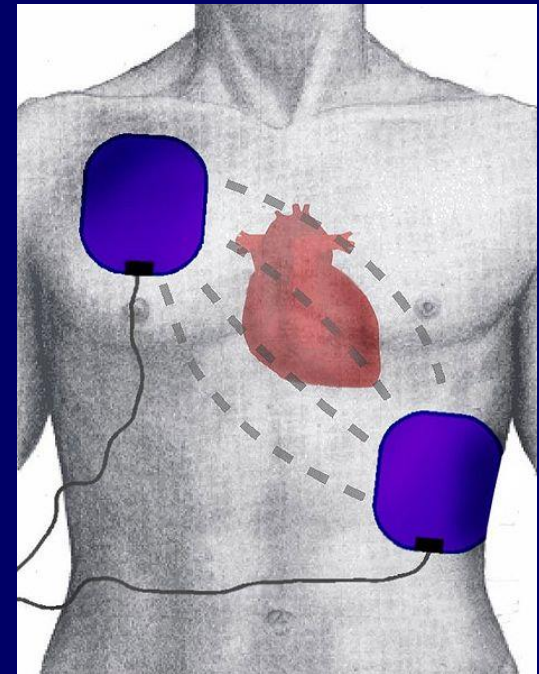


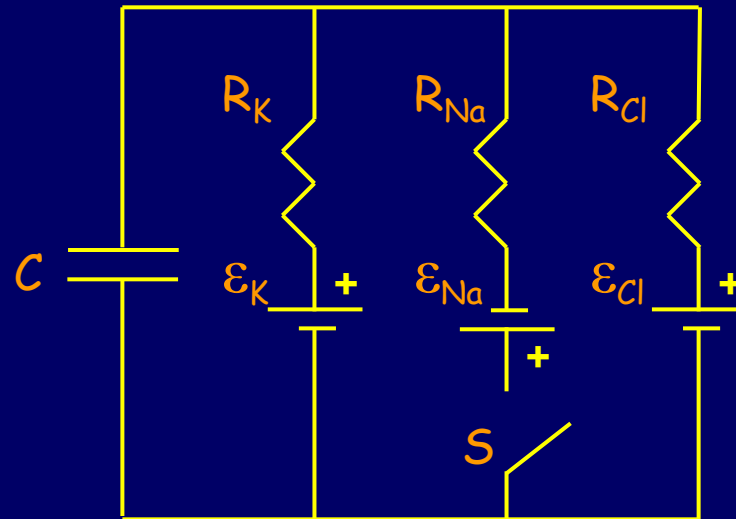
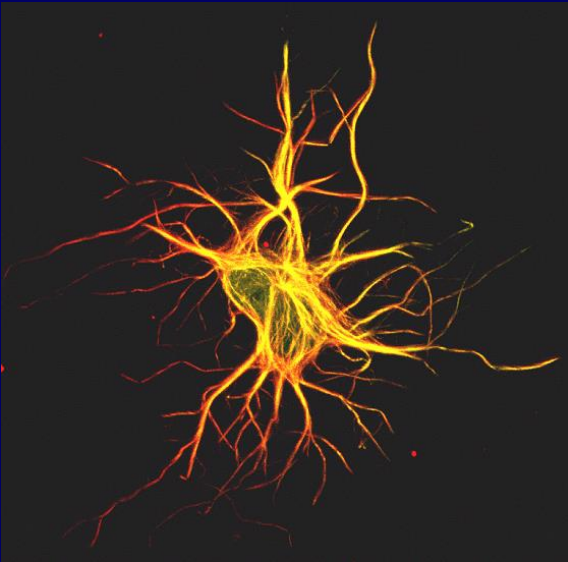
Physics 102: Lecture 7

RC Circuits



RC Circuits

- Circuits that have both resistors and capacitors:



- With **resistance** in the circuits, **capacitors** do not **charge** and **discharge** instantaneously – it takes time (even if only fractions of a second).

RC Circuits

Used to controllably store and release energy

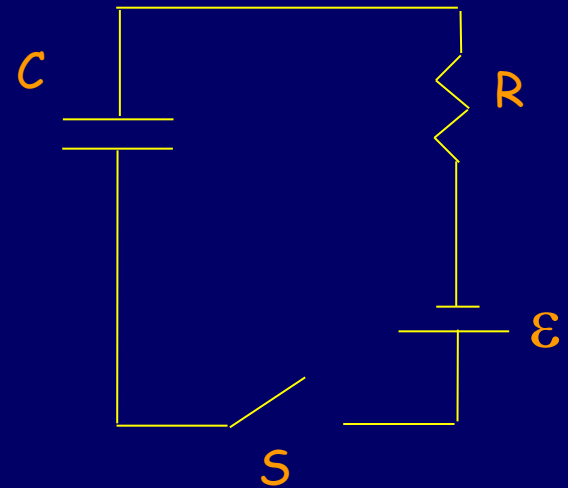
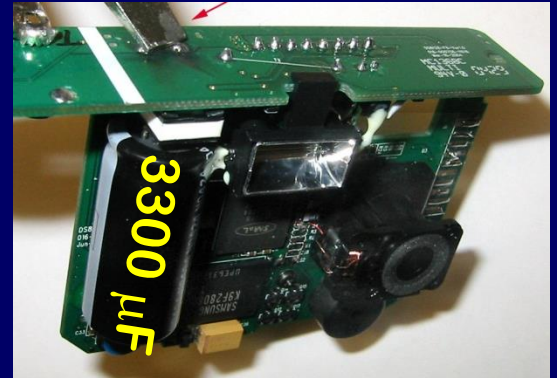
Today:

- RC Circuits
- Charging Capacitors
- Discharging Capacitors
- Intermediate Behavior

Charging Capacitors

Storing energy to use later

- Capacitor is initially uncharged and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
- What is current I_∞ in circuit a long time later?



