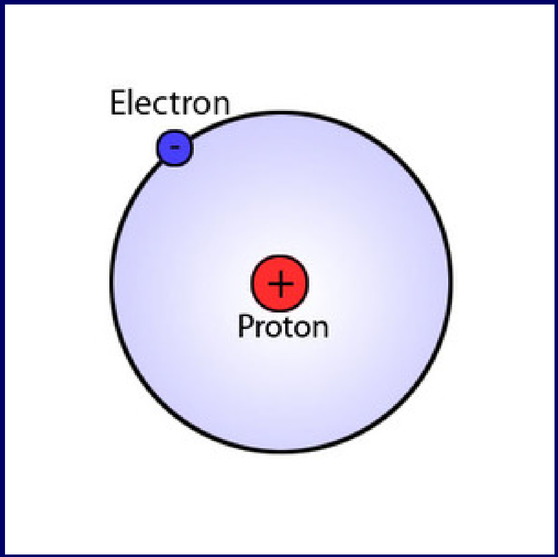
Physics 102: Lecture 24

Heisenberg Uncertainty Principle & Bohr Model of Atom



Heisenberg Uncertainty Principle

Recall: Quantum Mechanics tells us outcomes of individual measurements are uncertain

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

Planck's constant

$$\hbar = h/2\pi$$

"h-bar"

Uncertainty in momentum (along y)

Uncertainty in position (along y)

Rough idea: if we know momentum very precisely, we lose knowledge of location, and vice versa.

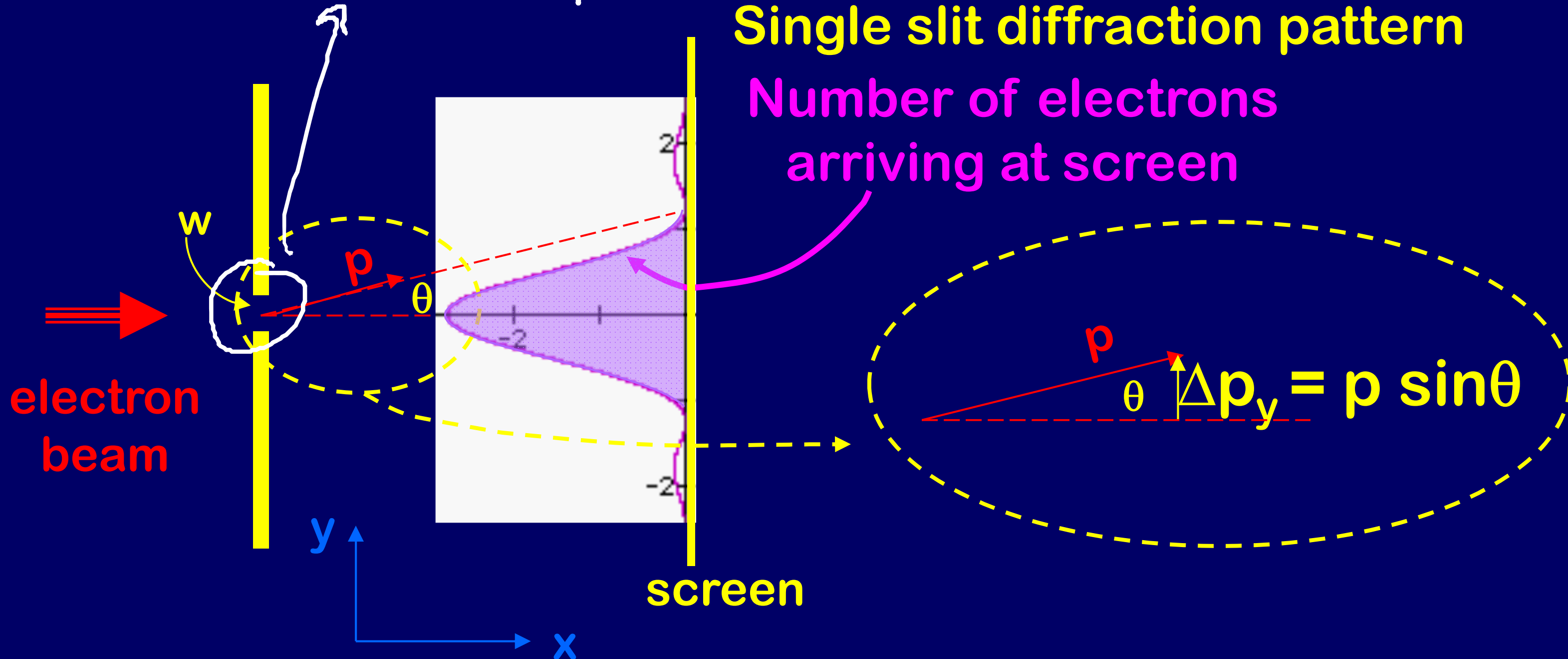
This "uncertainty" is fundamental: it arises because quantum particles behave like waves!

