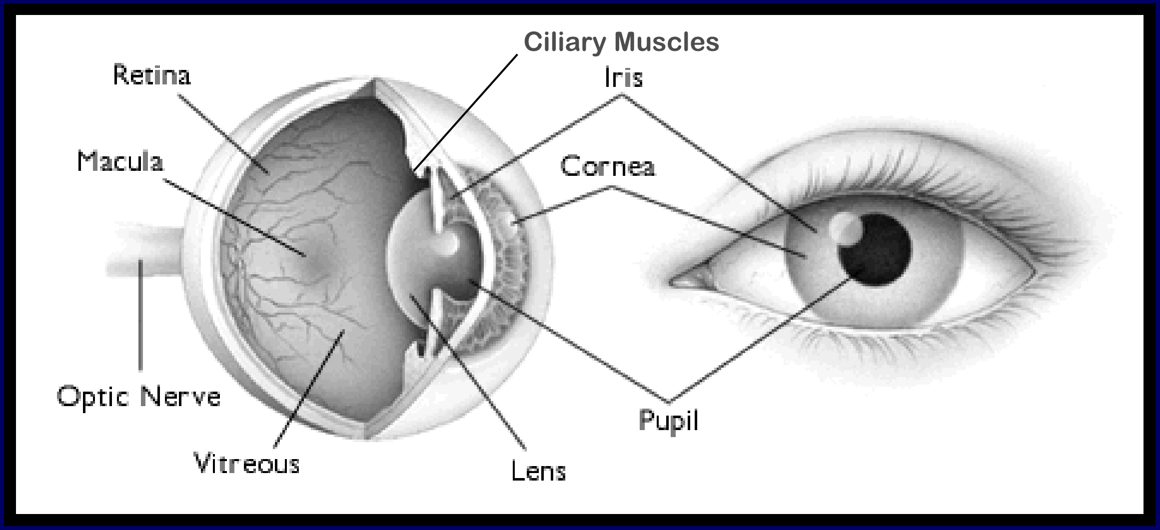
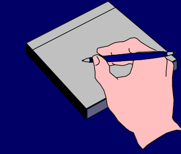
Physics 102: Lecture 19
Lenses and your EYE

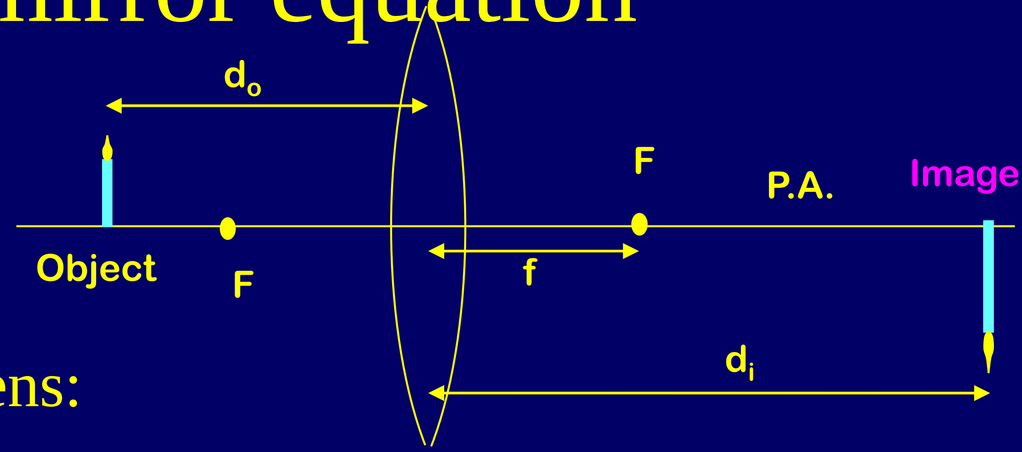


First: we need a quantitative approach to lenses!



Lens Equation

Same as mirror equation



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- d_o = distance object is from lens:
  - Positive: object before lens
  - Negative: object after lens

- d_i = distance image is from lens:
  - Positive: real image (after lens)
  - Negative: virtual image (before lens)

- f = focal length lens:
  - Positive: converging lens
  - Negative: diverging lens

