

Name: \_\_\_\_\_

DISC: \_\_\_\_\_

Score: \_\_\_\_ / 20

## Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- **You must show all of your work to receive credit for these problems**
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
5	10	5	5

1. For the electron and photon listed in the table fill in the remaining quantities:

**We will make use of the following conversions and constants:**

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$m_e = 511 \text{ keV}/c^2$$

SPEED OF LIGHT	$c = 3 \times 10^8 \text{ m/s}$
$h$	$6.626 \times 10^{-34} \text{ J s}$
$\hbar$	$1.0546 \times 10^{-34} \text{ J s}$
ELECTRON MASS	$m_e = 9.1 \times 10^{-31} \text{ kg}$
$hc$	$1240 \text{ eV nm}$
MOMENTUM	$p = h/\lambda$
PHOTON ENERGY	$E = hc/\lambda$
PARTICLE ENERGY	$E = p^2/2m$
UNCERTAINTY PRINCIPLE	$\Delta x \Delta p \geq \hbar/2$

ENERGY	WAVELENGTH $\lambda$	MOMENTUM $p$	POSITION UNCERTAINTY
photon: $E = 5 \text{ eV}$	<b>248 nm</b>	<b><math>2.67 \times 10^{-27} \text{ kg m/s}</math></b>	
electron: $E = 5 \pm 0.1 \text{ eV}$	<b>0.549 nm</b>	<b>2.26 keV/c</b>	<b><math>\geq 154.5 \text{ nm}</math></b>

Table (5 pts.):

**Step 1: Find the wavelength and momentum of the photon:**

$$\lambda = \frac{hc}{E} = \frac{(1240 \text{ eV nm})}{(5 \text{ eV})} = 248 \text{ nm}$$

$$p = \frac{h}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J s})}{248 \text{ nm}} = 0.0267 \times 10^{-25} = 2.67 \times 10^{-27} \text{ kg m/s}$$

**Step 2: Find the wavelength and momentum of the electron:**

$$p = \sqrt{2mE} = \sqrt{2(511 \text{ keV}/c^2)(5 \text{ eV})} = 2.26 \text{ keV}/c$$

$$\lambda = \frac{h}{p} = \frac{(6.626 \times 10^{-34} \text{ J s})}{(2.26 \text{ keV}/c)} \frac{1 \text{ eV}}{(1.602 \times 10^{-19} \text{ J})} = (1.830 \times 10^{-18} \text{ s}) \times \left(3 \times 10^{17} \frac{\text{nm}}{\text{s}}\right) = 0.549 \text{ nm}$$

**Step 3: Use the energy resolution to find the momentum resolution:**

$$p = \sqrt{2mE} = \sqrt{2(511 \text{ keV}/c^2)(0.1 \text{ eV})} = 0.320 \text{ keV}/c$$

**Step 4: Apply Uncertainty Relation:**  $\Delta x \geq \frac{\hbar}{2 \times 2 \times 0.320 \frac{\text{keV}}{c}} = \frac{1.0546 \times 10^{-34} \text{ J s}}{1.602 \times \frac{10^{-19} \text{ J}}{\text{eV}}} \frac{1}{1279 \frac{\text{eV}}{c}} = 0.000515 c \times 10^{-12} \text{ s}$

$$\Delta x \geq 0.000515 \left(3 \times 10^{17} \frac{\text{nm}}{\text{s}}\right) \times 10^{-12} \text{ s} = 0.001545 \times 10^5 \text{ nm} \geq 154.5 \text{ nm}$$

2. Remembering that the energy levels of a hydrogen-like atom can be calculated:  $E_n = -13.6 \text{ eV} \left( \frac{Z^2}{n^2} \right)$ , where  $n$  is an integer, and  $Z = 1$  for hydrogen. Calculate the following properties of the photon needed to make the following transitions in hydrogen:

**Step 1: Find the energy of the transitions:**  $\Delta E = (-13.6 \text{ eV}) \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$

