some terms:

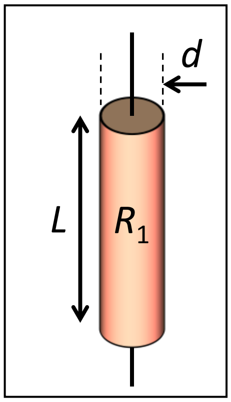
- I: the current (units A)
- ΔQ: the total charge passing through an area
- Δt: the time interval used to measure ΔQ

Batteries

A battery is a device that provides a source of charge to an electric circuit. Batteries are designed to maintain a stable potential difference. This potential difference is the *electromotive force* ε. The battery pushes charges through the circuit.

Resistance

A resistor is a device that prevents the flow of charge. Resistance is the tendency of the device to impede the flow of charges and is related to the conduction of heat through a material. It depends on the geometry and the material from which the resistor is made.



Resistance is calculated:

$$R = \rho\frac{L}{A}$$

Let's define some terms:

- R: the resistance of the resistor (Units: Ohms Ω).
- ρ: the resistivity of the resistive material (Units: Ohm-meter $\Omega \cdot m$).
- L: the length of the resistor.
- A: the cross-sectional area of the resistor.

Ohm's Law

Resistance is needed to produce a current. Ohm's Law tells us how to relate currents to the resistance of a device:

$$R = \frac{V_R}{I}$$

Let's define some terms:

- R: the resistance of the device.
- V_R: the potential difference (also called the voltage drop) across the terminals of a resistor.
- I: the current through the resistor.

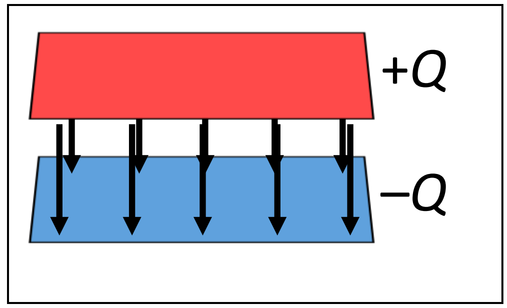
Power (P): Resistors dissipate energy as heat. They cannot store energy. The dissipation of energy is called power (units: Watt, $W = J/s$). The power of a resistor is calculated as follows:

$$P = I^2R = IV$$

For a complex network, R is replaced by Req [the equivalent resistance] to obtain the total power in the whole circuit. For power consumption by individual resistors, you need to know the current through that particular resistor.

Capacitor

A capacitor is a device which stores electrical energy by storing separated charge. A capacitor consists of two spatially separated conductors charged to $+Q$ and -Q.



The *capacitance* (C) is defined as the ratio of the charge on one conductor of the capacitor to the potential difference between the conductors. *It is a geometrical property of the device*. Capacitance is measured in *Farads (F)*:

$$C_0 = \frac{Q}{V}$$

The C_0 denotes the capacitance with vacuum filling the space between the plates.

Energy Stored (units: Joules):

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Let's define the terms:

- C: Capacitance
- V: Potential difference between capacitor plates
- Q: Charge on capacitor plates.

Dielectric Materials

Insulating material (called a dielectric) placed between the conductors *increases the capacitance* by the dielectric constant κ.

Examples:

- vacuum, $\kappa = 1$.
- water: $\kappa = 80.4$!

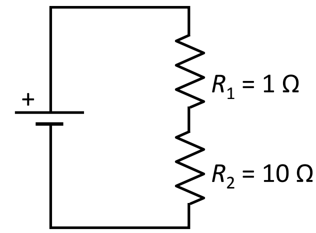
If you insert material between these plates, then $C = \kappa C_0$ gives the new, increased, capacitance.

Circuits

The three devices listed can be used in electric circuits. The two basic circuits are:

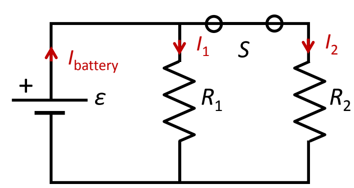
- **series circuits:** all of the circuit elements share a common current.

A series circuit with two resistors



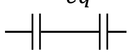
- **parallel circuits:** all of the circuit elements share a common voltage.

A parallel circuit with two resistors and a switch.

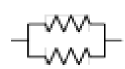
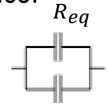


Series and parallel circuits can be combined to form very complex circuit networks. The following are the rules for circuit elements in series and parallel:

Series:

- **Resistors:** 
  - Current: $I_{eq} = I_1 = I_2$
  - Voltage: $V_{eq} = V_1 + V_2$
  - Resistance: $R_{eq} = R_1 + R_2$
- **Capacitors:** 
  - Current: $I_{eq} = I_1 = I_2$
  - Voltage: $V_{eq} = V_1 + V_2$
  - Capacitance: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Parallel:

- **Resistors:** 
  - Current: $I_{eq} = I_1 + I_2$
  - Voltage: $V_{eq} = V_1 = V_2$
  - Resistance: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
- **Capacitors:** 
  - Current: $I_{eq} = I_1 + I_2$
  - Voltage: $V_{eq} = V_1 = V_2$
  - Capacitance: $C_{eq} = C_1 + C_2$

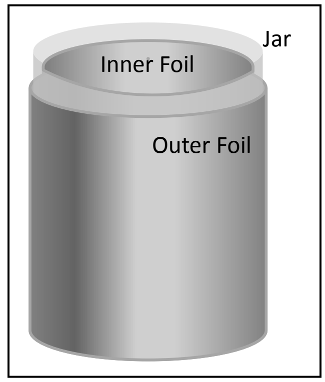
Situations for Lecture 6: Circuit Elements

Situation 1: The Peanut Butter Jar Capacitor

You are building a circuit to make your own radio. You find yourself out of the necessary capacitors to tune your favorite station. You remember you can build your own with common household materials. After searching around the house you are able to find the following items:

Item	Material	Dielectric Constant (κ)	Diameter	Height	Thickness
Peanut Butter Jar	Plastic	2.26	$10\ cm$	$12\ cm$	$2\ mm$
Peanut Butter Jar	Glass	4	$10\ cm$	$14\ cm$	$3\ mm$
Jam Jar	Glass	4	$5\ cm$	$8\ cm$	$3\ mm$
Aluminum Foil					

You take the aluminum foil and cut rectangle pairs, one pair for each jar. The foil rectangles are just large enough to fit around the inner and outer diameters of the jar. They have been cut so that *each foil in the pair has the same area*. You can treat this capacitor as a parallel-plate capacitor.



1) If the outer foil covers the outside of the jar, what is the area of the outer foil? What is the height of the inner foil? Fill in the table below.

Jar	Height of Inner Foil	Area of Outer Foil
Plastic Peanut Butter		
Glass Peanut Butter		
Glass Jam		

2) Where is the energy stored in each capacitor? Discuss this with your teammates and take notes on your discussion.

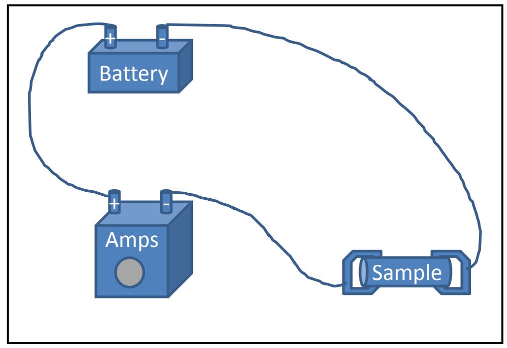
3) The capacitance of a parallel plate capacitor in vacuum depends only on the area, A, of the foils and their separation distance d: $C_0 = \varepsilon_0 A/d$ ($\varepsilon_0 = 8.85 \times 10^{-12} F/m$). Find the capacitance for each jar capacitor. Fill in the table below:

Jar	Capacitance
Plastic Peanut Butter	
Glass Peanut Butter	
Glass Jam	

4) Which capacitor would you choose to store the most charge? Discuss this with your teammates and take notes you your discussion.

Situation 2: Salty Water

You are given a set of water samples to test for salinity. Each sample is in a tube of diameter $0.5\ cm$ and length $2\ cm$. At each end of the tube are electrodes, which can be connected to a battery. An ammeter is used to measure the current which passes through the samples.



1) A voltage source supplies $6\ V$. Each sample is connected to the source and the current is measured. The results are given in the following table:

Sample	Current (A)	Resistance (Ω)	Resistivity ($\Omega \cdot m$)
1	0.6		
2	0.06		
3	6.7×10^{-7}		
4	6×10^{-5}		

What is the resistance of each sample? Fill in the values in the table.

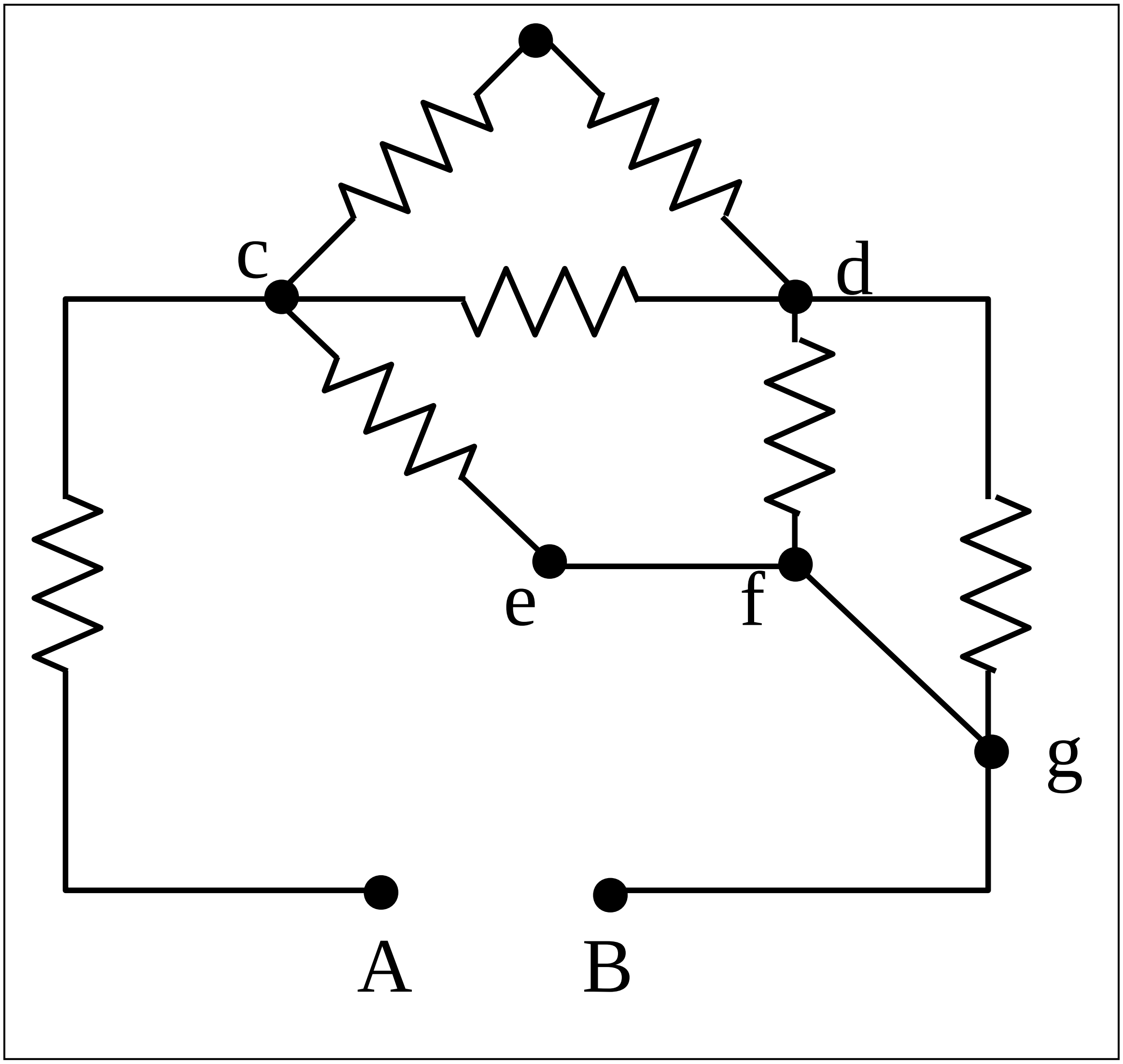
2) What is the resistivity, ρ, of each sample? Fill in the values in the table.

3) Which of the samples is(are) most likely sea water? Which is(are) most likely drinking water? Discuss with your teammates and take notes on the discussion.

Situations for Lecture 7: Series and Parallel Circuits

Situation 1: The Resistor Network

Consider the resistor network below. All resistors have the same resistance: $R = 3\ \Omega$



1) With your teammates discuss the circuit network. Is it only a series circuit, only a parallel circuit, both, neither? Make sure everyone gets the opportunity to explain how they interpret this circuit. Take notes on your discussion.

2) Simplify this resistor network and then compute the equivalent resistance between points **A** and **B.**

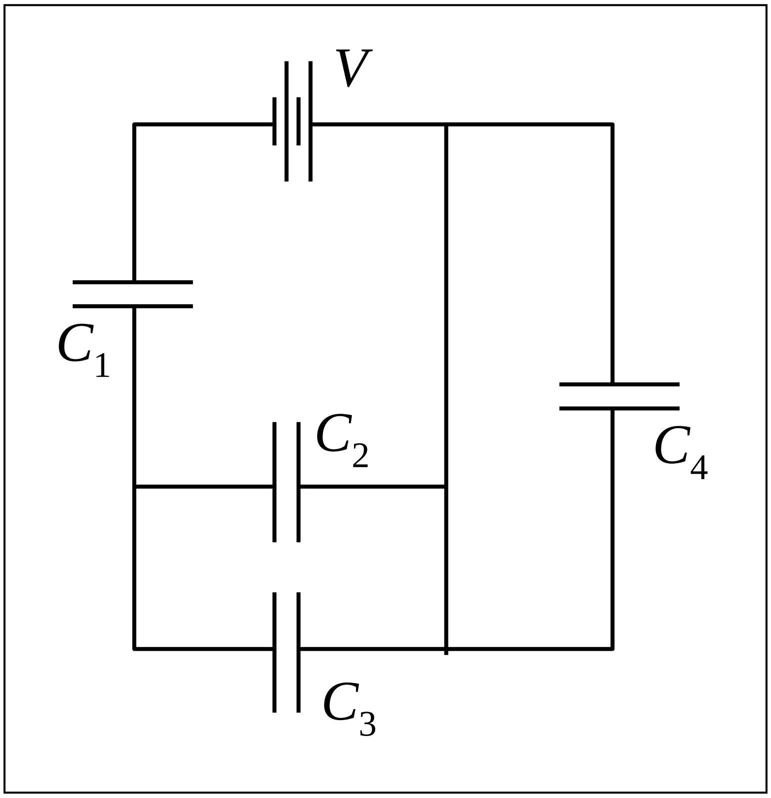
3) Suppose a $15\ V$ power supply was connected between **A** and **B**. What power would be dissipated in this network of resistors?

4) What is the potential difference between points **c** and **g**, *i.e.*, $V_c - V_g$?

5) If these resistors were all identical light bulbs, which one would be brightest? Which one would be the least bright?

Situation 2: The Capacitor Network

Consider the capacitor network, connected to a battery, shown in the figure below. You measured the quantities in the table:



Capacitor	Capacitance	Charge	Voltage	Energy
C_1	$6\ \mu F$	$6\ \mu C$		
C_2				
C_3	$0.5\ \mu F$			$25\ \mu J$
C_4				
Battery	Voltage			
V				

Your task will be to fill in the remaining values in the table.

1) Discuss with your teammates how this capacitor network is similar to and different from the resistor network in Situation 1. Be specific and take notes on your discussion.

2) Discuss your strategy for finding the remaining values in the table. What rules will you need to apply? Will you have to find any equivalent circuits along the way? Be specific and take notes on your discussion. Before you proceed make sure that everyone on your team understands the problem solving plan you are about to undertake.

3) Find the remaining values in the table. Remember to show your work. If you find it is not possible to find all of the values, explain why.

4) Find the equivalent capacitance of the entire network.

Week 6 Lectures 8 and 9—Kirchhoff's Rules and RC Circuits

Summary

Glossary

***Junction rule:** describes the behavior at the connection point of two or more current paths in a circuit. The current into a junction must be the same as the sum of all currents out of a junction.*

***Kirchhoff's Rules:** statements of conservation of energy and charge used to answer questions about the behavior of complex electrical circuits.*

***Loop rule:** describes the potential difference around a closed loop in an electric circuit.*

***Time constant (τ):** the characteristic time which describes the time needed for a capacitor in an RC circuit to charge or discharge to 1/e of its maximum charge*

Key Ideas

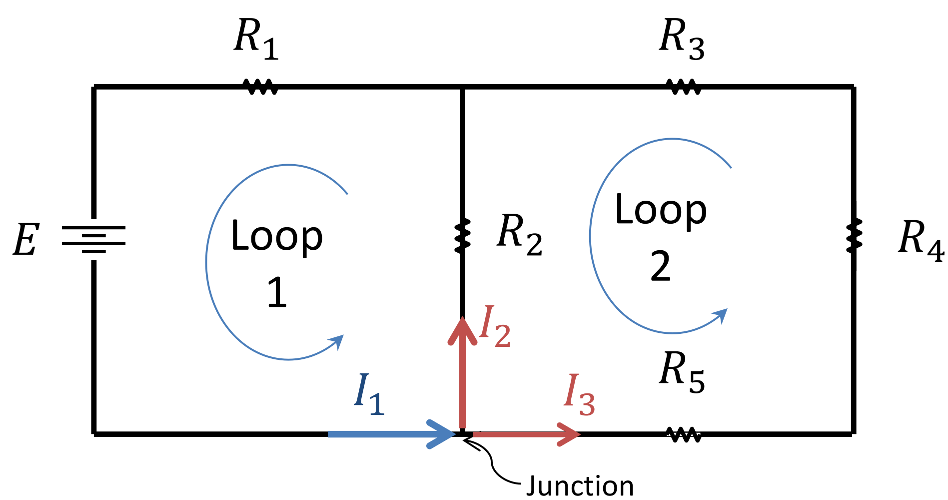
Kirchhoff's Rules

Kirchhoff's Rules, also called Kirchhoff's Loop Rules, are conservation rules. The physical quantities conserved are:

- Charge
- Energy

Kirchhoff's Rules are:

1. *Junction rule:* Current into a junction equals current out of a junction. A statement of the conservation of charge.
2. *Loop rule:* Around any closed loop, the sum of all potential drops equals the sum of all potential rises.



1. Junction Rule: $I_1 = I_2 + I_3$
2. Loop Rule:
    1. Loop 1: $\mathrm{E} - \mathrm{I_2 R_2} - \mathrm{I_1 R_1} = 0$
    2. Loop 2: $-I_3 R_3 + I_2 R_2 - I_3 R_5 - I_3 R_4 = 0$

Circuits with Resistors and Capacitors: Time Dependence

Charging or discharging a capacitor takes time. This time has a characteristic *time constant* $\tau = RC$. In complicated circuits, you may have to work to find out the value of R or C as an equivalent resistance or capacitance.

Charging:

When a capacitor charges both the charge and current change with time.

The charge on a capacitor:

$$q(t) = q_0(1 - e^{-\frac{t}{RC}})$$

Let's define some terms:

- q_0 is the final charge on the capacitor
- $q_0 = CV_0$
- V_0 is the final voltage across the capacitor.

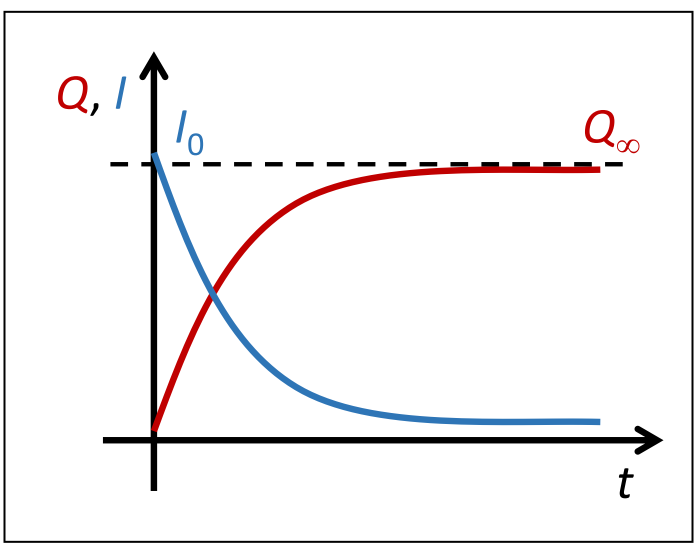
The current through a capacitor:

$$I(t) = I_0 e^{-t/RC}$$

Let's define some terms:

- I_0 is the initial current through the capacitor
- t is the time elapsed since the capacitor started charging
- RC is the time constant.

