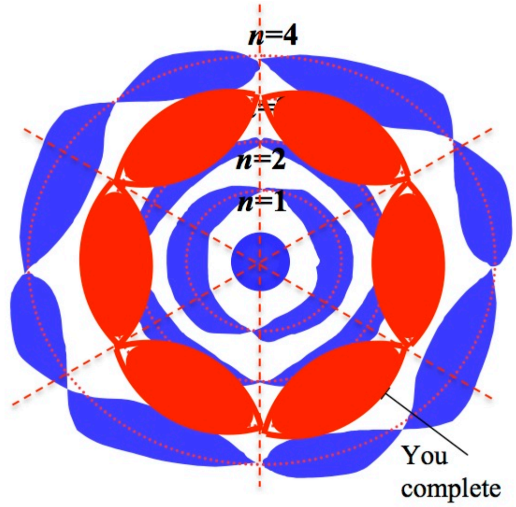
Week 13 Solutions

1. Atomic Physics



a) Complete the standing wave picture on page 2 of this packet for $n = 3$.

See the figure. The $n = 3$ wave has six nodes.

b) Write out the electron configuration of the ground state of Argon ($Z = 18$) in terms of the spectroscopic notation.

2 electrons in the $n = 1$, $l = 0$ shell.
2 electrons in the $n = 2$, $l = 0$ shell.
6 electrons in the $n = 2$, $l = 1$ shell.
2 electrons in the $n = 3$, $l = 0$ shell.
6 electrons in the $n = 3$, $l = 1$ shell.

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

c) When the outermost electron in an atom is in an excited state, the atom is more easily ionized than when the outermost electron is in the ground state. Discuss this and state why it is so.

Ionization of an atom means the removal of an electron from the atom. This is more easily accomplished if the electron starts out with more energy, because less energy needs to be added in order to pull it the rest of the way out. (It's easier to escape from a hole if you're already part way up.

d) Write out the possible quantum number assignments for each electron in a Neon (Z=10) atom. Hint: How many should you have if all electrons are present?

There are ten electrons. Here are the ground state quantum number assignments.

n	l	m_l	m_s	Shell	Subshell
1	0	0	-½	K	s
1	0	0	+½	K	s
2	0	0	-½	L	s
2	0	0	+½	L	s
2	1	-1	-½	L	p
2	1	-1	+½	L	p
2	1	0	-½	L	p
2	1	0	+½	L	p
2	1	+1	-½	L	p
2	1	+1	+½	L	p

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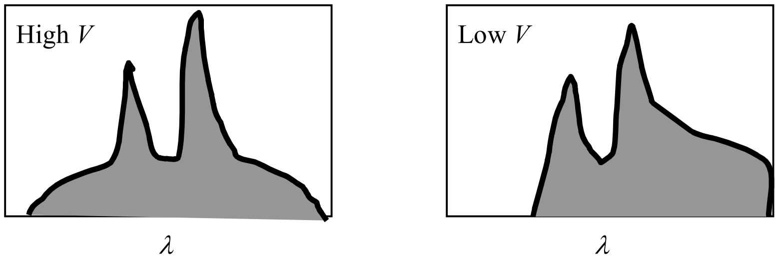
2. X-rays



a) Consider the two pictures of the X-ray spectra produced by an X-ray tube when the tube is operated at two different potential differences (V). (An X-ray tube shoots electrons whose kinetic energy is related to the potential.) Explain why the characteristic lines occur at the same wavelengths in the two spectra, while the cutoff wavelength (left side goes to zero) shifts to the right when a smaller voltage is used. Explain each feature of the curve, the two bumps and the hill they sit on.

The lines mean that a lot of the X-rays have the same energy (because the wavelength tells us the energy). These are produced when an atomic electron falls from a high energy level to a lower energy level in the atom. This energy difference is a property of the atom, and doesn't depend on the energy of the incoming electrons, as long as the incoming electron has enough energy to excite the atomic electron to the high energy level.

The smooth distribution (the part that is not in the peaks) is due to "bremsstrahlung," produced when an electron hits the atom and slows down. In this situation, the atomic electrons are not excited. The x-ray energy is simply equal to the amount of energy lost by the incoming electron. The maximum x-ray energy (the minimum wavelength) is just the incoming electron's energy, so lower energy electrons will have lower maximum x-ray energy (longer minimum wavelength).

b) The X-ray spectrum for some material, with a tube operated at some voltage looks like one of the above two diagrams except that both the $K\alpha$ and $K\beta$ lines are missing! Can you explain why this might be possible?

The simplest explanation is that the electrons do not have enough energy to excite the atomic electrons into the $n = 2$ or $n = 3$ energy levels. Remember that K_α is produced by the $n = 2 \rightarrow n = 1$ transition, and K_β by the $n = 3 \rightarrow n = 1$ transition.

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3. Even more assorted Atomic Physics and X-ray problems …

a) Use the Bohr model to estimate the wavelength of the K_α line in the X-ray spectrum of lead (Z = 82).

In the Bohr model, $E_n \approx -13.6 \text{ eV} \frac{Z^2}{n^2}$, where n is the principal quantum number.

K_α is produced by the $n = 2 \rightarrow n = 1$ transition, so use:

$E_{\text{photon}} = E_2 - E_1 = -2.29\times10^4$ eV - (-9.14×10^4 eV) = 6.85×10^4 eV.

$\lambda = hc/E_{\text{photon}}$ = 1240 nm·eV/6.85×10^4 eV = 0.018 nm.

b) Suppose an electron tube has a potential difference of 45,000 volts. What is the shortest-wavelength X-ray emitted from this tube if at most 25% of each electron's energy is converted into an X-ray? What kind of X-ray is this called?

The maximum E_{photon} is 11.25 keV. $\lambda_{\text{min}} = hc/E_{\text{max}}$ = 0.110 nm. This is a Bremsstrahlung x-ray, because it is produced by slowing the electron, not by an atomic transition.

"Bremsstrahlung" is German for "braking radiation".

c) Near UV photons are produced in the gas of a "fluorescent light" when the ionized gas de-excites. When these photons strike the coated walls of the tube, they are absorbed and re-emitted with other wavelengths, giving rise to a more "continuous" spectrum as you have observed in everyday life. Is it possible for a photon to be converted into a photon of a shorter wavelength? Choose YES or NO and explain your answer.

Shorter wavelength means higher energy, so this is not possible. It would violate conservation of energy.

d) Calculate the ionization energy of a hydrogen atom with an electron in the n = 3 state. Could such a photon of this energy knock a ground-state electron in a one-electron Helium atom to the n = 3 state? Answer YES or NO and EXPLAIN your reasoning either way.

The energy of an n = 3 electron in hydrogen is -13.6 eV / 3^2 = -1.51 eV. Ionizing the atom requires that the electron have at least zero energy (so it can escape to infinity). That is, the ionization energy (the amount that must be added to the atom to ionize it) is 1.51 eV.

Helium has Z = 2, so $E_3 - E_1$ = -6.0 eV - (-54.4 eV) = 48.4 eV. This is much larger than 1.51 eV, so that photon can't do the job.