Charging Capacitors: $t = 0$

- Capacitor is initially uncharged and switch is open.
What is current I_0 in circuit immediately after switch is closed?

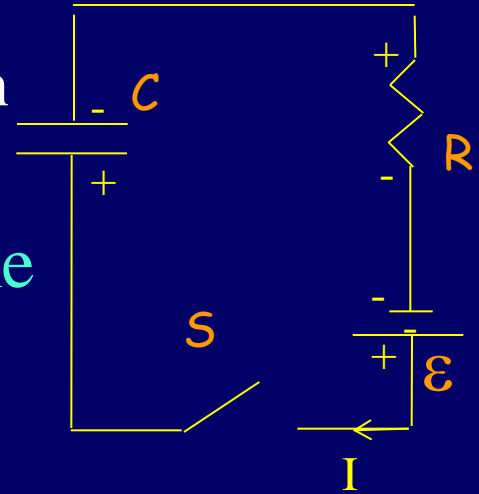
- Jargon (Immediately after, capacitor has same charge as immediately before)

- KLR

$$-V_C - IR + \varepsilon = 0$$

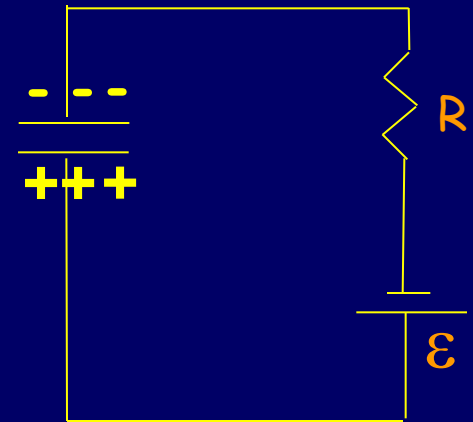
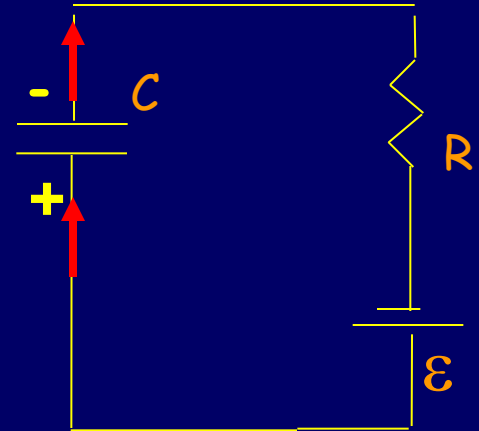
$$0/C - IR + \varepsilon = 0$$

$$I = \varepsilon / R$$



Charging Capacitors: $t > 0$

- $I_0 = \mathcal{E}/R$
- Positive charge flows
 - Onto bottom plate (+Q)
 - Away from top plate (-Q)
 - As charge builds up, V_C rises ($V_C = Q/C$)
 - Loop: $\mathcal{E} - V_C - IR = 0$
 - $I = (\mathcal{E} - V_C)/R$
 - Therefore I falls as Q rises
- When t is very large (∞)
 - $I_\infty = 0$: no current flow into/out of capacitor at long times



– Loop: $\mathcal{E} - V_C - 0 = 0$



ACT/CheckPoint 1

Both switches are initially open, and the capacitor is uncharged. What is the current through the battery just after switch S_1 is closed?

