

Your questions/comments

IMPORTANT ANNOUNCEMENTS:

EX3 has been graded. Raw mean = 70%, scaled = 75%

“Why did the physics student fall asleep in class? Because he thought it was Bohring!”

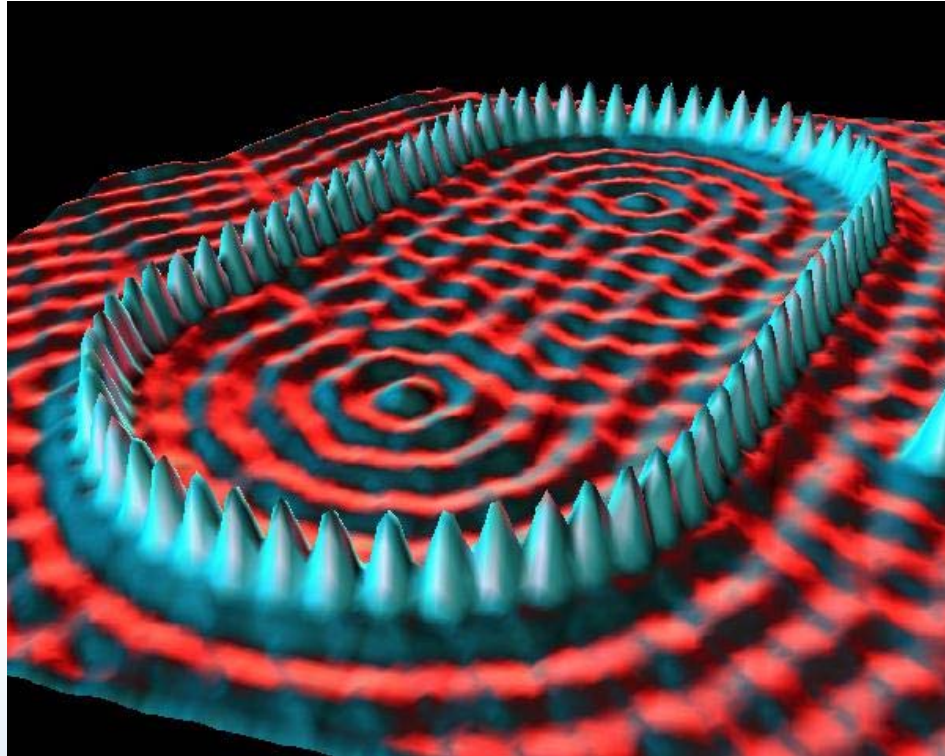
“Well this unit seems to be drastically more conceptual than previous ones. You have a lot of explaining to do.”

“Uncertainty principle is very uncertain”

“Pleeeeeease slow down! I feel like I barely got a grip on the diffraction interference stuff from last week. Just go slow, this stuff is kind of abstract and hard.”

“plz go over de Broccoli wavelength. & eat your vegetables.”

“Electrons are just trying to mess with our minds...”



Phys 102 – Lecture 24

The classical and Bohr atom

State of late 19th Century Physics

- Two great theories *“Classical physics”*
 - Newton’s laws of mechanics & gravity *Phys. 101*
 - Maxwell’s theory of electricity & magnetism, including EM waves *Phys. 102*
- But... some unsettling problems
 - Stability of atom & atomic spectra
 - Photoelectric effect
 - ...and others
- New theory required *Quantum mechanics*



ACT: Quick review

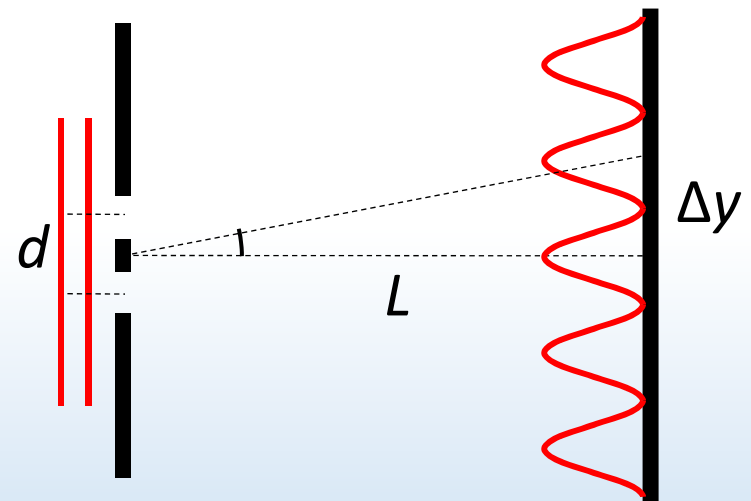
Consider the interference pattern from mono-chromatic light of wavelength λ passing through a double slit.

What happens to the spacing Δy between interference maxima if λ doubles?

