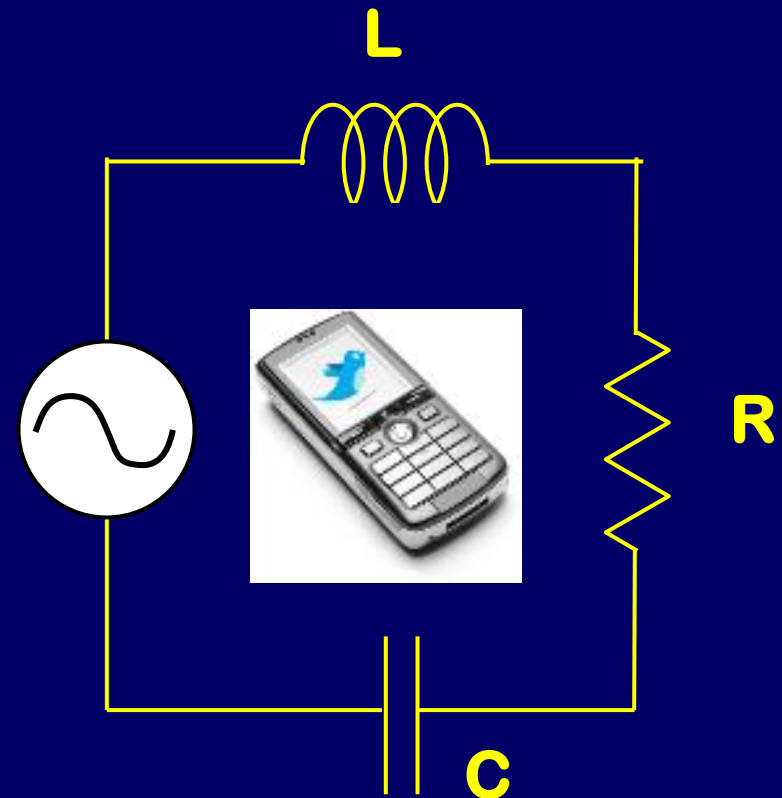
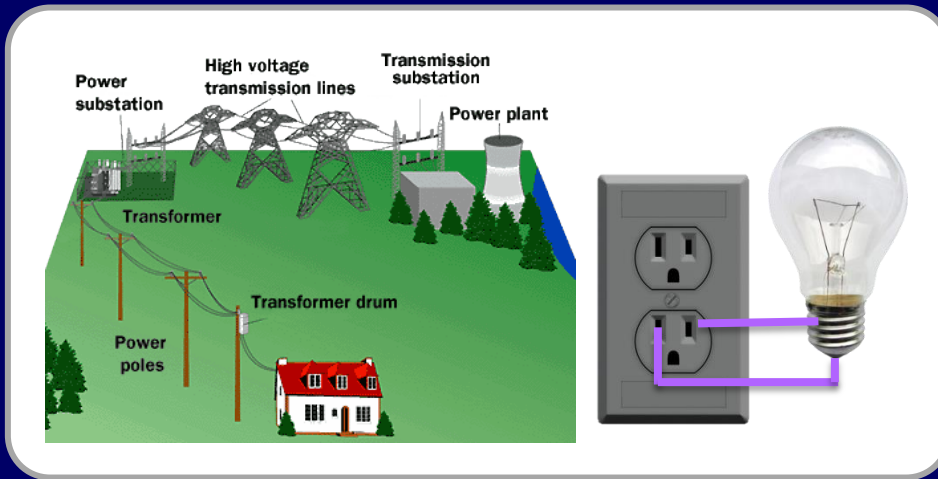


# Physics 102: Lecture 12

## AC & RLC Circuits

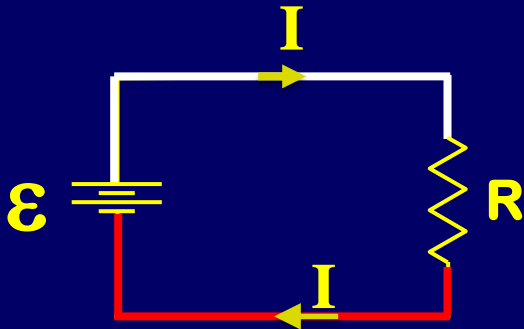
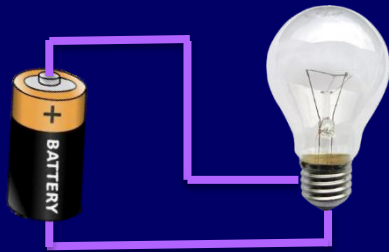


