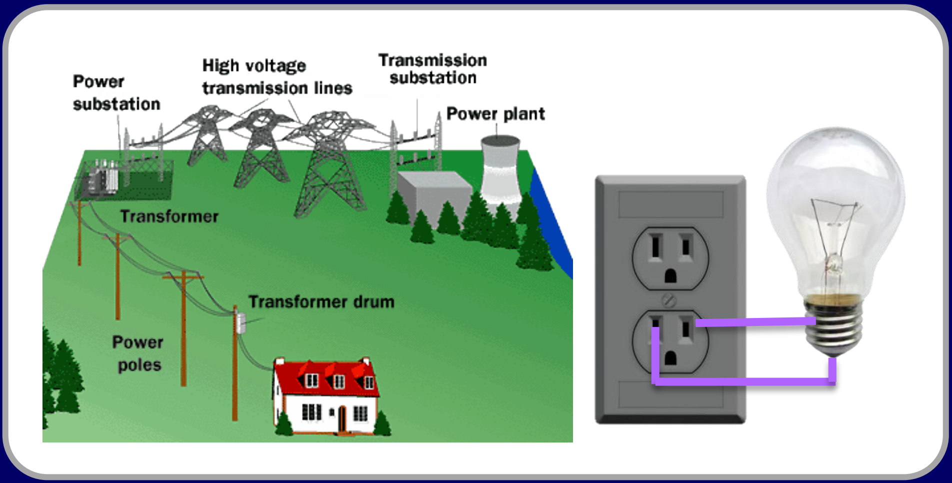
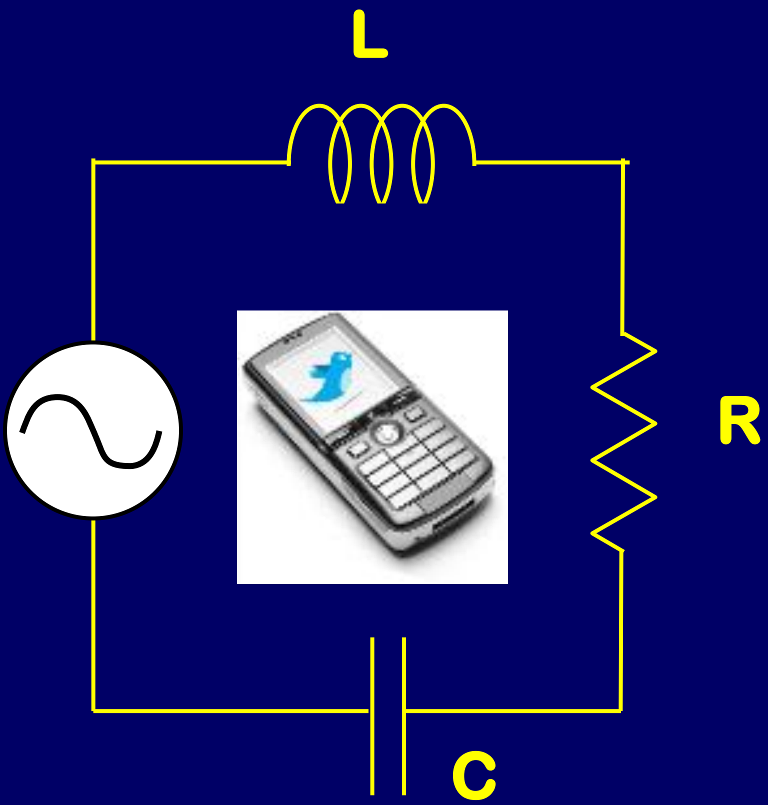
Physics 102: Lecture 12

