

Physics 102: Lecture 28

Special Relativity



End-of-semester info

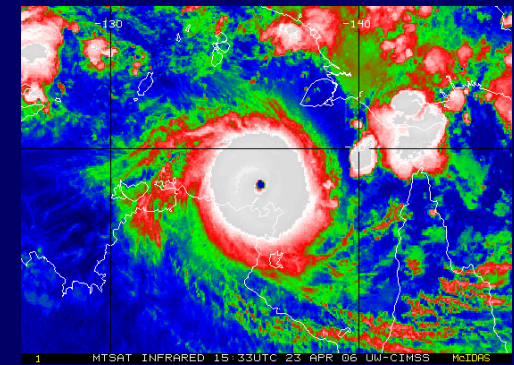
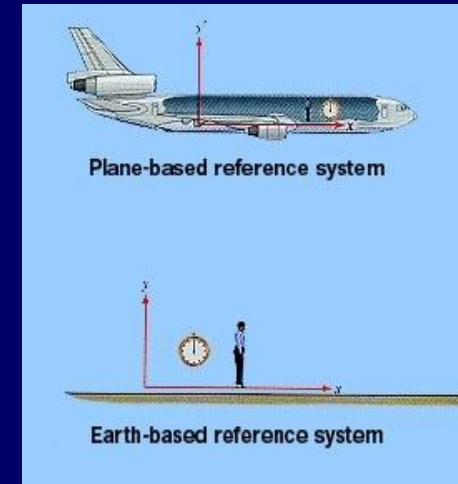
- Final exam info:
 - A1: Thursday, May 15, 1:30-4:30pm
 - A2: Friday, May 9, 1:30-4:30pm
 - Approximately 50 questions
 - Cumulative (all material from semester covered evenly)
 - Problem list from text available on home page.
- CHECK GRADEBOOK!
- Wednesday lecture is overview of course

Postulates of Relativity

- Laws of physics are the same in every inertial frame

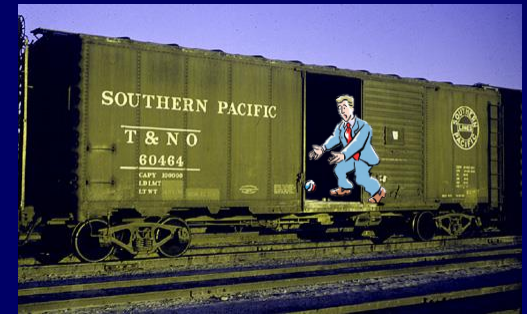
Inertial Reference Frame

- Frame which is in uniform motion (constant velocity)
 - No Accelerating
 - No Rotating
- Technically Earth is not inertial, but it's close enough.



Postulates of Relativity

- Laws of physics are the same in every inertial frame
 - the same laws of physics can be used in any inertial reference frame and the results will be consistent for that frame.
- Speed of light in vacuum is c for everyone
 - Measure $c=3 \times 10^8$ m/s if you are on train going east or on train going west, even if light source isn't on the train.



Weird!

Example

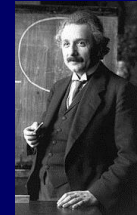
Relative Velocity (Ball)



- Justin Verlander throws baseball @90 mph.
How fast do I think it goes when I am:

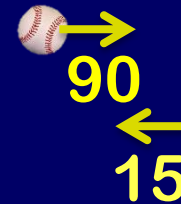
– Standing still?

90 mph



– Running 15 mph towards?

A) 75 mph B) 90 mph C) 105 mph



– Running 15 mph away?

