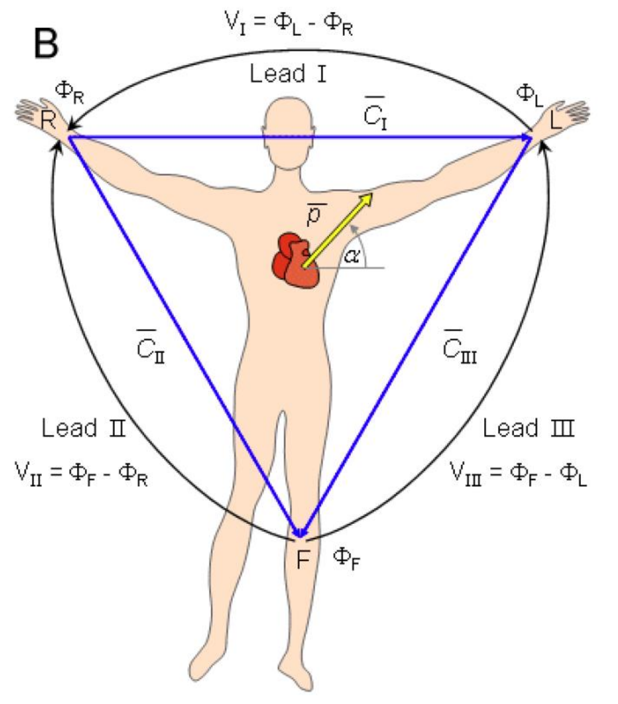
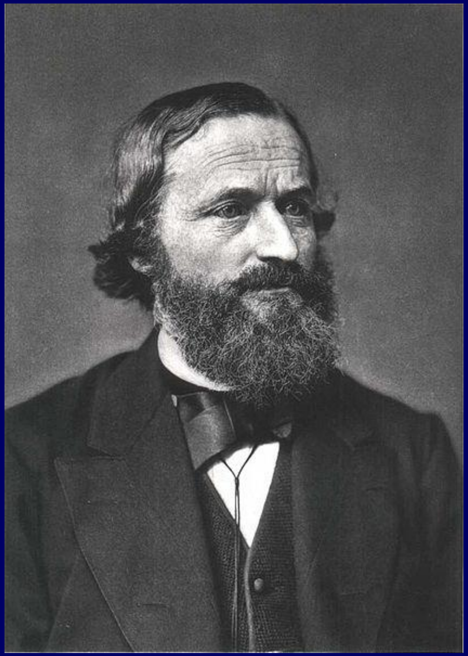
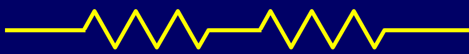
Physics 102: Lecture 06

Kirchhoff's Laws





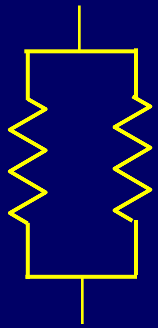
Last Time



Last Lecture

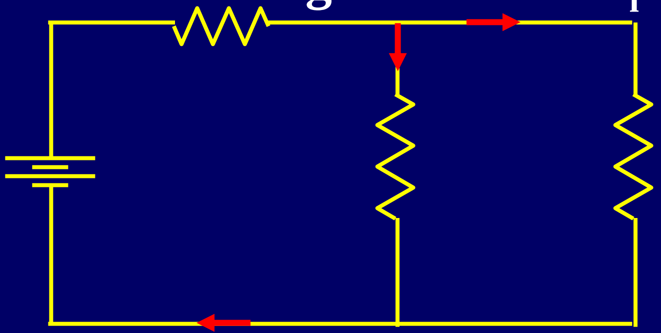
- **Resistors in series:** $R_{eq} = R_1 + R_2 + R_3 + \cdots$

  Current through each is same; Voltage drop is IR_i



- **Resistors in parallel:** $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

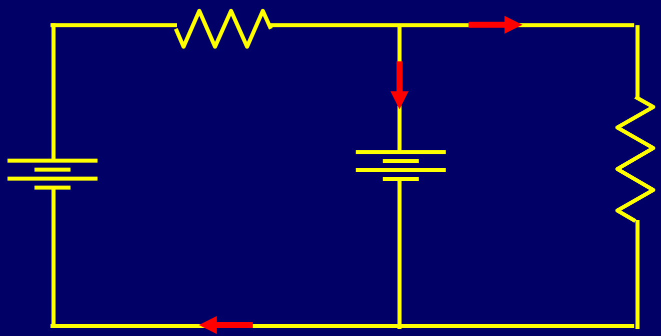
  Voltage drop across each is the same; Current through each is V/R_i