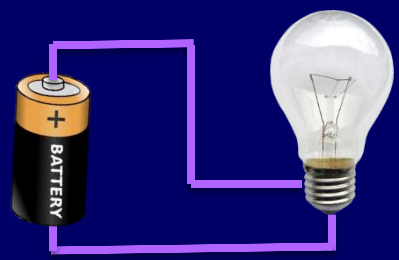
AC & RLC Circuits

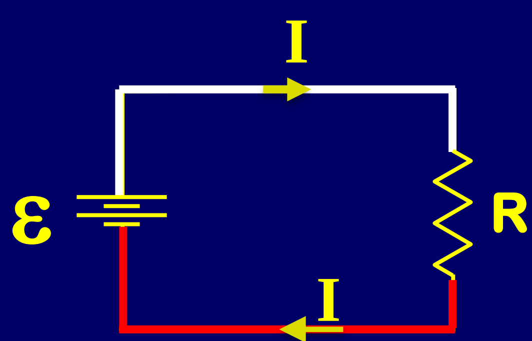




DC vs. AC circuits

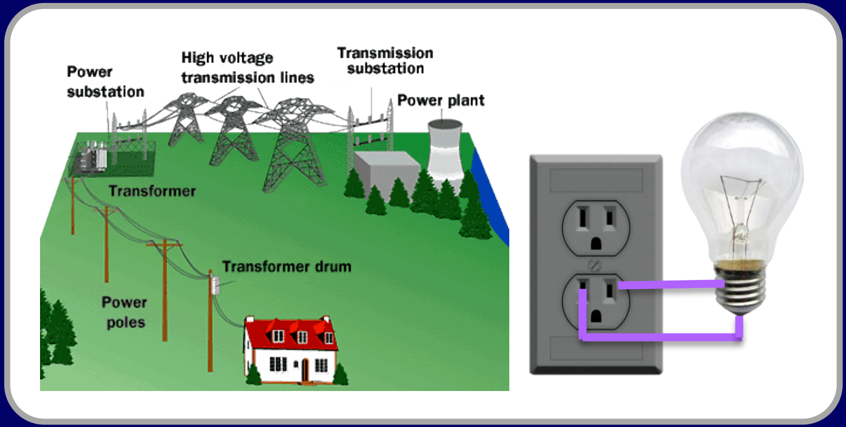
DC
Direct Current

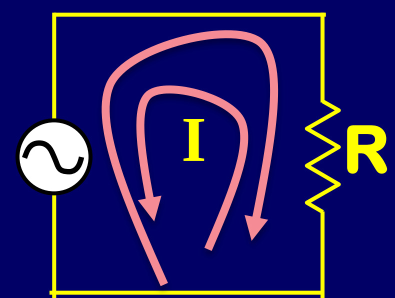


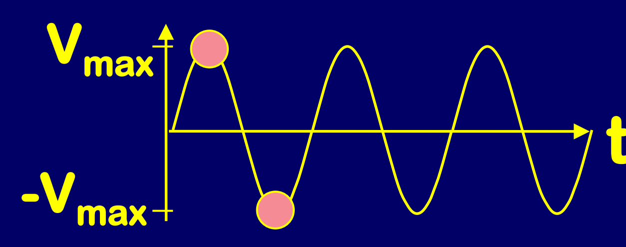


Direction of current is fixed

AC
Alternating Current

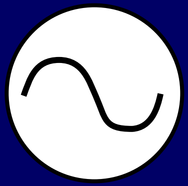






Direction of current alternates!

Review: Generators and EMF

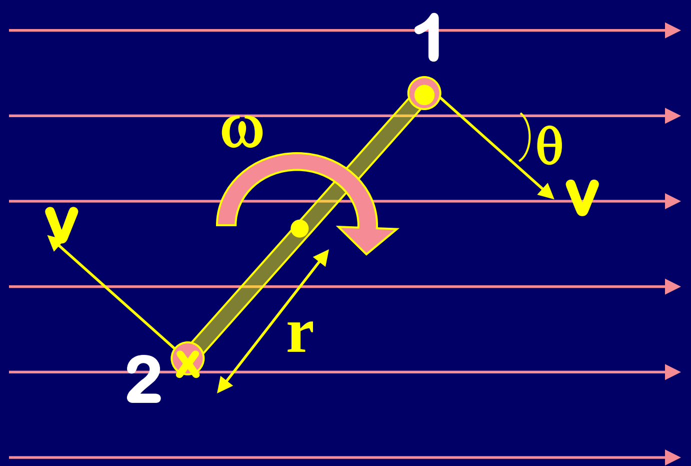


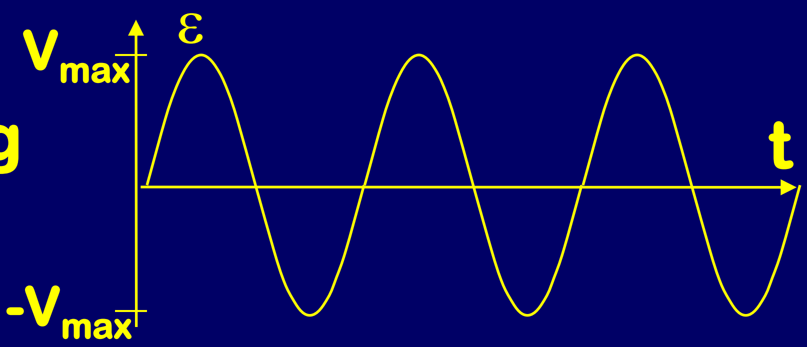
Voltage across generator:

