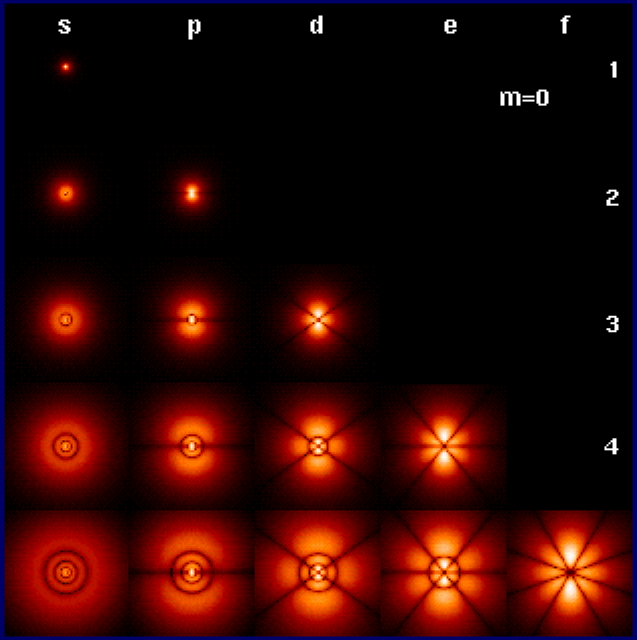
Physics 102: Lecture 25

Atomic Spectroscopy & Quantum Atoms



From last lecture – Bohr model

Angular momentum is quantized

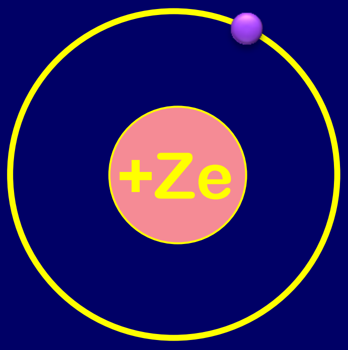
$L_n = nh/2\pi \qquad n = 1, 2, 3 \ldots$

Energy is quantized

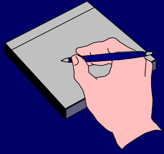
$$E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{Z^2}{n^2} \approx -\frac{13.6 \cdot Z^2}{n^2}\ \boldsymbol{eV}\,(\boldsymbol{where}\ \hbar \equiv h/2\pi)$$

Radius is quantized

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = (0.0529\ \boldsymbol{nm})\frac{n^2}{Z}$$



Velocity too!



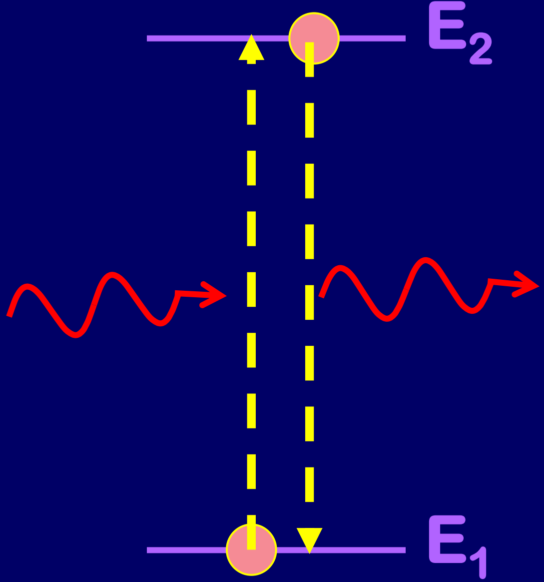
Transitions + Energy Conservation

- Each orbit has a specific energy:

$$\boxed{E_n = -13.6\, Z^2/n^2}$$

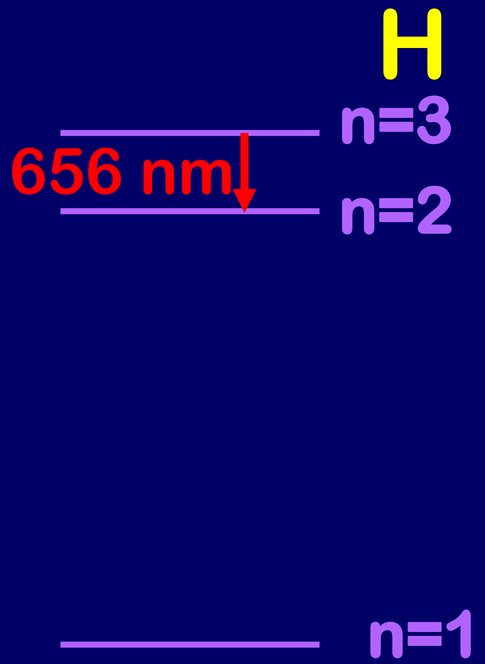
- Photon emitted when electron jumps from high energy to low energy orbit. Photon absorbed when electron jumps from low energy to high energy:

$$\boxed{E_2 - E_1 = h\,f = h\,c\,/\,\lambda}$$



Demo: Line Spectra

In addition to the continuous blackbody spectrum, elements emit a discrete set of wavelengths which show up as lines in a diffraction grating.



This is how neon signs & Na lamps work!

Spectra give us information on atomic structure

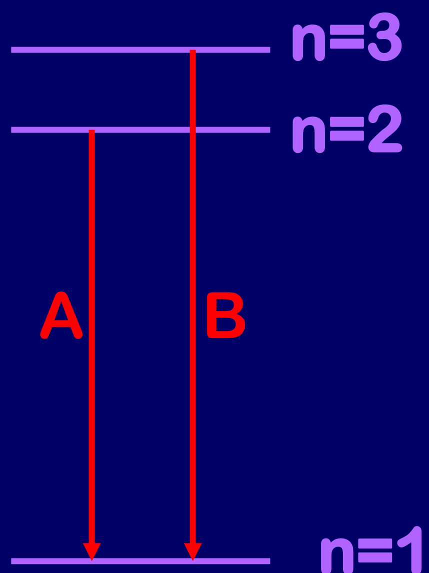
Checkpoint 1.1

Electron A falls from energy level n=2 to energy level n=1 (ground state), causing a photon to be emitted.

Electron B falls from energy level n=3 to energy level n=1 (ground state), causing a photon to be emitted.

Which photon has more energy?

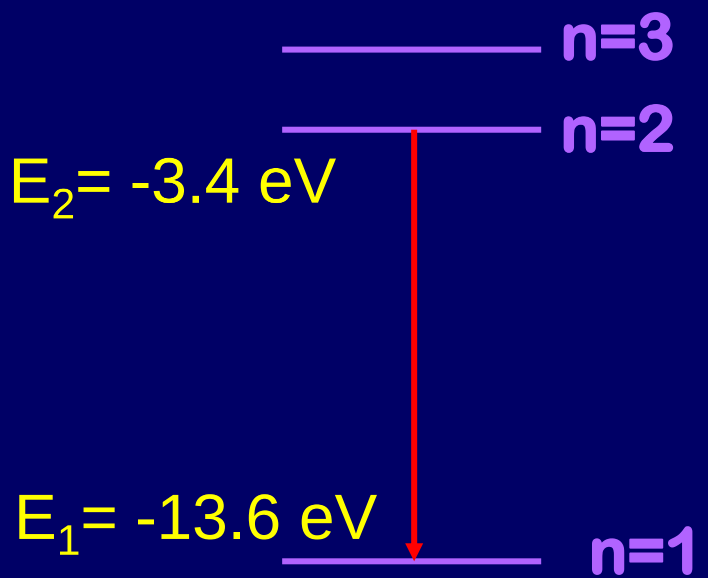
1) **Photon A**
2) **Photon B**



Example

Spectral Line Wavelengths

Calculate the wavelength of photon emitted when an electron in the hydrogen atom drops from the n=2 state to the ground state (n=1).



