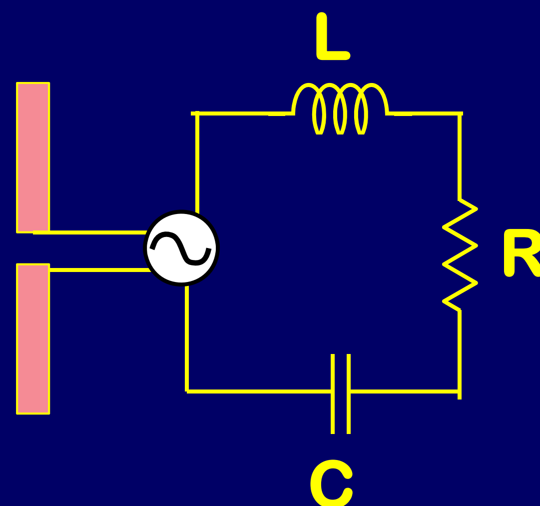
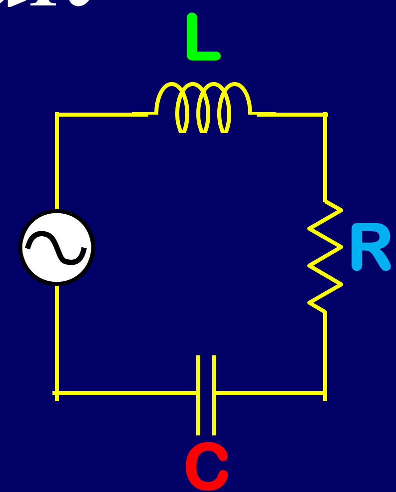
Physics 102: Lecture 13

RLC circuits & Resonance



Review: AC Circuit



- $I = I_{max}\sin(2\pi ft) = I_L = I_R = I_C$
- $V_R = I_{max}R\sin(2\pi ft)$

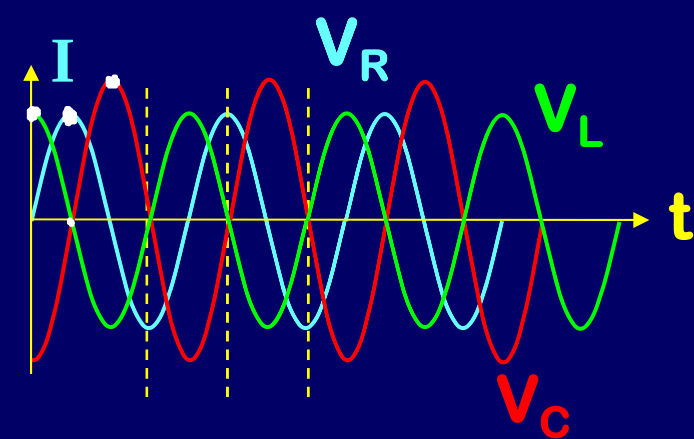
  V_R in phase with I
- $V_C = I_{max}X_C\sin(2\pi ft - \pi/2)$

  V_C lags I "ICE"

  $V_{C,max} = I_{max}X_C \quad X_C = 1/\omega C$
- $V_L = I_{max}X_L\sin(2\pi ft + \pi/2)$

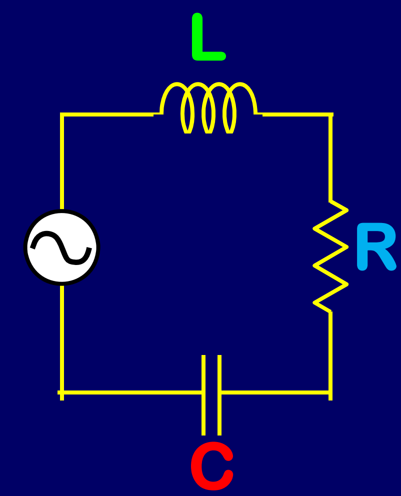
  V_L leads I "ELI"

  $V_{L,max} = I_{max}X_L \quad X_L = \omega L$

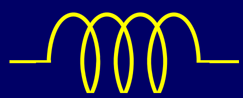


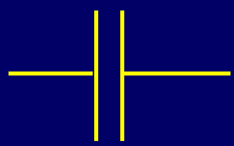
Goal:

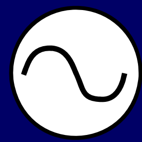
write down equations for all of the voltages



$V_R = I_{max} R \sin(2\pi f t)$ 

$V_L = I_{max} X_L \sin(2\pi f t + \pi/2)$ 

$V_C = I_{max} X_C \sin(2\pi f t - \pi/2)$ 

The only element left: generator! 

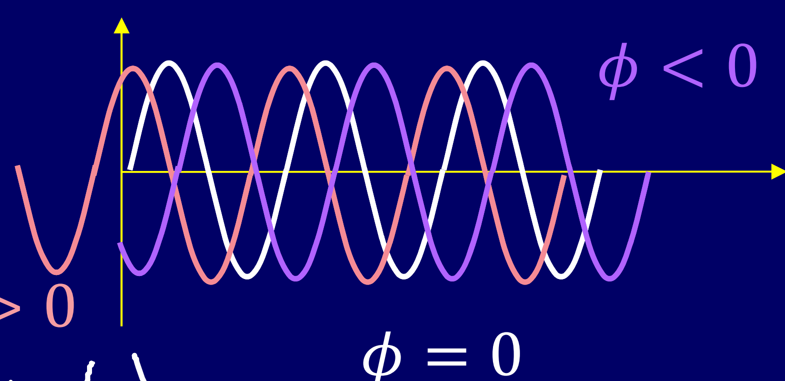
$$V_{gen} = \underline{V_{gen,max}} \sin(2\pi f t + \underline{\phi})$$

ϕ: "phase angle"

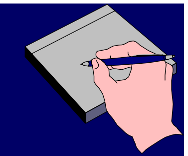
Like a shift in time!