Example

$$\frac{1}{15\ cm} + \frac{1}{d_i} = \frac{1}{10\ cm}$$

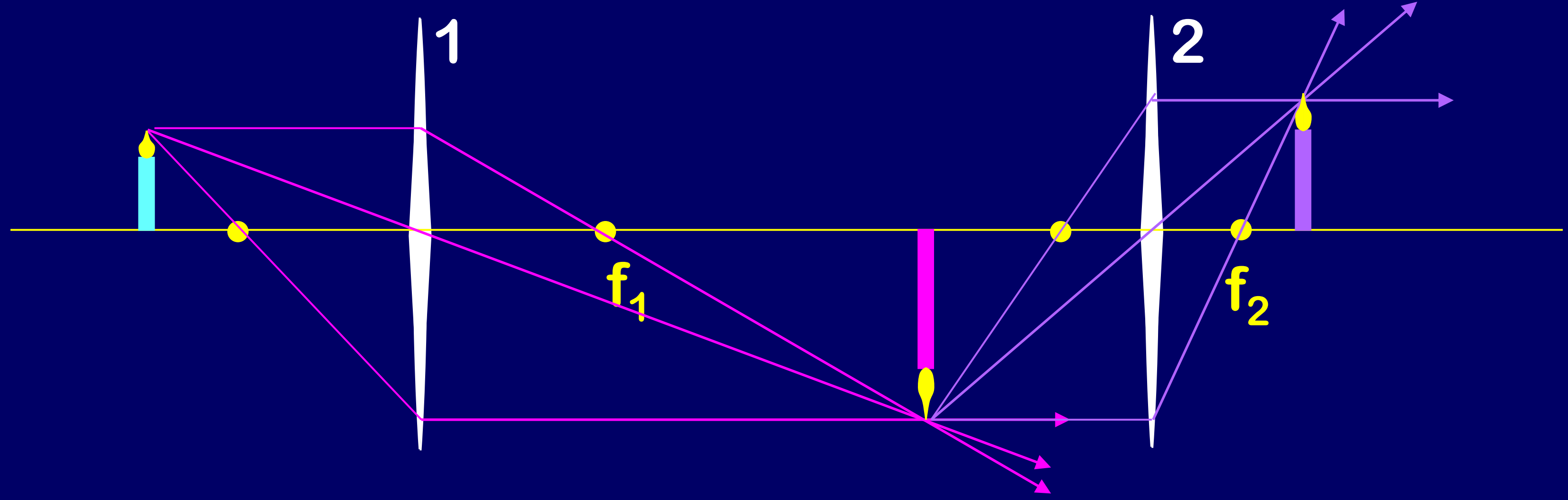
$$\Rightarrow d_i = 30\ cm$$

$$m = -\frac{d_i}{d_o} = -2$$

Multiple Lenses

Telescopes & Microscopes

Image from lens 1 becomes object for lens 2

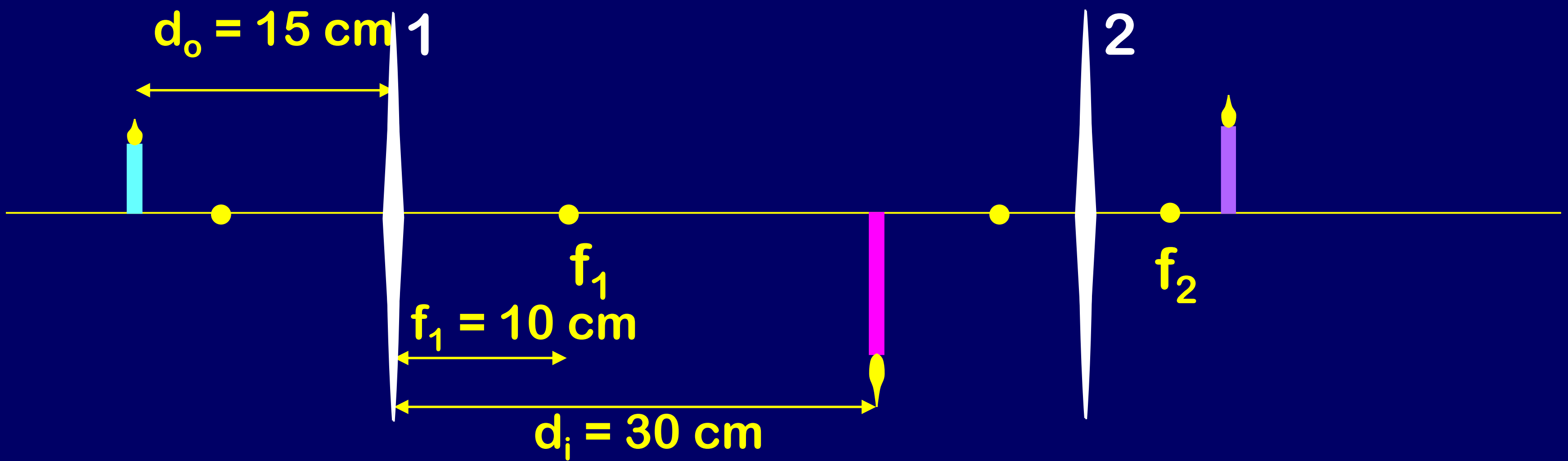


Lens 1 creates a real, inverted and enlarged image of the object.

Lens 2 creates a real, inverted and reduced image of the image from lens 1.

The combination gives a real, upright, enlarged image of the object.