$$\varepsilon = \omega A B \sin(\theta)$$

$$\varepsilon = \omega A B \sin(\omega t)$$

$$\varepsilon = V_{max} \sin(\omega t)$$



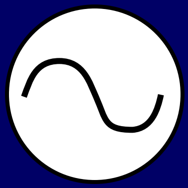


Frequency = How fast its spinning

Amplitude = Maximum voltage

Example

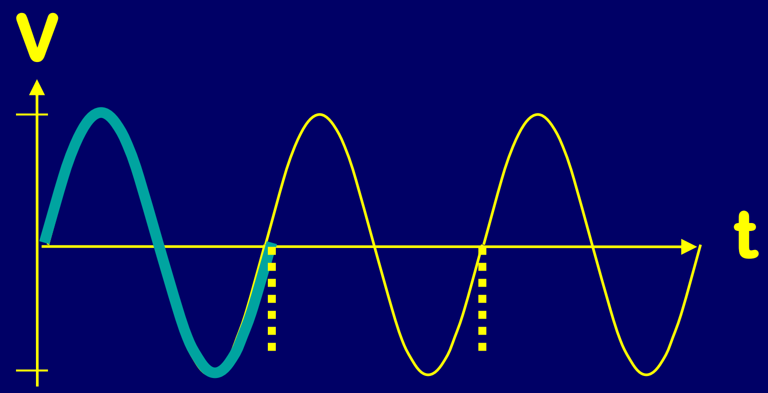
AC Source



$V(t) = V_{max} \sin(\omega t) = V_{max} \sin(2\pi f\, t)$

V_{max} = maximum voltage

f = frequency (cycles/second)



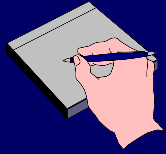
$V(t) = 24 \sin(8\pi\, t)$

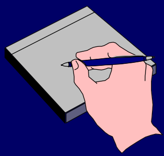
$2\pi f\, t =$

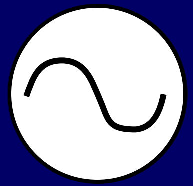
$f =$

T=(   )seconds/cycle

RMS: Root Mean Square $V_{rms} = V_{max}/\sqrt{2}$

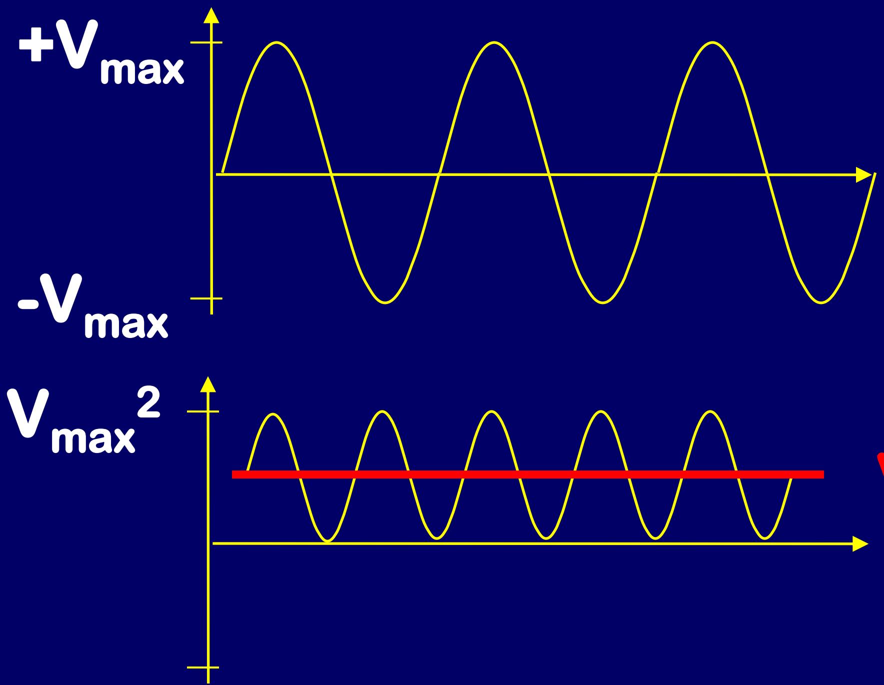






RMS?

$V(t) = V_{max} \sin(2\pi f\, t)$



Square:

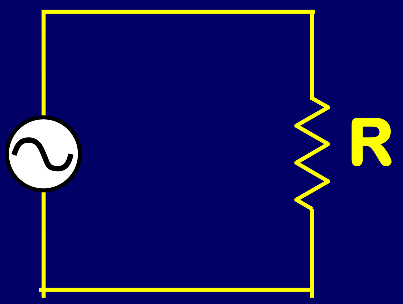
Mean:

$V_{max}^2 / 2$

square Root: $V_{max} / \sqrt{2}$

RMS: Root Mean Square $V_{rms} = V_{max}/\sqrt{2}$

CheckPoint 2.1, 2.2



$$I(t) = \frac{V_{max}}{R} sin(377t) = 10sin(377t)$$

Find I_{max}

Find I_{rms}

Average power dissipated: $P = I_{rms}V_{rms} = \frac{1}{2}I_{max}V_{max}$

(Only for an AC circuit with a resistor)