Example

Electron diffraction

Electron beam traveling through slit will diffract

What is y, p_y?



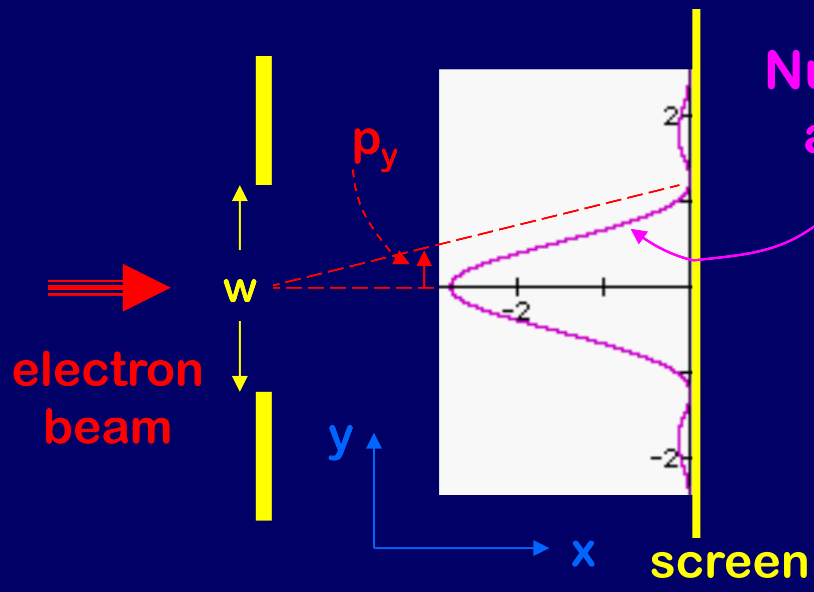
Recall single-slit diffraction 1st minimum:

$p = h/\lambda$

$\sin\theta = \lambda/w$ $\quad w = \lambda/\sin\theta \quad$ **$= \Delta y$**

$$\Delta p_y \Delta y = p \sin\theta \frac{\lambda}{\sin\theta} = \lambda p = h$$

Using de Broglie λ



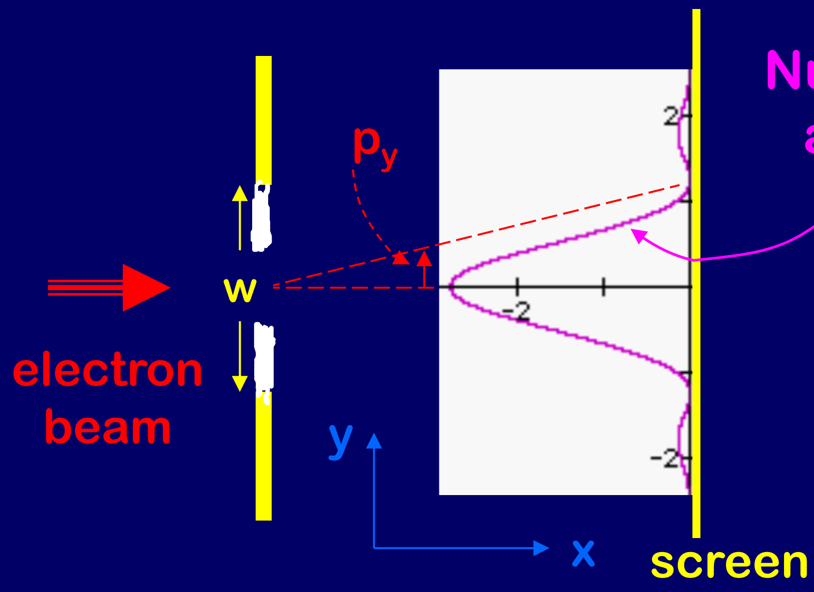
$$\Delta p_y \cdot w = h$$

Electron entered slit with momentum along x direction and no momentum in the y direction. When it is diffracted it acquires a p_y which can be as big as h/w.