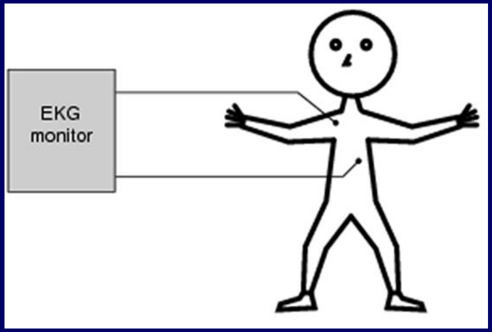
- **Solved Circuits**

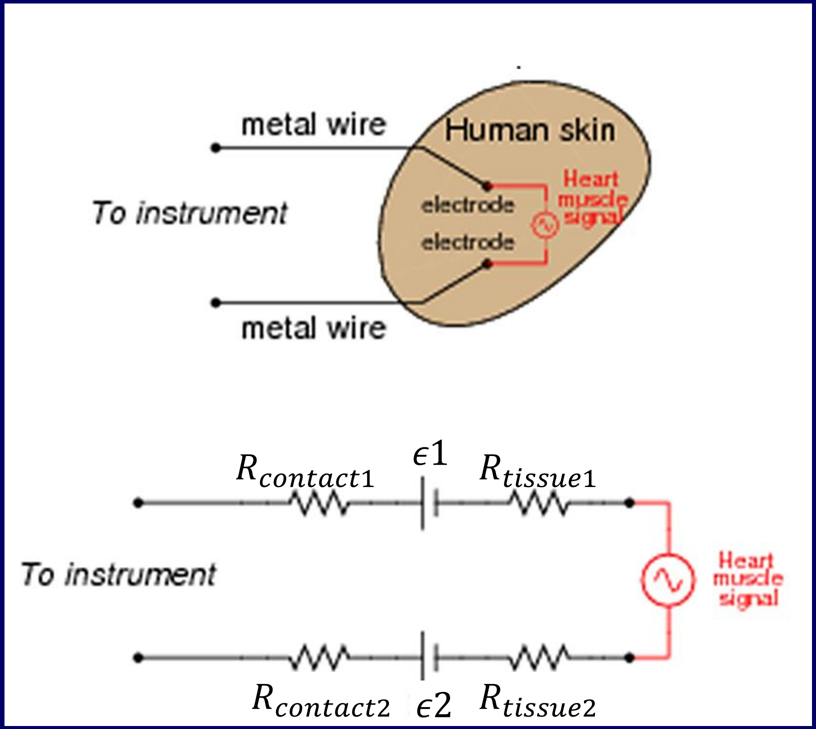
Today

- **What about this one?**



When would you ever find a circuit like this?!?!





$\epsilon 1$ and $\epsilon 2$ arise from surface electrochemistry!

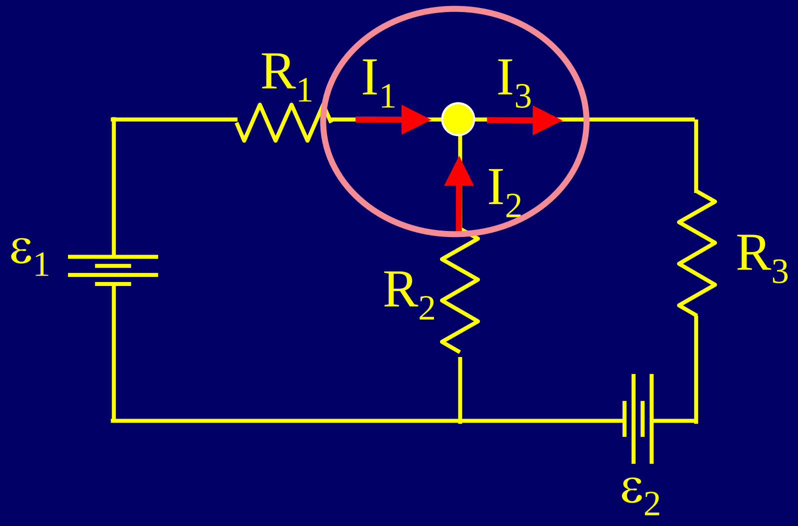
Kirchhoff's Rules

- Kirchhoff's Junction Rule (KJR):
  - Current going in equals current coming out.

- Kirchhoff's Loop Rule (KLR):
  - Sum of voltage drops around a loop is zero.

