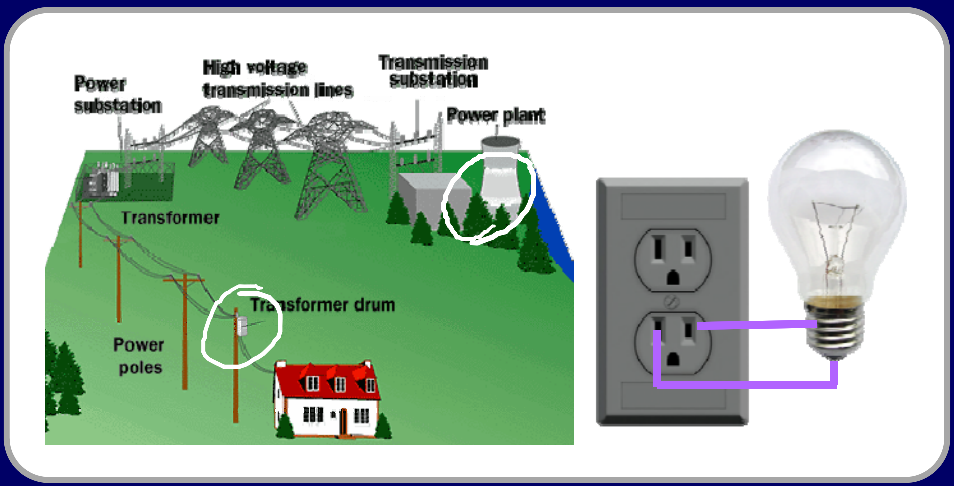
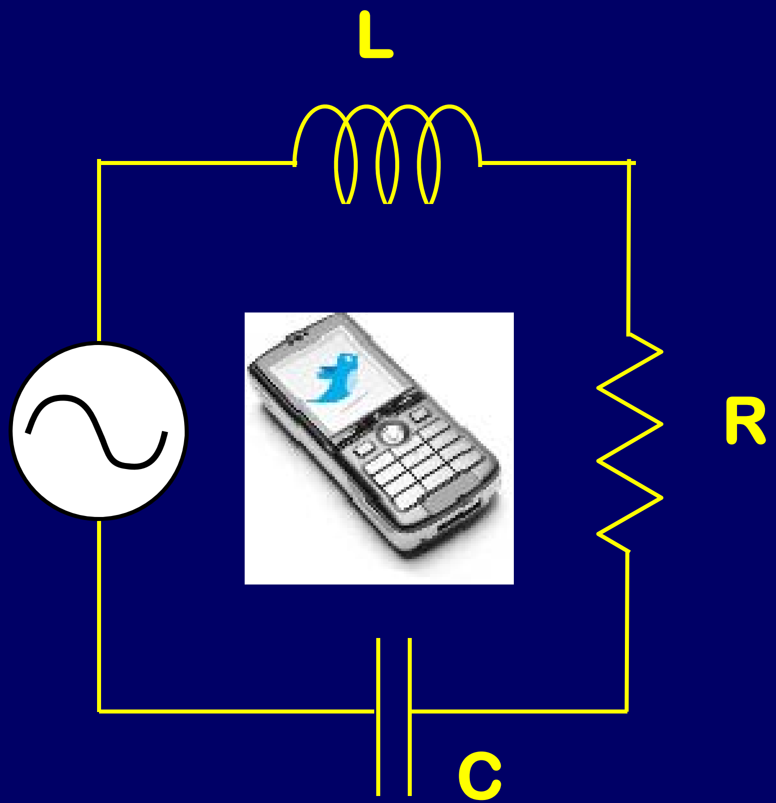
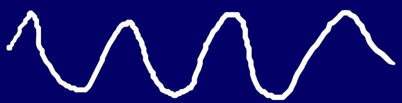
Physics 102: Lecture 12

