End-of-semester info

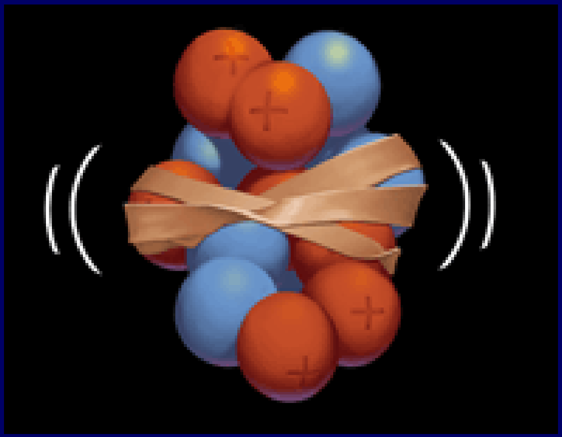
- Final exam info:
  - A1: Friday, Dec. 20, 8:00-11:00am
  - A2: Tuesday, Dec. 17, 7:00-10:00pm
  - Approximately 50 questions
  - Cumulative (all material from semester covered evenly)
- CHECK GRADEBOOK!
  - Missing iClicker points? – you need to show me proof you were in class!
- Online ICES evaluation!
- James Scholar projects
  - Due Thursday, Dec. 5 by email (ychemla@illinois.edu)

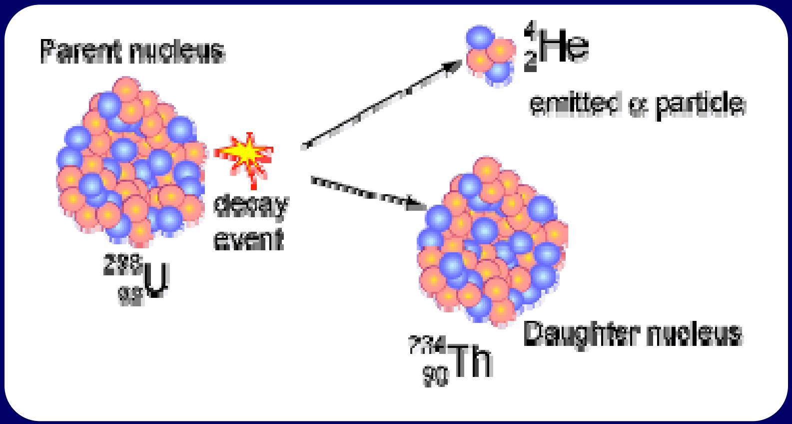
End-of-semester info

- Lectures next week:
  - Monday: Lect. 28 “Relativity”
  - Wednesday: Lect. 29 in-class review
- Office hours next week:
  - TAs will hold office hours until Monday next week (some may hold extra office hours?)
  - I will hold office hours Mondays 2-3pm through Dec. 16
- Extra study problems:
  - No homework next week, optional practice problems available on byteshelf
  - Extra “exam-type” problems to be posted online

Physics 102: Lecture 28

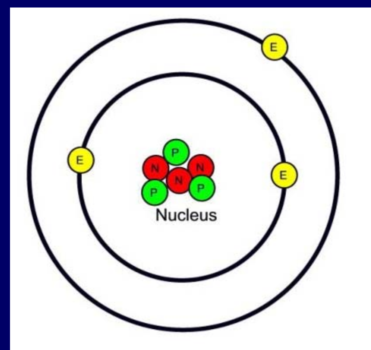
Nuclear Binding, Radioactivity



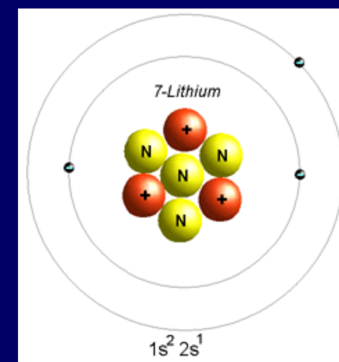


Nuclear Physics

A → Z → $^{6}_{3}\text{Li}$



$^{7}_{3}\text{Li}$



Z = proton number ("atomic number")

Gives chemical properties (and name)

N = neutron number (different "isotopes")

A = nucleon number (atomic mass number)

Gives you mass density of element

$$A=N+Z$$

Checkpoint 1.1

A material is known to be an isotope of lead.

Based on this information which of the following can you specify?

