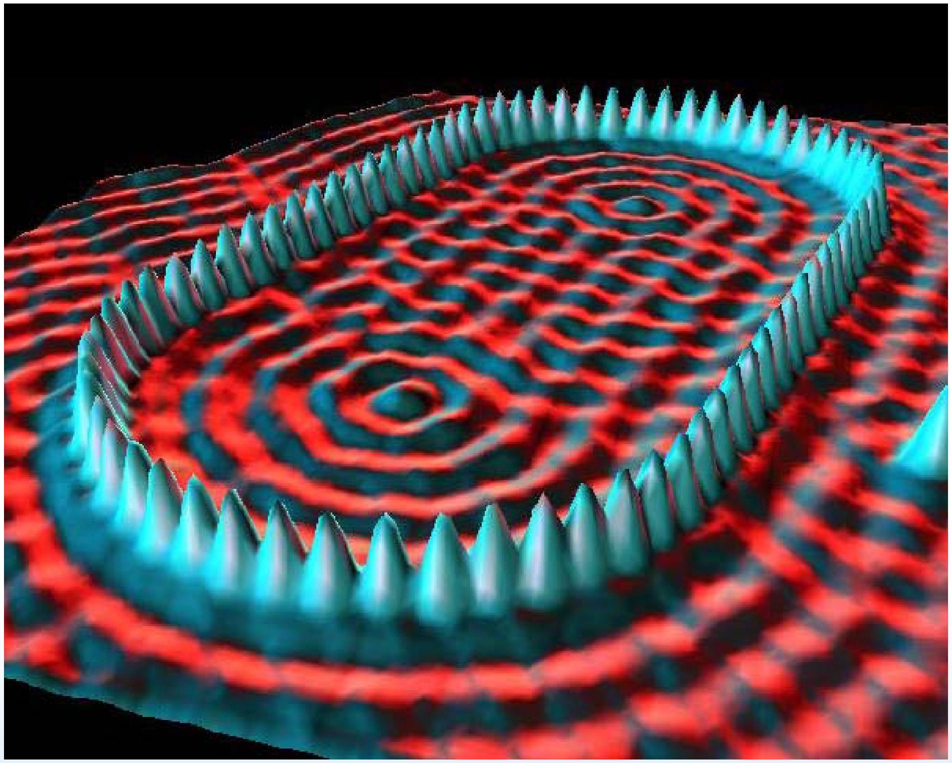


Phys 102 – Lecture 24

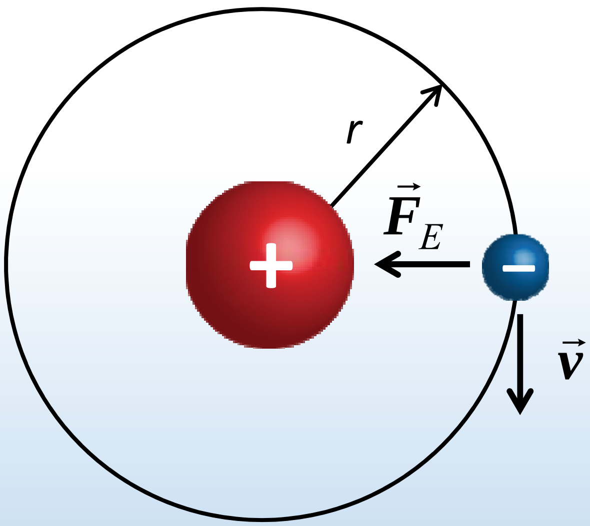
The classical and Bohr atom

State of late 19th Century Physics

- Two great theories *"Classical physics"*
  - Newton's laws of mechanics & gravity
  - Maxwell's theory of electricity & magnetism, including EM waves
- But... some unsettling problems
  - Stability of atom & atomic spectra
  - Photoelectric effect
  - ...and others
- New theory required *Quantum mechanics*

The "classical" atom

Negatively charged electron orbits around positively charged nucleus



Hydrogen atom

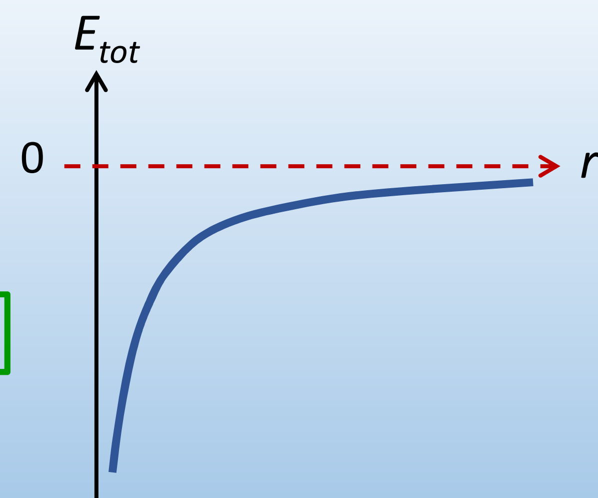
Orbiting e⁻ has centripetal acceleration:

$$F_E = k\frac{e^2}{r^2} = \frac{mv^2}{r} \qquad \text{so,} \quad \frac{ke^2}{r} = mv^2$$