1) $I_b = 0$

2) $I_b = \mathcal{E} / (3R)$

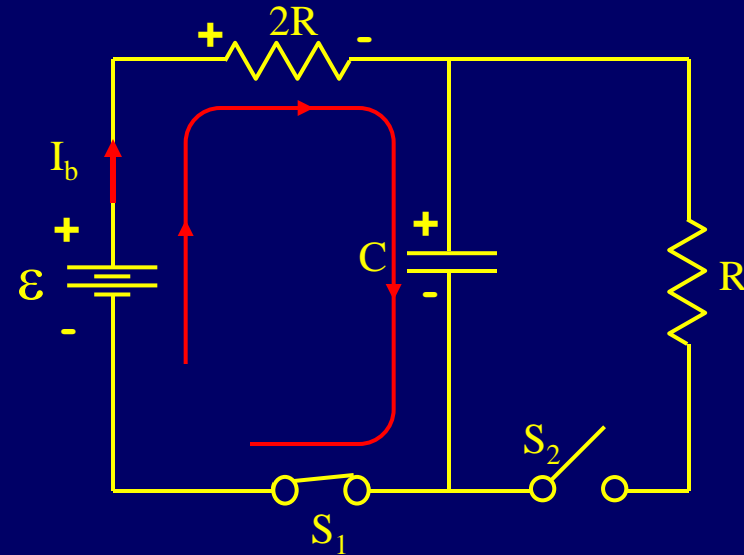
3) $I_b = \mathcal{E} / (2R)$

4) $I_b = \mathcal{E} / R$

$$-I(2R) - V_C + \mathcal{E} = 0$$

$$-I(2R) - 0 + \mathcal{E} = 0$$

$$I = \mathcal{E} / 2R$$





ACT/CheckPoint 3

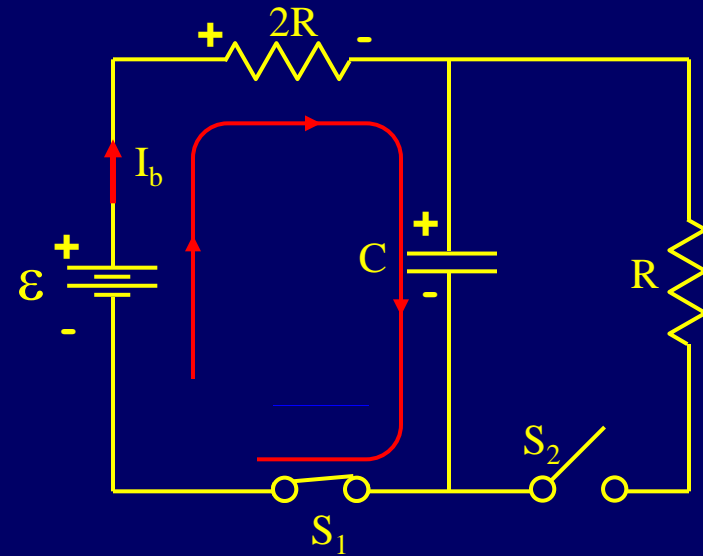
Both switches are initially open, and the capacitor is uncharged.
What is the current through the battery after switch 1 has been closed a long time?

1) $I_b = 0$

2) $I_b = \mathcal{E}/(3R)$

3) $I_b = \mathcal{E}/(2R)$

4) $I_b = \mathcal{E}/R$

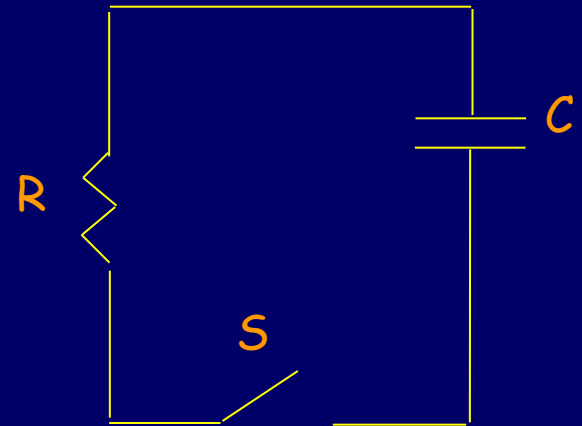
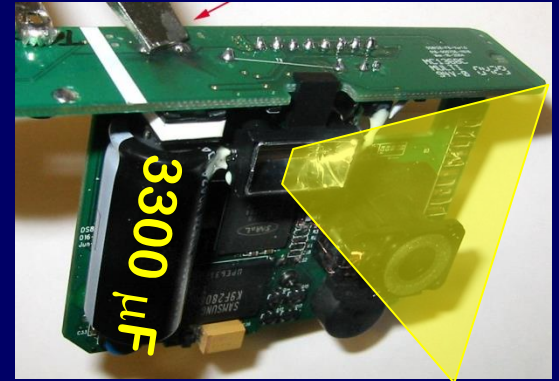


- Long time \Rightarrow current through capacitor is zero
- $I_b = 0$ because the battery and capacitor are in series.
- **KLR:** $\mathcal{E} - 0 - q_{\infty}/C = 0 \Rightarrow q_{\infty} = \mathcal{E}C$

Discharging Capacitors

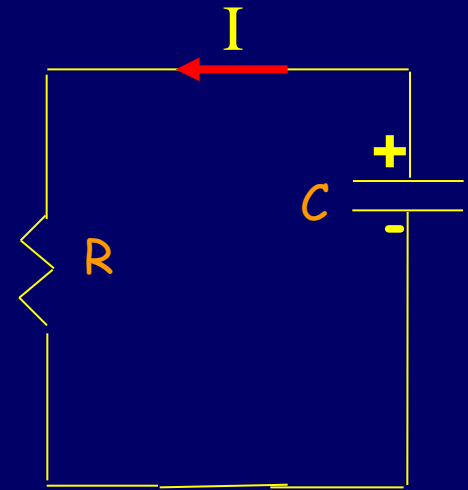
Time to use that stored energy!

- Capacitor is initially charged (Q) and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
- What is current I_∞ in circuit a long time later?