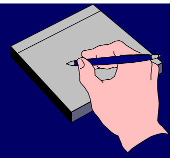
1) The atomic mass number

2) The neutron number

3) The number of protons

Chemical properties (and name) determined by number of protons (Z)

Lead → Z=82



Binding Energy

Einstein's famous equation $E = m c^2$

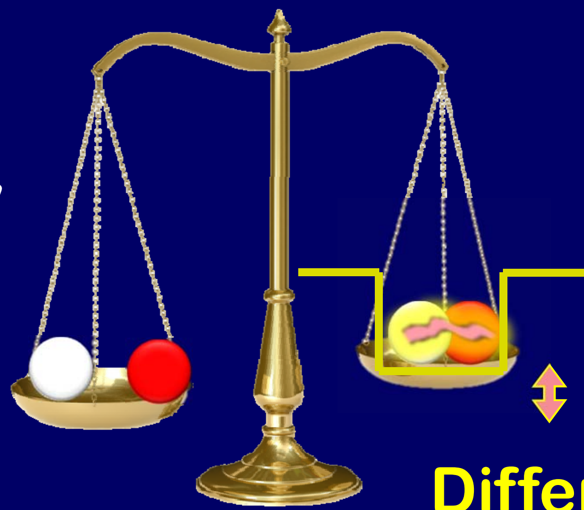
Example

proton:

$mc^2=(1.67\times10^{-27}\text{kg})(3\times10^{8}\text{ m/s})^2=1.50\times10^{-10}\text{ J}$

Proton:
$mc^2 = 938.3\text{MeV}$

Neutron:
$mc^2 = 939.5\text{MeV}$



Deuteron:
$mc^2 = 1875.6\text{MeV}$

Difference = binding energy = 2.2MeV

Adding these = 1877.8MeV

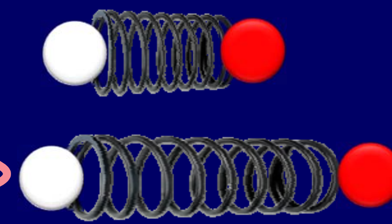
$$M_{Deuteron}\, c^2 = M_{Proton}\, c^2 + M_{Neutron}\, c^2 - |\text{Binding Energy}|$$

$$M_{Deuteron} = M_{Proton} + M_{Neutron} - |\text{Binding Energy}|/c^2$$

ACT: Binding Energy

Which system "weighs" more?

1) Two balls attached by a relaxed spring.
2) Two balls attached by a stretched spring.
3) They have the same weight.



$$M_1 = M_{balls} + M_{spring}$$

$$M_2 = M_{balls} + M_{spring} + E_{spring}/c^2$$

$$M_2 - M_1 = E_{spring}/c^2 \sim 10^{-16}\ \text{Kg}$$

Checkpoint 1.2

Where does the energy released in the nuclear reactions of the sun come from?