$$E_n = -13.6eV\frac{Z^2}{n^2}$$

$$hf = E_2 - E_1$$
$$= -3.4eV - (-13.6eV) = 10.2eV$$

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{10.2eV} = \frac{1240}{10.2} \approx 124nm$$

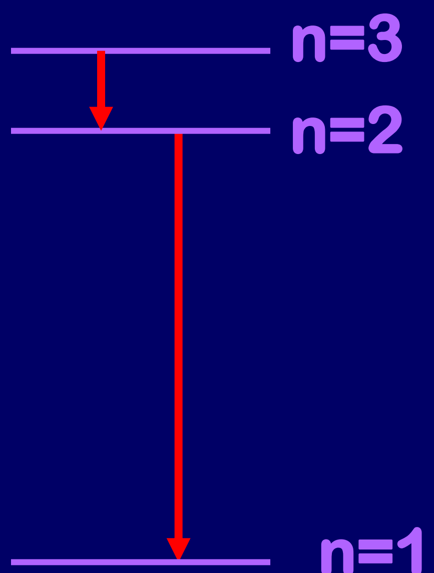
ACT: Spectral Line Wavelengths

Compare the wavelength of a photon produced from a transition from n=3 to n=2 with that of a photon produced from a transition n=2 to n=1.

(A) $\lambda_{32} < \lambda_{21}$

(B) $\lambda_{32} = \lambda_{21}$

(C) $\lambda_{32} > \lambda_{21}$



ACT/Checkpoint 1.2

The electrons in a large group of hydrogen atoms are excited to the n=3 level. How many spectral lines will be produced?

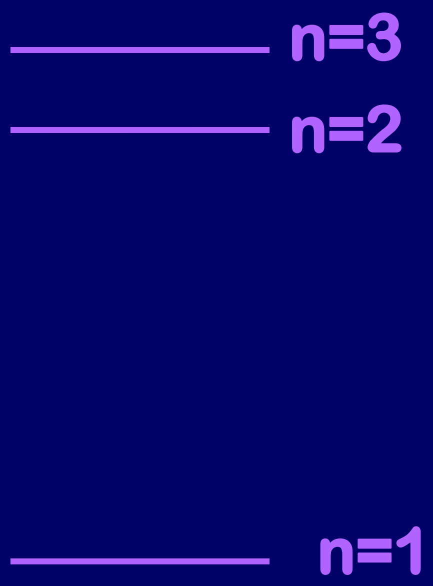
A. 1

B. 2

C. 3

D. 4

E. 5



The Bohr Model is incorrect!

To be consistent with the Heisenberg Uncertainty Principle, which of these properties cannot be quantized (have the exact value known)?

Electron Radius ← **Would know location**

Electron Energy

Electron Velocity ← **Would know momentum**

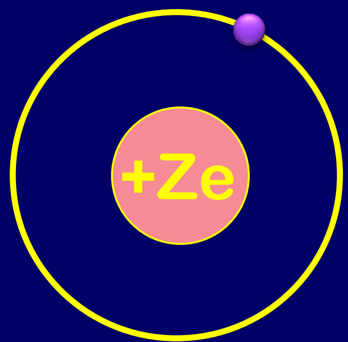
Electron Angular Momentum

But, in the Bohr model:

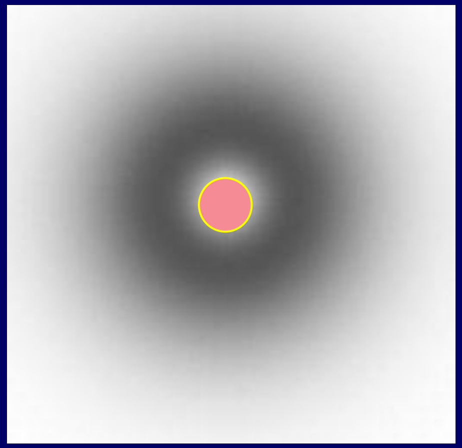
$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = (0.0529\ \boldsymbol{nm})\frac{n^2}{Z}$$

Quantized radii and velocities for electron orbitals

Checkpoint 2



Bohr Model



Quantum Atom

So what keeps the electron from “sticking” to the nucleus?

