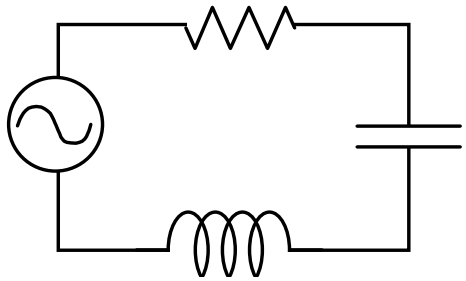
Week 7 Solutions

1. Series RLC Circuit



Generator: $V(t) = 200 \sin(500t)$ Volts
Resistor: $R = 100\ \Omega$
Capacitor: $C = 23\ \mu\text{F}$
Inductor: $L = 0.04$ H

a) What is the period, T, of the generator's oscillation?

$$T = \frac{2\pi}{500} = 1.26 \times 10^{-2} \text{ sec}$$

b) What is the rms generator voltage?

$$V_{\text{rms}} = \tfrac{1}{\sqrt{2}} V_{\text{max}} = 71 \text{ V}$$

c) Compute the inductive and capacitive reactances and finally the overall impedance of the circuit.

$$f = \frac{1}{T} = 79.6 \text{ Hz}$$

$$X_C = \frac{1}{2\pi f C} = 86.9\ \Omega; \qquad X_L = 2\pi f L = 20.0\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 120.0\ \Omega$$

d) What is the maximum current in the circuit?

$$I_{\text{max}} = V_{\text{max}} / Z = 1.67 \text{ A}$$

e) What is the phase angle φ? Is the current in the circuit ahead (leading) or behind (lagging) the voltage in the generator?

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = -33.8^\circ$$

Negative ϕ means the current leads the voltage.
ICE is more important than ELI when $X_C > X_L$.

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f) What is the average power (rms values) delivered by the generator?

$$P_{\text{rms}} = V_{\text{rms}} I_{\text{rms}} \cos\varphi = \tfrac{1}{2} V_{\max} I_{\max} \cos\varphi = 138.8 \text{ W}$$

g) At what frequency should this generator run to be on resonance?

$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} = 165.9 \text{ Hz}$$

h) If it did run on resonance, then what would the average power be?

On resonance:

$$X_L = X_C \quad \Rightarrow \quad Z = R \quad \Rightarrow \quad I_{\max} = V_{\max} / R \text{ and } \varphi = 0$$

$$\text{So, } P_{\text{rms}} = \frac{1}{2} V_{\max} I_{\max} = \frac{1}{2} \frac{V_{\max}^2}{R} = 200 \text{ W}$$

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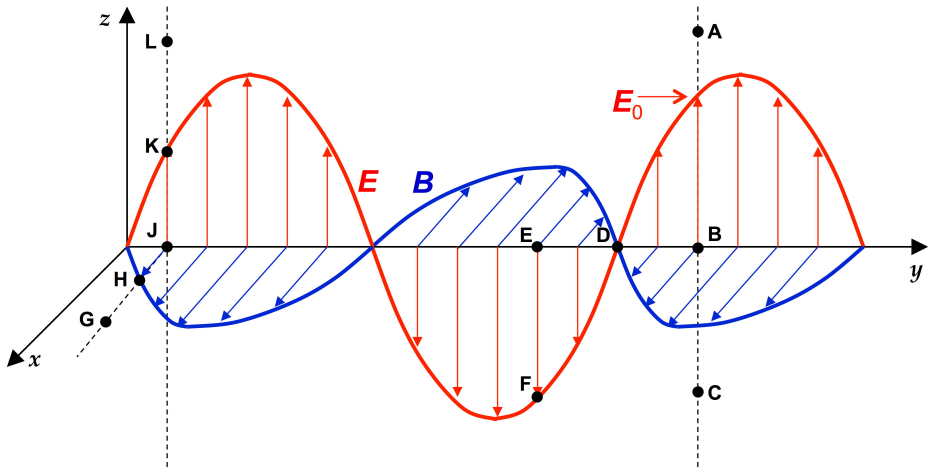
2: EM Waves