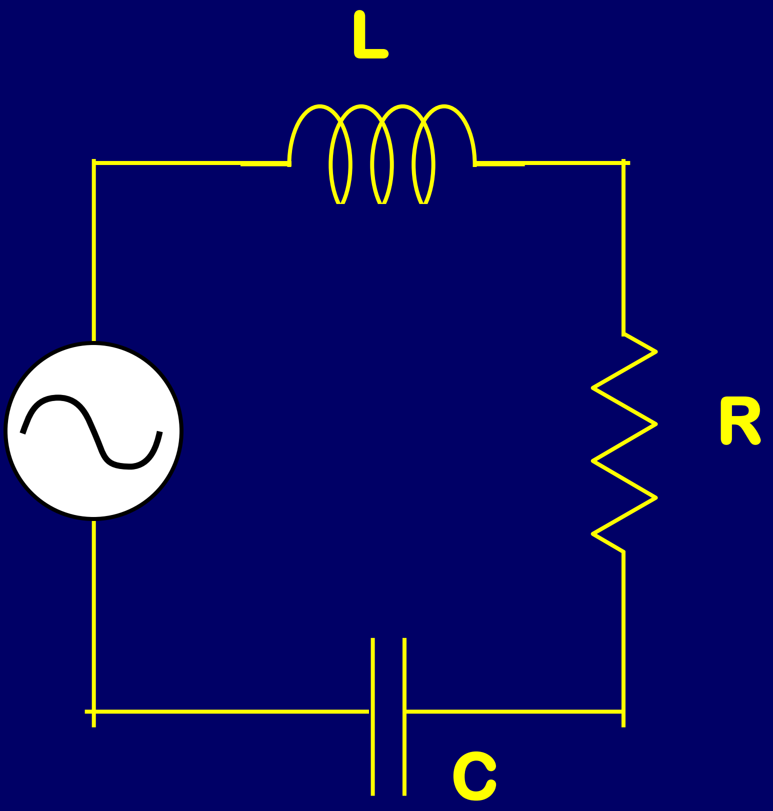
ACT: AC power dissipation

When your hair dryer is plugged in and running, it uses 1200 W of average power. If the rms voltage delivered by the wall outlet is 120 V, what is the rms current delivered to the hair dryer?

(A) 0.1 A **(B) 5 A** **(C) 10 A**

Inductors

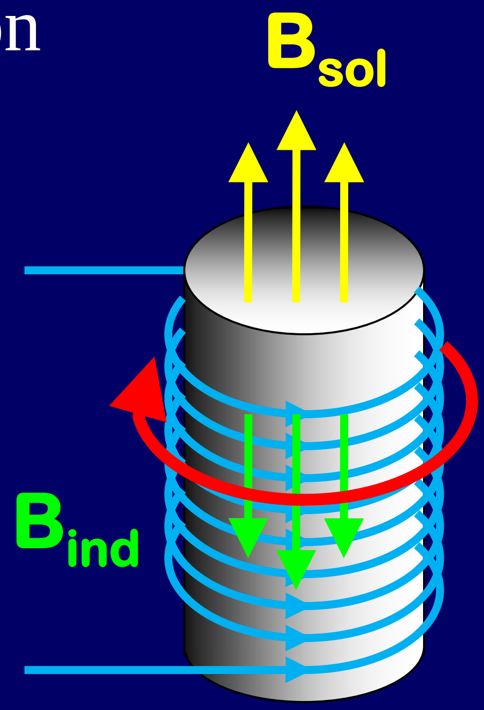
Inductors: a solenoid used as a circuit element



Inductors enable circuits to have special properties…

Self-Inductance

Recall from last time the solenoid cannon



- Changing current
- Changing B_{sol} field
- Changing Φ through itself!
  - Φ proportional to I:

$$\Phi = LI$$

"Inductance"

- Induced EMF (voltage)
  - Recall Faraday's law:

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta I}{\Delta t} = -L\frac{I_f - I_i}{t_f - t_i}$$

- Direction
  - Given by Lenz's Law
  - Opposes change in current!

Units: L = ε t / I
1 H = 1V-sec/amp

Physical Inductor

$$L \equiv \frac{\Phi}{I}$$

Recall: $\Phi = NBA$

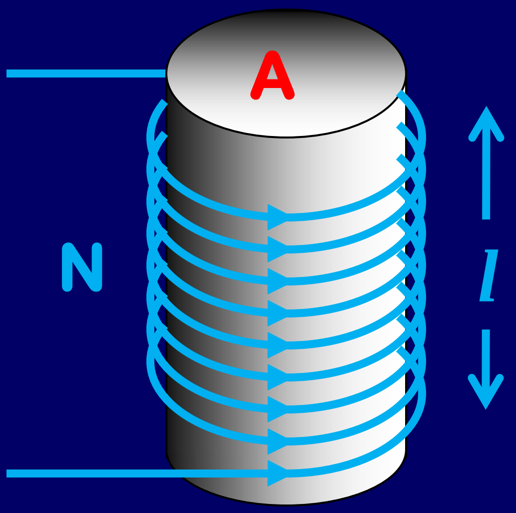
$$L = \frac{NBA}{I}$$

Recall: $B = \mu_o nI$

$$L = \frac{N\mu_0 n\cancel{I}A}{\cancel{I}}$$

$$L = N\mu_0 nA$$

$$L = \mu_0 n^2 lA$$



(# turns) = (# turns/meter) x (# meters)