Total energy of electron:

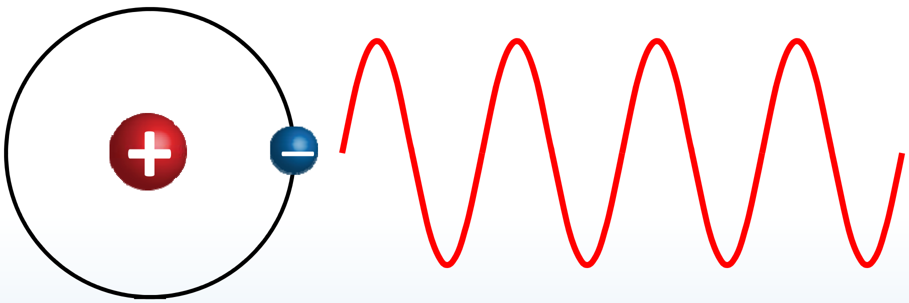
$$E_{tot} = K + U = \frac{1}{2}mv^2 - k\frac{e^2}{r} = -\frac{1}{2}\frac{ke^2}{r}$$

Recall Lect. 4



Stability of classical atom

Prediction – orbiting e^- is an oscillating charge & should emit EM waves in every direction



$$\bar{u}_{tot} = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2$$

EM waves carry energy, so e^- should lose energy & fall into nucleus!

Classical atom is NOT stable!

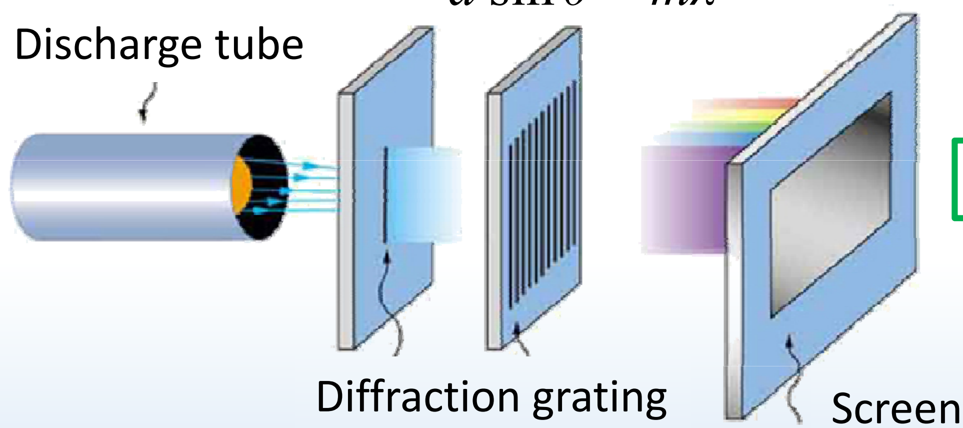
Lifetime of classical atom = 10^{-11} s

Reality – Atoms are stable

Atomic spectrum

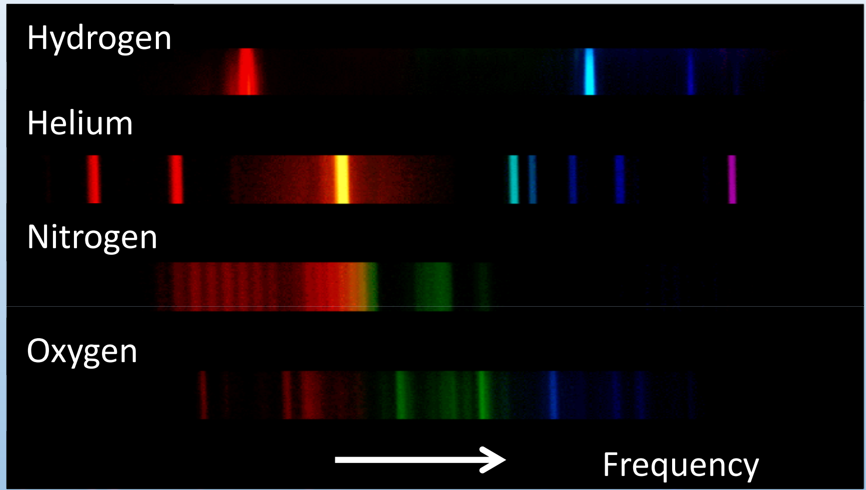
Prediction – e^- should emit light at whatever frequency *f* it orbits nucleus

$$d \sin\theta = m\lambda$$



Recall Lecture 22

Reality – Only <u>*certain*</u> frequencies of light are emitted & are different for different elements



Quantum mechanics

Quantum mechanics explains stability of atom & atomic spectra (and many other phenomena...)