The "Uncertainty in p_y" is $\Delta p_y \approx h/w$.

An electron passed through the slit somewhere along the y direction. The "Uncertainty in y" is $\Delta y \approx w$.

$$\Delta p_y \cdot \Delta y \approx h$$



$$\Delta p_y \cdot \Delta y \approx h$$

If we make the slit narrower (decrease w $=\Delta$y) the diffraction peak gets broader (Δp_y increases).

$$\Delta p_y \approx h/\Delta y$$

"If we know location very precisely, we lose knowledge of momentum, and vice versa."

ACT/Checkpoint 1

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

According to the H.U.P., if we know the x-position of a particle, we cannot know its:

(1) y-position

(2) x-momentum

(3) y-momentum

(4) Energy

to be precise...

Of course if we try to locate the position of the particle along the x axis to Δx we will not know its x component of momentum better than Δp_x, where

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

and the same for z.

Atoms

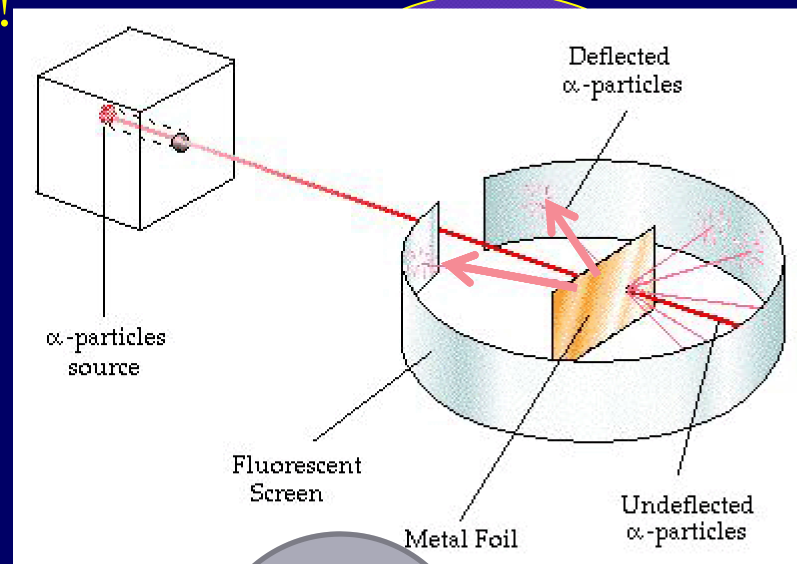
- Evidence for the nuclear atom
- Bohr model of the atom
- Spectroscopy of atoms
- Quantum atom

Today

Next lecture

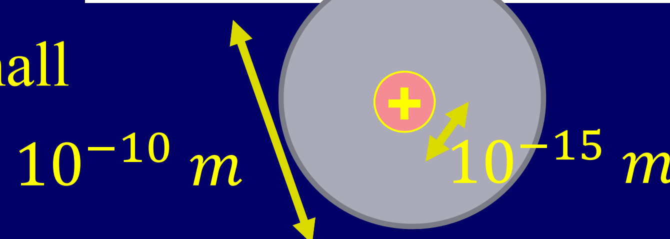
Rutherford Scattering

1911: Scattering He^{++} (an "alpha particle") atoms off of gold.
Mostly go through, some scattered back!



Plum pudding theory:
+ and – charges uniformly distributed → electric field felt by alpha never gets too large

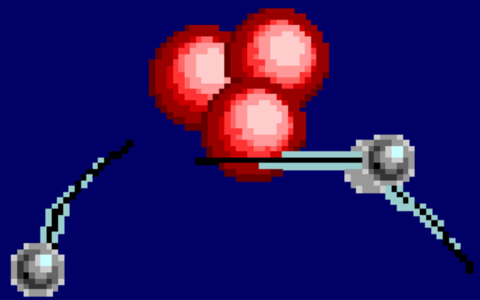
To scatter at large angles, need positive charge concentrated in small region (the nucleus)



Atom is mostly empty space with a small (r = 10^{-15} m) positively charged nucleus surrounded by "cloud" of electrons (r = 10^{-10} m)

Nuclear Atom (Rutherford)

Large angle scattering ⟶ Nuclear atom



Classic nuclear atom is not stable!

Electrons will radiate and spiral into nucleus

} Need quantum theory

Early "quantum" model: Bohr

Bohr Model is Science fiction

Nobel Prize 1922

The Bohr model is complete nonsense.