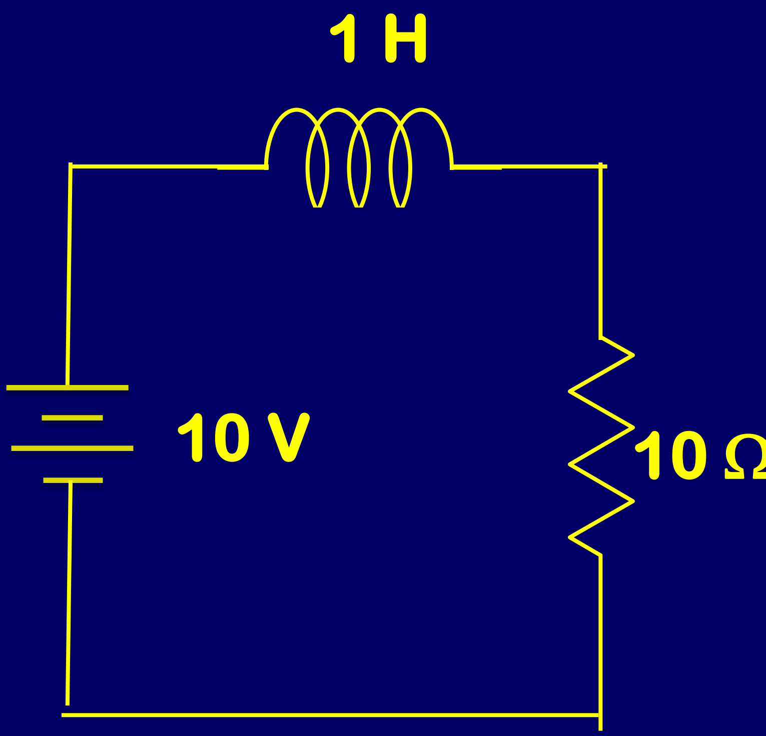
$$N = n\, l$$

Energy stored:

$$U = \tfrac{1}{2} LI^2$$

ACT: Inductors

A 10 W resistor is wired is series with a 10 V battery and a 1 H inductor. What is the voltage across the inductor?

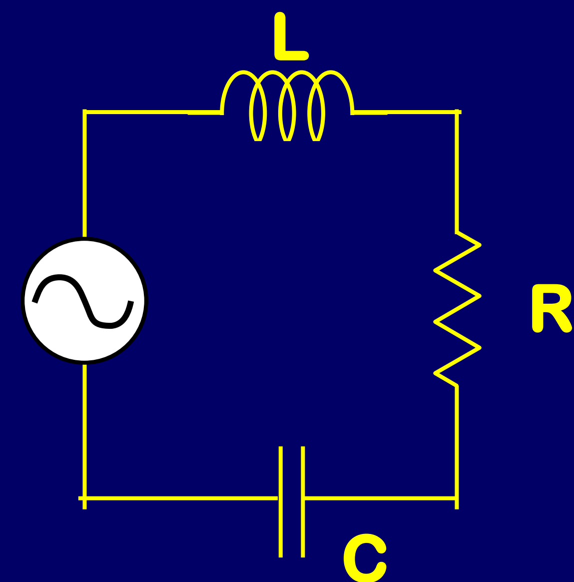


(A) 0 V **(B) 9 V** **(C) 10 V**

RLC circuits

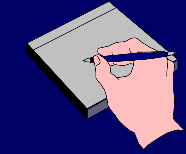
A circuit with an inductor, resistor, and capacitor in series!

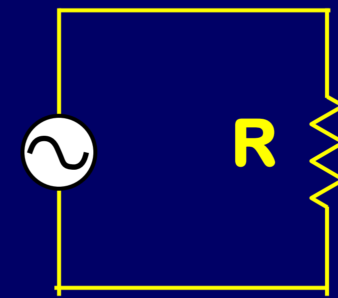
Used in:
- **Cell phones / radios**
- **Computers**
- **Watches / clocks**