QM is one of most successful and accurate scientific theories

- Predicts measurements to $<10^{-8}$ (ten parts per billion!)

Wave-particle duality – matter behaves as a wave

- Particles can be in many places at the same time
- Processes are probabilistic not deterministic
- Measurement changes behavior

Certain quantities (ex: energy) are *quantized*

Matter waves

Matter behaves as a wave with *de Broglie* wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

Wavelength of matter wave

Planck's constant

$$h = 6.626\times 10^{-34}\, J\cdot s$$

Momentum of particle

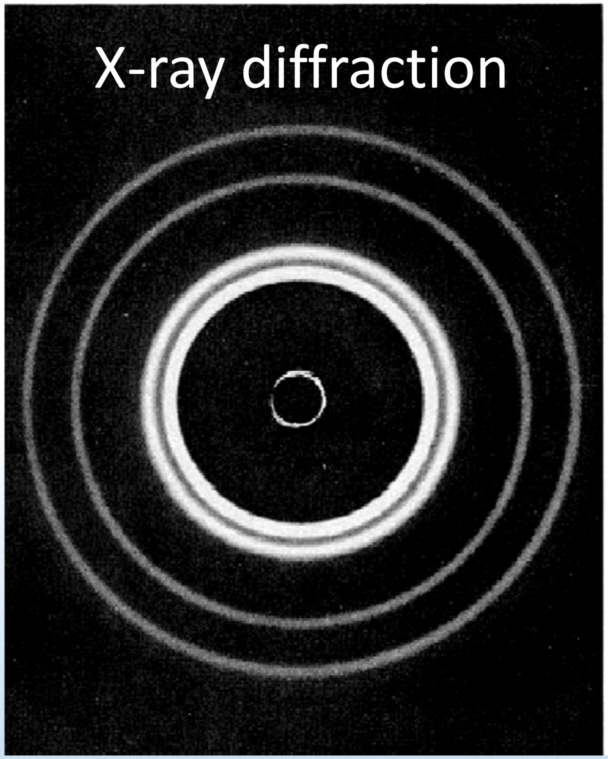
Ex: a fastball (m = 0.5 kg, v = 100 mph ≈ 45 m/s)

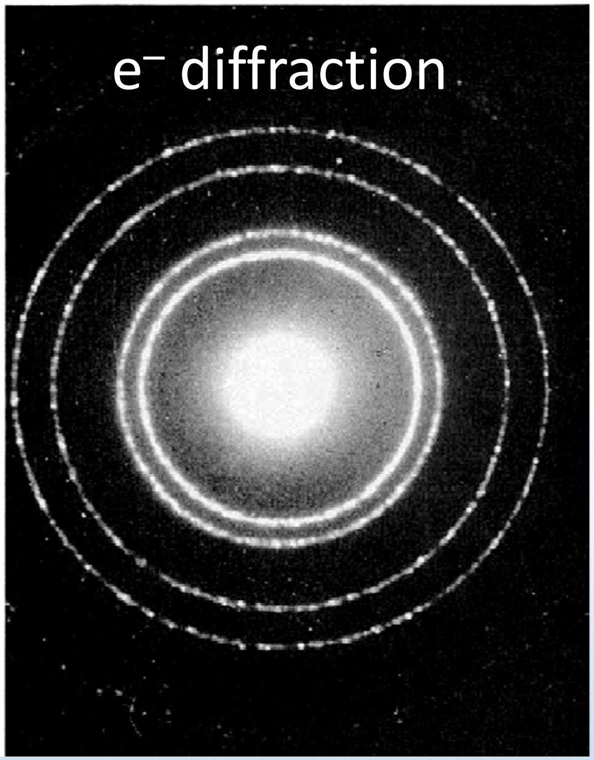
$$\lambda_{fastball} = \frac{h}{p}$$

Ex: an electron ($m = 9.1\times 10^{-31}$ kg, $v = 6\times 10^{5}$ m/s)

$$\lambda_{electron} = \frac{h}{p}$$

X-ray vs. electron diffraction



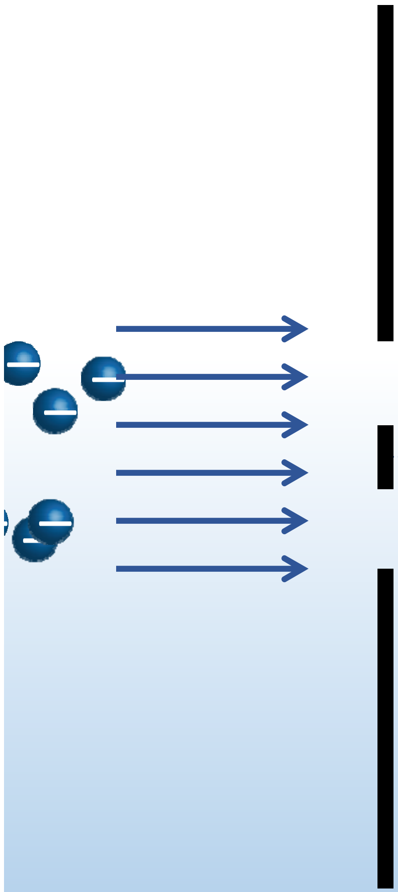


DEMO

Identical pattern emerges if de Broglie wavelength of e⁻ equals the X-ray wavelength!