Ch 21 Prob. 7, 13, 25, 35, 39, 53, 55

# DC vs. AC circuits

**DC**

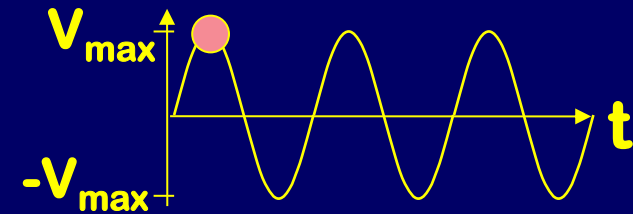
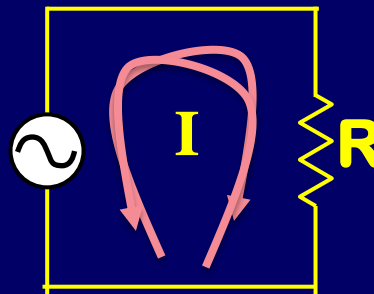
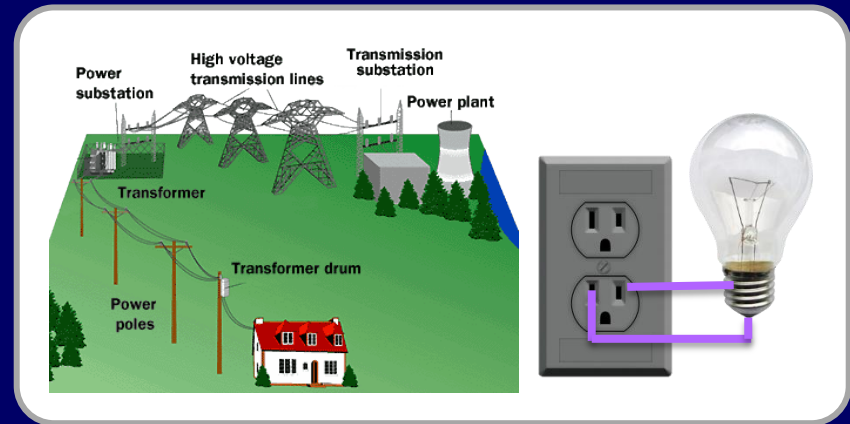
**Direct Current**



**Direction  
of current is fixed**

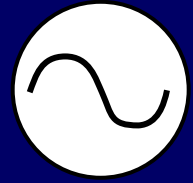
**AC**

**Alternating Current**



**Direction of current  
alternates!**

# Review: Generators and EMF



## Voltage across generator:

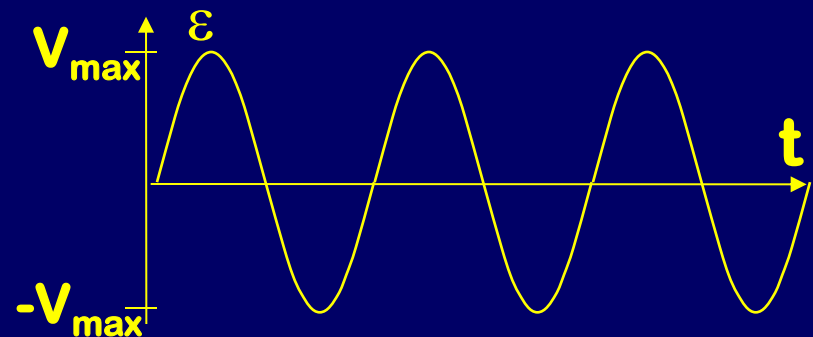
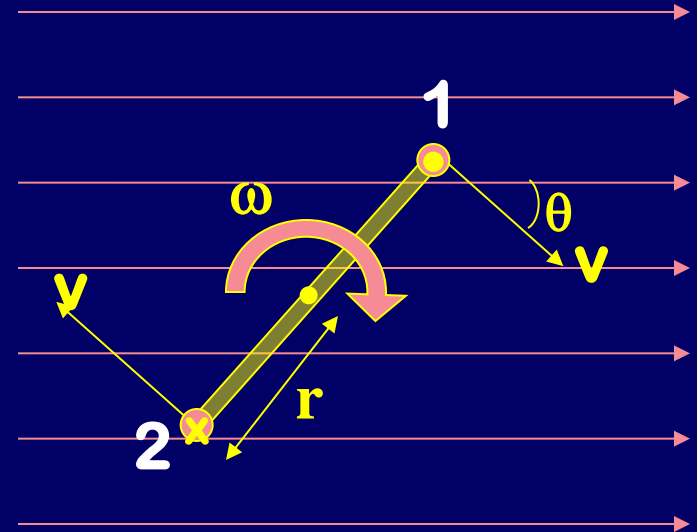
$$\varepsilon = \omega A B \sin(\theta)$$

$$\varepsilon = \omega A B \sin(\omega t)$$

$$\varepsilon = V_{\max} \sin(\omega t)$$

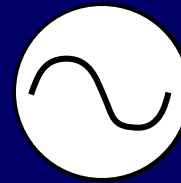
**$V_{\max}$  = Amplitude**

**= Maximum voltage**



# Example

## AC Source



$$V(t) = V_{\max} \sin(\omega t) = V_{\max} \sin(2\pi f t)$$

$V_{\max}$  = maximum voltage

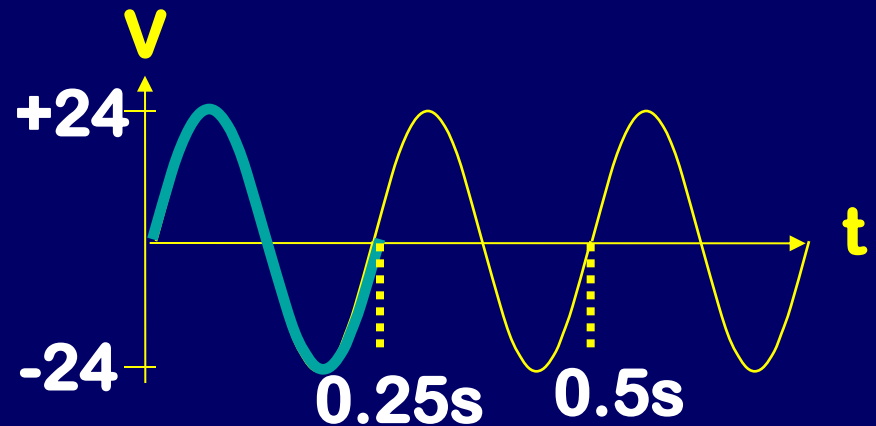
$f$  = frequency (cycles/second)

$$V(t) = 24 \sin(8\pi t)$$

$$2\pi f t = 8\pi t$$

$$f = 4 \text{ Hz}$$

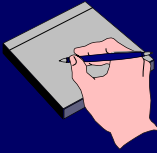
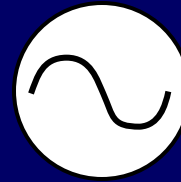
$$T = (1/4) \text{ seconds}$$



