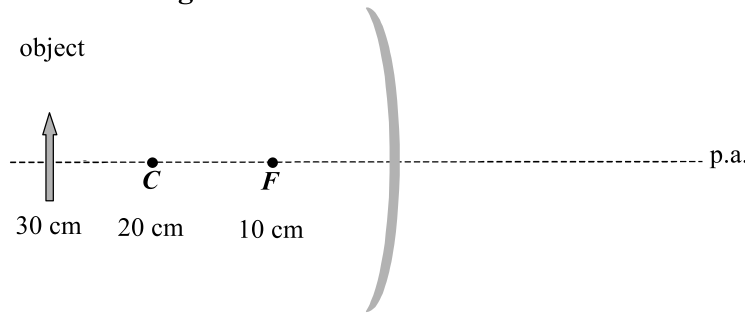
Week 9 Solutions

1. Mirrors ... revisited

Concave mirror with object well outside focal length

a) Given that F = 10 cm, and the object is located 20 cm to the left of F, use the mirror equation to locate the image position on the principal axis of the image. Does this agree with your ray tracing drawing of last week?



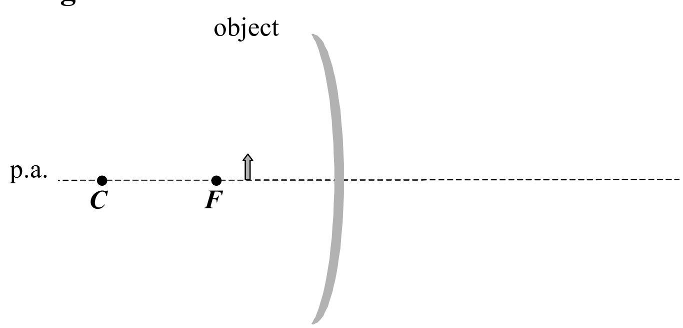
$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{d_i}+\frac{1}{(10+20)\text{ cm}}=\frac{1}{10\text{ cm}} \quad \Rightarrow \quad d_i = 15\text{ cm}$$

b) If the object has a height of 5 cm, how large is the image?

The magnification is $m=-\frac{d_i}{d_o}=-\frac{1}{2}$, so the image is 2.5 cm high (inverted).

Concave mirror with object "inside" focal length

a) The radius of curvature of this concave mirror is also 20 cm. Now the object is located a distance 7 cm in front of the mirror. Find the image using the mirror equation. Check with last week's example.



$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{d_i}+\frac{1}{7\text{ cm}}=\frac{1}{10\text{ cm}} \quad \Rightarrow \quad d_i = -23.3\text{ cm}$$

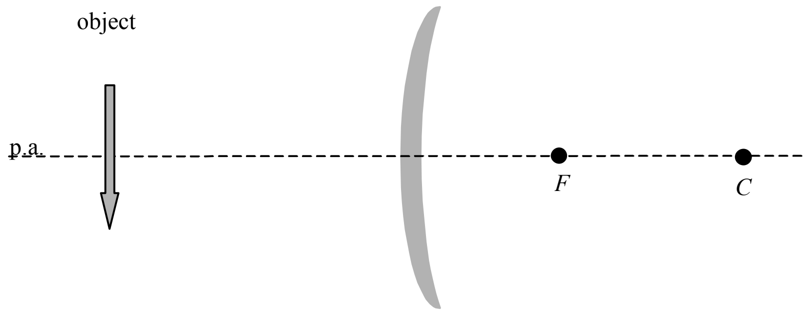
Negative d_i tells you that the image is behind the mirror (virtual).

b) If the object has a height of 3 cm, what is the image height, and is it upright or inverted?

The magnification is $m=-\frac{d_i}{d_o}=+3.33$, so the image is 10 cm high, upright.

Week 9 Solutions

2: Convex mirror revisited



The radius of curvature of this convex mirror is 40 cm. The object, of height 10 cm, is located a distance 35 cm in front of the mirror. Where is the image and how large is it? Again, compare to last week.

