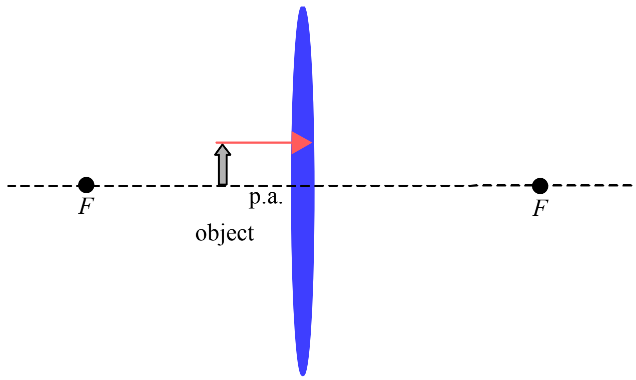
Week 10 Solutions

1. Magnifying Glass Revisited



a) The magnifying glass above has a focal length of 10 cm. You hold the magnifying glass 4 cm above some print on a page. What is the magnification of the observed text compared to the original text?

$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f} \quad \Rightarrow \quad d_i=-6.67 \text{ cm}; \quad m=\frac{-d_i}{d_0}=1.67$$

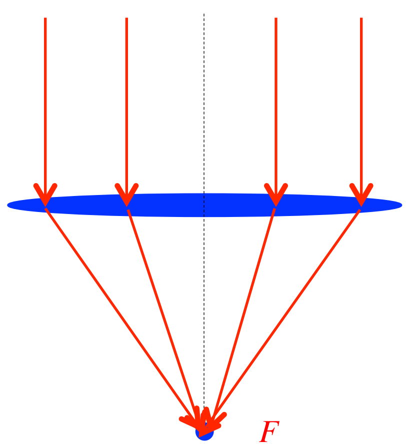
b) Where is the image in this problem? Is it real or virtual?

d_i = -6.67 cm tells us that it's a virtual image, 6.67 cm behind the lens (*i.e.*, on the same side as the object).

c) Suppose you used this lens to concentrate the sun's rays to burn a small hole in a dry leaf. How high above the leaf would you hold the lens?

When the object is very far away ($d_o \sim \infty$), then $d_i = f$. Hold the lens 10 cm above the leaf.

d) Make a quick sketch of the situation in part c).



Week 10 Solutions

2: At the Eye Doctor's …

a) A nearsighted person cannot see objects clearly beyond 25 cm (the far point). What kind of glasses is required: power and type?

We want objects that are very far away to appear to be (*i.e.*, have an image) 25 cm away from the person (*i.e.*, d_i = -25 cm). This means we want f = -25 cm. $f < 0$ requires a concave lens.

b) What is the near point for an unaided eye if a person wears lenses with a power of +1.5 diopters to read at 25 cm?

+1.5 diopters means f = (1 m)/1.5 = 66.7 cm. Assuming that the image is at the near point, we have: $d_i = 1/(1/66.7 - 1/25) = -40$ cm. The minus sign merely means that it's a virtual image. His near point is 40 cm.

c) At age 40, a man requires contact lenses (f = 65 cm) to read a book held 25 cm from his eyes. At age 45, he must now hold a book 29 cm from his eyes (using the same lenses).

i) By what distance has his near point changed?

His near point was: $d_i = 1/(1/65 - 1/25) = -40.6$ cm
It is now: $d_i = 1/(1/65 - 1/29) = -52.4$ cm
So, the near point has increased by 11.8 cm.

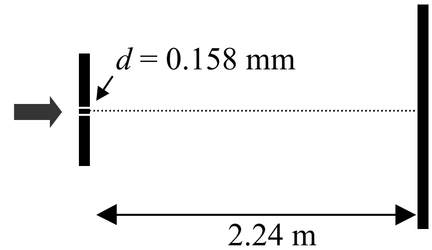
ii) What focal length lenses does he require at age 45 to read a book at 25 cm?

We want the image to be at -52.4 cm when the object is at 25 cm.
$f = 1/(1/25 - 1/52.4) = 47.8$ cm

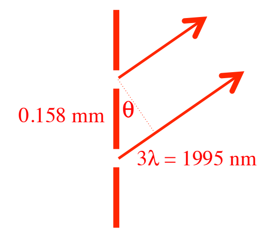
Week 10 Solutions

3. Young's Double Slit

a) Two slits are 0.158 mm apart. Red light (665 nm) strikes the screen. A flat screen is placed 2.24 m away. What is the distance on the screen between the central bright fringe and the 3rd order bright fringe?



A bright fringe appears when the path difference is a multiple of a wavelength. The third one appears at the angle for which it is 3λ. $\sin\theta = 1.995\ \mu\text{m} / 158\ \mu\text{m} = 0.0126$. $\theta = 0.723°$.