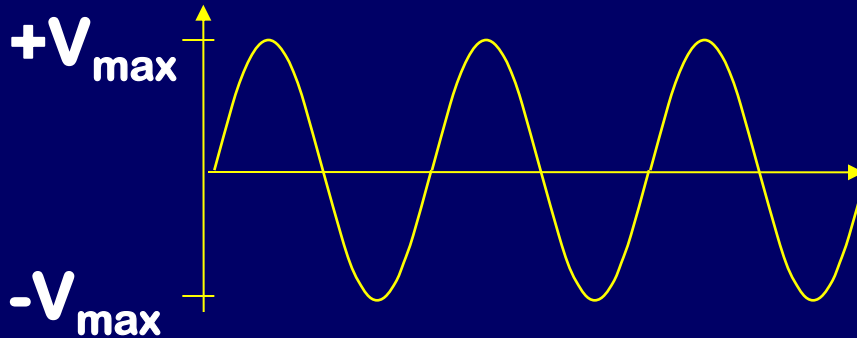
RMS: Root Mean Square

$$V_{\text{rms}} = V_{\max} / \sqrt{2}$$

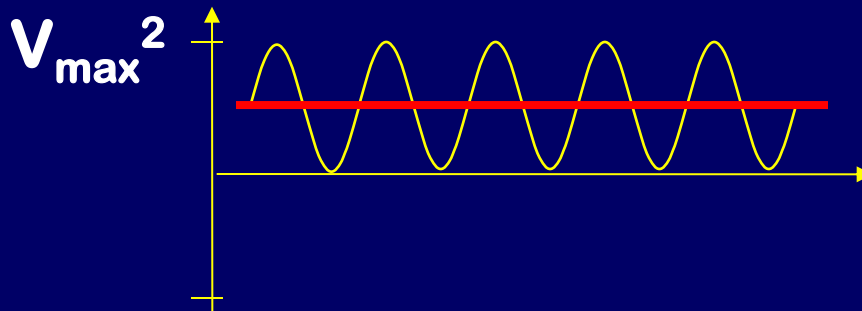
# RMS?



$$V(t) = V_{\max} \sin(2\pi f t)$$



**Square:**



**Mean:**

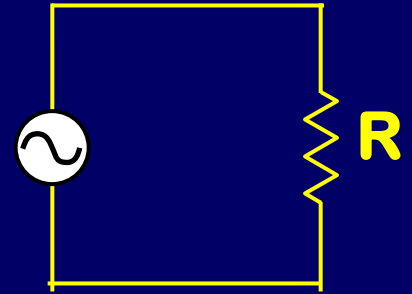
$$V_{\max}^2 / 2$$

**square Root:**  $V_{\max} / \sqrt{2}$

RMS: Root Mean Square  $V_{\text{rms}} = V_{\max} / \sqrt{2}$

# CheckPoint 2.1, 2.2

$$I(t) = \frac{V_{max}}{R} \sin(377t) = 10 \sin(377t)$$



Find  $I_{max}$

A) 20

B) 10

C) 7

Find  $I_{rms}$

A) 20

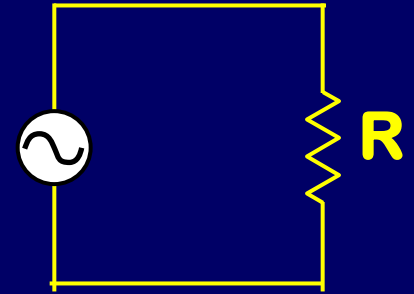
B) 10

C) 7

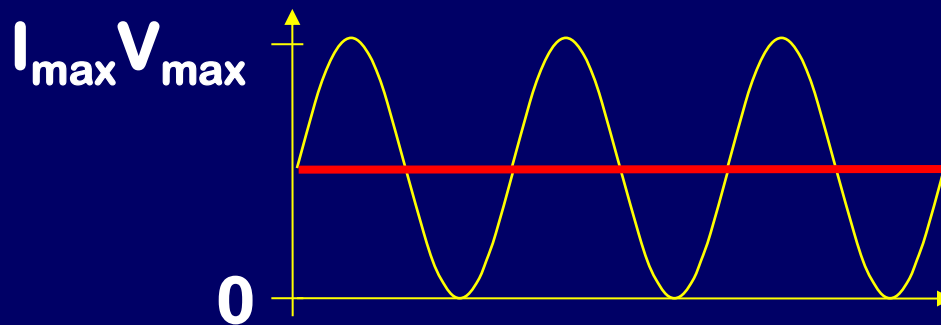
Just like  $V_{rms} = V_{max} / \sqrt{2} \dots$

$$I_{rms} = I_{max} / \sqrt{2} = 10 / \sqrt{2} \text{ A} = 7.07 \text{ A}$$

# Power dissipated



$$P(t) = I(t)V(t)$$



**Mean:**

$$I_{\max} V_{\max} / 2$$

Average power dissipated:  $\bar{P} = I_{rms} V_{rms} = \frac{1}{2} I_{\max} V_{\max}$   
(For an AC circuit with only a resistor)



# ACT: AC power dissipation

When your hair dryer is plugged in and running, it uses 1200 W of average power. If the rms voltage delivered by the wall outlet is 120 V, what is the rms current delivered to the hair dryer?