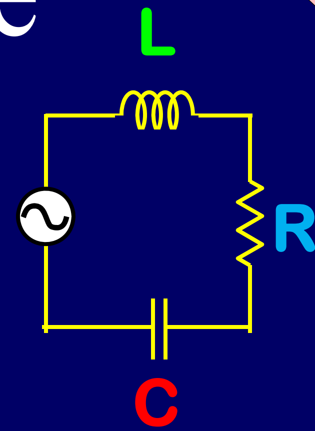
$\phi < 0$: shift forward lag (ex: C)

$\phi > 0$: shift backward lead (ex: L)

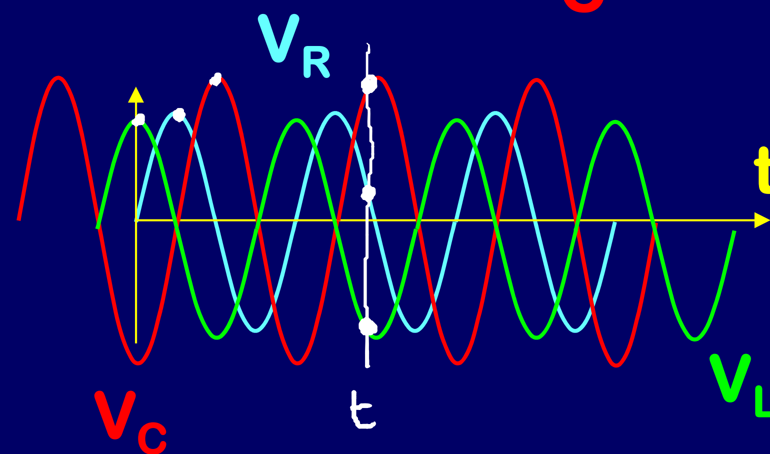


Kirchhoff: generator voltage





- Instantaneous voltage across generator (V_{gen}) must equal sum of voltage across all of the elements at all times:



$$V_{gen}(t) = V_R(t) + V_C(t) + V_L(t)$$

$$\cancel{V_{gen,max} \neq V_{L,max} + V_{R,max} + V_{C,max}}$$

What is $V_{gen,max}$?

Define <u>impedance</u> Z: $V_{gen,max} \equiv I_{max} Z$ **Like: V=IR**

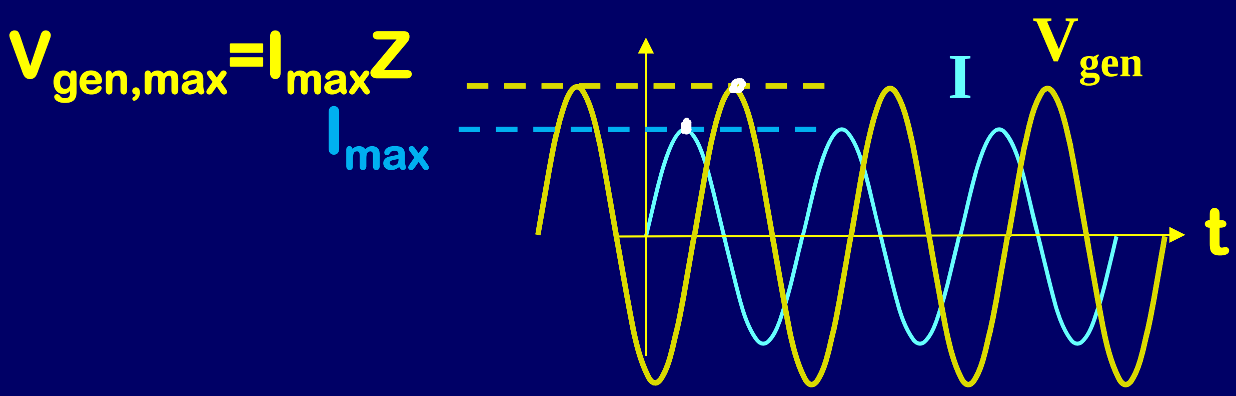
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

One last ingredient: is the generator voltage leading or lagging the current?

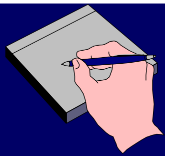
$$I = I_{max}\sin(2\pi f t)$$

$$V_{gen} = I_{max} Z \sin(2\pi f t + \phi)$$

Phase angle: $\tan(\phi) = \dfrac{(X_L - X_C)}{R}$



This example:
$\phi < 0$, **voltage is lagging**



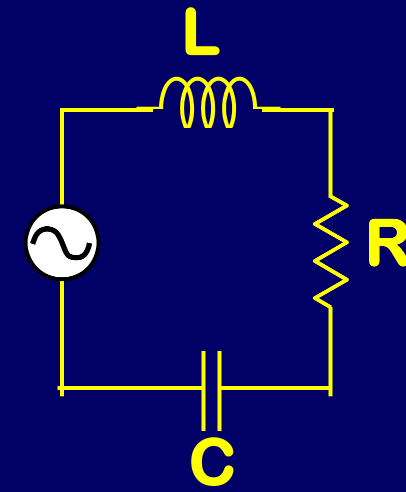
Example

Problem Time!