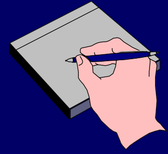
Multiple Lenses: Image 1



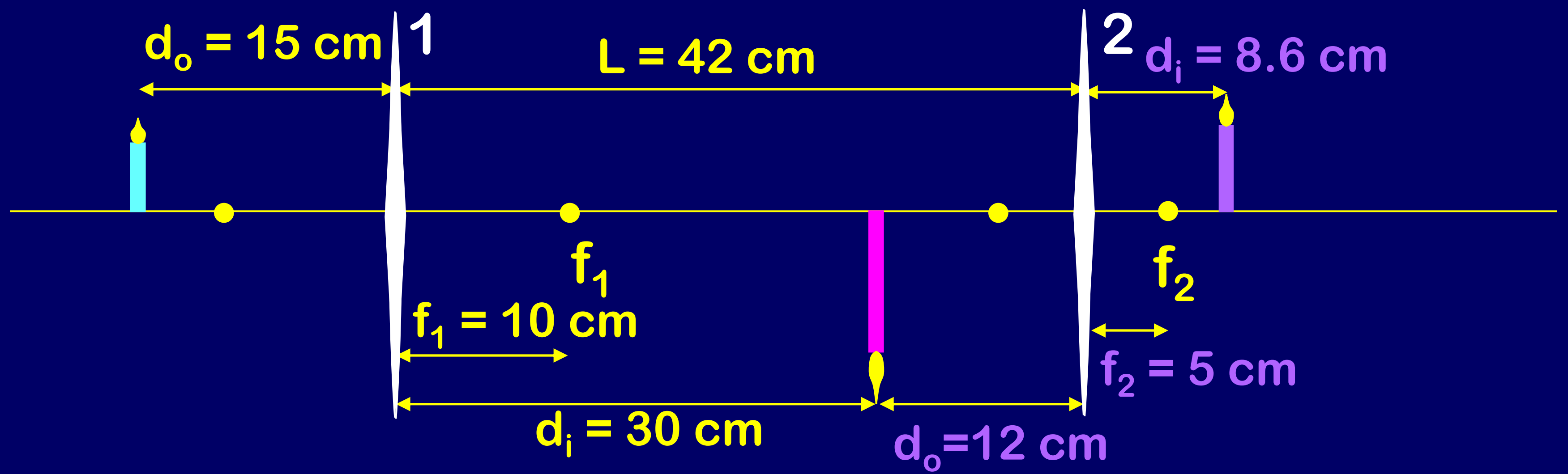
First find image from lens 1.

$$\frac{1}{15\,\text{cm}} + \frac{1}{d_i} = \frac{1}{10\,\text{cm}} \Rightarrow d_i = 30\,\text{cm}$$

Example



Multiple Lenses: Image 2

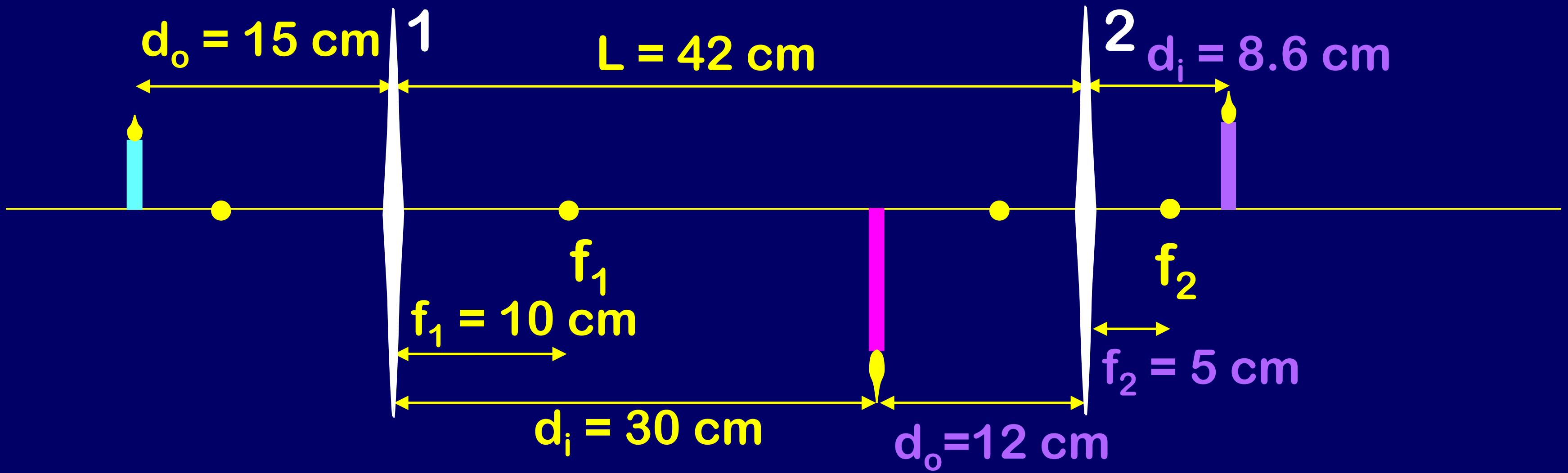


Now find image from lens 2.

$$\frac{1}{12\,\text{cm}} + \frac{1}{d_i} = \frac{1}{5\,\text{cm}} \quad \Rightarrow \quad d_i = 8.6\,\text{cm}$$

Notice that d_o could be negative for second lens!