(A) 0.1 A

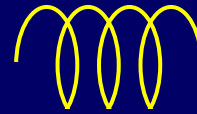
(B) 5 A

(C) 10 A

$$\overline{P} = I_{rms}V_{rms} = \frac{1}{2}I_{max}V_{max}$$



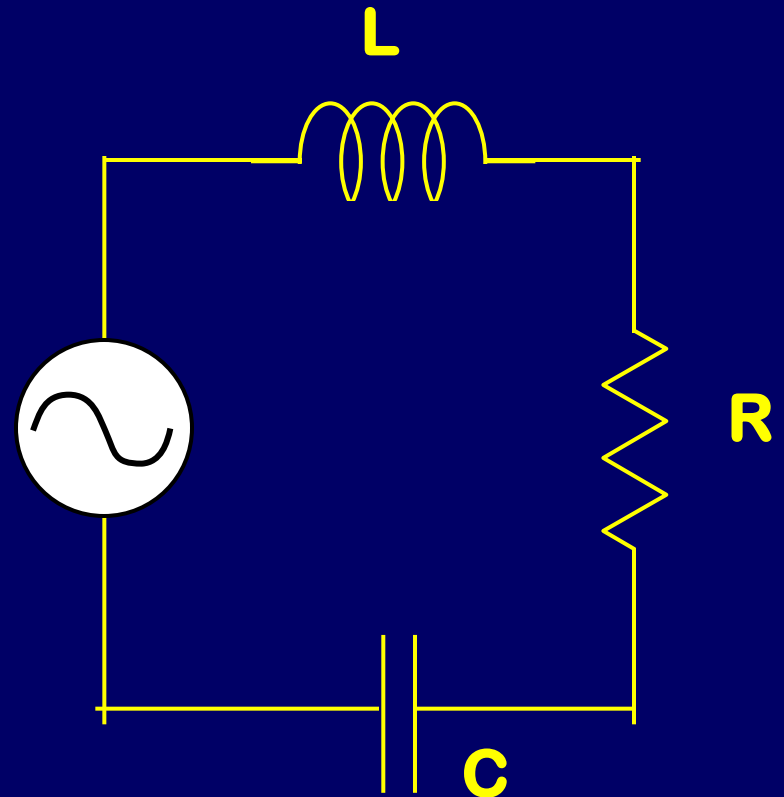
# Inductors



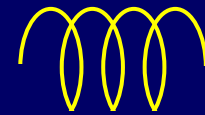
Inductors: a solenoid  
used as a circuit element



**Inductors enable circuits to  
have a resonance...**



# Self-Inductance



Recall the solenoid cannon

- Changing current
- Changing  $B_{\text{sol}}$  field
- Changing  $\Phi$  through itself!

–  $\Phi$  proportional to  $I$ :

$$\Phi = LI$$



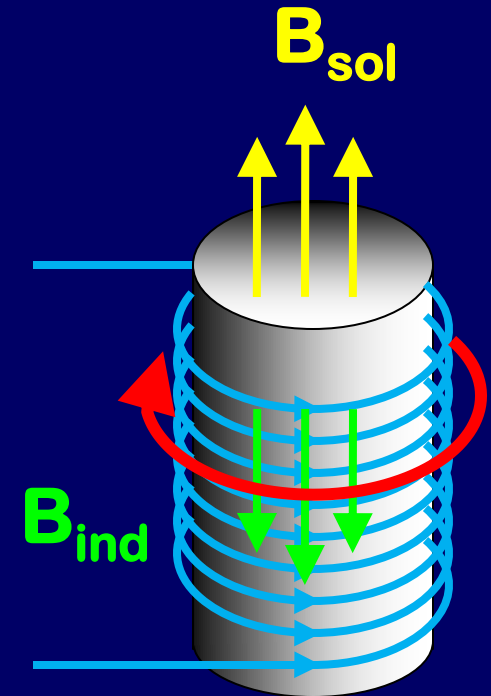
**“Inductance”**

- Induced EMF (voltage)

– Recall Faraday’s law:

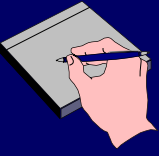
- Direction

- Given by Lenz’s Law
- Opposes change in current!



$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -L \frac{\Delta I}{\Delta t} = -L \frac{I_f - I_i}{t_f - t_i}$$

**Units:  $L = \varepsilon t / I$**   
**1 H = 1V-sec/amp**



# Physical Inductor

$$L \equiv \frac{\Phi}{I}$$

Recall:  $\Phi = NBA$

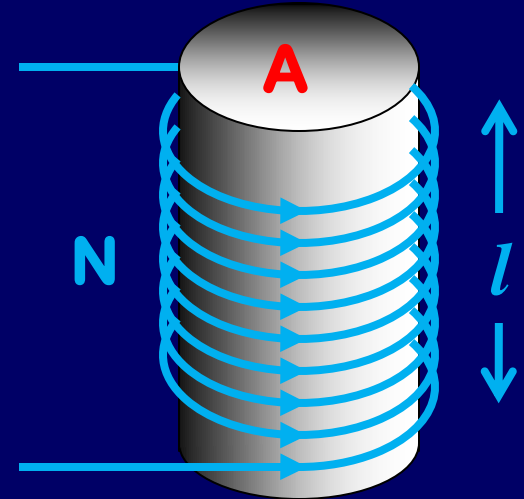
$$L = \frac{NBA}{I}$$

Recall:  $B = \mu_0 n I$

$$L = \frac{N \mu_0 n I A}{I}$$

$$L = N \mu_0 n A$$

$$L = \mu_0 n^2 l A$$



(# turns) = (# turns/meter) x (# meters)