An AC circuit with R = 2 Ω, C = 15 mF, and L = 30 mH is driven by a generator with voltage $V_{gen}(t)=2.5 \sin(8\pi t)$ Volts. Calculate the maximum current in the circuit, and the phase angle.

$X_L = 2\pi f L$

$X_C = 1/(2\pi f C)$



$I_{max} = V_{gen,max}/Z$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{2^2 + \left(8\pi \times .030 - \frac{1}{8\pi \times .015}\right)^2} = 2.76\Omega$$

$I_{max} = 2.5/2.76 = 0.91$ Amps

$$\tan(\phi) = \frac{X_L - X_C}{R} = \frac{\left(8\pi \times .030 - \frac{1}{8\pi \times .015}\right)}{2} \Rightarrow \varphi = -43.5^\circ$$

Power in RLC circuits

- The voltage generator supplies power.
  - Only resistor dissipates power.
  - Capacitor and Inductor store and release energy.
- $P(t) = I(t)V_R(t)$ oscillates so sometimes power loss is large, sometimes small.
- Average power dissipated by resistor:

$$\overline{P} = \tfrac{1}{2} I_{max} V_{R,max} = I_{rms} V_{R,rms}$$

$$= \tfrac{1}{2} I_{max} V_{gen,max} \cos(\phi)$$

$$= I_{rms} V_{gen,rms} \cos(\phi)$$

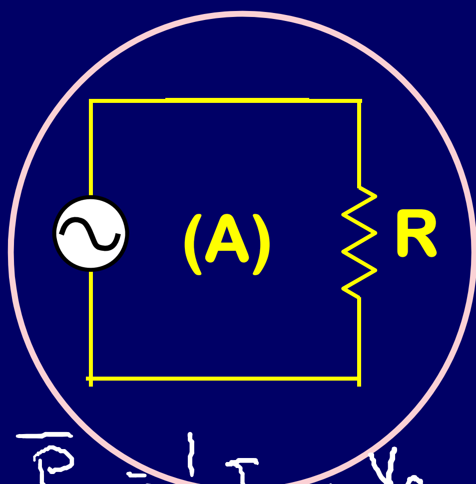
$$\tan\phi = \frac{X_L - X_C}{R}$$

If there is only a resistor, $\phi = 0$

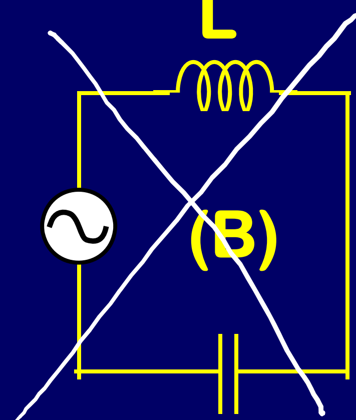
ACT: Power dissipation

Which one of these circuits dissipates the most average power?

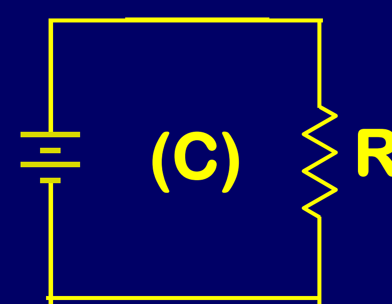
$V_{gen,max} = 10$ V



$V_{gen,max} = 100$ V



$\varepsilon = 1$ V



A) $\bar{P} = \frac{1}{2} I_{max} V_{R,max}$

$= \frac{1}{2} I_{max} V_{gen,max}$

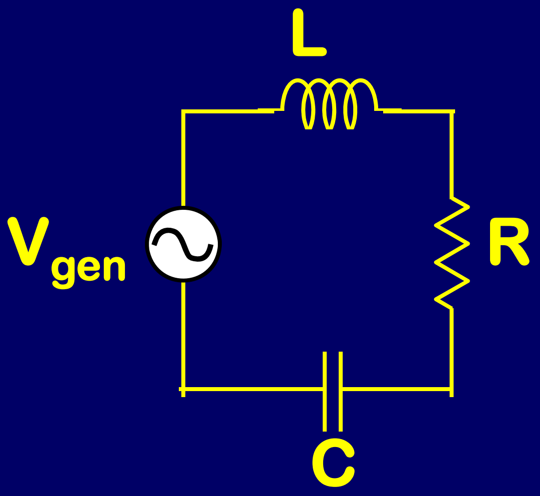
$= \frac{1}{2} \frac{V_{gen,max}^2}{R} = \frac{50}{R}$

C) $P = I\mathcal{E}$

$= \mathcal{E}^2/R$

$= 1/R$

Kirchhoff: generator voltage



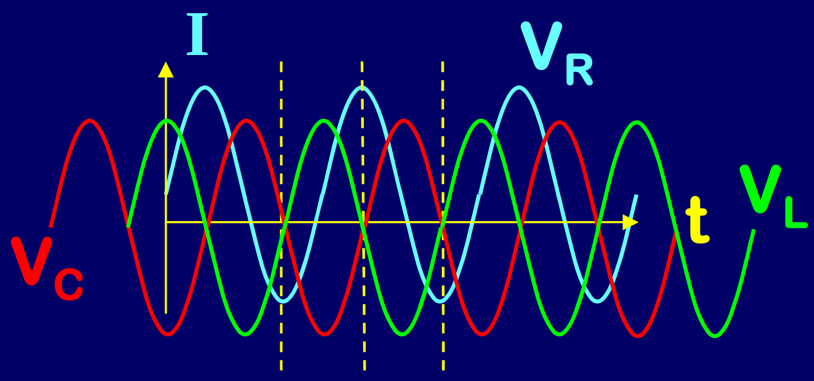
Write down Kirchhoff's Loop Equation:

$V_{gen}(t) = \cancel{V_L}(t) + V_R(t) + \cancel{V_C}(t)$ at every instant of time

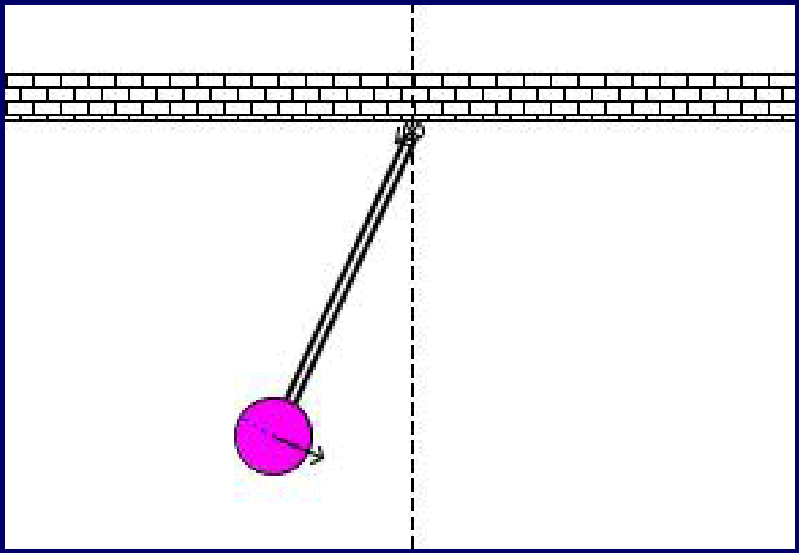
If the frequency is just right...

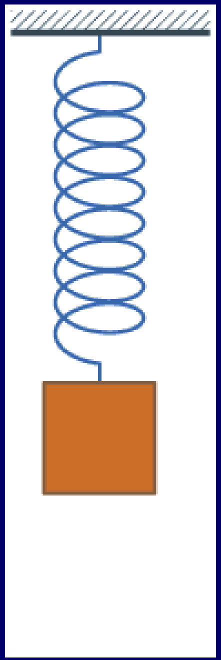
$$V_{gen}(t) = V_R(t)$$

Resonance!



Examples of mechanical resonance





Pendulum:

$$\omega_0 = \sqrt{g/l}$$

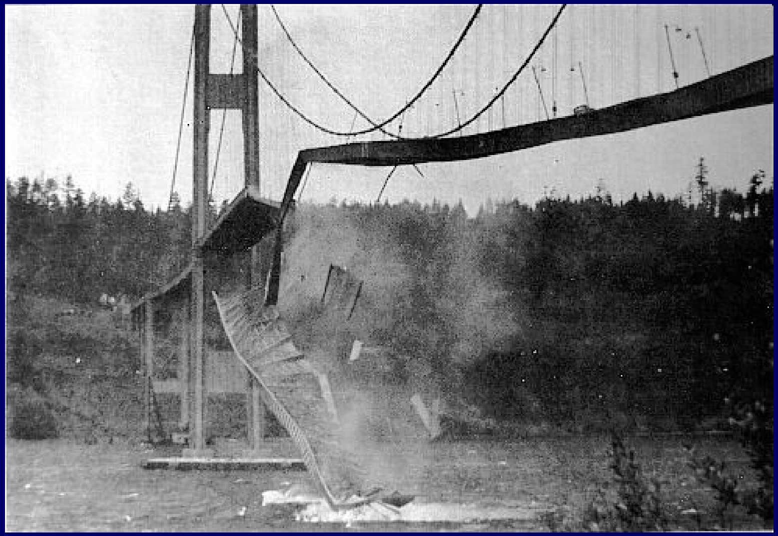
Mass on a spring:

$$\omega_0 = \sqrt{k/m}$$

Common features:

- **Energy converts between two forms (kinetic & potential)**
- **Weak input at ω_0 leads to a big response!**

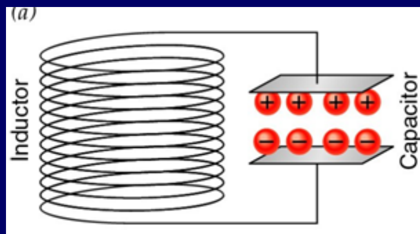
Resonance!