The graph of the time-dependence of a charging capacitor:



Discharging:

When a capacitor discharges both the charge and the current change as well:

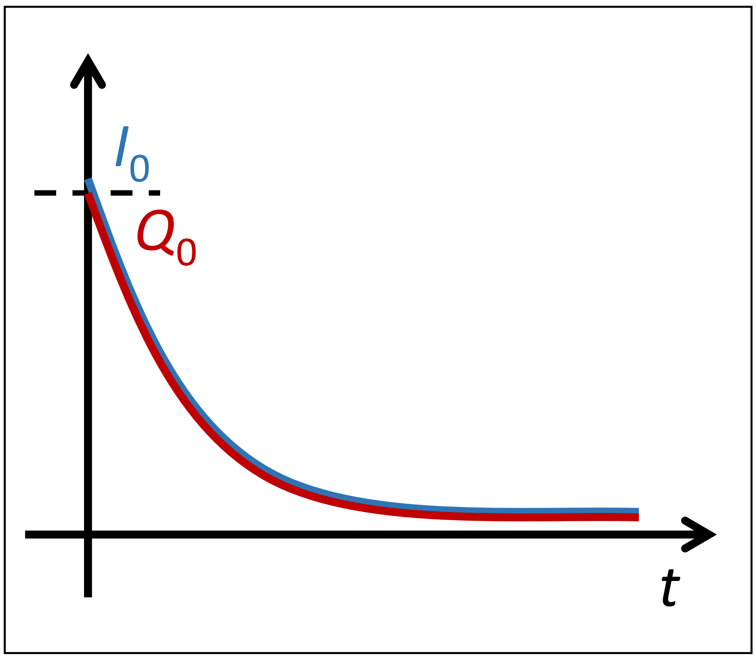
The charge on a capacitor: $$q(t) = q_0 e^{-t/RC}$$

This time, q_0 is the *initial charge* on the capacitor!

The current through a capacitor: $$I(t) = I_0 e^{-t/RC}$$

Notice, the expression for the time-dependence of the current does not change between charging and discharging!

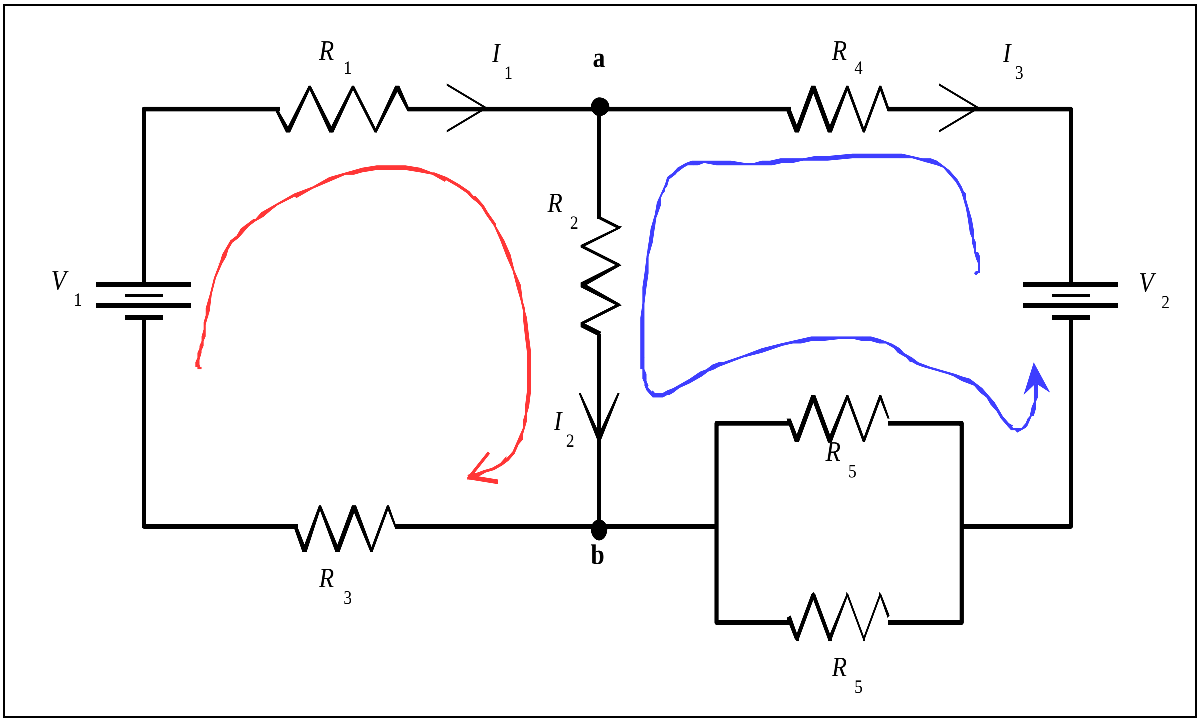
The graph of the time-dependence of a discharging capacitor:



Situations for Lecture 8: Kirchhoff's Rules

Situation 1: The Multi-Loop Circuit

Consider the circuit below.



1) Discuss with your teammates the features of this circuit. How will you work with them? Be specific and take notes on your discussion.

2) Consider the junction marked **b**. Write down the Junction Rule for this junction.

3) Consider the loop on the left-hand side of the circuit. Write down the Loop Rule for this loop of the circuit.

4) Now consider the right-hand loop of the circuit. Write down the loop Rule for this loop of the circuit. How is this loop different from the left-hand loop?

5) The table contains the values for the resistors and batteries. Use your results from the previous parts to find the currents requested. What does a negative current mean?

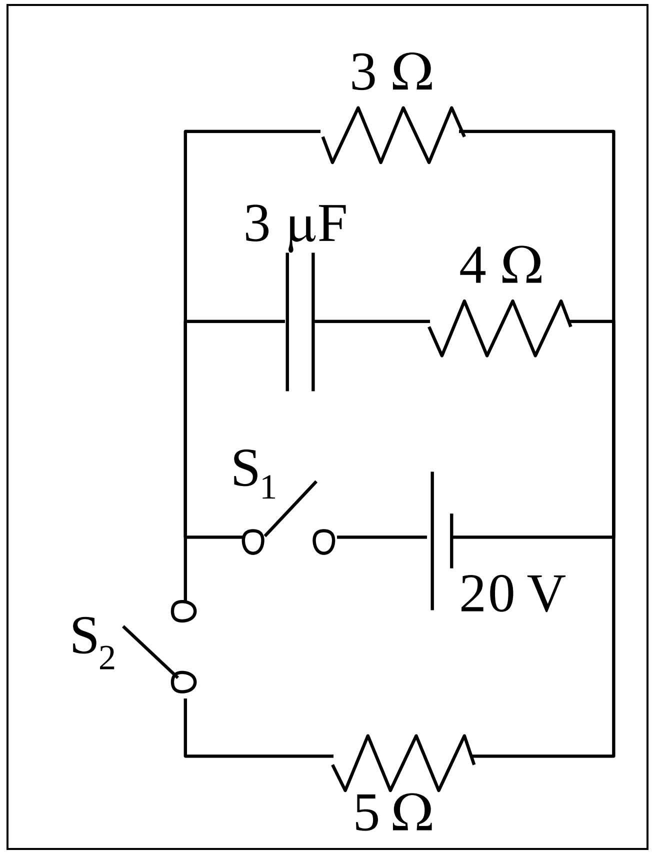
R_1	$1\ \Omega$	V_1	$4\ V$
R_2	$1\ \Omega$	V_2	$4\ V$
R_3	$1\ \Omega$		
R_4	$1\ \Omega$		
R_5	$2\ \Omega$		
R_6	$2\ \Omega$		

I_1	
I_2	
I_3	

Situations for Lecture 9: RC Circuits

Situation 1: A Complex Circuit with Resistors and Capacitors

Consider this circuit:



1) What are the important features of this circuit? Be specific and take notes on your discussion.

2) At time t_0, switch S_1 is closed (while S_2 remains open). What is the current drawn from the battery at the instant t_0?

3) What is the current through the battery a long time later?

4) How much energy is stored in the capacitor a long time after the switch is closed?

5) Would your answer change if the $3\ \Omega$ resistor at the top of the circuit were removed? Explain your reasoning.

6) Now S_1 is opened and S_2 is closed at exactly the same instant. How much current goes through the $4\ \Omega$ resistor immediately after this event?

7) Which direction does the current go through the $4\ \Omega$ resistor, to the left or right?

8) How long does it take for the current to fall to $1/e$ of the value you calculated through the $4\ \Omega$ resistor?

Week 7 Lectures 10 and 11—Magnetic Forces, Fields, Dipoles and Currents

Summary

Glossary

Magnetic field ($\vec{B}$): a property of moving charges (currents).
Magnetic force: the force experienced by a charged particle moving through a magnetic field.
Right-hand rule: a method of visualizing forces produced by products of vector quantities.

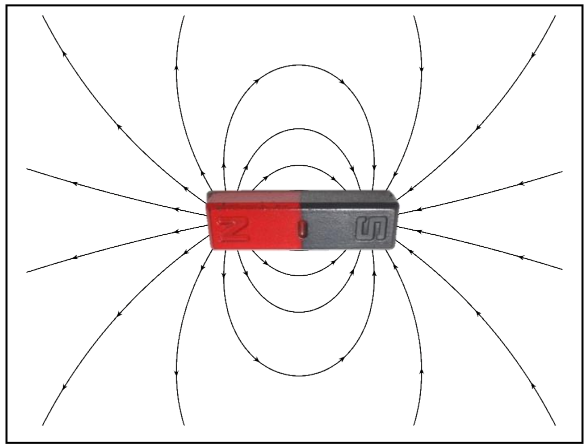
Key Ideas

Magnetic Fields come from *charges in motion*.

- Magnetic Materials: Microscopic motion
  - electron spin
  - orbital motion
- Electric Currents: Macroscopic motion.

Magnetic Field Lines

Analogous to electric field lines. They are *continuous*—they form closed loops. Lines emerge from the North pole and re-enter at the South pole. This is the direction of $\vec{\boldsymbol{B}}$, the magnetic field. The simplest field is a dipole as shown in the figure.



Magnetic Force

A charged particle moving through a magnetic field experiences a force:

$$|\vec{\boldsymbol{F}}| = qv|\vec{\boldsymbol{B}}|\sin\theta\text{; Units: } N$$

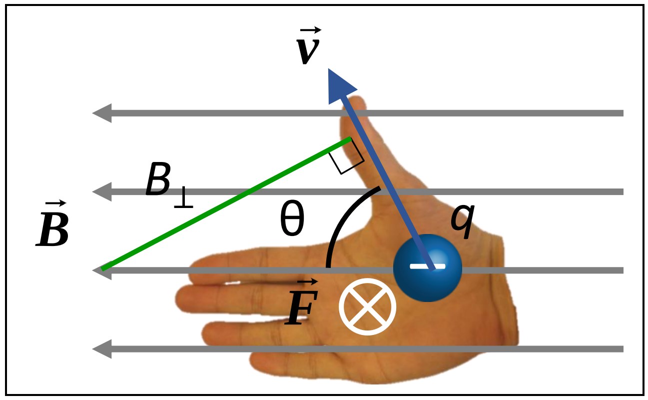
Let's define some terms:

- q: the electric charge of the particle
- v: the speed of the charged particle
- $|\vec{\boldsymbol{B}}|$: the magnitude of the magnetic field
- θ: the angle between the velocity vector $\vec{\boldsymbol{v}}$ and the magnetic field vector $\vec{\boldsymbol{B}}$.

Facts about the magnetic force:

- Magnetic force is a vector.
- Magnetic force only acts on *moving charges*.
- This force always *acts at right angles* to the velocity of the charged particle.
- This force does NO work. (Recall for work, the force is in the direction of the motion, not at right angles to it!).

The Right-Hand Rule



The right-hand rule is shown in the diagram:

Steps for the Right-Hand Rule:

- Step 1. Point your thumb along the $\vec{v}$ direction.
- Step 2. Point your fingers in the $\vec{B}$ direction.
- Step 3. The direction of the force, $\vec{F}$, is out of the palm of your hand.

NOTE: The angle θ is between the thumb and fingers, which is easily adjusted to anything less than $90°$. The force, $\vec{F}$, is always perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$, even if θ is not $90°$

Cyclotron Motion

A charged particle will exhibit circular motion in a magnetic field oriented perpendicular to its instantaneous velocity. An object undergoing circular motion experiences a centripetal force:

$$F_c = \frac{mv^2}{r}$$

Equating the centripetal force to the magnetic force yields another key result:

$$r = \frac{mv}{qB}$$

With this expression we can calculate the radius, r, of the circular path a charged particle will travel in a magnetic field. You should recognize that mv is also the particle's momentum.

Using $v = r\omega$, we can write the centripetal force in terms of the angular frequency:

$$F_c = mr\omega^2$$

The magnetic force is now:

$$F = qr\omega B \sin\theta$$

We can equate the centripetal force to the magnetic force and find:

$$\omega = \frac{qB}{m}; \text{ Remember: } \omega = 2\pi f.$$

Forces on Currents

Currents experience forces in a magnetic field. The magnitude of the force experienced by a current is:

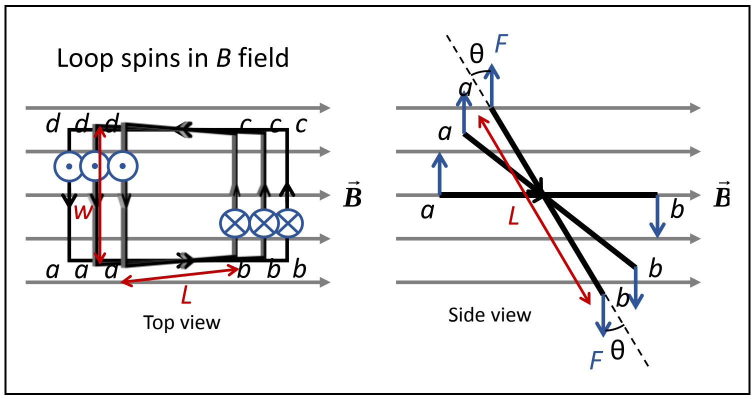
$$|\vec{\boldsymbol{F}}| = IL|\vec{\boldsymbol{B}}|\sin\theta$$

The direction is given by the Right-Hand Rule. The angle θ is the angle between the direction of the current and the direction of the magnetic field. For currents:

- The thumb points along the direction of $\vec{\boldsymbol{I}}$,
- fingers along $\vec{\boldsymbol{B}}$,
- direction of $\vec{\boldsymbol{F}}$ is out of the palm.

Torque on a Current-Carrying Wire Loop

Consider the 4 sides of a rectangular loop of current-carrying wire in a magnetic field as shown in the figure. The *left* and *right* sides experience equal and opposite forces which cause a torque about the pivot point. The front and back sides also have opposite forces. They tend to either squeeze the loop together or pull it apart. They do not add to the torque.



The magnitude of the torque is:

$$\tau_{loop} = IAN|\vec{\boldsymbol{B}}|\sin\theta$$

Let's define some terms:

- A is the area of the loop (any shape!)
- N is the number of wire turns in the loop
- I is the current running through each turn
- $|\vec{\boldsymbol{B}}|$ is the external field.

The angle θ is measured with respect to a line running perpendicular to the loop (the normal). Note that the normal vector could point *up* or *down* from the plane, but the correct sign is given by a *RHR* with the curling fingers along the current direction and the thumb in the direction of the normal.

Situations for Lecture 10: Magnetic Fields & Forces

Situation 1: The Particle in the Field

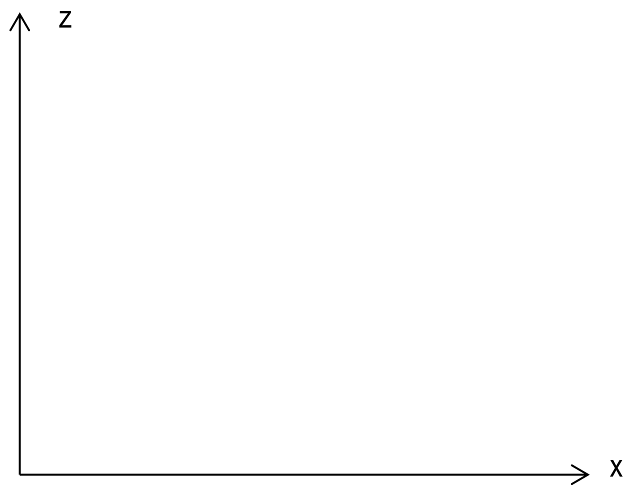
A charged particle with velocity $\vec{\boldsymbol{v}} = (2,0,0)\ m/s$ enters a magnetic field: $\vec{\boldsymbol{B}} = (0, 0, 3)\ T$.

1) What do you expect this particle to do in the electric field? Be specific. Discuss with your teammates and take notes on your discussion.

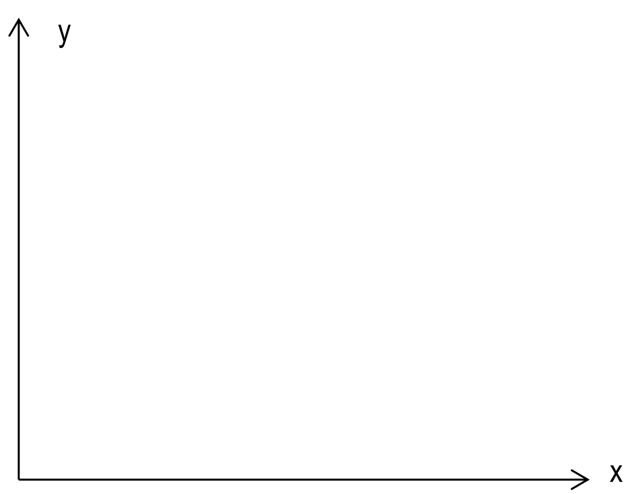
2) In the spaces below, sketch:

   a. The magnetic field.
   b. The velocity of the particle.
   c. The path(s) particle could take.

z vs x **plot**



y vs x **plot**



3) Calculate the radius. What information are you still missing, if any?

4) If the particle has charge $q = -6\ \mu C$, fill in the following table:

F_{xy}, the magnitude of the force in the xy-plane	F_z the magnitude of the force in the xz-plane	r the radius of curvature of the particle motion

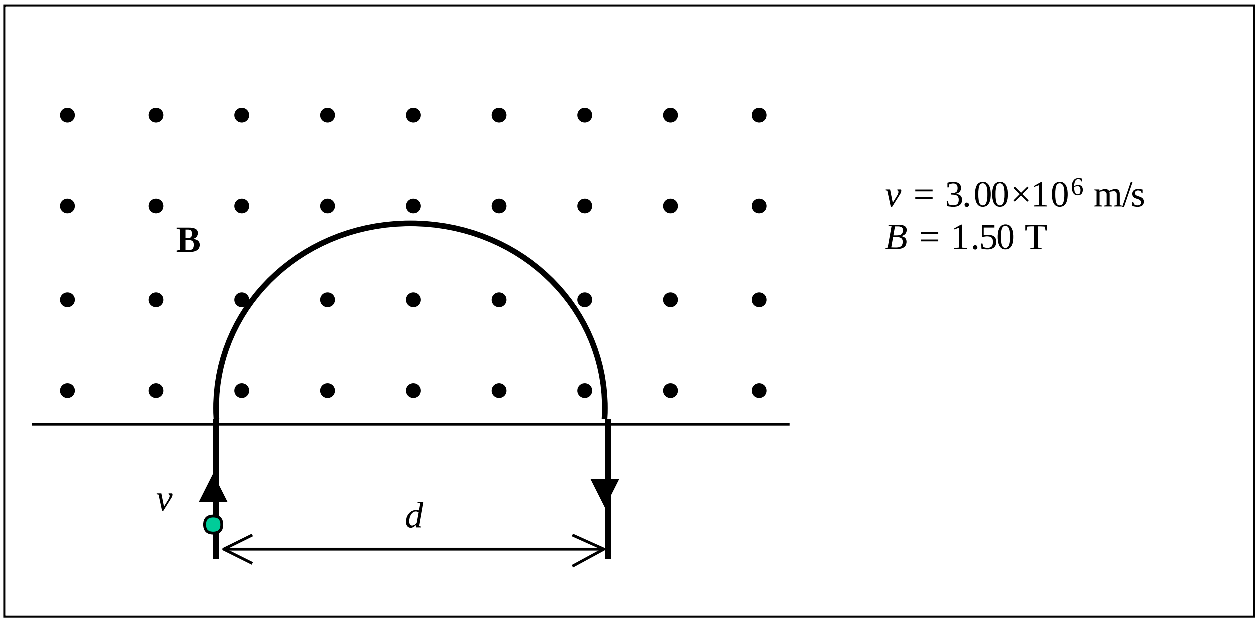
5) Will the particle travel clockwise or counter-clockwise as viewed from above the xy plane?