AC & RLC Circuits

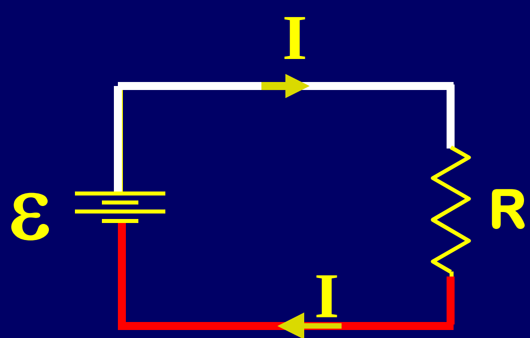






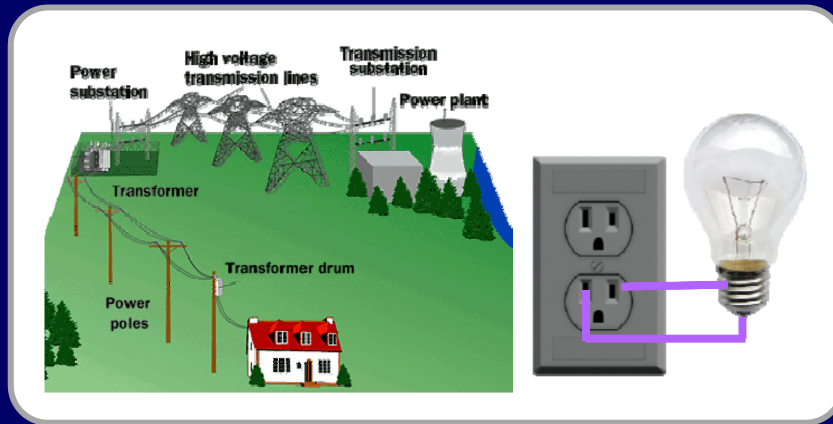
DC vs. AC circuits

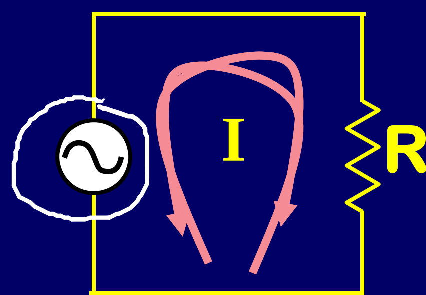
DC
Direct Current

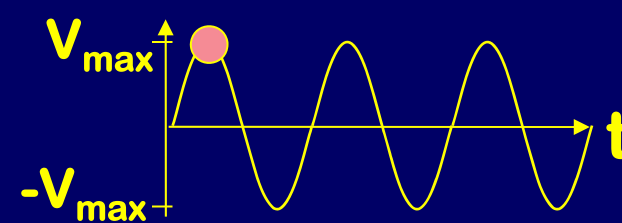


Direction of current is fixed

AC
Alternating Current

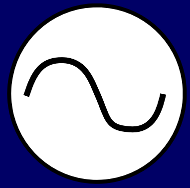






Direction of current alternates!

Review: Generators and EMF

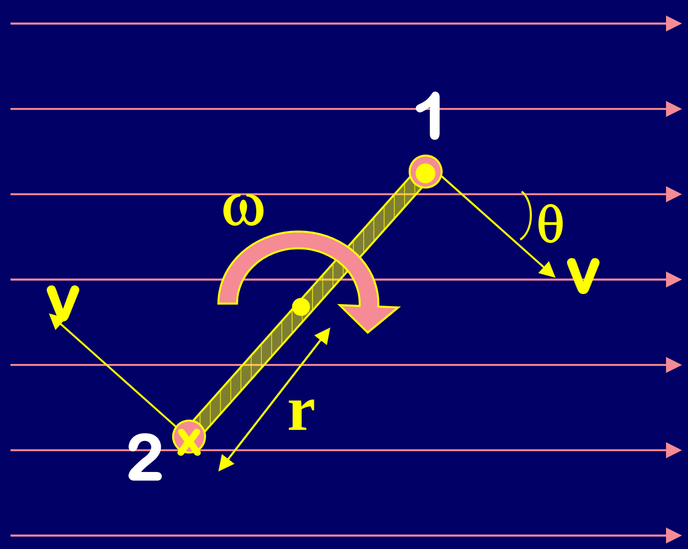


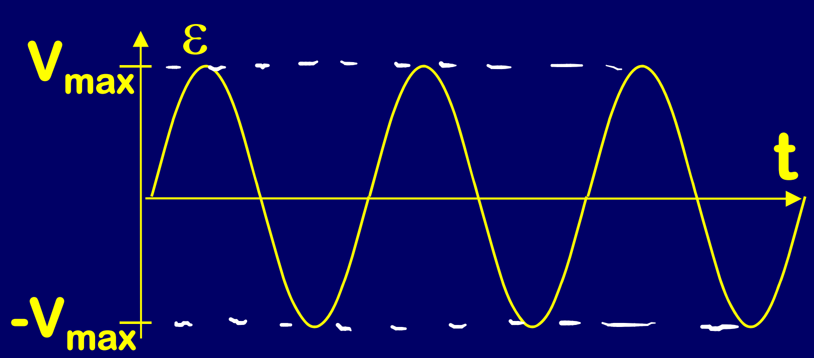
Voltage across generator:

$$\varepsilon = \omega A B \sin(\theta)$$

$$\varepsilon = \omega A B \sin(\omega t)$$

$$\varepsilon = V_{max} \sin(\omega t)$$

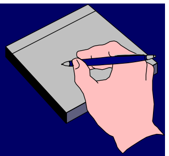




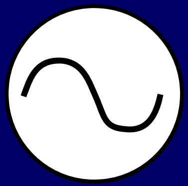
V_{max} = Amplitude

= Maximum voltage

Example



AC Source



$V(t) = V_{max} \sin(\omega t) = V_{max} \sin(2\pi f\, t)$

V_{max} = maximum voltage

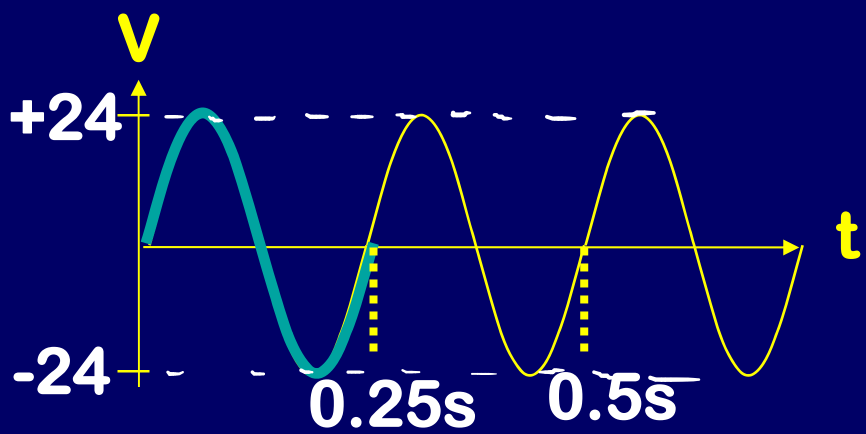
f = frequency (cycles/second)