A. Δy doubles

B. Δy remains the same

C. Δy is halved

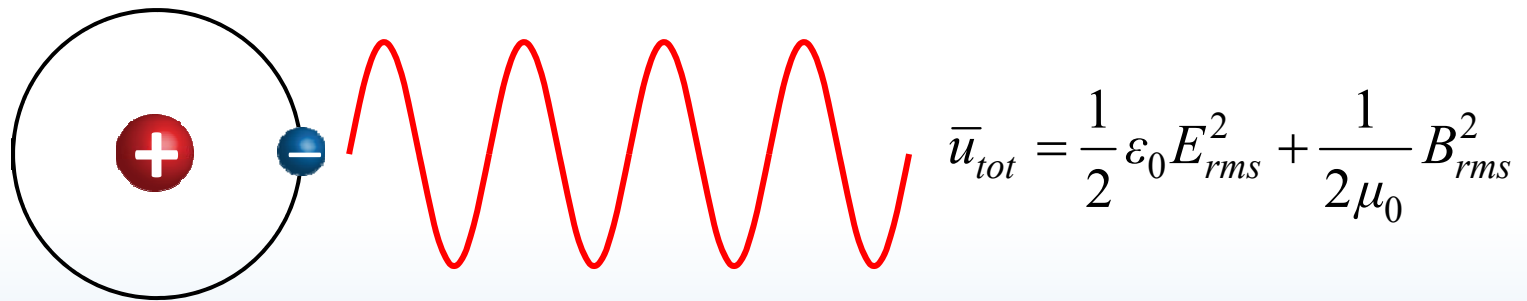


$$d \sin \theta = m\lambda \quad \Delta y \approx \frac{L}{d} \lambda$$

Stability of classical atom

Prediction – orbiting e^- is an oscillating charge & should emit EM waves in every direction

Recall Lect. 15 & 16



EM waves carry energy, so e^- should lose energy & fall into nucleus!

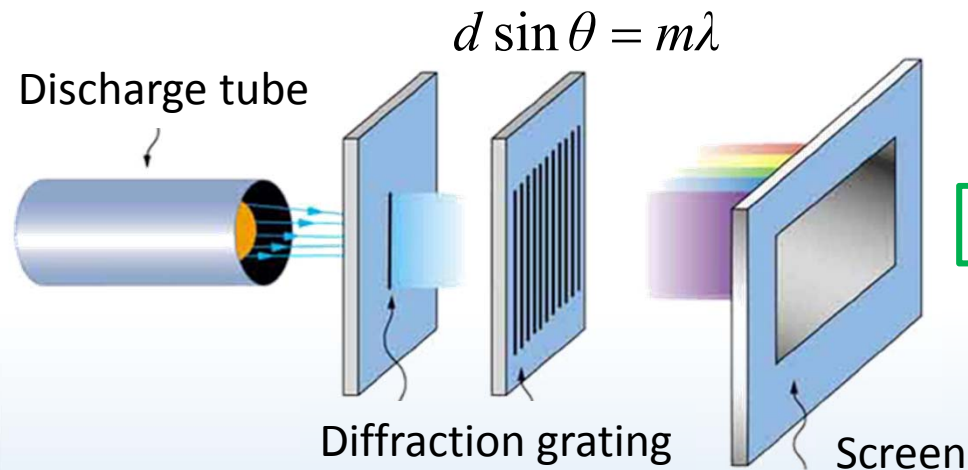
Classical atom is NOT stable!

Lifetime of classical atom = 10^{-11} s

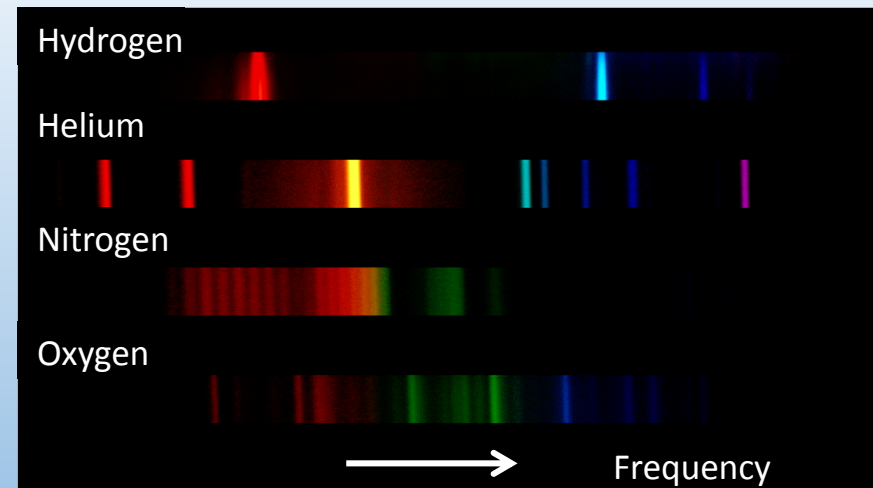
Reality – Atoms are stable

Atomic spectrum

Prediction – e^- should emit light at whatever frequency f it orbits nucleus



Reality – Only certain frequencies of light are emitted & are different for different elements



Quantum mechanics

Quantum mechanics explains stability of atom & atomic spectra (and many other phenomena...)

QM is one of most successful and accurate scientific theories

Predicts measurements to $<10^{-8}$ (ten parts per billion!)