$$N = n l$$

Checkpoint 1

Double N results in 4L

Energy stored:

$$U = \frac{1}{2} L I^2$$



# ACT: Inductors

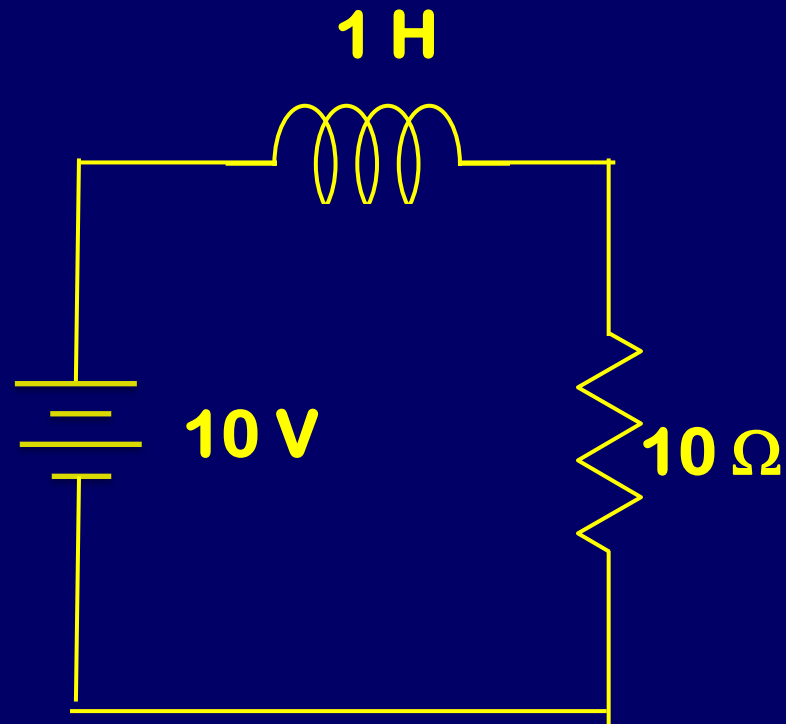
A  $10\ \Omega$  resistor is wired in series with a  $10\text{ V}$  battery and a  $1\text{ H}$  inductor. What is the voltage across the inductor?