(1) covalent bonds between atoms

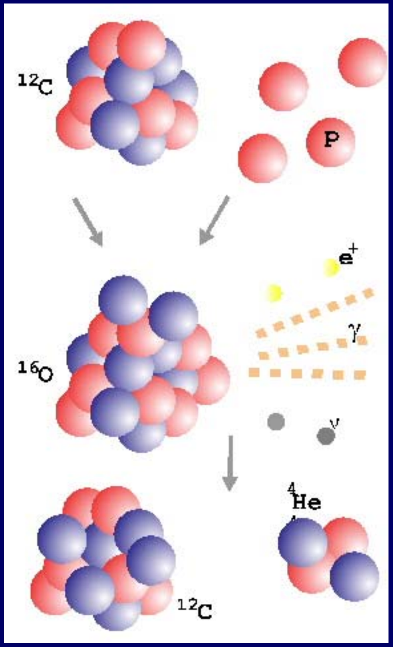
~1eV

(2) binding energy of electrons to the nucleus

~10eV – 1keV

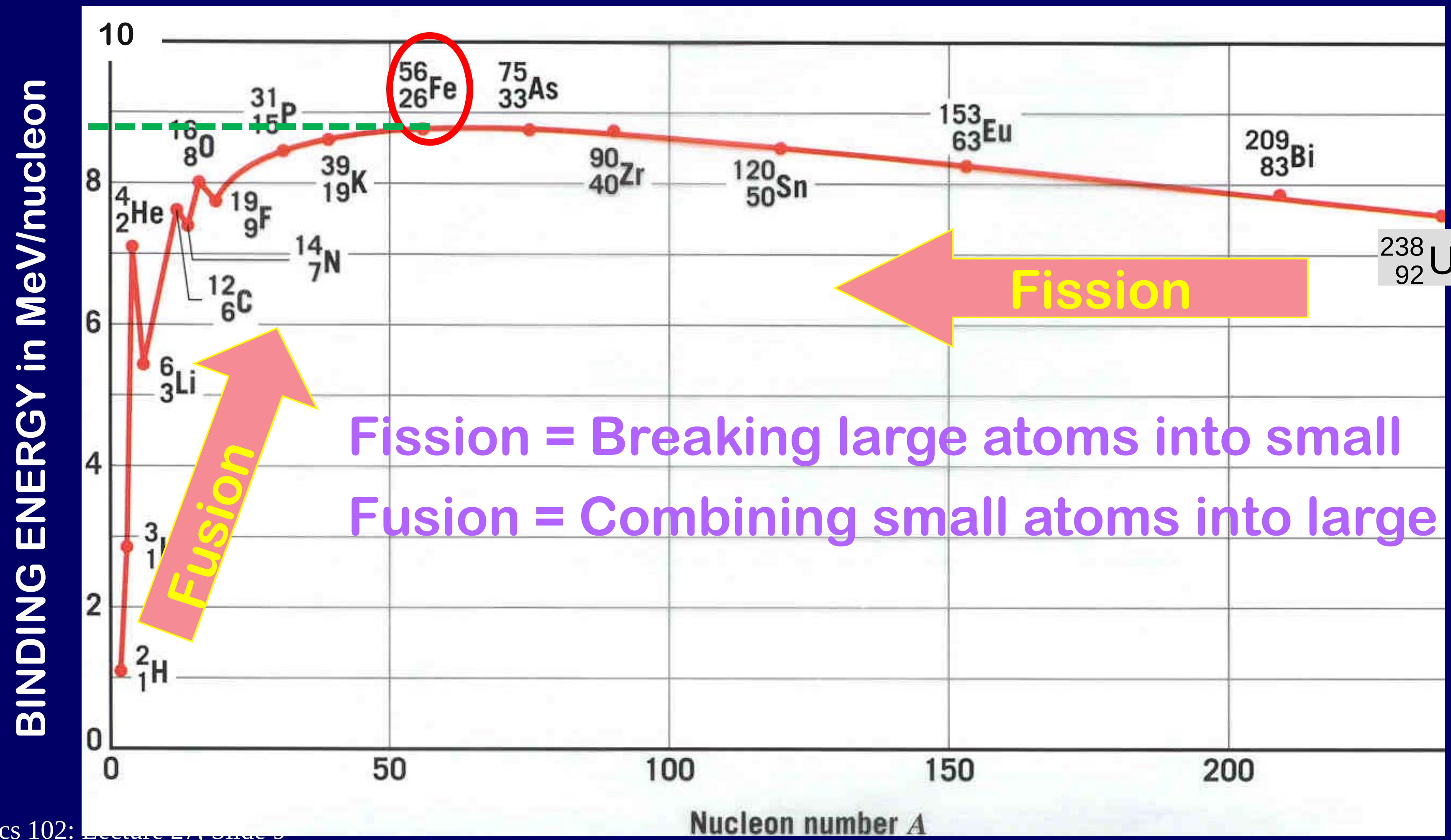
(3) binding energy of nucleons

~MeV



Binding Energy Plot

Iron (Fe) has most binding energy/nucleon!
(Checkpoint 2.1) A



Checkpoint 2.2

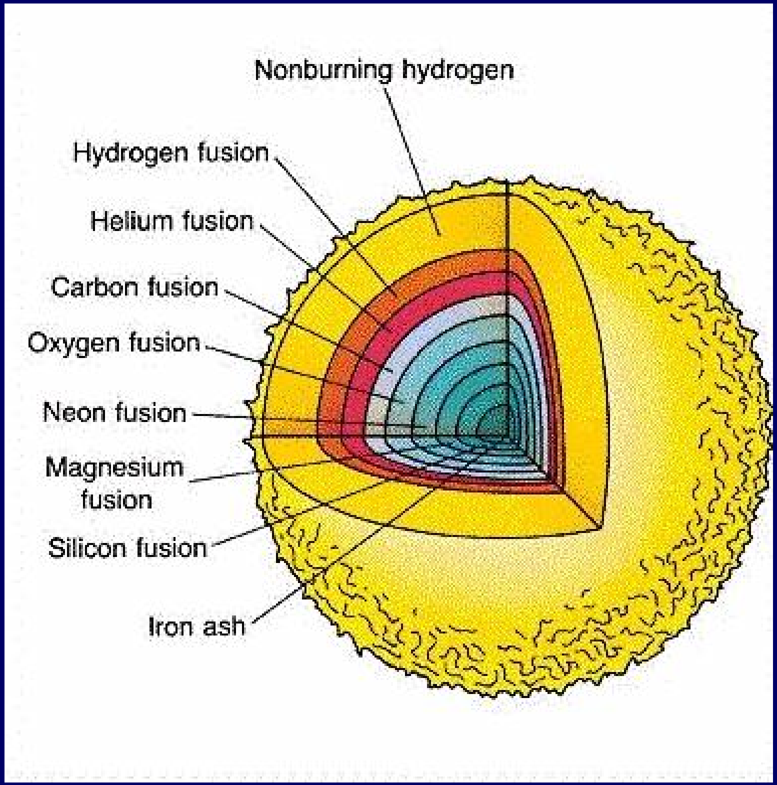
Which of the following is most correct for the total binding energy of an Iron nucleus (Z=26)?