Wave-particle duality – matter behaves as a wave

Particles can be in many places at the same time

Processes are probabilistic not deterministic

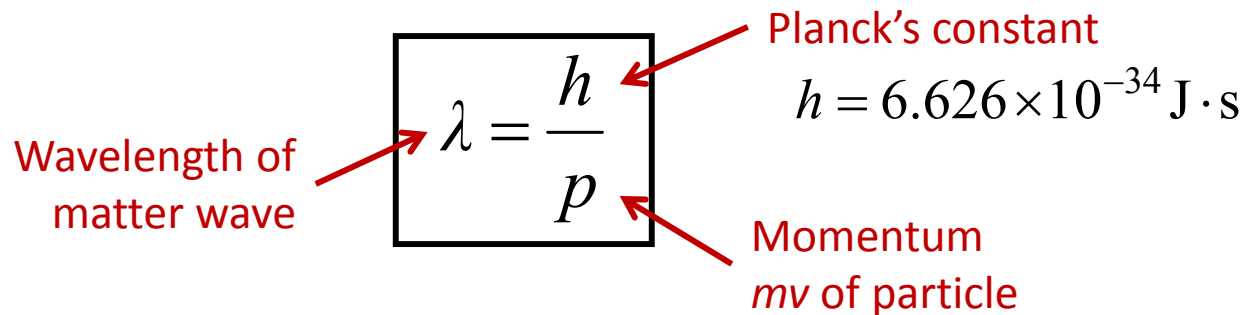
Measurement changes behavior

Certain quantities (ex: energy) are *quantized*

quanta

Matter waves

Matter behaves as a wave with *de Broglie* wavelength



The diagram shows the equation $\lambda = \frac{h}{p}$ enclosed in a black rectangular box. Three red arrows point from text labels to parts of the equation: one from 'Wavelength of matter wave' to λ , one from 'Planck's constant' to h , and one from 'Momentum mv of particle' to p . To the right of the box, the value of Planck's constant is given as $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$.

$$\lambda = \frac{h}{p}$$

Wavelength of matter wave

Planck's constant
 $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$

Momentum mv of particle

Ex: a fastball ($m = 0.5 \text{ kg}$, $v = 100 \text{ mph} \approx 45 \text{ m/s}$)

$$\lambda_{\text{fastball}} = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{0.5 \cdot 45} = 3 \times 10^{-35} \text{ m}$$

20 orders of magnitude smaller than the proton!

Ex: an electron ($m = 9.1 \times 10^{-31} \text{ kg}$, $v = 6 \times 10^5 \text{ m/s}$)

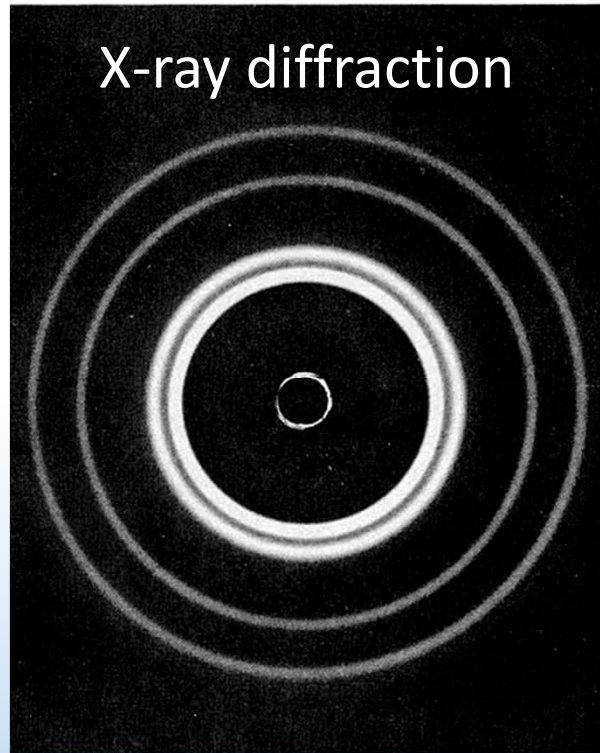
$$\lambda_{\text{electron}} = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \cdot 6 \times 10^5} = 1.2 \text{ nm}$$

X-ray wavelength

How could we detect matter wave?

Interference!

X-ray vs. electron diffraction



DEMO

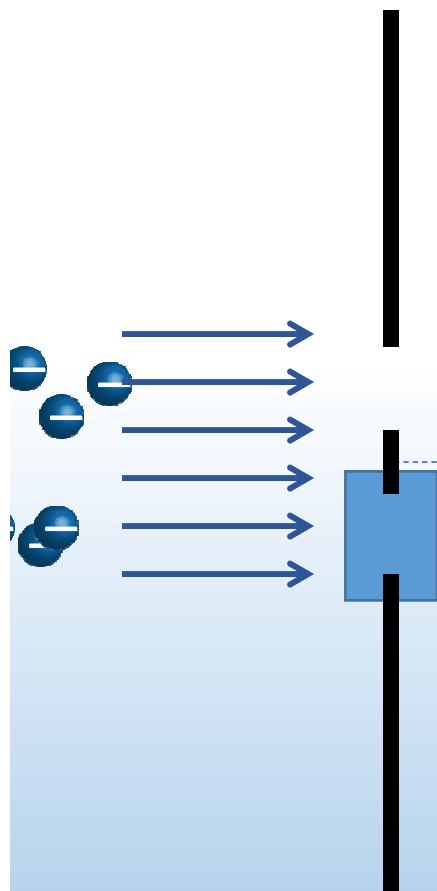
Identical pattern emerges if de Broglie wavelength of e^- equals the X-ray wavelength!

