Physics 102: Lecture 04

Capacitors
(& batteries)

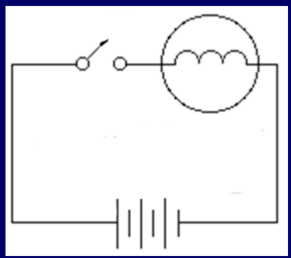
Physics 102 so far

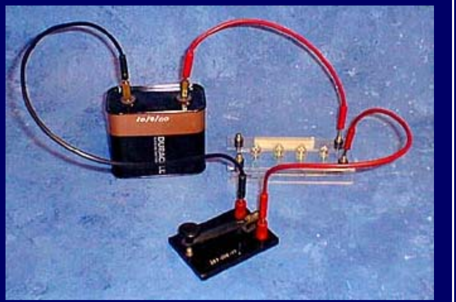
Basic principles of electricity

- Lecture 1 – electric charge & electric force
- Lecture 2 – electric field
- Lecture 3 – electric potential energy and electric potential

Applications of electricity – circuits

- Lecture 4 – capacitance
- Lecture 5 – resistance
- Lecture 6 – Kirchhoff's rules
- Lecture 7 – RC circuits
- Lecture 12 & 13 – AC circuits

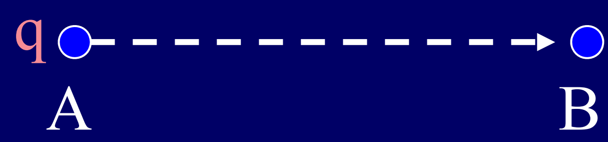




Recall from last lecture…..
Electric Fields, Electric Potential

Comparison:

Electric *Potential Energy* vs. Electric *Potential*



ΔV_{AB} : the difference in electric potential between points B and A

ΔU_{AB} : the change in electric potential energy of a charge q when moved from A to B

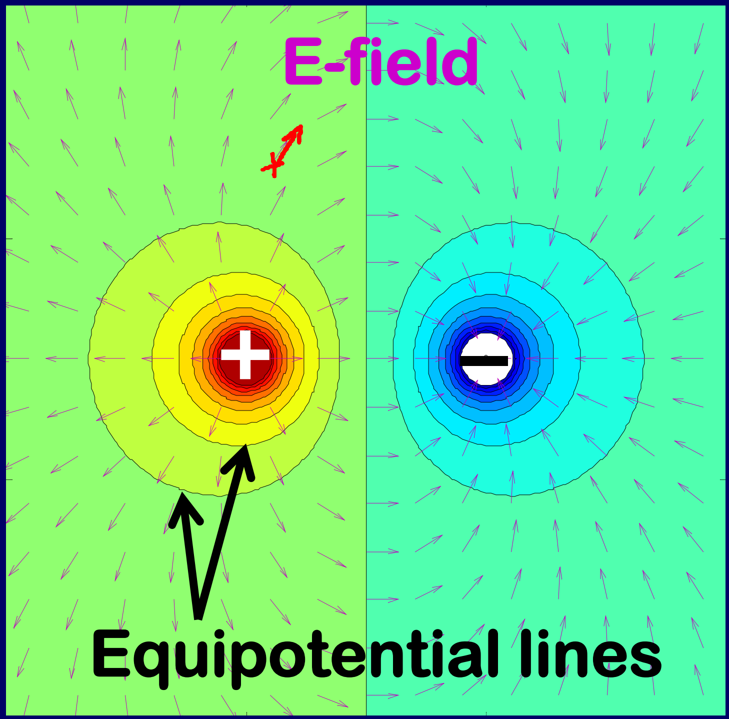
$$\Delta U_{AB} = q\,\Delta V_{AB}$$

Electric Potential: Summary

- E field lines point from higher to lower potential
- For positive charges, going from higher to lower potential is "downhill"

  Positive charges tend to go "downhill", from + to –

  Negative charges go in the opposite direction, from – to +



$$\Delta U_{AB} = q\,\Delta V_{AB}$$

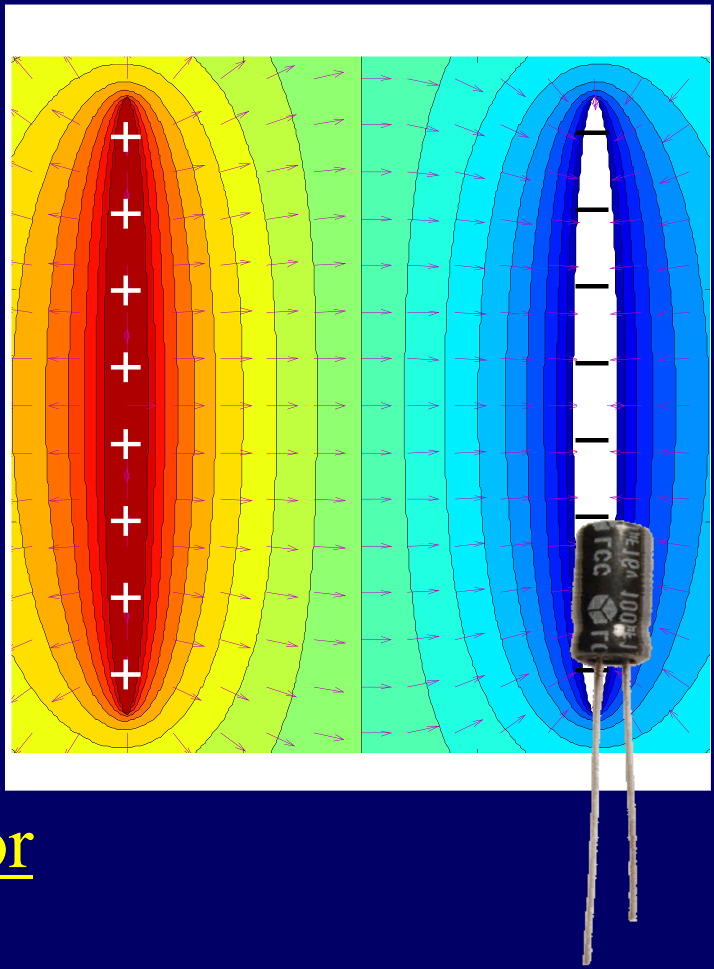
Uniform Electric Field: Important Special Case

Two large parallel conducting plates of area A

- +Q on one plate
- –Q on other plate

Then E is

- uniform between the two plates: $E=4\pi kQ/A$
- zero everywhere else
- This result is independent of plate separation