6) Will the particle travel in the positive or negative z-direction?

7) Sketch the three-dimensional path the particle will take as it travels in the magnetic field.

Situation 2: The Cyclotron

A proton (charge $e = 1.60 \times 10^{-19}\ C$ and mass $m = 1.67 \times 10^{-27}\ kg$), travelling with speed $v = 3.00 \times 10^{6}\ m/s$, enters normal to a region of uniform magnetic field $B = 1.50\ T$ as shown in the figure. (The field points out of the page.) After completing a semicircular path, the proton exits the field.



1) Calculate the distance d between the entrance and exit points on the proton's trajectory.

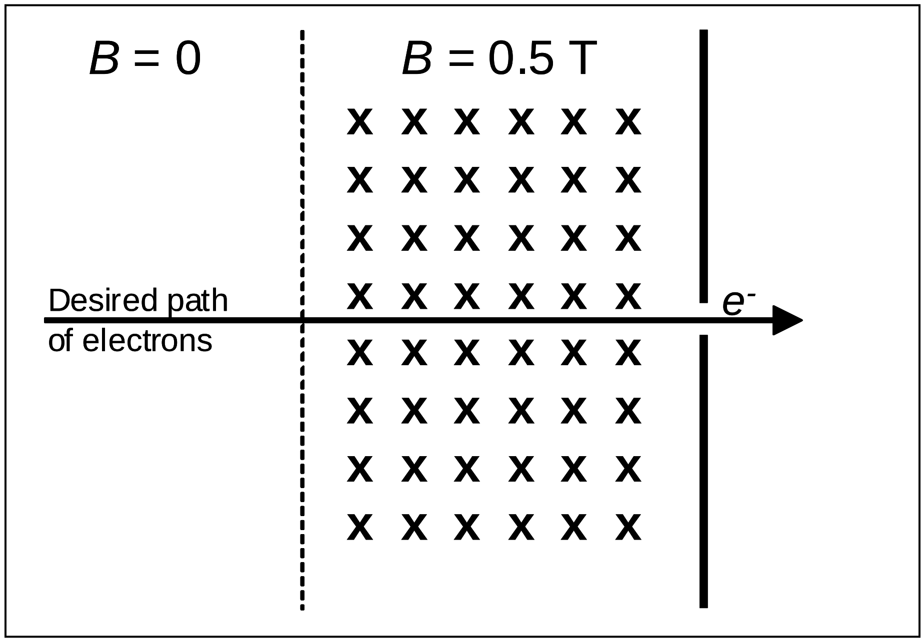
2) Calculate the amount of time Δt the proton spends inside the magnetic field.

3) If the particle was injected instead with velocity $2v_0$, it would:
   i. emerge at the same position X,but after a different amount of time $\Delta t'$
   ii. emerge at a different position X',but after the same amount of time Δt.
   iii. emerge at a different position X', and after a different amount of time $\Delta t'$.

Situation 3: The Velocity Selector

A uniform magnetic field of $0.5\ Tesla$ into the page is applied in the region shown below. A beam of electrons is directed through the region. Without an electric field, the path of the electrons will be a circle.

Electron Charge	Electron Mass
$-1.602 \times 10^{-19} C$	$9.109 \times 10^{-31} kg$



1) What direction will the electron beam bend when only the magnetic field is present? Draw an example trajectory on the figure.

2) What direction does the electric field need to point for the electron beam to travel in a straight line, as shown? Show this on the figure.

3) If we want a beam of electrons with speed $v = 600\ m/s$ to exit through the slit on the right, how strong should the field be?

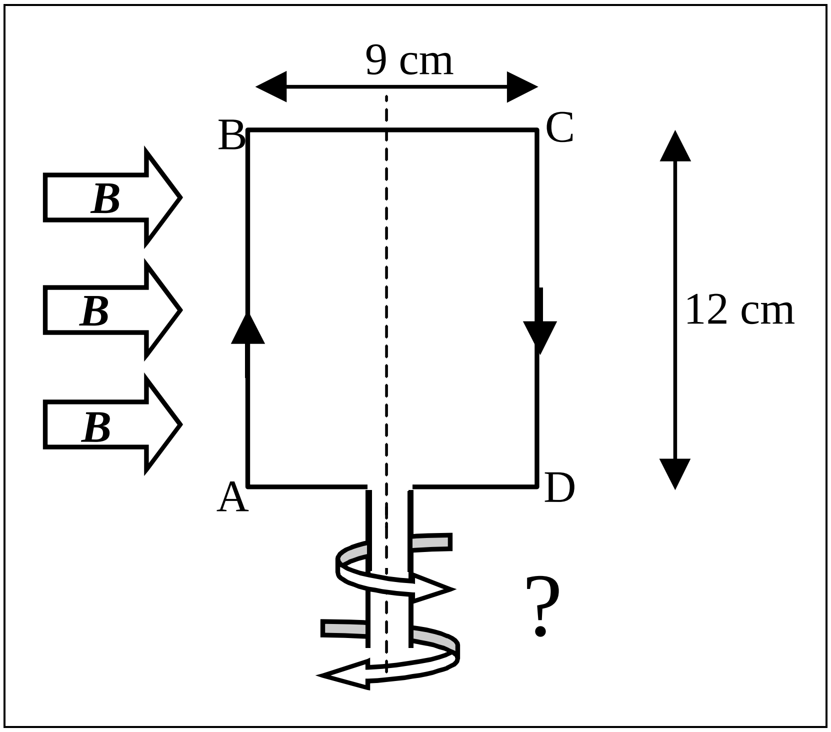
4) What happens if an electron in the beam has a different speed?

5) How would the answers change if the charge is positive, and all other quantities remain unchanged?

Situations for Lecture 11: Magnetic Dipoles & Current Loops

Situation 1: Torque on a Wire

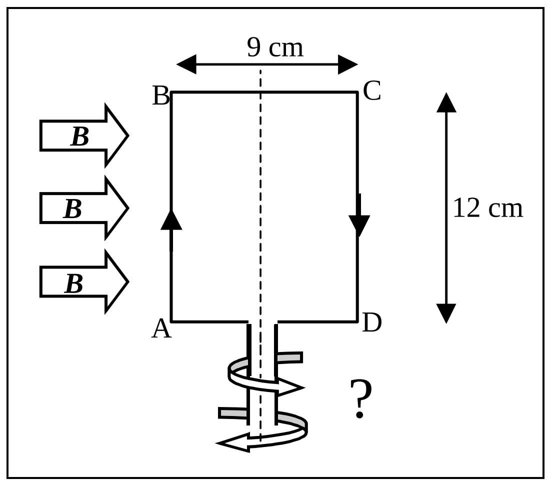
A wire loop with a current $I = 8\ A$ flowing through in the direction shown sits in a uniform magnetic field of strength $B = 3.3\ mT$. The initial orientation of the loop is flat on the page and the magnetic field runs left to right as indicated.



1) What is the magnitude and direction of the force on wire segment CD?

2) What is the magnitude of the torque on the loop in this position?

3) Which way does it twist? See the arrows and pick one.



4) If the magnetic field were instead oriented into the page, would the loop feel a torque? If so, describe the sense of the torque in this case.

Situation 2: The Mystery Metal

You found a piece of metal in the basement. Armed with only a compass and a pencil you decide to figure out the magnetic properties of the material. You remember the strength of the Earth's magnetic field is: $B_{Earth} = 5.329 \times 10^{-5} T = 53.29\ \mu\mathrm{T}$ –assume the Earth's magnetic field points due North.

1) You first move the compass around the piece of metal and observe that the needle always points in the same direction, North. What does this tell you about the metal? Discuss this with you teammates and take notes on your discussion.

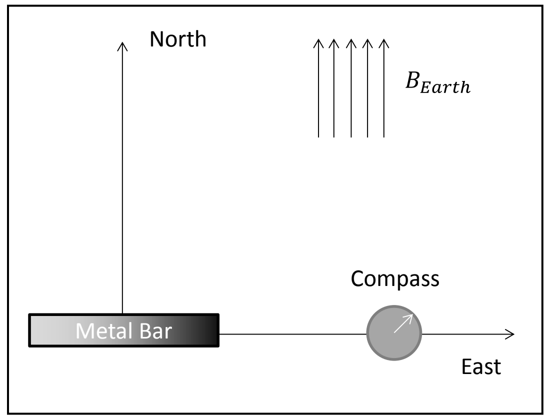
2) Despite the apparently non-magnetic result you obtained in your first trial you remain unconvinced that the metal is truly a non-magnetic metal. Ferromagnetic materials can be magnetized, at least somewhat, by vibrating the metal in a magnetic field.
   a. What magnetic field(s) do you have at your disposal to attempt to magnetize metal?

   b. How can you impart vibration to the metal?

3) You decide that dropping the metal repeatedly on the concrete floor will help you determine if the metal is ferromagnetic. You do this several times and re-measure for any residual field produced. The back of the compass has the *magnetic dipole moment* engraved on the case: $\mu = 0.5\ A \cdot m^2$, and a meter which can measure the torque experienced by the compass needle. Hint: $\tau = \mu B \sin\theta$

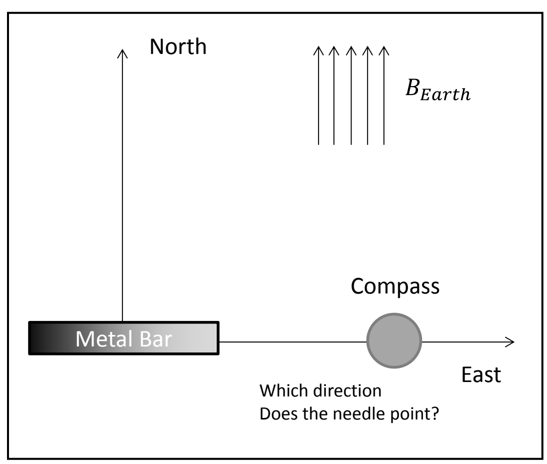
   a. After orienting the compass to North, you bring the metal close to the compass and notice the needle deflects to $20°$ East-of-North. The torque meter on the compass read $0.25\ N \cdot m$. Use this information and the sketch to calculate the values requested in the table. Assume the metal becomes a bar magnet.

Net Magnetic Field Strength B at the compass	
B_{m_x}: The x-component of the magnetic field from the metal	
B_{m_y}: The y-component of the magnetic field from the metal	



   b. You flip the piece of metal over. Find the quantities requested in the table.

Net Magnetic Field Strength B at the compass	
The angle θ of the net field direction at the compass	
The torque τ experienced by the compass needle due to the change in field.	



Week 8 Lectures 12 and 13—Currents, Magnetic Fields, and Lenz' Law

Summary

Glossary

Electromagnetic induction: *the process of producing an EMF across a conductor by variation of the magnetic field enclosed within a closed conducting loop.*

Induced magnetic field *(B_{ind}): the resulting magnetic field in response to an electromagnetically induced EMF.*

Magnetic flux: *the number of magnetic field lines through a closed area. Units Weber (Wb)*

Motional EMF *(ε): the EMF produced by continuously changing the area presented to a static magnetic field.*

Key Ideas

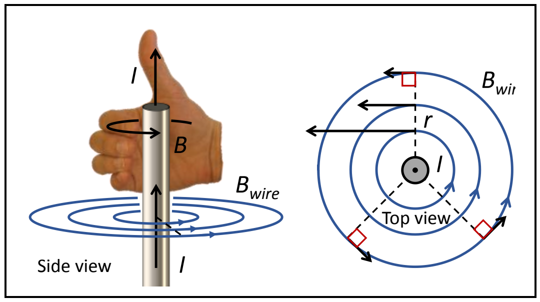
Currents and Magnetic Fields

Currents cause magnetic fields. The following help us determine the magnitude and direction of the magnetic field caused by different current geometries:

Field near a long, straight wire

- Thumb points along current $\vec{I}$,
- Curly fingers point in direction of $\vec{B}$.

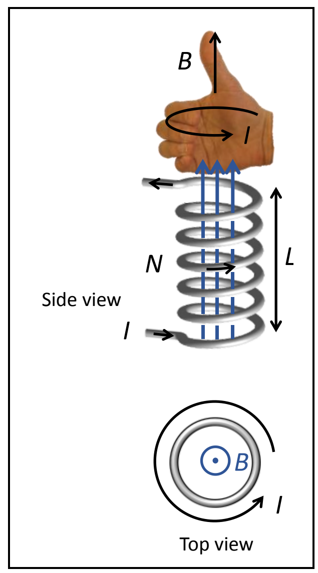
$$B = \frac{\mu_0 I}{2\pi r}$$



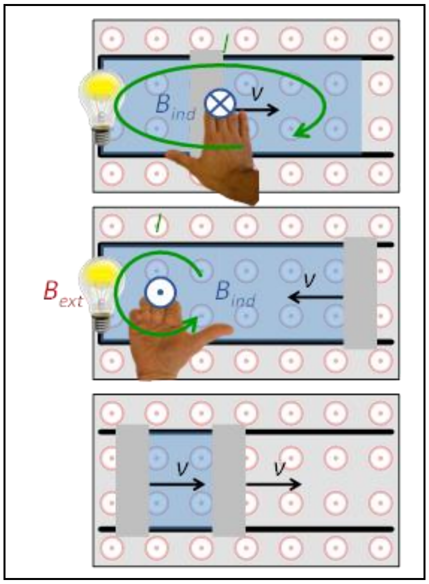
Field inside long solenoid with $n = N/L$ turns/meter

- Curl fingers around the solenoid in the direction of the current $\vec{I}$,
- Thumb points along $\vec{B}$, which is uniform inside the solenoid.

$$B = \mu_0 n I$$



Motional EMF



Magnetic fields can produce an electro-motive force (EMF) in a closed loop of conducting material. This process is *electromagnetic induction*.

To produce a *motional EMF,* ε, one or both of the following must happen:

- the area of the conducting loop in a static magnetic field must change:
    - Change the size of the loop.
    - Change the area of the loop containing lines of magnetic field.
    - Rotating a fixed area loop.
- the magnetic field must change:
    - in magnitude.
    - in direction.

The magnetic field lines contained within the area of a closed loop of conductor is the *magnetic flux:*

$$\Phi = BA\cos\varphi$$

Let's define some terms:

- Φ: The magnetic flux (Units: Weber (Wb)—$1\ Wb = 1\ T\ m^2$)
- B: The magnetic field strength
- A: The area of the conducting loop
- φ: The angle between the normal vector and the magnetic field.

Lenz' Law

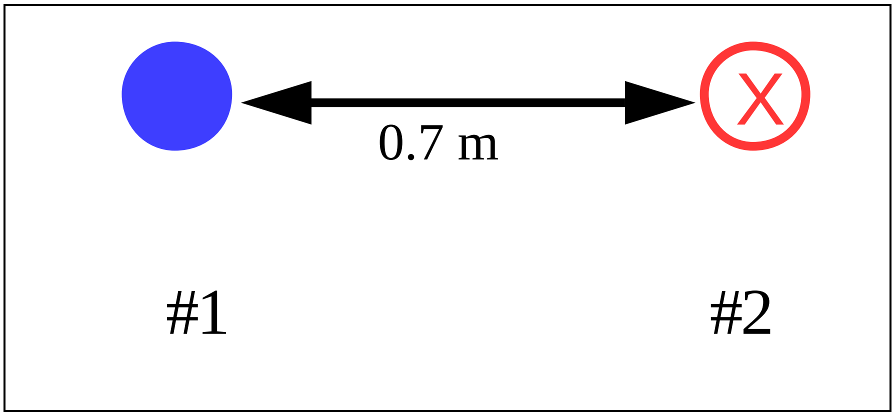
The induced EMF in a conductor produces a current. This current produces a new magnetic field. This magnetic field is the *induced magnetic field,* B_{ind}. This field direction is ***opposed the change in flux***. This is a statement of conservation of energy.

If the external magnetic flux is getting *stronger (more field lines in the area)*, an induced current is produced in a direction that generates a magnetic field that opposes the external flux. If the external flux is getting *weaker (fewer field lines in the area),* the induced current is produced in a direction that increases flux through the loop.

Situations for Lecture 12: Currents and Magnetic Fields

Situation 1: Forces on Wires

Two long straight wires ($15\ m$ each) are positioned perpendicular to the page as indicated in the figure. In wire #1, a current of $4\ A$ is coming out of the page. In wire #2, a current of $2\ A$ is going into the page.



1) What is the magnetic field at the location of wire #1 due to wire #2? Calculate the magnitude and put an arrow on the drawing to indicate the direction.

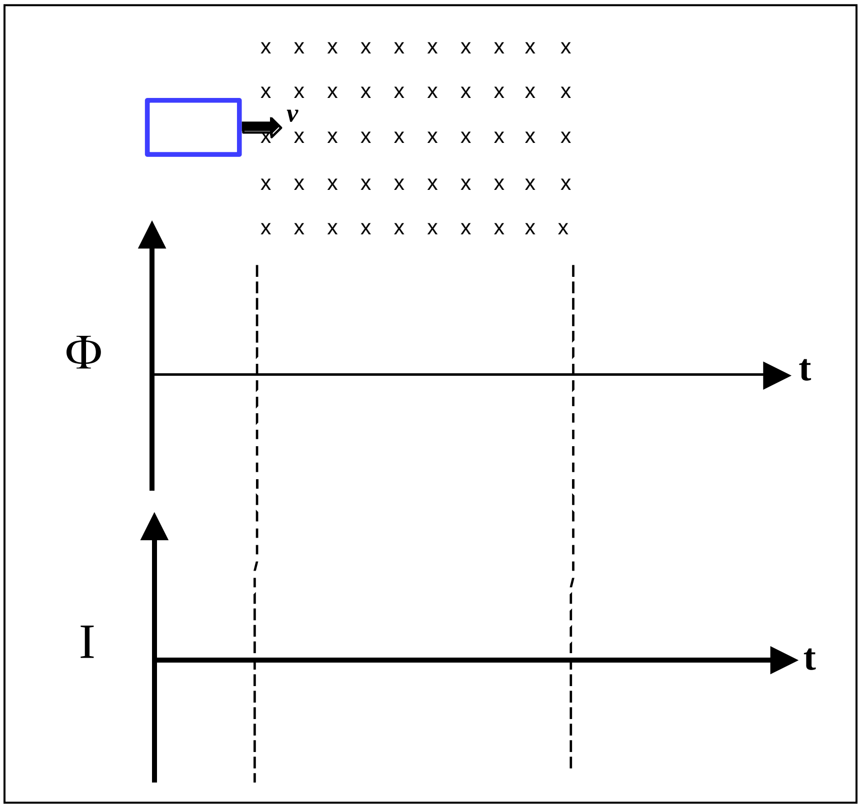
2) What is the magnitude and direction of the force on wire #1 caused by this field? Again, put an arrow on the drawing and label it.

3) Can you make any general conclusions about the force between two parallel wires when the currents in the wires run:

1. in opposite directions.
2. the same direction.

Situations for Lecture 13: Motional EMF and Lenz' Law

Situation 1: The Moving Loop



A rectangular loop has the following parameters:

- width $w = 3\ cm$
- length $l = 5\ cm$

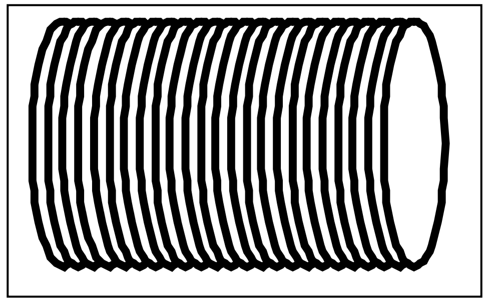
It is pulled at a uniform speed, $v = 1.5\ m/s$, into, through, and out of, a uniform magnetic field of strength, $B = 3.3\ mT$ (into the page). The picture shows this situation and the axes below the figure correspond to the position of the front edge of the loop:

- the first dashed line is at a time when the loop just begins to enter the magnetic field.
- The second dashed line is when it just begins to exit.

1) Plot the total flux versus time on the diagram. Pay attention to the scale of your plot.

2) If the loop has 15 turns of wire, with a resistance per unit length $R/l = 8\ \Omega/m$, what is the total resistance of the loop? Plot the induced current in the loop on the diagram paying attention to the vertical scale (in Amperes).