Electron diffraction

Beam of mono-energetic e^- passes through double slit

Interference pattern =
probability

$$d \sin \theta = m\lambda \quad \lambda =$$

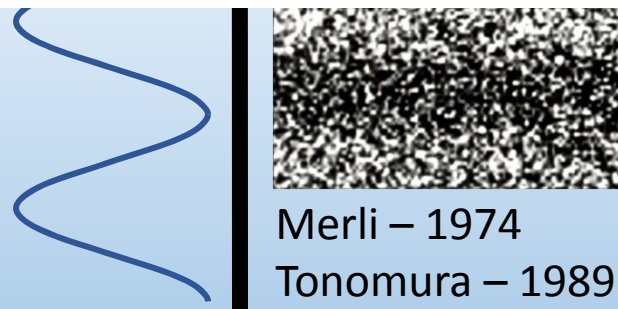
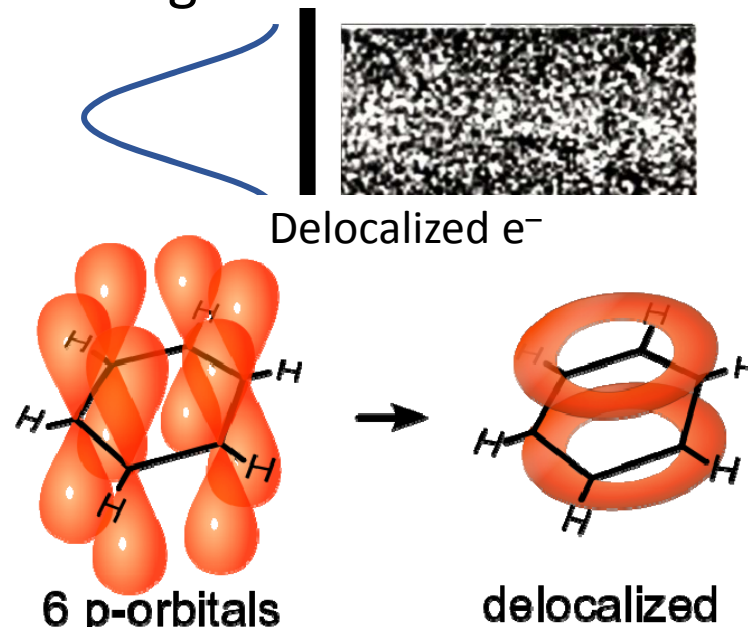


Wait! Does this mean e^-
passes through both slits?

Yes! e^- is in both places at once

What if we measure which
slit the e^- passes through?

Interference disappears!



Merli – 1974

Tonomura – 1989

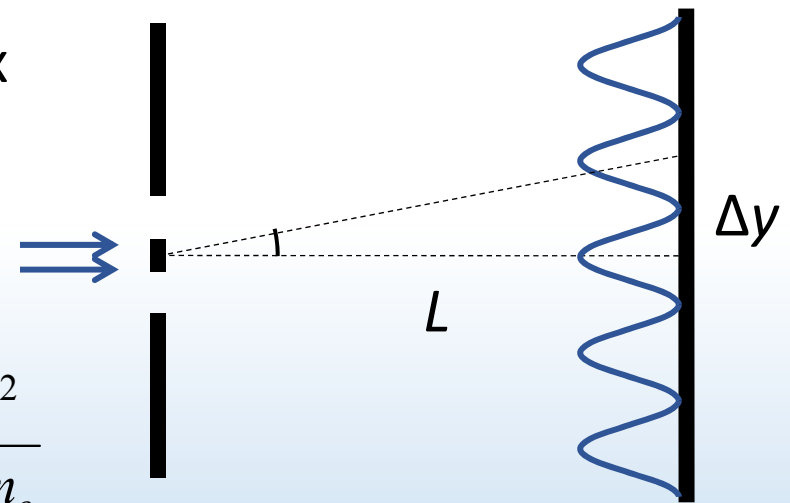


ACT: Double slit interference

Consider the interference pattern from a beam of mono-energetic electrons A passing through a double slit.

Now a beam of electrons B with 4x the energy of A enters the slits.

What happens to the spacing Δy between interference maxima?



A. $\Delta y_B = 4 \Delta y_A$ $K.E. = \frac{1}{2} m_e v^2 = \frac{p^2}{2m_e}$

B. $\Delta y_B = 2 \Delta y_A$

C. $\Delta y_B = \Delta y_A$

D. $\Delta y_B = \Delta y_A / 2$

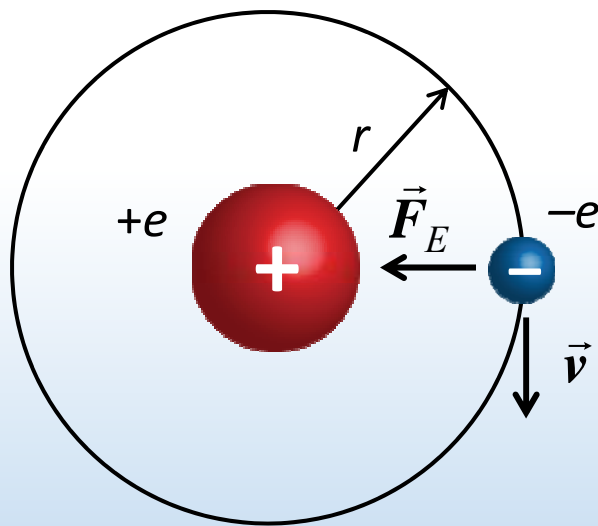
E. $\Delta y_B = \Delta y_A / 4$

$$d \sin \theta = m \lambda \quad \Delta y \approx \frac{L}{d} \lambda \quad \lambda = \frac{h}{p}$$