Kirchhoff's Junction Rule (KJR)

- Conceptual basis: conservation of charge
- At any junction in a circuit, the current that enters the junction equals the current that leaves the junction
- Example:

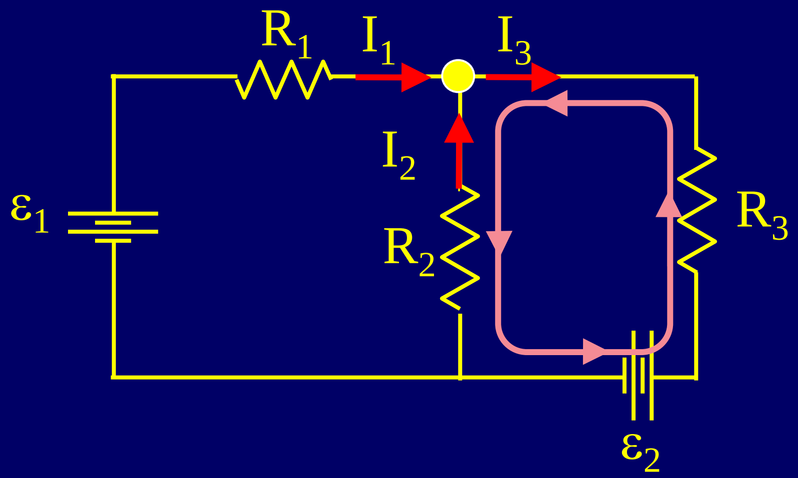


At junction:

$$I_1 + I_2 = I_3$$

Kirchhoff's Loop Rule (KLR)

- Conceptual basis: conservation of energy
- Going around any complete loop in a circuit, <u>the sum total of all the potential differences is zero</u>
- Example:



Around the right loop:

$$\varepsilon_2 + I_3R_3 + I_2R_2 = 0$$

Using Kirchhoff's Rules

(1) Label all currents

Choose any direction

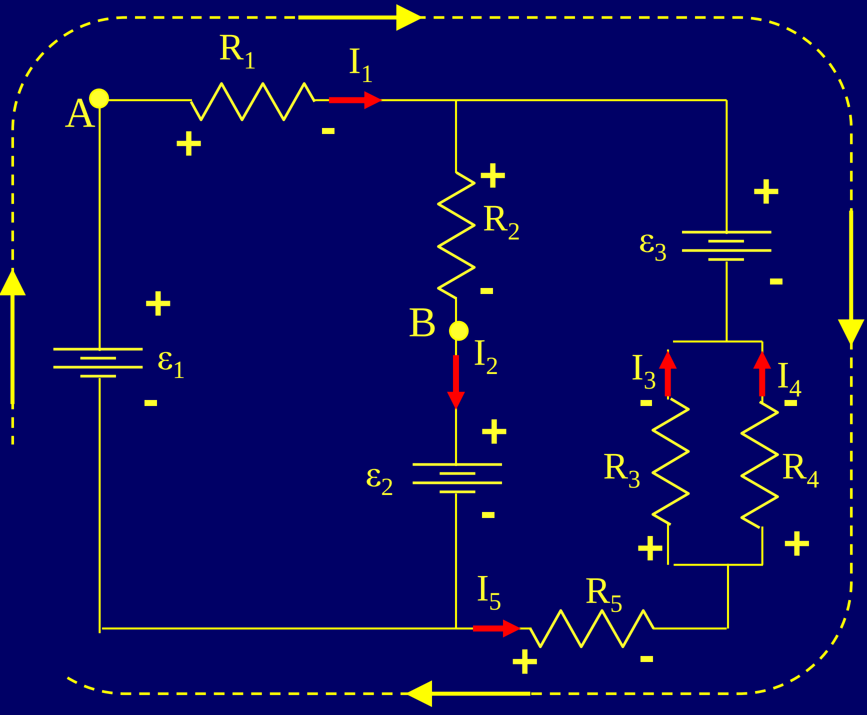
(2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