This is called a parallel plate capacitor

Parallel Plate Capacitor: Potential Difference

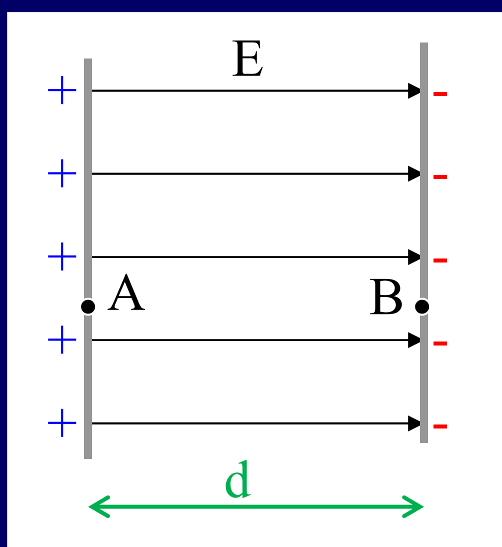
Charge Q on plates

Lect. 3

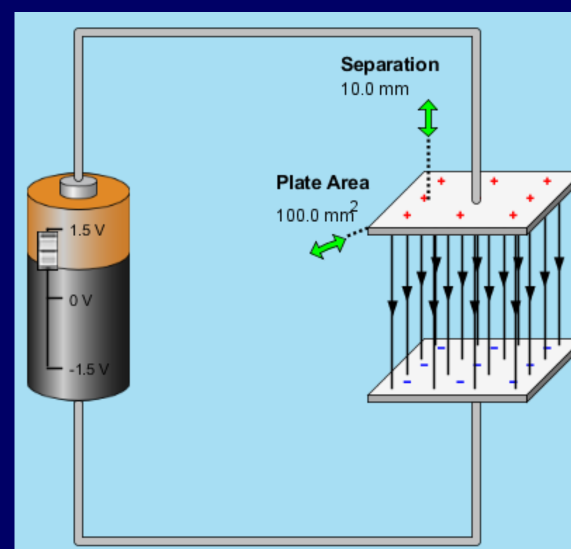
$V = V_A - V_B = +E\,d$ (like $W_{you} = qEd = \Delta U;\ \Delta V = \Delta U/q$)

$= 4\,\pi\,k\,Q\,d\,/\,A$

Voltage is proportional to the charge! $V \propto Q$



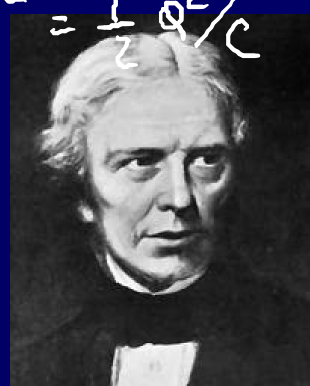
PhET Simulation

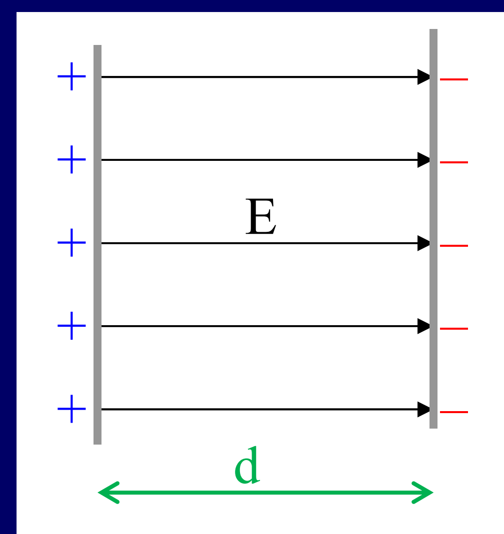


Capacitance: The ability to store separated charge $C \equiv Q/V$

- Any pair conductors separated by a small distance. (e.g. two metal plates)
- Capacitor stores separated charge $Q=CV$
  - Positive Q on one conductor, negative Q on other
  - Net charge is zero
- Stores Energy $U_c = (\frac{1}{2}) Q V$

$U_c = \frac{1}{2} C V^2 = \frac{1}{2} Q^2/C$



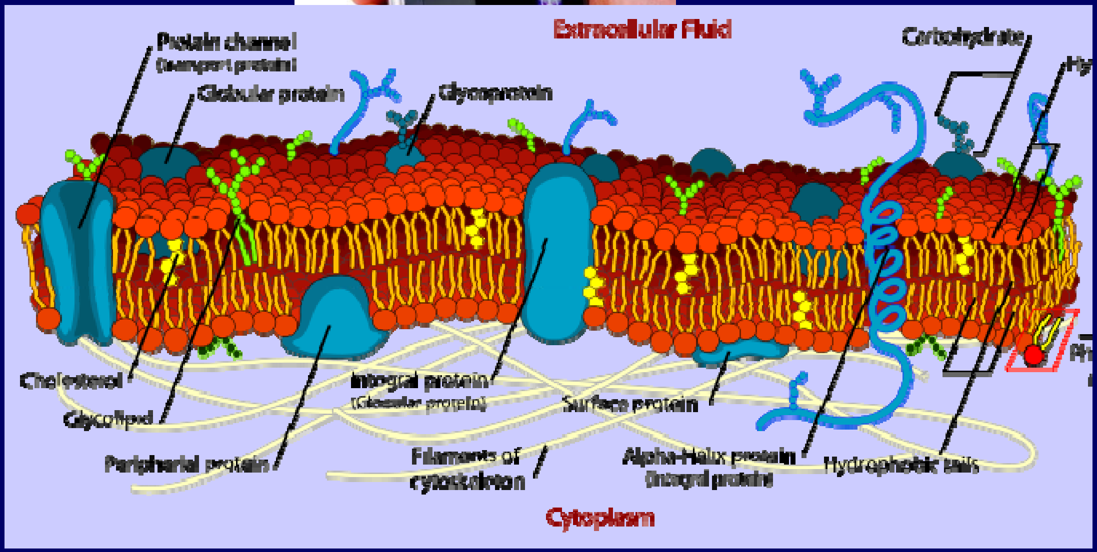


Units:
1 Coulomb/Volt
= 1 Farad (F)

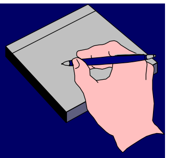
Why Separate Charge?

A way to store *and release* energy!

- Camera Flash
- Defibrillator
- AC → DC
- Tuners / resonant circuits
  – Radio
  – Cell phones
- Electronics
  – Touch screen
- Cell membranes



Capacitance of Parallel Plate Capacitor



$V = Ed \quad E = 4\pi kQ/A$

(Between two large plates)