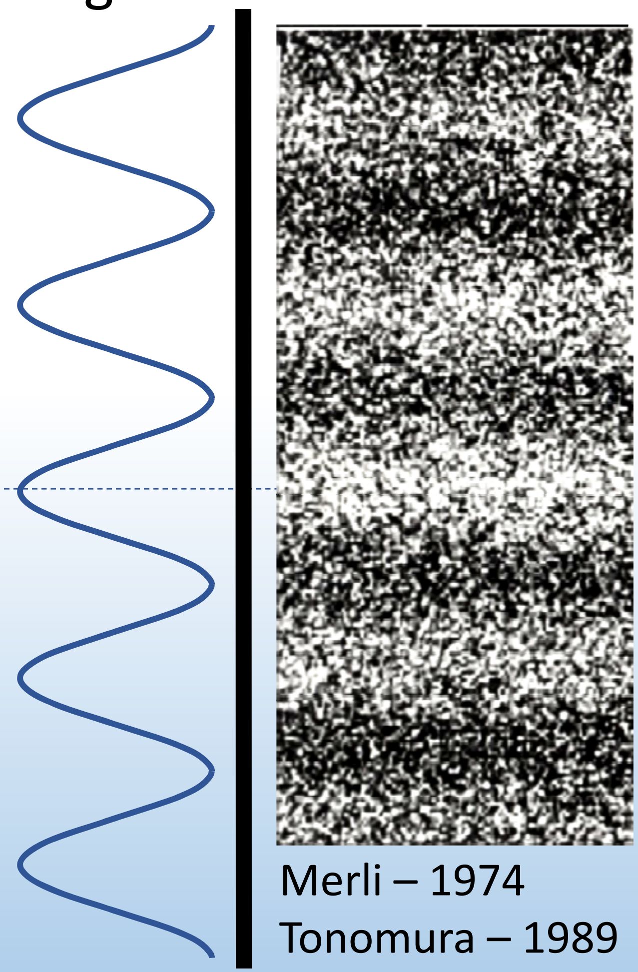
Electron diffraction

Beam of monoenergetic e^- passes through double slit



Interference pattern = probability

$$d \sin\theta = m\lambda \qquad \lambda = \frac{h}{p}$$

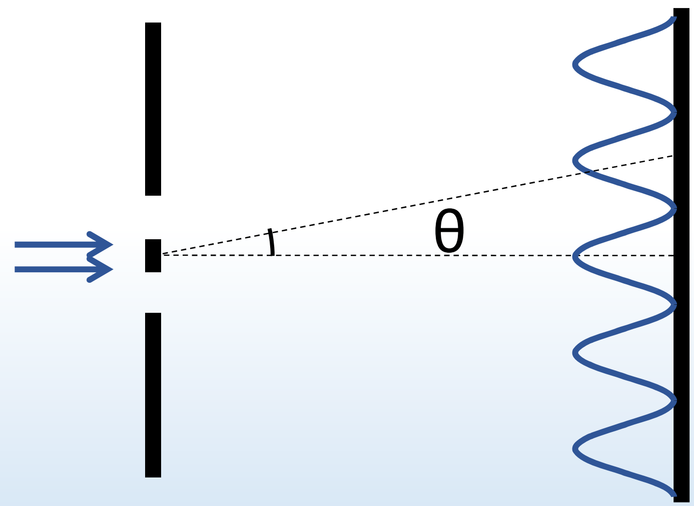


Merli – 1974
Tonomura – 1989

ACT: Double slit interference

Consider the interference pattern from a beam of mono-energetic electrons *A* passing through a double slit.