Centripetal Acceleration

Pauli Exclusion Principle

Heisenberg Uncertainty Principle

Quantum Mechanics

Theory used to predict probability distributio

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\lambda = h/p$$

$$\Delta x \Delta p \geq h/2$$

Quantum Mechanical Atom

- Predicts available energy states agreeing with Bohr.
- Don’t have definite electron position, only a probability function.
- Each orbital can have 0 angular momentum!
- Each electron state labeled by 4 numbers:

  n = principal quantum number (1, 2, 3, …)

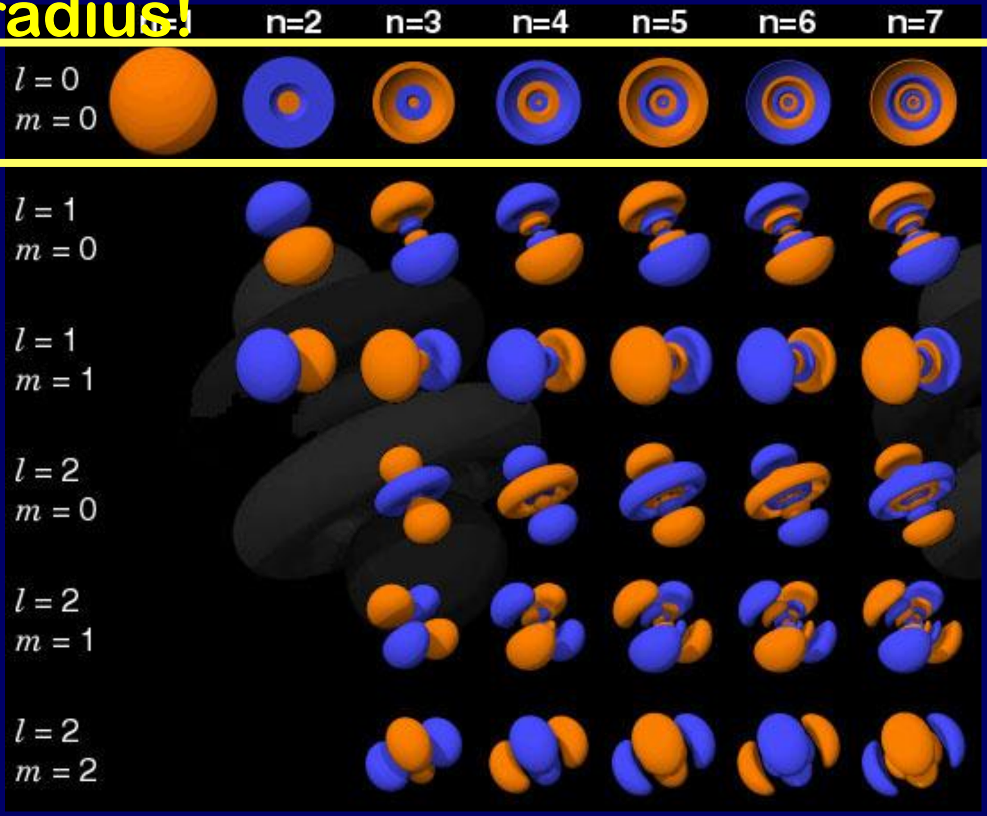
  ℓ = angular momentum (0, 1, 2, … n-1)

  m_ℓ = component of l ($-\ell < m_\ell < \ell$)

  m_s = spin (-½ , +½)

Quantum Mechanics (vs. Bohr)

Electrons are described by a probability function, not a definite radius!