If $K.E._B = 4 K.E._A$, $p_B = 2 p_A$ $\lambda_B = \lambda_A / 2$ $m \checkmark$

The “classical” atom

Negatively charged electron orbits around positively charged nucleus



Hydrogen atom

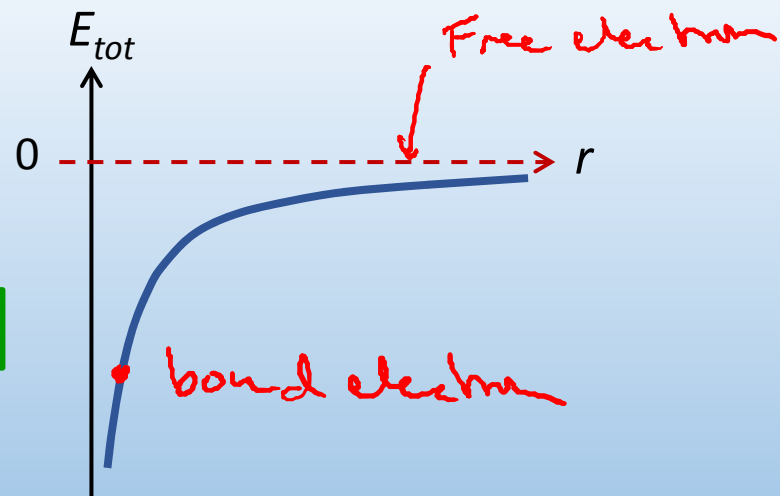
Recall Lect. 4

Orbiting e^- has centripetal acceleration:

$$F_E = k \frac{e^2}{r^2} = \cancel{ma_c} \frac{mv^2}{r} \quad \text{so, } \frac{ke^2}{r} = mv^2$$

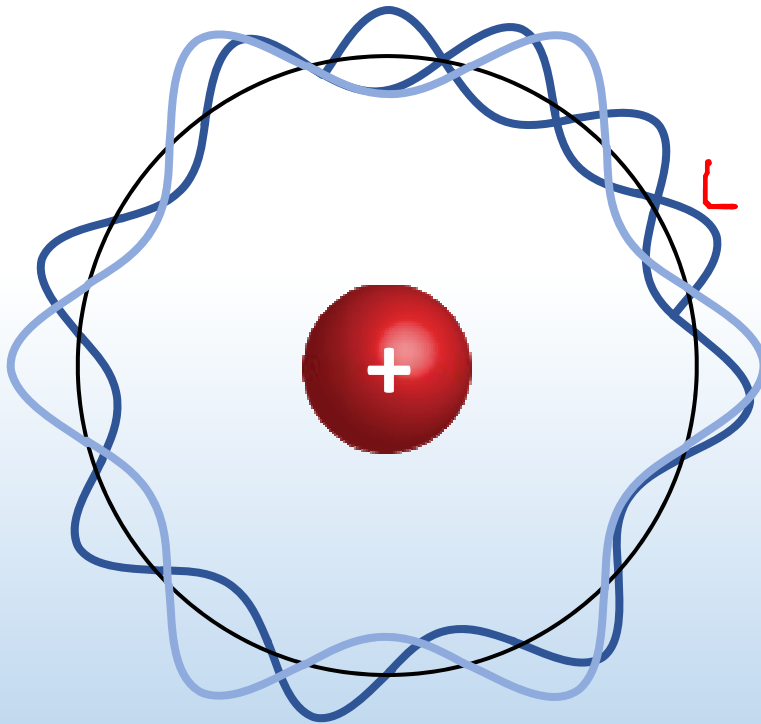
Total energy of electron:

$$E_{tot} = K + U = \frac{1}{2}mv^2 - k \frac{e^2}{r} = -\frac{1}{2} \frac{ke^2}{r}$$



The Bohr model

e^- behave as waves & only orbits that fit an integer number of wavelengths are allowed