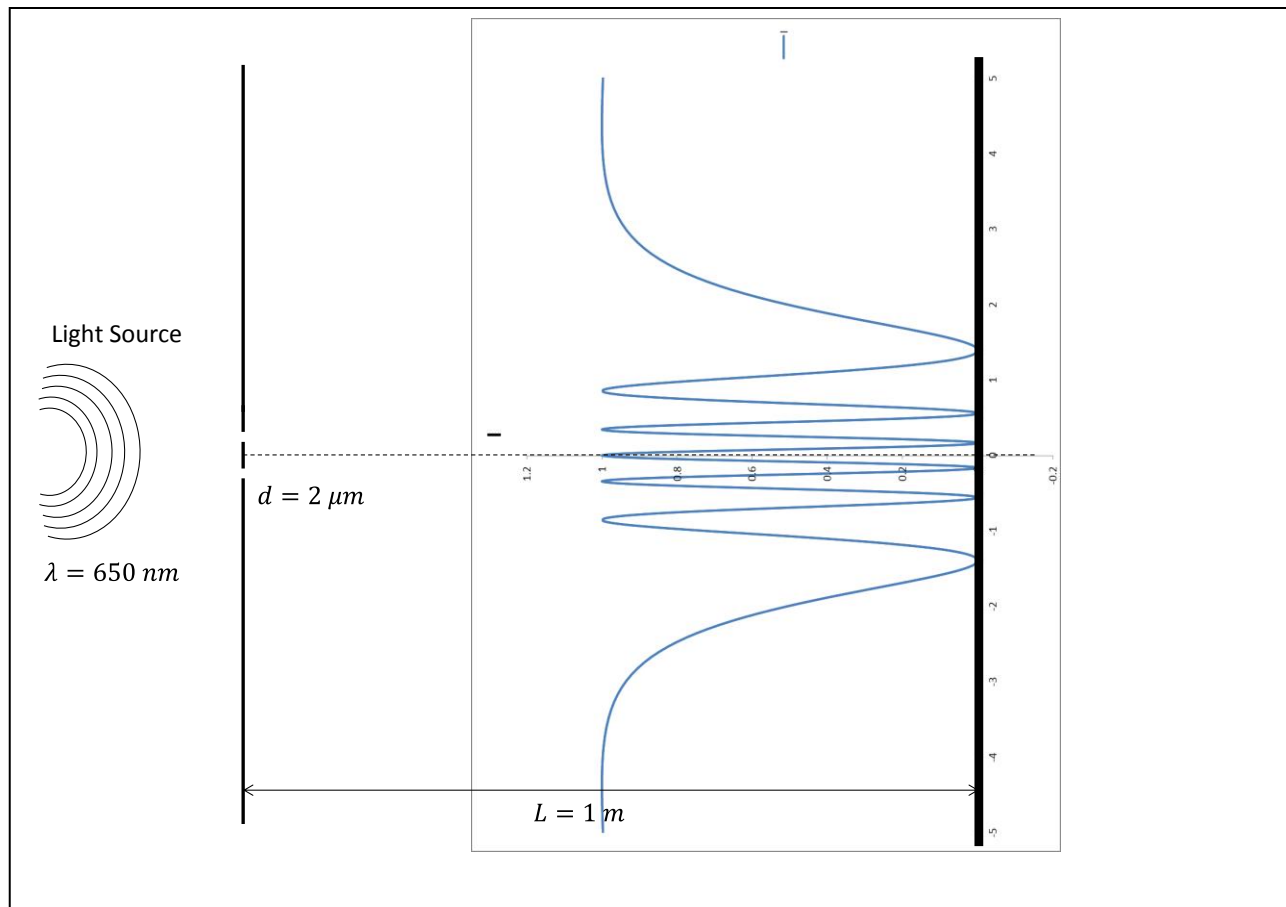
**Step 2: Find the wavelength associated with each transition:**  $\lambda = \frac{(1240 \text{ eV nm})}{\Delta E}$