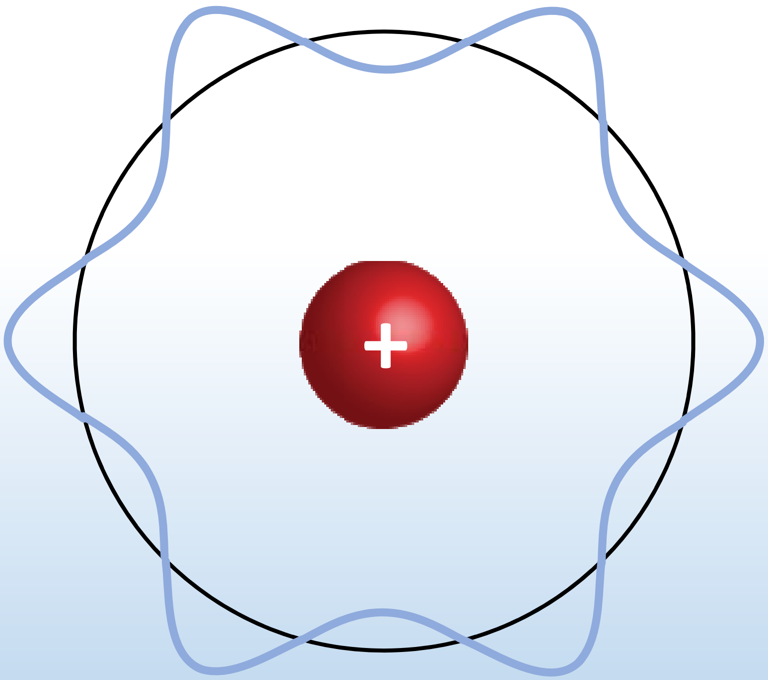
Now a beam of electrons *B* with 4x the energy of *A* enters the slits. What happens to the spacing θ between interference maxima?



A. $\theta_B = 4\ \theta_A$
B. $\theta_B = 2\ \theta_A$
C. $\theta_B = \theta_A$
D. $\theta_B = \theta_A / 2$
E. $\theta_B = \theta_A / 4$

The Bohr model

e^- behave as waves & only orbits that fit an integer number of wavelengths are allowed



Orbit circumference

$$2\pi r = n\lambda = n\frac{h}{p} \qquad n = 1, 2, 3\ldots$$

de Broglie wavelength

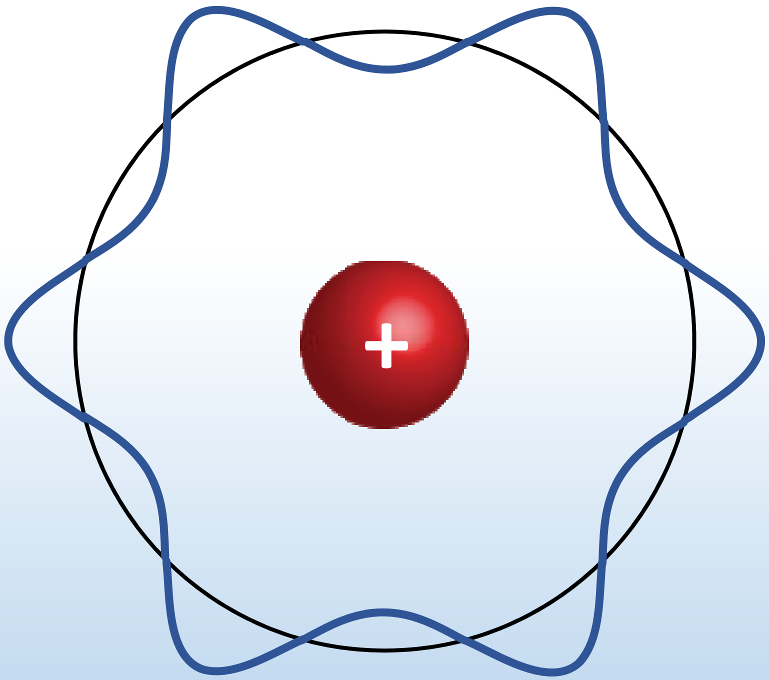
Angular momentum is *quantized*

$$L_n = n\hbar \qquad \hbar \equiv \frac{h}{2\pi}$$ "h bar"

"Quantum number"

ACT: Bohr model

What is the quantum number *n* of this hydrogen atom?