A) 75 mph B) 90 mph C) 105 mph

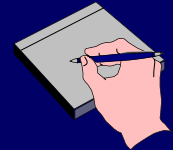


15

(Review 101 for help with Relative Velocities)

Example

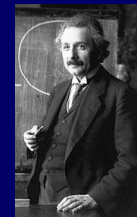
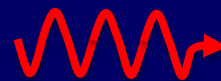
Relative Velocity (Light)



- Now he throws a photon ($c=3 \times 10^8$ m/s). How fast do I think it goes when I am:

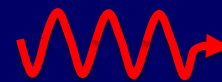
- Standing still

3×10^8 m/s



- Running 1.5×10^8 m/s towards

A) 1.5×10^8 m/s B) 3.0×10^8 m/s



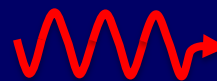
1.5



C) 4.5×10^8 m/s

- Running 1.5×10^8 m/s away

A) 1.5×10^8 m/s B) 3.0×10^8 m/s



1.5

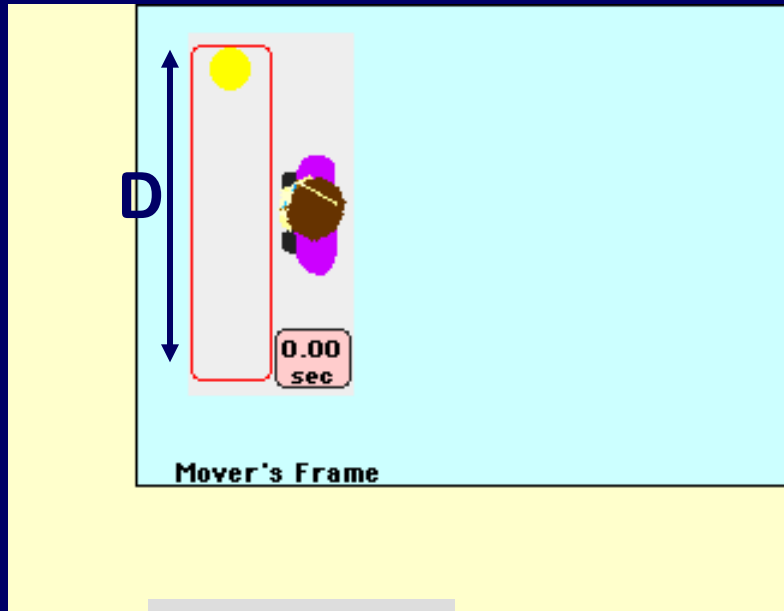


C) 4.5×10^8 m/s

Checkpoint 1

Strange but True!

Consequences: 1. Time Dilation

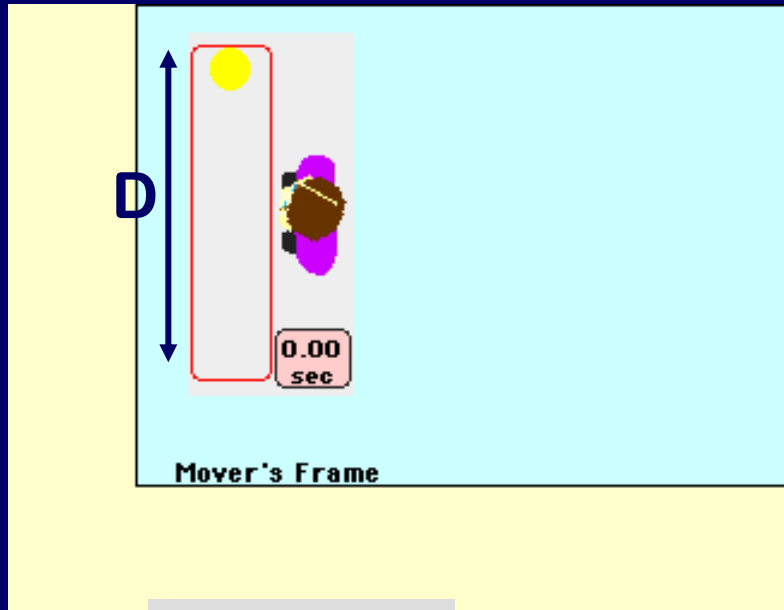


$$c \Delta t_0 = 2D$$

$$\Delta t_0 = \frac{2D}{c}$$

t_0 is called the “proper time”. Here it is the time between two events that occur at the same place, in the rest frame.