(3) Choose loop and direction

(4) Write down voltage <u>gains</u>

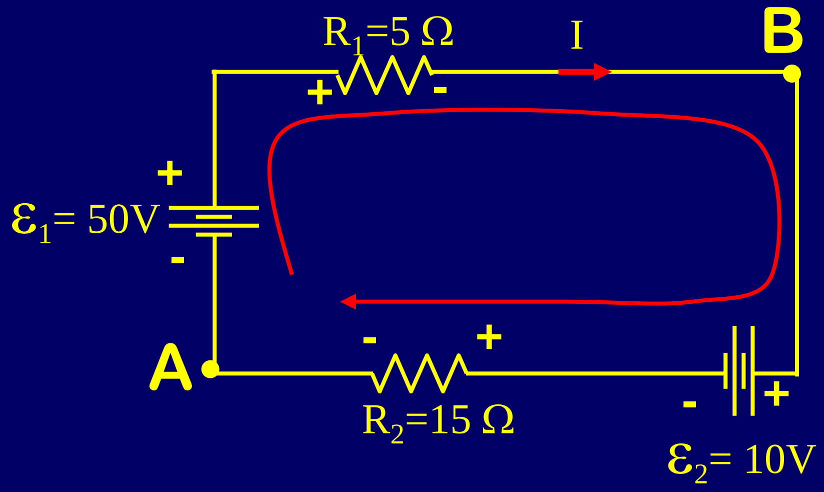
Be careful about signs



Loop Rule Practice

Find I:

1. Label currents
2. Label elements +/-
3. Choose loop
4. Write KLR



$$+\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

$$+50 - 5\,I - 10 - 15\,I = 0$$

$$I = +2\ \text{Amps}$$

What is the electric potential at V_B (assume $V_A = 0$):

$$V_A + \mathcal{E}_1 - IR_1 = V_B$$

$$0 + 50 - 2 \times 5 = V_B$$

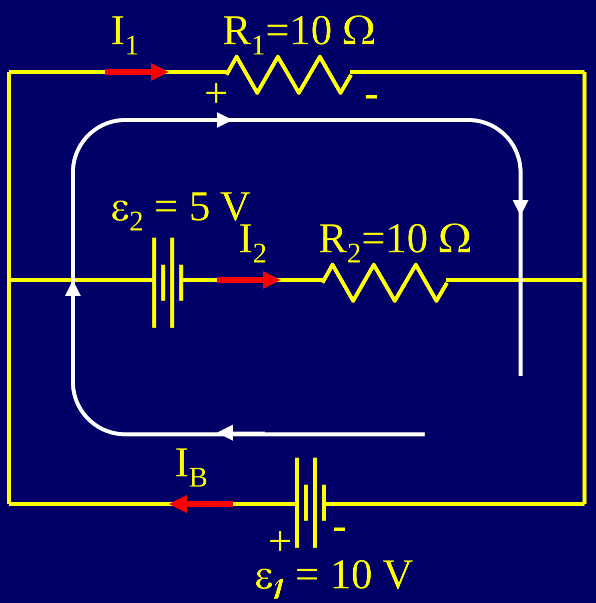
$$40V = V_B$$

Checkpoint 1

Calculate the current through R_1.

27%	57%	15%
1) $I_1 = 0.5$ A	2) $I_1 = 1.0$ A	3) $I_1 = 1.5$ A

$$\varepsilon_1 - I_1 R_1 = 0 \Rightarrow I_1 = \varepsilon_1 / R_1 = 1\text{A}$$

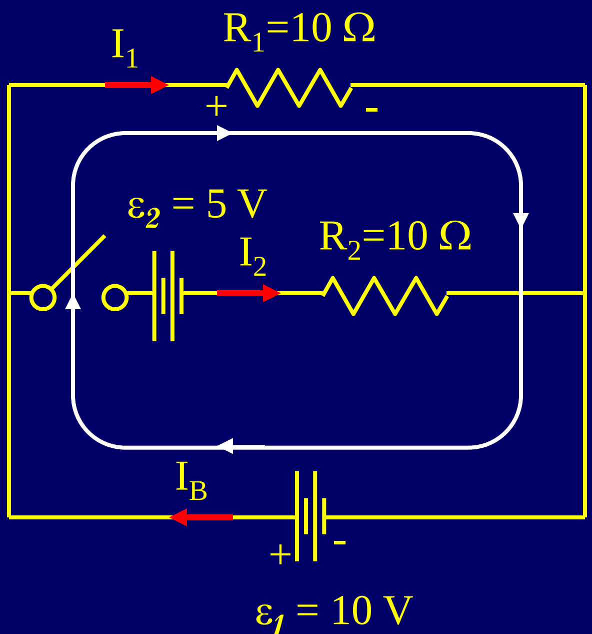


Checkpoint 1

Calculate the current through R_1.

33%	**48%**	**19%**
1) $I_1 = 0.5$ A	2) $I_1 = 1.0$ A	3) $I_1 = 1.5$ A

$$\varepsilon_1 - I_1 R_1 = 0 \Rightarrow I_1 = \varepsilon_1 / R_1 = 1\text{A}$$



ACT: Voltage Law

How would I_1 change if the switch was opened?