Discharging Capacitors

- Capacitor is initially charged (Q) and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
 - **KLR:** $Q/C - I_0 R = 0$
 - So, $I_0 = Q/RC$
- What is current I_∞ in circuit a long time later?
 - $I_\infty = 0$





ACT/CheckPoint 5

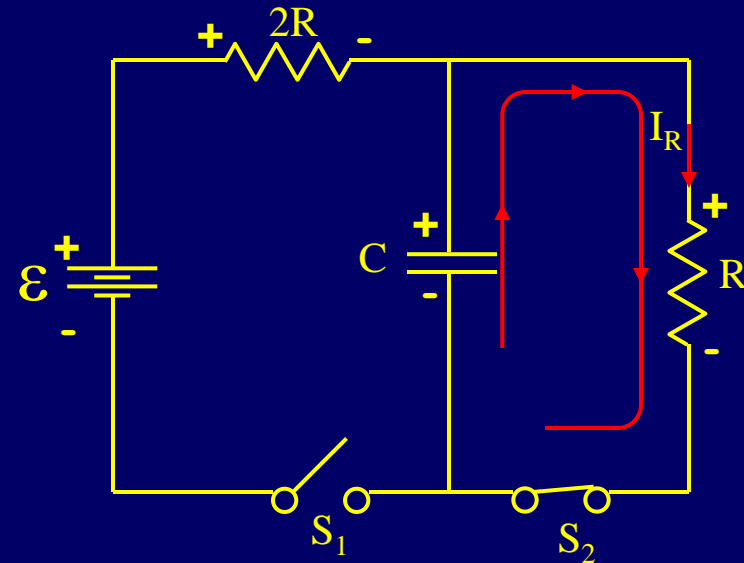
After switch 1 has been closed for a long time, it is opened and switch 2 is closed. What is the current through the right resistor just after switch 2 is closed?

1) $I_R = 0$

2) $I_R = \varepsilon / (3R)$

3) $I_R = \varepsilon / (2R)$

4) $I_R = \varepsilon / R$



KLR: $q_0/C - IR = 0$

Recall q_0 is charge on capacitor after charging:

$q_0 = \varepsilon C$ (since charged w/ switch 2 open!)

$\varepsilon - IR = 0 \Rightarrow I = \varepsilon / R$

Summary: charging & discharging

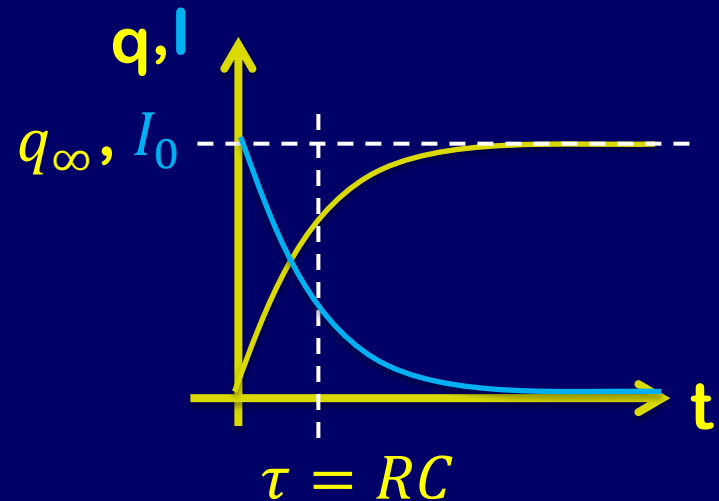
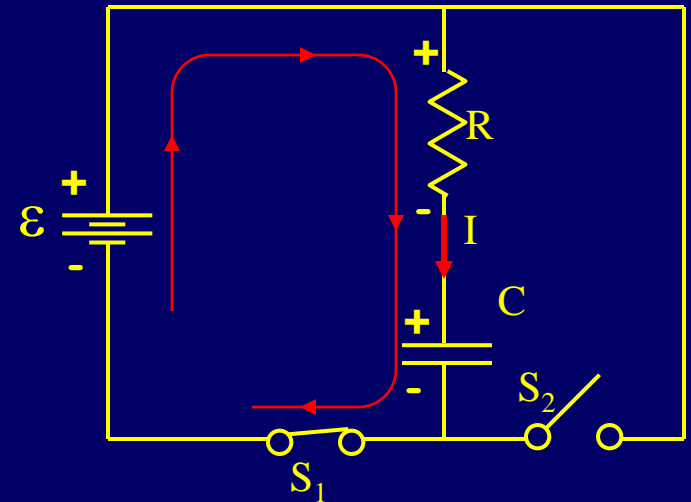
- Charge (and therefore voltage) on Capacitors cannot change instantly: remember $V_C = Q/C$
- Short term behavior of Capacitor:
 - Q is same as just before!
 - If the capacitor starts with no charge, it has no potential difference across it and acts as a wire
 - If the capacitor starts with charge, it has a potential difference across it and acts as a battery.
- Long term behavior of Capacitor: Current through a Capacitor is eventually zero.
 - $I = 0$ through capacitor
 - If the capacitor is charging, when fully charged no current flows and capacitor acts as an open circuit
 - If capacitor is discharging, potential difference is zero and no current flows

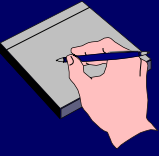
RC Circuits: Charging



The switches are originally open and the capacitor is uncharged. Then switch S_1 is closed.