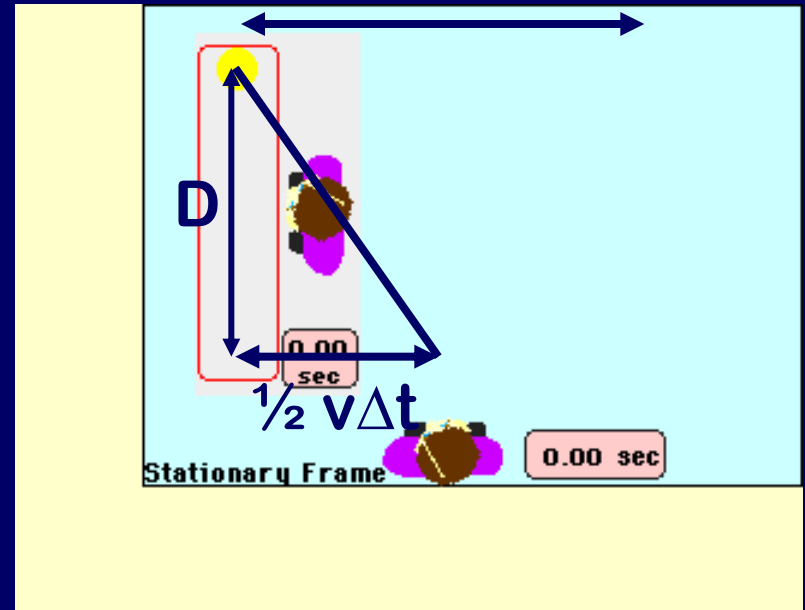
Time Dilation



$$c \Delta t_0 = 2D$$

$$\Delta t_0 = \frac{2D}{c}$$

t_0 is proper time
Because it is rest
frame of event



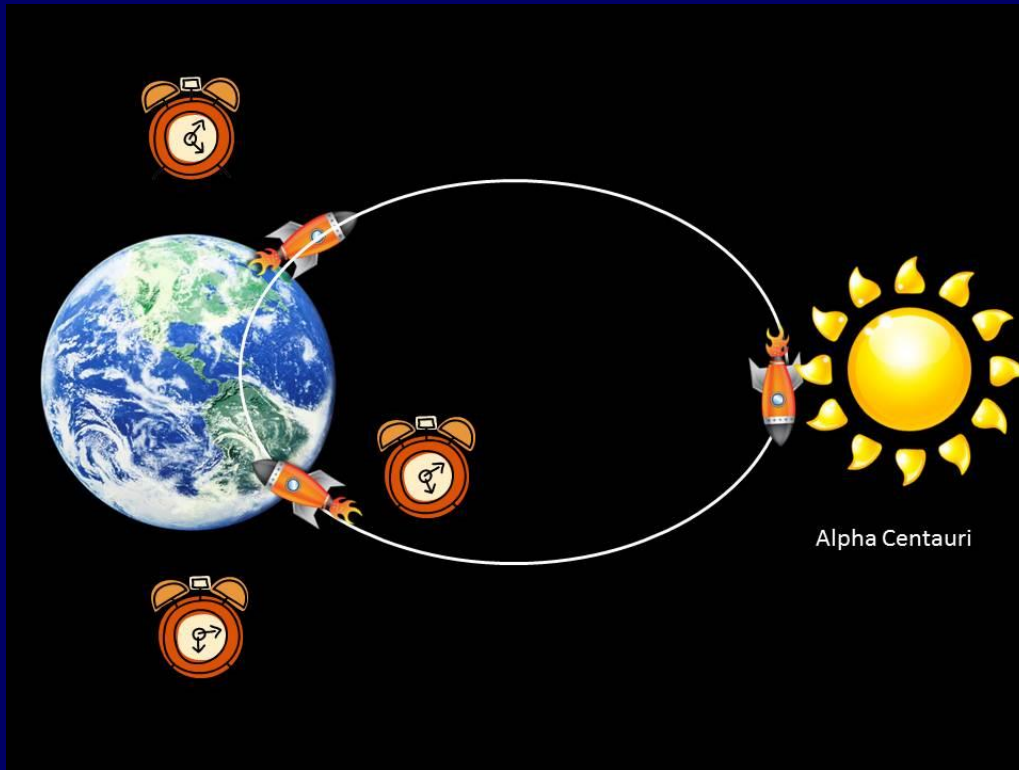
$$\left(c \frac{\Delta t}{2} \right)^2 = (D)^2 + \left(\frac{v \Delta t}{2} \right)^2$$

$$\Delta t = \frac{2D}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > \Delta t_0$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Importantly, when deriving time dilation the two events,

- 1) photon leaving right side of train and
 - 2) photon arriving back at right side of train,
- occur **at same point on the train**.