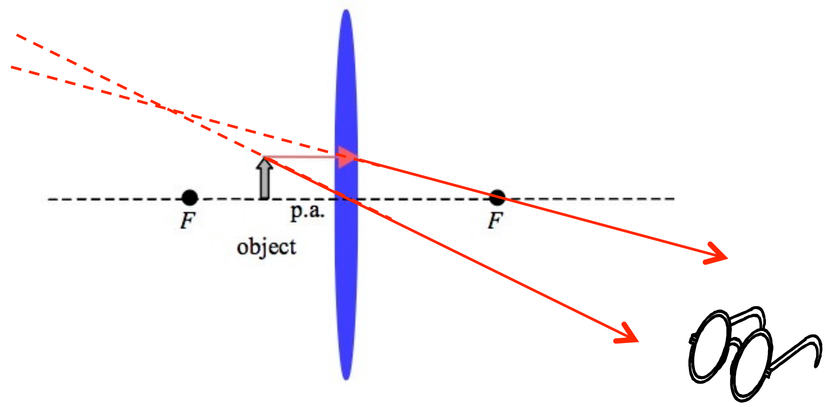
This mirror has a negative focal length, f = -20 cm.

$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{d_i}+\frac{1}{35\text{ cm}}=-\frac{1}{20\text{ cm}} \quad\Rightarrow\quad d_i=-12.7\text{ cm}$$ (behind the mirror).

The magnification is $m=-\frac{d_i}{d_o}=+0.36$, so the image is 3.6 cm high, upright.

Week 9 Solutions

3. Magnifying glass



a) The diagram above shows an observer (to the right indicated by the glasses) using a converging "magnifying" glass. The small object is located inside the focal length. The first part of the first ray of real light is drawn for you. Complete this ray the way a real light beam would travel.

See the diagram.

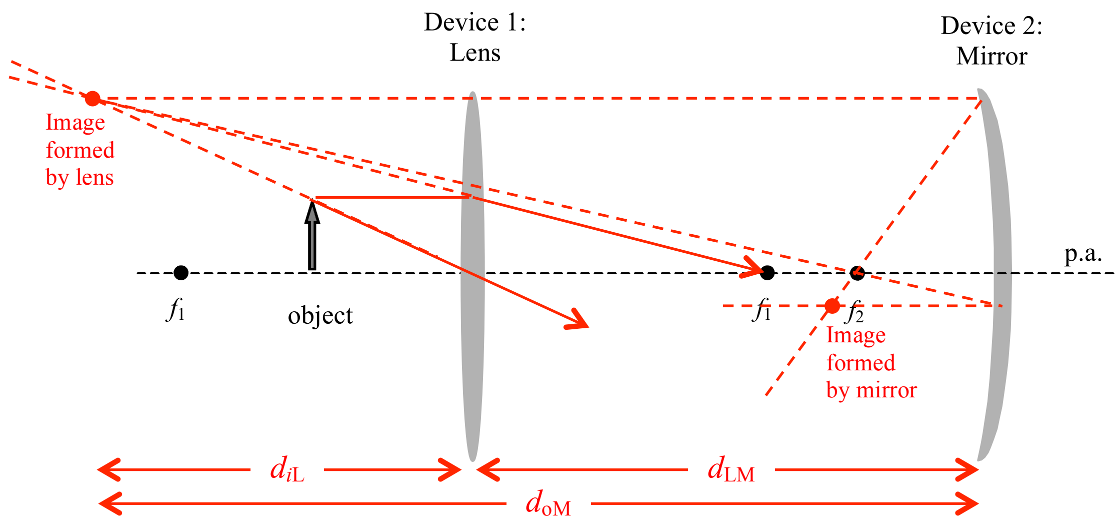
b) Now put in the ray that just passes through the lens center. Where do these rays converge? Is the image real or virtual?

See the diagram. The rays converge to the left of the lens. The image is virtual, because the light doesn't actually come from (pass through) the image.

Week 9 Solutions

4. Lens-Mirror Problem

Try this lens-mirror problem and find the exact location of the image produced by the mirror. The focal length of the lens is f_1, and the focal length of the mirror is f_2. The object is inside the lens's focal length, as depicted below.