$V(t) = 24 \sin(8\pi\, t)$

$2\pi f\, t = 8\pi t$

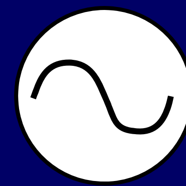
$f = 4$ Hz

$T = (1/4)$seconds

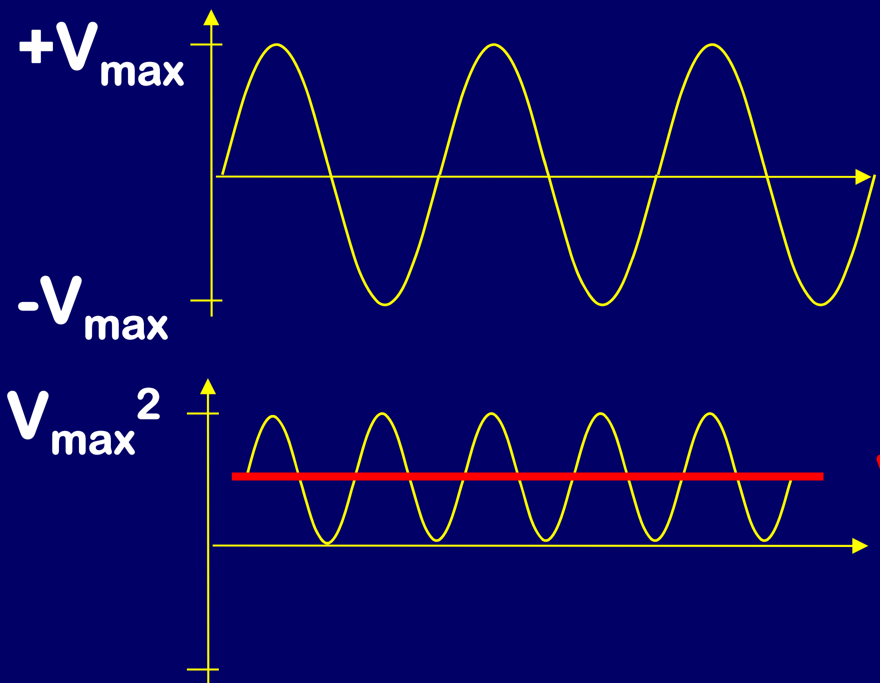


RMS: Root Mean Square $V_{rms} = V_{max}/\sqrt{2}$

RMS?



$V(t) = V_{max} \sin(2\pi f\, t)$



Square:

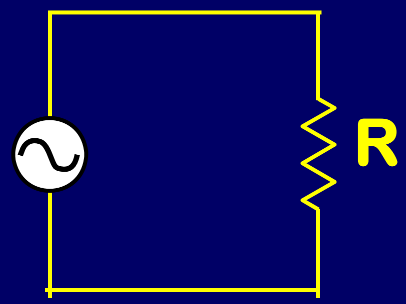
Mean: $V_{max}^2 / 2$

square Root: $V_{max} / \sqrt{2}$

RMS: Root Mean Square $V_{rms} = V_{max}/\sqrt{2}$

CheckPoint 2.1, 2.2

$$I(t) = \frac{V_{max}}{R} sin(377t) = 10sin(377t)$$



Find I_{max}

Well… We know that the maximum value of sin is 1. So the maximum current is 10!

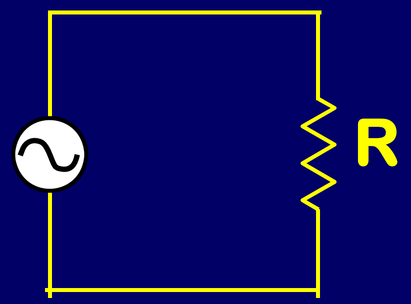
$I_{max} = 10\ A$

Find I_{rms}

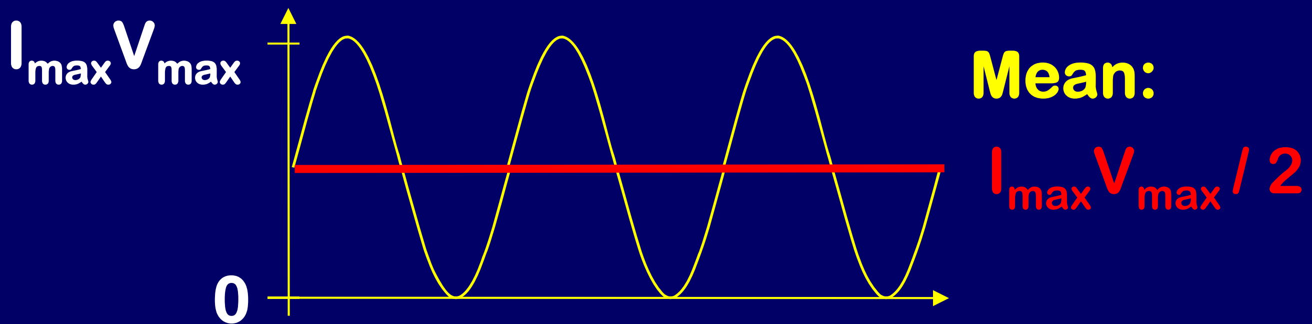
Just like $V_{rms}=V_{max}/\sqrt{2}$ …

$I_{rms}= I_{max}/\sqrt{2} = 10/\sqrt{2}\ A = 7.07\ A$

Power dissipated



P(t) = I(t)V(t)



Average power dissipated: $\overline{P} = I_{rms}V_{rms} = \frac{1}{2}I_{max}V_{max}$

(For an AC circuit with only a resistor)

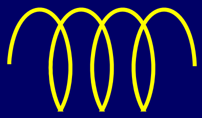
ACT: AC power dissipation

When your hair dryer is plugged in and running, it uses 1200 W of average power. If the rms voltage delivered by the wall outlet is 120 V, what is the rms current delivered to the hair dryer?