$n = 1 \rightarrow n = 3$	$\frac{1}{3^2} - \frac{1}{1^2} = -0.889$	$E = -13.6 \text{ eV} \times -0.889 = 12.09 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{12.09 \text{ eV}} = 102.56 \text{ nm}$
$n = 1 \rightarrow n = 2$	$\frac{1}{2^2} - \frac{1}{1^2} = -0.74$	$E = -13.6 \text{ eV} \times -0.74 = 10.06 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{10.06 \text{ eV}} = 123.26 \text{ nm}$
$n = 4 \rightarrow n = 5$	$\frac{1}{5^2} - \frac{1}{4^2} = -0.0225$	$E = -13.6 \text{ eV} \times -0.0225 = 0.306 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{0.306 \text{ eV}} = 4052.3 \text{ nm}$
$n = 2 \rightarrow n = 3$	$\frac{1}{3^2} - \frac{1}{2^2} = -0.139$	$E = -13.6 \text{ eV} \times -0.139 = 1.890 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{1.890 \text{ eV}} = 656.1 \text{ nm}$
$n = 3 \rightarrow n = 4$	$\frac{1}{4^2} - \frac{1}{3^2} = -0.0486$	$E = -13.6 \text{ eV} \times -0.0486 = 0.66 \text{ eV}$	$\frac{(1240 \text{ eV nm})}{0.66 \text{ eV}} = 1876 \text{ nm}$

Table (10  
pts.):

TRANSITION	WAVELENGTH $\lambda$	ENERGY $E$
$n = 1 \rightarrow n = 3$	<b>102.56 nm</b>	<b>12.09 eV</b>
$n = 1 \rightarrow n = 2$	<b>123.26 nm</b>	<b>10.06 eV</b>
$n = 4 \rightarrow n = 5$	<b>4052.3 nm</b>	<b>0.306 eV</b>
$n = 2 \rightarrow n = 3$	<b>656.1 nm</b>	<b>1.890 eV</b>
$n = 3 \rightarrow n = 4$	<b>1876 nm</b>	<b>0.66 eV</b>

3. A beam of red light  $\lambda = 650 \text{ nm}$  impinges a screen with two slits spaced  $d = 2 \mu\text{m}$  apart.



Sketch (1  
pts.):  
Table (4 pts.):

- a) On the diagram sketch the interference pattern you expect from two-slit interference.  
b) Fill in the following table for the interference minima ( $d \sin \theta = (m + \frac{1}{2}) \lambda$ ):

Minimum	Angle	Position
$m = 1$	<b>29.2°</b>	<b>0.0349 m</b>
$m = 2$	<b>54.4°</b>	<b>1.397 m</b>
$m = 3$	<b>undefined</b>	<b>undefined</b>
$m = 4$	<b>undefined</b>	<b>undefined</b>

**Step 1: Find the angle associated with each  $m$ :  $\theta = \text{asin} \left[ \frac{(m + \frac{1}{2}) \lambda}{d} \right] = \text{asin} \left[ \frac{(m + \frac{1}{2})(650 \text{ nm})}{2 \mu\text{m}} \right]$**

$$m = 1: \theta = \text{asin} \left[ \frac{\left(\frac{3}{2}\right)(650 \text{ nm})}{2 \mu\text{m}} \right] = \text{asin}(487.5 \times 10^{-3}) = 29.2^\circ$$

$$m = 2: \theta = \text{asin} \left[ \frac{\left(\frac{5}{2}\right)(650 \text{ nm})}{2 \mu\text{m}} \right] = \text{asin}(812.7 \times 10^{-3}) = 54.4^\circ$$

$$m = 3: \theta = \text{asin} \left[ \frac{\left(\frac{7}{2}\right)(650 \text{ nm})}{2 \mu\text{m}} \right] = \text{asin}(1137.8 \times 10^{-3}) = \text{undefined}$$

**The argument is greater than 1. That's OK. This just means that we cannot get any interference minima after the second minimum. Remember, the interference pattern is geometry dependent.**

**Step 2: Using  $L = 1\text{ m}$  and our trig functions we can locate in space the interference minima on the screen:**

$$m = 1: y = L \tan 29.2^\circ = 0.558\text{ m}$$

$$m = 2: y = L \tan 54.4^\circ = 1.397\text{ m}$$