Situation 2: The Solenoid

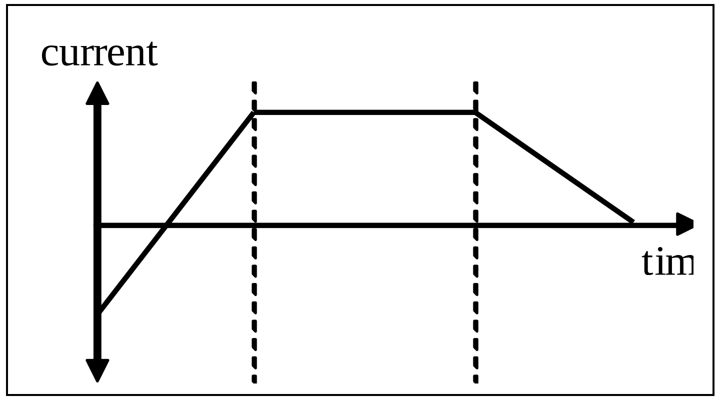


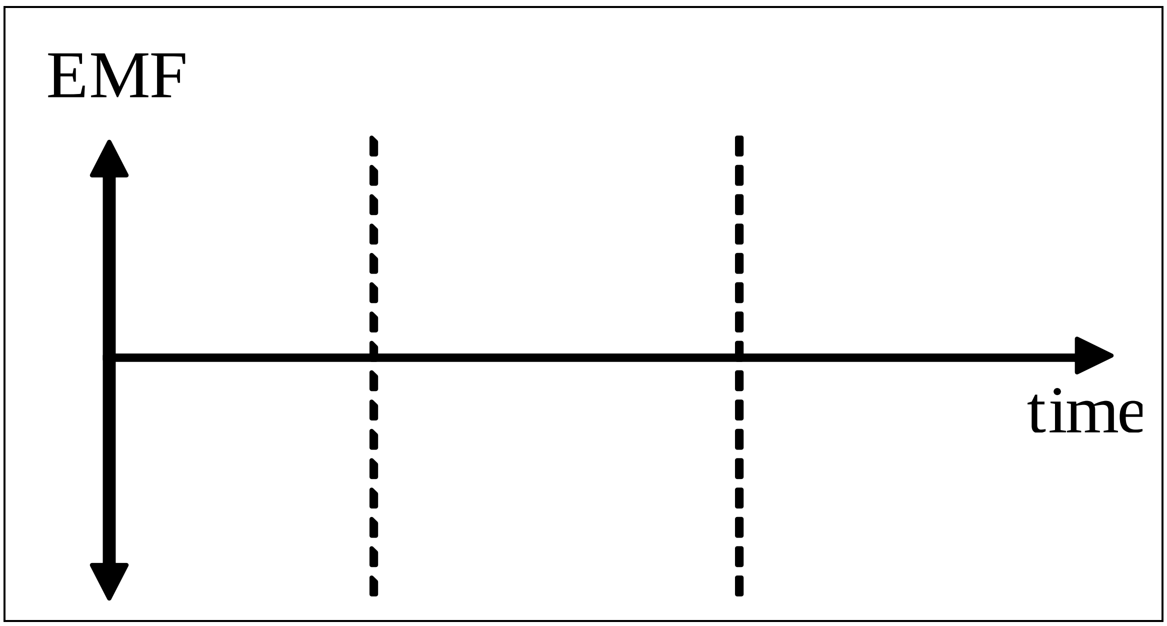
A 100 turn solenoid has a radius $1\ cm$ and length $10\ cm$.

1) What is the magnetic field strength inside the solenoid if a current of $1\ Amp$ is flowing? First, write a general equation, then calculate a value.

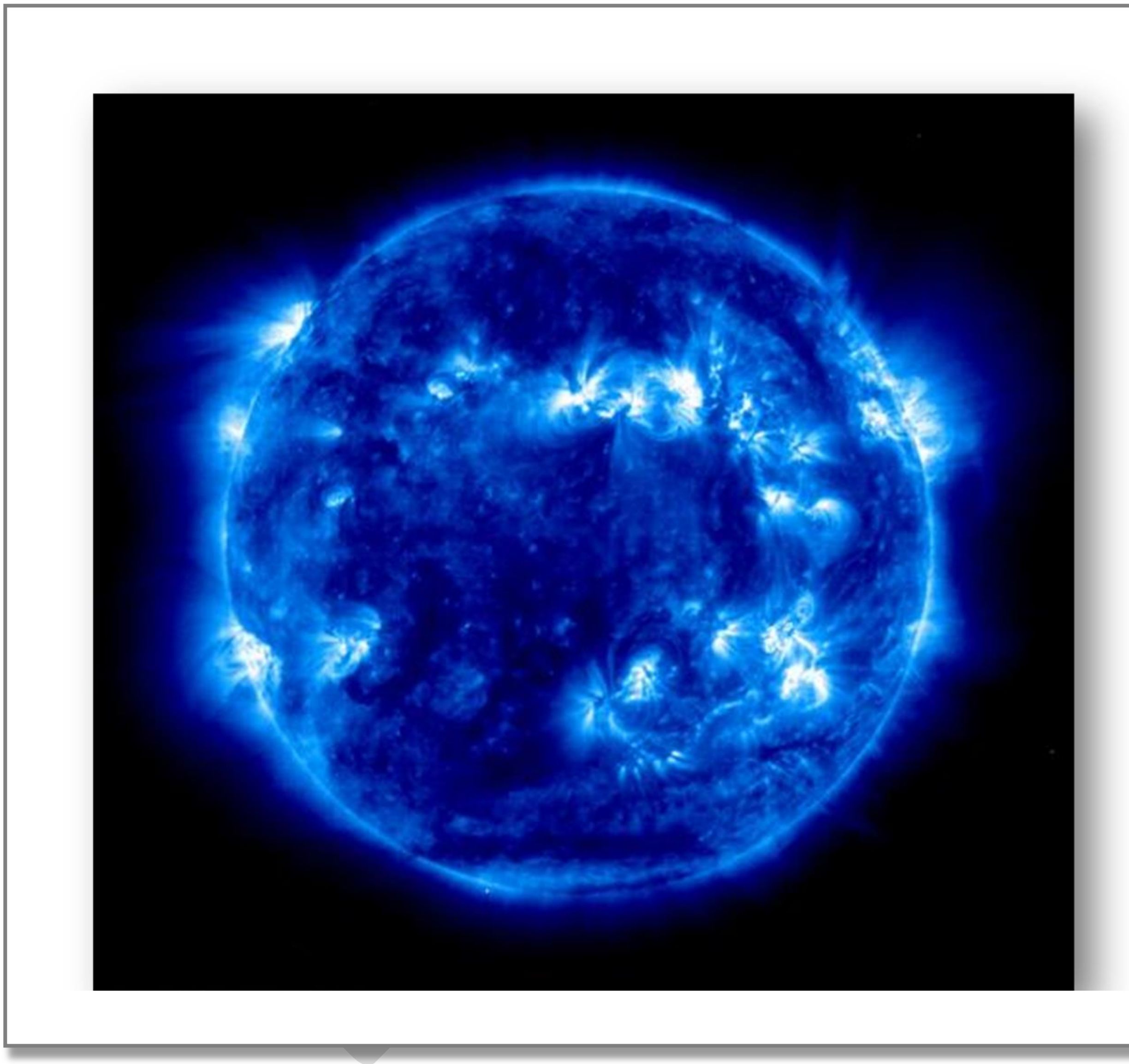
2) What flux does the magnetic field from part 1) generate through one turn of wire? Again, write a general equation, then calculate a value.

3) If the current shown (i.e., it varies with time) flows through the solenoid, sketch the EMF of the solenoid on the plot below.





Week 9 Lectures 14 and 15—Faraday's Law, Electromagnetic Waves and Polarization



Summary

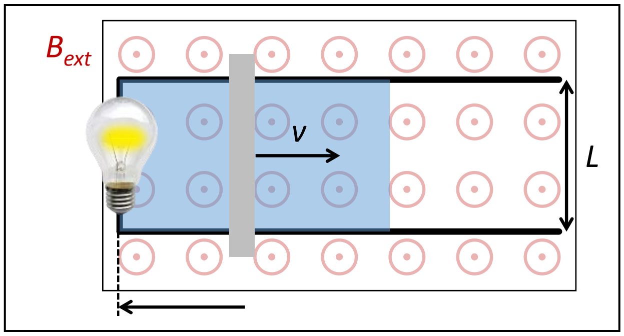
Glossary

Electromagnetic waves: *the result of oscillating electromagnetic fields. Electromagnetic waves travel at the speed of light.*

Transformer: *an electrical device which uses Faraday's Law to adjust voltages.*

Key Ideas
Faraday's Law

Magnetic fields can produce an electro-motive force (EMF) in a closed loop of conducting material. This is *electromagnetic induction.* Faraday's Law helps determine the strength of the EMF produced by induction. Faraday's Law is:



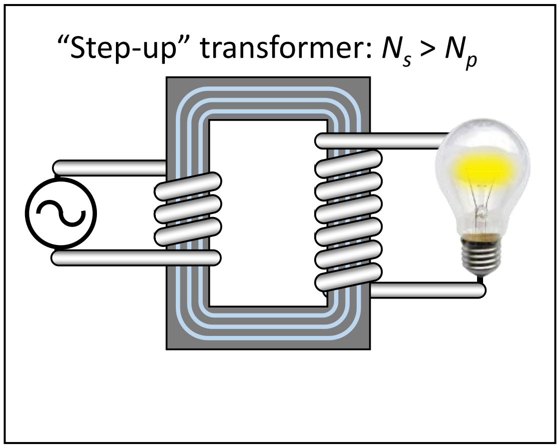
$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

Let's define the terms:

- ε EMF induced around a loop
- N number of turns of wire in a loop
- $\frac{\Delta\Phi}{\Delta t}$ change in the magnetic flux through the loop
- Δt time over which this change took place
- The overall negative sign comes from Lenz' Law.

Transformer

The transformer is an electrical device that uses Faraday's Law to alter a voltage. It has *primary* (P) and *secondary* (S) independent wire loops. The wire loops are wrapped around a conducting (usually iron) core. The core confines the magnetic field with in it. Energy conservation, and the flux from the primary going through the secondary, leads to the following ratios



$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

These relationships state that the ratio of secondary to primary currents is equal to the ratio of primary to secondary loops (N_P and N_S). Note: This only happens for changing currents. DC currents do not make an induced EMF in the second coil! Why?

Electromagnetic Waves

Accelerating charges produce electromagnetic waves. Sources include:

- oscillating charges
- sine-wave generators
- antennae
- decaying atoms or nuclei
- many other things.

An electromagnetic wave has a frequency associated with the source and the frequency range is staggering:

- $10^4\ Hz$ or less for certain radio communications (of submarines)
- $10^{24}\ Hz$ for the light bursts associated with the decays of excited nuclei.

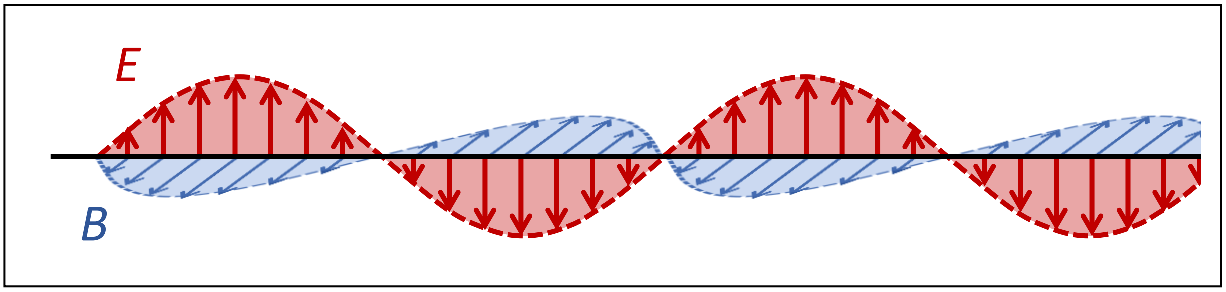
More common frequencies:

- FM radio and TV in the 100 MHz range
- visible light, all in a narrow band from $4\ to\ 7.9 \times 10^{14}\ Hz$.

All EM waves travel at the same speed in vacuum, the *speed of light*. They have a wavelength, λ, given by the relation:

$$\boxed{\lambda = c/f}$$

Visualizing Electromagnetic Waves

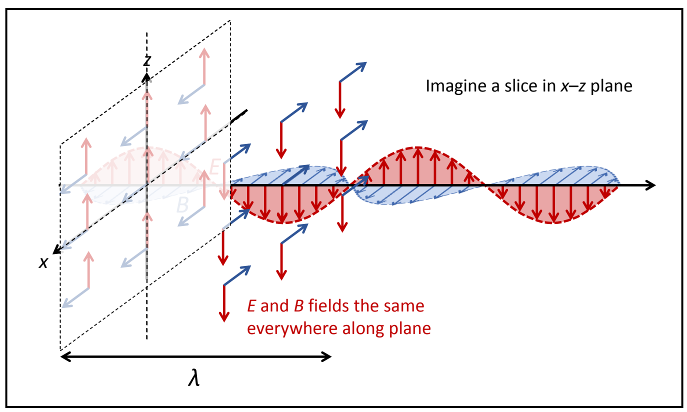


This figure is commonly used to illustrate an EM wave moving to the right (in vacuum). It illustrates these important facts:

1. The E and B fields are perpendicular.
2. They are in phase. When one is big the other is too.
3. They go in the same direction.
4. There is a fixed relationship between their strengths (amplitudes in the picture above): $E = cB$.

This figure fails to show how the E and B fields vary as one moves **perpendicular** to the direction of motion.

For a plane EM wave, E and B are constant over an entire infinite plane perpendicular to the direction of motion. Examine the figure at the right. Notice the electric and magnetic fields are the same at each point on the test plane (dashed rectangle).

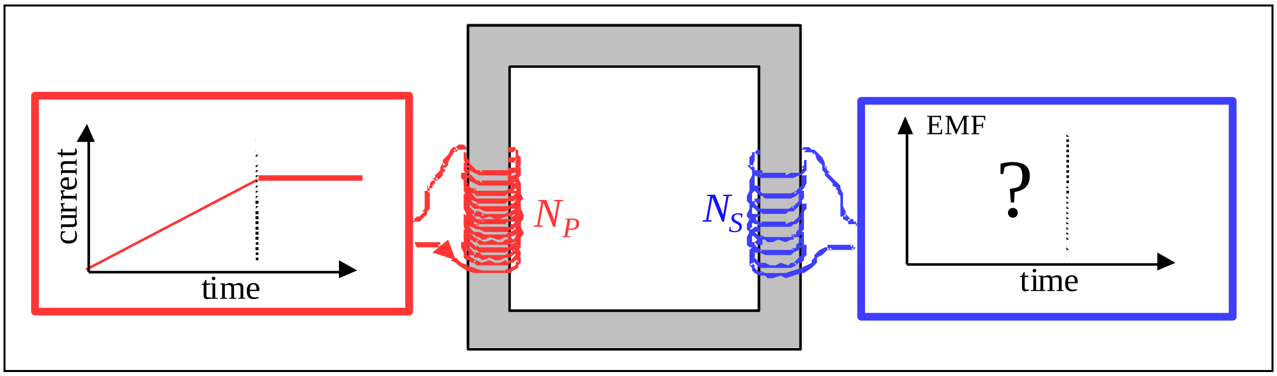


NOTE: If we move the plane along the direction of motion, at the same speed as the wave, then the E and B picture in that plane will remain unchanged in time.

Situations for Lecture 14: Faraday's Law of Induction

Situation 1: The Simple Transformer

Consider a simple transformer as shown in the diagram.



The primary coils are connected to a box that produces a current following the red (left) graph. Notice that as it turns on, the current increases at a constant rate until it reaches a fixed final value.

1) In the space below draw a curve of the magnetic field inside the red (primary) coils versus time. Label the graph.

2) On the same plot, draw the magnitude of the magnetic field through the blue (secondary) coils versus time. Use a different color pencil or pen and mark it. In what ways are these the same or different?

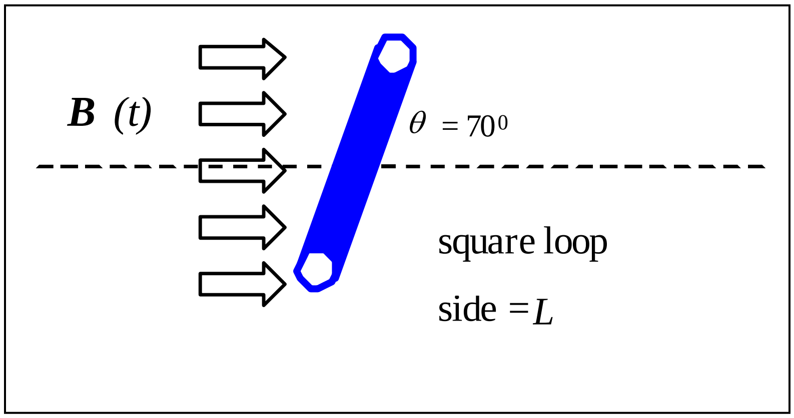
3) How would your curve look in part 2) if the vertical axis was magnetic flux rather than magnetic field? Draw this curve in the space below (label the graph):

4) When the flux is constant through the secondary coils, what is the induced EMF in the coil?

5) Make a plot of EMF in the secondary versus time in the space below (label the graph):

Situation 2: The Changing Field

A square loop with side $L = 0.25\ m$ is shown in a side-on view. A horizontal magnetic field has the time-dependent magnitude: $B(t) = [3.5 - 0.02t]\ T$.



1) What is the flux through the loop at $t = 10\ s$?

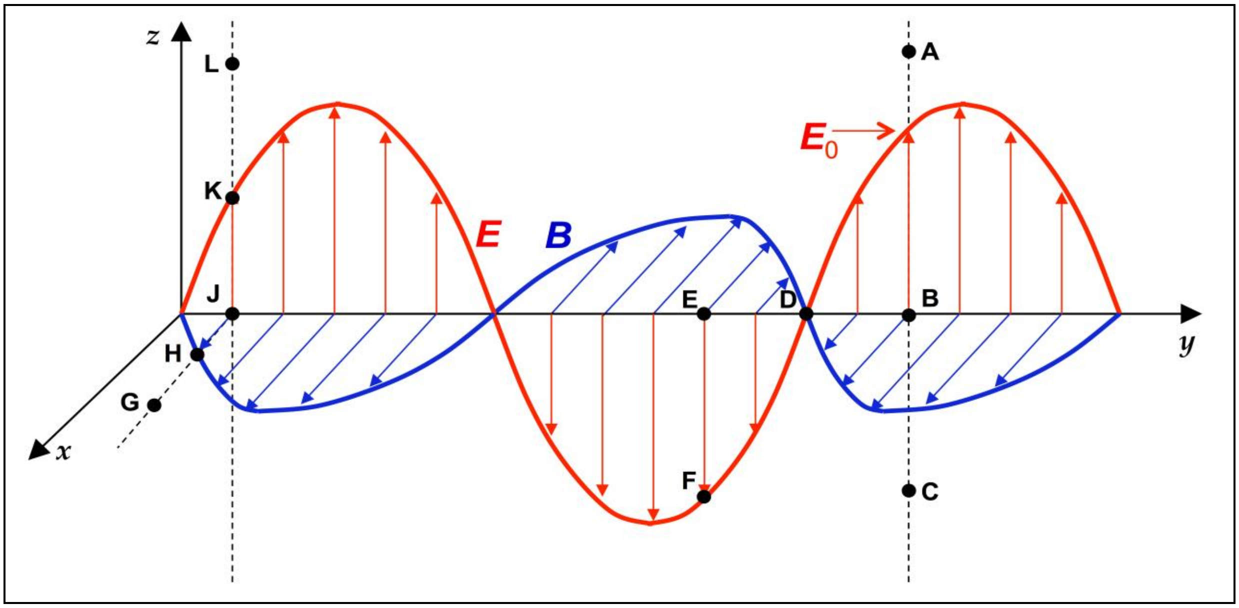
2) At what time is the flux through the loop zero? At this time what is the magnitude of the EMF around the loop?

3) If the net resistance of the loop is $15\ \Omega$ what is the induced current as a function of time? Start with $t = 0$. Make a sketch of the induced current versus time for this problem and then Mark an (X) or a dot (·) on the figure to indicate the direction of the induced current.

Situations for Lecture 15: Electromagnetic Waves

Situation 1: The Traveling Electromagnetic Wave

Consider the electromagnetic wave shown in the diagram below:



This wave is moving in the +y direction. Several points are indicated in the figure. They are referred to in the question.

1) Is this wave polarized and if so, in what direction? Discuss this with your teammates and take notes on your discussion.

2) Compare the strength of the electric field at points **A**, **B** and **C**. Which statement is true? Discuss with your group until you all agree and take notes on your discussion.
   a. A > B > C
   b. A > 0; B = 0; C < 0
   c. A = 0; B = E0; C = 0
   d. A = B = C = E0

3) Which of the following statements are true about the relationship between the electric field at the following points? (Several could be correct; justify your answers.) Discuss with your teammates and take notes on your discussion.
   a. E = D = B
   b. E = F
   c. F = -A
   d. D = 0

4) What is the relationship between the electric fields at the series of points **G**, **H**, **J**, **K**, **L**? What is the relationship between the magnetic fields there? Remember to take notes on your team discussion.