**“Moving
clocks
run
slowly”**

**Checkpoint 2:
“twin paradox”**

Twin Paradox

You discover that you have a long-lost twin who's been on a high-speed ($0.5c$) spaceship for the last 10 years. When your twin returns to earth she will be

A) Younger

B) Same

C) Older

Travelling forward in time is science, not science fiction!

Really!

I'm serious!

Example



Time Dilation

A π^+ (pion) is an unstable elementary particle. It may decay into other particles in 10 nanoseconds (when at rest)

Suppose a π^+ is created at Fermilab with a velocity $v=0.99c$. How long will it live before it decays?

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10 \text{ ns}}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} = \frac{10 \text{ ns}}{\sqrt{1 - (0.99)^2}} = 71 \text{ ns}$$

- If you are moving with the pion, it lives 10 ns
- In lab frame where it has $v=0.99c$, it lives 7.1 times longer
- Both are right!
- This is not just “theory.” It has been verified experimentally (many times!)

Example

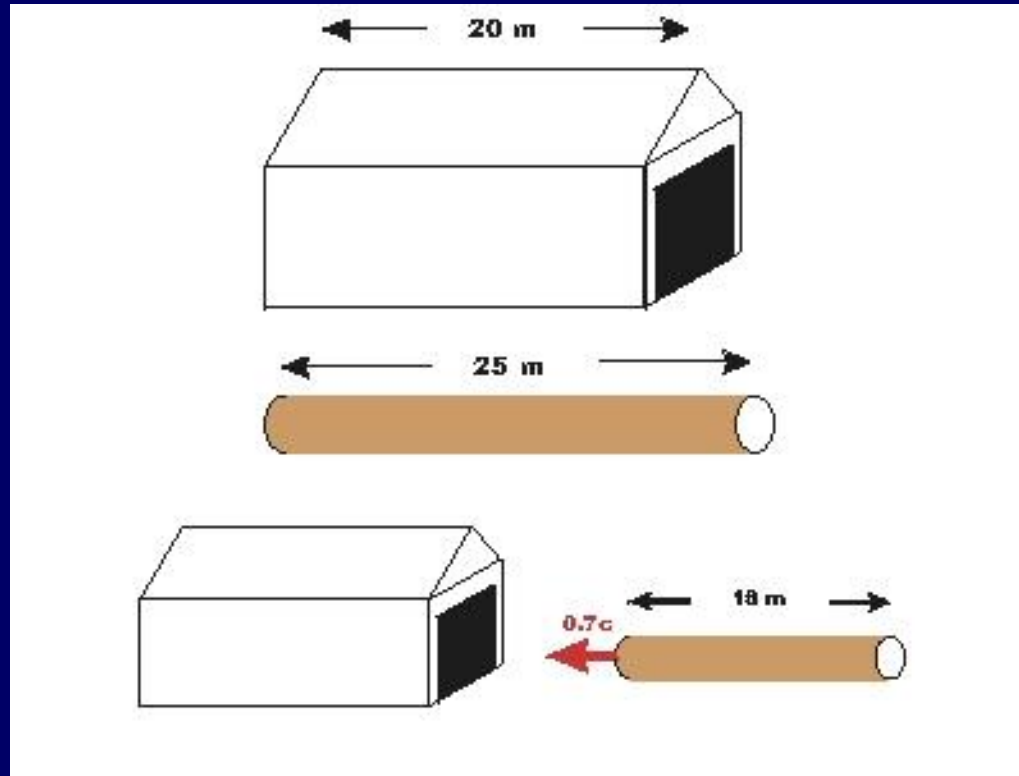
Time Dilation



v/c	γ
0.1	1.005
0.2	1.021
0.5	1.155
0.9	2.294
0.99	7.089
0.999	22.366
0.9999	70.712
0.99999	223.607
0.999999	707.107
0.9999999	2236.068

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t = \gamma \Delta t_0$$

Consequences: 2. Length contraction



On board a Train



send photon to end
of boxcar & back:

$$\Delta t_0 = 2 \Delta x_0 / c$$

Δt_0 = time for light to travel to
front and back of train for the
observer on the train.