It takes 4 numbers to describe the electron

Bohr: just n

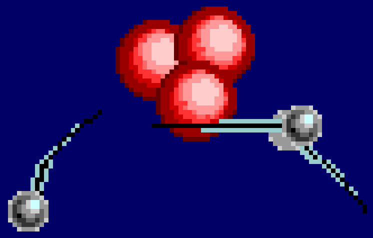
Each orbital n can have 0 angular momentum

Bohr: $L_n = n\,\hbar$

$n = 1,2,3 \ldots$

Quantum Numbers

Each electron in an atom is labeled by 4 #'s



n = Principal Quantum Number (1, 2, 3, …)

- Determines the Bohr energy

ℓ = Orbital Quantum Number (0, 1, 2, … n-1)

- Determines angular momentum
- $\ell < n$ always true!

$$L = \sqrt{\ell(\ell+1)}\,\frac{h}{2\pi}$$

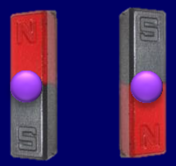
m_ℓ = Magnetic Quantum Number ($-\ell$, … 0, … ℓ)

- Component of ℓ
- $|m_\ell| \le \ell$ always **true!**

$$L_z = m_\ell \frac{h}{2\pi}$$

m_s = Spin Quantum Number (-½ , +½)

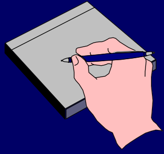
- "Up Spin" or "Down Spin"



ACT: Quantum numbers

For which state of hydrogen is the orbital angular momentum required to be zero?

1. n=1
2. n=2
3. n=3



Spectroscopic Nomenclature

“Shells”

n=1 is “K shell”

n=2 is “L shell”

n=3 is “M shell”

n=4 is “N shell”

n=5 is “O shell”

“Subshells”

ℓ=0 is “s state”

ℓ=1 is “p state”

ℓ=2 is “d state”

ℓ=3 is “f state”

ℓ=4 is “g state”

Example

1 electron in ground state of Hydrogen:

