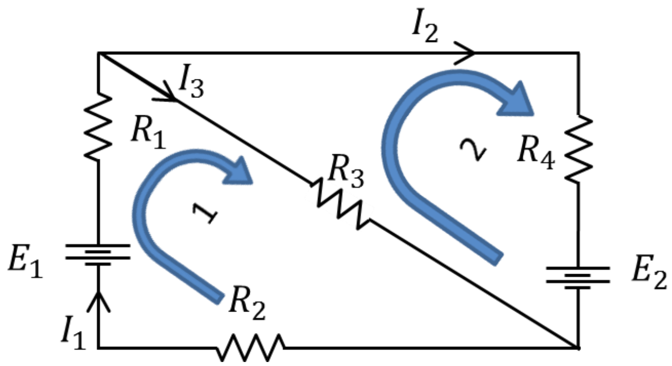
Name: ______________________ DISC: __________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
5	5	10	5

1. Consider the resistor network:



SERIES	$R_{eq} = R_1 + R_2$
PARALLEL	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
OHM'S LAW	$V = I\,R$
CONTINUITY OF CURRENT AT A JUNCTION	$I_3 = I_1 + I_2$

Useful Information

a. The table contains the values for the resistors and batteries. Now let's use this information to find the current I_1:

R_1	R_2	R_3	R_4	E_1	E_2
20 Ω	20 Ω	10 Ω	10 Ω	40 V	20 V

Loop 1 (1 pts):
Loop 2 (1 pts):
I_1 (3 pts):

i. Write the loop rule for loop 1:

$$\boldsymbol{E_1 - I_1(R_1 + R_2) - I_3R_3 = 0}$$

ii. Write the loop rule for loop 2:

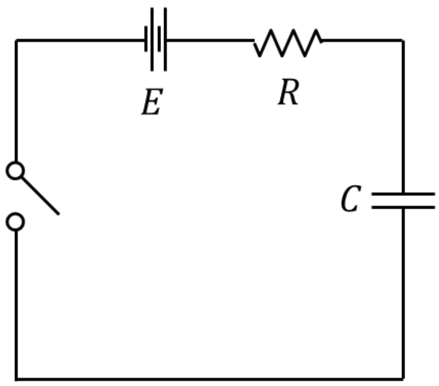
$$\boldsymbol{I_3R_3 - I_2R_4 - E_2 = 0}$$

iii. Using your loop rules above, solve for I_1: $\boxed{\boldsymbol{I_1 = \frac{2}{3}A}}$

(Use the attached scratch paper, but put you answer in the box above.)