Δx_0 = length of train according
to the observer on the train.

Train traveling to right with speed v



$$\Delta x$$

the observer on the ground sees :

$$\Delta t_{\text{forward}} = (\Delta x + v \Delta t_{\text{forward}}) / c$$

$$\Delta t_{\text{backward}} = (\Delta x - v \Delta t_{\text{backward}}) / c$$

$$\Delta t_{\text{total}} = \Delta t_{\text{forward}} + \Delta t_{\text{backward}} = (2 \Delta x / c) \gamma^2$$

(Using time dilation: $\Delta t_{\text{total}} = \Delta t_0 \gamma$)

$$2 \Delta x_0 = 2 \Delta x \gamma$$

$\Delta x = \Delta x_0 / \gamma$ the length of the moving train is
contracted!



Length Contraction

“Moving objects shrink!”

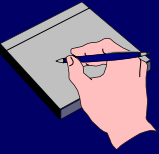
Length in
moving
frame

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Length in
object's rest
frame

Example

Length Contraction



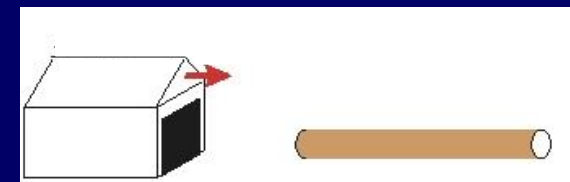
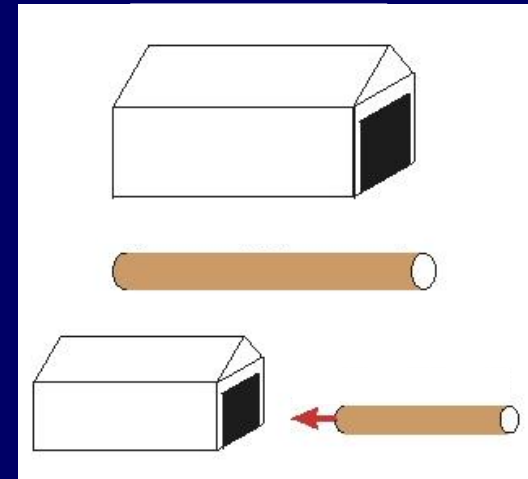
Sue is carrying a pole 10 meters long. Paul is on a barn which is 8 meters long. If Sue runs quickly $v=0.8c$, can she ever have the entire pole in the barn?

Paul: The barn is 8 meters long, and the pole is only

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = 10 \sqrt{1 - .8^2} = 6 \text{ meters}$$

Sue: No way! This pole is 10 meters long and that barn is only

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = 8 \sqrt{1 - .8^2} = 4.8 \text{ meters}$$



Who is right?

A) Paul

B) Sue

C) Both

What Really Happens?

Lets have the door closed at the far end, Paul can open the back door, once the pole is completely inside the front door.

A) The pole smashes into the back door

B) The pole does not smash into the back door

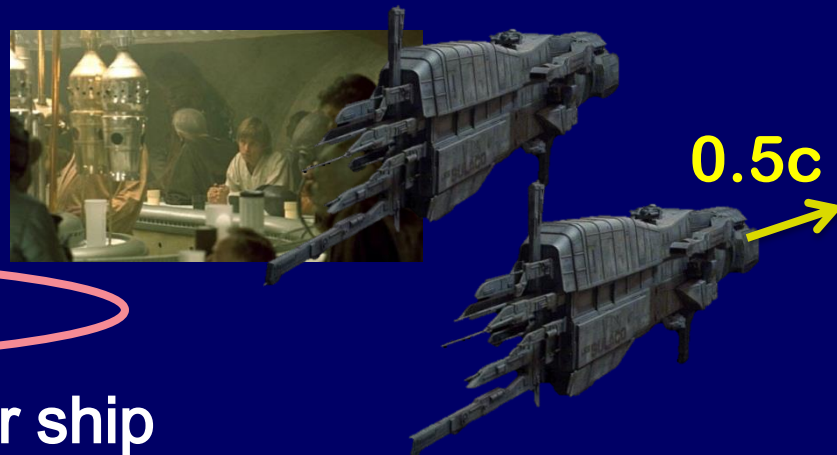
Sue says that Paul cheated and opened the back door before he closed the front door.