Week 10 Lectures 16 and 17—Electromagnetic Wave Energy and Ray Optics

Summary

Glossary

Index of refraction (n): *the quantity that describes the speed of light in a material relative to the speed of light in vacuum.*

Law of Malus: *the relationship describing the intensity of an electromagnetic wave as it passes through as series of linear polarizers.*

Linearly polarized: *the electric field oscillates along a single axis.*

Object: *the source of light rays*

Principal Axis: *the perpendicular line intersecting the midpoint of a lens or mirror.*

Rays: *the graphical representation of light propagation.*

Real Image: *the physical location of the convergence of light rays.*

Reflection: *light that changes direction and returns to the original medium after interacting with the interface between two media.*

Refraction: *the bending of light after passing through a material interface.*

Snell's law: *the relationship which describes how much light will bend as it passes through an interface between to materials of different index of refraction.*

Virtual Image: *the apparent location of the convergence of light rays.*

Key Ideas

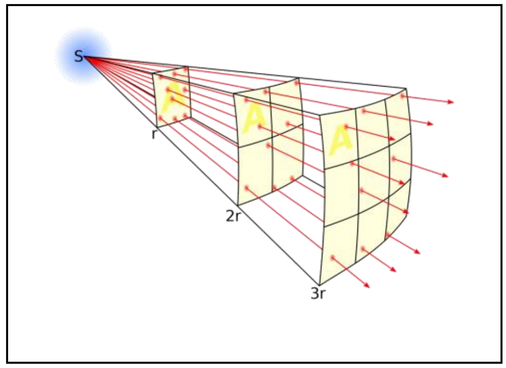
Electromagnetic intensity

EM waves carry energy, for example, the heat you feel when the sun shines on your skin. Intensity is the power per unit area:

$$S = cu = c\varepsilon_0 E^2 = \frac{c}{\mu_0} B^2, \text{ Units: } W/m^2$$
$$E \text{ and } B \text{ are rms values.}$$

Peak and rms amplitudes are related:

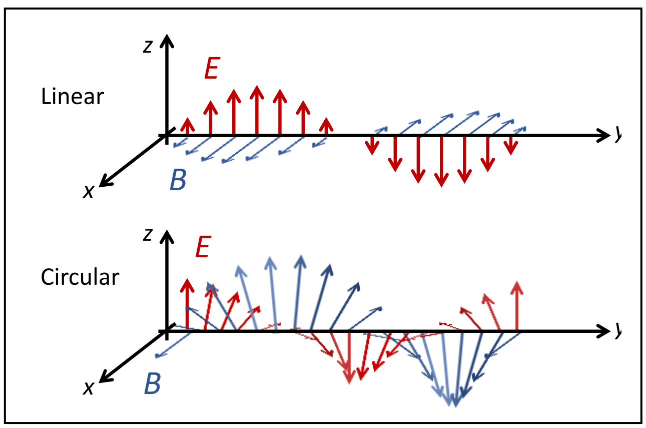
$$E_{rms} = \frac{1}{\sqrt{2}} E_0$$



The total energy absorbed from an EM wave depends on:

1) the exposure time.
2) effective intercepting area.

Polarization in EM Waves



The illustrations of waves show that EM waves are *transverse waves*. The examples shown are also *linearly polarized*, the electric field always oscillates along the same axis (here up/down). We use the electric field to define the direction of polarization, if there is one.

Polarized light can be produced from unpolarized light by selective absorption. [A common light bulb, for instance, produces unpolarized light.] For example, certain materials only pass light with an electric field oscillating along a fixed direction. Polarized sunglasses are a common example. The lenses only pass vertical electric fields. This reduces two features:

- 50% of the intensity
- all light polarized horizontally, such as glare.

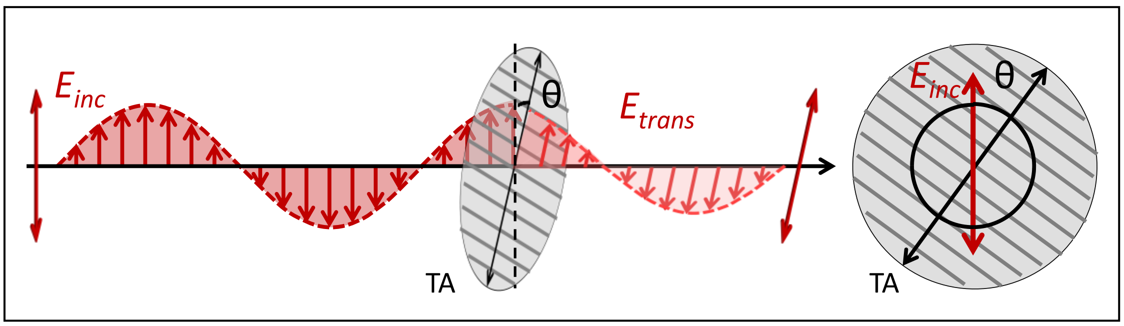
Polarized EM waves are made by broadcast antennas. For example AM transmitters are tall vertical poles. FM transmitters are much shorter horizontal antennas. Which way waves are they polarized? To receive them, which way you should orient your radio antenna?

Law of Malus

In the figure below, unpolarized light of intensity I_0 is directed through a linear polarizer with a vertical transmission axis (TA). No matter which way the TA is pointed, 50% of the light gets through. This light, with intensity I_1, passes through a second linear polarizer whose TA is oriented at angle θ with respect to the first TA. The intensity of light getting through is reduced by the factor $\cos^2\theta$which leads to the *Law of Malus:*

$$I_{trans} = I_{inc}\cos^2\theta$$

(Note, sometimes we call intensity S and sometimes we call it I. They are the same thing!) The figure below illustrates the Law of Malus or two (2) linear polarizers:



Ray Optics

The study of light uses geometry. We also need some new vocabulary:

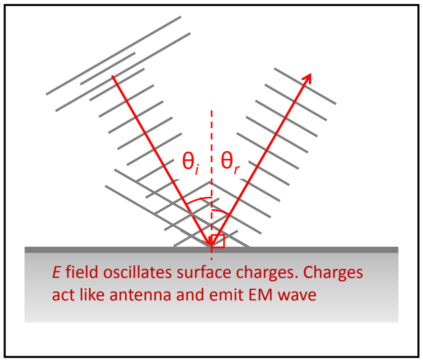
- *Rays*: Arrows indicating the direction of propagation of a light wave. Rays always appear to travel in straight lines.
- *Object*: Term used to indicate the source from which light rays appear to originate.
- *Real Image*: Location of convergence of actual light rays in either mirror or lens problem. You can put a physical screen where a real image is located.
- *Virtual Image*: Place where light rays appear to converge. Real light beams do not converge in this situation. For example, images behind a mirror or in front of a lens.
- *Principal Axis*: Straight line which intersects the midpoint of a mirror or lens at $90°$.

Interaction with Matter

Light interacts with matter in a variety of ways. At the boundary between two different materials light waves can undergo:

- **Reflection:** light that changes direction and returns to the original medium after interacting with the interface between two media.
- **Refraction:** the bending of light after passing through a material interface.
- **Absorption:** energy deposition in a medium after light has interacted with it.

Law of Reflection: Angle of Incidence equals Angle of Reflection:

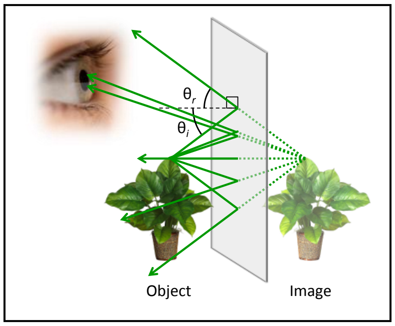


$$\theta_i = \theta_r$$

To keep things straight, always measure this angle with respect to the normal (the dashed line) to the surface!

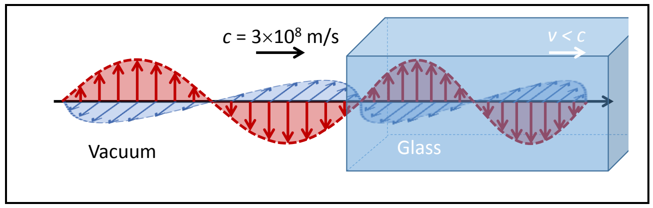
Flat Mirror

An upright virtual image of same size as object is located as far "behind" mirror as the object is in front. Use law of reflection to find upright, same-sized, image.



Refraction

Light travels at different speeds in different materials. This causes the bending of light rays at the interface between materials--*refraction*. This is the principle that drives all optical instruments. It is characterized by the **index of refraction**:



$$n = \frac{c}{v} = \frac{[speed\ of\ light\ in\ vacuum]}{[speed\ of\ light\ in\ material]}$$

Notice:

- n is always greater than or equal to 1.0
- $c = 3 \times 10^8\ m/s.$

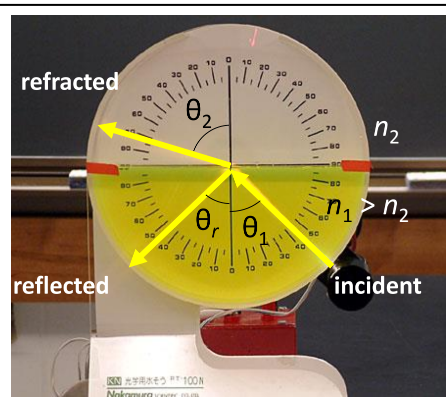
Snell's law

Snell's law describes how light passes through the interface of two materials of different *index of refraction.*

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Let's define some terms:

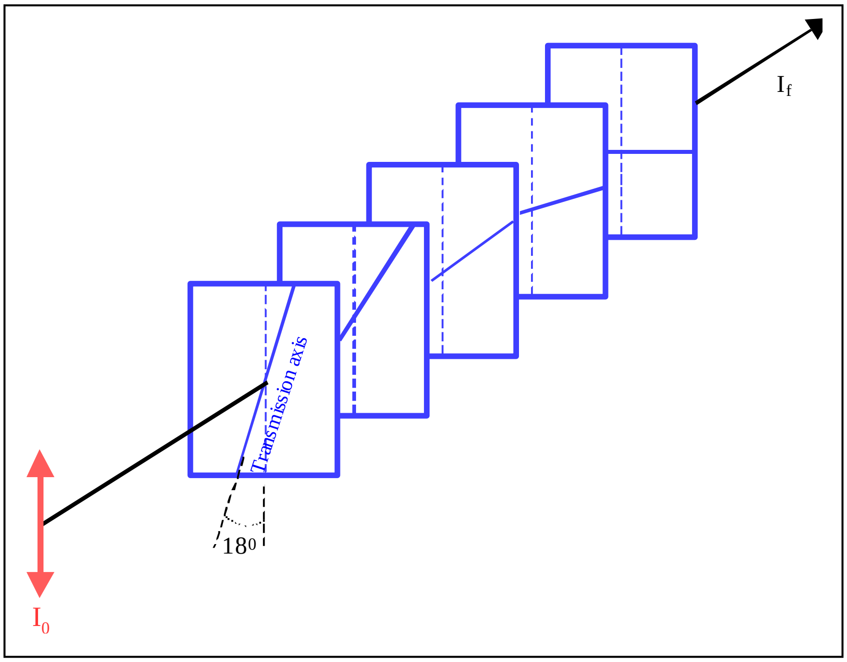
- Subscript 1: the incident ray
- Subscript 2: the refracted ray
- n_1: the index of refraction for the material associated with the incident ray
- n_2: the index of refraction for the material associated with the refracted ray
- θ_1: the angle of incidence with respect to the normal to the interface
- θ_2: the angle of refraction



Situations for Lecture 16: Electromagnetic Wave Energy & Polarization

Situation 1: A Sequence of Polarizers

Consider the series of linear polarizers shown:



Vertically polarized light, of intensity I_0, is incident on a series of 5 linear polarizers, each with its Transmission Axis tilted by the same amount relative to the previous filter, $\theta = 18^0$

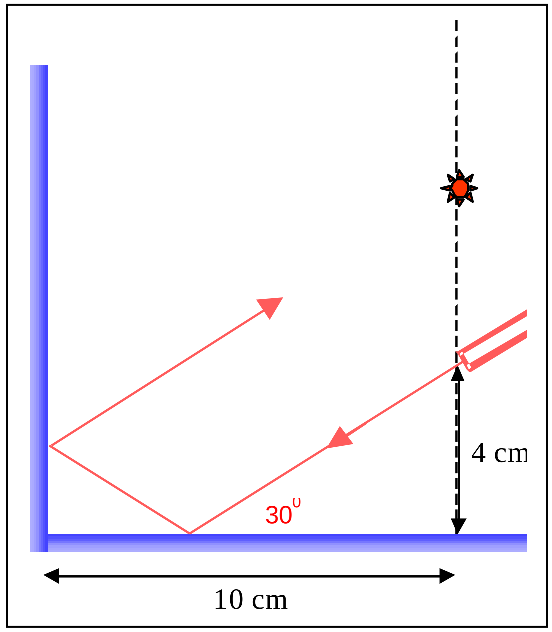
1) What is the intensity of the light after the 1st polarizer?

2) What is the final intensity I_f in terms of I_0?

3) Suppose *vertically polarized* light starts at the end (where the black arrow is) and propagated in the opposite direction through the polarizers. What is the light intensity after *three* polarizers?

Situations for Lecture 17: Reflections

Situation 1: Flat Mirrors!



Consider the following situation:

A laser beam located $4\ cm$ above a horizontal surface shoots a beam of light at a flat mirror with a $30°$ angle with respect to the horizontal. The reflected beam then strikes a second flat mirror which is at a right angle with respect to the first. The final reflected beam hits the star somewhat above the laser and in the same plane as the laser.

1) What is the important information in this problem? Discuss this with your teammates and take notes on the discussion.

2) What is the angle of incidence of the initial beam on the horizontal flat mirror according to our common convention?

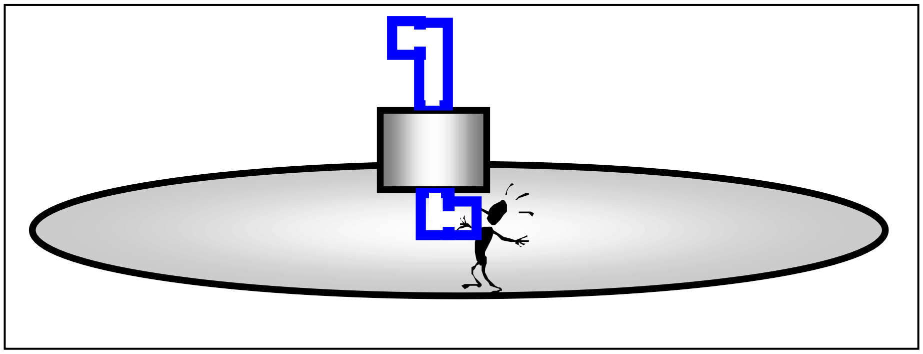
3) What is the angle of incidence of the reflected beam when it strikes the second flat mirror?

4) The final beam hits the starred spot. How high is the star above the horizontal surface?

Situation 2: The Periscope

Consider the following situation:

Our little friend is using a submarine's periscope to peek outside the sub. Use this page to make a large sketch of the periscope with internal mirrors and rays to indicate how this device works. Using flat mirrors is the simple way to go!



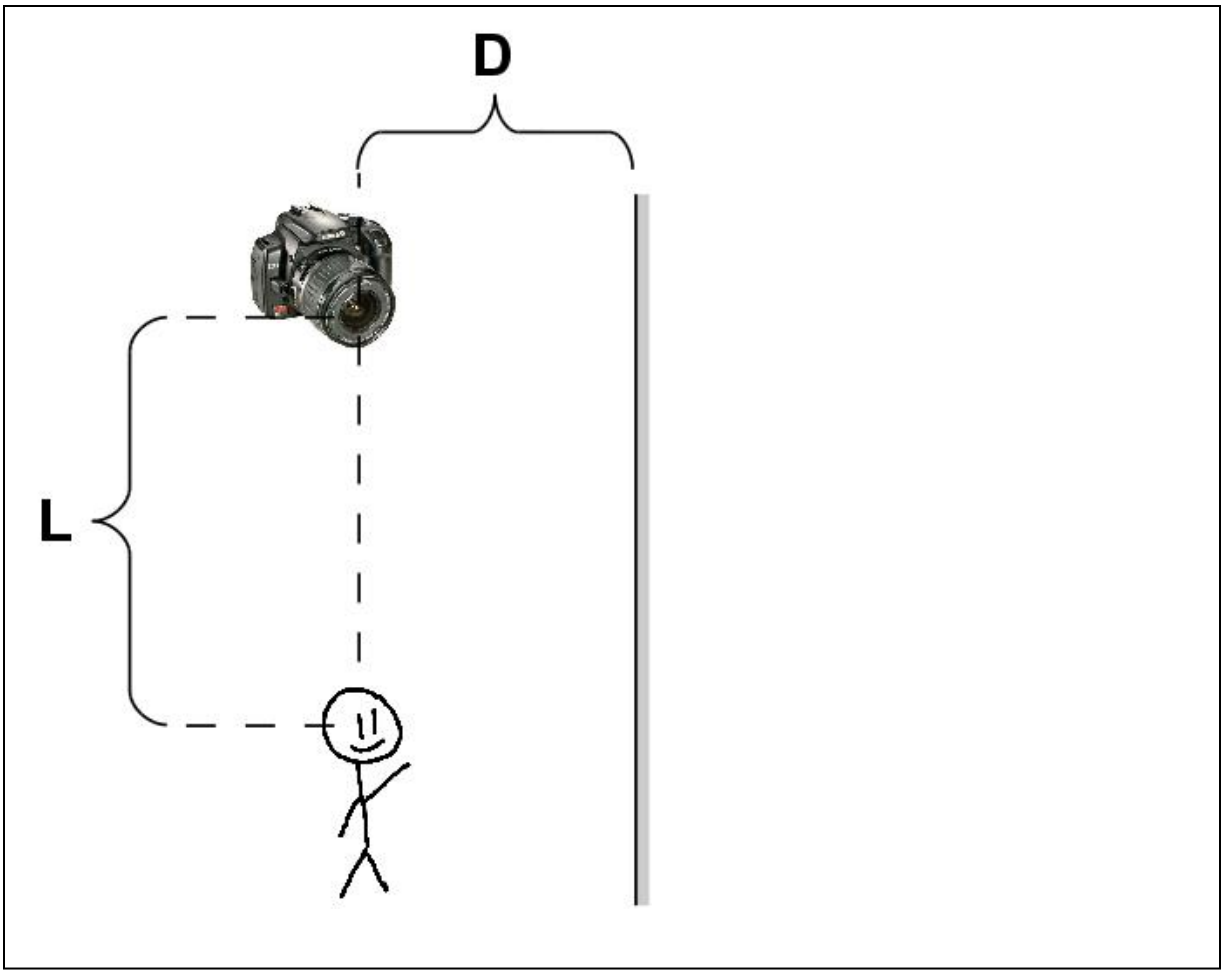
CHALLENGE: Will the final image (the image that our little friend sees) be upright or inverted? Will it look magnified? Justify your answer to your TA with a drawing.

Situation 3: The Camera and the Mirror

Your friend decides to take a picture of you using a flat mirror.

Instead of taking a picture of you directly, your friend focuses the camera on the image of you formed behind the mirror. You are standing a distance $L = 0.85\ m$ away from the camera. You are both a distance $D = 0.30\ m$ away from the face of the mirror, as shown in the diagram.

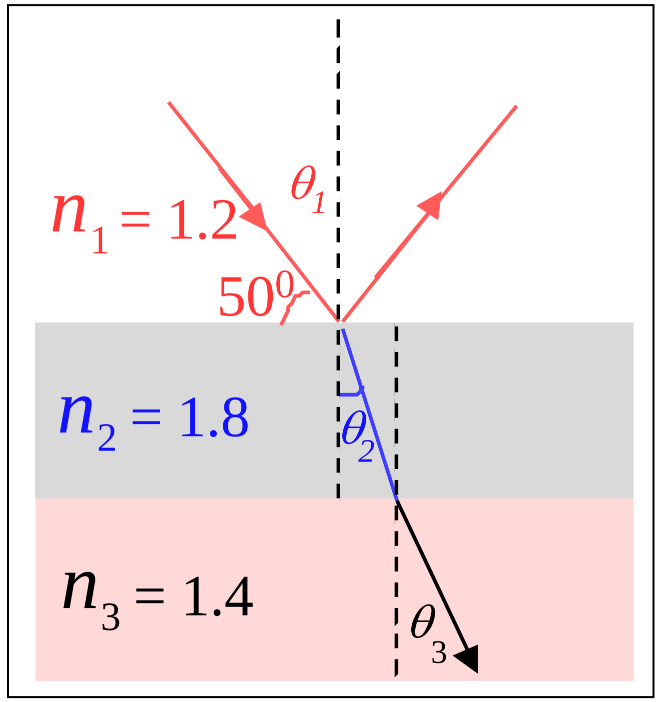
1) On the diagram draw two rays to locate the image of you that your friend holding the camera sees in the mirror. Remember, principal rays begin at the object (you) and end at the image (you behind the mirror).
2) On the diagram indicate the exact location (with numerical distances) and draw a picture of your image behind the mirror.
3) Is the image REAL or VIRTUAL?
4) Is the image UPRIGHT or INVERTED?