Orbit circumference

$$L = mvr = 2\pi r = n\lambda \quad n = 1, 2, 3 \dots$$

de Broglie wavelength

Angular momentum is *quantized*

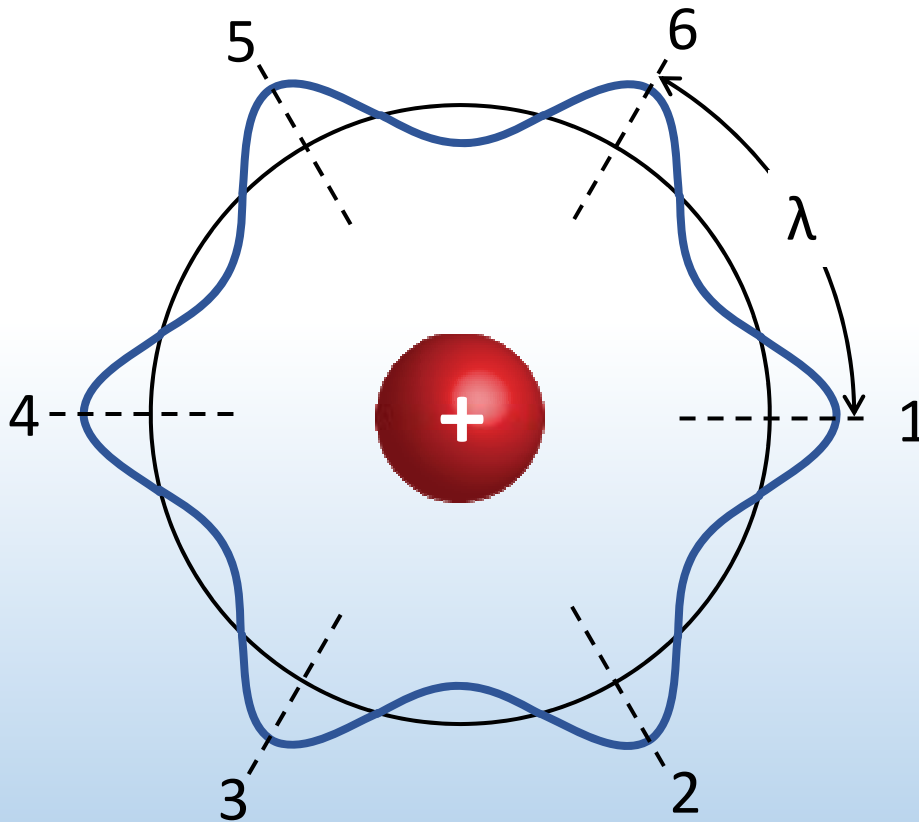
$$L_n = n\hbar \quad \hbar \equiv \frac{h}{2\pi} \quad \text{"h bar"}$$

"Quantum number"



ACT: Bohr model

What is the quantum number n of this hydrogen atom?



A. $n = 1$

B. $n = 3$

C. $n = 6$

D. $n = 12$

n is the number of $e^- \lambda$ that fit around orbit circumference

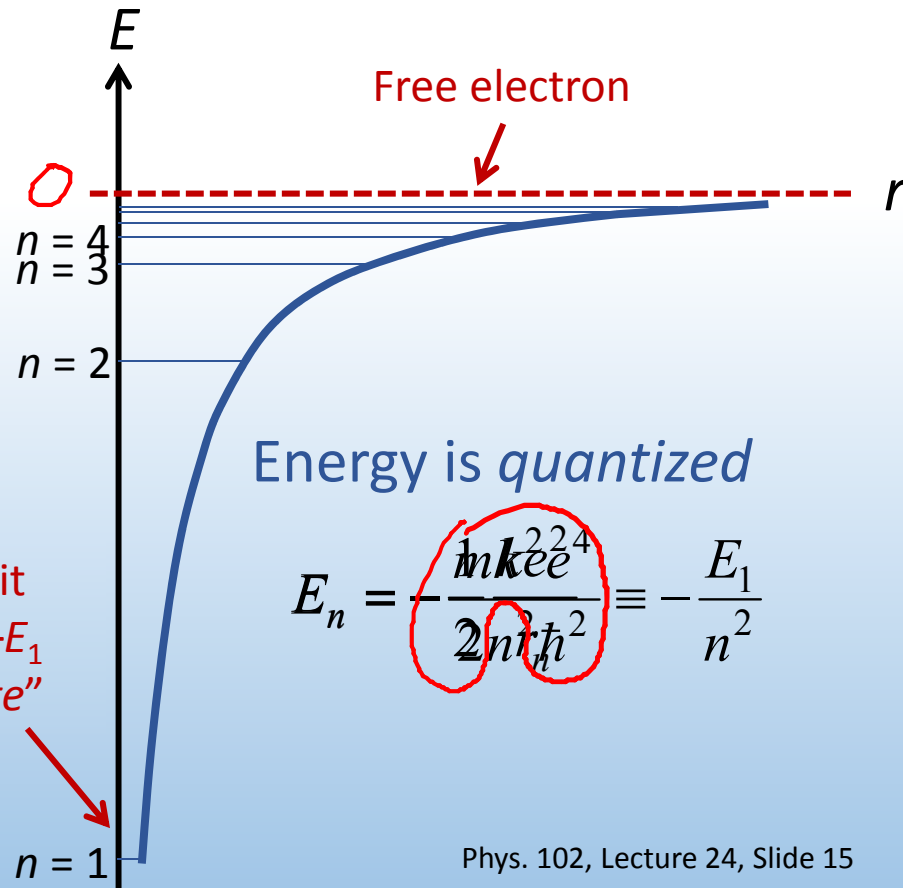
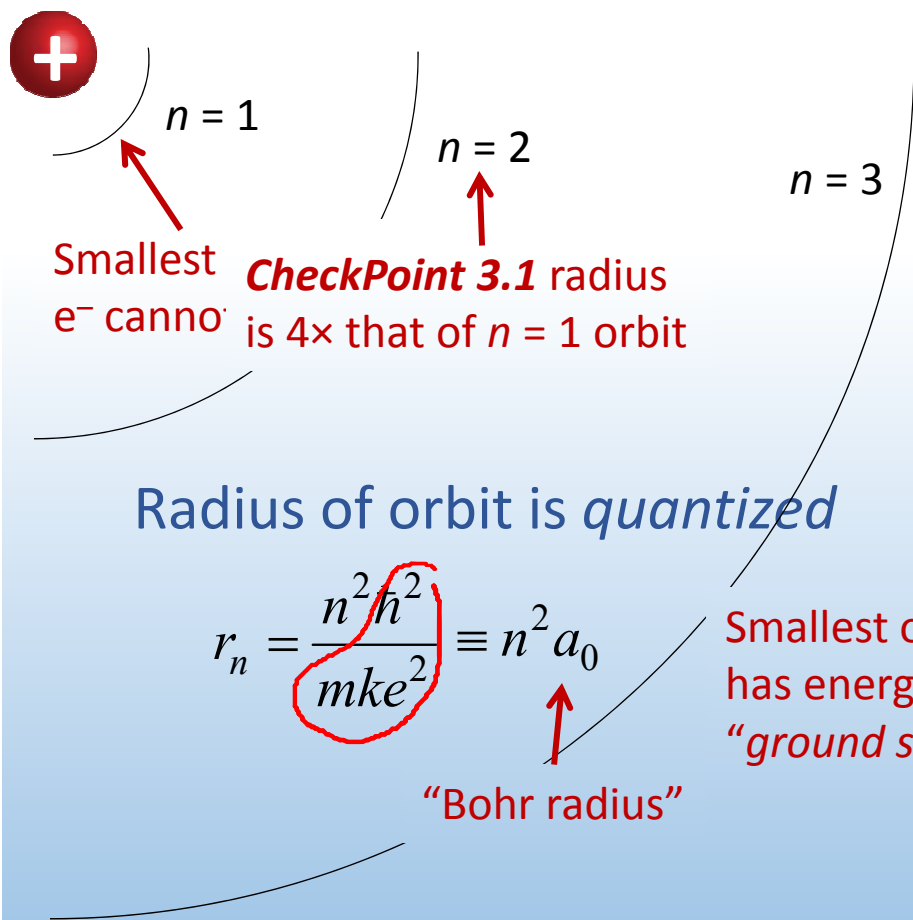
Energy and orbit quantization