Electrons do not circle the nucleus in little planet-like orbits.

The assumptions injected into the Bohr model have no basis in physical reality.

BUT the model does get some of the numbers right for SIMPLE atoms…

Hydrogen-Like Atoms

single electron with charge -e

nucleus with charge +Ze
(Z protons)

$$\boxed{e = 1.6 \times 10^{-19}\ \text{C}}$$

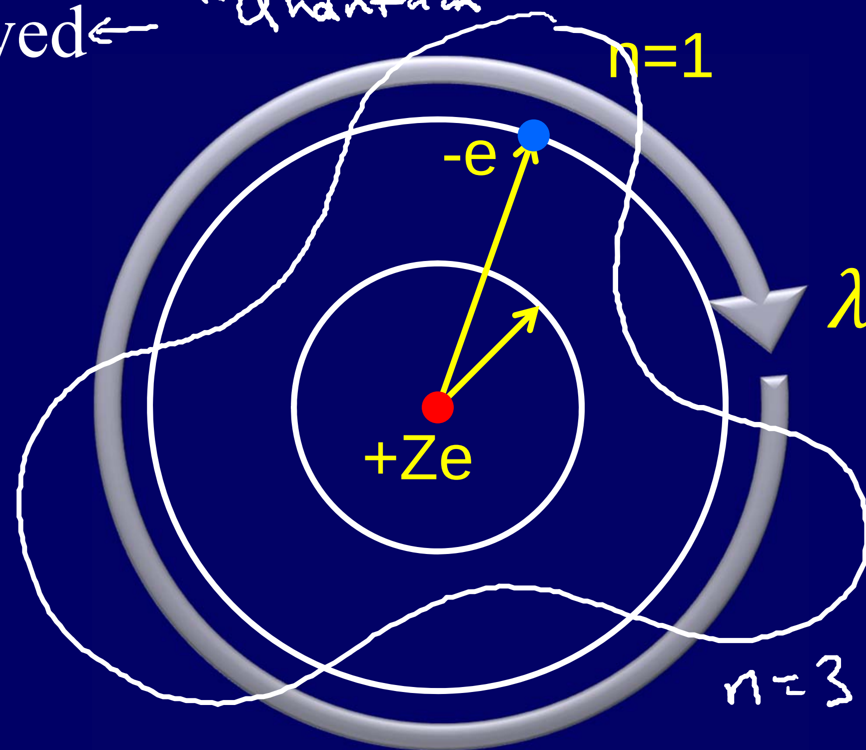
Ex: H (Z=1), He^{+} (Z=2), Li^{++} (Z=3), etc

The Bohr Model

Electrons circle the nucleus in orbits ← "Classical"

Only certain orbits are allowed ← "Quantum"