Multiple Lenses: Magnification



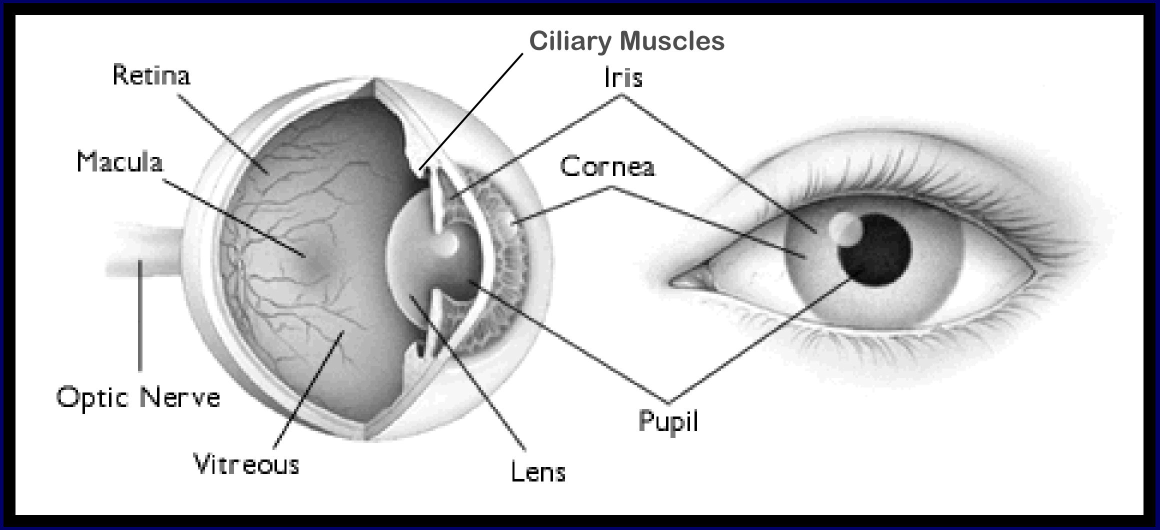
Net magnification: $\boxed{m_{net} = m_1\, m_2}$

$$m_1 = -\frac{30}{15} = -2 \qquad m_2 = -\frac{8.6}{12} = -.72$$

$$m_{net} = m_1 m_2 = +1.43$$

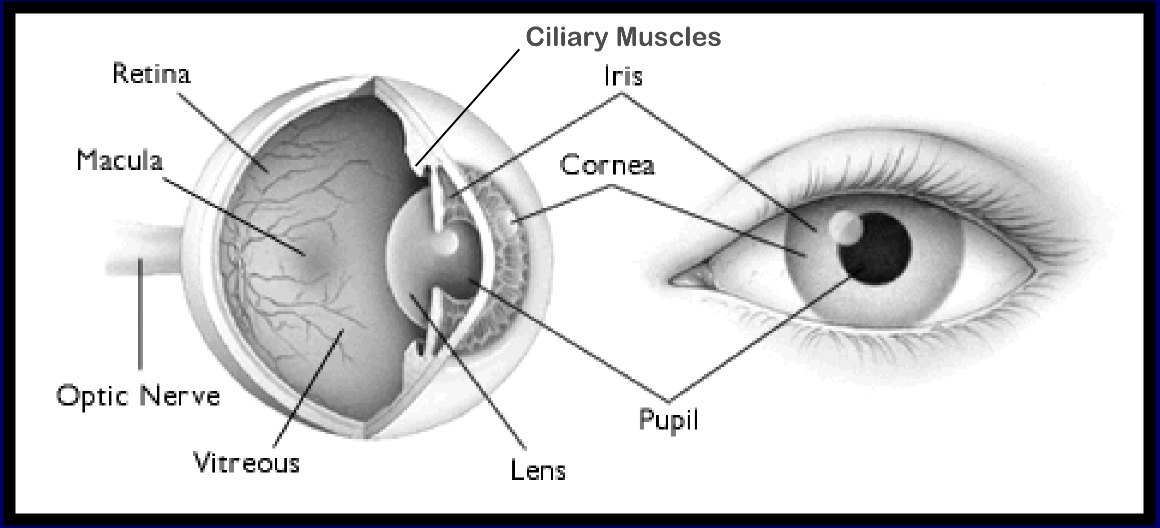
The Eye

- One of first organs to develop.
- ~100 million Receptors
- Sensitive to a few photons!



