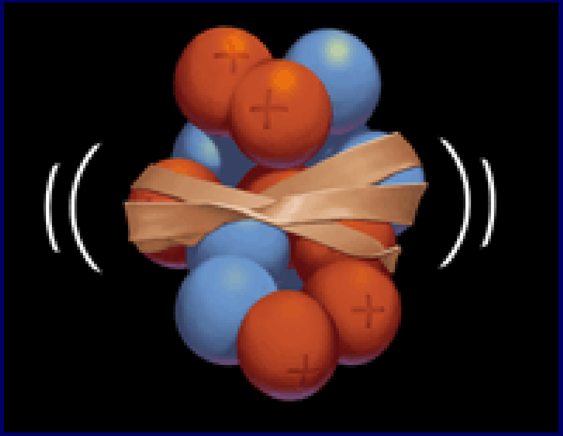
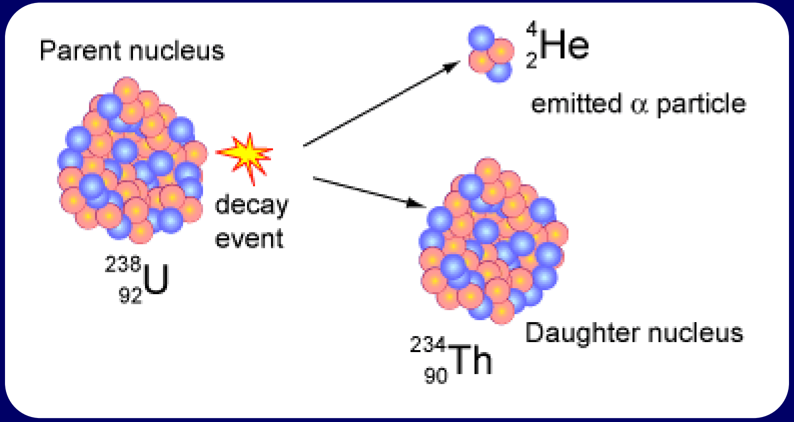
Physics 102: Lecture 28