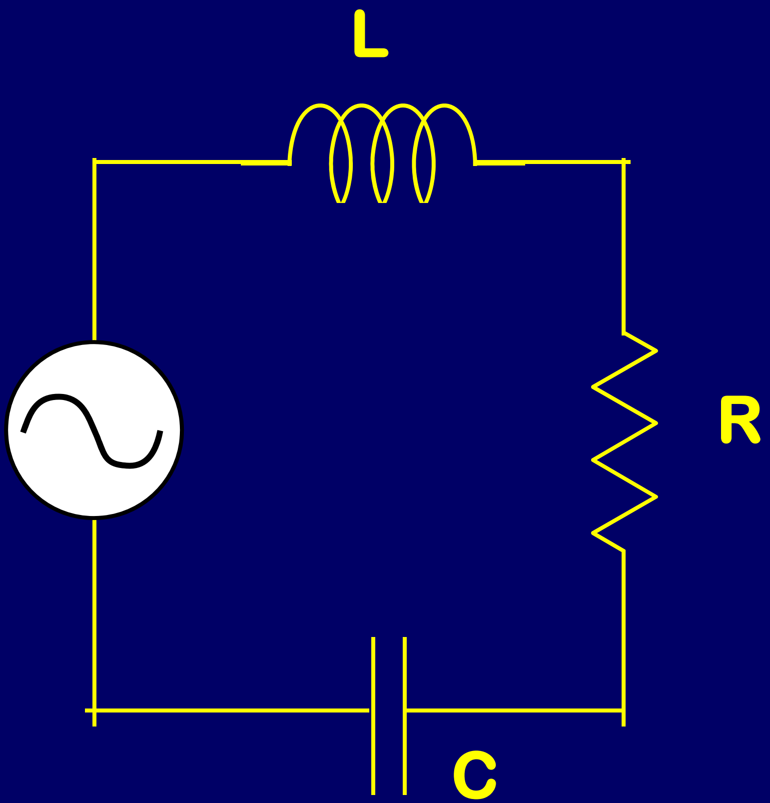
(A) 0.1 A **(B) 5 A** **(C) 10 A**

$$\bar{P} = I_{rms} V_{rms}$$

Inductors



Inductors: a solenoid used as a circuit element



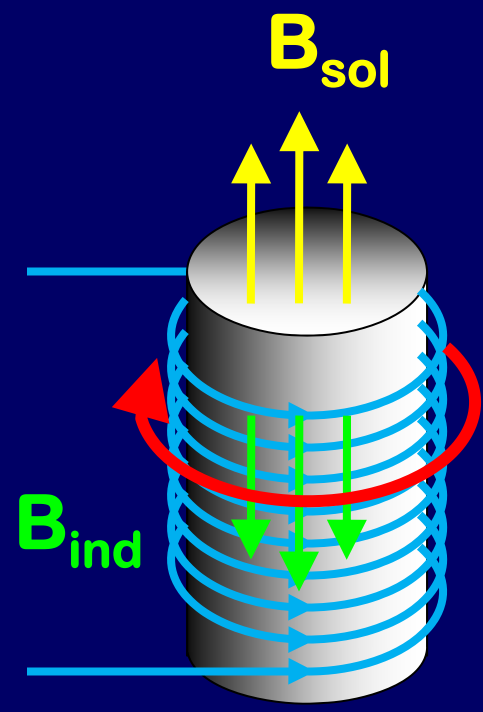
Inductors enable circuits to have a resonance…

Self-Inductance

Recall the solenoid cannon

- Changing current
- Changing B_{sol} field $\Phi \propto B_{sol} \propto I$
- Changing Φ through itself!
  - Φ proportional to I: $\Phi = LI$

"Inductance"



- Induced EMF (voltage)
  - Recall Faraday's law:

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta I}{\Delta t} = -L\frac{I_f - I_i}{t_f - t_i}$$

- Direction
  - Given by Lenz's Law
  - Opposes change in current!

Units: L = ε t / I
1 H = 1V-sec/amp

Physical Inductor

$$L \equiv \frac{\Phi}{I}$$

Recall: $\Phi = NBA \cos\phi$

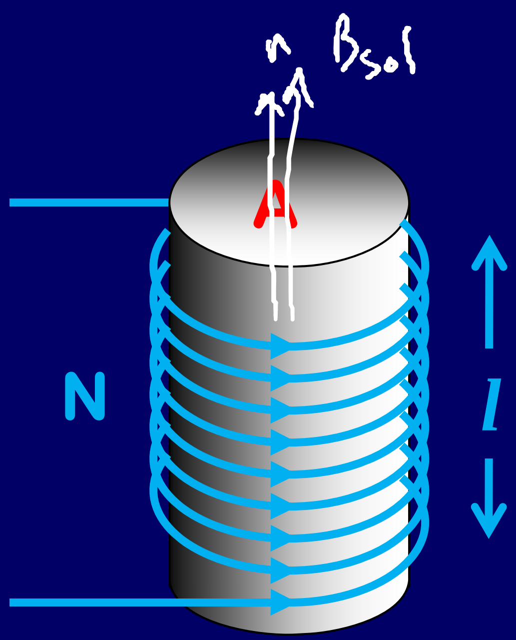
$$L = \frac{NBA}{I}$$

Recall: $B = \mu_o nI$

$$L = \frac{N\mu_0 nIA}{I}$$

$$L = N\mu_0 nA$$

$$\boxed{L = \mu_0 n^2 lA}$$



(# turns) = (# turns/meter) x (# meters)