n=1, ℓ=0 is denoted as: $1s^1$

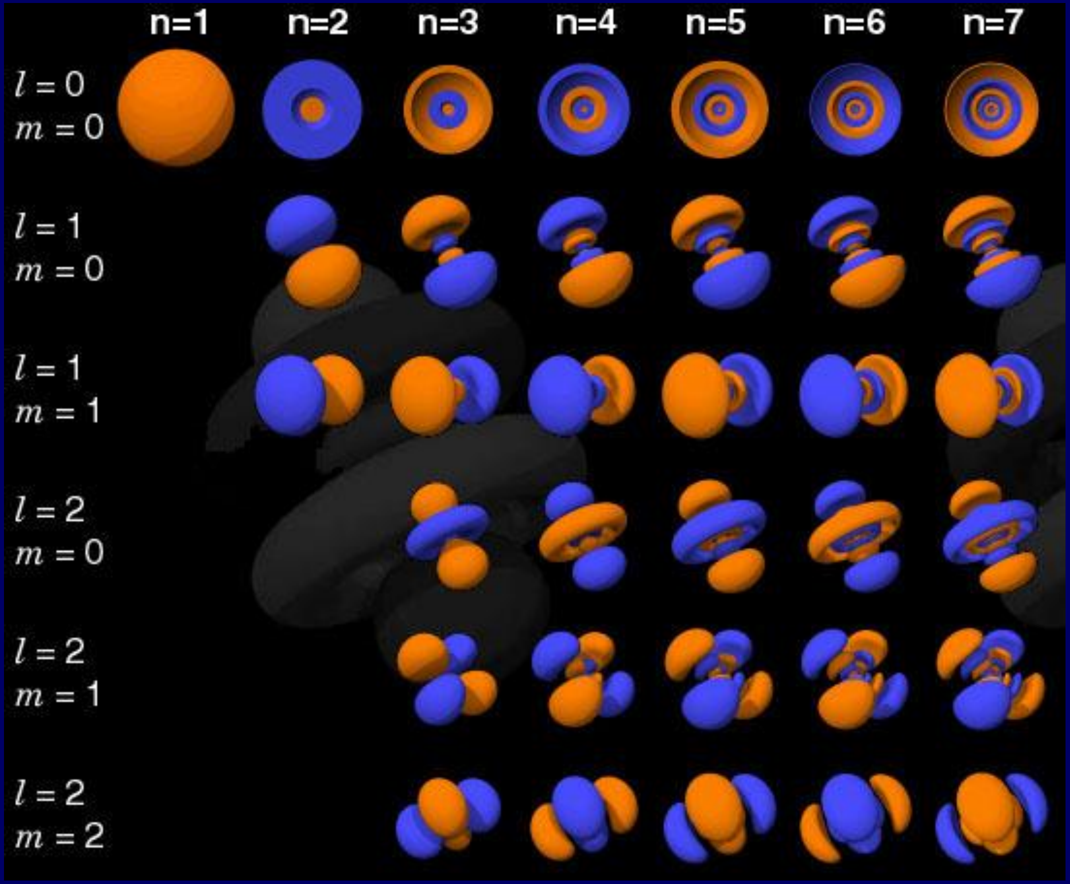
n=1

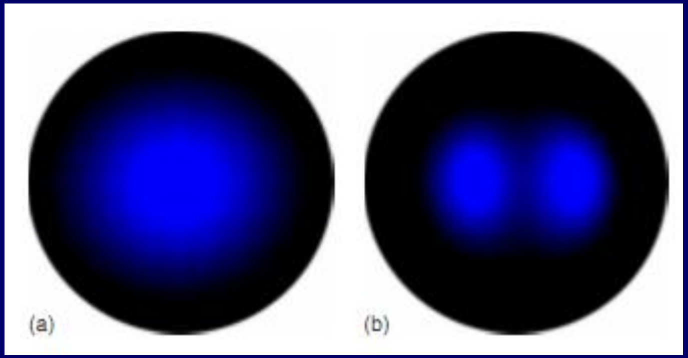
ℓ=0

1 electron

Electron orbitals

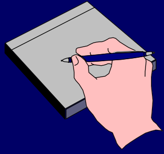
In correct quantum mechanical description of atoms, positions of electrons not quantized, orbitals represent probabilities





Carbon orbitals imaged in 2009 using electron microscopy!

Example



Quantum Numbers

How many unique electron states exist with n=2?