1) Increase

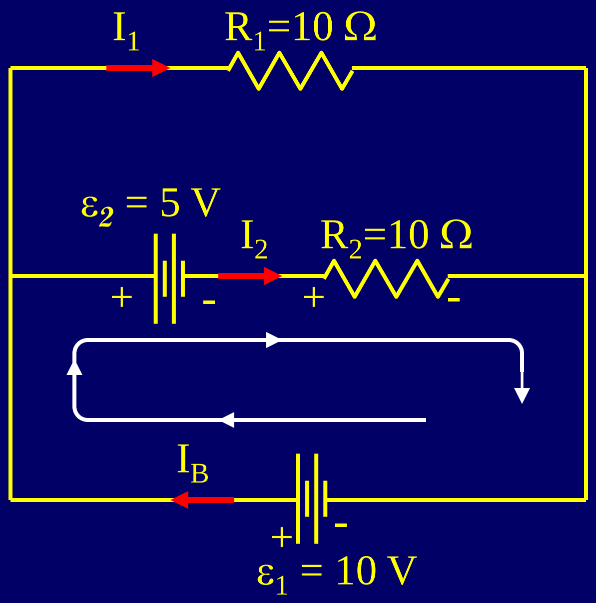
2) No change

3) Decrease

ACT/Checkpoint 2

Calculate the current through R_2.

55%	27%	18%
A) $I_2 = 0.5$ A	B) $I_2 = 1.0$ A	C) $I_2 = 1.5$ A

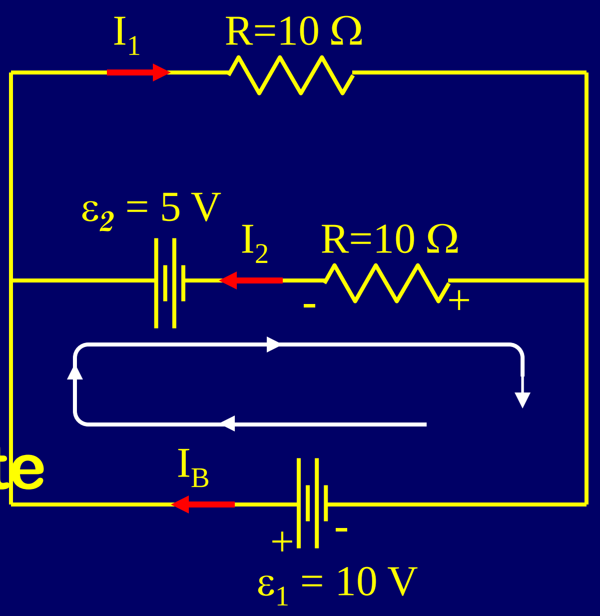


$$\varepsilon_1 - \varepsilon_2 - I_2 R_2 = 0$$

$$\Rightarrow I_2 = 0.5\text{A}$$

Checkpoint 2

"How do I know the direction of I_2?"
It doesn't matter. Choose whatever direction you like. Then solve the equations to find I_2.
If the result is positive, then your initial guess was correct. If result is negative, then actual direction is opposite to your initial guess.



Work through Checkpoint with opposite sign for I_2:

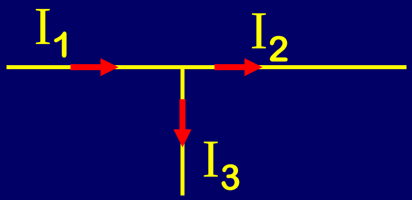
$+\varepsilon_1 - \varepsilon_2 + I_2 R_2 = 0$ Note the sign change from last slide

$\Rightarrow I_2 = -0.5A$ Answer has same magnitude as before but opposite sign. That means current goes to the right, as we found before.

Kirchhoff's Junction Rule

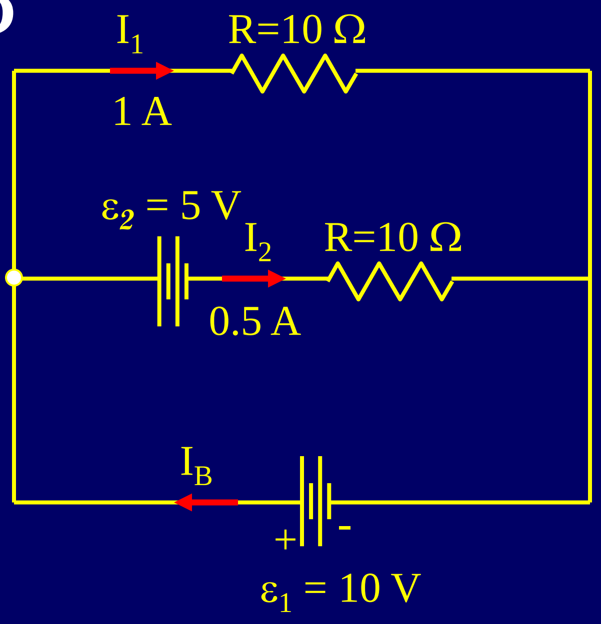
Current Entering = Current Leaving

$I_1 = I_2 + I_3$



ACT/Checkpoint 3

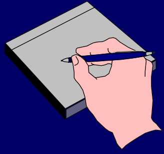
Calculate the current through the 10V battery.