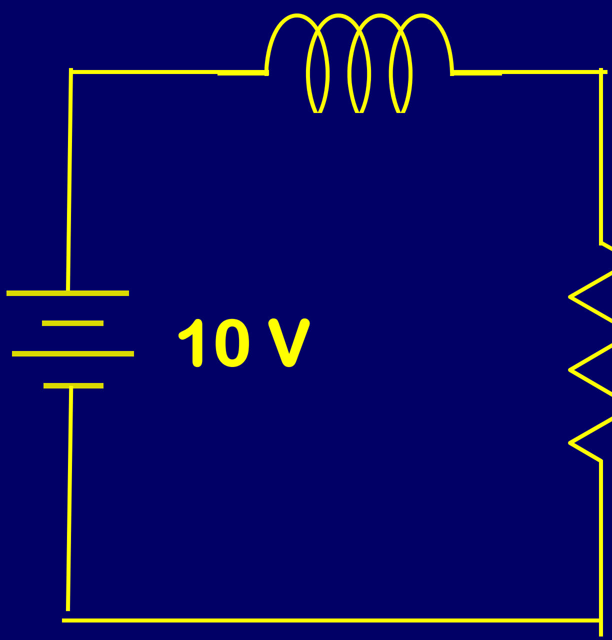
$$N = n\,l$$

Energy stored:

$$\boxed{U = \tfrac{1}{2} LI^2}$$

ACT: Inductors

A 10 Ω resistor is wired is series with a 10 V battery and a 1 H inductor. What is the voltage across the inductor?



(A) 0 V (B) 0.1V (C) 10 V

$$|V_L| = L\frac{\Delta I}{\Delta t} = 0$$

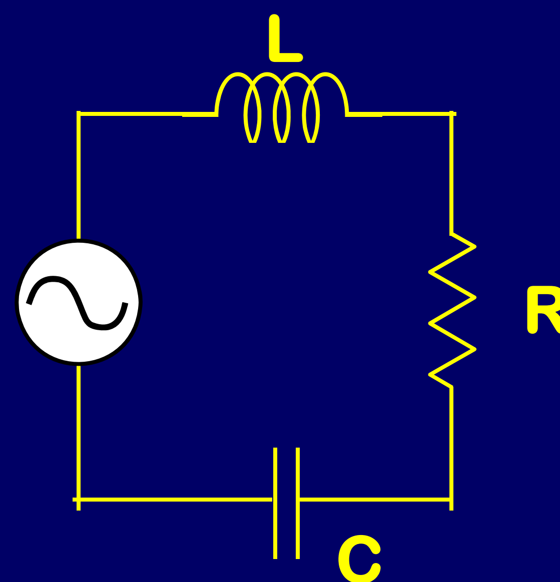
RLC circuits

A circuit with an inductor, resistor, and capacitor in series!

Used in:

- **Cell phones / radios**
- **Computers**
- **Watches / clocks**

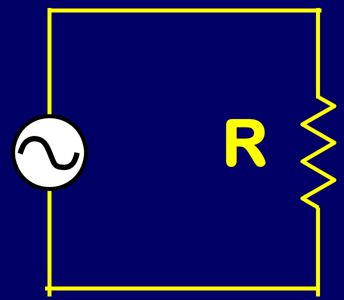


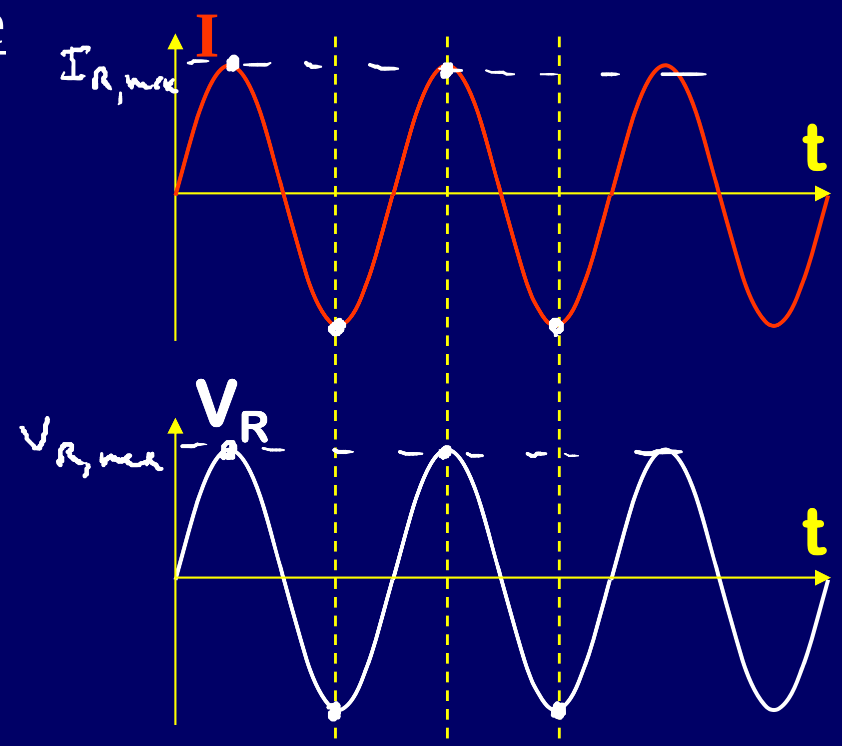
First: Understand each element individually

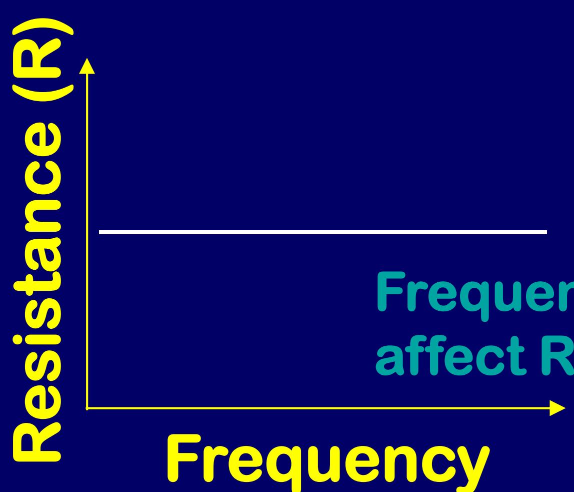
Resistors in AC circuit

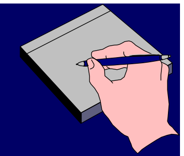
$V_R = I\,R$ always true – Ohm's Law



- $V_{R,max} = I_{max} R$ $\quad V_{R,rms} = I_{rms} R$
- Voltage across resistor is "IN PHASE" with current.
  - V_R goes up and down <u>at the same times</u> as I does.