13% 9 MeV

39% 234 MeV

31% 270 MeV

17% 504 Mev

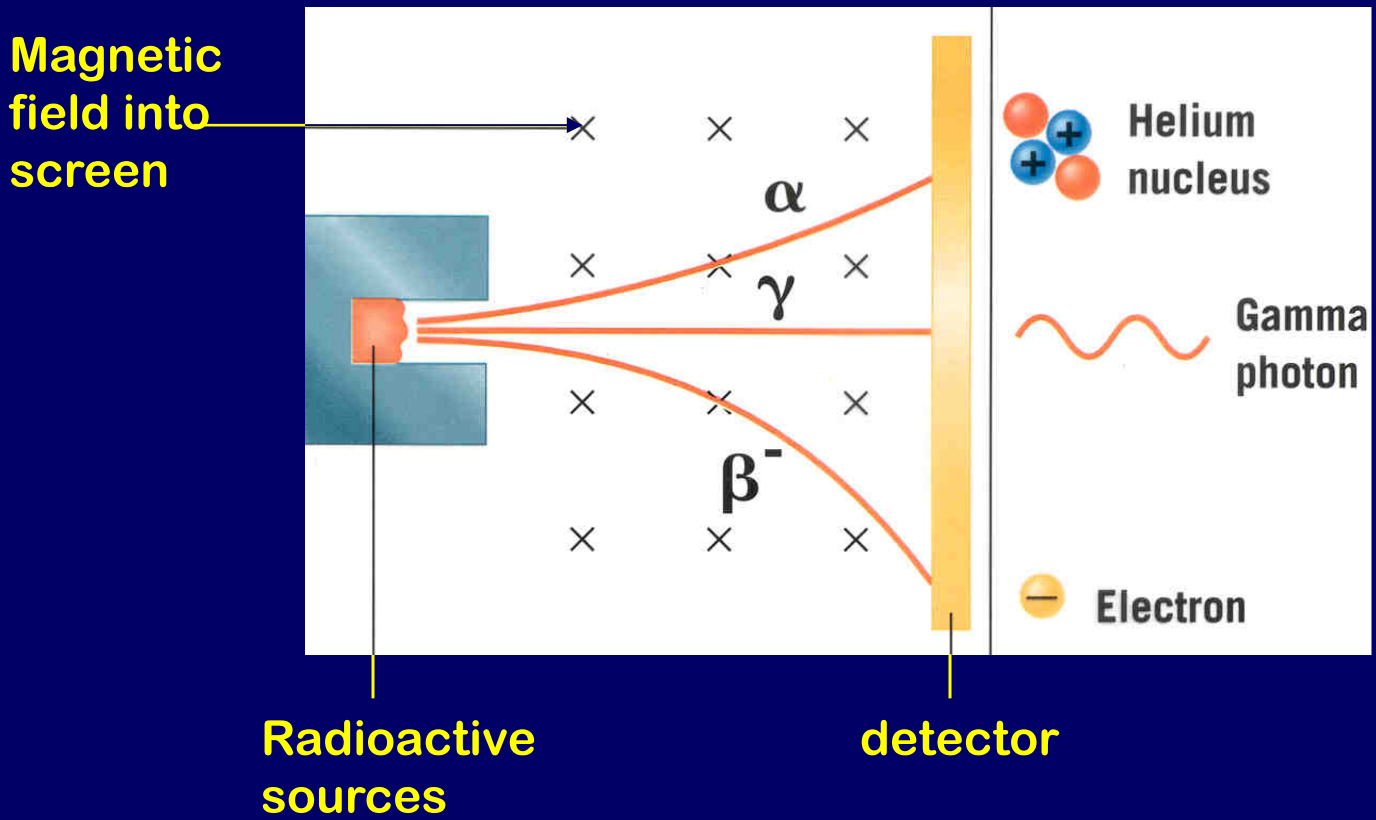


For Fe, B.E./nucleon ≈ 9MeV

$^{56}_{26}Fe$ **has 56 nucleons**

Total B.E ≈ 56x9=504 MeV

3 Types of Radioactivity



α particles: ${}^{4}_{2}He$ nuclei — Easily Stopped

β^- particles: electrons — Stopped by metal

γ : photons (more energetic than x-rays) penetrate!

