Week 14 Solutions

1. Nuclear Physics

a) What do you think might prevent the protons in a vat of liquid from undergoing spontaneous fusion in order to arrive at a lower overall binding energy? In other words, why doesn't this "just happen?"

In order to fuse, nuclei must come into contact. Because of the large, repulsive electrical force, this will only happen when the nuclei approach each other with a lot of kinetic energy. There was a homework problem about the electric potential energy of two nuclei (protons) in contact.

b) (Mass/Energy) Complete the calculation that an ordinary 1-lb object has the energy to run a 100 MW power plant for 13 years.

The energy in a 1 lb (0.45 kg) object is $E = mc^2 = 4\times10^{16}$ J. In ten years, the power plant generates $E = (10^8 \text{ J/s})\times(3.2\times10^7 \text{ s/yr})\times(10 \text{ yr}) = 3.2\times10^{16}$ J.

Note: Unfortunately, this energy is not available to us, unless we can annihilate the object with antimatter.

c) (Nuclear Reaction Rules) Complete the following decay processes by stating what the symbol X represents, (*i.e.*, X = α, β^{-}, or γ).

- If Z changes by one unit, a singly charged article (*e.g.*, electron) must have been emitted. A will remain unchanged.
- If Z changes by two units, it was a doubly charged particle (*e.g.*, α). A will decrease by four units (the mass of the emitted helium nucleus)
- If Z is unchanged, then the particle was neutral (a photon).

1. ${}^{211}_{82}\text{Pb} \rightarrow {}^{211}_{83}\text{Bi} + \beta^{-}$

2. ${}^{231}_{90}\text{Th} \rightarrow {}^{231}_{90}\text{Th} + \gamma$

3. ${}^{234}_{90}\text{Th} \rightarrow {}^{230}_{88}\text{Bi} + \alpha$

4. ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + \alpha$

d) (Nuclear Reaction Rules) Why do α decay and β decay produce new elements but γ decay does not?

Elements are determined by the number of protons in the nucleus. In α decay two protons leave. In β decay, a neutron is converted to a proton (to conserve electric charge). In γ decay, the internal energy changes, but the number of protons is unchanged.

Week 14 Solutions

2. Binding Energy / Fission / Fusion

Discuss each of the following statements with your group and decide whether it is true or false. Note, have each group member be responsible for defending a statement.

a) In nuclear fission, the binding energy per nucleon of the byproduct nuclei increases compared to the original nucleus.

True. Energy has been released, so the total energy of the nuclei decreases. Remember that binding energy is the negative of the total energy (which is negative), so it increases.

b) In a fusion reaction, two lighter nuclei are merged to form a heavier nucleus + a byproduct + extra kinetic energy.

True. Again, energy is released. In this case, nucleons in the heaver nucleus are more tightly bound (E is more negative), which is the reason that energy is released.

c) The binding energy per nucleon is greatest for a 1-nucleon nucleus and steadily decreases for increasing atomic number.

False. Not only is there no binding energy for a single proton (it's not bound to anything), but also the binding energy per nucleon increases with Z for all nuclei lighter than iron ($Z = 26$). That's why fusion occurs for light nuclei, and fission occurs for heavy nuclei.

d) The mass per nucleon is greatest when the binding energy per nucleon is the least.

True. Remember that $m = E_{tot}/c^2$. Smaller binding energy means larger E_{tot}.

Solve the following problem

e) If the average energy released in a fission event is 208 MeV, find the total number of fission events required to operate a 100-W light bulb for 1.0 h.

We need $E = (100\ \text{J/s})(3600\ \text{s/hr}) = 3.6\times10^5$ J.
One fission event releases $(2.08\times10^8\ \text{eV/event})(1.6\times10^{-19}\ \text{J/eV}) = 3.3\times10^{-11}$ J/event.
So, we need $(3.6\times10^5\ \text{J})/(3.3\times10^{-11}\ \text{J/event}) = 1.1\times10^{16}$ events.

Week 14 Solutions

3. Radioactivity …

a) The ^{14}C content decreases after the death of a living system with a half-life of 5730 years. If an archaeologist finds an ancient fire pit containing partially consumed firewood and the ^{14}C content of the wood is only 12.5% that of an equal carbon sample from a present-day tree, what is the age of the fire pit?

$0.125 = (½)^3$, so the wood is three half-lives = 17,190 years old.

b) In 9.0 days, the number of radioactive nuclei decreases to one-eighth the number present initially. What is the half-life (in days) of the material?

Again, $1/8 = (½)^3$, so the half-life is three days.

Week 14 Solutions

4. Special Relativity

a) Muons from cosmic rays are measured in a special detector and the relativistic γ factor is determined to be approximately 30. How fast are they going with respect to c? Give your answer as a fraction of c.

The γ factor is $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$, so $\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = 0.9989$.

b) If muons at rest have a lifetime of 2197 ns, what is the lifetime of such a cosmic ray muon? How far would it go in one lifetime?

The lifetime is larger by a factor of 30: 65,910 ns = 6.591×10^{-5} s. In that time, the muon travels $(6.591\times10^{-5}\ \text{s})(3\times10^{8}\ \text{m/s}) = 2\times10^{4}$ m.

c) Which of the following quantities will two observers always measure to be the same, regardless of the relative velocity between the observers? Explain your answers.

(i) The time interval between two events

No. Each observer sees the other person's clock running slow (by the γ factor).

(ii) The length of an object

No. The observed length of an object is contracted (by the γ factor) when it is moving with respect to the observer. Different speeds give different contractions.

(iii) The speed of light in a vacuum

Yes. This is the empirical basis of special relativity.

(iv) The relative speed between them

Yes. It is required if all observers are to be equivalent. This result can be derived - it's not an assumption.

Week 14 Solutions

5. Even more fun with Special Relativity

a) You are packing for a trip to another star, to which you will be traveling at 0.99c. Should you buy smaller sizes of your clothing, because you will be skinnier on the trip?

No. You do not observe yourself shrink. You're not moving.

Can you sleep in a smaller cabin than usual, because you will be shorter when you lie down?

No. You're not moving with respect to the cabin, so it doesn't shrink.

You are paid according to the time spent traveling in space. After you return, you open up your pay envelope. What will be your reaction?

This is the twin paradox. According to him, you've been away much longer than you think, by a factor of $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 7.09$. Your paycheck will be surprisingly large.

b) (classic!) You watch as a very fast farmer picks up a 15 m long ladder and runs toward his barn, which you realize is only 10 m long. As the farmer gets going faster and faster, you watch as he heads toward the barn, carrying the ladder lengthwise. At one instant, you observe that the farmer and the ladder have disappeared in the barn, then out they come again on the other side. Hmmm... you would have thought that you could have always seen part of the ladder, right? What is the minimum speed that the farmer was running?

The moving ladder must have a γ factor of 15/10 = 1.5 so that it will have shrunk enough to fit into the stationary barn. So, $\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = 0.75$. The farmer will win an Olympic gold medal.