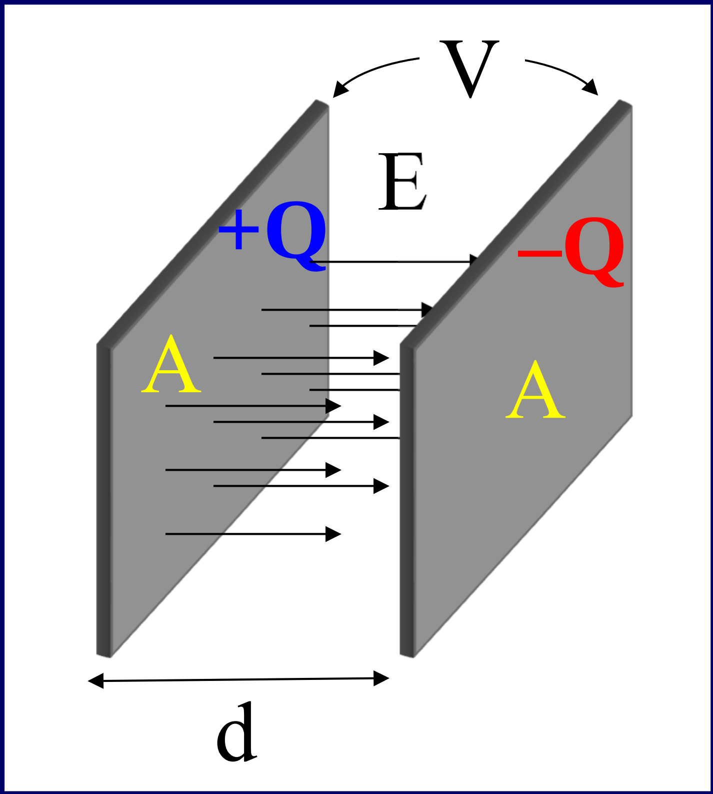
So: $V = 4\pi kQd/A$

Recall: $C \equiv Q/V$

So: $C = A/(4\pi kd)$

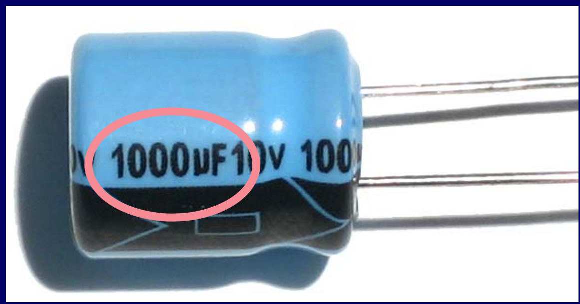
Recall:

$\varepsilon_0 = 1/(4\pi k) = 8.85 \times 10^{-12}\ C^2/Nm^2$



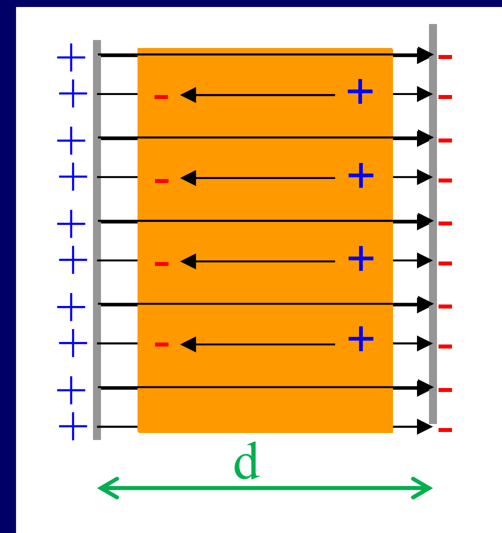
$C = \varepsilon_0 A/d$

Parallel plate capacitor



Induced dipole lect. 1

Dielectric



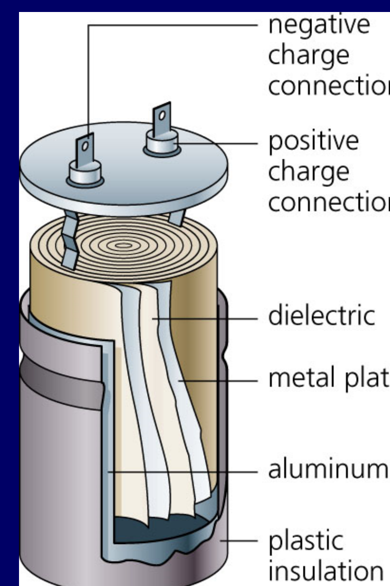
- Placing a dielectric (insulator) between the plates **increases** the capacitance.

Dielectric constant ($\kappa > 1$)

$$C = \kappa\, C_0$$

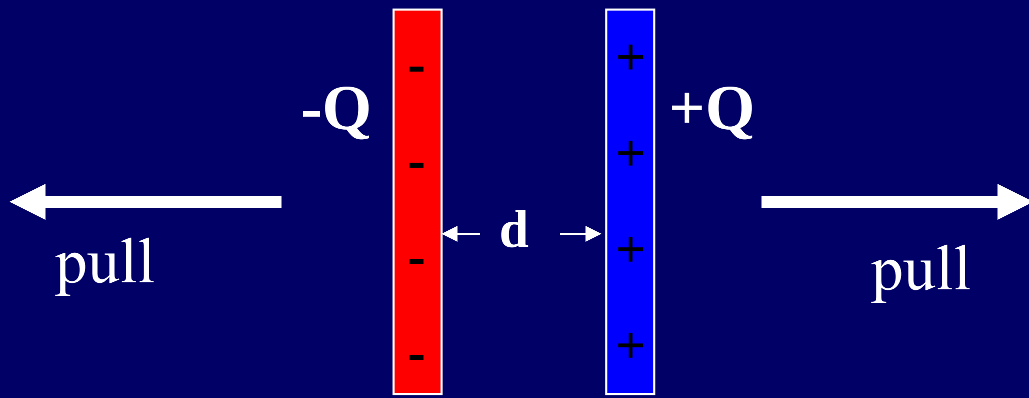
Capacitance **with** dielectric

Capacitance **without** dielectric



For same charge Q, E (and V) is reduced so C = Q/V increases

ACT: Parallel Plates



A parallel plate capacitor given a charge Q. The plates are then pulled a small distance further apart. What happens to the charge Q on each plate of the capacitor?

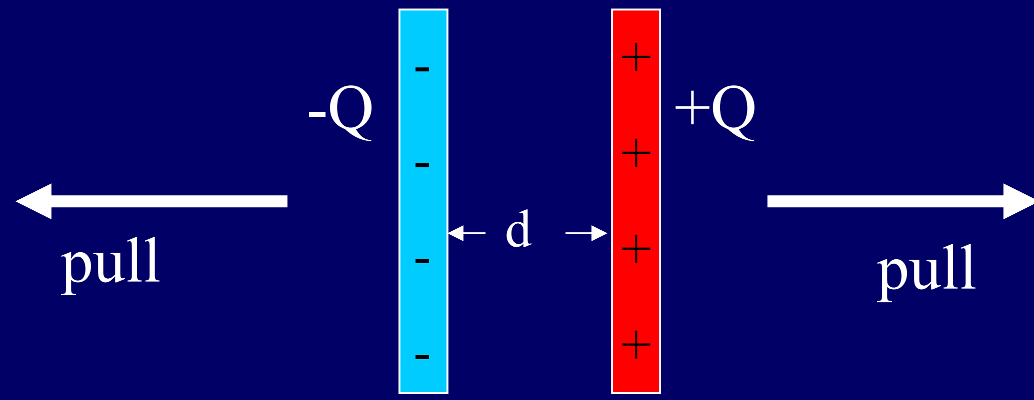
A) Increases

B) Constant

C) Decreases

Remember charge is real/physical. There is no place for the charges to go.

CheckPoint 4.1



A parallel plate capacitor given a charge Q. The plates are then pulled a small distance further apart. Which of the following apply to the situation after the plates have been moved?