18%	36%	46%
1) $I_B = 0.5$ A	2) $I_B = 1.0$ A	3) $I_B = 1.5$ A

$I_B = I_1 + I_2 = 1.5$ A

Kirchhoff's Laws



(1) Label all currents

Choose any direction

(2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

(3) Choose loop and direction

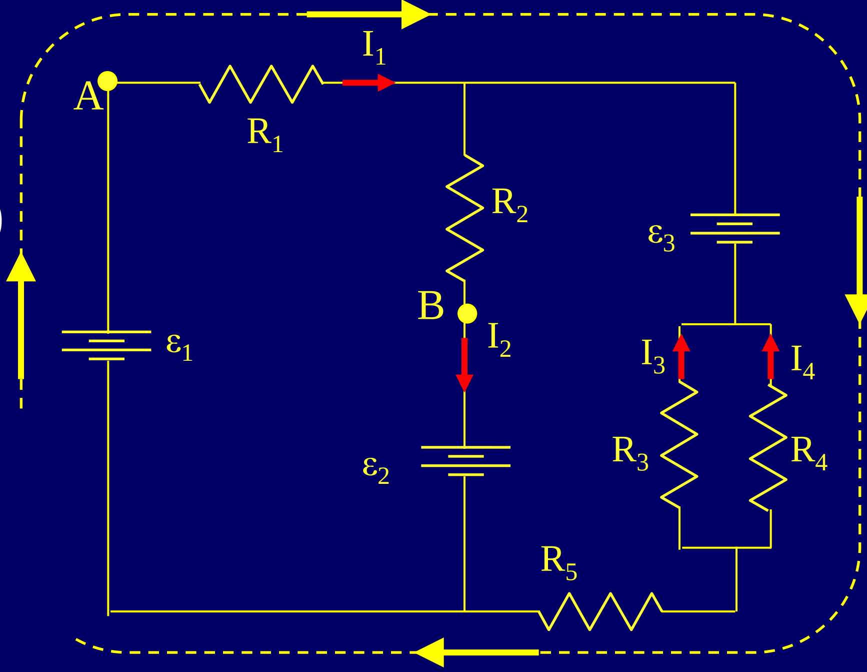
Your choice!

(4) Write down voltage gains

Follow any loops

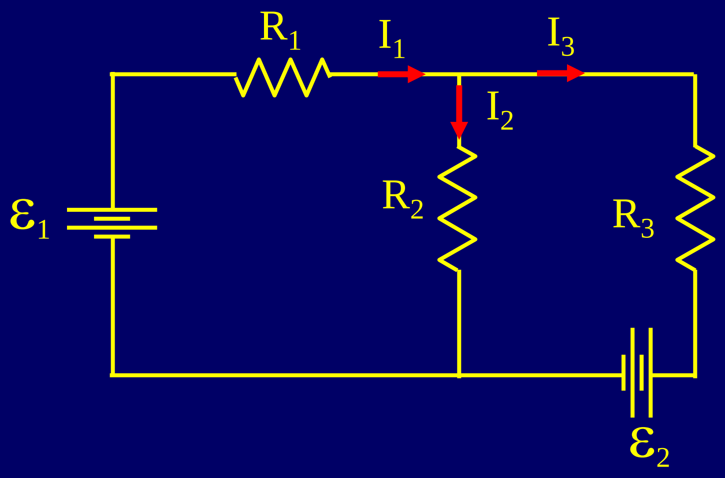
(5) Write down junction equation

$I_{in} = I_{out}$



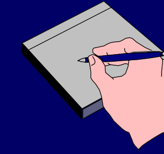
You try it!

In the circuit below you are given ε_1, ε_2, R_1, R_2 and R_3. Find I_1, I_2 and I_3.



Example

You try it!



In the circuit below you are given $\mathcal{E}_1$, $\mathcal{E}_2$, R_1, R_2 and R_3. Find I_1, I_2 and I_3.

✓ **1. Label all currents** (Choose any direction)

✓ **2. Label +/- for all elements** (Current goes + ⇒ - for resistor)

✓ **3. Choose loop and direction** (Your choice!)