Angular momentum is *quantized*

$$L_n \equiv pr = mvr = n\hbar \quad n = 1, 2, 3 \dots$$

From classical atom:

$$mvr = \frac{ke^2}{r} \Rightarrow mvr^2 = \frac{ke^2}{v} \Rightarrow mv^2 r^2 = ke^2$$



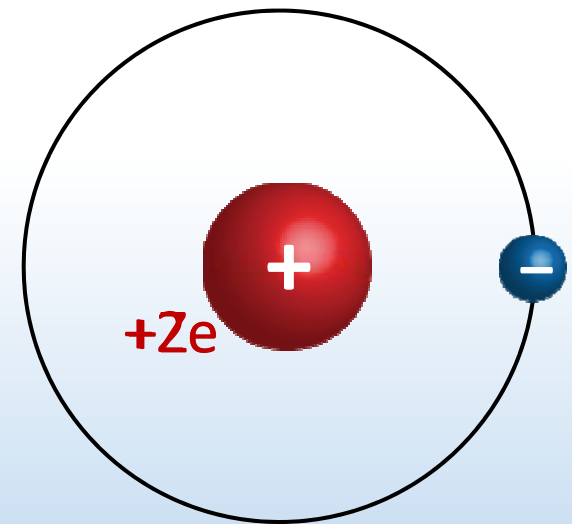


ACT: CheckPoint 3.2

Suppose the charge of the nucleus is doubled ($+2e$), but the e^- charge remains the same ($-e$). How does r for the ground state ($n = 1$) orbit compare to that in hydrogen?

For hydrogen: $r_n = \frac{m \hbar^2 n^2}{m k Z e^2} = \frac{n^2}{Z} a_0$

One factor of e comes from e^- charge, Z^2
second factor from nuclear charge Z^2



A. 1/2 as large

46%

B. 1/4 as large

36%

C. the same

18%

Works for any single e^- atom (ex: H, He^+ , Li^{++})



ACT: CheckPoint 3.3

There is a particle in nature called a *muon*, which has the same charge as the electron but is 207 times heavier. A muon can form a hydrogen-like atom by binding to a proton.

$$r_n = \frac{n^2 \hbar^2}{m k e^2}$$

How does the radius of the ground state ($n = 1$) orbit for this hydrogen-like atom compare to that in hydrogen?

A. 207× larger
27%

B. The same
37%

C. 207× smaller
36%

Atomic units

At atomic scales, Joules, meters, kg, etc. are not convenient units

“Electron Volt” – energy gained by charge $+1e$ when accelerated by 1 Volt: $U = qV$ $1e = 1.6 \times 10^{-19} \text{ C}$, so $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Planck constant: $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$

Speed of light: $c = 3 \times 10^8 \text{ m/s}$

$$hc \approx 2 \times 10^{-25} \text{ J}\cdot\text{m} = 1240 \text{ eV}\cdot\text{nm}$$

Electron mass: $m = 9.1 \times 10^{-31} \text{ kg}$ $mc^2 = 8.2 \times 10^{-13} \text{ J} = 511,000 \text{ eV}$