ACT / Checkpoint 3

You're at the interstellar café having lunch in outer space - your spaceship is parked outside. A speeder zooms by in an identical ship at half the speed of light. From your perspective, their ship looks:

- (A) longer than your ship
- (B) shorter than your ship
- (C) exactly the same as your ship



$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Always < 1

$$L_0 > L$$

In the speeder's reference frame

In your reference frame

Last physics 102 checkpoint heck yeah!
Sorry, I'm guessing on this one :/

Time Dilation vs. Length Contraction

- Time intervals between same two events:

Consider only those intervals which occur at one point in rest frame “on train”.

- Δt_0 is in the reference frame at rest, “on train”. “proper time”
- Δt is measured between same two events in reference frame in which train is moving, using clock that isn’t moving, “on ground”, in that frame.

$$\Delta t_0 = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$



$$\Delta t > \Delta t_0$$

Time seems longer
from “outside”

- Length intervals of same object:

- L_0 is in reference frame where object is at rest “proper length”
- L is length of moving object measured using ruler that is not moving.

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$



$$L_0 > L$$

Length seems shorter
from “outside”

Consequence: Simultaneity depends on reference frame

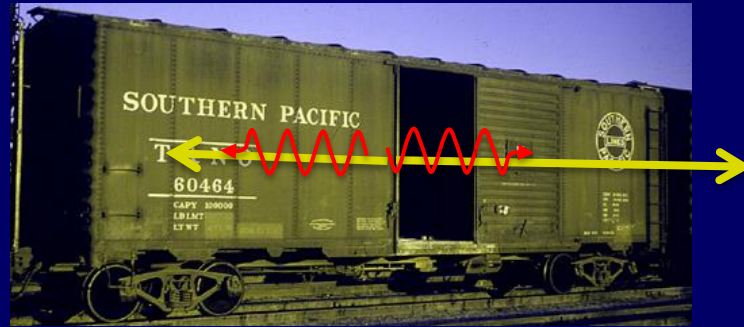
Aboard train



Light is turned on and arrives at the front and back of the train at the same time.

The two events,
1) light arriving at the front of the train and
2) light arriving at the back of the train,
are simultaneous.

Train traveling to right with speed v



the observer on the ground sees :

Light arrives first at back of the train and then at the front of the train, because light travels at the same speed in all reference frames.

The same two events,
1) light arriving at the front of the train and
2) light arriving at the back of the train,
are NOT simultaneous.

**Therefore whether two events
occur at the same time
-simultaneity-
depends on reference frame.**

Relativistic Momentum

Relativistic Momentum

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Note: for $v \ll c$ $p = mv$

Note: for $v = c$ $p = \text{infinity}$

Relativistic Energy

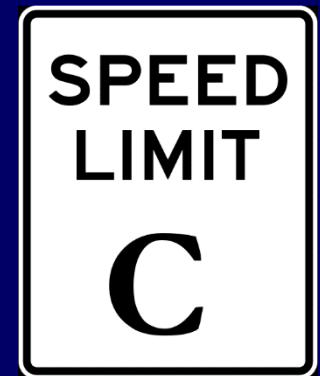
$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Note: for $v = 0$ $E = mc^2$

Note: for $v \ll c$ $E = mc^2 + \frac{1}{2}mv^2$

Note: for $v = c$ $E = \text{infinity}$ (if m is not 0)

Objects with mass always have $v < c$!



Summary

- Laws of physics work in any inertial frame
 - “Simultaneous” depends on frame
 - **Considering those time intervals that are measured at one point in the rest frame,**
Time dilates relative to proper time
or moving clocks run slow.
 - Length contracts relative to proper length
or moving objects are shorter.
 - Energy/Momentum conserved
- Laws of physics for $v \ll c$ reduce to Newton's Laws