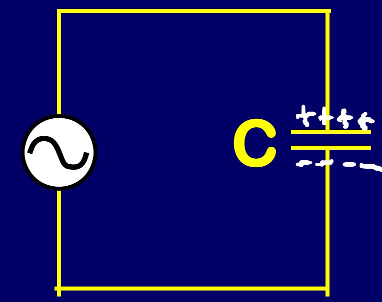




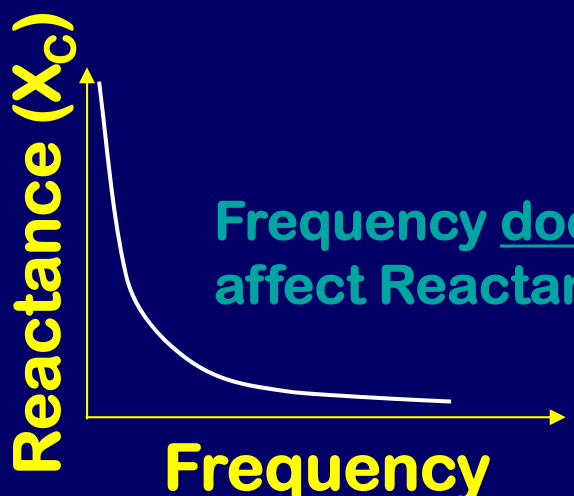
Capacitors in AC circuit

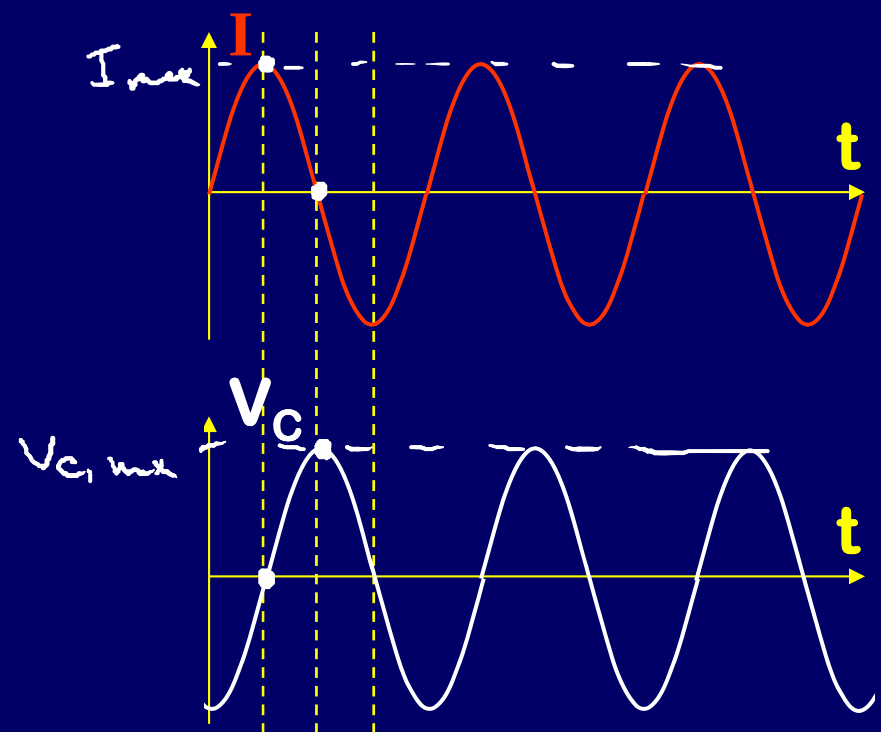
$V_C = Q/C$ always true

- $V_{C,max} = I_{max} X_C$
- Capacitive Reactance: $X_C = 1/(2\pi f C)$
- Voltage across capacitor "LAGS" current.
  - V_C goes up and down <u>just after</u> I does.

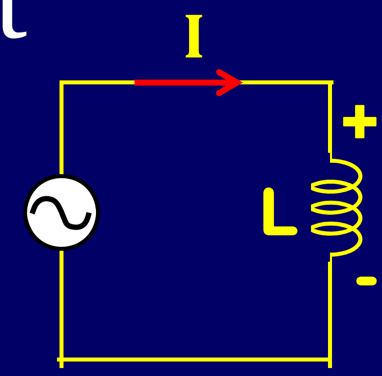


"ICE"