Nuclear Binding, Radioactivity





$$E=mc^2$$

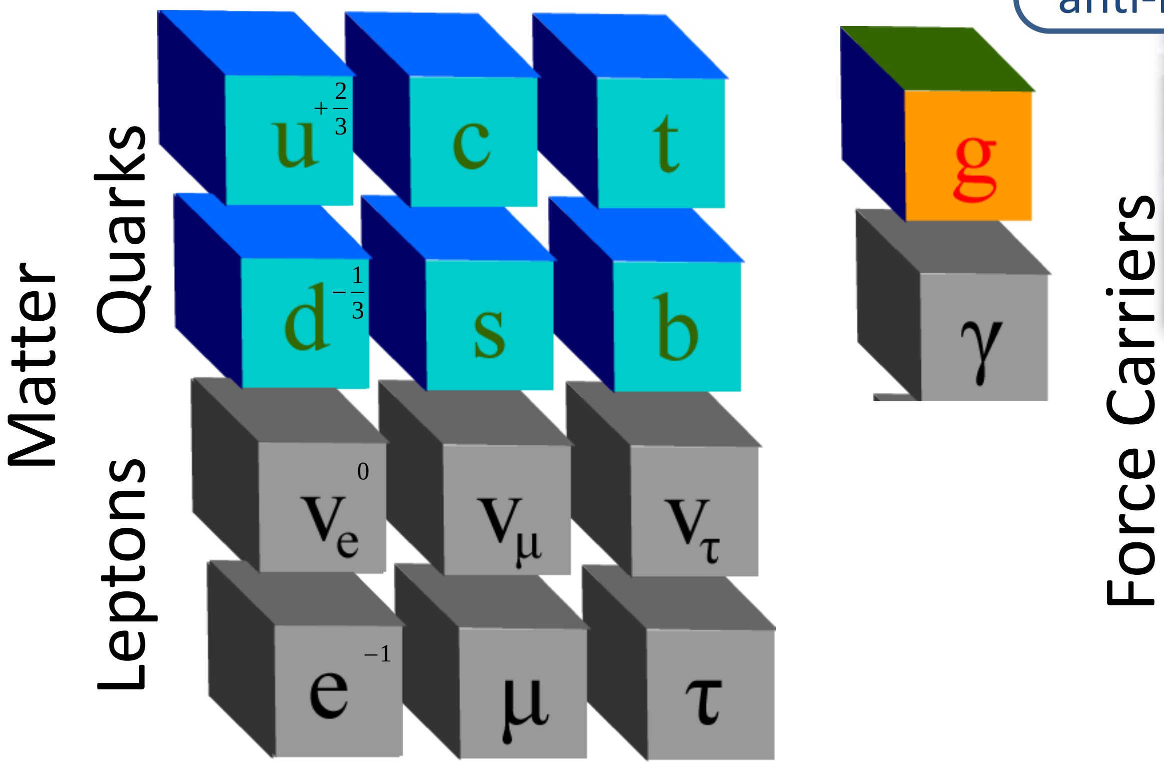
End-of-semester info

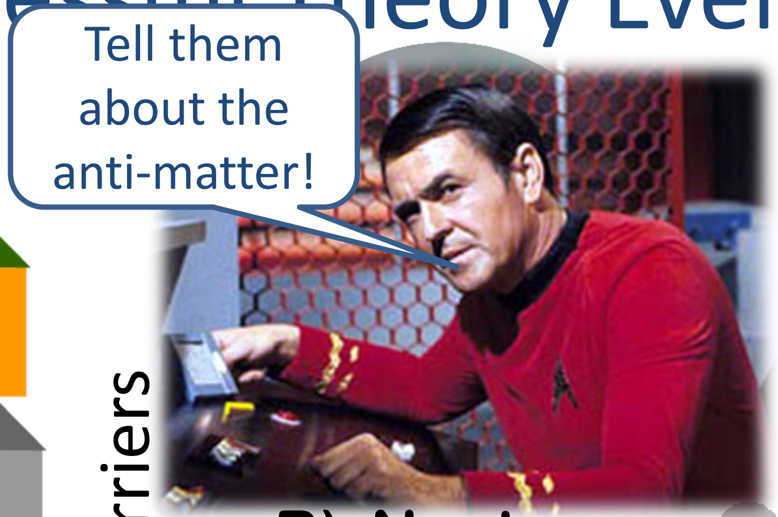
- Final exam info:
  - A1: Thursday, May 15, 1:30-4:30pm
  - A2: Friday, May 9, 1:30-4:30pm
  - Approximately 50 questions
  - Cumulative (all material from semester covered evenly)
  - Problem list from text available on home page.

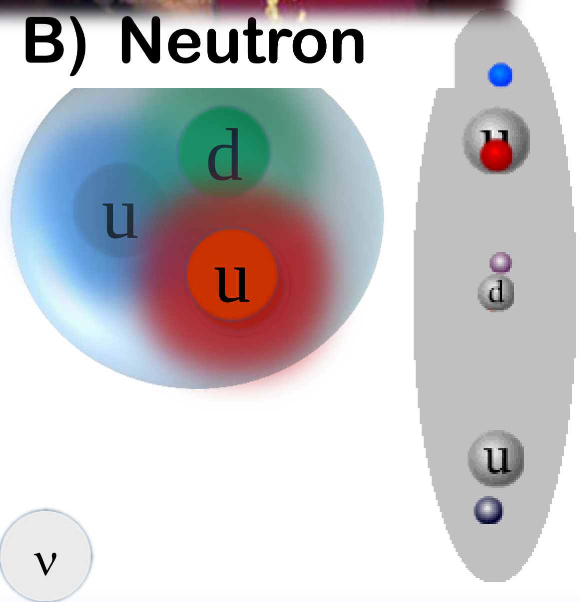
- CHECK GRADEBOOK!

Standard Model

a.k.a. The Most Successful Theory Ever!





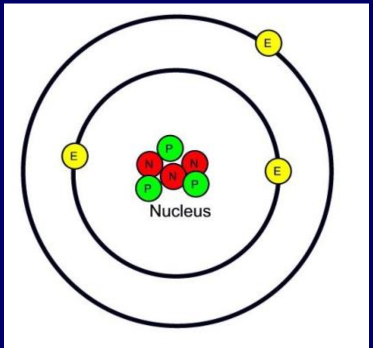


Back to our regularly scheduled lecture!

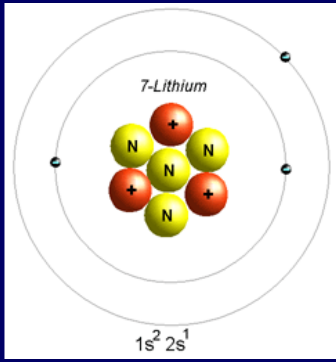
Nuclear Physics

A → 6
Z → 3

$^{6}_{3}\mathrm{Li}$



$^{7}_{3}\mathrm{Li}$



Z = proton number ("atomic number")
Gives chemical properties (and name)

N = neutron number (different "isotopes")

A = nucleon number (atomic mass number)
Gives you mass density of element

$$A=N+Z$$

Checkpoint 1.1

A material is known to be an isotope of lead.

Based on this information which of the following can you specify?

1) The atomic mass number

2) The neutron number

3) The number of protons

Chemical properties (and name) determined by number of protons (Z)

Lead → Z=82

Binding Energy

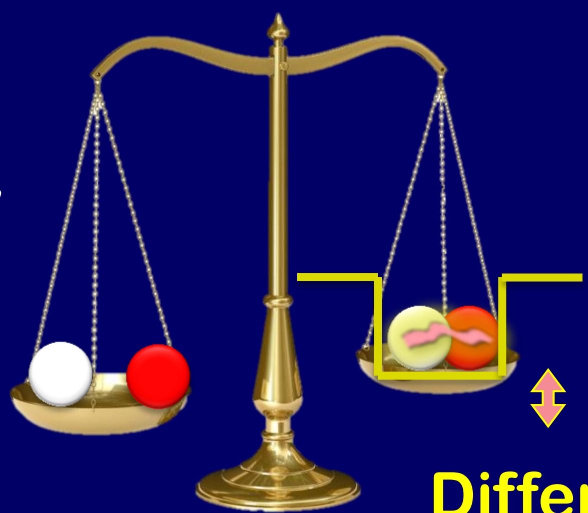
Einstein's famous equation $E = m c^2$

Example

proton:
$mc^2 = (1.67\times10^{-27}\text{kg})(3\times10^{8}\text{ m/s})^2 = 1.50\times10^{-10}\text{ J}$