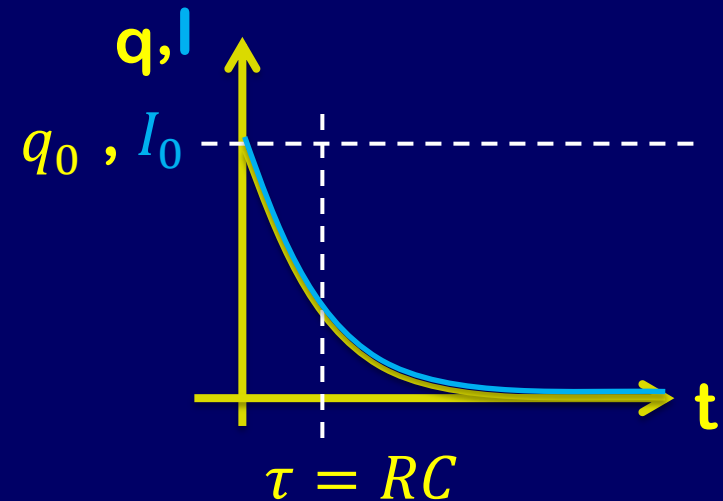
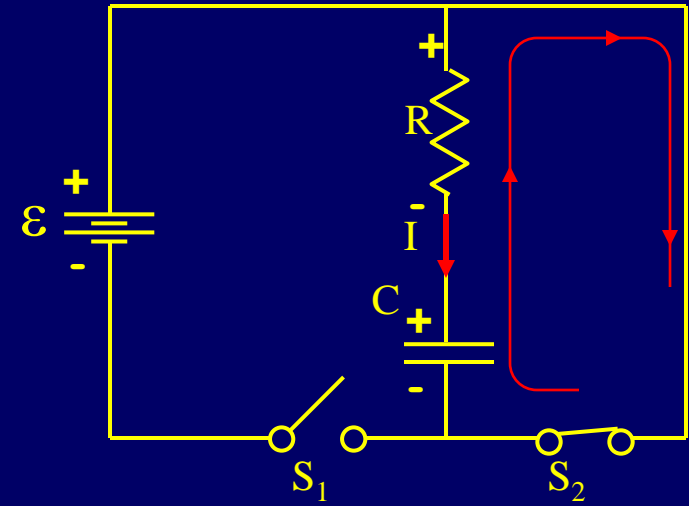
- **Loop:** $\varepsilon - I(t)R - q(t) / C = 0$
- **Just after...:** $q = 0$
 - Capacitor is uncharged
 - $\varepsilon - I_0 R = 0 \Rightarrow I_0 = \varepsilon / R$
- **Long time after:** $I = 0$
 - Capacitor is fully charged
 - $\varepsilon - q_\infty / C = 0 \Rightarrow q_\infty = \varepsilon C$
- **Intermediate (more complex)**
 $q(t) = q_\infty (1 - e^{-t/RC})$
 $I(t) = I_0 e^{-t/RC}$





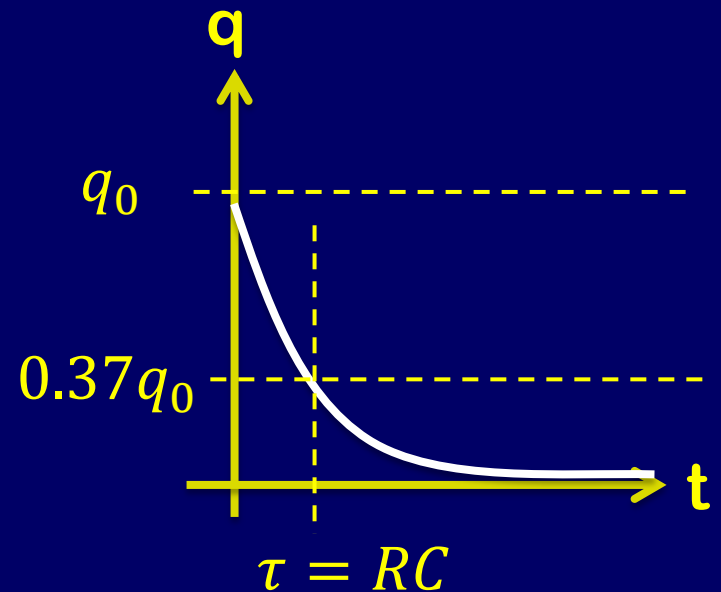
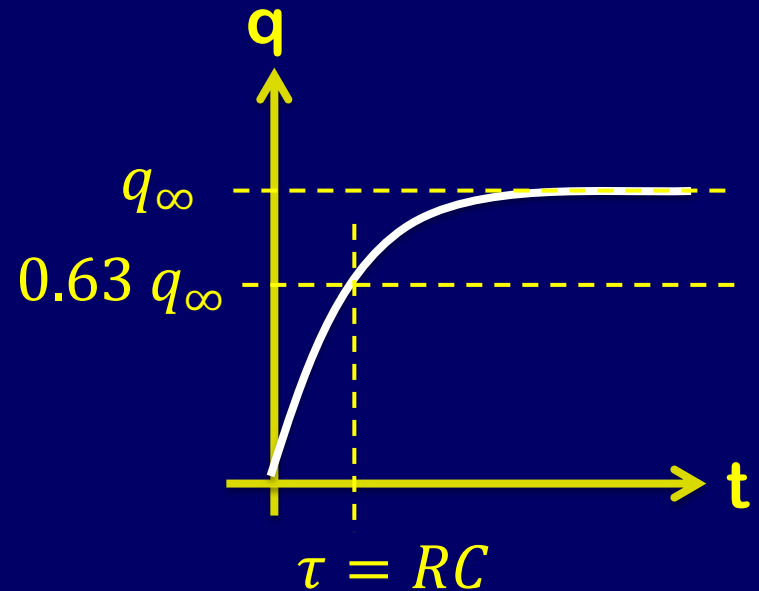
RC Circuits: Discharging