After traveling 2.24 m to the right, the light has moved away from the forward direction by a distance, $y = 2.24\ \text{m} \times \tan\theta = 0.0283$ m.

b) Now, in addition to the red light, a yellow-green light (565 nm) also impinges on the slits. What is the distance between the 3rd order red fringe and the 3rd order yellow-green fringe?

The same calculation for yellow light yields $\theta = 0.615°$ and y = 0.0240 m.
The separation between the two fringes is 0.0043 m.

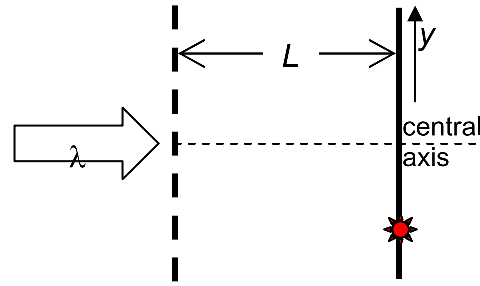
c) Do the two colors interfere either constructively or destructively at any points on the screen?

No. Because the frequencies are different, there is no place where the phase difference between the waves remains constant.

Week 10 Solutions

4. Diffraction Grating

a) A diffraction grating is used to analyze a light source. It is first calibrated with a 680 nm red laser beam. The first bright fringe below the central line is observed at y = 5 cm below the central axis on a screen placed L = 25 cm from the grating. How many lines per cm does this grating have?



Just like the double slit, we see bright fringes when the path difference between adjacent slits is a multiple of a wavelength. For the first fringe, the path difference must be λ = 680 nm. Here, $\tan\theta$ = (5 cm) / (25 cm) = 0.2. θ = 11.3°. Call the separation between grating slits d. $\sin\theta = 0.196 = \lambda/d$, so d = 3.47 μm. The number of slits ("lines") per cm is N = (0.01 m) / (3.47×10^{-6} m) = 2881.

b) An unknown light source has a spot exactly 4.18 cm above the central axis. What is its wavelength? Is the answer unique? Why or why not?

$\sin\theta = \lambda/d$. Using $\tan\theta$ = (4.18 cm) / (25 cm), a bit of algebra yields λ = 572 nm. The answer is not unique, because it might not be the first spot. That is, perhaps $m \neq 1$ in $\sin\theta = m\lambda/d$.

c) Does the source in part b) also have a spot below the central axis?

Yes. The pattern is symmetrical about the central axis.

d) Where is the second principal maximum for the red (680nm) light?

Now, $\sin\theta = 2\lambda/d = 0.392$. So, $\tan\theta = 0.426$. So, the position of the second maximum is at y = (25 cm) $\times \tan\theta$ = 10.65 cm.

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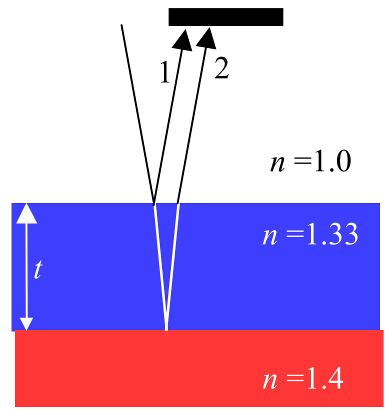
5. Thin Film Interference



a) Red light (λ_0 = 700 nm) hits a thin film of water on top of a layer of glass as shown. The reflected light destructively interferes so the screen on the figure to the right shows no reflected red light. What is the minimal thickness of the water?

For destructive interference, the path difference must be a half-integer multiple of a wavelength. The minimal thickness means the path difference has the smallest of these values, namely $\lambda/2$.

Don't forget: The wavelength in water is shorter by the index of refraction: $\lambda_{H20} = \lambda_0/1.33 = 526$ nm.

The path difference is twice the film thickness, $2t$, so we want the film thickness to be $t = \lambda_{H20}/4 = 132$ nm thick.

b) Consider the thickness of the film to be the thickness found in the previous part. Now white light (all wavelengths) hits this film. What is the next wavelength that, on reflection through this system, is also in complete destructive interference?

"Next" means next shorter. There is no destructive interference for longer wavelengths, because the path difference is less than half the wavelength. The next destructive interference occurs at $\lambda = \lambda_0/3$, for which the path difference is $3\lambda/2$.

c) If water is added to increase the thickness so that the red light (700 nm) is now seen on the screen at its maximal brightness, how thick is the layer of water now?

This means the thickness is $\lambda_{H20}/2$, to make the path difference one wavelength. So, $t = 263$ nm, twice as thick as in part a).

d) If the material above the film had an index of refraction of 1.1 instead of 1.0, would any of your answers change?

λ_0 is the wavelength in the material above the film. The wavelength in water is reduced by the ratio of the indices of refraction, which is now 3.1/1.1 = 1.18, not 1.3. Parts a) and c), which depend on λ_{H2O} (which has become 592 nm) will be changed. Part b) does not depend on λ_{H2O}.