A. $n = 1$
B. $n = 3$
C. $n = 6$
D. $n = 12$

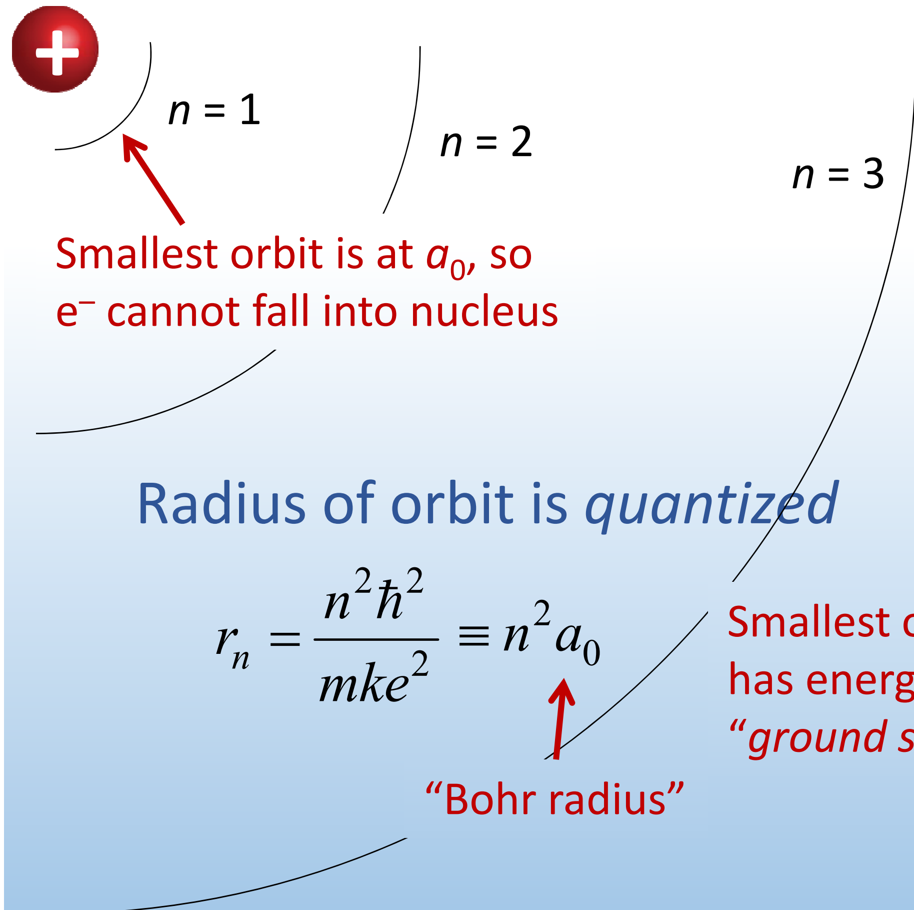
Energy and orbit quantization

Angular momentum is *quantized*

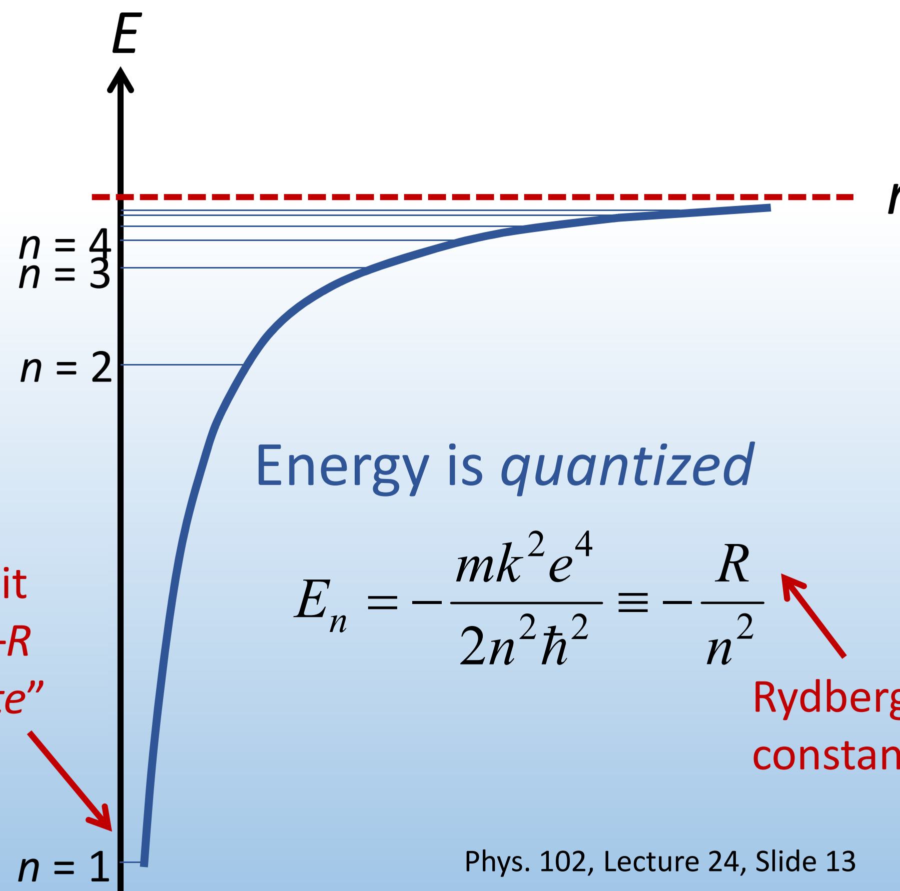
$$L_n \equiv pr = mvr = n\hbar$$

From classical atom:

$$\frac{ke^2}{r} = mv^2$$



Smallest orbit is at a_0, so e^- cannot fall into nucleus



Radius of orbit is *quantized*

$$r_n = \frac{n^2\hbar^2}{mke^2} \equiv n^2 a_0$$

"Bohr radius"