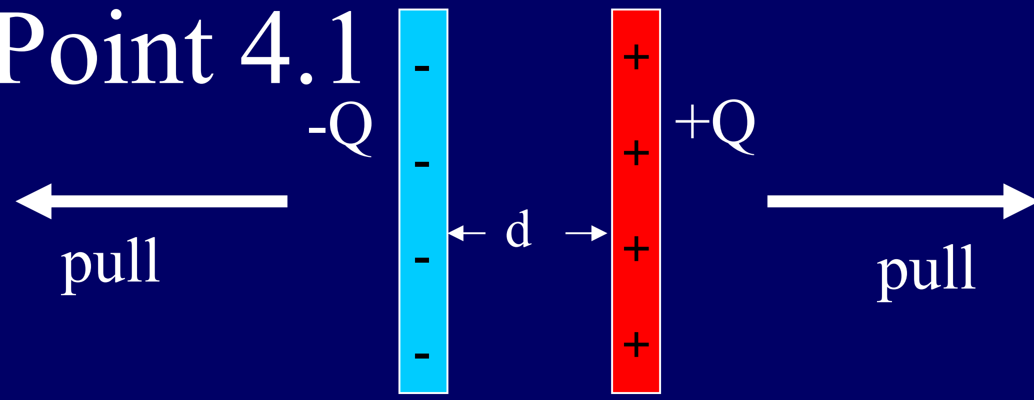
1)The capacitance increases	87%	True	False (circled)
2)The electric field increases	92%	True	False (circled)
3)The voltage between the plates increases	19%	True (circled)	False

After item 1: $C = \varepsilon_0 A/d$ — C decreases!

After item 2: $E = Q/(\varepsilon_0 A)$ — Constant

After item 3: $V = Ed$

ACT/CheckPoint 4.1



A parallel plate capacitor given a charge Q. The plates are then pulled a small distance further apart. Which of the following apply to the situation after the plates have been moved?

The energy stored in the capacitor

A) increases

B) constant

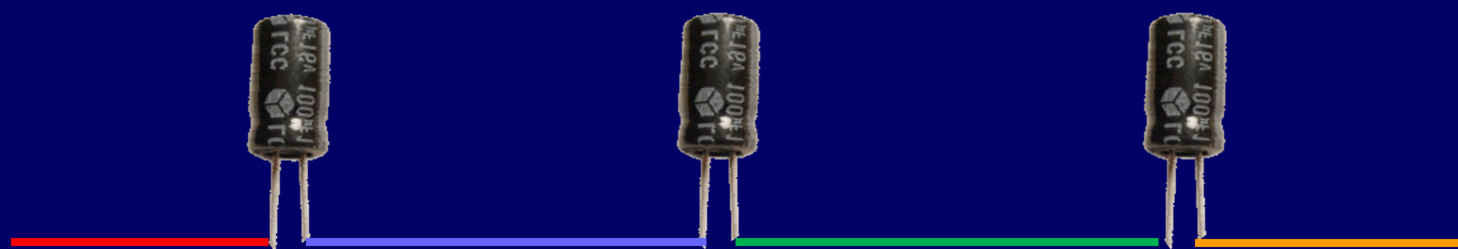
C) decreases

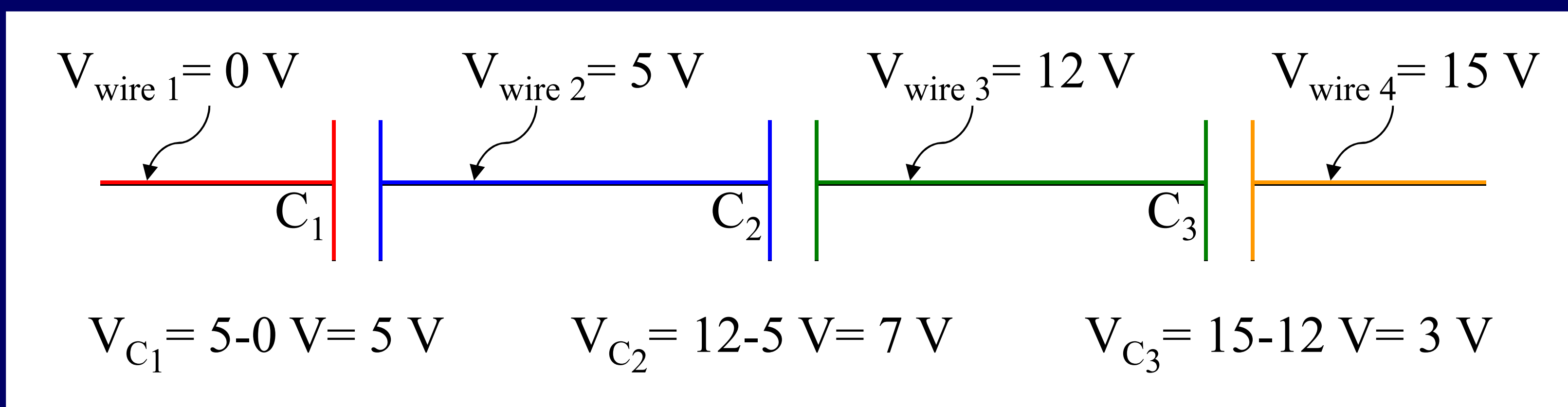
$U = ½ QV$ Q constant, V increased

Plates are attracted to each other, you must pull them apart, so the potential energy of the plates increases. $W_{you} = \Delta U$

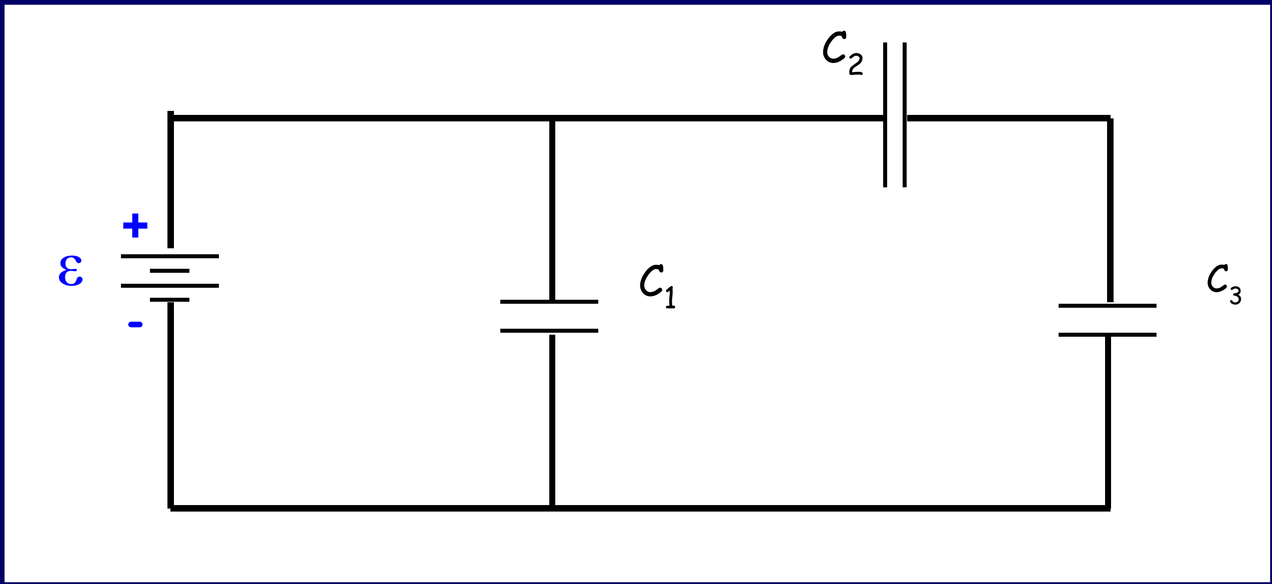
Capacitors are used in circuits!