Proton:
$mc^2 = 938.3\text{MeV}$

Neutron:
$mc^2 = 939.5\text{MeV}$



Deuteron:
$mc^2 = 1875.6\text{MeV}$

Difference = binding energy = 2.2MeV

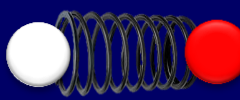
Adding these = 1877.8MeV

$$M_{Deuteron}\, c^2 = M_{Proton}\, c^2 + M_{Neutron}\, c^2 - |\text{Binding Energy}|$$

$$M_{Deuteron} = M_{Proton} + M_{Neutron} - |\text{Binding Energy}|/c^2$$

ACT: Binding Energy

Which system "weighs" more?

1) Two balls attached by a relaxed spring. 
2) Two balls attached by a stretched spring. 
3) They have the same weight.

$$M_1 = M_{balls} + M_{spring}$$

$$M_2 = M_{balls} + M_{spring} + E_{spring}/c^2$$

$$M_2 - M_1 = E_{spring}/c^2 \sim 10^{-16}\ \text{Kg}$$

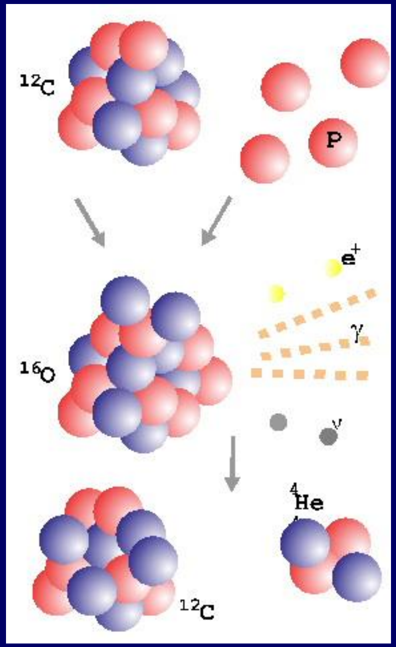
Checkpoint 1.2

Where does the energy released in the nuclear reactions of the sun come from?

(1) covalent bonds between atoms

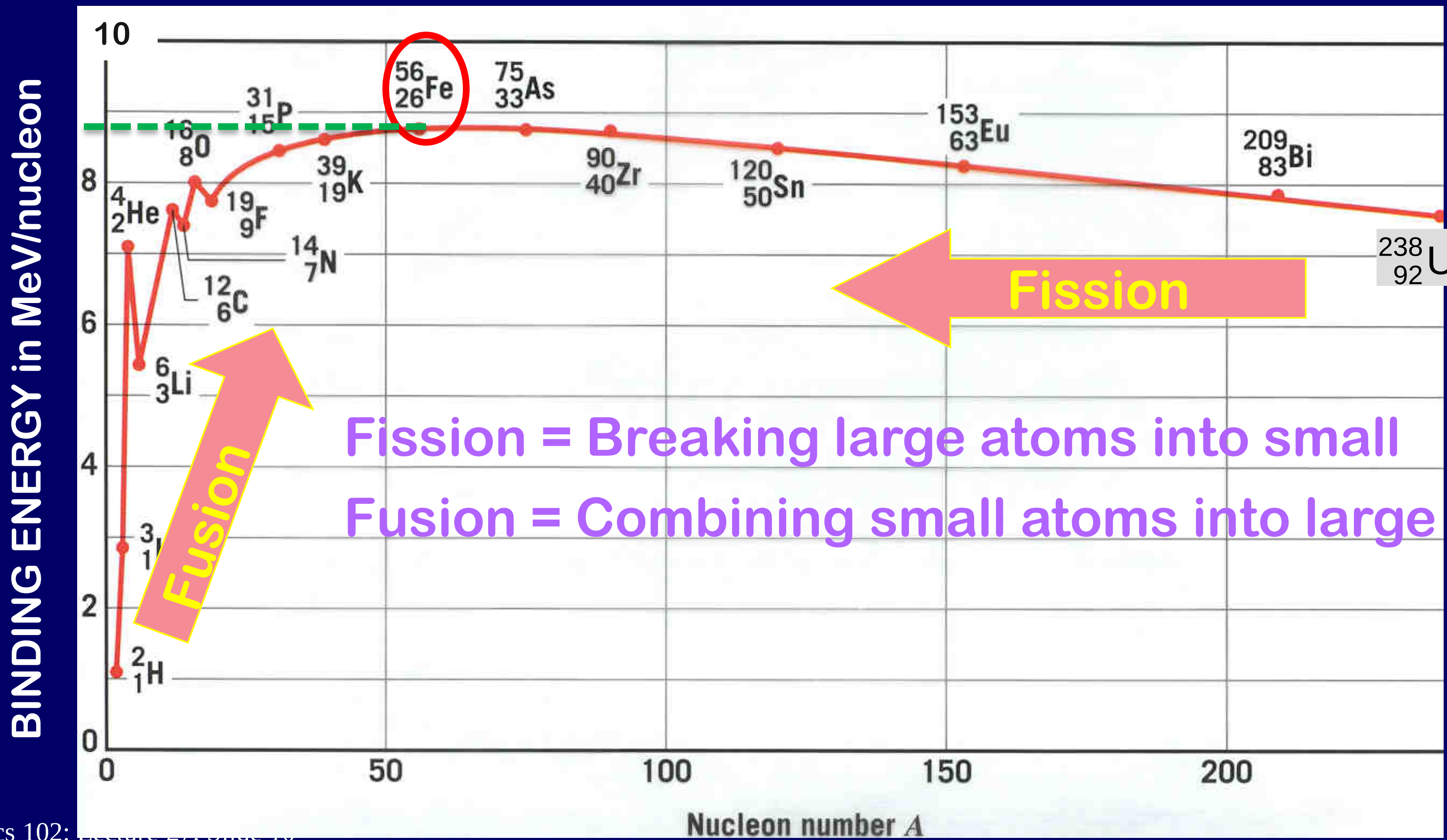
(2) binding energy of electrons to the nucleus