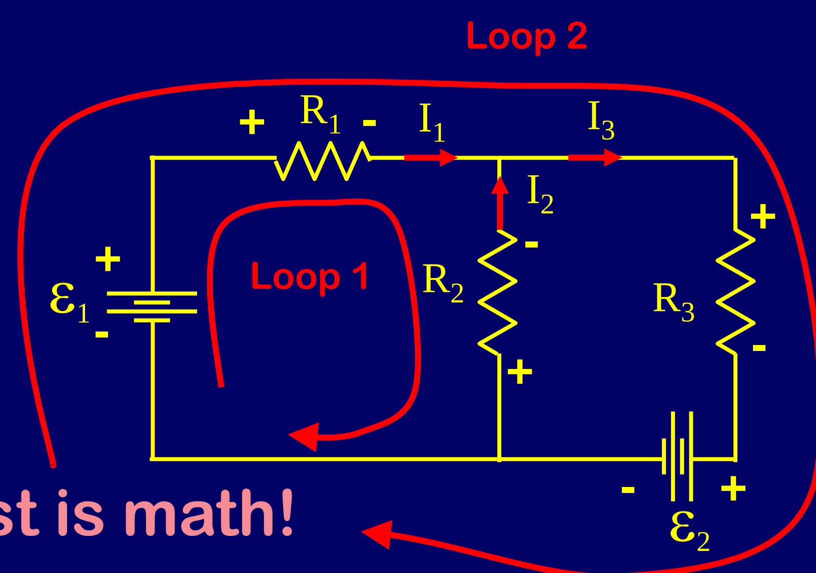
✓ **4. Write down voltage drops** (Potential increases or decreases?)

Loop 1: $+\mathcal{E}_1 - I_1R_1 + I_2R_2 = 0$

Loop 2: $+\mathcal{E}_1 - I_1R_1 - I_3R_3 - \mathcal{E}_2 = 0$

✓ **5. Write down junction equation**

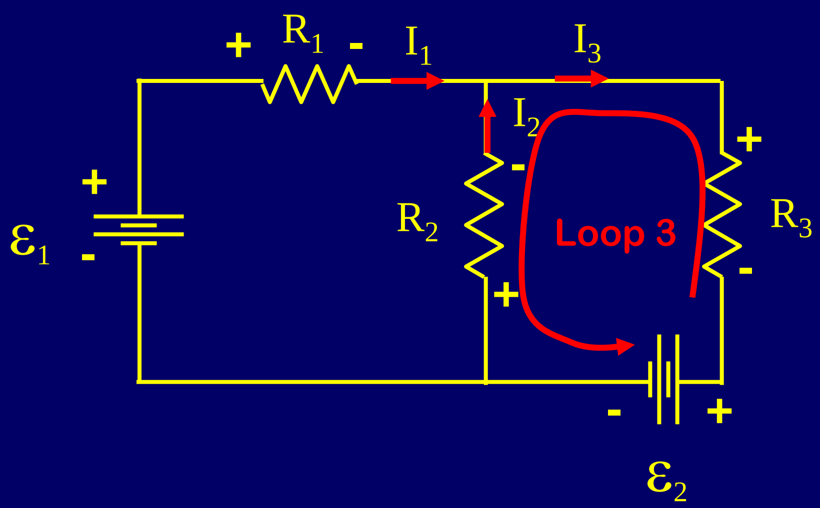
Node: $I_1 + I_2 = I_3$



3 Equations, 3 unknowns the rest is math!

ACT: Kirchhoff loop rule

What is the correct expression for “Loop 3” in the circuit below?



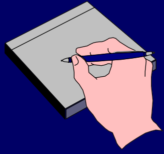
1) $+\varepsilon_2 - I_3R_3 - I_2R_2 = 0$