ACT: Focusing and the Eye

Cornea	n= 1.38
Lens	n = 1.4
Vitreous	n = 1.33

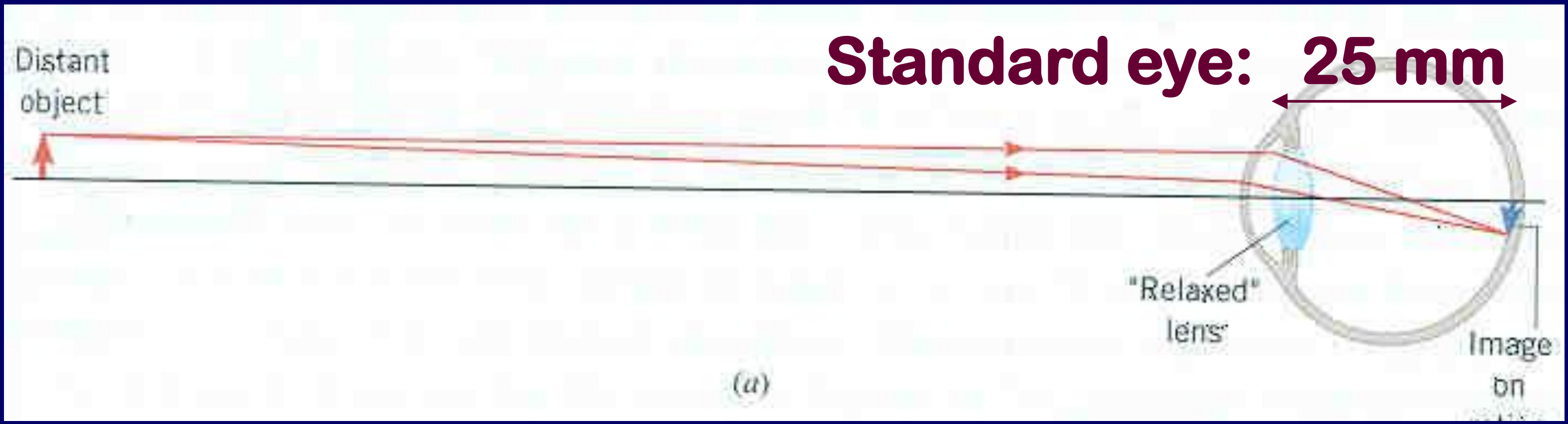


Which part of the eye does most of the light bending?

A) Lens B) Cornea C) Retina D) Cones

Eye (Relaxed)

Example



Determine the focal length of your eye when looking at an object far away.

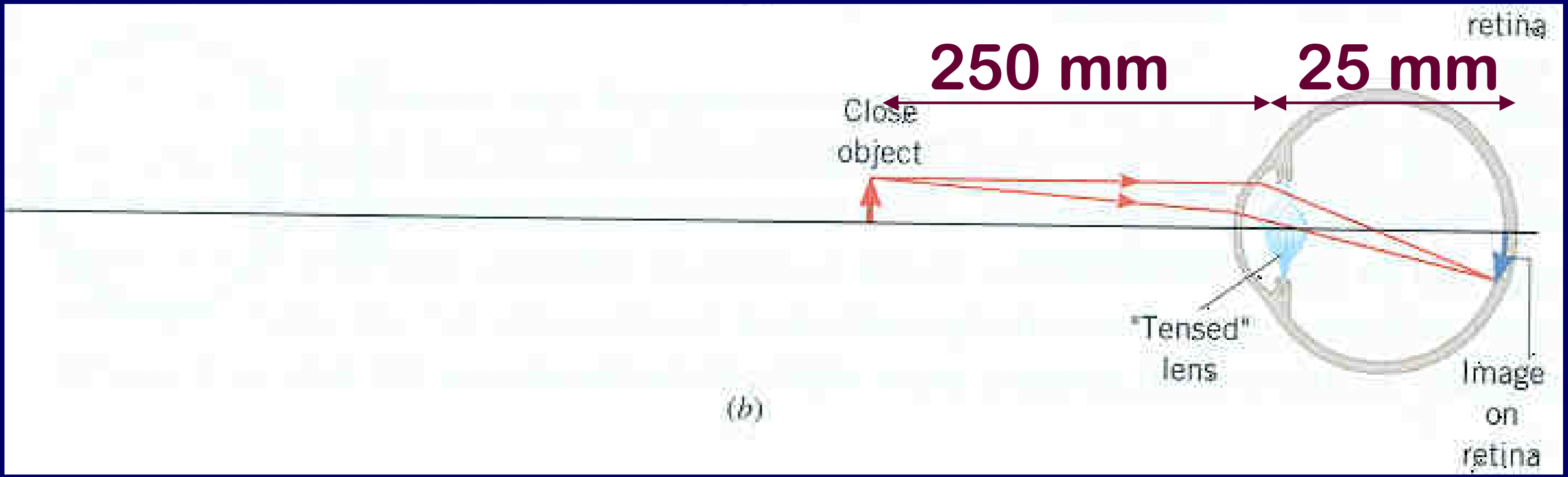
Object is far away: $d_o = \infty$

$$\frac{1}{\infty} + \frac{1}{25\ \boldsymbol{mm}} = \frac{1}{f}$$

Want image at retina: $d_i = 25mm$

$$f_{relaxed} = 25\ \boldsymbol{mm}$$

Eye (Tensed)



Determine the focal length of your eye when looking at an object up close (25 cm).

$$\frac{1}{250\ mm} + \frac{1}{25\ mm} = \frac{1}{f}$$

Object is up close:

$$d_o = 25cm = 250mm$$

$$f_{tense} = 22.7\ mm$$

Want image at retina: $d_i = 25mm$

$$f_{relaxed} = 25\ mm$$

Near Point, Far Point

- Eye's lens changes shape (changes *f*)
  - Object at any d_o should have image be at retina (d_i = approx. 25 mm)
- Can only change shape so much
- "Near Point"
  - Closest d_o where image can be at retina
  - Normally, ~25 cm (if far-sighted then further)
- "Far Point"
  - Furthest d_o where image can be at retina
  - Normally, infinity (if near-sighted then closer)

ACT/Checkpoint 1

A person with almost normal vision (near point at 26 cm) is standing in front of a plane mirror.

What is the closest distance to the mirror where the person can stand and still see himself in focus?