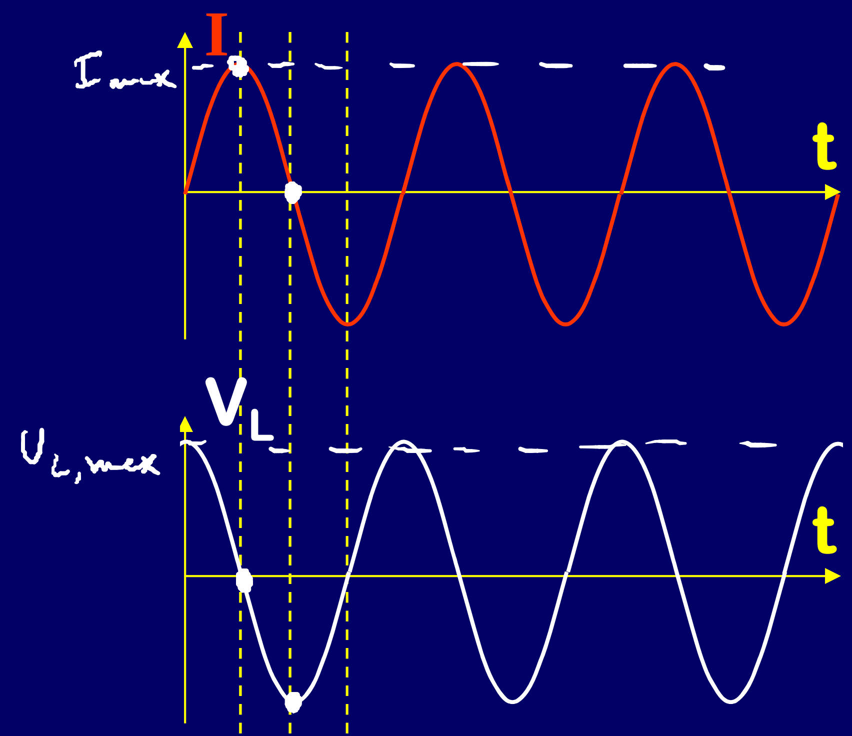
Inductors in AC circuit

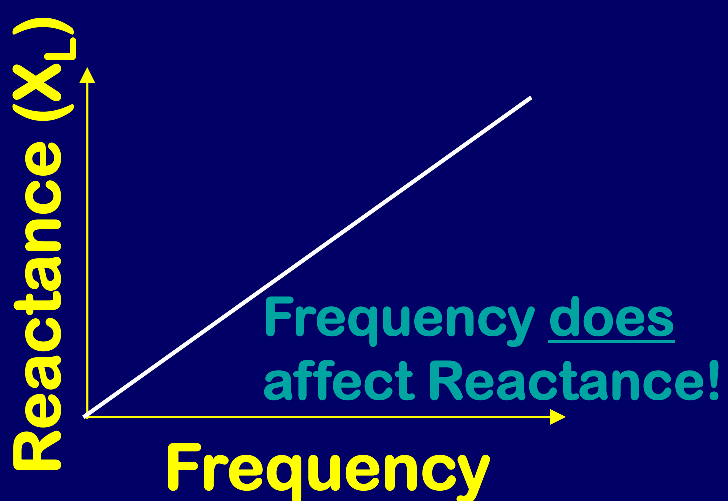


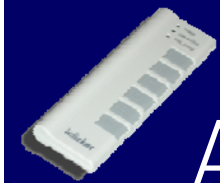
$V_L = +L(\Delta I)/(\Delta t)$ always true

- $V_{L,max} = I_{max} X_L$
- Inductive Reactance: $X_L = 2\pi f L$
- Voltage across inductor "LEADS" current.
  - V_L goes up and down <u>just before</u> I does.

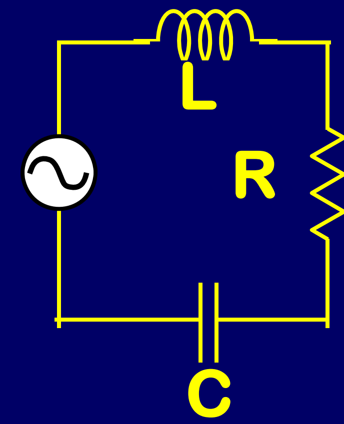
"ELI"







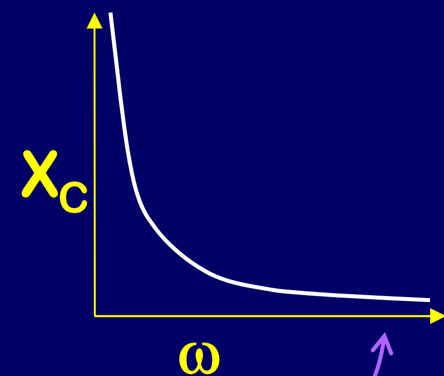
ACT/CheckPoints 3.1, 3.2



The capacitor can be ignored when…

(a) frequency is very large

(b) frequency is very small

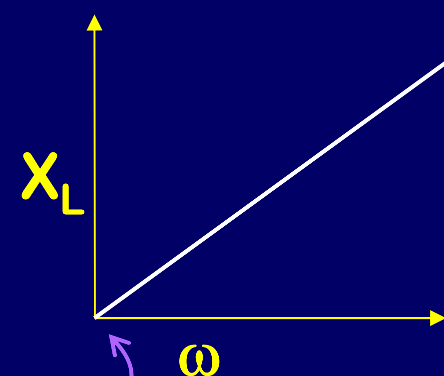


very large ω gives very small X_C

The inductor can be ignored when…

(a) frequency is very large

(b) frequency is very small

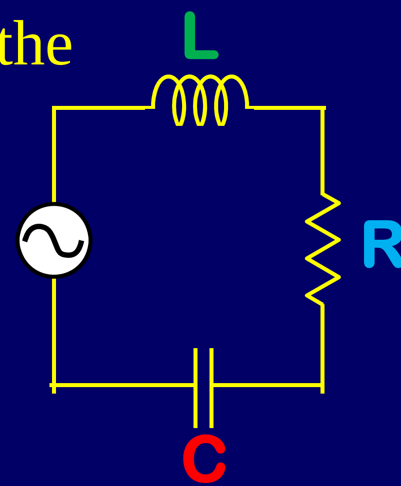


very small ω gives very small X_L

Example

AC Circuit Voltages

An AC circuit with R = 2 Ω, C = 15 mF, and L = 30 mH has a current I(t) = 0.5 sin(8π t) amps. Calculate the maximum voltage across R, C, and L.