First: Understand each element individually

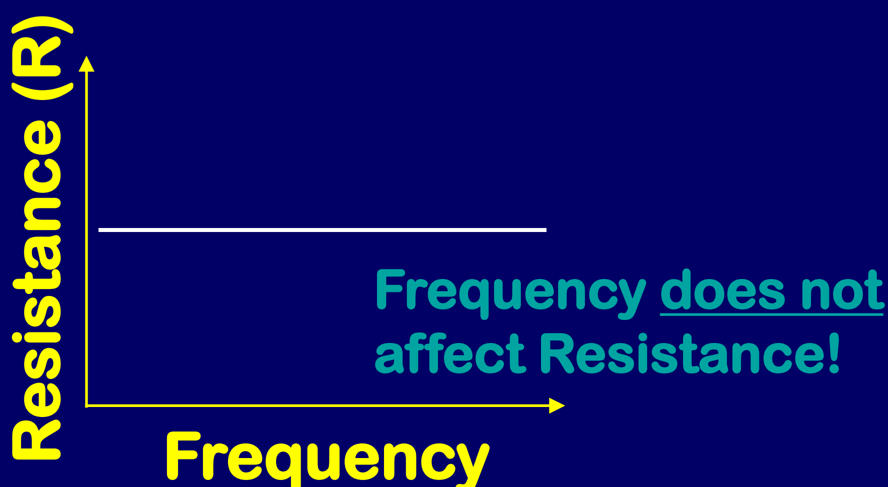
Resistors in AC circuit

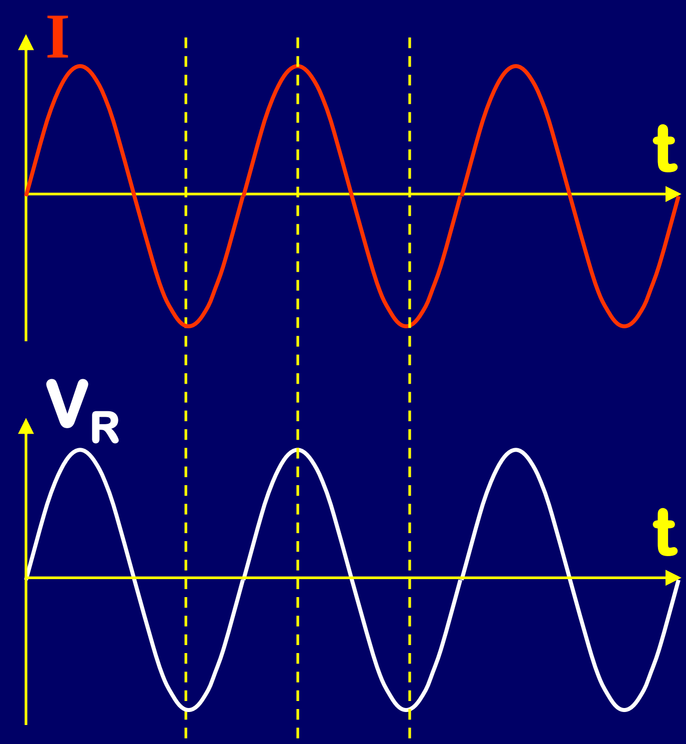


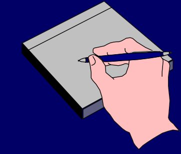


$V_R = I\,R$ always true – Ohm's Law

- $V_{R,max} = I_{max}R$
- Voltage across resistor is "IN PHASE" with current.
  - V_R goes up and down at the same times as I does.

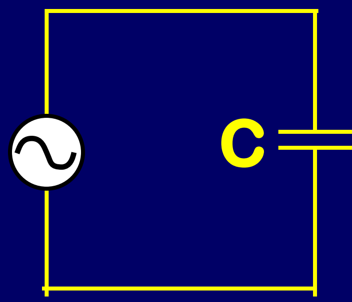






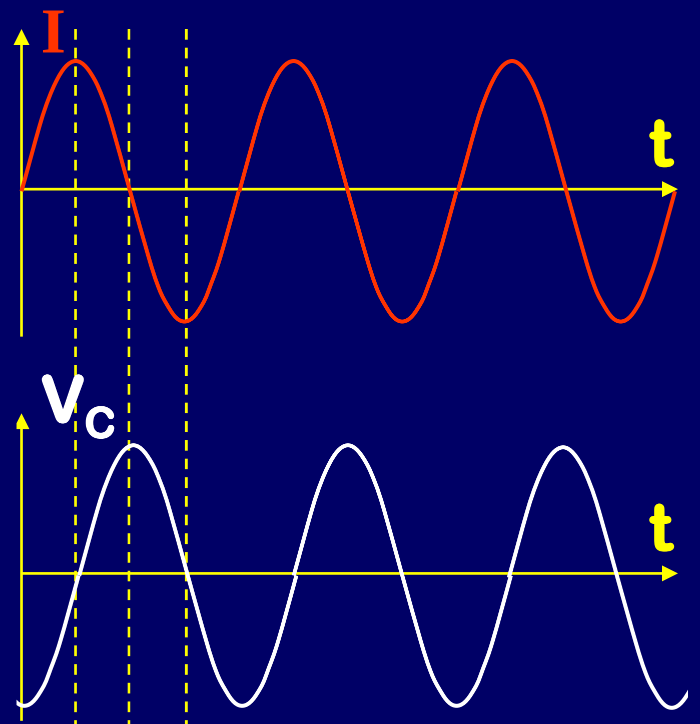
Capacitors in AC circuit

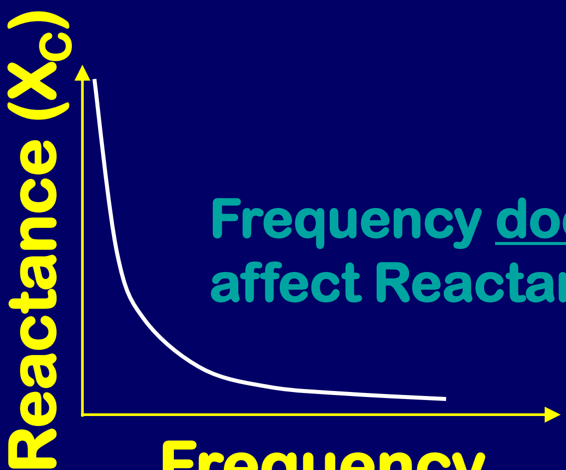
$V_C = Q/C$ always true



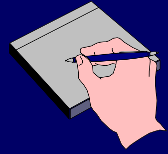
"ICE"