(3) binding energy of nucleons



Binding Energy Plot

Iron (Fe) has most binding energy/nucleon!
(Checkpoint 2.1)



Checkpoint 2.2

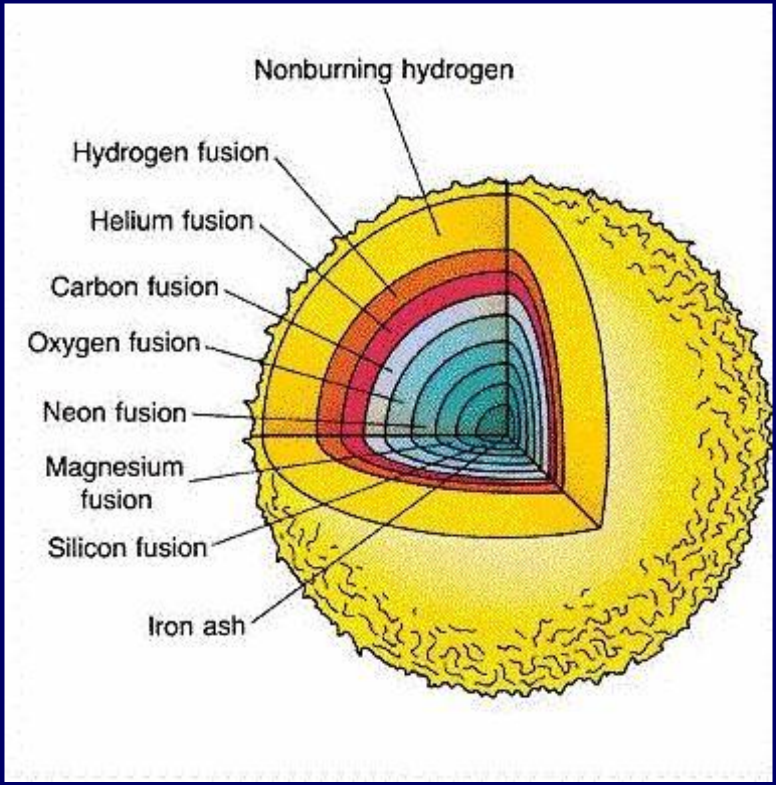
Which of the following is most correct for the total binding energy of an Iron nucleus (Z=26)?

