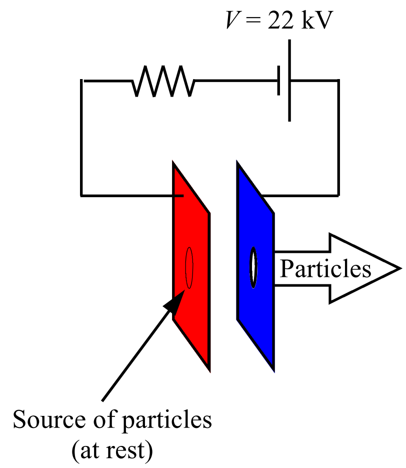
# Week 12 Solutions

## 1. deBroglie Wavelength




a) An electron is accelerated by the electric field between two charged plates. What is the de Broglie wavelength of these electrons?

$$KE = \text{ 22 keV } = \frac{p^2}{2m_e} = \left(\frac{h}{\lambda}\right)^2 \frac{1}{2m_e}$$

$$\lambda = \frac{h}{\sqrt{2m_e KE}} = 8.28\times10^{-12}\text{ m} = 0.00828\text{ nm}$$

b) If the plate voltages were reversed and protons were introduced and accelerated through the gap, what would be the de Broglie wavelength for the protons?

Replace $m_e$ with $m_p$. $\lambda = 1.93\times10^{-13}$ m = 0.000193 nm.

c) *Review*. If the separation between the plates is 1.5 mm, what is the strength of the electric field between the plates?

$E = V/d = 1.47\times10^7$ V/m $= 1.47\times10^7$ N/C

d) If the electrons strike a crystal normal to a regular interatomic layer spacing of $d$ = 1.5Å (which acts like the regular slits of a diffraction grating), through which non-zero angle do the greatest number of electrons scatter?

This is just a diffraction grating. The first maximum occurs at $d\sin\theta = \lambda$.
$\sin\theta = 0.0552$, so $\theta = 3.16°$.

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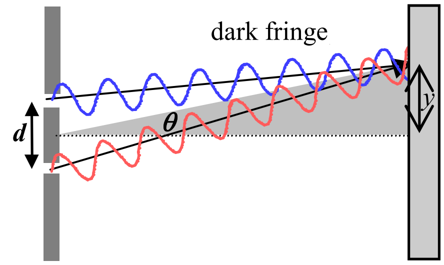
## 2. de Broglie Double Slit

In the figure to the right, which we first saw for the double- slit scattering of ordinary light, a dark spot occurs on the screen.




a) Write an expression for the location of this first dark spot in terms of t he angle $\theta$, the slit separation $d$, and the de Broglie wavelength, $\lambda$ of a particle beam.

$d\sin\theta = \frac{\lambda}{2}$. It's just like light waves. You want destructive interference.

b) If the screen were placed a distance $L$ = 2 m away from an effective slit separation $d = 10^{-9}$ m, the dark spot occurs at a distance of $y$ = 6 mm from the central bright spot, and the particle beam is made of electrons, what kinetic energy do they have?

Use: $p = h/\lambda$ and $KE = p^2/2m_e$.
$KE$ = 41.8 keV. Be sure to use consistent units in your calculation.

c) What is the kinetic energy of a neutron ($m_n = 1.67 \times 10^{-27}$ kg) that has the same de Broglie wavelength?

Use $m_n$ instead of $m_e$. $KE_n$ = 22.8 eV.

## 3. Uncertainty Principle

A proton is confined to a nucleus whose diameter is $5.5 \times 10^{-15}$ m. If this distance is considered to be the uncertainty in the position of the proton, what is the minimum uncertainty in its momentum?

The uncertainty principle is: $(\Delta p_y)(\Delta y) \geq \frac{h}{4\pi}$.
If $\Delta y = 5.5\times10^{-15}$ m, then $\Delta p_y$ must be at least $h/(4\pi\Delta y) = 9.6\times10^{-21}$ kg·m/s. (SI units)

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## 4. Bohr Atom

a) Calculate the "Bohr radius" corresponding to an electron in the ground state of hydrogen. How much larger is the n = 3 radius?

This problem is done in lecture 24, slide 9: $r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = 0.0529\text{ nm}\frac{n^2}{Z}$.

Hydrogen has $Z = 1$, and the ground state is $n = 1$, so $r_1 = 0.0529$ nm.
The n = 3 state is 9× larger: $r_3 = 0.4761$ nm.

b) A hydrogen atom in the ground-state absorbs a photon and is excited to the n = 4 level. What is the energy (in eV) of this photon?

$E_n = \frac{-13.6\text{ eV}}{n^2}$, so $E_4 - E_1 = -0.85\text{ eV} + 13.6\text{ eV} = 12.75\text{ eV}$.

## 5. Don't get Bohred yet …

a) Calculate the ionization energy of a hydrogen atom with an electron in the n = 3 state.

$E_3 = -13.6\text{ eV} / 9 = -1.51\text{ eV}$. You must give the electron 1.51 eV to eject it from the atom. That's the ionization energy.

b) A muonic-gold atom consists of a single negative muon (mass = 207 $m_e$) captured in hydrogen-like levels around a gold (Z=79) nucleus. Treat the muon as a very heavy electron. (Be careful of the mass factor in the energy levels. You can use the mass ratio to make your calculations quicker.)

*i*) Calculate the ground-state binding energy of the muonic-gold atom.

The energy is proportional to the mass (see lecture 24, slide 19), so the ground state energy is 207×(-13.6 eV) = -2815 eV.

*ii*) Calculate the wavelength and energy (in eV) of the photon produced when the atom de-excites from the $n = 5$ to $n = 4$ level.

The energy of the photon is $E = (2815\text{ eV}) \times (1/4^2 - 1/5^2) = 63.3\text{ eV}$.

Its wavelength is $\lambda = hc/E = (1240\text{ eV}\cdot\text{nm}) / (63.3\text{ eV}) = 19.6\text{ nm}$.