(A)  $0\text{ V}$

(B)  $0.1\text{ V}$

(C)  $10\text{ V}$

$$|V_L| = L \frac{\Delta I}{\Delta t} = 0$$

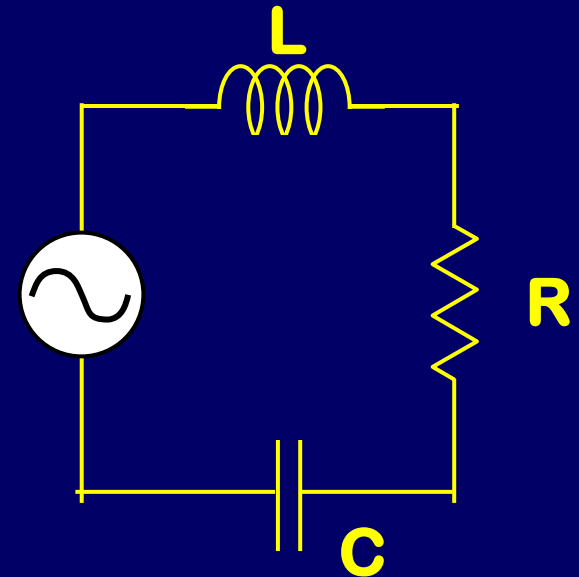


# RLC circuits

**A circuit with an inductor, resistor, and capacitor in series!**

**Used in:**

- **Cell phones / radios**
- **Computers**
- **Watches / clocks**



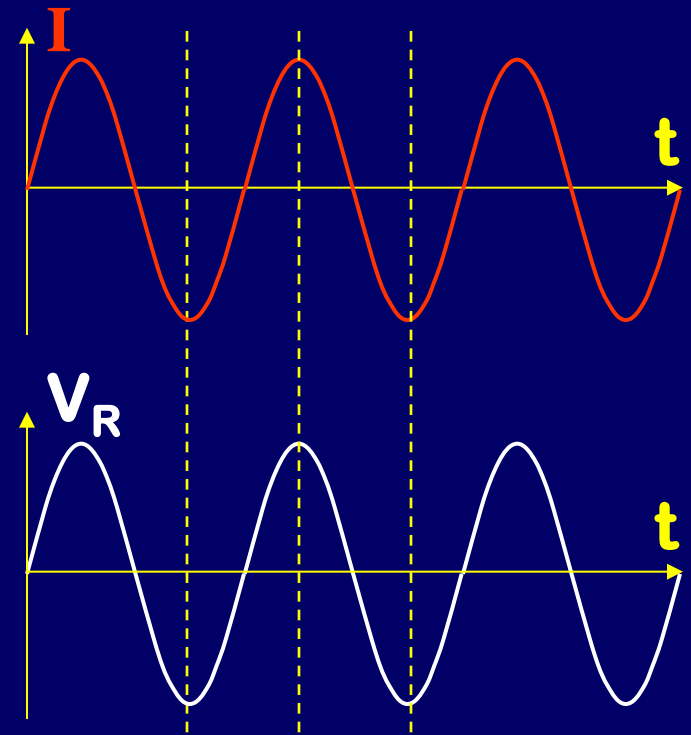
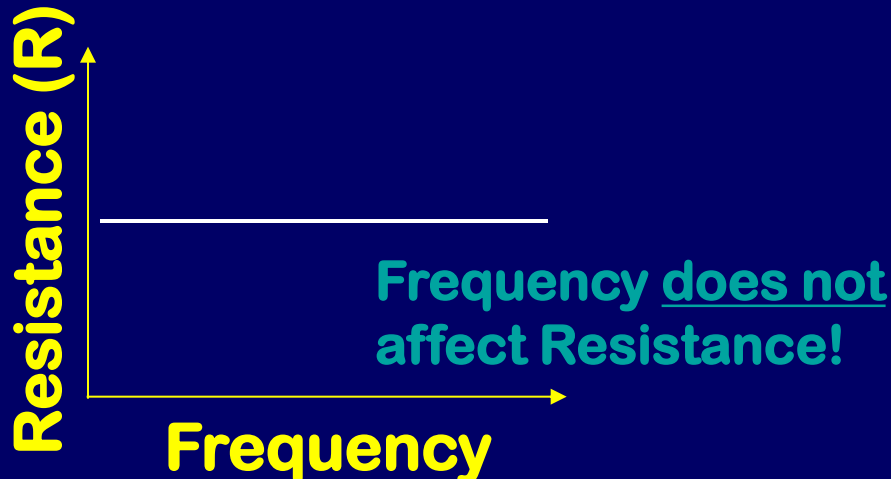
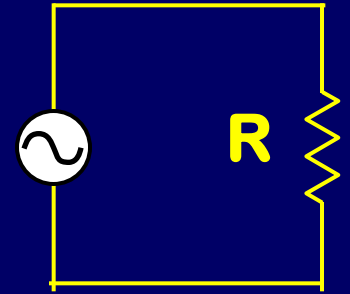
**First: Understand each element individually**

# Resistors in AC circuit



$V_R = I R$  always true – Ohm's Law

- $V_{R,\max} = I_{\max} R$
- Voltage across resistor is “IN PHASE” with current.
  - $V_R$  goes up and down at the same times as  $I$  does.

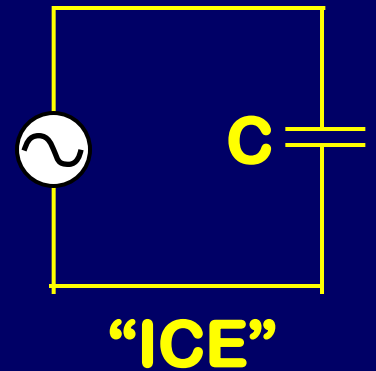


# Capacitors in AC circuit

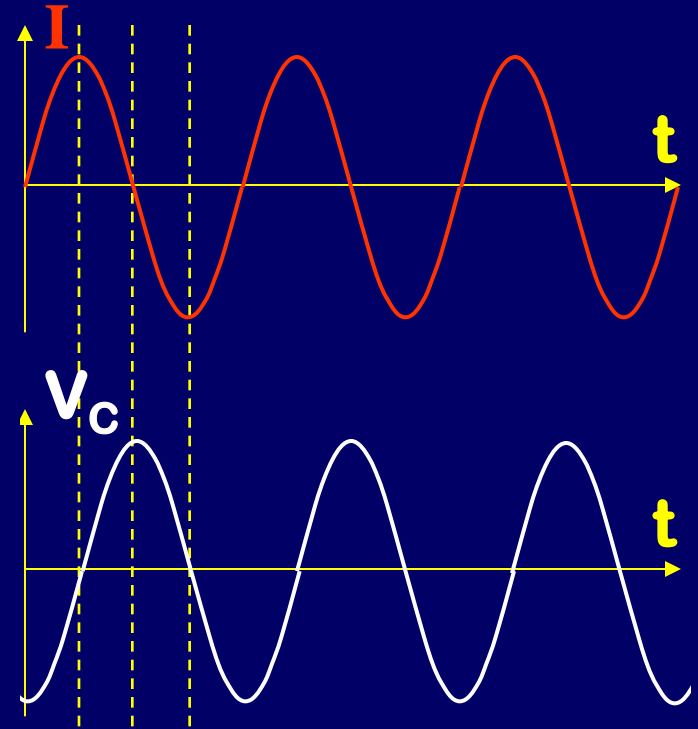
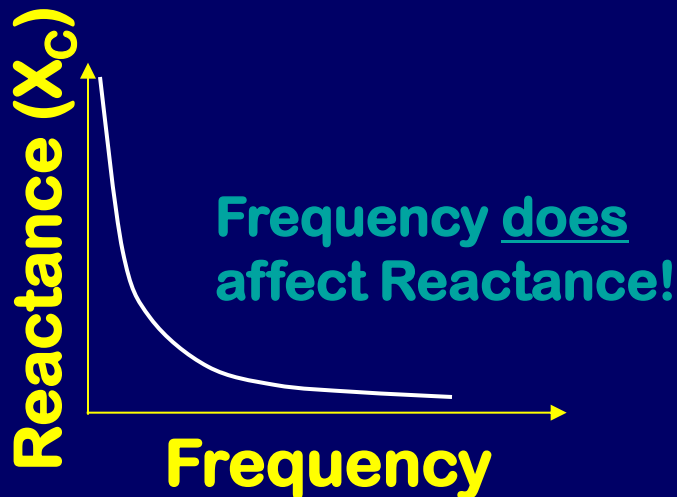


$$V_C = Q/C \text{ always true}$$

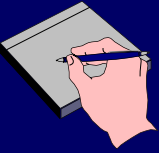
- $V_{C,\max} = I_{\max} X_C$
- Capacitive Reactance:  $X_C = 1/(2\pi fC)$
- Voltage across capacitor “LAGS” current.