Tacoma Narrows Bridge

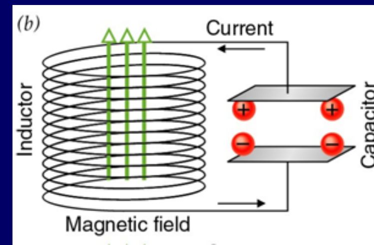
Millennium Bridge

Electric energy

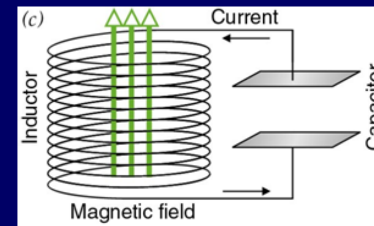


$V_C = V_{C,max}$

$V_L = -V_{L,max}$

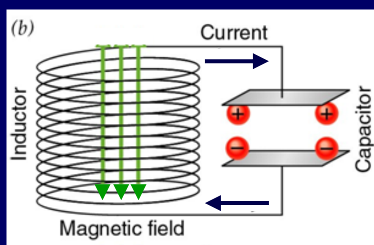


Magnetic energy



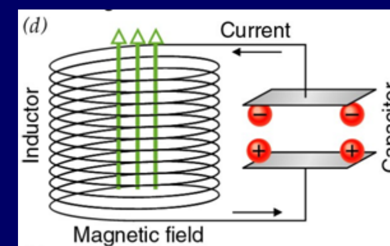
$V_C = 0$

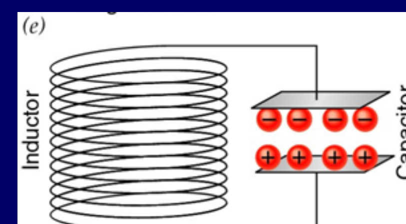
$V_L = 0$



RLC circuit: Electrical resonance

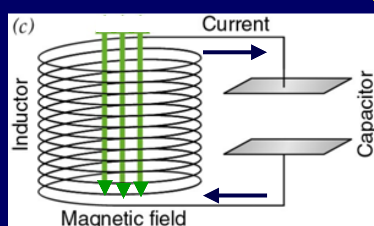
f_0

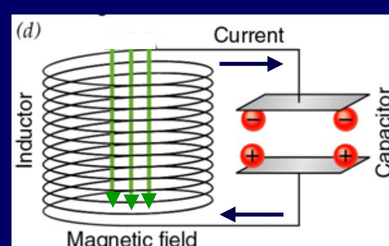




$V_C = 0$

$V_L = 0$





Magnetic energy

Electric energy

$V_C = -V_{C,max}$

$V_L = V_{L,max}$

Resonance

R is independent of f

X_L increases with f

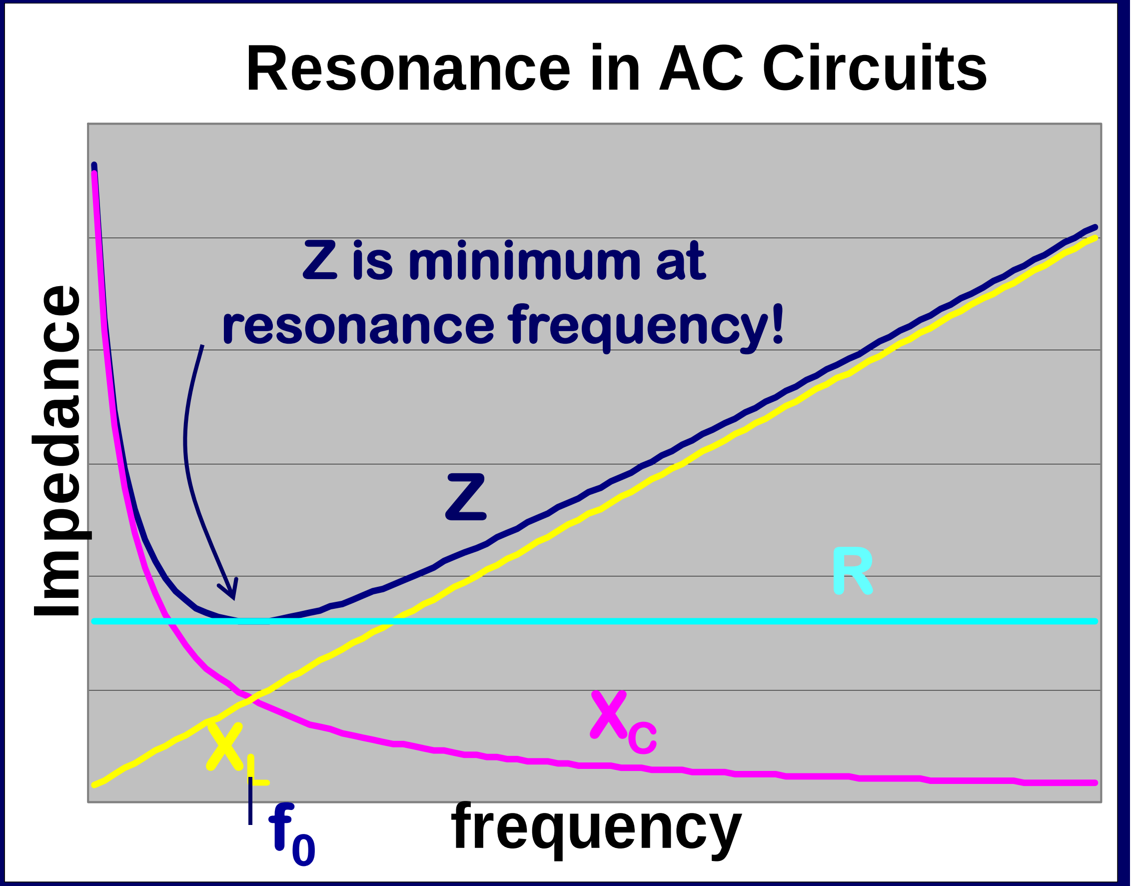
$$X_L = 2\pi f L$$

X_C decreases with f

$$X_C = 1/(2\pi f C)$$

Z: X_L and X_C subtract

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Resonance: $X_L = X_C$

$V_{L,max} = V_{C,max}$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$\tan\phi = \frac{X_L - X_C}{R} = 0$$