See Work on Last Page

2. A simple RC circuit is shown below:

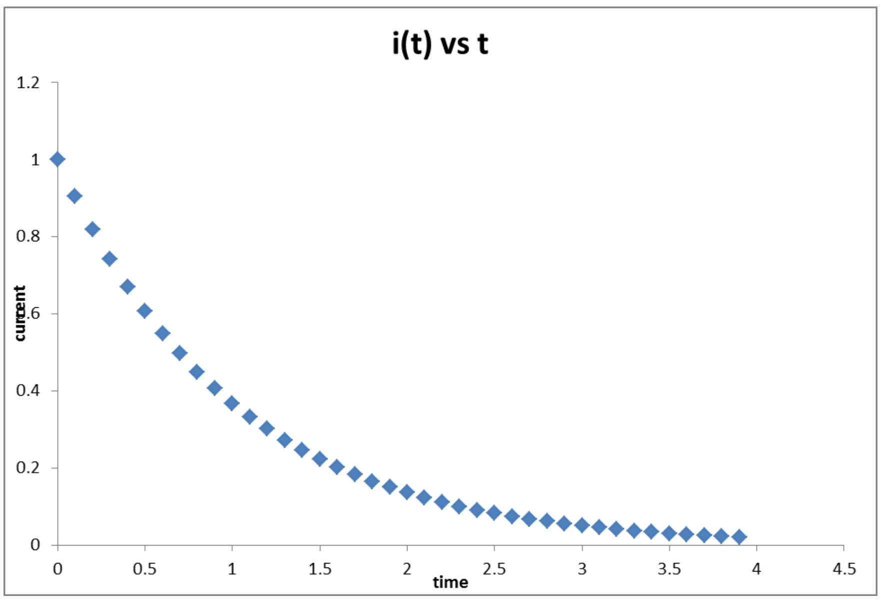


TIME CONSTANT	$\tau = RC$
R	$5\ \Omega$
C	$3\ \mu F$
E	$9\ V$
CAPACITANCE	$C = Q/V$
$Q(t) = Q_0\left(1 - e^{-\frac{t}{\tau}}\right)$	$Q(t) = Q_0\left(e^{-\frac{t}{\tau}}\right)$

Useful Information

Labels (1 pt):
Form (1 pt):

a. Initially the switch is open and the capacitor is uncharged. After the switch is closed, the capacitor starts charging. Sketch the current through the circuit as a function of time. Don't forget to correctly label your sketch.

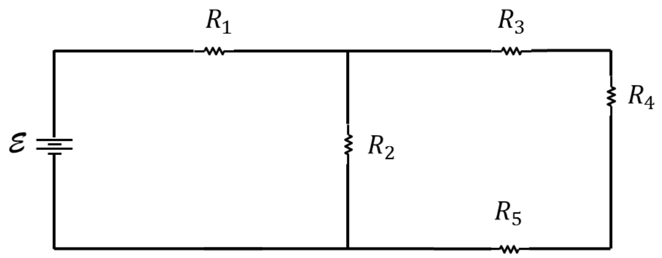


b. After the capacitor is fully charged, what is the maximum charge Q the capacitor will achieve?

$$CV = Q = 3 \times 10^{-6} \times 9 = 27\ \mu C$$

Set-up (1 pt):
Algebra (1 pt):
Charge (1 pt):

3. Consider the resistor network:



SERIES	$R_{eq} = R_1 + R_2$
PARALLEL	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
OHM'S LAW	$V = I\,R$
CONTINUITY OF CURRENT AT A JUNCTION	$I_3 = I_1 + I_2$

Useful Information

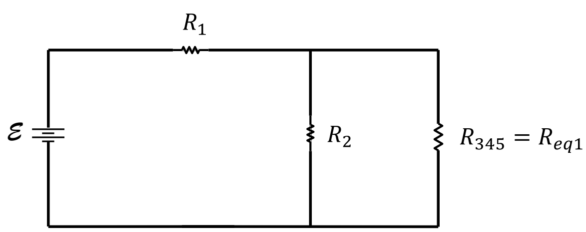
a. Let all resistors have resistance $R = 5\,\Omega$ and $\varepsilon = 6\,V$. Find the effective resistances requested and calculate the current flowing through each:

Table (6 pts):

STEP	ACTION	RESULT	CURRENT
1	EFFECTIVE RESISTANCE $R_{345} = R_{eq1}$:	$\mathbf{15\,\Omega}$	$\mathbf{0.171\,A}$
2	EFFECTIVE RESISTANCE $R_{2345} = R_{eq2}$:	$\mathbf{3.75\,\Omega}$	$\mathbf{0.686\,A}$
3	EFFECTIVE RESISTANCE $R_{12345} = R_{eq3}$:	$\mathbf{8.75\,\Omega}$	$\mathbf{0.686\,A}$

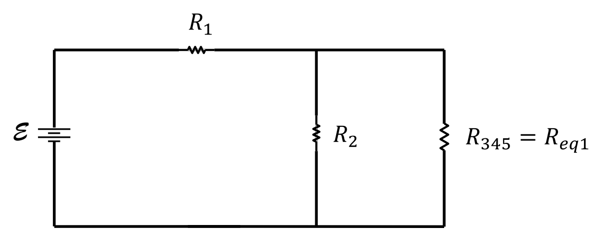
b. For each step in the above table, draw the effective circuit:

