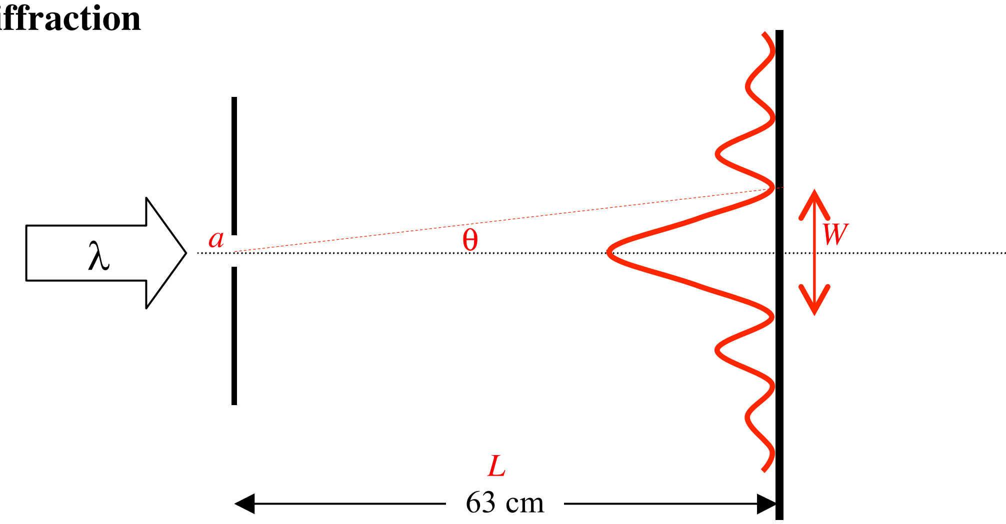
Week 11 Solutions

1. Single Slit Diffraction



a) Light of 665 nm is incident on a single slit. Draw the expected intensity pattern above in a qualitative manner. That is, don't expect to draw it to scale. Show at least five brightest peaks. See the figure.

b) Indicate the width of the central bright fringe on your drawing. How would you calculate the width?
See the figure. The width is determined by the deflection angle, θ, to the first minimum of the diffraction pattern and by the distance, L, from slit to screen. Angles to minima are described by this equation: $\sin\theta = \frac{m\lambda}{a}$, where a is the slit width. Here, $m = 1$. W is then calculated using: $\tan\theta = \frac{W}{2L}$. (Look at the long, skinny triangle in the figure.)

c) If the width of the central bright fringe is 2.6 cm on the flat screen, what is the width of the slit?
We are told W, so work backward.

Plug into the second equations: $\theta = \tan^{-1}\left(\frac{W}{2L}\right) = 1.182^\circ$.

Plug into the first equation: $a = \frac{\lambda}{\sin\theta} = 32.2\ \mu\text{m}$

d) If the light had a wavelength of $\lambda = 425$ nm instead, now what would the width of the bright fringe be?
You could plug in as in part c), but there is a simpler solution.
Because θ is small, $\sin\theta = \tan\theta$, and $\frac{m\lambda}{a} = \frac{W}{2L}$.
W is proportional to λ, so, the new width is $W_{new} = (425/665) \times 2.6\text{ cm} = 1.7\text{ cm}$.

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2. Blackbody Effect

a) What is the surface temperature of Betelgeuse, a red giant star in the constellation Orion, which radiates with a peak wavelength of about 970 nm? Now, repeat this calculation for Rigel, a bluish-white star in Orion, with a peak wavelength of 145 nm.

Use the relation between peak wavelength and surface temperature

$\lambda_{\max} T = 0.2898 \times 10^{-2}\ \text{m} \cdot \text{K}$.

$T_{\text{Betelgeuse}} =$ 2,988 K (a cool star)
T_{Rigel} = 19,986 K (a very hot star)

b) Assume that the tungsten filament of a light bulb is a blackbody. Determine its peak wavelength if its temperature is 2900 K.

$\lambda_{\max} = \left(0.2898 \times 10^{-2}\ \text{m} \cdot \text{K}\right)/\text{T} = 1.00\ \mu\text{m}$

c) Now, look at your answer to b) and comment on why this suggests that more of the light bulb's energy goes into heat than into light. Why is this evident?

The peak wavelength is in the infrared, which we can't see. The bulb gets hot, because glass absorbs infrared light.

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3. Photoelectric Effect Practice

a) The work function for a sodium surface is 2.28 eV. What is the maximum wavelength (in nm) that an electromagnetic wave can have and still eject electrons from this surface?

The photon energy must equal or exceed 2.28 eV. Use $E = (hc)/\lambda$.

Constants: $h = 6.626\times10^{-34}$ J·s $= 4.14\times10^{-15}$ eV·s
$hc = 1.24\times10^{-6}$ eV·m $= 1240$ eV·nm

So, we need $\lambda \leq 1240 / 2.28 = 543$ nm.

b) An owl has good night vision because its eyes can detect a light intensity as small as 5.0×10^{-13} W/m^2. What is the minimum number of photons per second that an owl eye can detect if its pupil has a diameter of 8.5 mm and the light has a wavelength of 510 nm?

1 Watt = 1 J/s = 1/1.6$\times10^{-19}$ = 6.25×10^{18} eV/s.
Each 510 nm photon has E = $(hc)/\lambda = 2.43$ eV.
An intensity of $I = 5\times10^{-13}$ W/m^2 means that the rate at which energy enters the owl's eye is
$IA = 5\times10^{-13} \times (\pi*0.00425^2) = 2.84\times10^{-17}$ W = 177 eV/s.
So, the number of photons per second is 178 / 2.43 = 73.

c) A laser emits 1.3×10^{18} photons per second in a beam of light that has a diameter of 2.00 mm and a wavelength of 514.5 nm. Determine the average electric field strength and the average magnetic field strength for the electromagnetic wave that constitutes the beam. You will have to look back to Problem Solver unit #7 for part of this.

The problem asks for E and B in SI units, so we need to calculate the energy in Joules.

Each photon has $E = (hc)/\lambda = 1240 / 514.5 = 2.41$ eV $= 3.86\times10^{-19}$ J.
The laser beam intensity is $1.3\times10^{18} * 3.86\times10^{-19} /(\pi*0.001^2) = 1.60\times10^5$ W/m^2,

From PS #7: The average intensity in an EM wave is $S = c\varepsilon_0 E_{rms}^2$.
So, $E_{rms} = 7760$ J/C, and $B_{rms} = E_{rms}/c = 2.59\times10^{-5}$ T.