2) $+\varepsilon_2 - I_3R_3 + I_2R_2 = 0$

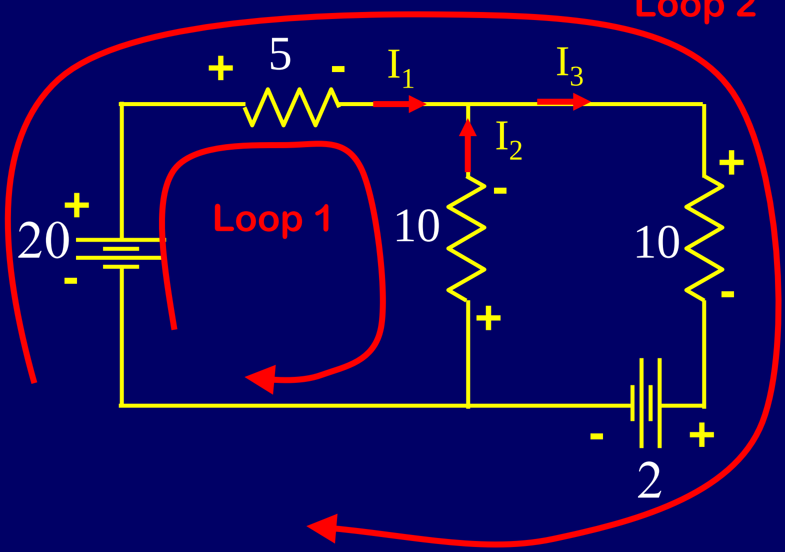
3) $+\varepsilon_2 + I_3R_3 + I_2R_2 = 0$

Example

Let's put in real numbers



In the circuit below you are given $\mathcal{E}_1$, $\mathcal{E}_2$, R_1, R_2 and R_3. Find I_1, I_2 and I_3.



1. **Loop 1: $20 - 5I_1 + 10I_2 = 0$**
2. **Loop 2: $20 - 5I_1 - 10I_3 - 2 = 0$**
3. **Junction: $I_3 = I_1 + I_2$**

solution: substitute Eq.3 for I_3 in Eq. 2:

$20 - 5I_1 - 10(I_1+I_2) - 2 = 0$

rearrange: $15I_1 + 10I_2 = 18$

rearrange Eq. 1: $5I_1 - 10I_2 = 20$

Now we have 2 eq., 2 unknowns. Continue on next slide

$15I_1+10I_2 = 18$
$5I_1 - 10I_2 = 20$

Now we have 2 eq., 2 unknowns.

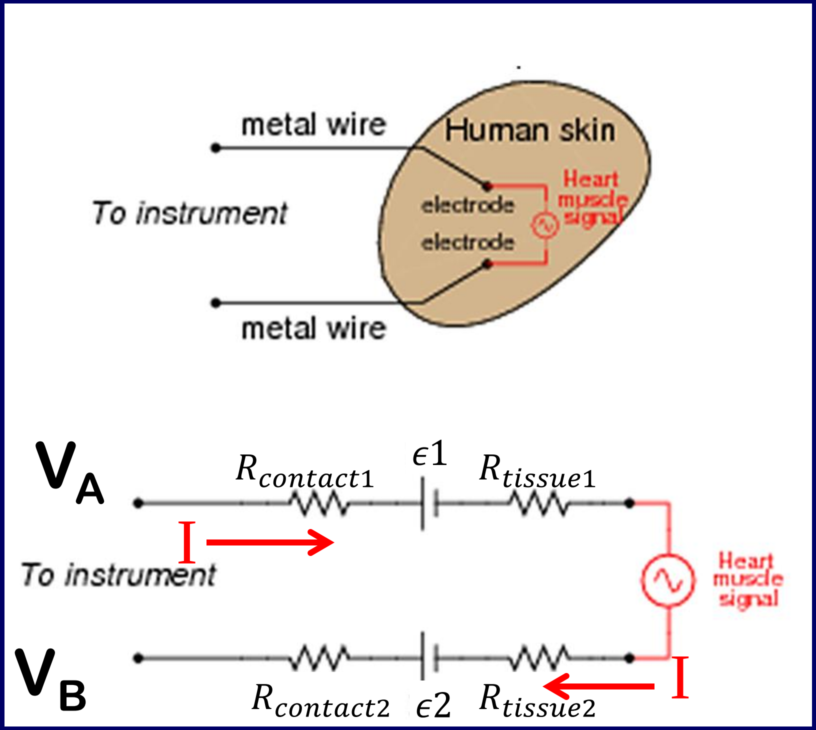
Add the equations together:
$20I_1=38$ $I_1=1.90$ A

Plug into bottom equation:
$5(1.90)-10I_2 = 20$ $I_2=-1.05$ A
note that this means direction of I_2 is opposite to that shown on the previous slide

Use junction equation (eq. 3 from previous page)
$I_3=I_1+I_2 = 1.90-1.05$
$I_3 = 0.85$ A

We are done!

Back to where we started…



$$V_{EKG} = V_A - V_B$$

$$V_A - IR_{c1} - \epsilon 1 - IR_{t1} + V_{sig} - IR_{t2} + \epsilon 2 - IR_{c2} = V_B$$

$$V_A - V_B = IR_{c1} + \epsilon 1 + IR_{t1} - V_{sig} + IR_{t2} - \epsilon 2 + IR_{c2}$$