Smallest orbit has energy $-R$ *"ground state"*

Energy is *quantized*

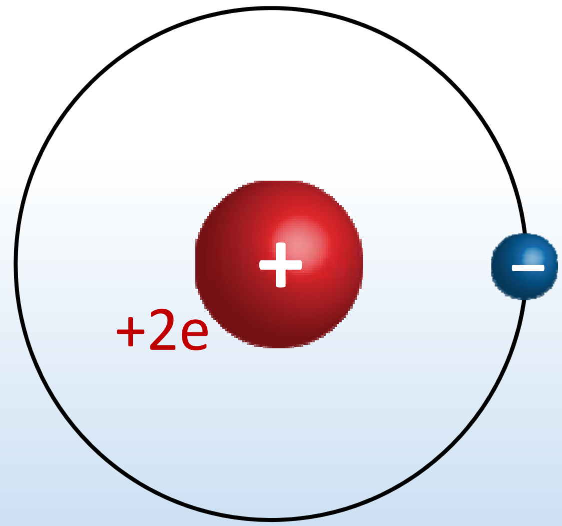
$$E_n = -\frac{mk^2e^4}{2n^2\hbar^2} \equiv -\frac{R}{n^2}$$

Rydberg constant

ACT: CheckPoint 3.2

Suppose the charge of the nucleus is doubled (+2e), but the e^- charge remains the same (–e). How does *r* for the ground state (*n* = 1) orbit compare to that in hydrogen?

For hydrogen: $r_n = \dfrac{n^2\hbar^2}{mke^2}$



A. 1/2 as large

B. 1/4 as large

C. the same

ACT: CheckPoint 3.3

There is a particle in nature called a *muon*, which has the same charge as the electron but is 207 times heavier. A muon can form a hydrogen-like atom by binding to a proton.

How does the radius of the ground state (n = 1) orbit for this hydrogen-like atom compare to that in hydrogen?

A. 207× larger

B. The same

C. 207× smaller

ACT: Bohr model momentum

According to the Bohr model, how is the electron ***linear momentum*** $p = mv$ quantized?

A. p is not quantized
B. p is quantized as $1/n$
C. p is quantized as n

Atomic units

At atomic scales, Joules, meters, kg, etc. are not convenient units

"Electron Volt" – energy gained by charge +1e when accelerated by 1 Volt: $U = qV$

1e = 1.6×10^{-19} C, so 1 eV = 1.6×10^{-19} J

Planck constant: $h = 6.626 \times 10^{-34}$ J·s
Speed of light: $c = 3 \times 10^{8}$ m/s

$$hc \approx 2\times10^{25}\,\mathrm{J\cdot m} = 1240\,\mathrm{eV\cdot nm}$$

Electron mass: $m = 9.1 \times 10^{-31}$ kg

$$mc^2 = 8.2\times10^{-13}\,J = 511{,}000\,\mathrm{eV}$$

Since $U = \dfrac{ke^2}{r}$, ke^2 has units of eV·nm like hc

$$ke^2 \approx 1.44\,\mathrm{eV\cdot nm}$$

$$\frac{ke^2}{\hbar c} = 2\pi\frac{ke^2}{hc} \approx \frac{1}{137}$$

"Fine structure constant" (dimensionless)

Rydberg constant & Bohr radius

Energy of electron in H-like atom (1 e⁻, nuclear charge +Ze):

$$E_n = -\frac{Z^2}{n^2} R = -\frac{mk^2 Z^2 e^4}{2n^2 \hbar^2} = -\frac{Z^2}{2n^2} mc^2 \left(\frac{ke^2}{\hbar c} \right)^2 = -\frac{511,000\, eV}{2 \cdot 137^2} \frac{Z^2}{n^2}$$

$$= -13.6\, eV \frac{Z^2}{n^2}$$

Radius of electron orbit:

$$r_n = \frac{n^2 \hbar^2}{mkZe^2} = \frac{n^2}{Z} a_0 = \frac{n^2}{Z} \frac{\hbar c}{mc^2} \left(\frac{\hbar c}{ke^2} \right) = \frac{n^2}{Z} \frac{1240\, eV \cdot nm}{2\pi \cdot 511,000\, eV} 137$$

