

Name: _____

DISC: _____

Score: ____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- **You must show all of your work to received credit for these problems**
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
5	10	5	5

1. For the electron and photon listed in the table fill in the remaining quantities:

We will make use of the following conversions and constants:

$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

$m_e = 511 \text{ keV}/c^2$

SPEED OF LIGHT	$c = 3 \times 10^8 \text{ m/s}$
h	$6.626 \times 10^{-34} \text{ J s}$
\hbar	$1.0546 \times 10^{-34} \text{ J s}$
ELECTRON MASS	$m_e = 9.1 \times 10^{-31} \text{ kg}$
hc	1240 eV nm
MOMENTUM	$p = h/\lambda$
PHOTON ENERGY	$E = hc/\lambda$
PARTICLE ENERGY	$E = p^2/2m$
UNCERTAINTY PRINCIPLE	$\Delta x \Delta p \geq \hbar/2$

ENERGY	WAVELENGTH λ	MOMENTUM p	POSITION UNCERTAINTY
photon: $E = 5 \text{ eV}$	248 nm	$2.67 \times 10^{-27} \text{ kg m/s}$	
electron: $E = 5 \pm 0.1 \text{ eV}$	0.549 nm	2.26 keV/c	$\geq 154.5 \text{ nm}$

Table (5 pts.):

Step 1: Find the wavelength and momentum of the photon:

$$\lambda = \frac{hc}{E} = \frac{(1240 \text{ eV nm})}{(5 \text{ eV})} = 248 \text{ nm}$$

$$p = \frac{h}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J s})}{248 \text{ nm}} = 0.0267 \times 10^{-25} = 2.67 \times 10^{-27} \text{ kg m/s}$$

Step 2: Find the wavelength and momentum of the electron:

$$p = \sqrt{2mE} = \sqrt{2(511 \text{ keV}/c^2)(5 \text{ eV})} = 2.26 \text{ keV}/c$$

$$\lambda = \frac{h}{p} = \frac{(6.626 \times 10^{-34} \text{ J s})}{(2.26 \text{ keV}/c)} \frac{1 \text{ eV}}{(1.602 \times 10^{-19} \text{ J})} = (1.830 \times 10^{-18} \text{ s}) \times \left(3 \times 10^{17} \frac{\text{nm}}{\text{s}}\right) = 0.549 \text{ nm}$$

Step 3: Use the energy resolution to find the momentum resolution:

$$p = \sqrt{2mE} = \sqrt{2(511 \text{ keV}/c^2)(0.1 \text{ eV})} = 0.320 \text{ keV}/c$$

Step 4: Apply Uncertainty Relation: $\Delta x \geq \frac{\hbar}{2 \times 2 \times 0.320 \frac{\text{keV}}{c}} = \frac{1.0546 \times 10^{-34} \text{ J s}}{1.602 \times \frac{10^{-19} \text{ J}}{\text{eV}}} \frac{1}{1279 \frac{\text{eV}}{c}} = 0.000515 c \times 10^{-12} \text{ s}$

$$\Delta x \geq 0.000515 \left(3 \times 10^{17} \frac{\text{nm}}{\text{s}}\right) \times 10^{-12} \text{ s} = 0.001545 \times 10^5 \text{ nm} \geq 154.5 \text{ nm}$$

2. Remembering that the energy levels of a hydrogen-like atom can be calculated: $E_n = -13.6 \text{ eV} \left(\frac{Z^2}{n^2}\right)$, where n is an integer, and $Z = 1$ for hydrogen. Calculate the following properties of the photon needed to make the following transitions in hydrogen:

Step 1: Find the energy of the transitions: $\Delta E = (-13.6 \text{ eV}) \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

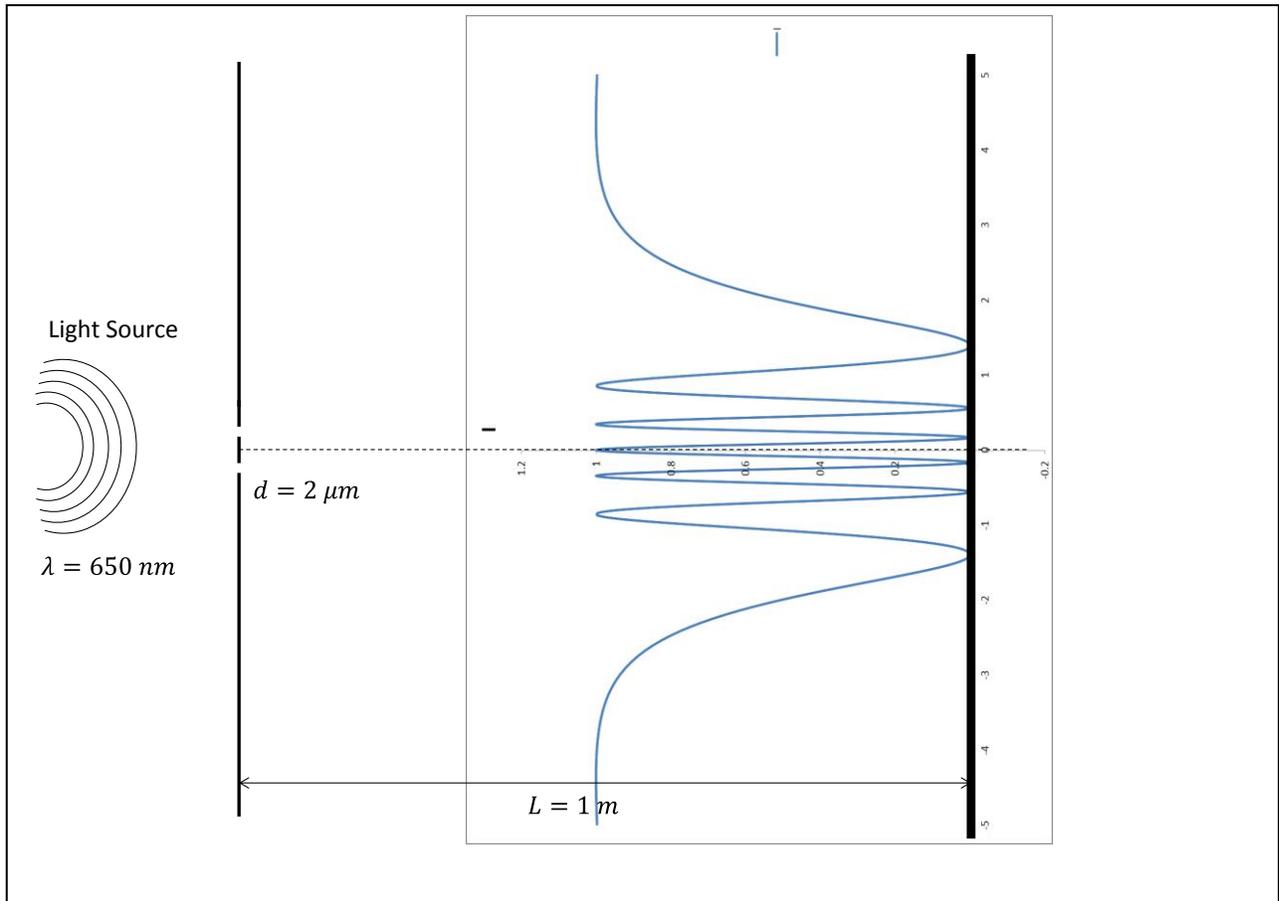
Step 2: Find the wavelength associated with each transition: $\lambda = \frac{(1240 \text{ eV nm})}{\Delta E}$