- **Loop:** $q(t) / C + I(t) R = 0$
- **Just after...:** $q=q_0$
 - Capacitor is still fully charged
 - $q_0 / C + I_0 R = 0 \Rightarrow I_0 = -q_0 / (RC)$
- **Long time after:** $I=0$
 - Capacitor is discharged
 - $q_{\infty} / C = 0 \Rightarrow q_{\infty} = 0$
- **Intermediate (more complex)**
 $q(t) = q_0 e^{-t/RC}$
 $I(t) = I_0 e^{-t/RC}$



What is the time constant?

- The time constant $\tau = RC$.
- Given a capacitor starting with no charge, the **time constant** is the amount of time an RC circuit takes to charge a capacitor to about **63%** of its final value.
- The **time constant** is the amount of time an RC circuit takes to discharge a capacitor to about **37%** of its original value.



Example

Time Constant Demo

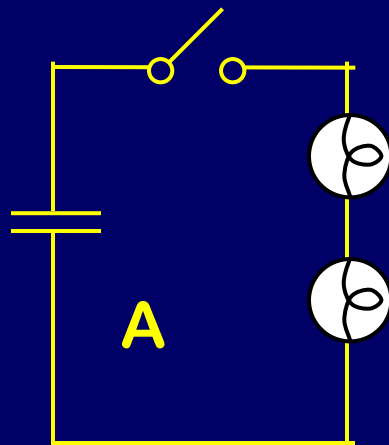
Each circuit has a 1 F capacitor charged to 100 Volts.
When the switch is closed:

- Which system will be brightest?
- Which lights will stay on longest?
- Which lights consumes more energy?

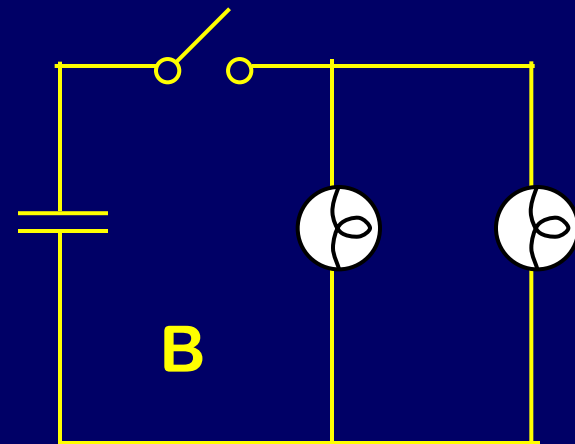
(B) $I = V / (R/2)$

(A) $\tau = 2RC$

(C) Same
 $U = 1/2 CV^2$



$\tau = 2RC$



$\tau = RC/2$

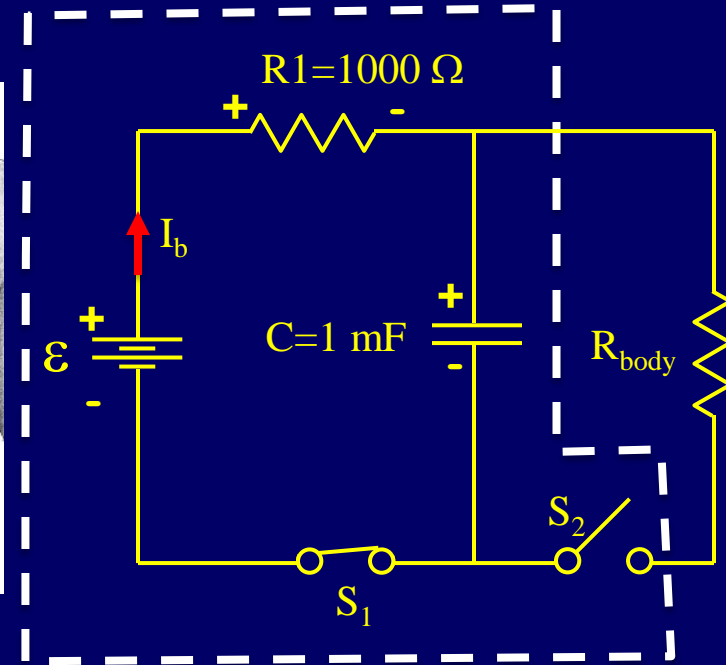
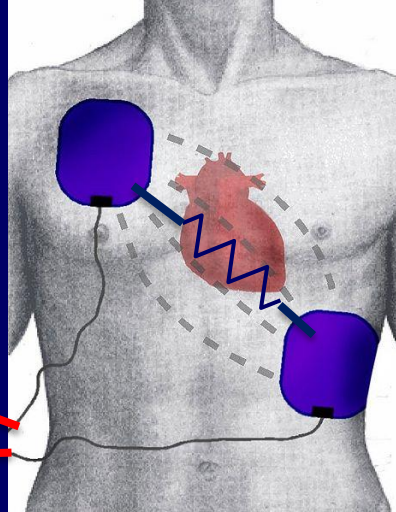
C=Same