$$= 0.0529 nm \frac{n^2}{Z}$$

ACT: Hydrogen-like atoms

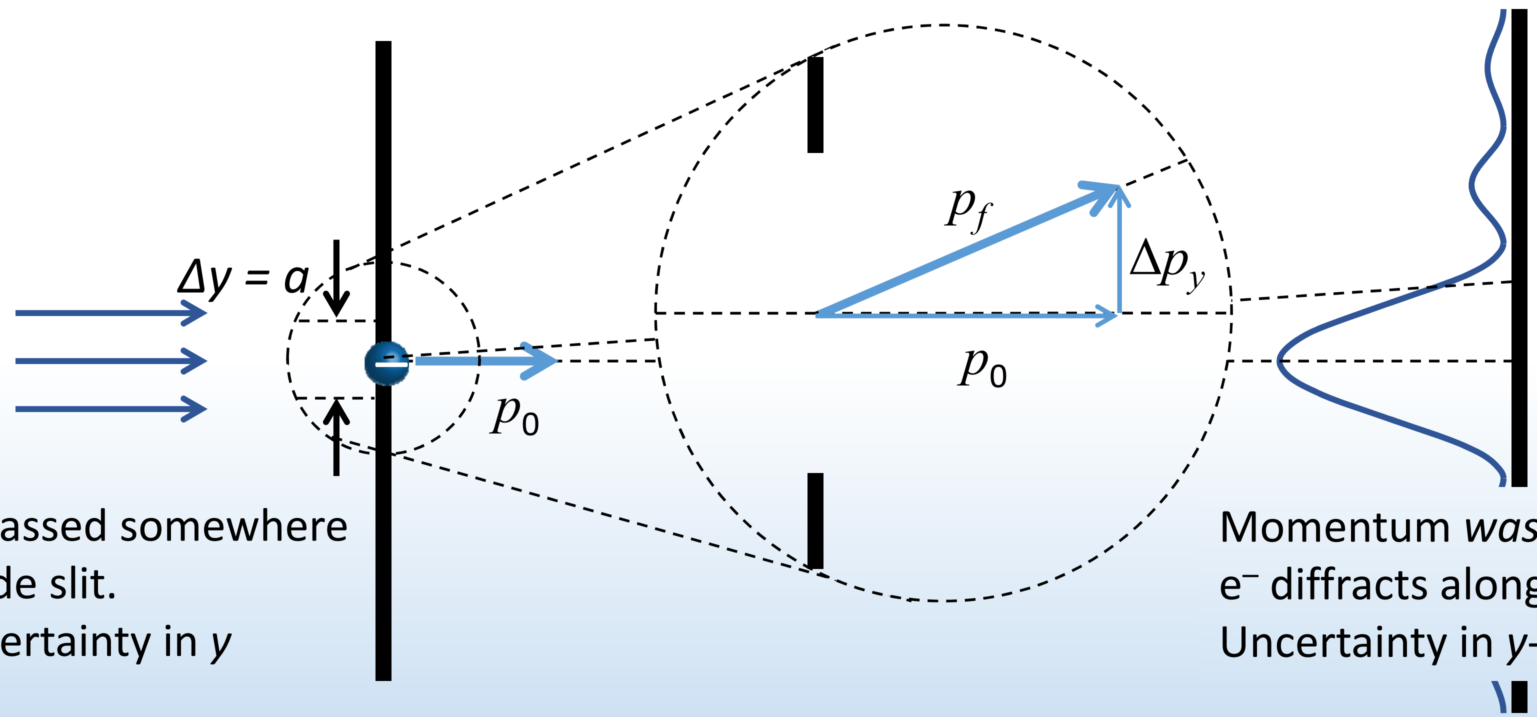
Consider an atom with a nuclear charge of +2e with a single electron orbiting, in its ground state (n = 1), i.e. He^{+}.

How much energy is required to ionize the atom totally?

A. 13.6 eV
B. 2 × 13.6 eV
C. 4 × 13.6 eV

Heisenberg Uncertainty Principle

e^- beam passing through slit of width *a* will diffract



e^- passed somewhere inside slit.
Uncertainty in *y*

Momentum *was* along *x*, but e^- diffracts along *y*.
Uncertainty in *y*-momentum

If slit narrows, diffraction pattern spreads out

Uncertainty in *y*

Uncertainty in *y*-momentum

$$\boxed{\Delta y \cdot \Delta p_y \geq \frac{\hbar}{2}}$$

ACT: CheckPoint 4

The Bohr model cannot be correct! To be consistent with the Heisenberg Uncertainty Principle, which of the following properties *cannot* be quantized?

1. Energy is quantized $E_n = -\frac{R}{n^2}$
2. Angular momentum is quantized $L_n = n\hbar$
3. Radius is quantized $r_n = n^2 a_0$
4. Linear momentum & velocity are quantized $p_n = \frac{\hbar}{n a_0}$

A. All of the above

B. #1 & 2

C. #3 & 4

Summary of today's lecture

- Classical model of atom

  Predicts unstable atom & cannot explain atomic spectra

- Quantum mechanics

  Matter behaves as waves

  Heisenberg Uncertainty Principle

- Bohr model

  Only orbits that fit *n* electron wavelengths are allowed

  $$\underbrace{\text{Explains the stability of the atom; Energy quantization correct for single } e^- \text{ atoms } (H, He^+, Li^{++})\text{; However, it is } \textit{fundamentally} \text{ incorrect}}_{\text{Need complete quantum theory (Lect. 26)}}$$