Situation 4: Snell's Law for 3 Materials

Consider the following:

A light ray travels through a material with index of refraction n_1. It strikes an interface at a $50°$ angle with respect to the horizontal. It passes into material 2, then into material 3. The index of refraction of each material is shown.



1) What is the value of θ_2 ?

2) The slab of material with index of refraction n_2 is $3\ cm$ thick. How far from the center line does the ray exit the slab with index of refraction n_2?

3) What is the value of angle θ_3 ?

4) If the middle slab were removed, would θ_3 remain the same? Discuss this first, then check your intuition by making an explicit calculation. Take notes on your discussion.

Week 11 Lectures 18 and 19—Spherical Mirrors, Total Internal Reflection and Lenses

Summary

Glossary

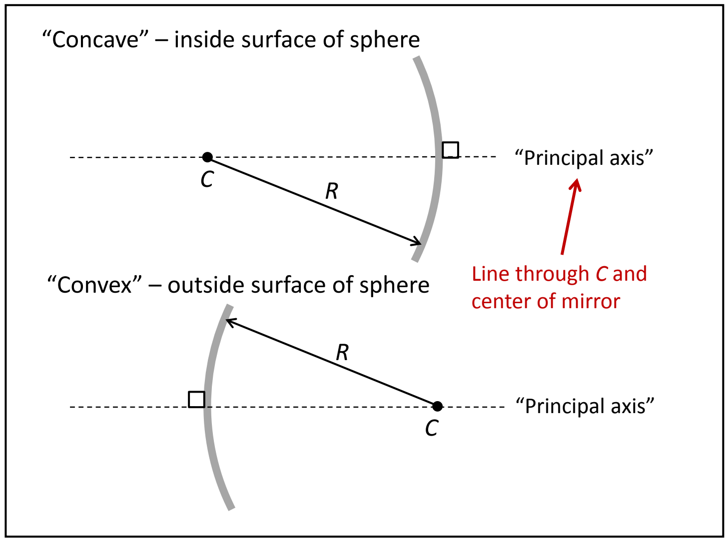
Concave Mirror: *a spherical mirror of positive focal length*
Convex Mirror: *a spherical mirror of negative focal length*
Critical angle (θc): *the incident angle at which light refracted at an interface will be bent $90°$.*
Focal length: *in lenses and mirror, the measure of the ability of the lens or mirror to converge or diverge parallel rays of light.*
Focal point(f): *or focus is the point where light rays converge.*
Total internal reflection: *the result of light incident at an interface which exceeds the critical angle.*

Key Ideas

Spherical Mirrors

A spherical mirror has a surface cut from a sphere of radius R. The two types of spherical mirror are:

- Concave Mirror: positive focal length; converging
- Convex Mirror: negative focal length; diverging

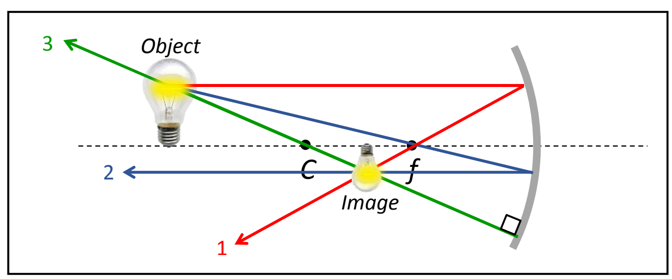


The focal length is $f = R/2$. The following techniques are used to locate an image when rays are near to the principal axis (small angles).

Start by trying finding an image of the top of the object The intersection of any two of the three rays tells you where the image is located.

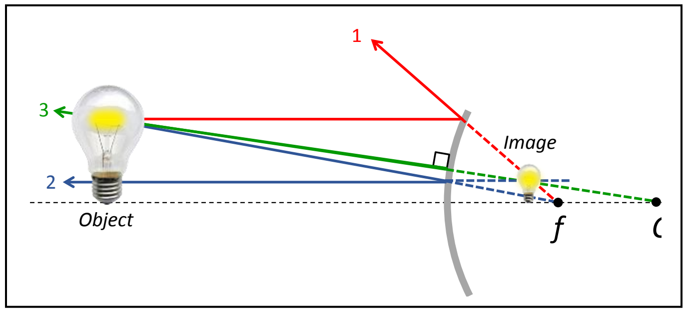
- Solid lines indicate the path of light.
- Dashed lines indicate the apparent path of light (virtual rays).
- Convergence (intersection) of solid lines produces a real image.
- Convergence of dashed lines produces a virtual image.

Concave Mirror



1) Ray 1 begins parallel to the principal axis (p.a.), hits the mirror, and reflects through the focal point, f.
2) Ray 2 passes through the focal point and is reflected parallel to the p.a.. If the object is located further than the focal point, this is easy. If the object is "inside" the focal point, then extend the starting ray backwards until it passes through the focal point.
3) Ray 3 goes through the center of the sphere (C), touching the tip of the object, hits the mirror and reflects back on itself. If the object is close to the center, this ray is hard to draw.

Convex Mirror (note: negative focal length $f = -R/2$)



1) Ray 1 begins parallel to the principal axis (p.a.), reflects out at an angle as if it had come from the focal point.
2) Ray 2 heads toward the focal point and is reflected parallel to the p.a.. Its extension behind the mirror shows the convergence with ray 1 at the location of the virtual image.
3) Ray 3 travels toward the center of curvature and reflects directly back on itself. As before, extension behind the mirror shows the convergence with the other virtual rays.

The mirror and magnification equations are used to find the image location and size in spherical mirrors. Important symbols:

- f: Focal length. Note: concave is positive; convex is negative; flat is infinite
- d_0: Object distance. Measured from mirror to object. Note: It will be negative if behind the Mirror (virtual; can only happen in multiple mirror/lens problems)
- d_i: Image distance. Measured from mirror to image. Note: It will be negative if behind the mirror (virtual).
- h_0: Object height.
- h_i: Image height.
- m: Magnification. Ratio, h_i/h_0, of image height to object height

Mirror equation

Be careful with the signs. Note: d_i may be positive or negative corresponding to *in front of the mirror* or *behind the mirror*, respectively.

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

Magnification equation

$$m = \frac{h_i}{h_0} = -\frac{d_i}{d_0}$$

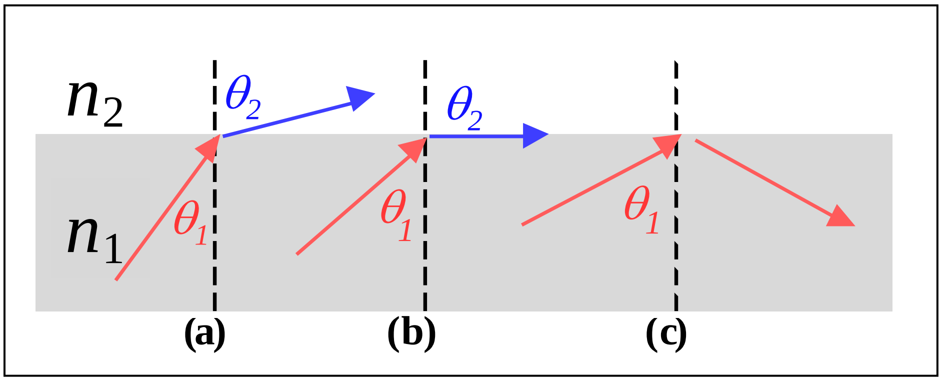
Note the minus sign here. If m is negative, it means the image is inverted. If it is positive, it is upright. Values of $|m| > 1$ imply increased size, value $|m| < 1$ imply smaller images. The strategy normally is to find the image location, then compute the magnification.

Total Internal Reflection

Critical angle

If $n_1 > n_2$ (ray travels from dense to less dense material), there is an angle of incidence which corresponds to a $90°$ *angle of refraction*. This incident angle is called the *critical angle*.

Consider the following figure:



- Incident angles less than this *critical* angle look like case (a).
- At the critical angle the refracted light goes right along the interface boundary as in case (b).
- At incident angles greater than the critical angle, all the light just gets reflected back into the material of type 1, a condition we call *total internal reflection*, case (c).

Total internal reflection is important for keeping light inside a material, for example in fiber optic applications. The *critical angle* is the incident angle at which this effect just begins to take place. It is given by:

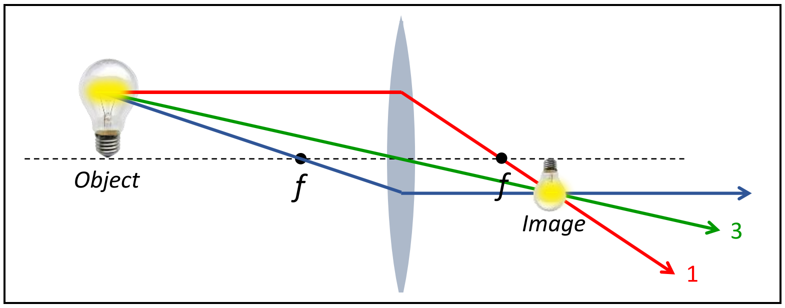
$$\sin\theta_c = \frac{n_2}{n_1};\ n_1 > n_2$$

Thin Lenses

Ray Tracing

Mark a focal point (f) on both sides of these lenses. Light passes through the lens. *Converging lenses* are thicker in the middle than they are at the edge. *Diverging lenses* are thinner in the middle compared to the edge.

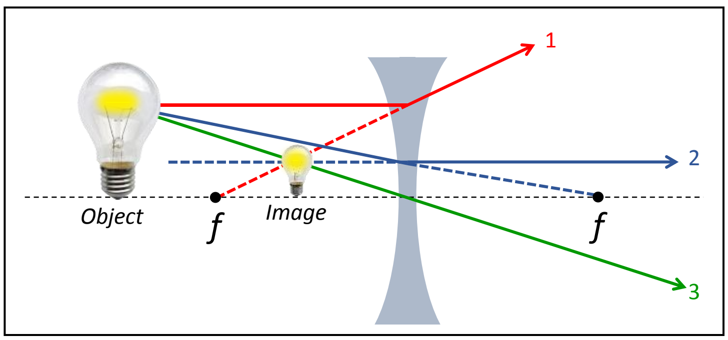
Converging Lens (typical)



1) Ray 1 begins parallel to the p.a., bends through opposite focus
2) Ray 2 passes through the near focal point and is bent out parallel to the p.a. on the opposite side of the lens.
3) Ray 3 goes straight through the center of the lens undeflected. This is the easiest ray to draw!

Diverging lens (typical)

The object is *always virtual*. The focal length is negative. You will have to extend rays (using dashed lines) on the object side of the lens to find the image.



1) Ray 1 begins parallel to the p.a. then refracts away from the p.a. as if it had come from the focal point on the near side of the lens.
2) Ray 2 (subtle one) ray starts from the object toward opposite focus, but strikes lens along the way at which point it is bent parallel to the p.a. To find where this *appears* to have come from extend this parallel line backwards.
3) Ray 3 still the easy one, travels right through the center of the lens undeflected.

The thin lens and magnification equations are exactly the same as those for mirrors. We usually look at the problem from *left to right* with a side-on view as shown in this tiny image. A review of symbols:

- f: focal length. Converging is positive; diverging is negative.
- d_0: object distance. Measured from lens to object. In our conventions, d_0 is *positive* if the object is to the *left* of the lens and *negative* if the *virtual object* is to the *right* of the lens (this only occurs in multiple lens problems).
- d_i: image distance. Measured from the lens to the image. This will be *positive* if the image is on the *right* side of the lens and *negative* if the image is on the *left* side of the lens.
- h_0: object height.

- h_i: image height.
- m: magnification. Ratio of image height to object height

Thin-lens equation

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

Be careful with the signs.

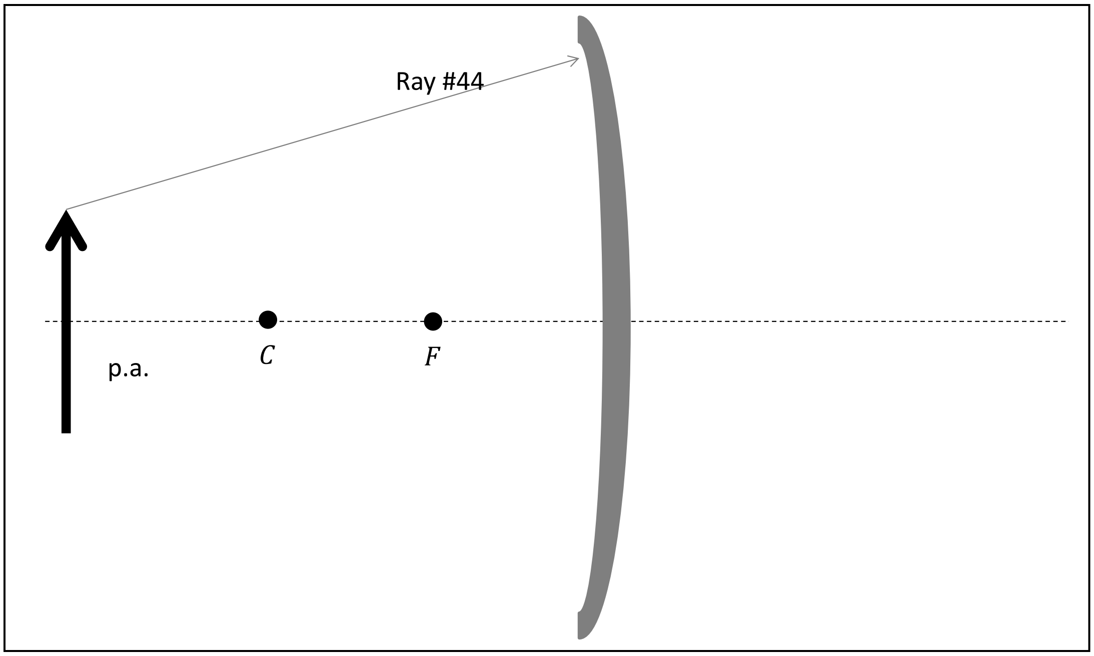
Magnification equation

$$m = \frac{h_i}{h_0} = -\frac{d_i}{d_0}$$

If m is *negative*, the image is *inverted*. If m is *positive*, the image is *upright*. If the magnitude of $m > 1$, the image is *larger* while $m < 1$ implies *smaller* images.

Situations for Lecture 18: Spherical Mirrors

Situation 1: Concave mirror with object well outside focal length



1) Use ray tracing to find the location of the image. Make sure you image the entire object. Notice it spans the principal axis. (Ignore the ray labeled #44.)

2) Describe the characteristics of this image by circling the right choices:

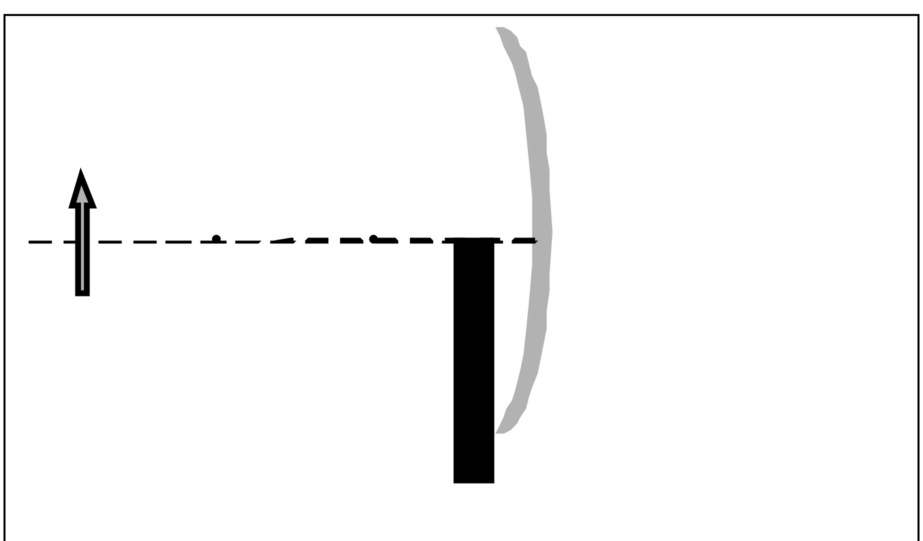
REAL	VIRTUAL	
UPRIGHT	INVERTED	
ENLARGED	REDUCED	SAME SIZE

3) Complete "Ray #44" in the diagram. Where does it go after reflecting off the mirror?

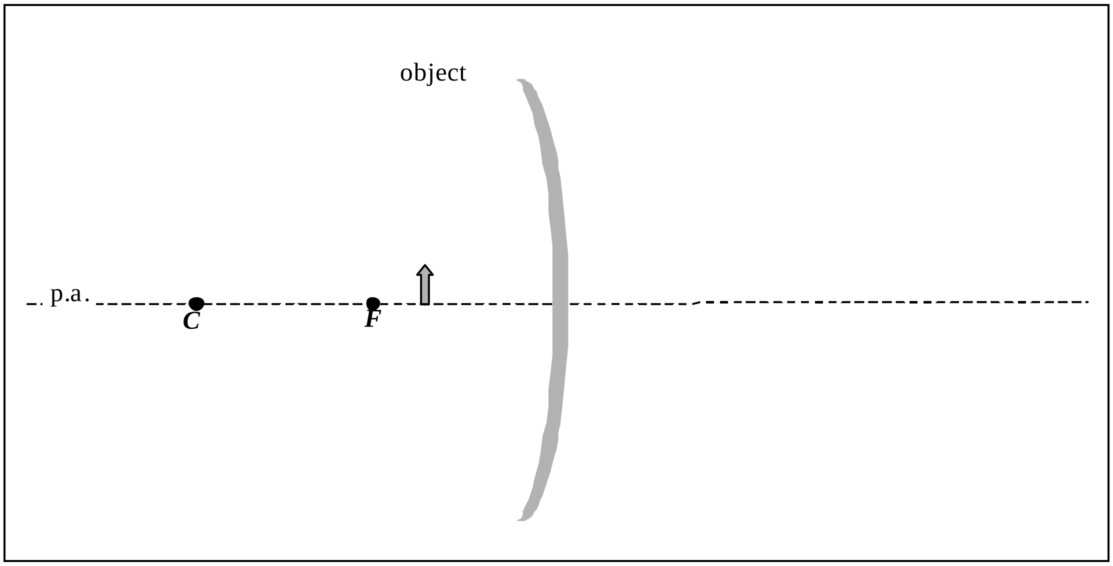
4) Given $F = 10\ cm$, and the object is located $20\ cm$ to the left of F, use the mirror equation to locate the image position on the principal axis of the image. Does this agree with your ray tracing?

5) If the object has a height of $5\ cm$, how large is the image?

6) Suppose somebody covers the bottom half of the mirror as shown in the picture. Will the image still form? Discuss this a bit and then draw some lines to support your conclusions.



Situation 2: Concave mirror with object inside the focal length



1) Use ray tracing to find the location of the image. Hint: you might think about extending your rays behind the mirror.

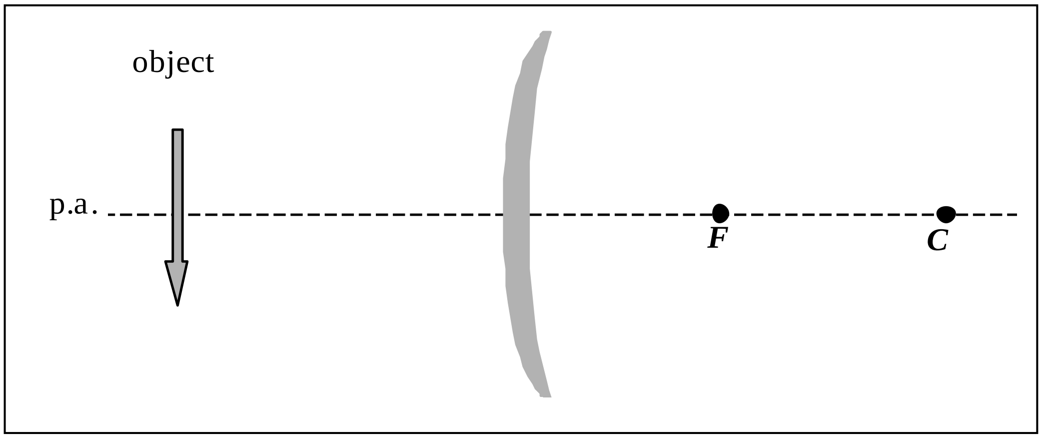
2) Describe the characteristics of this image by circling the right choice:

REAL	VIRTUAL	
UPRIGHT	INVERTED	
ENLARGED	REDUCED	SAME SIZE

3) The radius of curvature of this concave mirror is $20\ cm$. The object is located a distance $7\ cm$ in front of the mirror. Find the image using the mirror equation. Check with your ray tracing drawing.