Resonance

R is independent of f

X_L increases with f

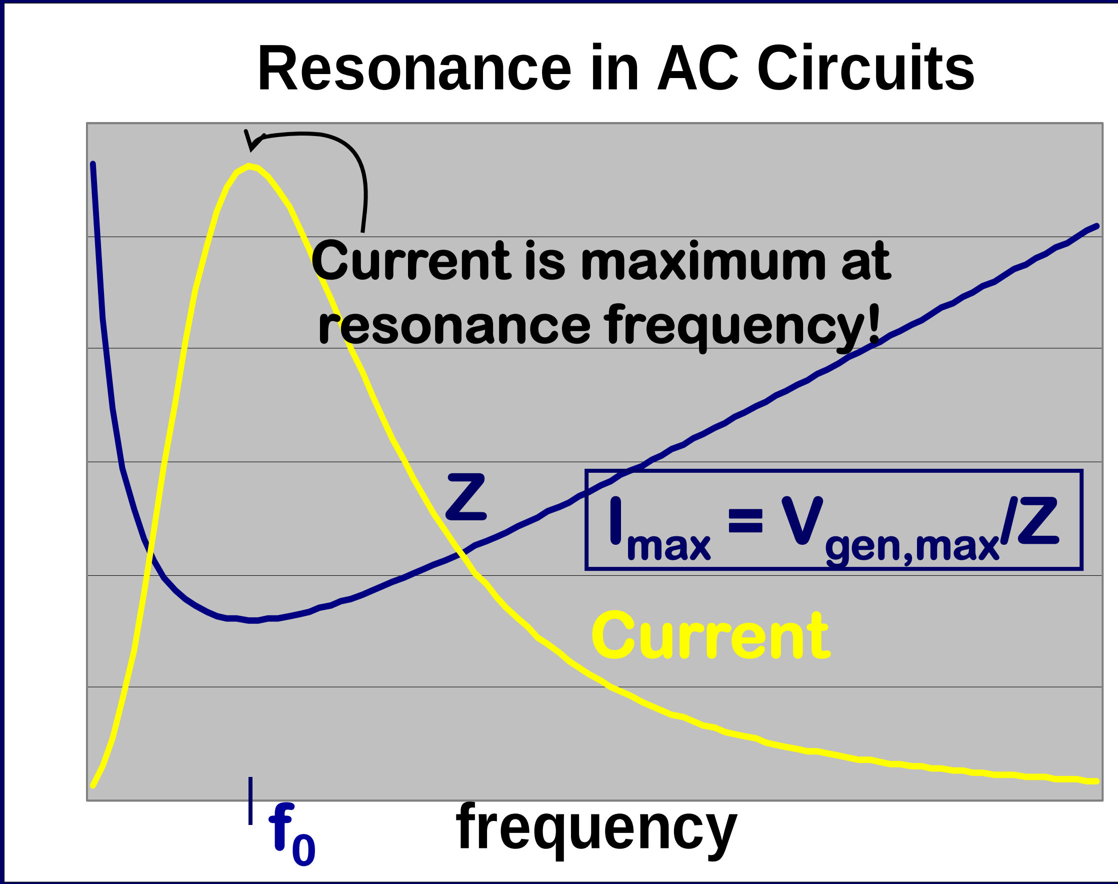
$$X_L = 2\pi f L$$

X_C decreases with f

$$X_C = 1/(2\pi f C)$$

Z: X_L and X_C subtract

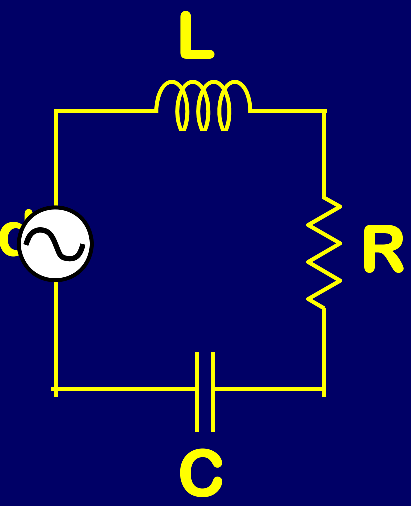
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Resonance: $X_L = X_C$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

CheckPoint 14.1

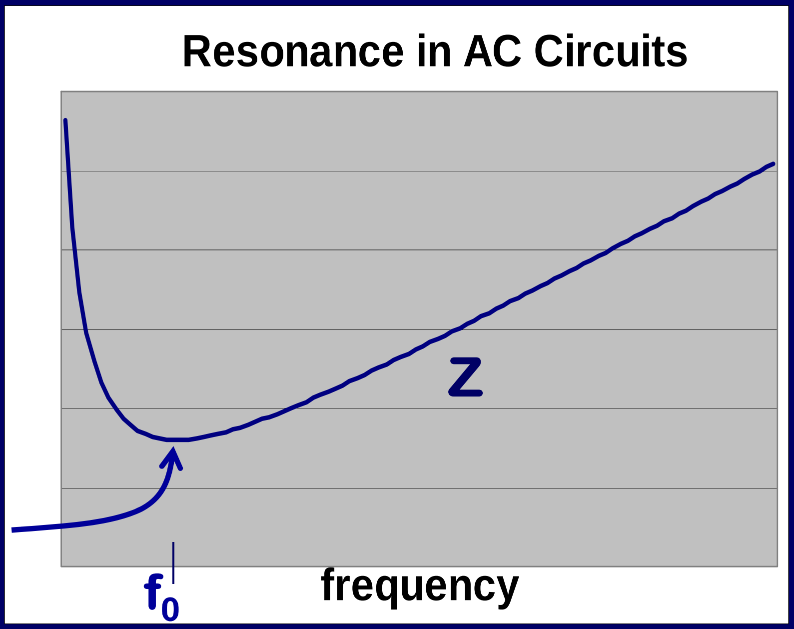


As the frequency of the circuit is either raised above or lowered below the resonant frequency, the impedance of the circuit:

48% Always increases

27% Only increases for lowering the frequency

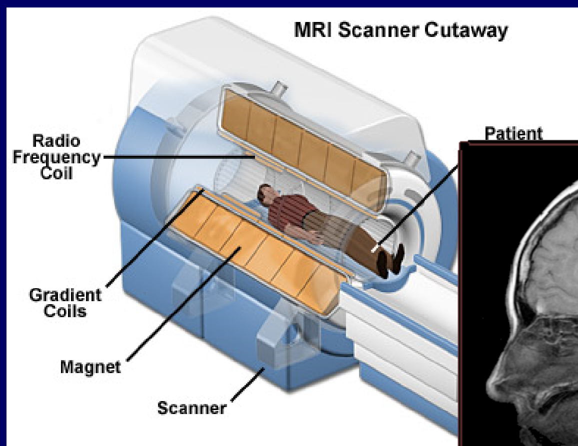
25% Only increases for raising the frequency

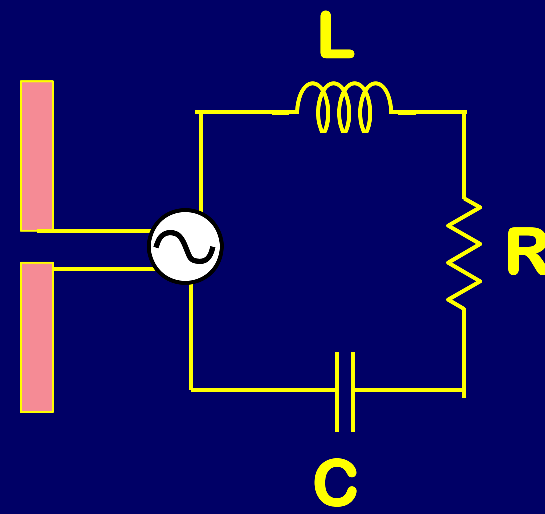


Z is minimum at f_o!

Any other frequency will have higher Z!

What is it good for?

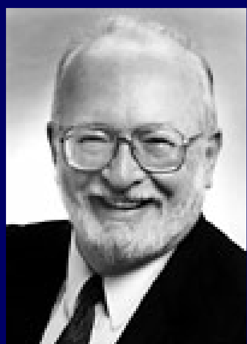




- Current through circuit depends on frequency (maximum at resonance frequency f_o)
  - Radio receiver – Picks out radio station freq f_o
  - NMR/MRI – Picks out signal from protons at f_o