13% 9 MeV

23% 234 MeV

28% 270 MeV

36% 504 Mev



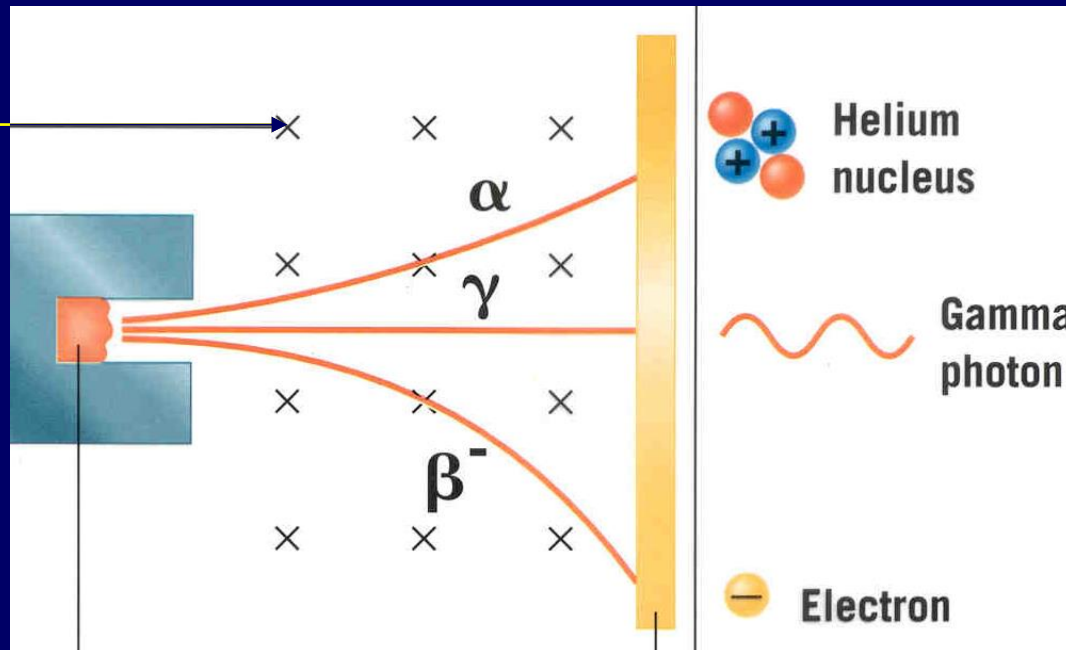
For Fe, B.E./nucleon ≈ 9MeV

$^{56}_{26}\mathrm{Fe}$ **has 56 nucleons**

Total B.E ≈ 56x9=504 MeV

3 Types of Radioactivity

Magnetic field into screen



Radioactive sources

detector

α particles: $^{4}_{2}He$ nuclei — Easily Stopped

β^{-} particles: electrons — Stopped by metal

γ : photons (more energetic than x-rays) penetrate!

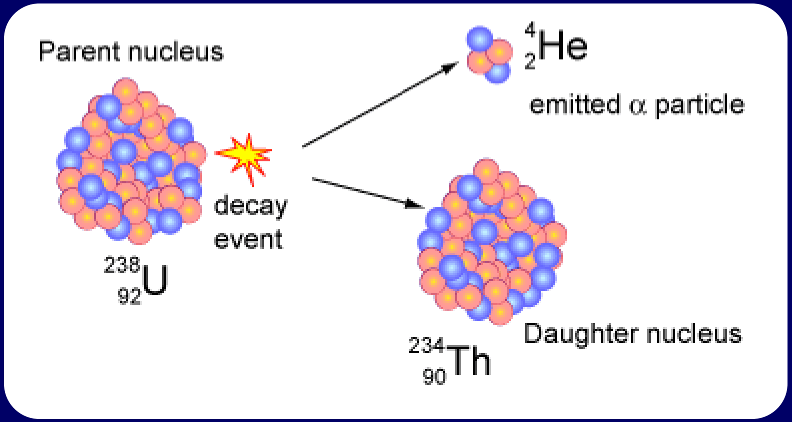
Decay Rules

Example

1) Nucleon Number (A) is conserved.
2) Charge is conserved.
3) Energy and momentum are conserved.

α: example $$^{238}_{92}U \rightarrow ^{234}_{90}Th + \alpha$$ recall $$^{4}_{2}He = \alpha$$

1) 238 = 234 + 4 Nucleon number conserved
2) 92 = 90 + 2 Charge conserved



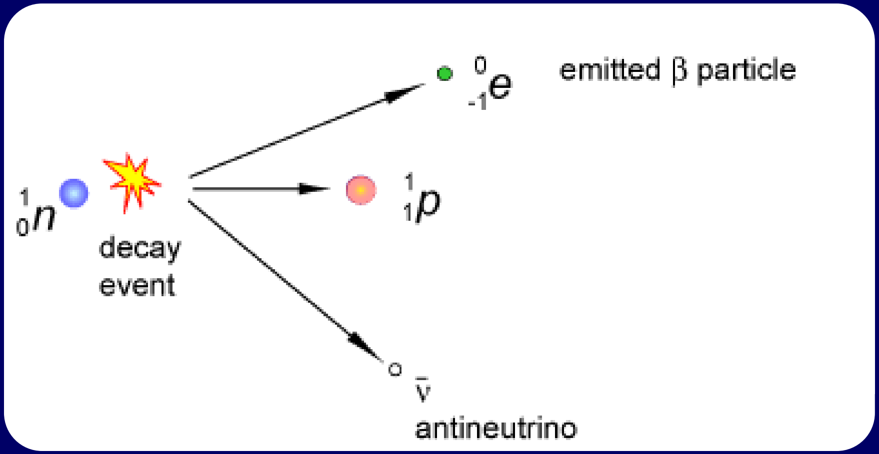
Example

Decay Rules

1) **Nucleon Number (A) is conserved.**
2) **Charge is conserved.**
3) **Energy and momentum are conserved.**

β: **example** $$ {}_{0}^{1}\mathrm{n} \rightarrow {}_{1}^{1}\mathrm{p} + {}_{-1}^{0}e^{-} + {}_{0}^{0}\bar{\nu} $$

anti-neutrino needed to conserve momentum.