An electromagnetic wave is moving in the +*y* direction. The gray shaded area highlights the *y*-*z* plane. Several points are indicated in the figure and referred to in the questions below.

a) Is this wave polarized and if so, in what direction?
The polarization is the direction of electric field oscillation: *z*-polarization.

b) Compare the strength of the electric field at points A, B and C. Which statement is true? Discuss with your group until you all agree!!

i. A > B > C
ii. A > 0; B = 0; C < 0
iii. $A = 0$; $B = E_0$; $C=0$
iv. $A = B = C = E_0$ The strength of *E* only depends on the y position.

c) Which of the following statements are true about the relationship between the electric field at the following points? (Several could be correct; justify your answers.)
i. $E = D = B$
ii. $E = F$ The same *y*.
iii. $F = -A$ They are on opposite sides of the zero point.
iv. $D = 0$ $E = 0$ here.

d) What is the relationship between the electric fields at the series of points G,H,J,K,L? What is the relationship between the magnetic fields there?

They are all at the same *y* position. Therefore the *E*'s and *B*'s are all equal.

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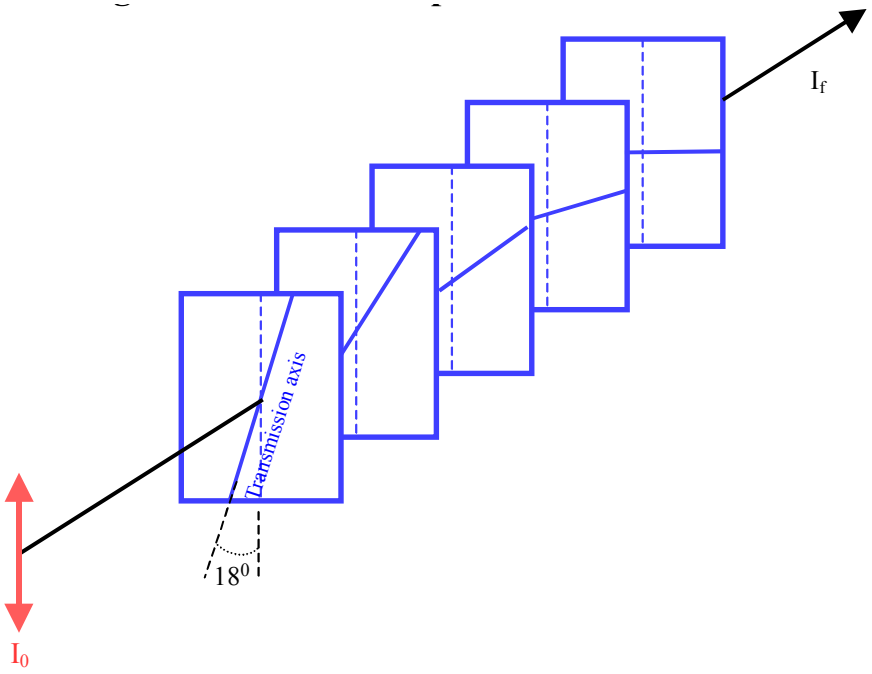
3. Transmission through a series of linear polarizers



Vertically polarized light, of intensity I_0, runs through a series of 5 linear polarizers, each with its transmission axis tilted by the same amount relative to the previous filter, namely at 18°.

a) What is the intensity of the light after the 1st polarizer?

$I_1 = I_0 (\cos 18°)^2 = 0.905\ I_0$

b) What is the final intensity I_f in terms of I_0?

$I_f = I_0 (\cos 18°)^{10} = 0.605\ I_0$

c) Suppose **vertically polarized** light starts at the end (where the black arrow is) and propagated in the opposite direction through the polarizers. What is the light intensity after three polarizers?

$I_3 = 0$. It's absorbed by the first polarizer it hits.