$$2\pi r = n\lambda$$
$$= nh/p$$



$n = 1,2,3 \ldots$

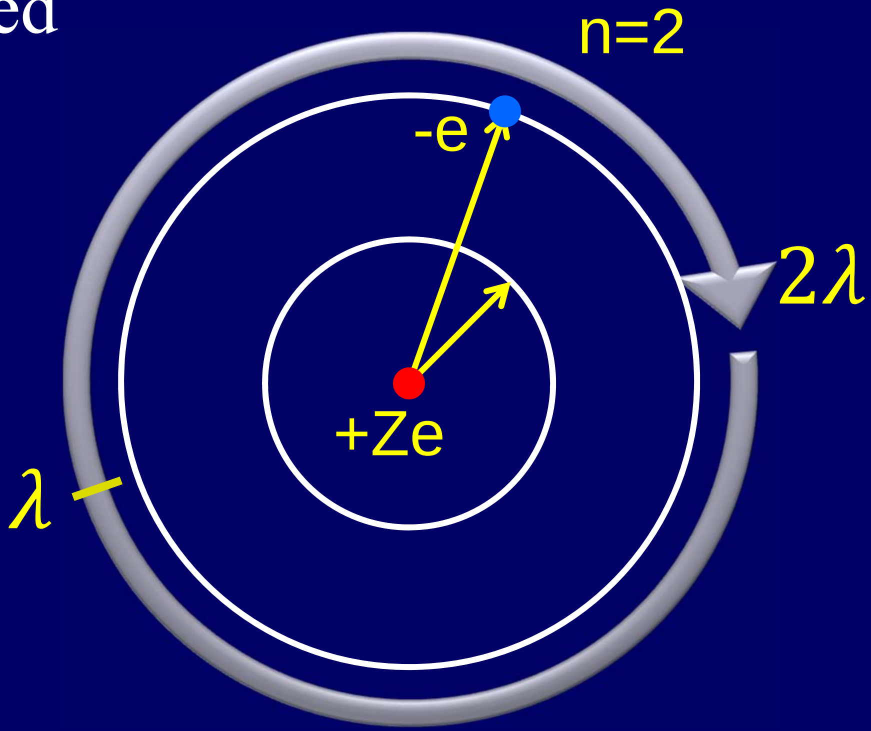
The Bohr Model

Electrons circle the nucleus in orbits

Only certain orbits are allowed

$$2\pi r = n\lambda$$
$$= nh/p$$

$$L = pr = nh/2\pi = n\hbar$$



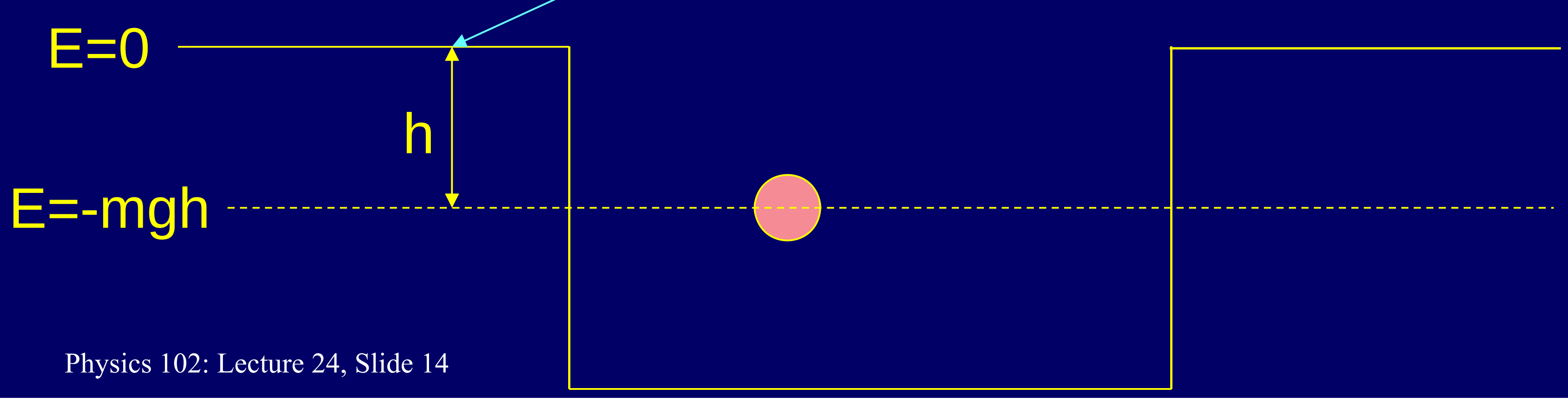
Angular momentum is quantized

Energy is quantized: $E = -13.6\ eV\ Z^2/n^2$ $\quad n = 1,2,3\ ...$

v is also quantized in the Bohr model!