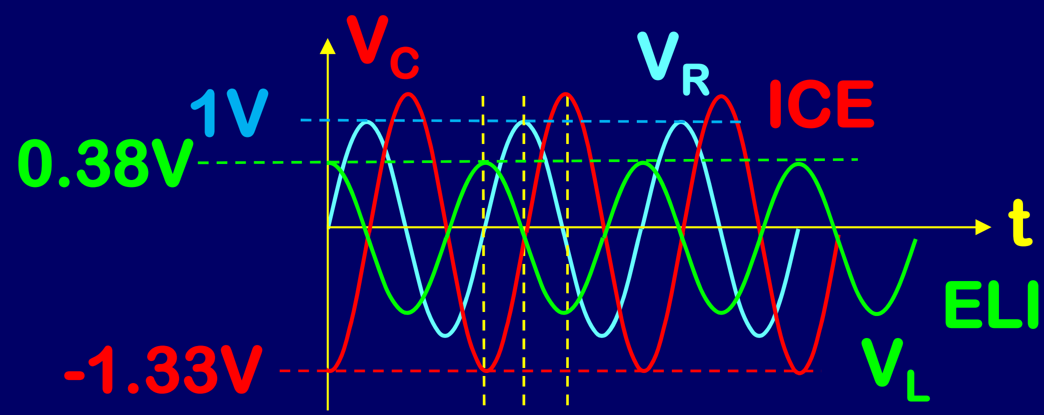
$V_{R,max} = I_{max} R = 0.5 \times 2 = 1$ Volt

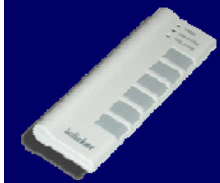
$V_{C,max} = I_{max} X_C = 0.5 \times 1/(8\pi \times 0.015) = 1.33$ Volts

$V_{L,max} = I_{max} X_L = 0.5 \times 8\pi \times 0.03 = 0.38$ Volts

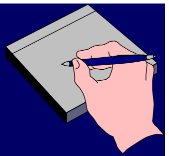
$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

$$X_L = 2\pi f L = \omega L$$

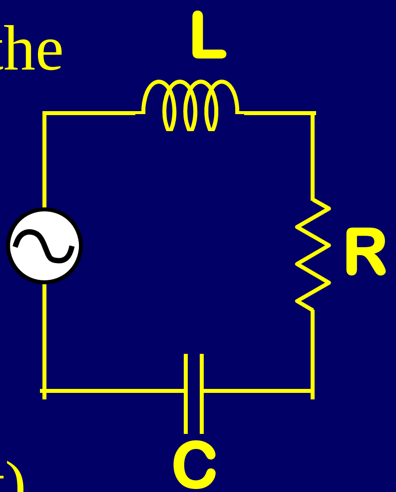




ACT: AC Circuit Voltages



An AC circuit with $R = 2\ \Omega$, $C = 15$ mF, and $L = 30$ mH has a current $I(t) = 0.5 \sin(8\pi\ t)$ amps. Calculate the maximum voltage across R, C, and L.



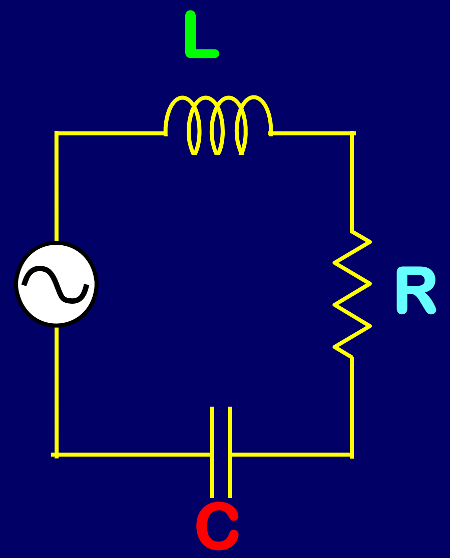
Now the frequency is <u>increased</u> so $I(t) = 0.5 \sin(16\pi\ t)$. Which element's maximum voltage <u>decreases?</u>

1) $V_{R,max}$ **Stays same: R doesn't depend on f**

2) $V_{C,max}$ **Decreases: $X_C = 1/(2\pi fC)$**

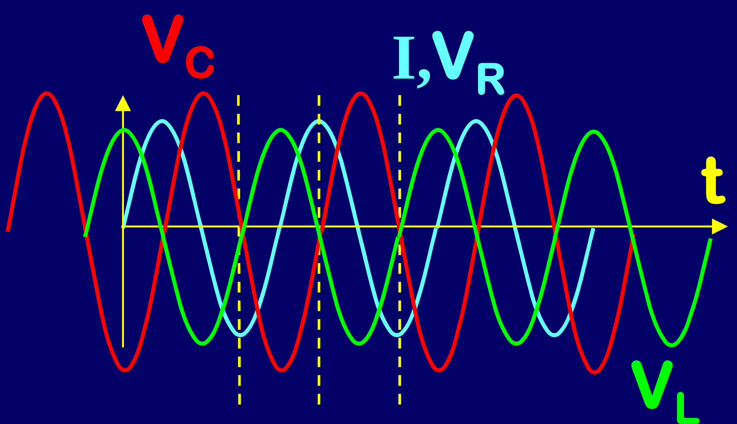
3) $V_{L,max}$ **Increases: $X_L = 2\pi f\ L$**

Summary so far…



- $I = I_{max}\sin(2\pi ft)$
- $V_R = I_{max}R\sin(2\pi ft)$
  - V_R in phase with I
- $V_C = I_{max}X_C\sin(2\pi ft - \pi/2)$
  - V_C lags I $\quad X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$

"ICE"



- $V_L = I_{max}X_L\sin(2\pi ft + \pi/2)$
  - V_L leads I $\quad X_L = 2\pi f L = \omega L$

"ELI"