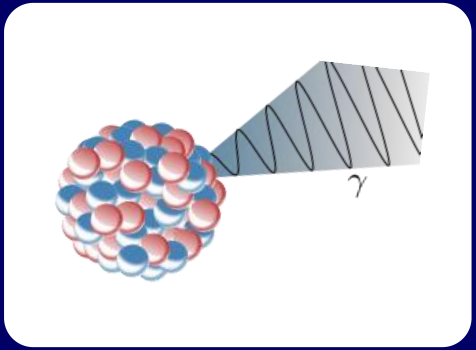


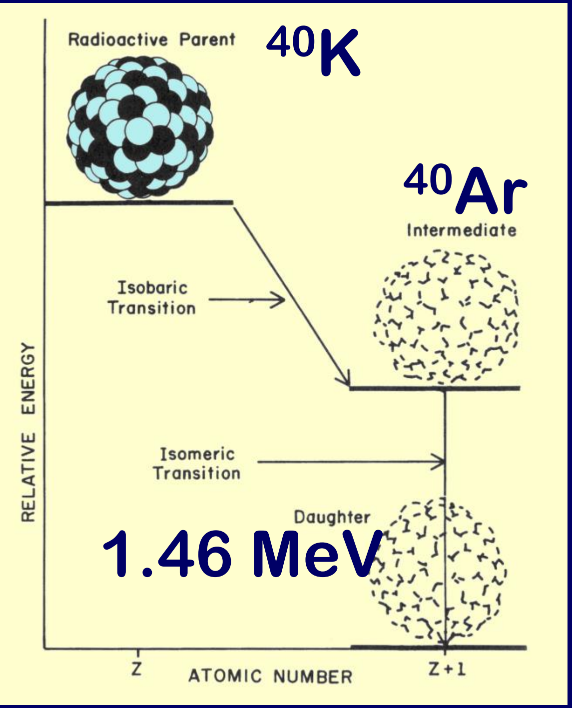
Decay Rules

1) Nucleon Number (A) is conserved.
2) Charge is conserved.
3) Energy and momentum are conserved.

γ: example $${}_{Z}^{A}P^{*} \rightarrow {}_{Z}^{A}P + {}_{0}^{0}\gamma$$

"nuclear isomer": excited state





ACT: Checkpoint 3

A nucleus undergoes α decay. Which of the following is FALSE?

A. Nucleon number decreases by 4 — 25%

B. Neutron number decreases by 2 — 32%

C. Charge on nucleus increases by 2 — 43%

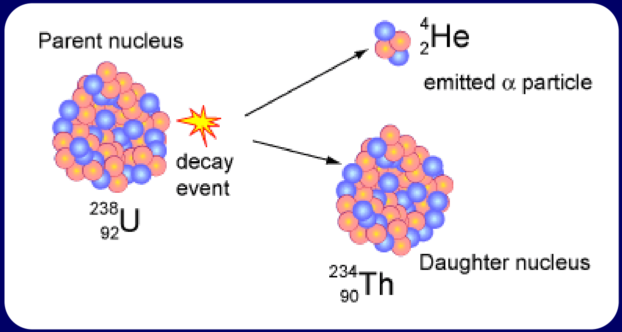
α decay is the emission of $^{4}_{2}\mathrm{He} = \alpha$

A decreases by 4

Ex. $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

Z decreases by 2

(charge decreases!)



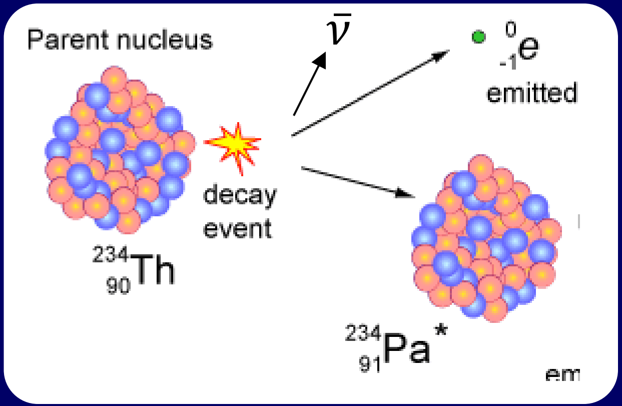
Checkpoint 4

The nucleus ${}^{234}_{90}\mathrm{Th}$ undergoes β^- decay.

Which of the following is true?

A. The number of protons in the daughter nucleus increases by one.

B. The number of neutrons in the daughter nucleus increases by one.



β^- decay is accompanied by the emission of an electron: creation of a charge -e.

$${}^{234}_{90}\mathrm{Th} \rightarrow {}^{???}_{??}\mathrm{X} + {}^{0}_{-1}e^- + {}^{0}_{0}\bar{\nu}$$

$${}^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}e^- + {}^{0}_{0}\bar{\nu}$$

In fact, $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$ inside the nucleus, and the electron and neutrino "escape."