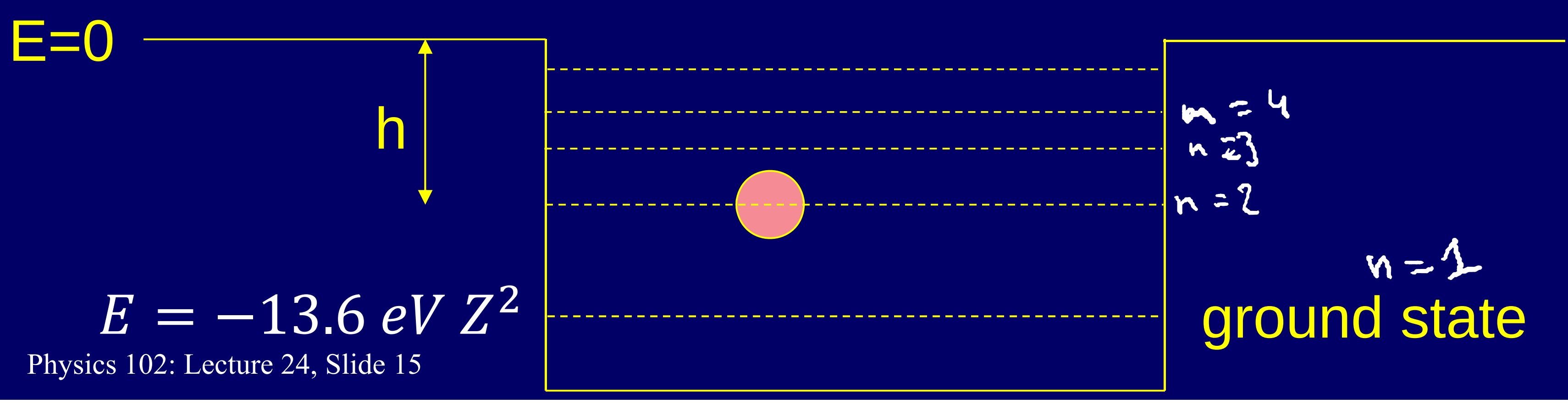
An analogy: Particle in Hole

- The particle is trapped in the hole
- To free the particle, need to provide energy mgh
- Relative to the surface, energy = -mgh
  - – a particle that is “just free” has 0 energy



An analogy: Particle in Hole

- Quantized: only fixed discrete heights of particle allowed
- Lowest energy (deepest hole) state is called the "ground state"



$E = -13.6\ eV\ Z^2$

Some (more) numerology

- 1 eV = kinetic energy of an electron that has been accelerated through a potential difference of 1 V

  $1\ \text{eV} = q\Delta V = 1.6 \times 10^{-19}\ \text{J}$

- h (Planck's constant) = $6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$

  $hc = 1240\ \text{eV}\cdot\text{nm}$

- m = mass of electron = 9.1×10^{-31} kg

  $mc^2 = 511{,}000\ \text{eV}$

- $U = ke^2/r$, so ke^2 has units eV·nm (like hc)

  $\frac{ke^2}{\hbar c} = 2\pi ke^2/(hc) = 1/137$ (dimensionless)

  "fine structure constant"

For Hydrogen-like atoms:

$L_n = n\hbar \quad n = 1, 2, 3 \ldots$ $\qquad E_{tot} = \frac{1}{2}mv^2 - k\frac{Ze^2}{r}$

Energy levels (relative to a "just free" E=0 electron):

$$\boxed{E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{Z^2}{n^2} \approx -\frac{13.6 \cdot Z^2}{n^2}\ \text{eV}\ \left(\text{where } \hbar \equiv h/2\pi\right)}$$

$$-\frac{1}{2}mc^2\left(\frac{ke^2}{\hbar c}\right)^2 = -\frac{1}{2}\frac{511{,}000\,\text{eV}}{137^2} = -13.6\ \text{eV}$$

Radius of orbit:

"Bohr radius"

$$\boxed{r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = \left(0.0529\ \text{nm}\right)\frac{n^2}{Z}}$$

p_n or v_n also quantized

Checkpoint 2

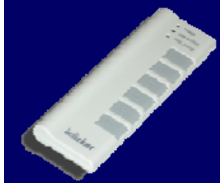
$$r_n = (\frac{h}{2\pi})^2 \frac{1}{mke^2}\frac{n^2}{Z} = \underbrace{(0.0529nm)}_{\text{Bohr radius}}\frac{n^2}{Z}$$

If the electron in the hydrogen atom was 207 times heavier (a muon), the Bohr radius would be

1) 207 Times Larger
2) Same Size
3) 207 Times Smaller

$$\text{Bohr Radius} = (\frac{h}{2\pi})^2 \frac{1}{mke^2}$$

This "m" is electron mass!



ACT/Checkpoint 3

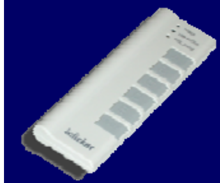
A single electron is orbiting around a nucleus with charge +3. What is its ground state (n=1) energy? (Recall for charge +1, E= -13.6 eV)

1) E = 9 (-13.6 eV)
2) E = 3 (-13.6 eV)
3) E = 1 (-13.6 eV)

$3^2/1 = 9$

$$E_n = -13.6\,eV\,\frac{Z^2}{n^2}$$

Note: This is LOWER energy since negative!



ACT: What about the radius?

Z=3, n=1

1. larger than H atom
2. same as H atom
3. smaller than H atom

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = (0.0529nm)\frac{n^2}{Z}$$

Summary

- Bohr's Model gives accurate values for electron energy levels... for H-like atoms
- But Quantum Mechanics is needed to describe electrons in atom.
- Next time: electrons jump between states by emitting or absorbing photons of the appropriate energy.