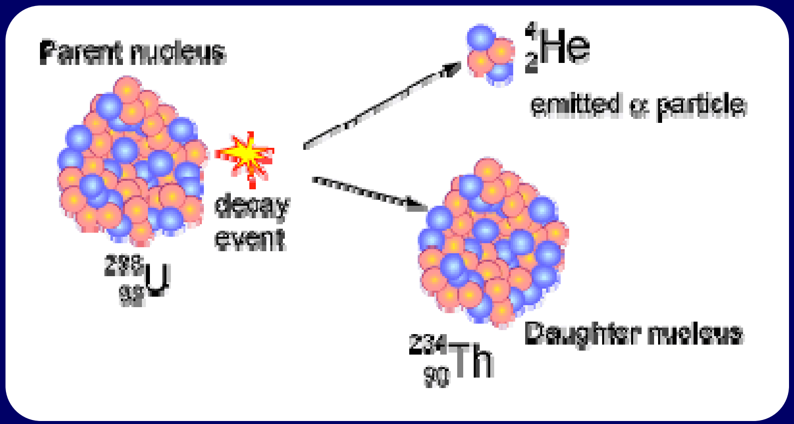
Example

Decay Rules

1) **Nucleon Number (A) is conserved.**
2) **Atomic Number (Z) is conserved.**
3) **Energy and momentum are conserved.**

α: **example** $^{238}_{92}U \rightarrow {}^{234}_{90}Th + \alpha$ **recall** $^{4}_{2}He = \alpha$

1) 238 = 234 + 4 Nucleon number conserved
2) 92 = 90 + 2 Charge conserved



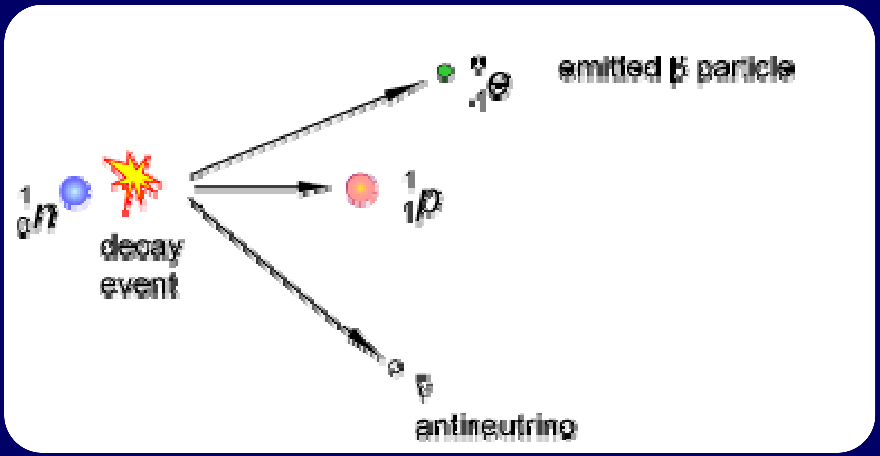
Example

Decay Rules

1) Nucleon Number (A) is conserved.
2) Atomic Number (Z) is conserved.
3) Energy and momentum are conserved.

β: example $\;{}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}e^{-} + {}^{0}_{0}\overline{\nu}$

anti-neutrino needed to conserve momentum.



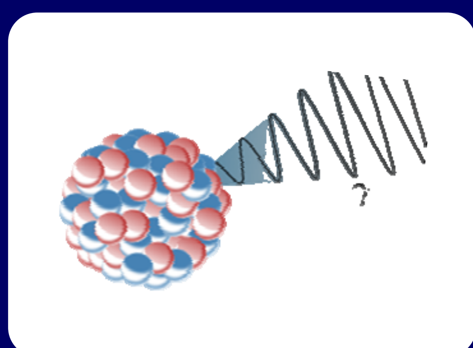
Example

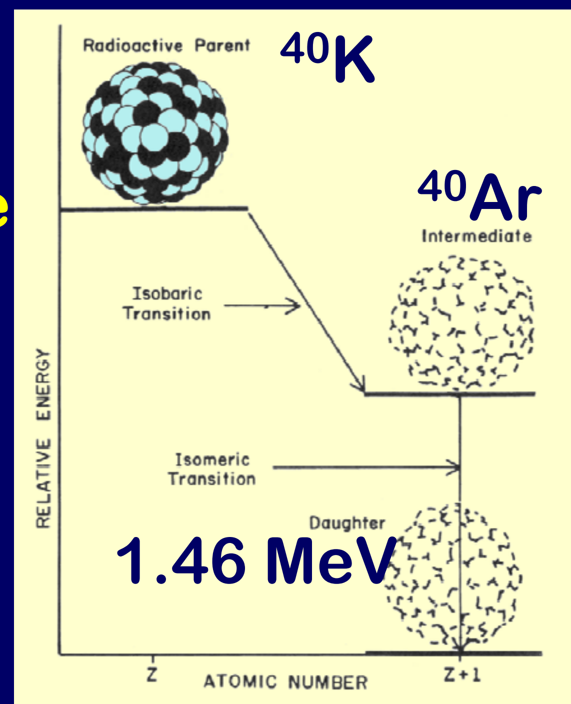
Decay Rules

1) **Nucleon Number (A) is conserved.**
2) **Atomic Number (Z) is conserved.**
3) **Energy and momentum are conserved.**