- $V_{C,max} = I_{max} X_C$
- Capacitive Reactance: $X_C = 1/(2\pi f C)$
- Voltage across capacitor "LAGS" current.
  - V_C goes up and down <u>just after</u> I does.



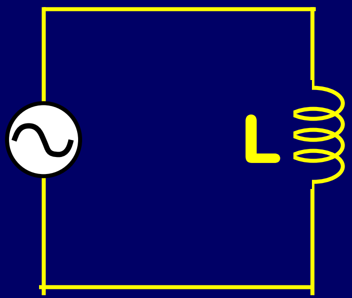


Frequency does affect Reactance!



Inductors in AC circuit

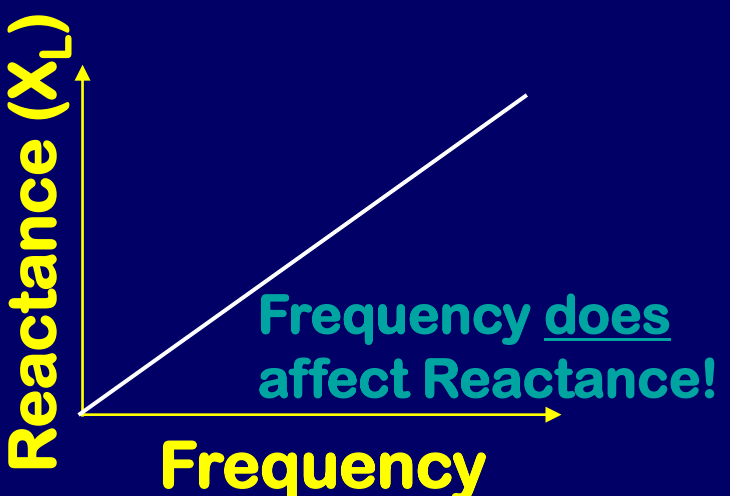
$V_L = +L(\Delta I)/(\Delta t)$ always true

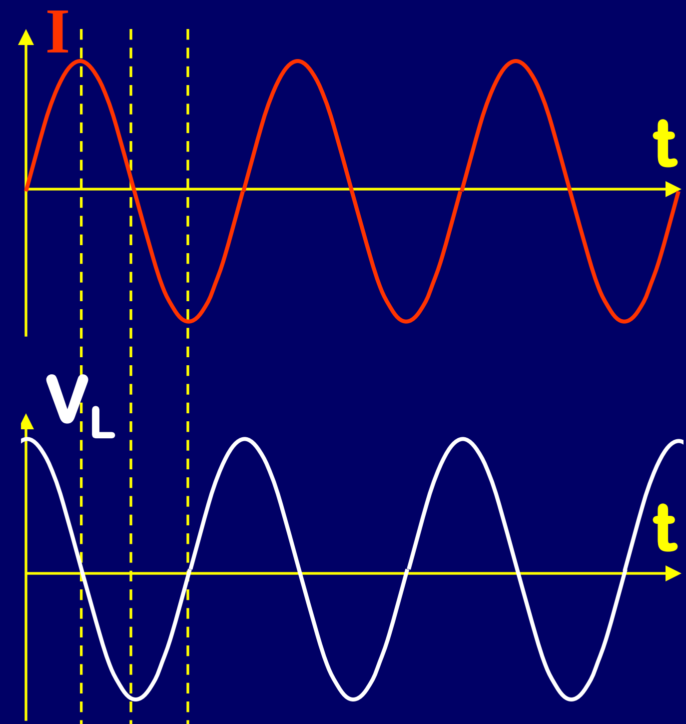


- $V_{L,max} = I_{max} X_L$
- Inductive Reactance: $X_L = 2\pi f L$
- Voltage across inductor "LEADS" current.

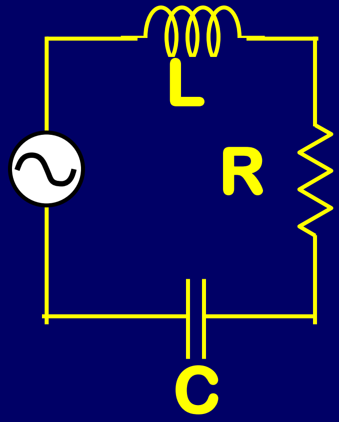
"ELI"

  – V_L goes up and down just before I does.