ACT: Decay

Which of the following decays is NOT allowed?

A $^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \alpha$

238 = 234 + 4

92 = 90 + 2

B $^{214}_{84}\mathrm{Po} \rightarrow {}^{210}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

214 = 210 + 4

84 = 82 + 2

C $^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma$

14 = 14+0

$6 \neq 7+0$

D $^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca} + {}^{0}_{-1}e^{-} + {}^{0}_{0}\bar{\nu}$

40 = 40+0+0

19 = 20-1+0

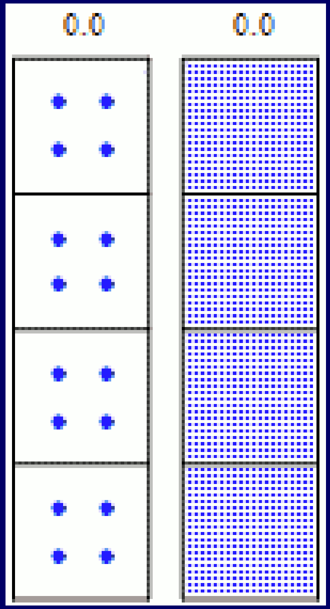
Radioactive decay rates

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Decays per second or "activity"

No. of nuclei present

decay constant



Checkpoint 5

If the number of radioactive nuclei present is cut in half, how does the activity change?