- $V_C$  goes up and down just after  $I$  does.

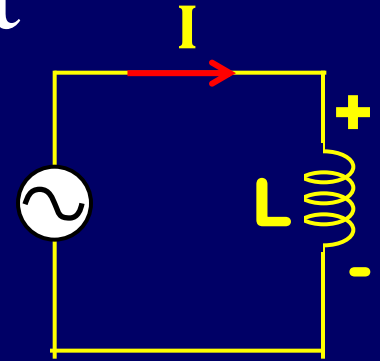


# Inductors in AC circuit



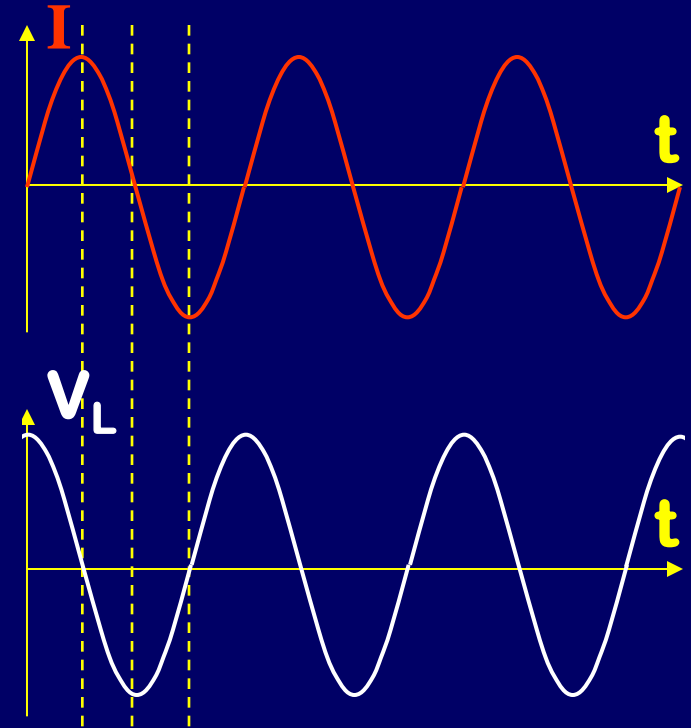
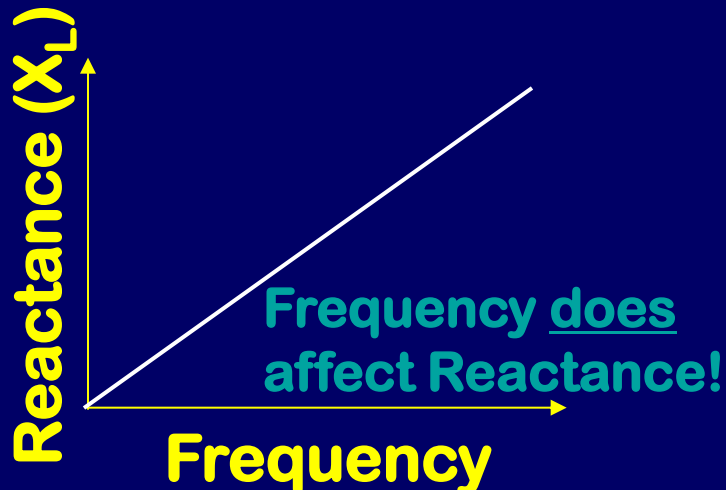
$$V_L = +L(\Delta I)/(\Delta t) \text{ always true}$$

- $V_{L,\max} = I_{\max} X_L$
- Inductive Reactance:  $X_L = 2\pi fL$
- Voltage across inductor “LEADS” current.



“ELI”

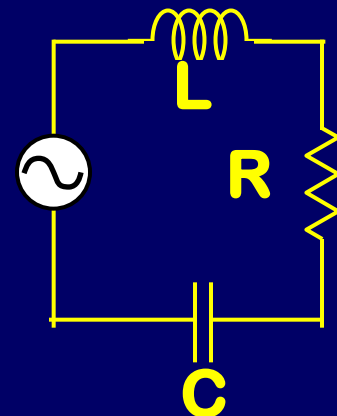
- $V_L$  goes up and down just before  $I$  does.







# ACT/CheckPoints 3.1, 3.2

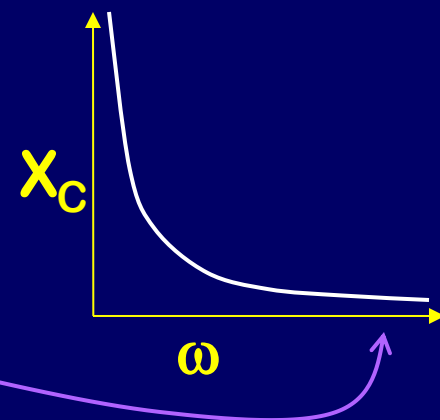


The **capacitor** can be ignored when...