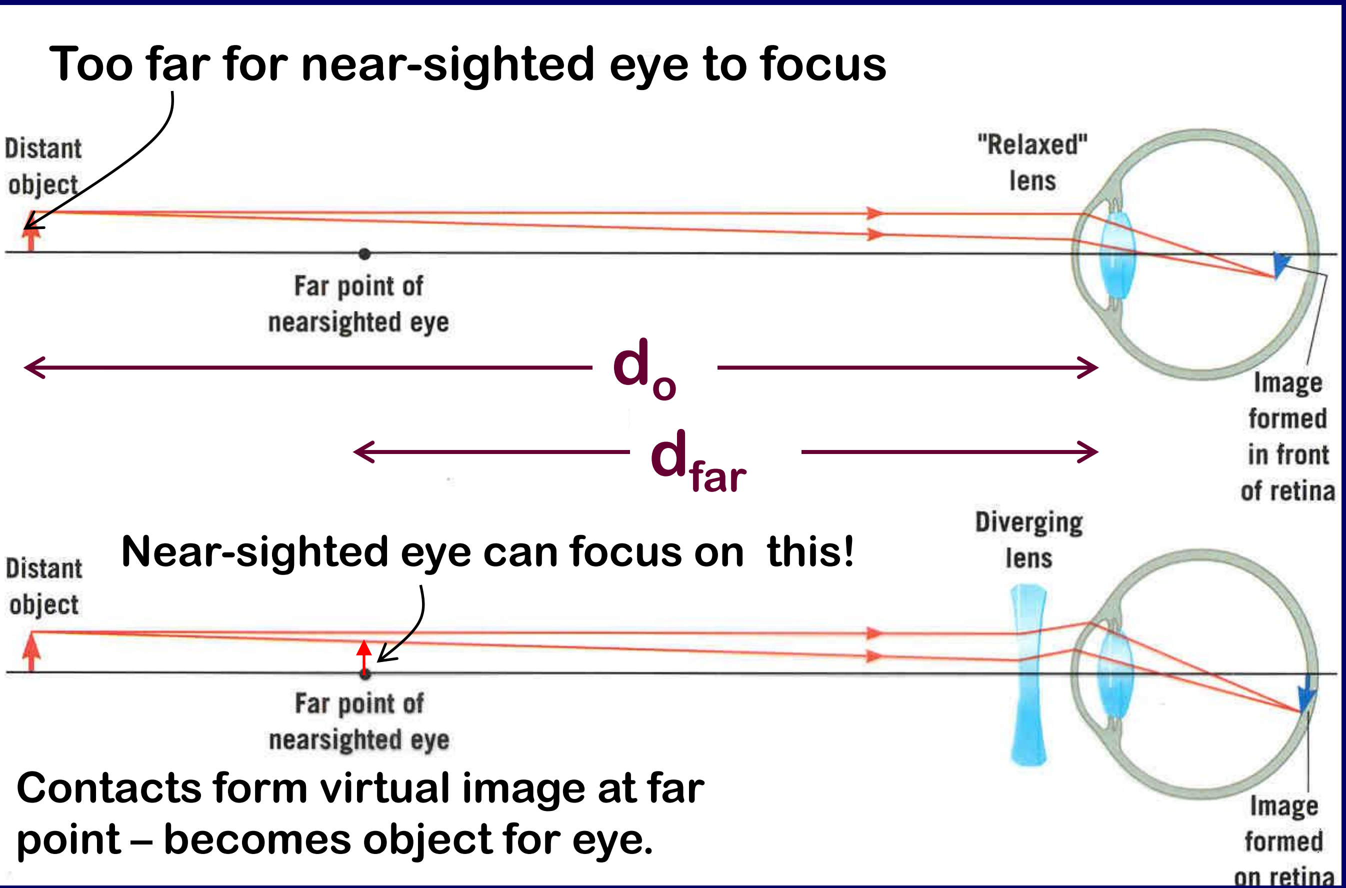
A) 13 cm

B) 26 cm

C) 52 cm

If you are nearsighted...

(far point is too close)



Example

$$\frac{1}{d_o} + \frac{1}{-d_{far}} = \frac{1}{f_{lens}}$$

$$\frac{1}{\infty} + \frac{1}{-d_{far}} = \frac{1}{f_{lens}}$$

$$\boxed{f_{lens} = -d_{far}}$$

Want to have (virtual) image of distant object, $d_o = \infty$, at the far point, $d_i = -d_{far}$.

Refractive Power of Lens

Diopter = 1/f

where f is focal length of lens in meters.