γ: **example** $${}_{Z}^{A}P^{*} \rightarrow {}_{Z}^{A}P + {}_{0}^{0}\gamma$$

"nuclear isomer": excited state





ACT: Checkpoint 3

A nucleus undergoes α decay. Which of the following is FALSE?

1. Nucleon number decreases by 4 — 27%
2. Neutron number decreases by 2 — 39%
3. Charge on nucleus increases by 2 — 34%

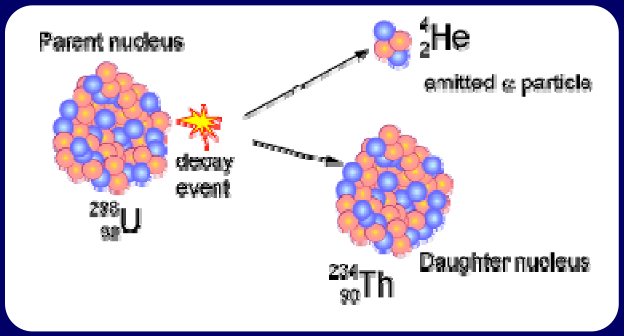
α decay is the emission of $^{4}_{2}\text{He} = \alpha$

A decreases by 4

Ex. $$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}He$$

Z decreases by 2

(charge decreases!)



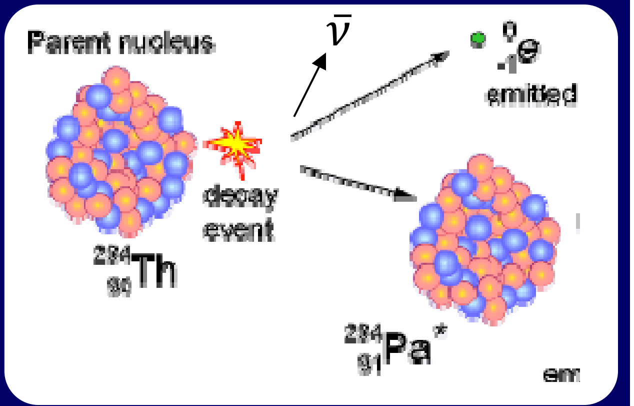
Checkpoint 4

The nucleus $^{234}_{90}\mathrm{Th}$ undergoes β^- decay.

Which of the following is true?

1. The number of protons in the daughter nucleus increases by one.

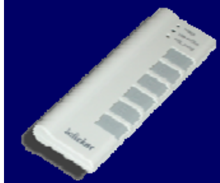
2. The number of neutrons in the daughter nucleus increases by one.



β^- decay is accompanied by the emission of an electron: creation of a charge -e.

$$^{234}_{90}\mathrm{Th} \rightarrow\, ^{???}_{??}\mathrm{X} + \,^{0}_{-1}e^- + \,^{0}_{0}\bar{\nu} \qquad ^{234}_{90}\mathrm{Th} \rightarrow\, ^{234}_{91}\mathrm{Pa} + \,^{0}_{-1}e^- + \,^{0}_{0}\bar{\nu}$$

In fact, $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$ inside the nucleus, and the electron and neutrino "escape."



ACT: Decay

Which of the following decays is NOT allowed?

1 $^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \alpha$

238 = 234 + 4
92 = 90 + 2

2 $^{214}_{84}\mathrm{Po} \rightarrow {}^{210}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

214 = 210 + 4
84 = 82 + 2

3 $^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma$

14 = 14+0
$6 \neq 7+0$

4 $^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca} + {}^{0}_{-1}e^{-} + {}^{0}_{0}\bar{\nu}$

40 = 40+0+0
19 = 20-1+0

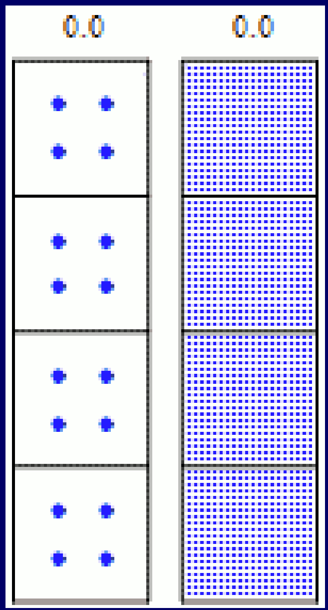
Radioactive decay rates

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Decays per second or "activity"

decay constant

No. of nuclei present



Checkpoint 5

If the number of radioactive nuclei present is cut in half, how does the activity change?