A) It remains the same 22%

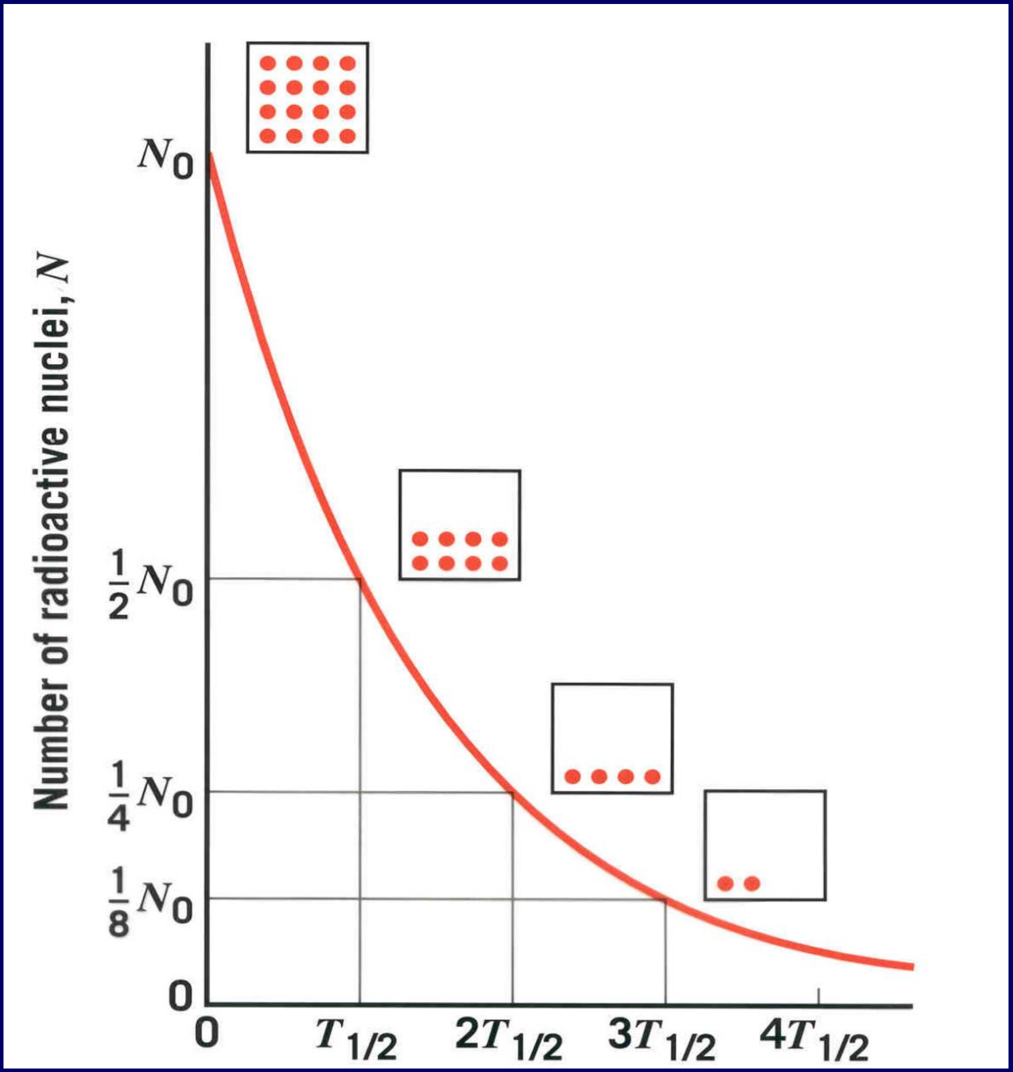
B) It is cut in half 55%

C) It doubles 23%

1 "becquerel" (Bq) =1 decay/s

1 "curie" Ci =3.7×10^{10} Bq

Decay Function



time ⟶

$$N(t) = N_0 e^{-\lambda t} = N_0 \cdot 2^{-\frac{t}{T_{1/2}}}$$

# of nuclei present at time t

we started with at t=0

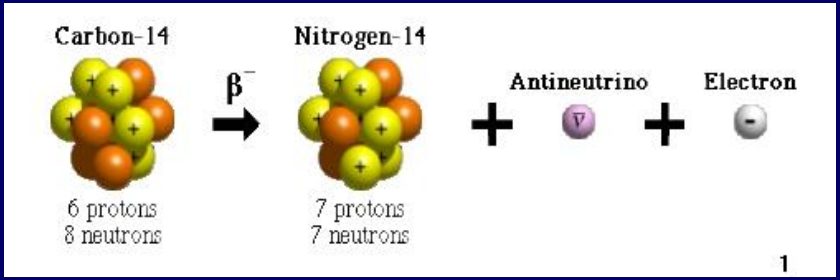
$$T_{1/2} = \frac{0.693}{\lambda}$$

Half of the remaining atoms are lost after a "half life" that lasts $T_{1/2}$; the activity also is reduced by one half every $T_{1/2}$

Example

You are radioactive!

One in 8.3×10^{11} carbon atoms is ^{14}C which β^- decays with a ½ life of 5730 years. Determine # of decays/s per gram of Carbon.



$$N_{14} = \left(\frac{1.0}{12}\frac{\text{mole}}{g}\right)\left(6.02 \times 10^{23}\right)\left(\frac{1}{8.3 \times 10^{11}}\right) = 6 \times 10^{10}\ \frac{\text{atoms}}{\text{g}}$$

$$\lambda = \frac{.693}{T_{1/2}} = \frac{.693}{5730 \times 365 \times 24 \times 60 \times 60} = 3.83 \times 10^{-12}\,\text{s}^{-1}$$

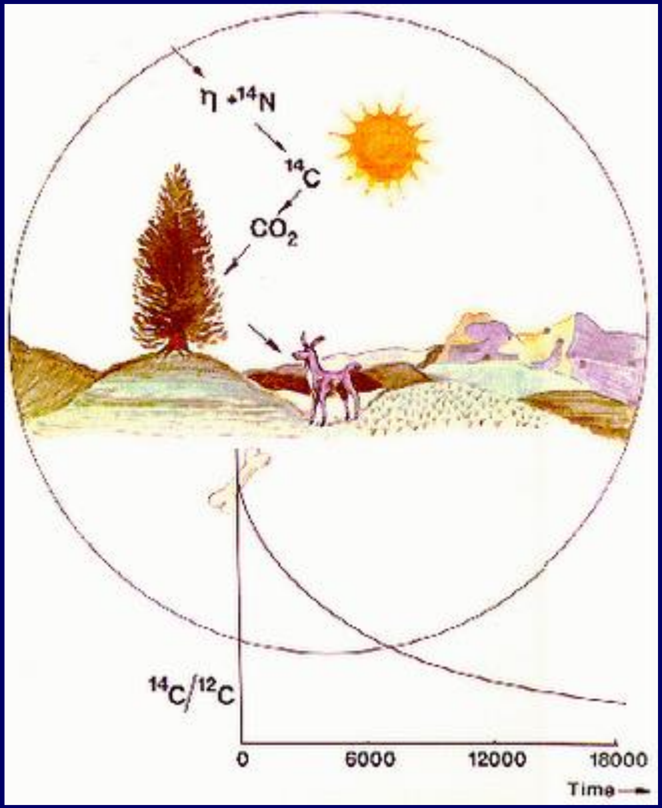
$$\frac{\Delta \text{N}}{\Delta \text{t}} = -\lambda \text{N} = 0.23\ \text{decays/s}$$

Example

Carbon Dating

We just determined that living organisms should have a decay rate of about 0.23 events/s per gram of carbon.

The bones of a man from the last ice age are found to have a decay rate of 0.23/2 events/s per gram. We can estimate he died about 6000 years ago.



ACT/Checkpoint 6

The half-life for beta-decay of ^{14}C is ~6,000 years. You test a fossil and find that only 25% of its ^{14}C is un-decayed. How old is the fossil?

A. 3,000 years

B. 6,000 years

C. 12,000 years

At 0 years: 100% remains

At 6,000 years: 50% remains

At 12,000 years: 25% remains

Summary

- Nuclear Reactions
  - Nucleon number conserved
  - Charge conserved
  - Energy/Momentum conserved
  - α particles = $^{4}_{2}He$ nuclei
  - β^- particles = electrons
  - γ particles = high-energy photons

Activity: $$\frac{\Delta N}{\Delta t} = -\lambda N$$ **Survival:** $$N(t) = N_0 e^{-\lambda t}$$ $$T_{1/2} = \frac{0.693}{\lambda}$$

- Decays
  - Half-Life is time for ½ of atoms to decay