$\ell = 0$: $2s^2$

$m_\ell = 0$: $m_s = ½ , -½$ 2 states

$\ell = 1$: $2p^6$

$m_\ell = +1$: $m_s = ½ , -½$ 2 states

$m_\ell = 0$: $m_s = ½ , -½$ 2 states

$m_\ell = -1$: $m_s = ½ , -½$ 2 states

There are a total of 8 states with n=2

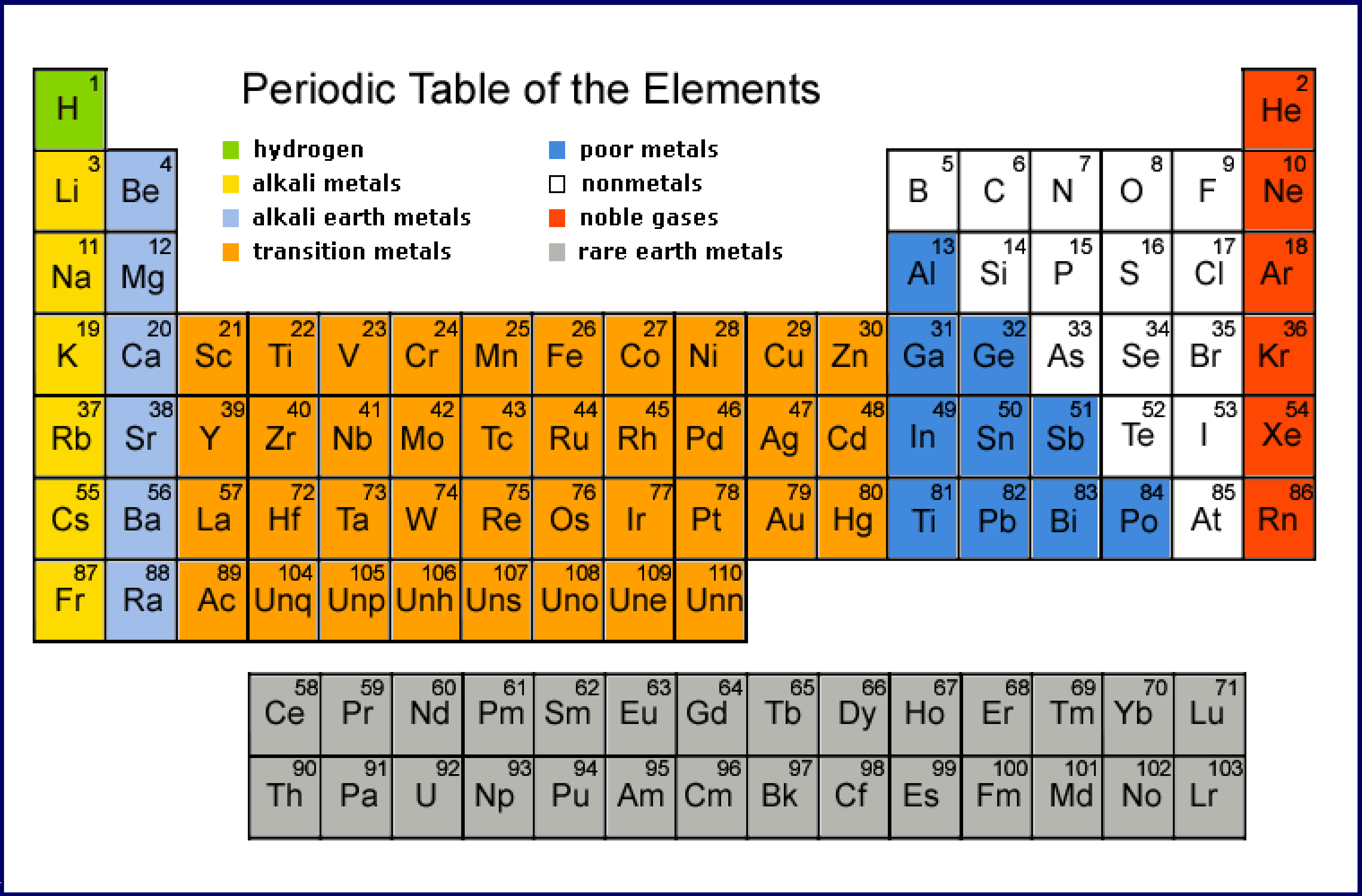
ACT: Quantum Numbers

How many unique electron states exist with n=5 and $m_\ell = +3$?

A) 0　　B) 4　　C) 8　　D) 16　　E) 50

Pauli Exclusion Principle

In an atom with many electrons only one electron is allowed in each quantum state (n, ℓ, m_ℓ, m_s).