- In circuits, elements are connected by wires.
- Any connected region of wire has the same potential. $E = 0$, $V = \text{const.}$
- The potential difference across an element is the *element's* "voltage."

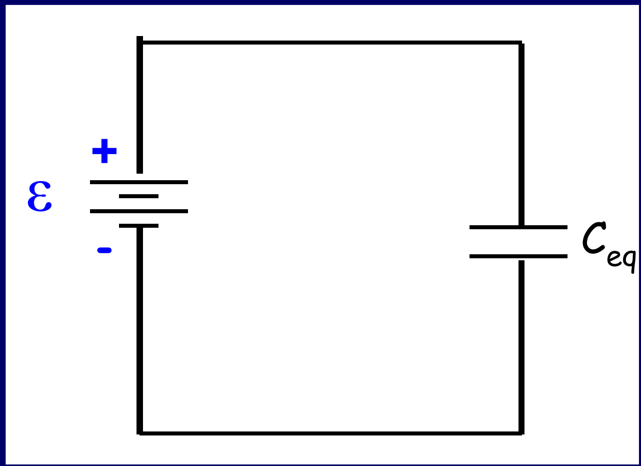




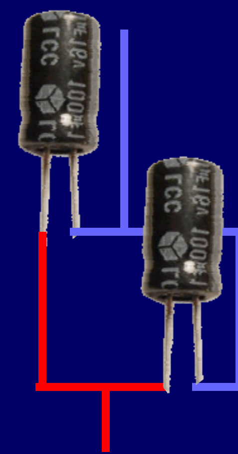
To understand complex circuits…



…treat capacitors in series and parallel as a fictitious equivalent capacitor!

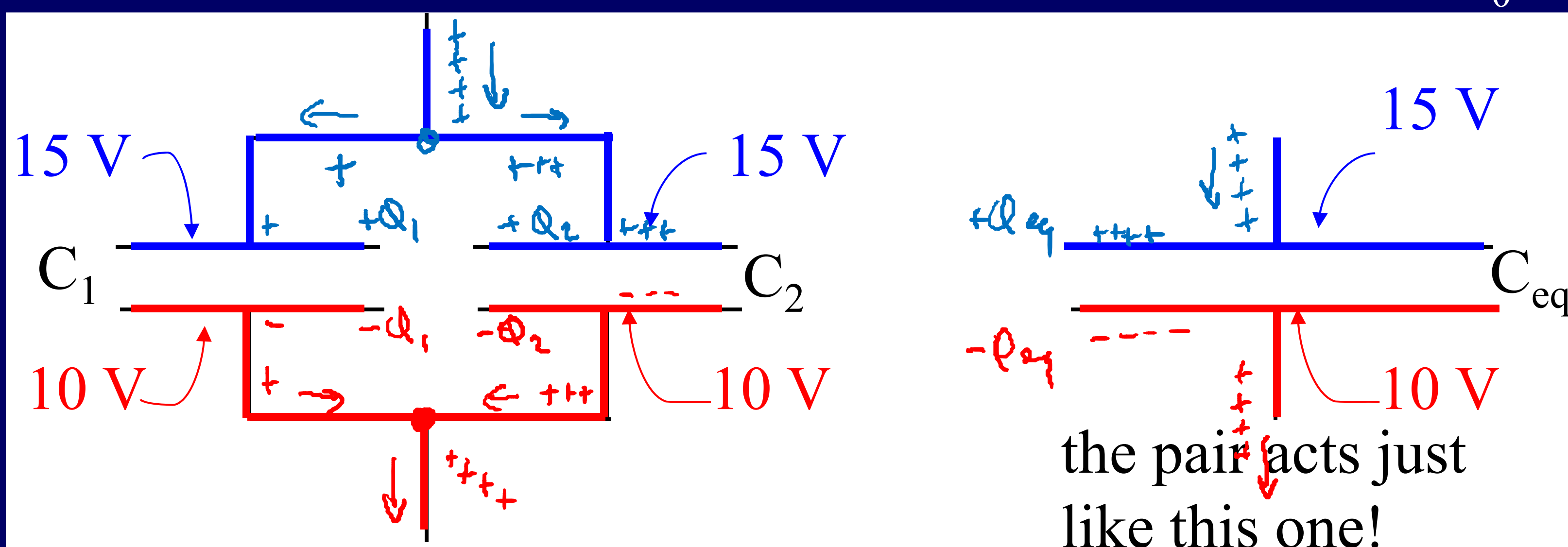


Capacitors in Parallel

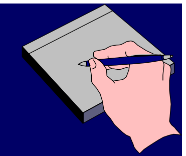


- Both ends connected together by wire
- Same voltage: $V_1 = V_2 = V_{eq}$
- Share Charge: $Q_{eq} = Q_1 + Q_2$
- Equivalent C: $C_{eq} = C_1 + C_2$