Ray tracing becomes cumbersome and inaccurate in complicated situations. One can solve the problem exactly using the mirror/lens equation twice, first to find the location, d_{iL}, of the image formed by the lens. Then, use the fact that this image is the object for the mirror (*i.e.*, $d_{iL} + d_{LM}$ becomes d_{oM}), to find the location, d_{iM}, of the image formed by the mirror. We must add d_{LM}, because the image is farther from the mirror than it is from the lens.

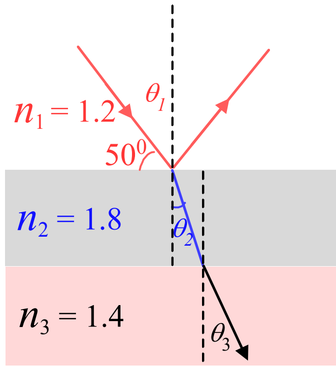
$$d_{iL} = 1/\left(1/f_1 - 1/d_o\right)$$

$$d_{iM} = 1/\left(1/f_2 - 1/d_{oM}\right)$$

$$= 1/\left(1/f_2 - 1/\left(d_{iL} + d_{LM}\right)\right)$$

Week 9 Solutions

5. Fun with Snell's Law



A light ray starts at the top of the picture above and strikes an interface at a 50° angle with respect to the horizontal. It passes into material 2, then into material 3. The index of refraction of each material is shown.

a) What is the value of θ_2?

$$\theta_1 = 40°. \quad n_2 \sin\theta_2 = n_1 \sin\theta_1 \quad \Rightarrow \quad \theta_2 = 25.4°$$

b) If the slab of material with index of refraction n_2 is 3 cm thick, how far is the exit point of the ray at the bottom of the slab of index of refraction n_2 with respect to the center dashed vertical line?

$$d = (3 \text{ cm})\tan(\theta_2) = 1.42 \text{ cm}$$

c) What is the value of angle θ_3?

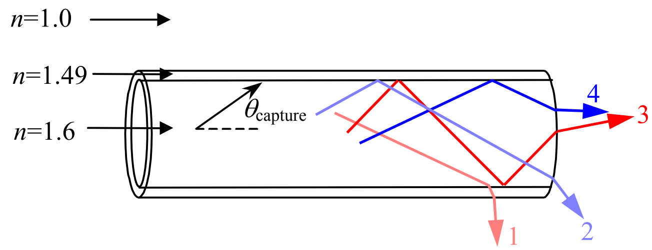
$$n_3 \sin\theta_3 = n_2 \sin\theta_2 \quad \Rightarrow \quad \theta_3 = 33.5°$$

d) If the middle slab were removed, would θ3 remain the same? Discuss this first, then check your intuition by making an explicit calculation.

Yes: $n_3 \sin\theta_3 = n_2 \sin\theta_2 = n_1 \sin\theta_1$, so θ_3 doesn't depend on n_2.

Week 9 Solutions

6. Fiber Optics



a) The optical fiber shown above has a core of polystyrene with index of refraction n_{core} = 1.6, a cladding of acrylic with $n_{cladding}$ = 1.49, and it is surrounded by air. Rays of light start from inside the fiber at the angles shown. Which one of these rays looks correct?

Ray 1 can't be correct, because the light should bend away from the normal when it passes into a medium with smaller index of refraction. Ditto for rays 3 and 4 (at the end of the fiber). So, only ray 2 looks correct.

b) If this same fiber were embedded inside a material with index of refraction $n_{outside}$ = 1.8, would your answer remain the same? What happens now?

We have replaced n = 1 with n = 1.8. The outside index of refraction is larger than inside, so rays 3 and 4 are now correct, and ray 2 is not. Ray 1 remains incorrect, because the bend is wrong at the core-cladding interface.

c) What is the maximum angle, $\theta_{capture}$, that a light ray can have and still stay entirely within the fiber?

The critical angle for total internal reflection is given by:
$n_1 \sin\theta_c = n_2$, that is when $\sin\theta_2 = 1$.
For the given indices of refraction: $\theta_{capture} = 90° - \sin^{-1}(1/1.6) = 51.4°$.
Subtract from 90°, due to the definition of $\theta_{capture}$ in the figure.
This keeps the light out of the air (n = 1). If we want to keep it out of the cladding, we need:
$\theta_{capture} = 90° - \sin^{-1}(1.49/1.6) = 21.4°$.