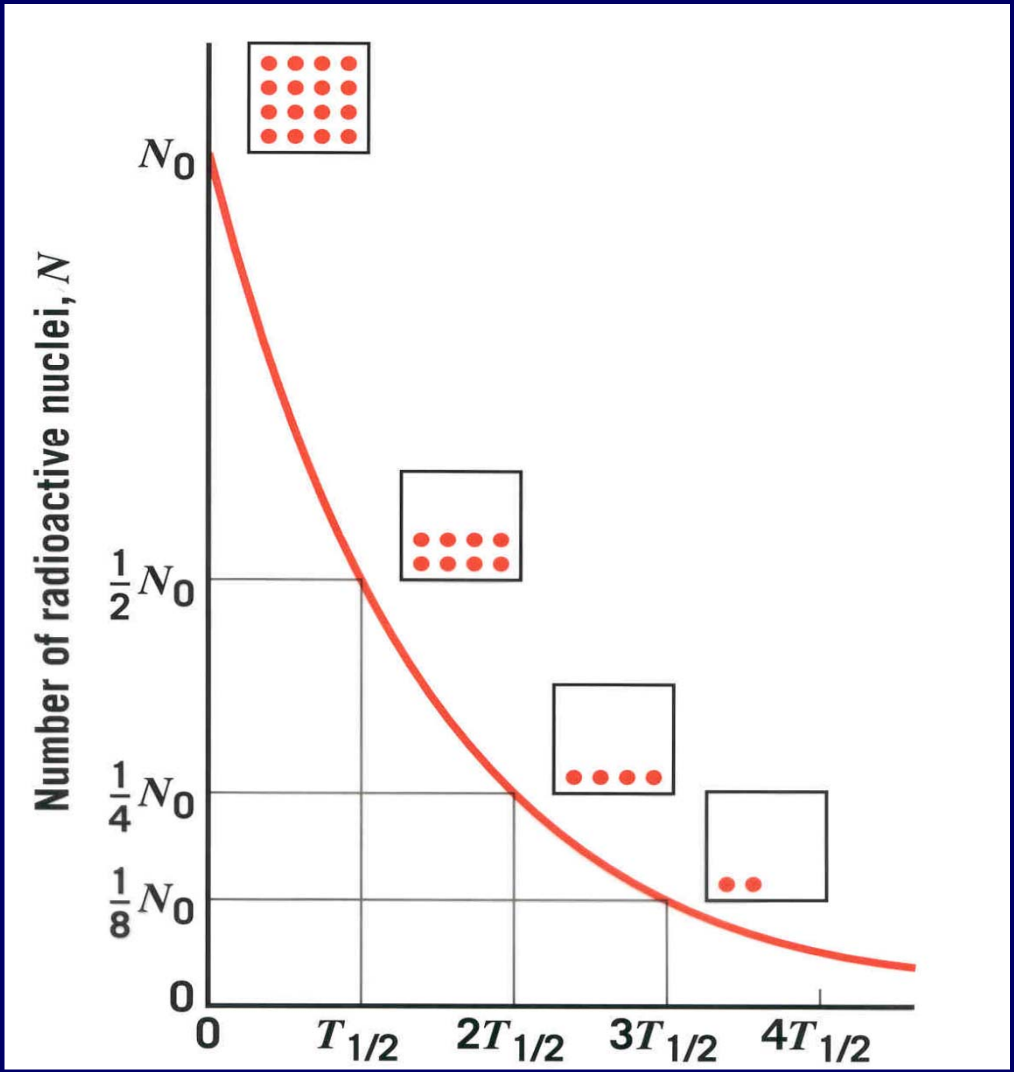
1 It remains the same	26%
2 It is cut in half	58%
3 It doubles	16%