Add areas – remember $C = \varepsilon_0 A/d$



Example



Parallel Practice

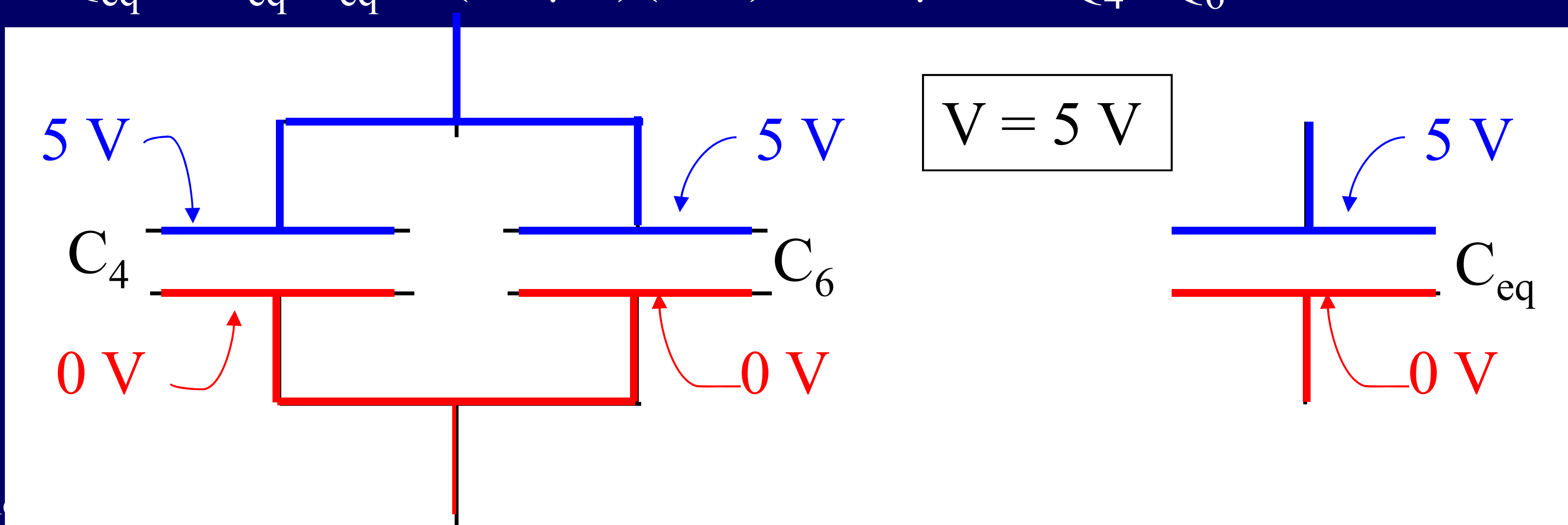
A 4 μF capacitor and 6 μF capacitor are connected in parallel and charged to 5 volts. Calculate C_{eq}, and the charge on each capacitor.

$$C_{eq} = C_4 + C_6 = 4\ \mu F + 6\ \mu F = 10\ \mu F$$

$$Q_4 = C_4 V_4 = (4\ \mu F)(5\ V) = 20\ \mu C$$

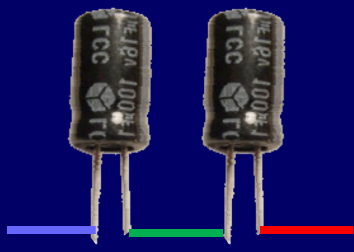
$$Q_6 = C_6 V_6 = (6\ \mu F)(5\ V) = 30\ \mu C$$

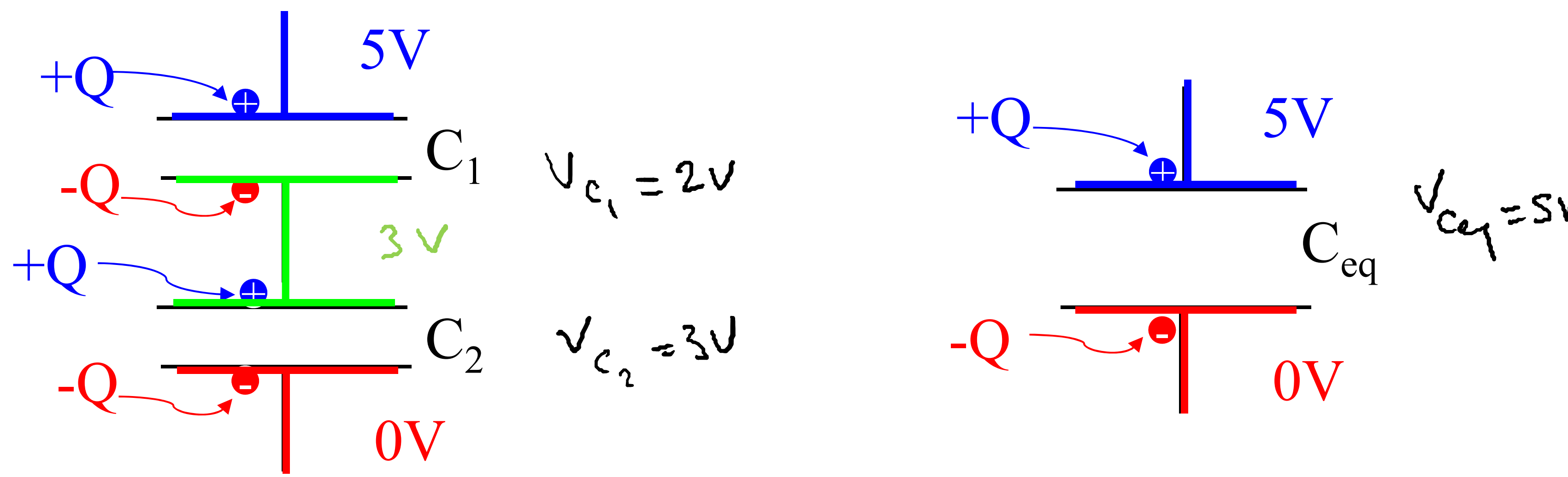
$$Q_{eq} = C_{eq} V_{eq} = (10\ \mu F)(5\ V) = 50\ \mu C = Q_4 + Q_6$$



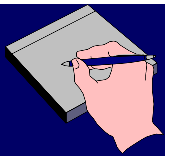
Capacitors in Series

- Connected end-to-end with NO other exits
- Same Charge: $Q_1 = Q_2 = Q_{eq}$
- Share Voltage: $V_1 + V_2 = V_{eq}$
- Equivalent C: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ Add d – remember $C = \varepsilon_0 A/d$