4) If the object has a height of $3\ cm$, what is the image height, and is it upright or inverted?

Situation 3: The Convex Mirror



1) Use ray tracing to find the location of the image. Make sure you image the entire object. Notice it spans the principal axis.

2) Describe the characteristics of this image by circling the right choice:

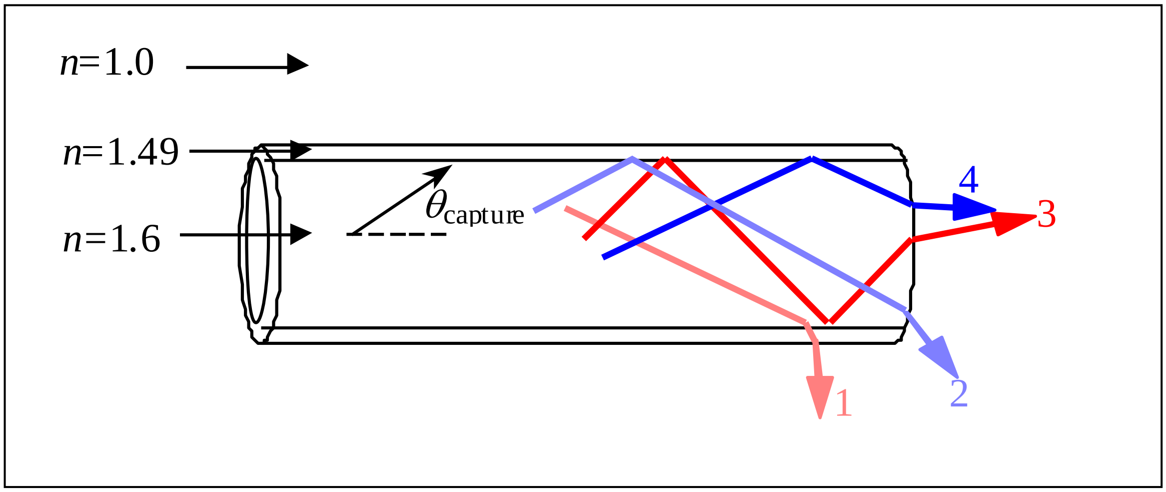
REAL	VIRTUAL	
UPRIGHT	INVERTED	
ENLARGED	REDUCED	SAME SIZE

3) The radius of curvature of this convex mirror is $40\ cm$. The object, of height $10\ cm$, is located a distance $35\ cm$ in front of the mirror. Where is the image and how large is it? Again, compare to your ray tracing drawing.

Situations for Lecture 19: Total Internal Reflection and Lenses

Situation 1: Fiber Optics

A fiber optic cable is shown:



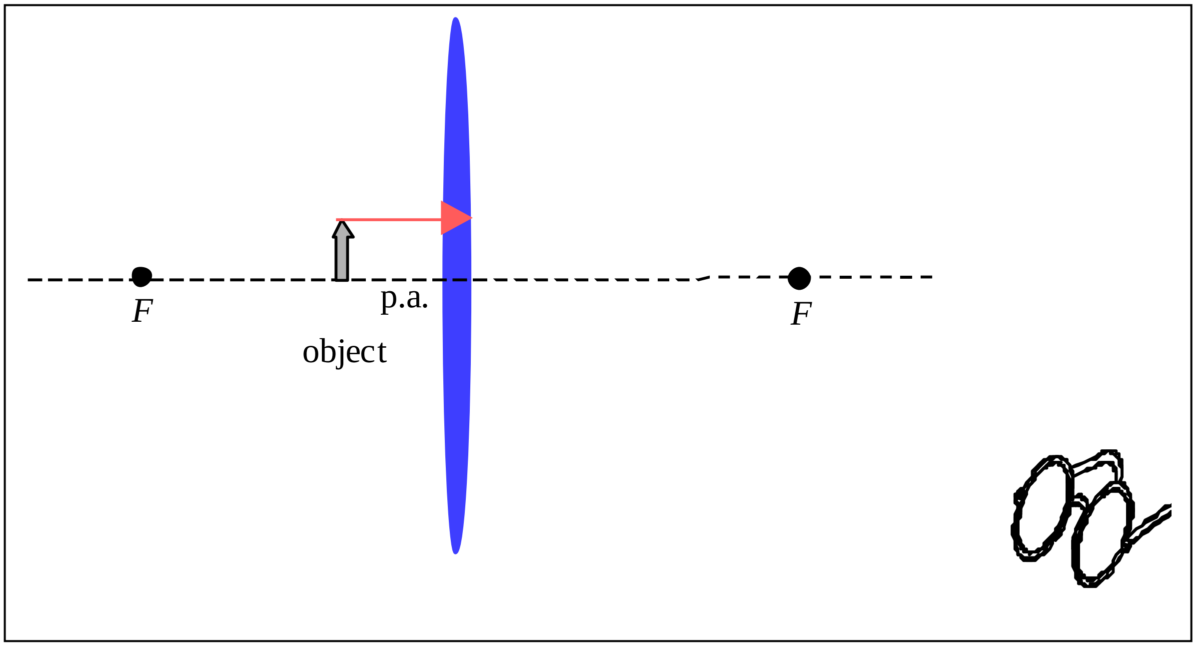
1) The core is polystyrene with index of refraction $n_{core} = 1.6$. The cladding (outer layer) is acrylic with $n_{cladding} = 1.49$. It is surrounded by air. Rays of light start from inside the fiber at the angles shown. Which of these rays looks correct? Explain your reasoning.

2) If this same fiber were embedded inside a material with index of refraction $n_{outside} = 1.8$, would your answer remain the same? What happens now? Explain your reasoning

3) What is the maximum angle, $\theta_{capture}$, that a light ray can have and still stay entirely within the fiber?

Situation 2: The Magnifying Glass

Consider the magnifying glass below:



The diagram above shows an observer (to the right indicated by the glasses) using a converging magnifying glass. The small object is located inside the focal length. The first part of the first ray of real light is drawn for you.

1) Complete this ray the way a real light beam would travel.

2) Now put in the ray that just passes through the lens center. Where do these rays converge? Is the image real or virtual?

3) The magnifying glass above has a focal length of $10\ cm$. You hold the magnifying glass $4\ cm$ above some print on a page. What is the magnification of the observed text compared to the original text?

4) Where is the image in this problem? Is it real or virtual?

5) Suppose you used this lens to concentrate the sun's rays to burn a small hole in a dry leaf. How high above the leaf would you hold the lens?

6) Make a quick sketch of how you would position the lens to burn the leaf.

Week 12 Lectures 20 and 21—The Eye, Corrective Lenses, and Optical Instruments

Summary

Glossary

Accommodation: *the process by which the eye changes the focal length of the lens.*

Angular magnification: *the increase in the observed size of an image from an optical instrument compared to the size of the image with the unaided eye.*

Dispersion of light: *the breaking of white light into its component wavelengths through refraction.*

Far point: *the furthest distance the eye can accommodate.*

Farsighted (hyperopic): *the condition which occurs when the relaxed eye produces an image behind the retina.*

Near point: *the closest distance the eye can accommodate and produce a clear image.*

Nearsighted (myopic): *the condition which occurs when the focal length of the relaxed eye produces an image in front of the retina.*

Refractive power: *a measure of a lens' ability to focus—typically used in eye correction.*

Key Ideas

The human eye and corrective lenses

To see an object, the eye must focus an image on the retina. The eye varies the focal length, f, of its lens by *accommodation*. For a normal eye, the *far point* is essentially at infinity and rays from a far-away object arrive at the eye parallel to the principle axis. The relaxed eye bends the rays to form an image on the retina.

Nearsighted (myopic) eyes focus too strongly when relaxed and the image of a far-away object forms in front of the retina. An additional diverging lens helps correct myopia. For example:

> The far point of a patient's eye is at $50\ cm$ (this is the furthest the person can see before things look blurry). What lens is needed to make the image sharp?

Solution steps:

- use the lens equation with and consider the object at infinity.
    - We want the image to form at the far point (where the patient can see clearly) which is located in front of the lens at the position – $50\ cm$.
    - We use

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}\rightarrow\frac{1}{\infty}+\frac{1}{-50}=\frac{1}{f};f=\ -50\ cm$$

- The negative sign implies the lens is divergent (thinner in the middle).

The closest object that can be clearly focused on the retina is at the *near point* (25 cm is normal). The tense eye bends rays from such an object to form an image on the retina.

Farsighted (hyperopic) eyes do not refract enough; therefore, very close objects have their images formed *behind* the retina. An additional converging (thicker in middle) lens, as found in reading glasses, helps correct hyperopia. For example:

> The near point of an eye is at 100 cm. What lens is needed to read a book at 25 cm?

Solution steps:

- The object is at 25 cm. We want the virtual image to appear at –100 cm.
- The result is a converging lens. Use

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}\rightarrow\frac{1}{25}+\frac{1}{-100}=\frac{1}{f};f=\ 33.3\ cm$$

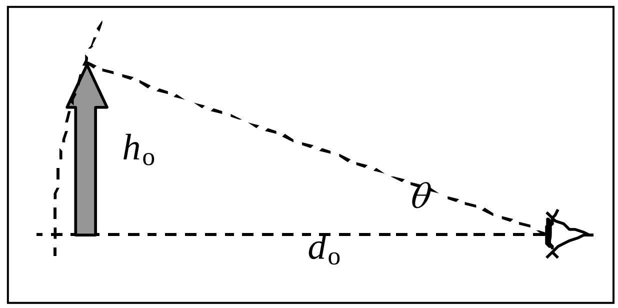
Refractive Power

1 diopter = $1/f$ with f *measured in meters*.

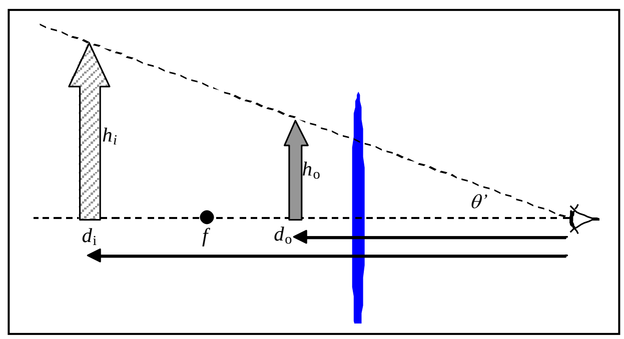
In our examples, the divergent lens had a refractive power of -2.0 diopters and the converging lens had a power of +3 diopters.

Angular Magnification

Angular magnification is the increased size of an image on your retina due to an optical instrument compared to the size of the image with only the unaided eye. The magnification, M, is equal to θ'/θ in general for an optical instrument, where $\theta \approx \frac{h}{d}$. Here h is the object height and d is the distance to the object.



Because our unaided eye has a near point, N, θ has its maximum value, when $d_0 = N$. We can bring the object (a bug, perhaps) closer, if we use a lens to produce an image that is beyond the near point. The light entering our eye appears to be coming from the image, so our eye can focus on it.



We can have d_o less than N as long as d_i is larger than N. The magnification, M, is the ratio of the new angular size, θ' to the unaided size, θ:

$$M = \frac{\theta'}{\theta} = \frac{h_0/d_0}{h_0/N} = N/d_0$$

You can control he magnification by moving the lens closer to or farther from the object. What's the best you can do?

Start with the thin lens equation:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

Set d_i equal to $-N$. We need the minus sign because the image is virtual (behind the lens). If the image is farther away than this, we don't get the biggest magnification, and if it's closer, we can't focus on it.

Solve for d_0. That's the distance from the lens to the object that maximizes M:

$$d_0 = \frac{fN}{f+N}$$

For this d_0,

$$M = \frac{f+N}{f} = 1 + \frac{N}{f}$$

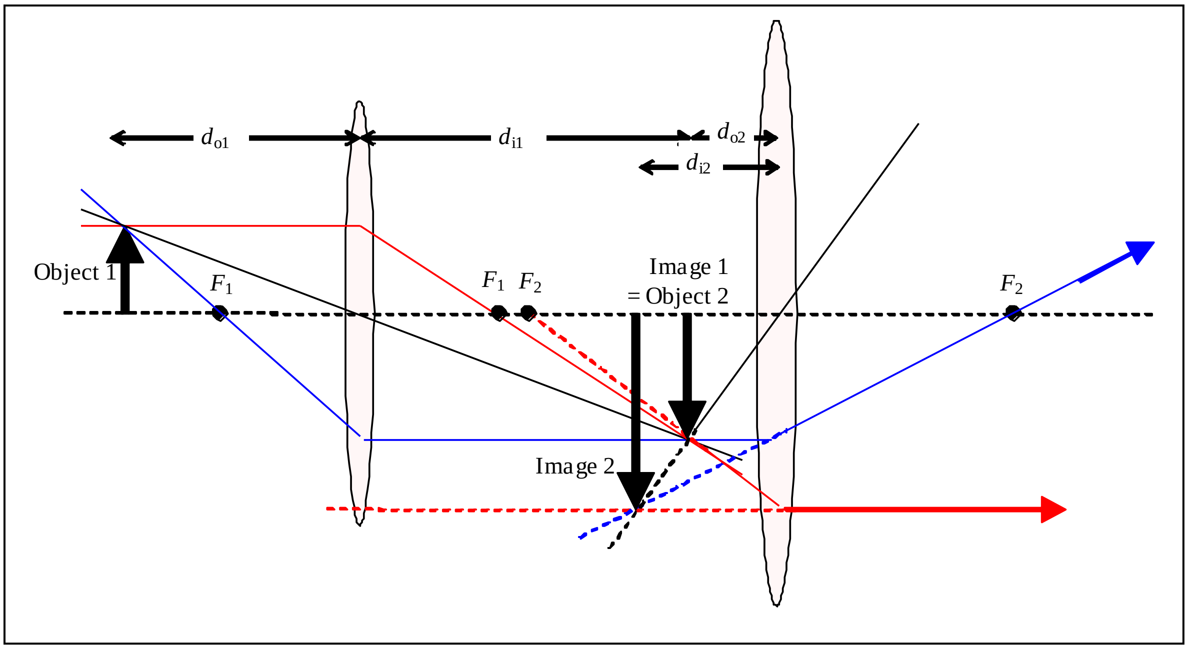
Strong lenses (short focal lengths) magnify more.

In the photo M is about 2.5. What is the focal length of the lens (assuming that M has been maximized)? Note that M seems to vary across the image.

Calculating Image Quantities for Multiple Lenses

Systems of multiple lenses, or lenses and mirrors are common. To solve these systems work them in steps. Let's look at an example:

1. Find the image from the first lens.
2. This image becomes the object for the second lens.
    - It is *inside* the focal length of the second lens so we have to be careful with our rays.
    - Follow the path of the real light: for the red and blue rays you have to extend it *backwards* using dashed lines to find the intersection point.



Quantitatively use the lens equations. Look at the diagram on the next page and follow these steps in order.

1) Determine the image location d_{i1} from the first lens alone. Ignore the presence of lens #2 even if the image from the first lens seems to be on the other side of it. The lightly shaded arrow in the figure represents this *intermediate image*.

2) Determine the magnification from the first lens, termed m_1.

3) Measure the distance from this image to the position of the second lens.

- If the image *is to the left* of the second lens, the measure will be a positive number.
- If the image happens to be to the right of the second lens, then the measured distance will be negative.
- This new distance is the object distance, d_{02}, for solving the lens equation to the second lens.

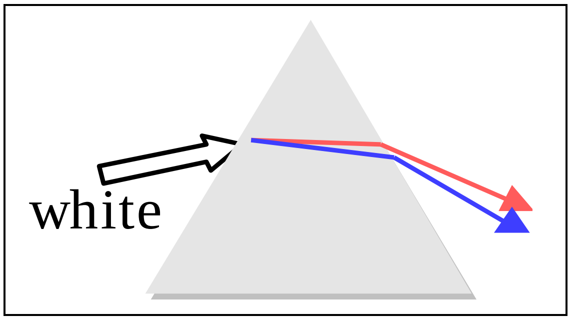
4) Solve the lens equation for the second lens alone. The image distance here, d_{i2}, is the location of the actual final image. In this example, it would be a negative quantity because it is to the left of the second lens.

5) Calculate m_2, the magnification of the second lens as $m_2 = -(d_{i2})/(d_{o2})$ and be careful with the signs of these quantities.

7) The final magnification is the product $m_{final} = m_1 m_2$

Dispersion of light

The index of refraction is different for different frequencies (colors) of light. In practice, the index is higher for the blue end of the rainbow (shorter wavelengths) compared to the red end (longer wavelengths). The consequence is that blue light bends more at the interface. This leads to the common examples of prisms and rainbows. Question: Is the speed of light the same for red light and blue light in glass? Answer: no.



Situations for Lecture 20: The Eye and Corrective Lenses

Situation 1: Correcting Vision

1) A nearsighted person cannot see objects clearly beyond $25\ cm$ (the far point). What kind of glasses are required, power and type?

2) What is the near point for an unaided eye if a person wears lenses with a power of $+1.5\ diopters$ to read at $25\ cm$?

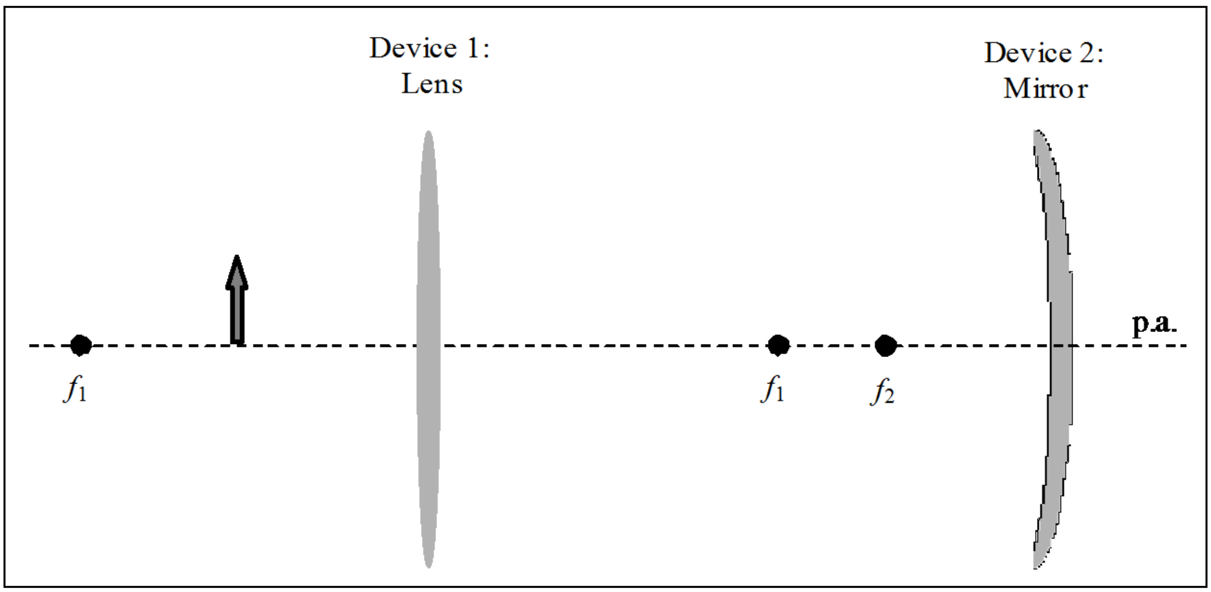
3) At age 40, a man requires contact lenses ($f = 65\ cm$) to read a book held $25\ cm$ from his eyes. At age 45, he must now hold a book $29\ cm$ from his eyes (using the same lenses).
   a. By what distance has his near point changed?

   b. What focal length lenses does he require at age 45 to read a book at $25\ cm$?