1 "becquerel" (Bq) =1 decay/s

1 "curie" Ci =3.7×10^{10} Bq

Decay Function

$e^{-\lambda t} = e^{-\ln 2 \lambda t/\ln 2} = 2^{-\lambda t/\ln 2}$



time →

$$N(t) = N_0 e^{-\lambda t} = N_0 \cdot 2^{-\frac{t}{T_{1/2}}}$$

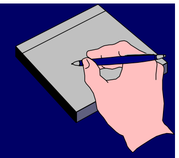
# of nuclei present at time t

 # we started with at t=0

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

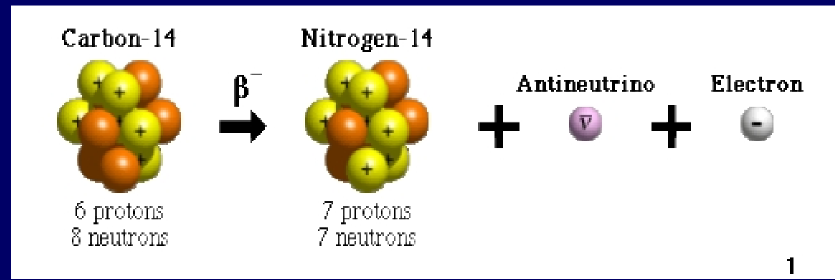
Half of the remaining atoms are lost after a "half life" that lasts $T_{1/2}$; the activity also is reduced by one half every $T_{1/2}$

Example



You are radioactive!

One in 8.3×10^{11} carbon atoms is ^{14}C which β^- decays with a ½ life of 5730 years. Determine # of decays/s per gram of Carbon.



C^{14}

$$N_{14} = \left(\frac{1.0}{12}\frac{\text{mole}}{g}\right)\left(6.02\times10^{23}\right)\left(\frac{1}{8.3\times10^{11}}\right) = 6\times10^{10}\ \frac{\text{atoms}}{\text{g}}$$

$$\lambda = \frac{.693}{T_{1/2}} = \frac{.693}{5730\times365\times24\times60\times60} = 3.83\times10^{-12}\,\text{s}^{-1}$$

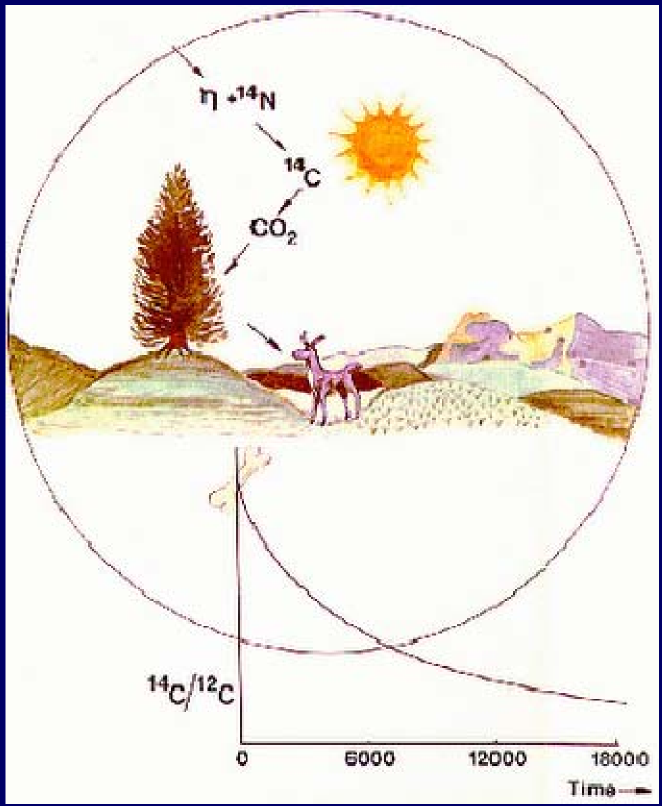
$$\frac{\Delta N}{\Delta t} = -\lambda N = 0.23\ \text{decays/s}$$

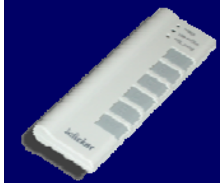
Carbon Dating

Example

We just determined that living organisms should have a decay rate of about 0.23 events/s per gram of carbon.

The bones of a man from the last ice age are found to have a decay rate of 0.23/2 events/s per gram. We can estimate he died about 6000 years ago.





ACT/Checkpoint 6

The half-life for beta-decay of ^{14}C is ~6,000 years. You test a fossil and find that only 25% of its ^{14}C is un-decayed. How old is the fossil?

1. 3,000 years
2. 6,000 years
3. 12,000 years

At 0 years: 100% remains

At 6,000 years: 50% remains

At 12,000 years: 25% remains

Summary

- Nuclear Reactions
  - Nucleon number conserved
  - Charge conserved
  - Energy/Momentum conserved
  - α particles = ${}^{4}_{2}He$ nuclei
  - β^- particles = electrons
  - γ particles = high-energy photons

Activity: $\frac{\Delta N}{\Delta t} = -\lambda N$ **Survival:** $N(t) = N_0 e^{-\lambda t}$ $T_{1/2} = \frac{0.693}{\lambda}$

- Decays
  - Half-Life is time for ½ of atoms to decay