Example



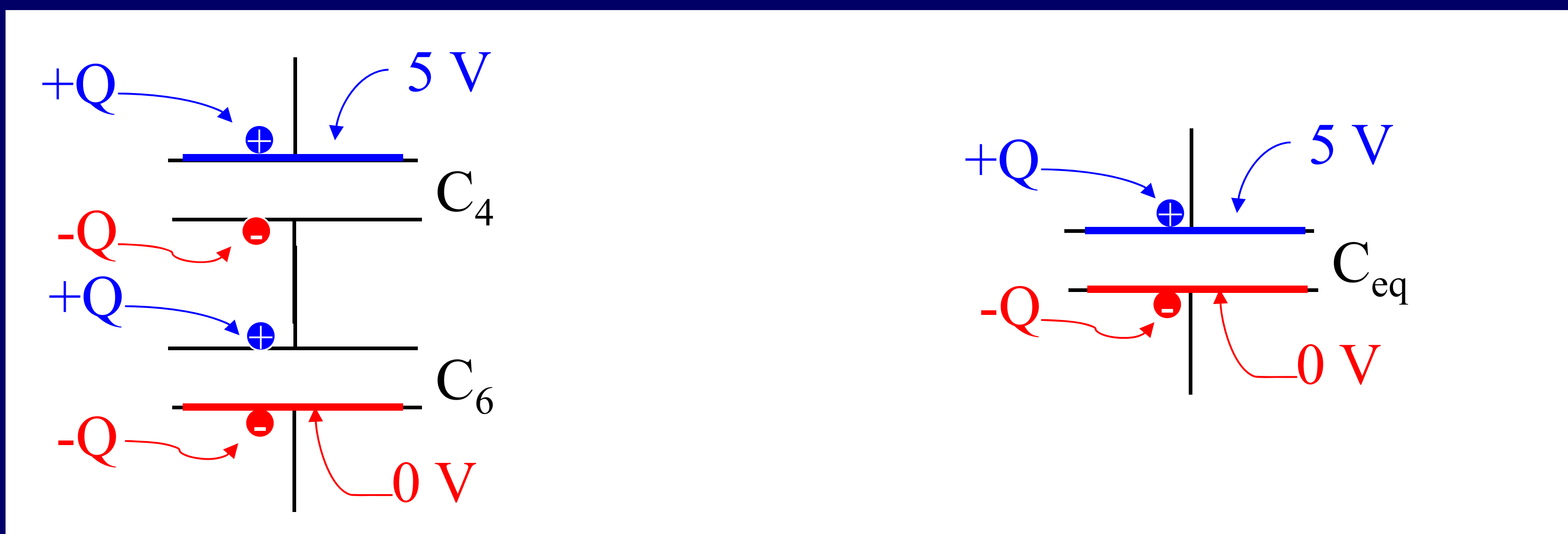
Series Practice

A 4 μF capacitor and 6 μF capacitor are connected in series and charged to 5 volts. Calculate C_{eq}, and the charge on the 4 μF capacitor.

$$C_{eq} = \left(\frac{1}{C_4} + \frac{1}{C_6}\right)^{-1} = \left(\frac{1}{4\mu F} + \frac{1}{6\mu F}\right)^{-1} = 2.4\mu F$$

$Q = CV$

$$Q_4 = Q_6 = Q_{eq} = C_{eq}V = (2.4\mu F)(5V) = 12\mu C$$



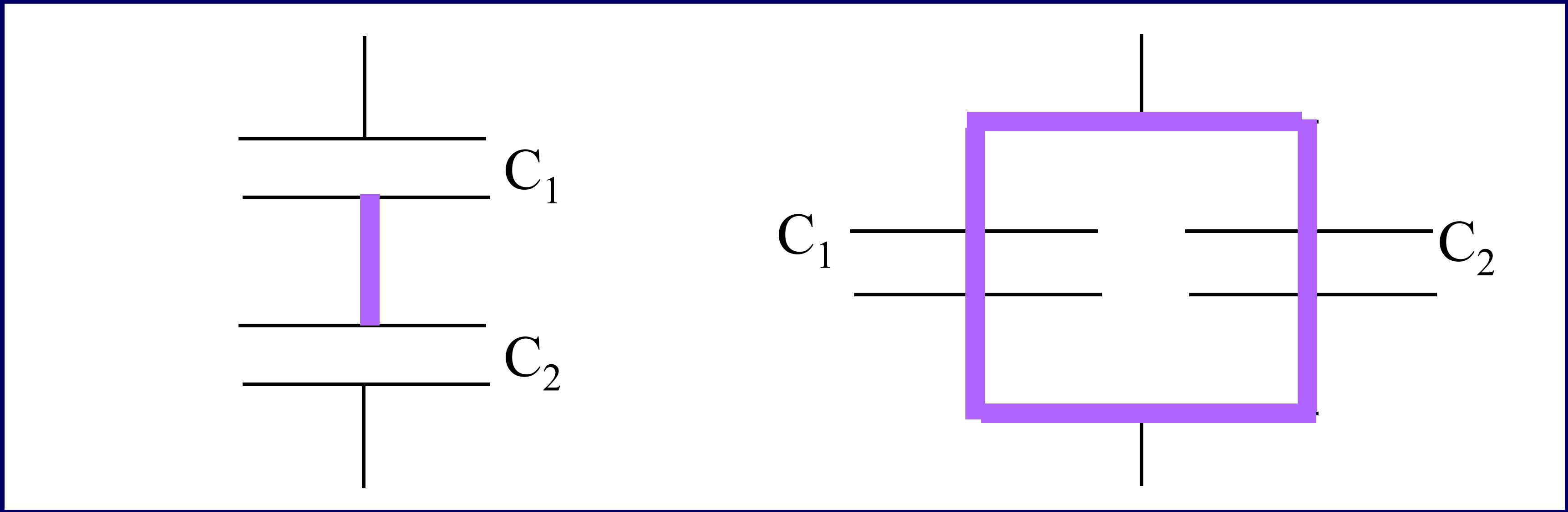
Comparison: *Series* vs. *Parallel*

Series

- Can follow a wire from one element to the other with no branches in between.

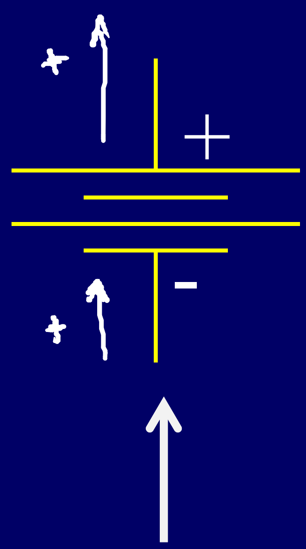
Parallel

- Can find a loop of wire containing both elements but no others (may have branches).



Electromotive Force

- Battery
  - Maintains constant potential difference V (electromotive force – emf ε)
  - Does NOT produce or supply charges, just "pushes" them.



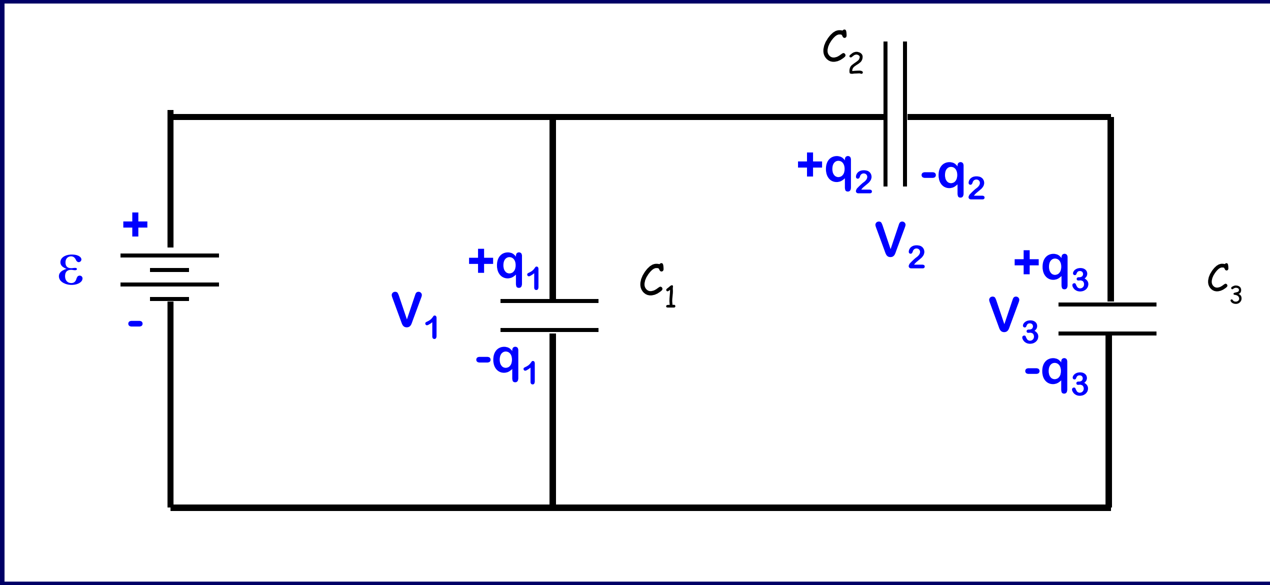
Usually "0V" by convention

Like a pump for charge!