$n = 1 \rightarrow n = 3$	$\frac{1}{3^2} - \frac{1}{1^2} = -0.889$	$E = -13.6 \text{ eV} \times -0.889 = 12.09 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{12.09 \text{ eV}} = 102.56 \text{ nm}$
$n = 1 \rightarrow n = 2$	$\frac{1}{2^2} - \frac{1}{1^2} = -0.74$	$E = -13.6 \text{ eV} \times -0.74 = 10.06 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{10.06 \text{ eV}} = 123.26 \text{ nm}$
$n = 4 \rightarrow n = 5$	$\frac{1}{5^2} - \frac{1}{4^2} = -0.0225$	$E = -13.6 \text{ eV} \times -0.0225 = 0.306 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{0.306 \text{ eV}} = 4052.3 \text{ nm}$
$n = 2 \rightarrow n = 3$	$\frac{1}{3^2} - \frac{1}{2^2} = -0.139$	$E = -13.6 \text{ eV} \times -0.139 = 1.890 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{1.890 \text{ eV}} = 656.1 \text{ nm}$
$n = 3 \rightarrow n = 4$	$\frac{1}{4^2} - \frac{1}{3^2} = -0.0486$	$E = -13.6 \text{ eV} \times -0.0486 = 0.66 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{0.66 \text{ eV}} = 1876 \text{ nm}$

TRANSITION	WAVELENGTH λ	ENERGY E
$n = 1 \rightarrow n = 3$	102.56 nm	12.09 eV
$n = 1 \rightarrow n = 2$	123.26 nm	10.06 eV
$n = 4 \rightarrow n = 5$	4052.3 nm	0.306 eV
$n = 2 \rightarrow n = 3$	656.1 nm	1.890 eV
$n = 3 \rightarrow n = 4$	1876 nm	0.66 eV

Table (10 pts.):

3. A beam of red light $\lambda = 650 \text{ nm}$ impinges a screen with two slits spaced $d = 2 \mu\text{m}$ apart.



Sketch (1 pts.):
Table (4 pts.):

- a) On the diagram sketch the interference pattern you expect from two-slit interference.
b) Fill in the following table for the interference minima ($d \sin \theta = (m + \frac{1}{2}) \lambda$):

Minimum	Angle	Position
$m = 1$	29.2°	0.0349 m
$m = 2$	54.4°	1.397 m
$m = 3$	undefined	undefined
$m = 4$	undefined	undefined

Step 1: Find the angle associated with each m : $\theta = \text{asin} \left[\frac{(m+\frac{1}{2})\lambda}{d} \right] = \text{asin} \left[\frac{(m+\frac{1}{2})(650 \text{ nm})}{2 \mu\text{m}} \right]$

$$m = 1: \theta = \text{asin} \left[\frac{\left(\frac{3}{2}\right) (650 \text{ nm})}{2 \mu\text{m}} \right] = \text{asin}(487.5 \times 10^{-3}) = 29.2^\circ$$

$$m = 2: \theta = \text{asin} \left[\frac{\left(\frac{5}{2}\right) (650 \text{ nm})}{2 \mu\text{m}} \right] = \text{asin}(812.7 \times 10^{-3}) = 54.4^\circ$$

$$m = 3: \theta = \text{asin} \left[\frac{\left(\frac{7}{2}\right) (650 \text{ nm})}{2 \mu\text{m}} \right] = \text{asin}(1137.8 \times 10^{-3}) = \text{undefined}$$

The argument is greater than 1. That's OK. This just means that we cannot get any interference minima after the second minimum. Remember, the interference pattern is geometry dependent.

Step 2: Using $L = 1\text{ m}$ and our trig functions we can locate in space the interference minima on the screen:

$$m = 1: y = L \tan 29.2^\circ = 0.558\text{ m}$$

$$m = 2: y = L \tan 54.4^\circ = 1.397\text{ m}$$