ACT/CheckPoints 3.1, 3.2



The capacitor can be ignored when…

(a) frequency is very large

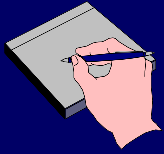
(b) frequency is very small

The inductor can be ignored when…

(a) frequency is very large

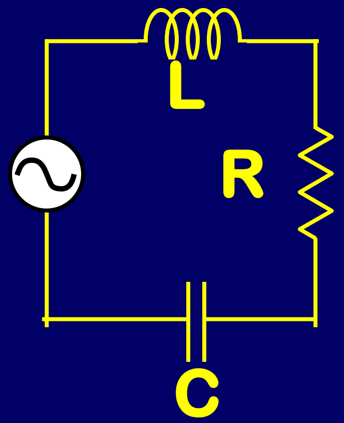
(b) frequency is very small

Example



AC Circuit Voltages

An AC circuit with R= 2 Ω, C = 15 mF, and L = 30 mH has a current I(t) = 0.5 sin(8π t) amps. Calculate the maximum voltage across R, C, and L.



$V_{R,max} = I_{max}\,R =$

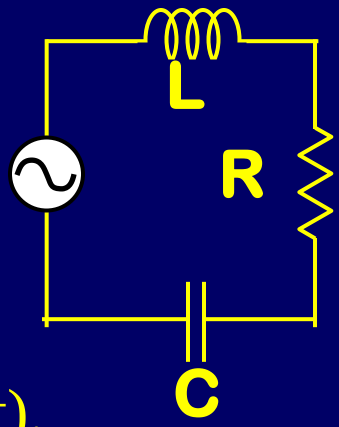
$V_{C,max} = I_{max}\,X_C =$

$V_{L,max} = I_{max}\,X_L =$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} \qquad X_L = 2\pi f L = \omega L$$

ACT: AC Circuit Voltages

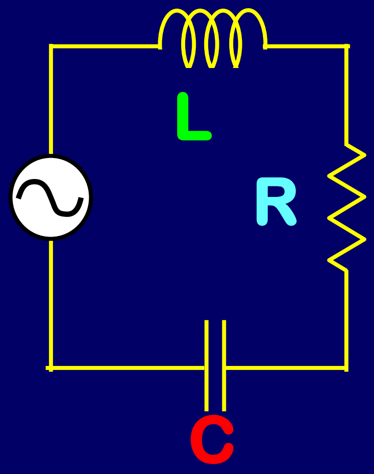
An AC circuit with R= 2 Ω, C = 15 mF, and L = 30 mH has a current I(t) = 0.5 sin(8π t) amps. Calculate the maximum voltage across R, C, and L.



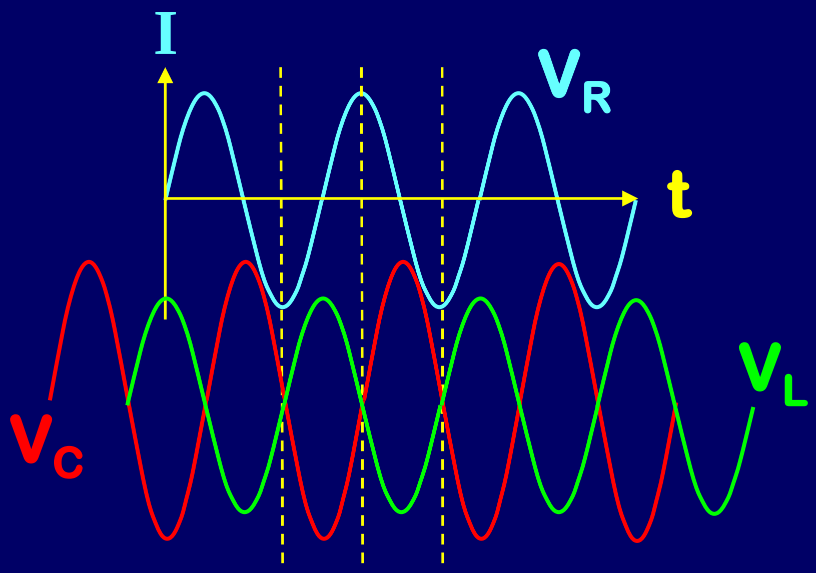
Now the frequency is <u>increased</u> so I(t) = 0.5 sin(16π t). Which element's maximum voltage <u>decreases?</u>

1) $V_{R,max}$
2) $V_{C,max}$
3) $V_{L,max}$

Summary so far…



- $I = I_{max}\sin(2\pi ft)$
- $V_R = I_{max}R\sin(2\pi ft)$
  - V_R in phase with I



- $V_C = I_{max}X_C\sin(2\pi ft - \pi/2)$
  - V_C lags I $\quad X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{\omega C}$

"ICE"

- $V_L = I_{max}X_L\sin(2\pi ft + \pi/2)$
  - V_L leads I $\quad X_L = 2\pi fL = \omega L$

"ELI"