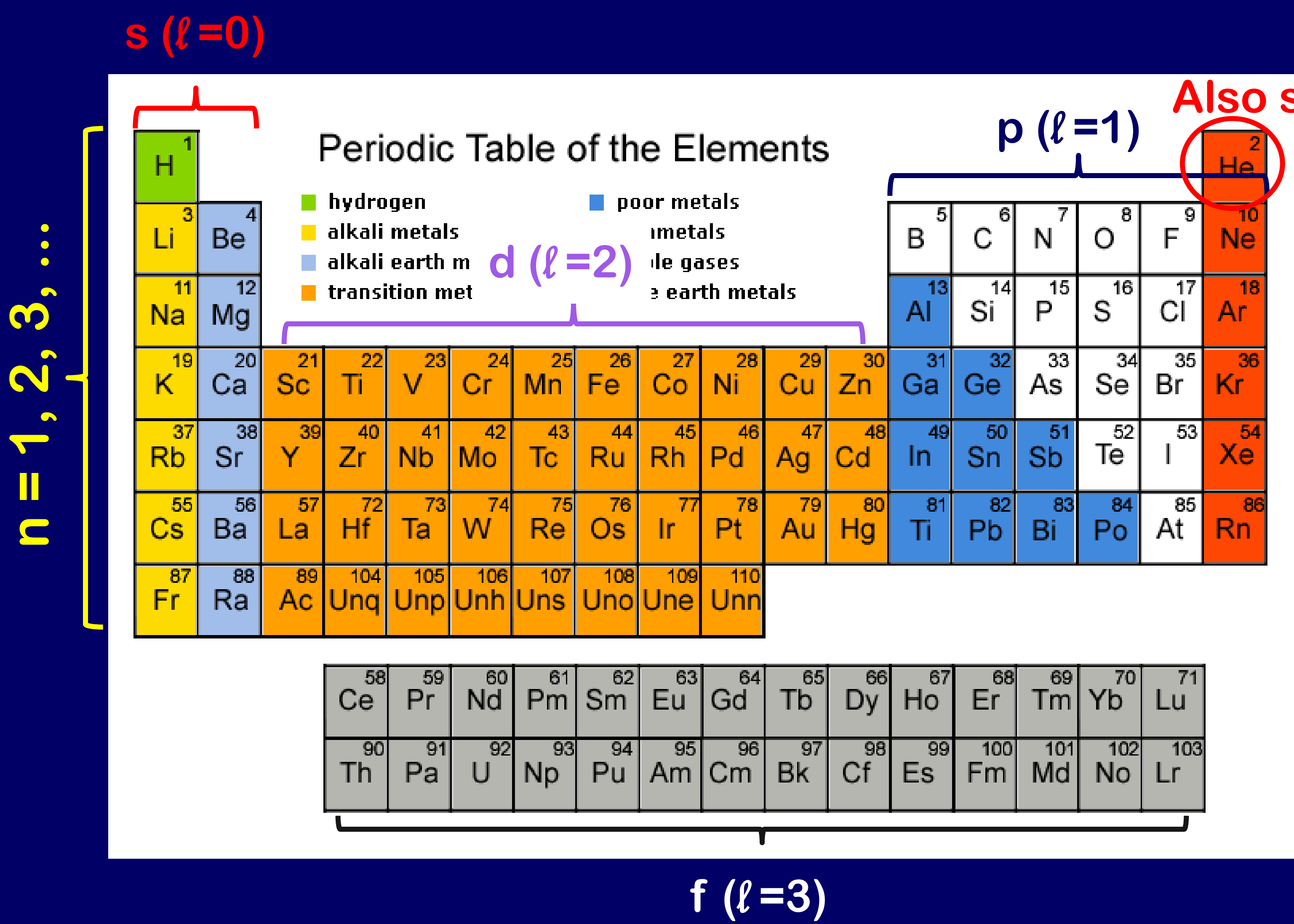
Electron Configurations

# electrons	Atom	Configuration		
1	H	$1s^1$		
2	He	$1s^2$	1s shell filled	(n=1 shell filled - noble gas)
3	Li	$1s^2 2s^1$		
4	Be	$1s^2 2s^2$	2s shell filled	
5	B	$1s^2 2s^2 2p^1$		
etc				
10	Ne	$1s^2 2s^2 2p^6$	2p shell filled	(n=2 shell filled - noble gas)

s shells hold up to 2 electrons

p shells hold up to 6 electrons

The Periodic Table



What determines the sequence? Pauli exclusion & energies

Summary

- Each electron state labeled by 4 numbers:

  n = principal quantum number (1, 2, 3, …)

  ℓ = angular momentum (0, 1, 2, … n-1)

  m_ℓ = component of l ($-\ell < m_\ell < \ell$)

  m_s = spin ($-½$, $+½$)

- Pauli Exclusion Principle explains periodic table
- Shells fill in order of lowest energy.