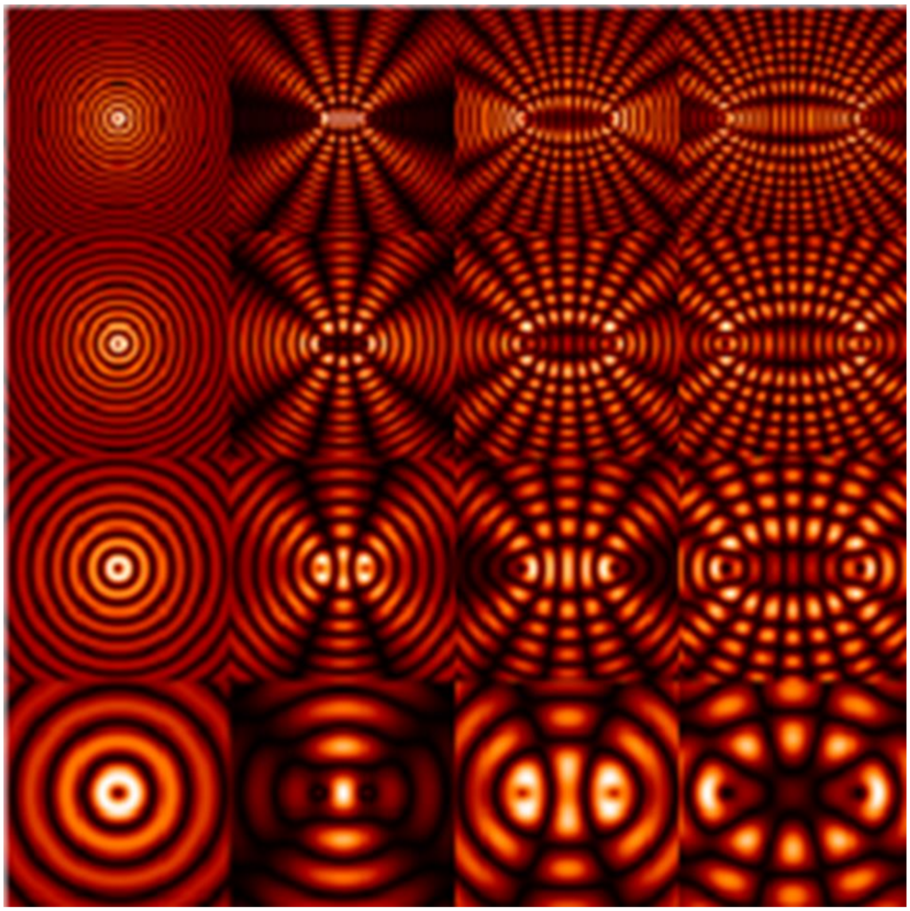
Situations for Lecture 21: Optical Instruments

Situation 1: Lens and Mirror

Try this lens-mirror problem and find the location of the image produced by the mirror. The focal length of the lens is f_1, and the focal length of the mirror is f_2. The object is inside the lens's focal length, as depicted below.



Week 13 Lectures 22 and 23—Interference and Diffraction



http://commons.wikimedia.org/wiki/File:Wavepanel.png

Summary

Glossary

Coherent: *for two or more light sources of the same wavelength the light is emitted with the same phase.*

Constructive: *the addition of two waves when the waves are in phase.*

Destructive: *the addition of two waves when the waves are out of phase*

In phase: *occurs in two waves of the same wavelength when the peaks of the waves align in position.*

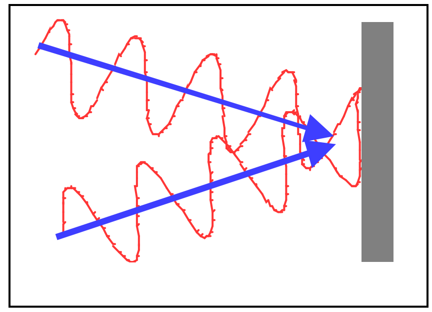
Out of phase: *occurs in two waves of the same wavelength when the peaks of the waves do not align in position.*

Path length: *the distance light travels between two points.*

Small angle approximation: $\sin\theta \approx \tan\theta \approx \theta$

Key Ideas

Light is a wave



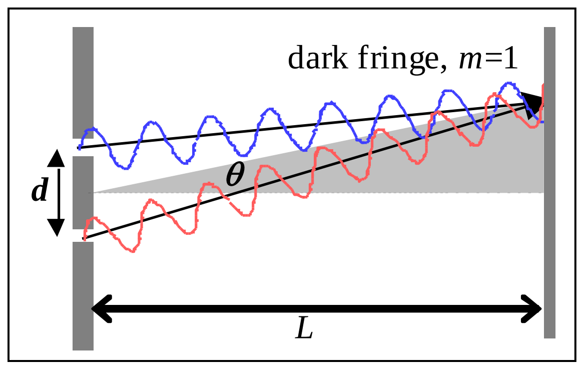
Now, we consider the wave nature of light.

When two *coherent* waves arrive at a point, the amplitudes add either constructively (as shown in the diagram) or destructively depending on the difference in the respective path lengths to that point on the screen.

If the difference is a whole integer number of their common wavelength, e.g., $\Delta = 0\lambda, 1\lambda, 2\lambda$, the waves are in phase and the amplitudes add *constructively*.

If the difference is a half-integer number of wavelengths, $\Delta = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$, etc., the amplitudes are out of phase and cancel. This is *destructive interference*.

Young's double-slit experiment



Light with a single wavelength λ hits a barrier with two slits separated by distance d. The two slits act as two point-like sources. They emit the same wavelength light with the same phase. This is coherence. The light hits a screen a distance L away from the slits. A pattern of bright and dark fringes appear on the screen. Using the diagram, we see:

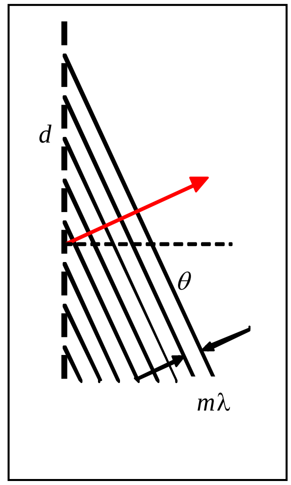
Bright Fringes (constructive):

$$\sin\theta = m\frac{\lambda}{d}; m = 0,1,2,3, \ldots$$

Dark Fringes (destructive):

$$\sin\theta = (m + \frac{1}{2})\frac{\lambda}{d}; m = 0,1,2,3, \ldots$$

Diffraction grating



A diffraction grating is a sheet with a large number of parallel, closely spaced slits. Each acts as an coherent source. A bright fringe (constructive interference) is produced for a particular wavelength (color) of light when the path difference between adjacent slits is an integral multiple of the wavelength. The path difference clearly depends on the angle of observation, θ, and the slit separation, d. From the figure one can derive:

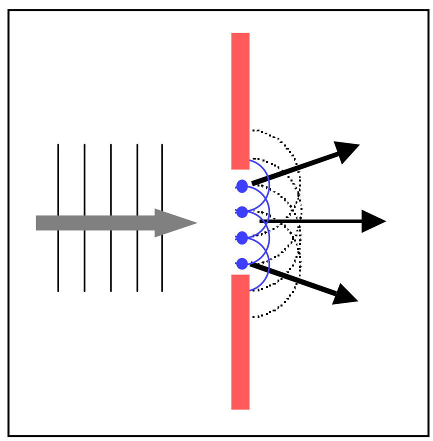
$$\sin\theta = m\frac{\lambda}{d}\text{; for any integer } m$$

This tells us the angle at which bright maxima appear for wavelength, λ. It is the same as the double slit formula. A diffraction grating gives very narrow bright spots, enabling accurate angle (and thus, wavelength) measurements.

Typically, a grating is specified by giving the number of lines per centimeter. So, 5000 lines per centimeter means: $\frac{1}{5000}cm = 2\times 10^{-4}\,cm = 2\times 10^{-6}\,m$. *Be sure to use the same units for all quantities when working problems*! Blue light: $\lambda = 450\,nm = 450\times 10^{-9}\,m$. Red light: $\lambda = 600\,nm$.

Our grating formula tells us that, for this grating, the first (m = 1) diffraction spots appear at $\theta = 13°$ (blue) and $\theta = 17°$ (red).

Diffraction



Diffraction is the *bending* of waves around obstacles or edges of an opening. It depends on the size of the wavelength, λ, compared to the size of the obstacle or opening, a.

Huygens' principle

Every point on a wave front acts as a source of wavelets that move forward with the same speed as the wave. The wave front at a later instant is tangent to the wavelets as shown in the figure to the right.

A typical problem might ask for the minimum aperture (diameter) in order to resolve two flashlights emitting $500\,nm$ light, separated by $5\,cm$ at a distance of $1609\,m$ (a mile). Here the *small angle approximation,* $\sin\theta \approx \tan\theta \approx \theta$, yields the result that the aperture must be greater than $2\,cm$ in order to see two distinct light sources.

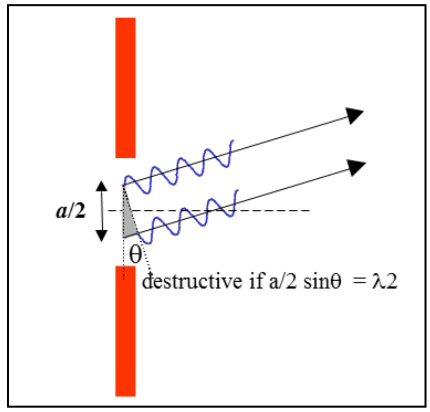
The *Resolving power* and the *Rayleigh criterion:*

$$\sin\theta \approx \theta \approx \frac{d}{L} \approx 1.22\frac{\lambda}{D}$$

are two ways of discussing how close two objects can be and still be distinguished. If λ is the wavelength of the light and D is the diameter of a circular opening through which the light must pass (e.g., a lens or an iris), then θ_{min} is the minimum opening angle to the two objects in order to just have the interference dark fringe of one on top of the bright central maximum of the other.

Single-slit diffraction

Following the path of each wavelet to a screen placed a large distance from a single slit of width a, we see that at some angle θ the wavelets shown arrive alternately out of phase by λ/2This causes a dark fringe to appear on the screen. The central, $\theta = 0$ region is bright and is called the central maximum.

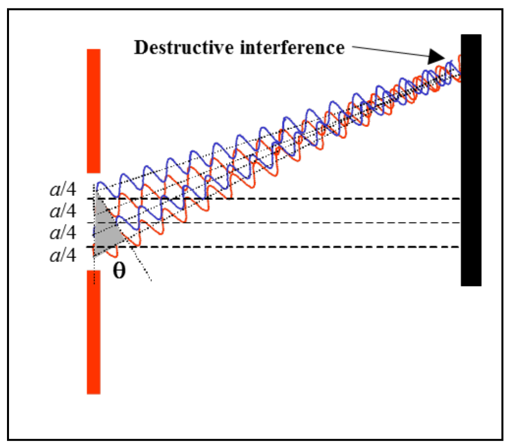


Dark fringes appear regularly according to the following expression:

$a\sin\theta = m\lambda$, where m is any integer except 0.

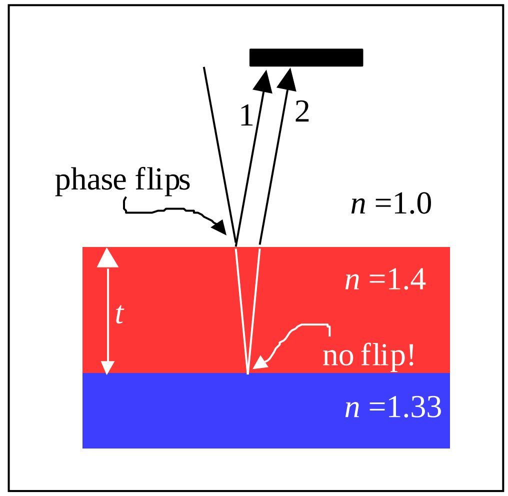
To see why consider the single slit to be made of two "half slits" separated by $a/2$ as shown in the figure. Each is a coherent source. When the paths from the top and bottom half differ by $\frac{\lambda}{2}$, destructive interference occurs and we get a dark fringe.

If you further divide the figure into 4 pieces, you arrive at the second-order dark fringes ($m = 2$). Examine the figure to the right for an example. You can continue this construction for all the dark fringes.



Thin-film interference

Interference also occurs when light reflects from the top and bottom surfaces of a thin, transparent, film such as a thin layer of oil on water or a coating on an optical lens. At the interface:



1. The phase of a wave flips by $180°$ when it reflects off a material with a *higher index of refraction*. For example, this will occur when light in air reflected off any solid material. The reflected wave is out of phase by $\frac{1}{2}\lambda$ with respect to the incoming wave.

2. A reflected wave does not flip its phase if it reflects off a material with a smaller index of refraction. For example, light reflected from a glass-to-air boundary, does not flip its phase.

It is important to remember that the wavelength of the light in the film is not the same as in air: $\lambda_{film} = \lambda_{vac}/n_{film}$ and the light travels through twice the real thickness of a film—down and back. So, what is $2t$ in terms of λ_{film}?

The path difference to the screen is computed in units of the wavelength in vacuum by considering that

Ray 1: Extra # wavelengths due to phase flip: $+\frac{1}{2}$

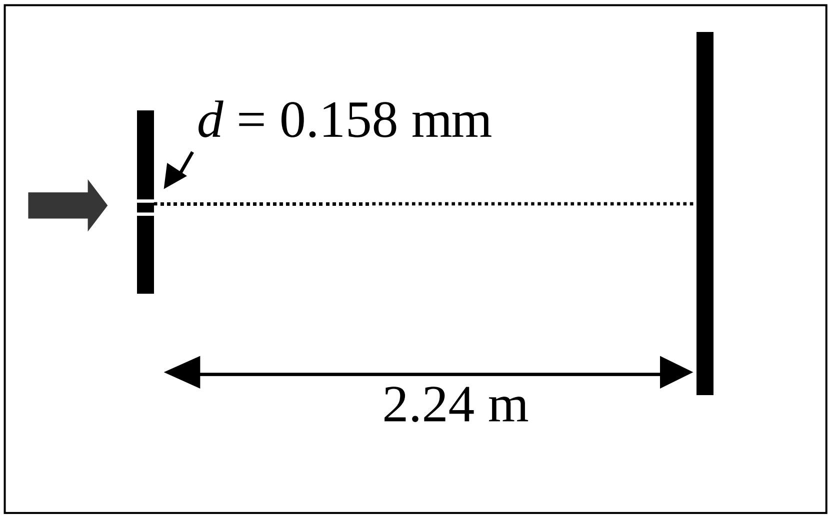
Ray 2: Extra # wavelengths due to $2t$ in the film $-\frac{2t}{\lambda_{film}}$

Because the bottom material has a lower index compared to the film, Ray 2 does NOT have an extra $\frac{1}{2}\lambda$ phase flip. If the bottom material was glass, $n = 1.5$, then it would flip at the boundary.

We'll get a bright reflection when $\frac{1}{2} - \frac{2t}{\lambda_{film}}$ equals a whole integer. Notice that the answer depends on the wavelength of the incoming light, because λ_{film} enters the expression. The degree of constructive or destructive interference depends on the color of light and the thickness of the thin film.

Situations for Lecture 22: Interference

Situation 1: Young's Double Slit



1) Two slits are $0.158\ mm$ apart. Red light ($665\ nm$) strikes the slits. A flat screen is placed $2.24\ m$ away. What is the distance at the screen between the central bright fringe and the 3rd order bright fringe?

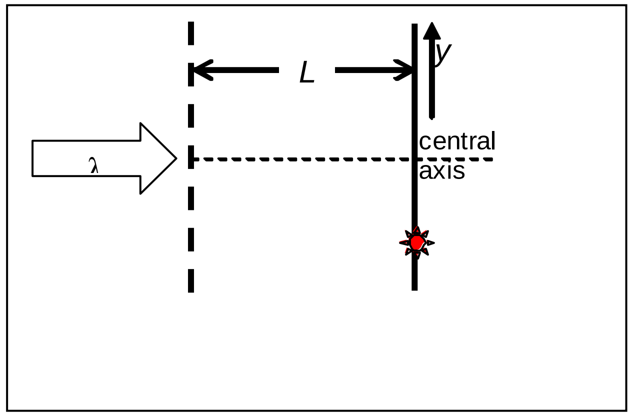
2) A yellow-green light ($565\ nm$) also impinges on the slits. What is the distance between the 3rd order red fringe and the 3rd order yellow-green fringe?

3) Do the two colors interfere either constructively or destructively at any points on the screen?

Situations for Lecture 23: Diffraction

Situation 1: The Diffraction Grating

1) A diffraction grating is used to analyze a light source. It is first calibrated with a $680\ nm$ red laser beam. The first bright fringe below the central maximum is observed at $y = 5\ cm$ below the central axis on a screen placed $L = 25\ cm$ from the grating. How many lines per cm does this grating have?

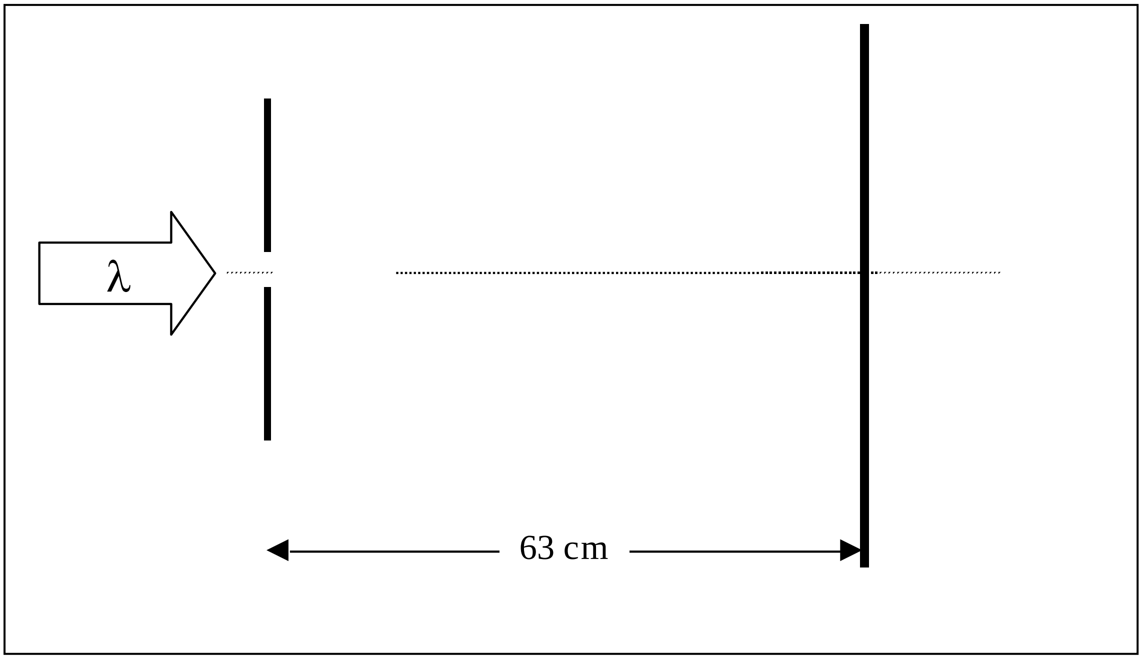


2) An unknown light source has a spot exactly $4.18\ cm$ above the central axis. What is its wavelength? Is the answer unique? Why or why not?

3) Does the source in part 2) also have a spot below the central axis?

4) Where is the second diffraction maximum for the red (680 nm) light?

Situation 2: Single Slit Diffraction



1) Light of $665\ nm$ is incident on a single slit. Draw the expected intensity pattern above in a qualitative manner. That is, don't expect to draw it to scale. Show at least five brightest peaks.

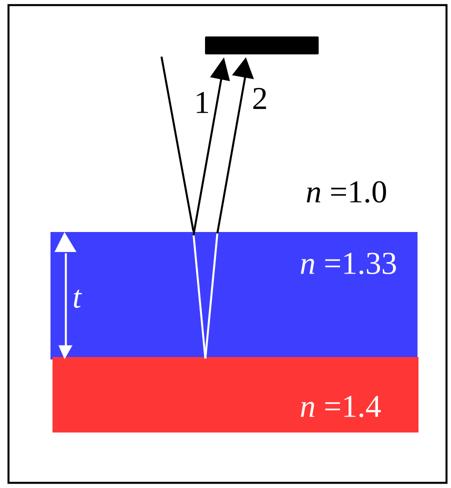
2) Indicate the width of the central bright fringe on your drawing. How would you calculate the width?

3) If the width of the central bright fringe is $2.6\ cm$ on the flat screen, what is the width of the slit?

4) If the light had a wavelength of $\lambda = 425\ nm$ instead, now what would the width of the bright fringe be?

Situation 3: Thin Film Interference

1) Red light ($\lambda_0 = 700\ nm$) hits a thin film of water on top of a layer of glass as shown. The reflected light destructively interferes so the screen on the figure to the right shows no reflected red light. What is the minimal thickness of the water?



2) Use the thickness calculated in part 1). White light (all wavelengths) hits this film. What is the next wavelength which, on reflection through this system, is also in complete destructive interference?

3) If water is added to increase the thickness so that the red light ($700\ nm$) is now seen on the screen at its maximal brightness, how thick is the layer of water now?

4) If the material above the film had an index of refraction of 1.1 instead of 1.0, would any of your answers change?