(a) frequency is very large

(b) frequency is very small

very large  $\omega$  gives very small  $X_C$

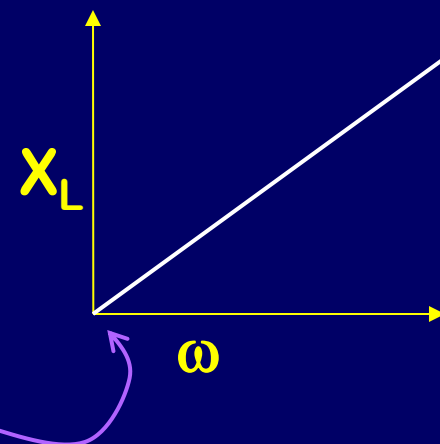


The **inductor** can be ignored when...

(a) frequency is very large

(b) frequency is very small

very small  $\omega$  gives very small  $X_L$

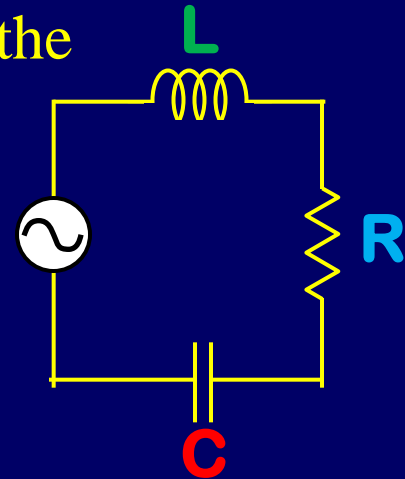


# Example

## AC Circuit Voltages



An AC circuit with  $R = 2 \Omega$ ,  $C = 15 \text{ mF}$ , and  $L = 30 \text{ mH}$  has a current  $I(t) = 0.5 \sin(8\pi t)$  amps. Calculate the maximum voltage across  $R$ ,  $C$ , and  $L$ .



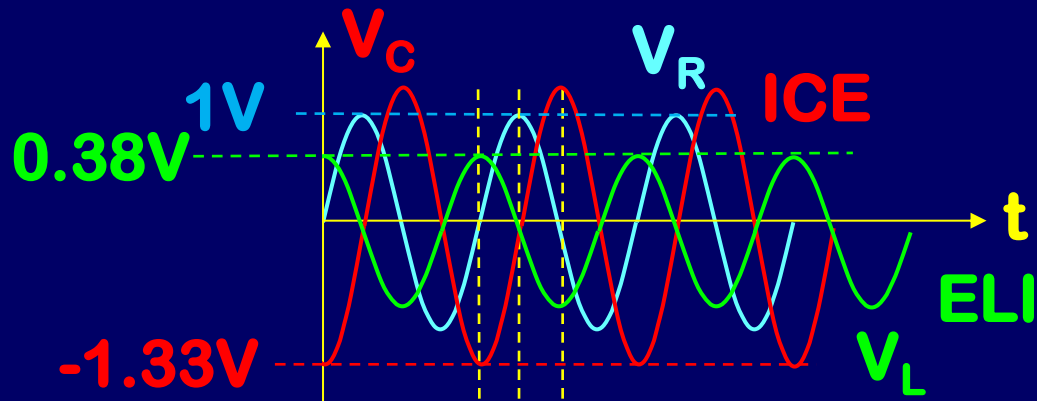
$$V_{R,\max} = I_{\max} R = 0.5 \times 2 = 1 \text{ Volt}$$

$$V_{C,\max} = I_{\max} X_C = 0.5 \times 1/(8\pi \times 0.015) = 1.33 \text{ Volts}$$

$$V_{L,\max} = I_{\max} X_L = 0.5 \times 8\pi \times 0.03 = 0.38 \text{ Volts}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

$$X_L = 2\pi f L = \omega L$$

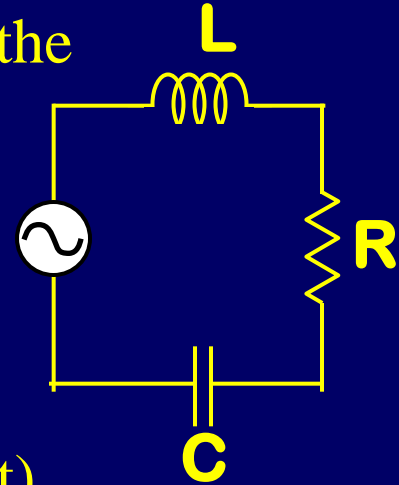




# ACT: AC Circuit Voltages



An AC circuit with  $R = 2\ \Omega$ ,  $C = 15\ \text{mF}$ , and  $L = 30\ \text{mH}$  has a current  $I(t) = 0.5 \sin(8\pi t)$  amps. Calculate the maximum voltage across  $R$ ,  $C$ , and  $L$ .



Now the frequency is increased so  $I(t) = 0.5 \sin(16\pi t)$ . Which element's maximum voltage decreases?

1)  $V_{R,\max}$

2)  $V_{C,\max}$

3)  $V_{L,\max}$

**Stays same:  $R$  doesn't depend on  $f$**

**Decreases:  $X_C = 1/(2\pi f C)$**

**Increases:  $X_L = 2\pi f L$**

# Summary so far...

- $I = I_{\max} \sin(2\pi ft)$
- $V_R = I_{\max} R \sin(2\pi ft)$ 
  - $V_R$  in phase with  $I$
- $V_C = I_{\max} X_C \sin(2\pi ft - \pi/2)$ 
  - $V_C$  lags  $I$      $X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$

**“ICE”**

- $V_L = I_{\max} X_L \sin(2\pi ft + \pi/2)$ 
  - $V_L$  leads  $I$      $X_L = 2\pi f L = \omega L$

**“ELI”**