CheckPoint 4.4

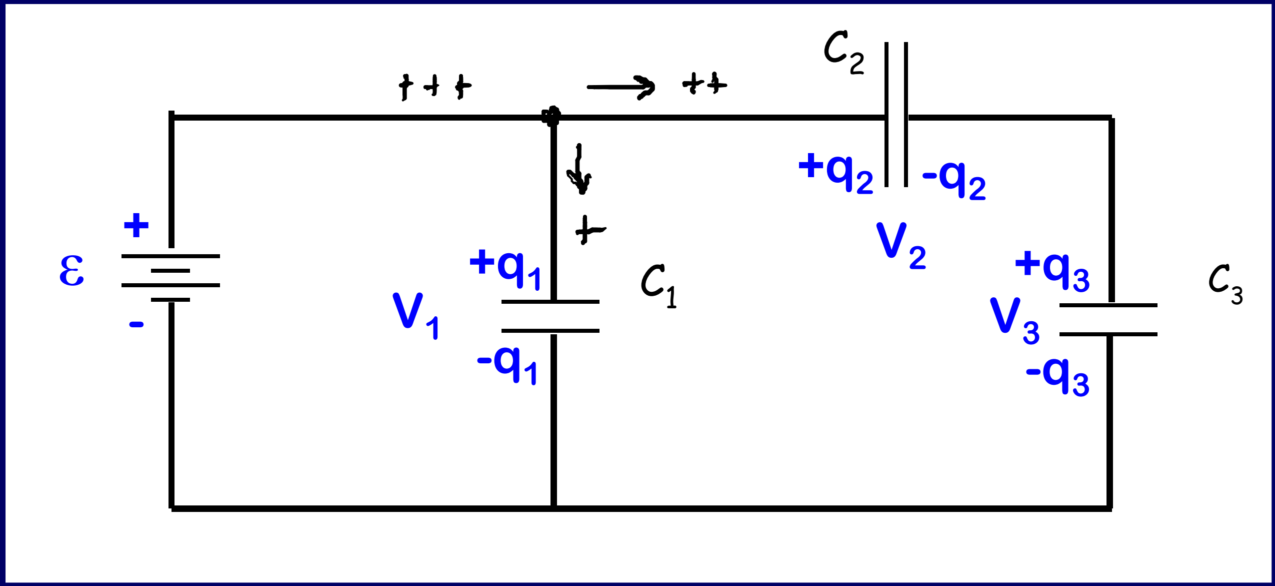
A circuit consists of three initially uncharged capacitors C_1, C_2, and C_3, which are then connected to a battery of emf ε. The capacitors obtain charges q_1, q_2, q_3, and have voltages across their plates V_1, V_2, and V_3. C_{eq} is the equivalent capacitance of the circuit. Which of these are true?

1) $q_1 = q_2$
2) $q_2 = q_3$
3) $V_2 = V_3$
4) $\varepsilon = V_1$
5) $V_1 < V_2$



ACT/CheckPoint 4.4: Which is true?

A circuit consists of three initially uncharged capacitors C_1, C_2, and C_3, which are then connected to a battery of emf ε. The capacitors obtain charges q_1, q_2, q_3, and have voltages across their plates V_1, V_2, and V_3. C_{eq} is the equivalent capacitance of the circuit.

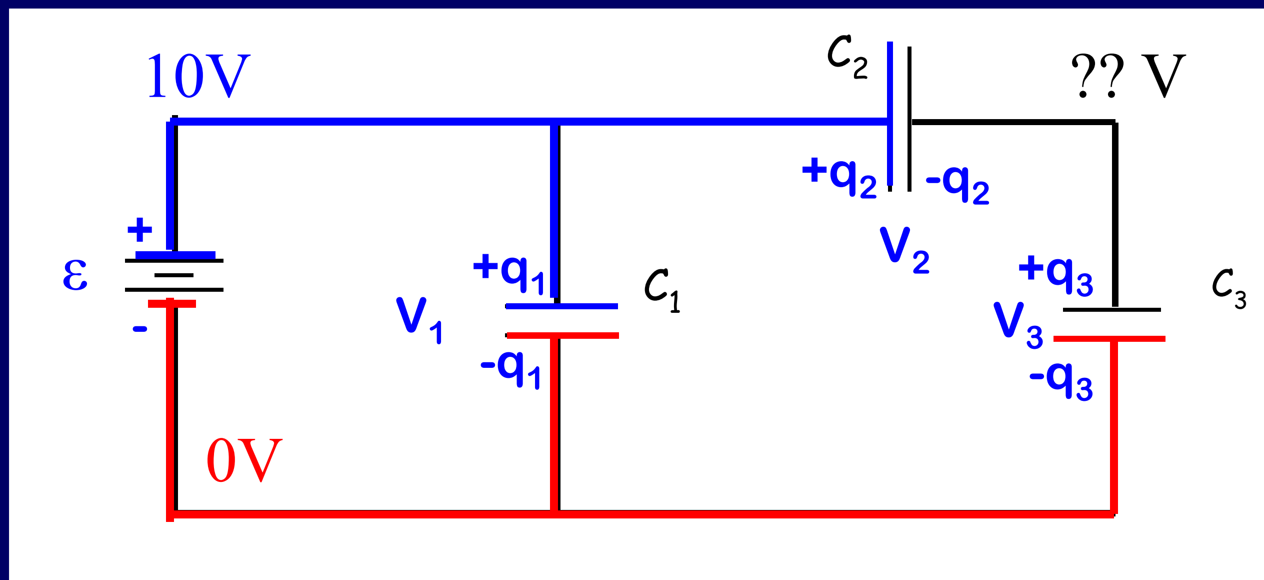


1) $q_1 = q_2$ Not necessarily. C_1 and C_2 are NOT in series.

2) $q_2 = q_3$ Yes! C_2 and C_3 are in series.

ACT/CheckPoint 4.4: Which is true?

A circuit consists of three initially uncharged capacitors C_1, C_2, and C_3, which are then connected to a battery of emf ε. The capacitors obtain charges q_1, q_2, q_3, and have voltages across their plates V_1, V_2, and V_3. C_{eq} is the equivalent capacitance of the circuit.



1) $V_2 = V_3$ Not necessarily, only if $C_2 = C_3$

2) $\varepsilon = V_1$ Yes! Both ends are connected by wires

Recap of Today's Lecture

- Capacitance $C = Q/V$
- Parallel Plate: $C = \varepsilon_0 A/d$
- Capacitors in parallel: $C_{eq} = C_1+C_2$
- Capacitors in series: $1/C_{eq} = 1/C_1+1/C_2$
- Batteries provide fixed potential difference