Step 1:



$\boldsymbol{R_{eq1} = 3 \times 5 = 15\,\Omega\; V_{eq1} = 6\,V - 5\Omega \times 0.686 = 2.57\,V;}$
$\boldsymbol{I_1 = \frac{V_{eq1}}{R_{eq1}} = = \frac{2.57V}{15\Omega} = 0.171\,A}$

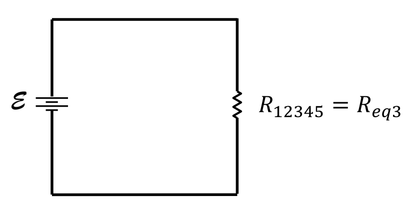
Step 2:



$\boldsymbol{\frac{1}{R_{eq2}} = \frac{1}{15\,Ohms} + \frac{1}{5\,Ohms} = \frac{4}{15\,Ohms}\; R_{eq2} = 3.75\,\Omega;\; I_2}$ **is the same as** $\boldsymbol{I_3}$ **because** $\boldsymbol{R_1}$ **is in series with** $\boldsymbol{\varepsilon}$**.**

Step 1 (2 pts):
Step 2 (1 pts):
Step 3 (1 pts):

Step 3:



$\boldsymbol{R_{eq3} = 5 + 3.75 = 8.75\,\Omega;\; I_3 = \frac{6\,V}{8.75\,\Omega} = 0.686\,A}$

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Kirchhoff's Loop Rules:

$$E_1 - I_1(R_1 + R_2) - I_3R_3 = 0$$
$$I_3R_3 - I_2R_4 - E_2 = 0$$

(0.5 pts.) **Junction Rule for this network** ***(OK if this is in a combined step as long as it is done correctly)***

$$I_1 = I_2 + I_3.$$

[Note: This is a variation of the version given in the "Useful Information" box above]

(0.5 pts) **Making some value substitutions (see table in problem 1):**

$$40\,V - I_1(20\,\Omega + 20\,\Omega) - I_3(10\,\Omega) = 0$$
$$I_3(10\,\Omega) - I_2(10\,\Omega) - 20\,V = 0$$

Simplify the arithmetic a little bit:

$$40\,V - I_1(40\,\Omega) - I_3(10\,\Omega) = 0$$
$$I_3(10\,\Omega) - I_2(10\,\Omega) - 20\,V = 0$$

(1 pt) **I will now use the junction rule to eliminate** I_2: $I_2 = I_1 - I_3$ ***(Full point for any other correct substitution, substituted correctly)***

$$40\,V - I_1(40\,\Omega) - I_3(10\,\Omega) = 0$$
$$I_3(10\,\Omega) - (I_1 - I_3)(10\,\Omega) - 20\,V = 0$$

Simplify again:

$$40\,V - I_1(40\,\Omega) - I_3(10\,\Omega) = 0$$
$$I_3(20\,\Omega) - I_1(10\,\Omega) - 20\,V = 0$$

And reorganize terms:

$$-I_1(40\,\Omega) - I_3(10\,\Omega) = -40\,V$$
$$-I_1(10\,\Omega) + I_3(20\,\Omega) = 20\,V$$

(1 pt) **I'm now going to multiply the top equation by 2 and add the two equations to each other to eliminate** I_3: ***(or any other acceptable technique, done correctly, for 2 equations and 2 unknowns)***

$$\begin{array}{rr} & -I_1(80\,\Omega) - I_3(20\,\Omega) = -80\,V \\ (+) & \underline{-I_1(10\,\Omega) + I_3(20\,\Omega) = 20\,V} \\ & -I_1(90\,\Omega) = -60\,V \end{array}$$

So we finish the arithmetic and find that $I_1 = \frac{2}{3}A$