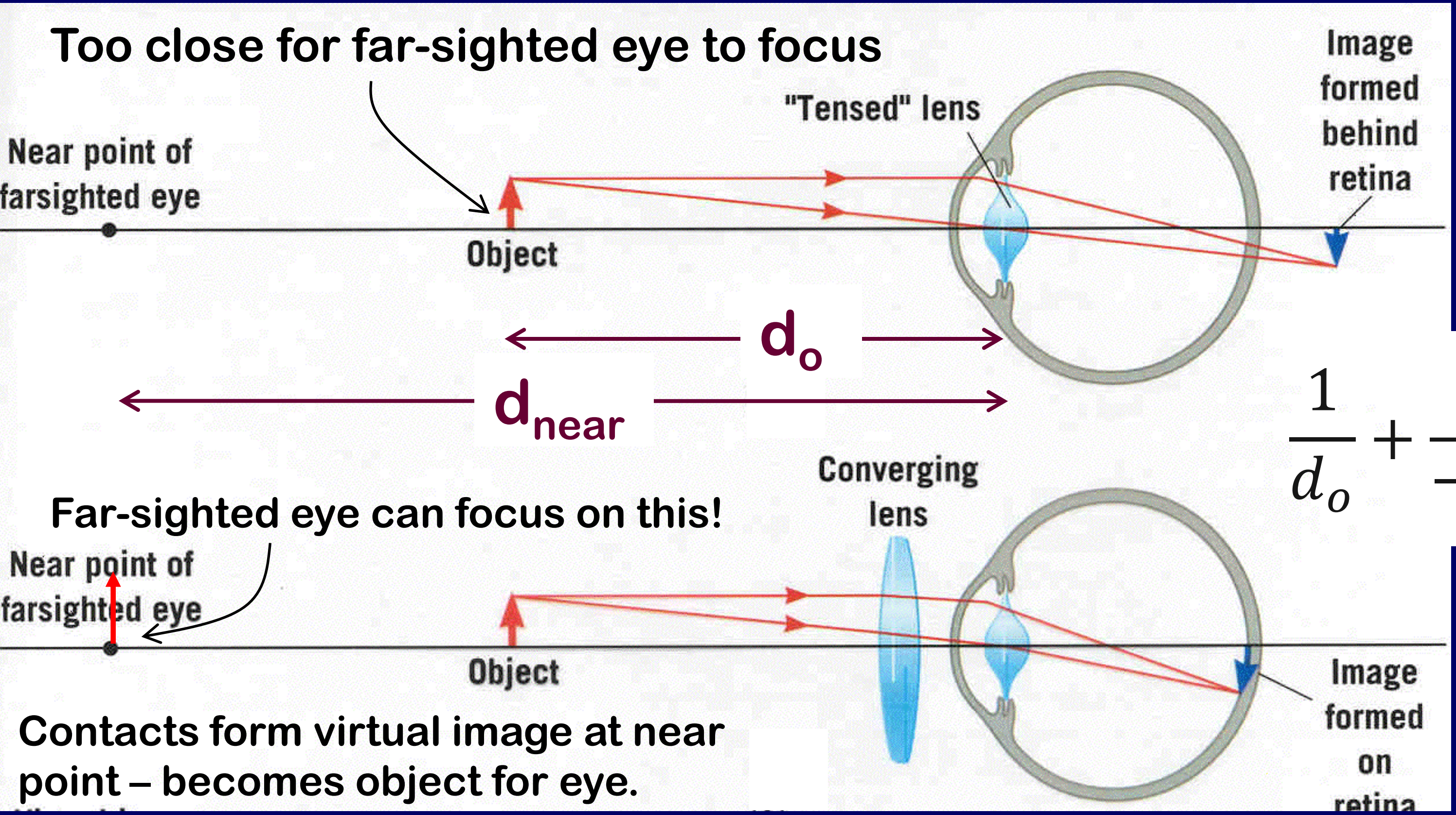
Example:

- Prescription reads -6.5 diopters
- f_{lens} = -1/6.5 = -0.154 m = -15.4 cm (a diverging lens)
- d_{far} = 15.4 cm (!)

$$f_{lens} = -\,d_{far}$$

If you are farsighted...

(near point is too far)



$$\frac{1}{d_o} + \frac{1}{-d_{near}} = \frac{1}{f_{lens}}$$

Example

When object is at d_o, lens must create an (virtual) image at $-d_{near}$

$$\frac{1}{25\ cm} + \frac{1}{-50\ cm} = \frac{1}{f}$$

$$f = 50\ cm$$

Example

Farsightedness

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_{lens}}$$

- Near point d_{near} > 25 cm
- To correct, produce virtual image of object at d_0 = 25 cm to the near point ($d_i = d_{near}$)

$$\frac{1}{d_o} + \frac{1}{-d_{near}} = \frac{1}{f_{lens}} \Rightarrow \frac{1}{25} + \frac{1}{-d_{near}} = \frac{1}{f_{lens}}$$

Example:

- **Near prescription reads +2.5 diopters**
- **f_{lens} = +1/2.5 = 0.4 m = 40 cm**
- **therefore d_{near} = 67 cm**

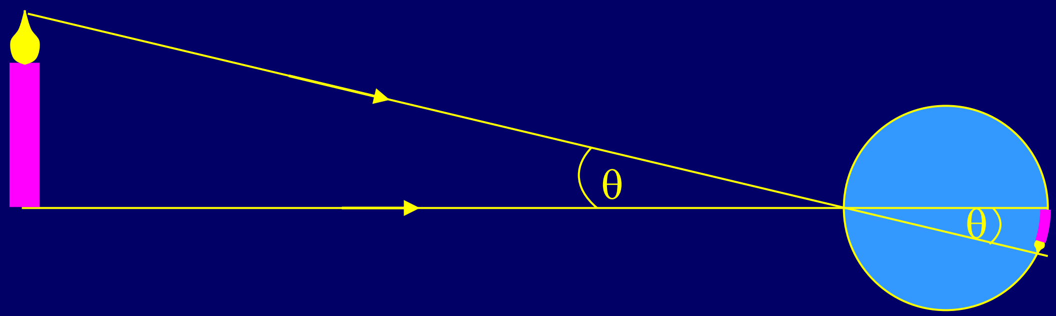
ACT/Checkpoint 2

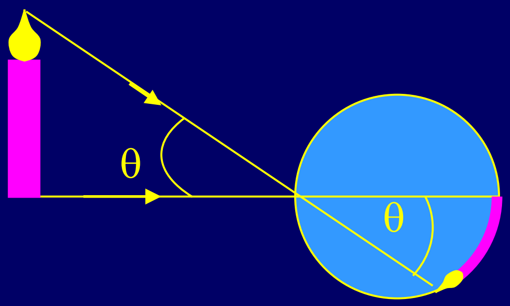
Two people who wear glasses are camping. One of them is nearsighted and the other is farsighted. Which person’s glasses will be useful in starting a fire with the sun’s rays?