Summary of Concepts

- Charge (and therefore voltage) on Capacitors cannot change instantly: remember $V_C = Q/C$
- Short term behavior of Capacitor:
 - If the capacitor starts with no charge, it has no potential difference across it and acts as a wire
 - If the capacitor starts with charge, it has a potential difference across it and acts as a battery.
- Long term behavior of Capacitor: Current through a Capacitor is eventually zero.
 - If the capacitor is charging, when fully charged no current flows and capacitor acts as an open circuit.
 - If capacitor is discharging, potential difference is zero and no current flows.
- Intermediate behavior: Charge and current exponentially approach their long-term values $\tau = RC$

Example

Practice: defibrillator



A 500 V battery is used to charge the 1 mF capacitor for 2 seconds. How much charge is stored on the capacitor?

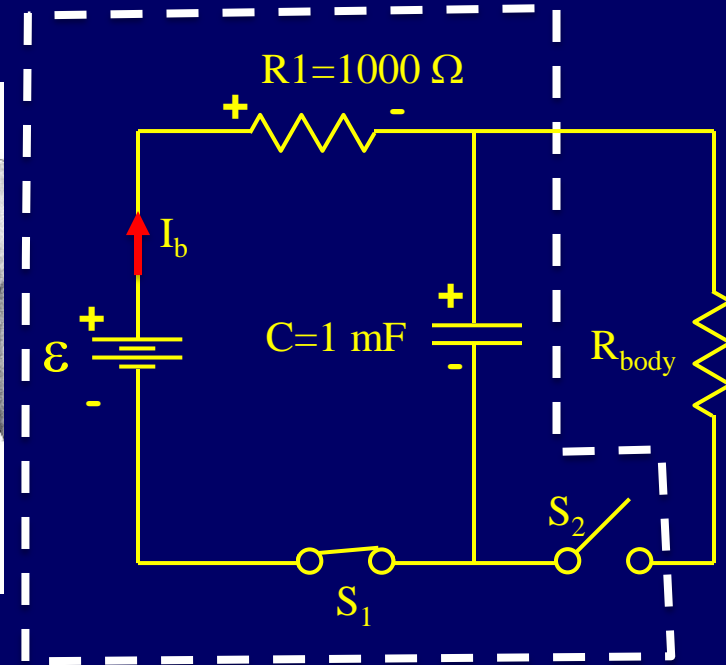
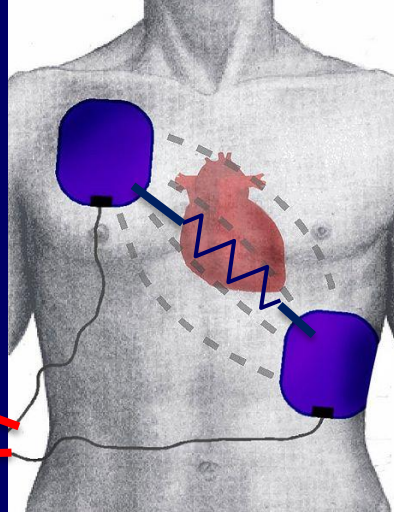
$$q(t) = q_{\infty}(1 - e^{-t/RC}) \quad q_{\infty} = C\mathcal{E} = (0.001 \text{ F})(500 \text{ V}) = 0.5 \text{ C}$$

$$q(2 \text{ s}) = 0.5 \text{ C} \left(1 - e^{-2\text{s}/1000\Omega \times 0.001\text{F}}\right) = 0.43 \text{ C}$$