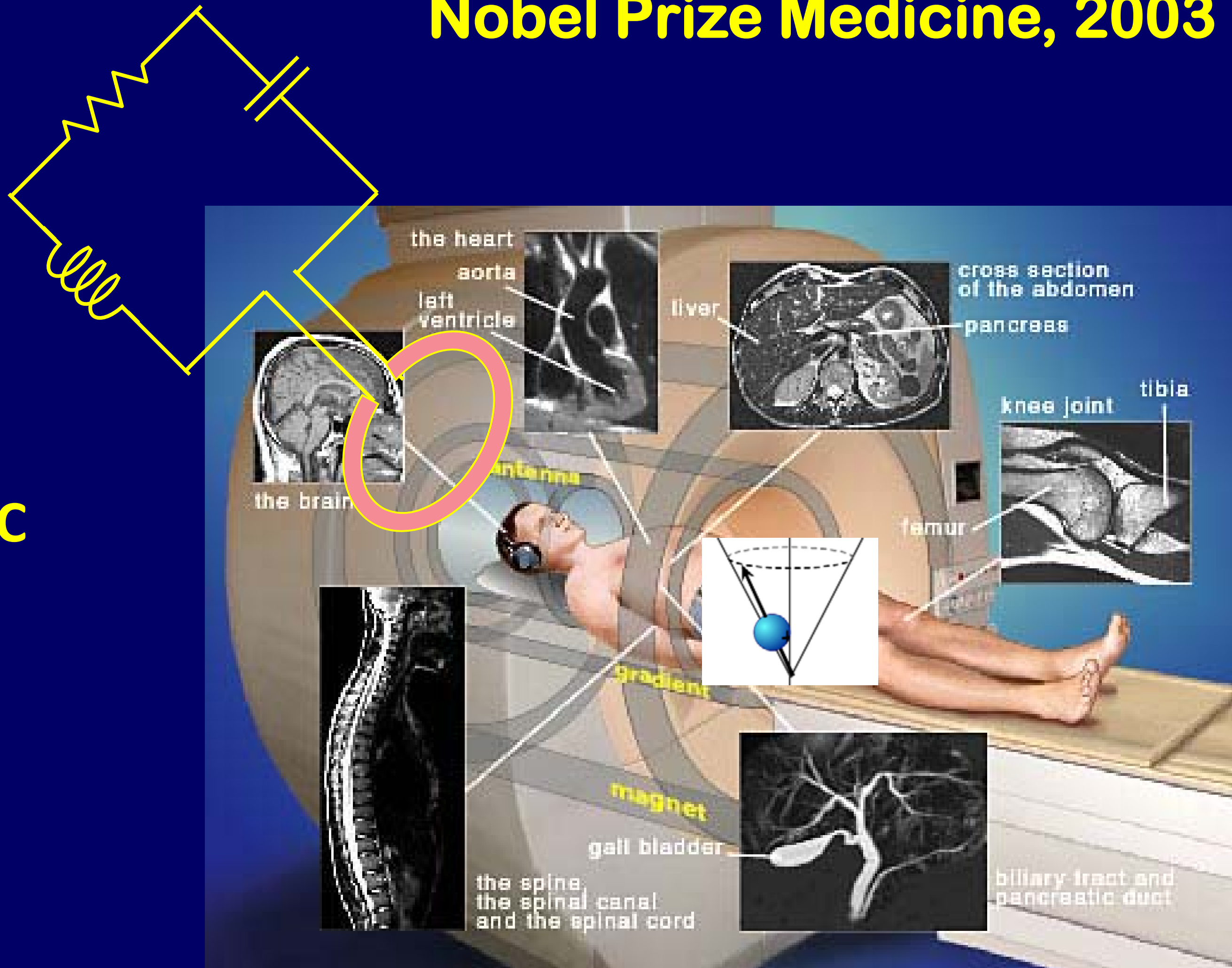
Magnetic Resonance Imaging

Nobel Prize Medicine, 2003



Lauterbur, UIUC

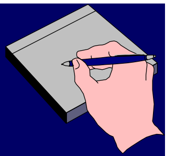
Mansfield

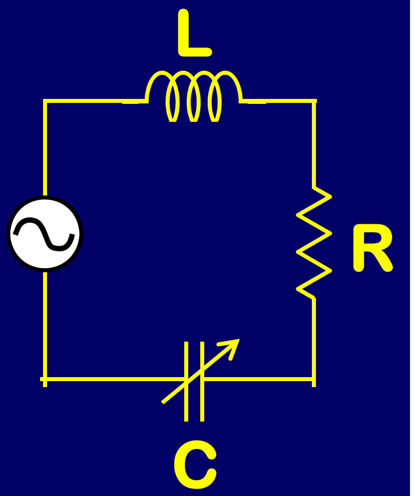


Faraday's Law + RLC circuit!

Example

Resonance in Radios





An AC circuit with R = 2 Ω, L = 0.30 μH and variable capacitance is connected to an antenna to receive radio signals at the resonance frequency. If you want to listen to music broadcasted at 96.1 MHz, what value of C should be used?

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{\left(2\pi \times 96.1\times10^{6}\right)^2 \times 0.30\times10^{-6}} = 9.1\times10^{-12}\,F$$

ACT: Radios

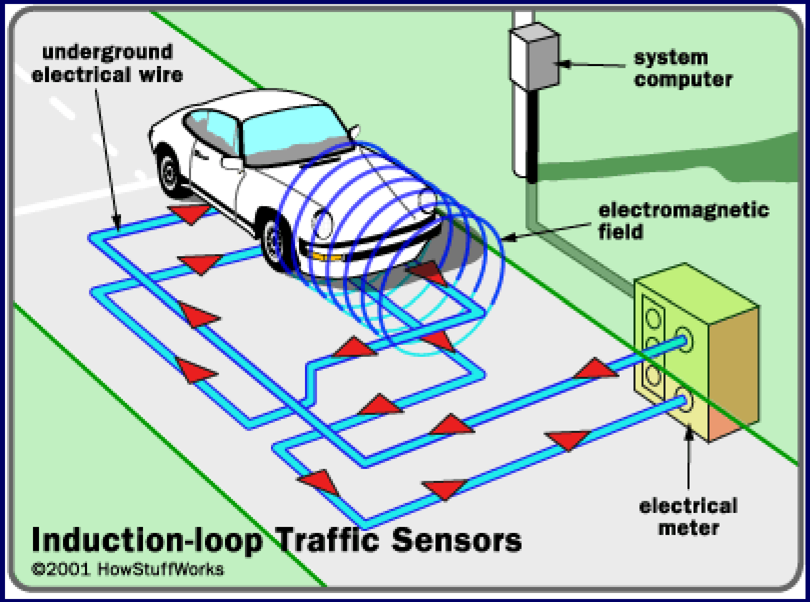
Your radio is tuned to FM 96.1 MHz and want to change it to FM 105.9 MHz, which of the following will work.

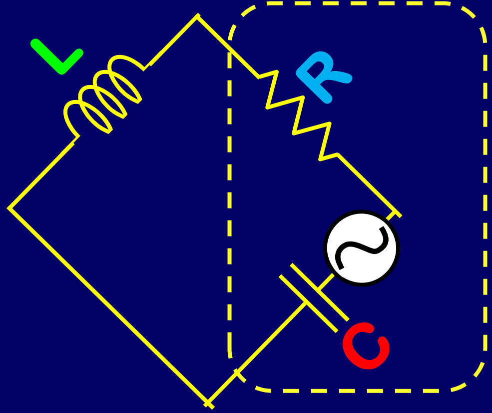
1. Increase Capacitance
2. Decrease Capacitance
3. Neither, you need to change R

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Higher frequency needs smaller capacitor so it can develop voltage quicker.

Another use for RLC circuits: traffic sensors





AC Summary

Resistors: $V_{R,max}=I_{max}\,R$

In phase with I

Capacitors: $V_{C,max}=I_{max}\,X_C$ $\quad X_c = 1/(2\pi f\,C)$

Lags I

Inductors: $V_{L,max}=I_{max}\,X_L$ $\quad X_L = 2\pi f\,L$

Leads I

Generator: $V_{gen,max}=I_{max}\,Z$ $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

Can lead or lag I $\quad \tan(\phi) = (X_L - X_C)/R$

Power is only dissipated in resistor:

$$\bar{P} = \tfrac{1}{2} I_{max} V_{gen,max} \cos(\phi)$$