A. nearsighted
B. farsighted

Angular Size
Checkpoint 3.1,3.2

Both are same size, but nearer one looks bigger.

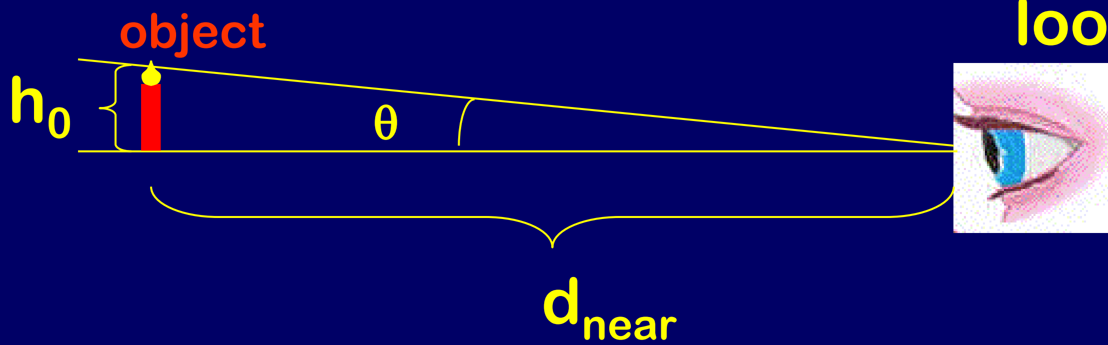




- Angular size tells you how large the image is on your retina, and how big it appears to be.

Angular size: Unaided Eye

How big the object looks with unaided eye.

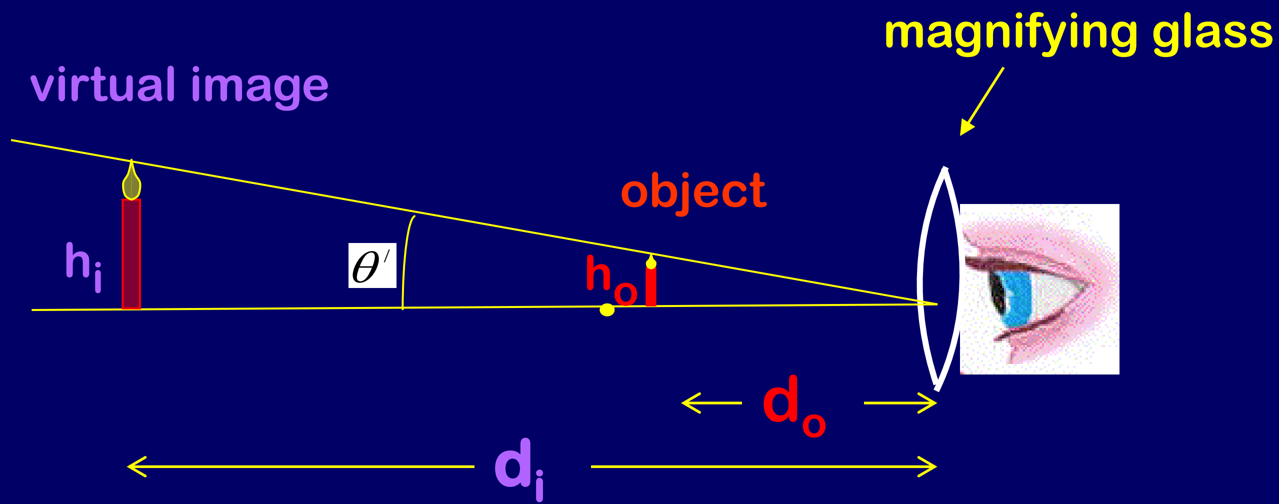


Bring object as close as possible (to near point d_{near})

$$\tan(\theta) = \frac{h_o}{d_{near}} \quad \Rightarrow \quad \theta \approx \frac{h_o}{d_{near}}$$

If θ is small and expressed in radians.

Magnifying Glass



Magnifying glass produces virtual image behind object, allowing you to bring object to a closer d_o, and larger θ'

Typically, $d_o = f$ **and** $d_i = \infty$

$\theta' = h_o/f$ **Compare to unaided eye:** $\theta' = h_o/d_{near}$

Ratio of the two angles is the angular magnification M:

$$M = \theta'/\theta = d_{near}/f$$

Summary

- Lenses
  - Lens equation & magnification
  - Multiple lenses
- The eye
  - Near & far point
  - Nearsightedness & farsightedness & corrective lenses
  - Angular magnification