$$V_C = Q/C = 0.43\text{C}/0.001\text{F} = 430 \text{ V}$$

Example

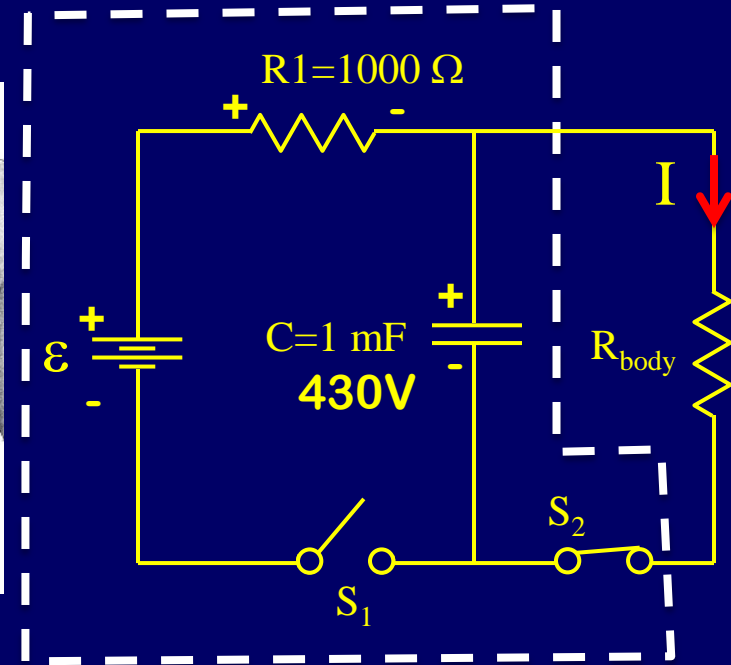
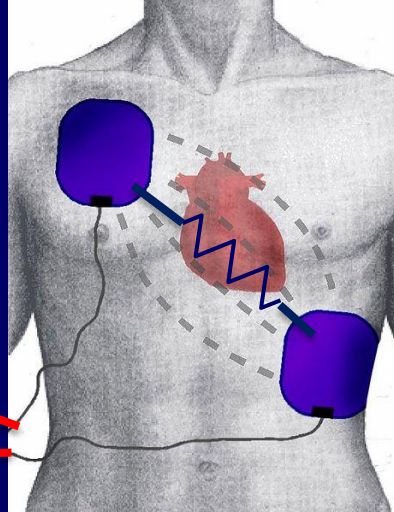
Practice: defibrillator



A 500 V battery is used to charge the 1 mF capacitor for 2 seconds. How much energy is stored in the capacitor?

$$U = Q^2 / 2C = (0.43C)^2 / (2 \times 0.001 \text{ F}) = 92 \text{ J}$$

ACT: defibrillator



After charging for 2 seconds, S_1 is opened; What is the current through the patient right after S_2 is closed if $R_{\text{body}} = 100 \Omega$?

(A) 0 A

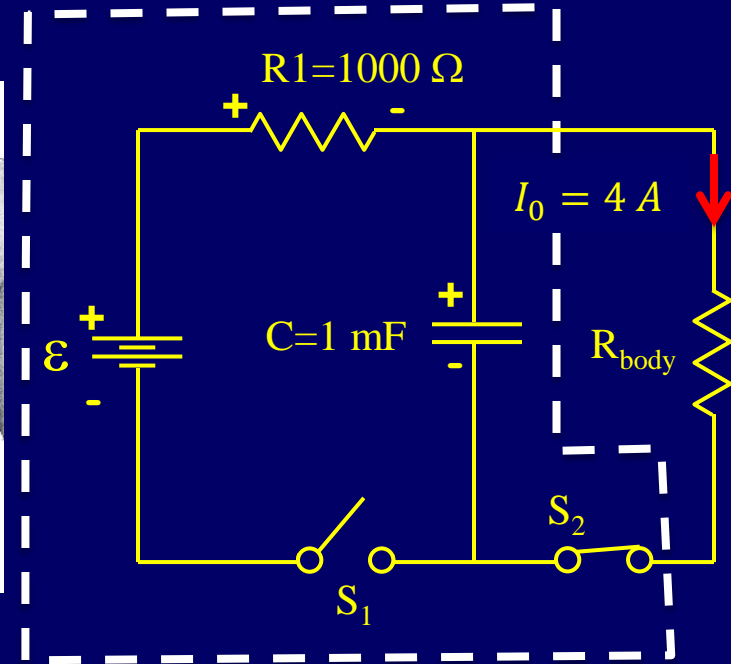
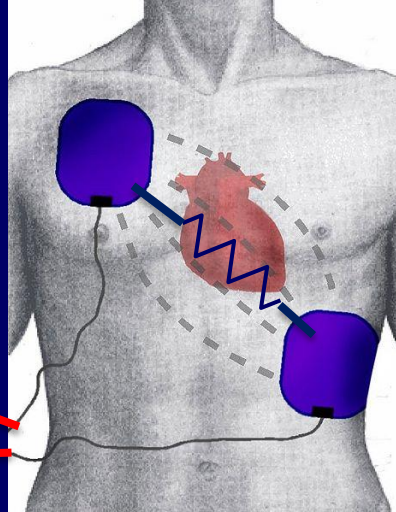
(B) 4.3 A

(C) 0.25 A

$$I = V/R = 430V/100\Omega = 4 A$$

Example

Practice: defibrillator



After charging for 2 s, S_1 is opened. What is the current through the patient 0.1 s after S_2 is closed if $R_{\text{body}} = 100 \Omega$?

$$I(0.1\text{s}) = I_0 e^{-t/RC} = 4.3 \text{ A } e^{-0.1 \text{ s} / 100 \Omega \times 0.001 \text{ F}} = 1.6 \text{ A}$$



RC Summary

Charging

$$q(t) = q_{\infty}(1 - e^{-t/RC})$$

$$V(t) = V_{\infty}(1 - e^{-t/RC})$$

$$I(t) = I_0 e^{-t/RC}$$

Discharging

$$q(t) = q_0 e^{-t/RC}$$

$$V(t) = V_0 e^{-t/RC}$$

$$I(t) = I_0 e^{-t/RC}$$

Time Constant $\tau = RC$

Large τ means long time to charge/discharge

Short term: Charge doesn't change (often zero or max)

Long term: Current through capacitor is zero.