Since $U = \frac{ke^2}{r}$, ke^2 has units of $\text{eV}\cdot\text{nm}$ like hc $ke^2 \approx 1.44 \text{ eV}\cdot\text{nm}$

$$\frac{ke^2}{\hbar c} = 2\pi \frac{ke^2}{hc} \approx \frac{1}{137} \quad \text{“Fine structure constant” (dimensionless)}$$

Calculation: energy & Bohr radius

Energy of electron in H-like atom (1 e⁻, nuclear charge +Ze):

$$E_n = -\frac{mk^2Z^2e^4}{2n^2\hbar^2} = -\frac{Z^2}{2n^2}mc^2\left(\frac{ke^2}{\hbar c}\right)^2 = -\frac{511,000\text{eV}}{2\cdot 137^2}\frac{Z^2}{n^2}$$
$$= -13.6\text{eV}\frac{Z^2}{n^2}$$

Radius of electron orbit:

$$r_n = \frac{n^2}{Z}a_0 = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2}{Z}\frac{\hbar c}{mc^2}\left(\frac{\hbar c}{ke^2}\right) = \frac{n^2}{Z}\frac{1240\text{eV}\cdot\text{nm}}{2\pi\cdot 511,000\text{eV}}137$$
$$= 0.0529\text{nm}\frac{n^2}{Z}$$

Bohr radius



ACT: Hydrogen-like atoms

Consider an atom with a nuclear charge of $+2e$ with a single electron orbiting, in its ground state ($n = 1$), i.e. He^+ .

How much energy is required to ionize the atom totally?

A. 13.6 eV

B. 2×13.6 eV

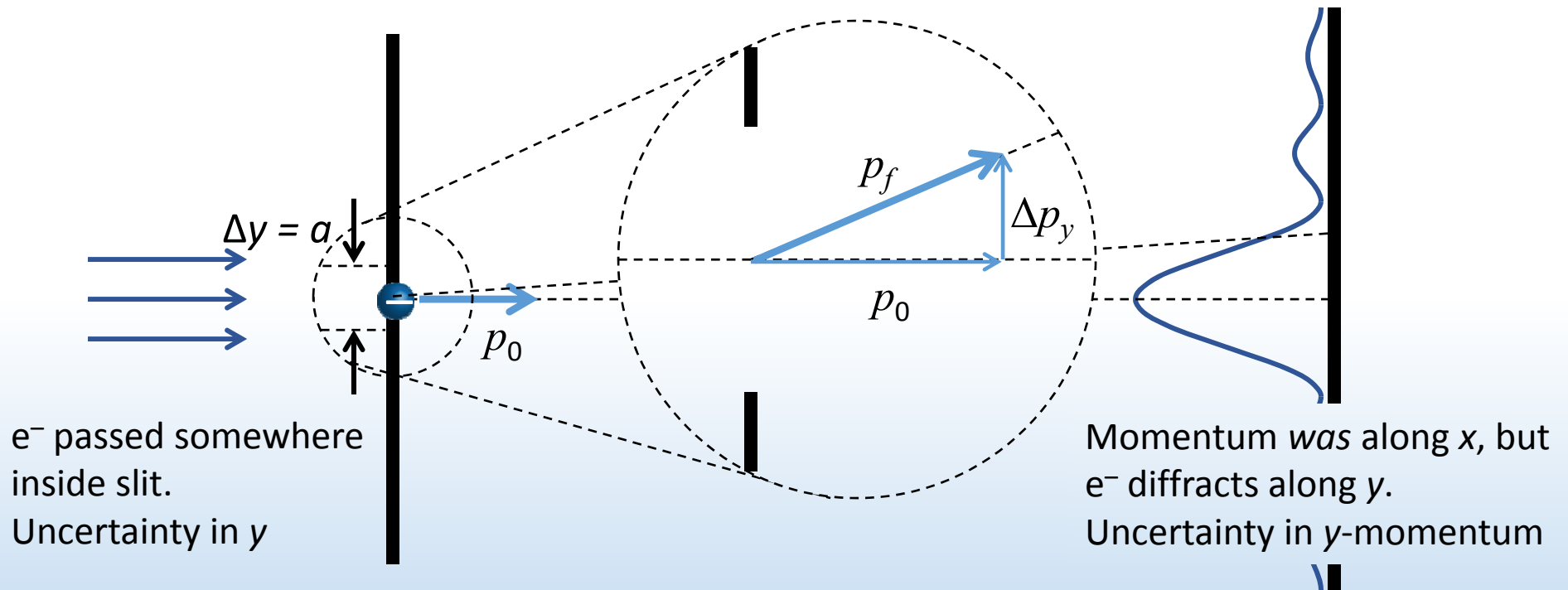
C. 4×13.6 eV

$$E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

Energy measured relative to
free electron ($E = 0$)

Heisenberg Uncertainty Principle

e^- beam with momentum p_0 passes through slit will diffract



If slit narrows, diffraction pattern spreads out $a \sin \theta = m\lambda$

Uncertainty in y

Uncertainty in
y-momentum

$$\Delta y \cdot \Delta p_y \gtrsim \frac{\hbar}{2}$$

CheckPoint 2.1



ACT: CheckPoint 4

The Bohr model cannot be correct! To be consistent with the Heisenberg Uncertainty Principle, which of the following properties *cannot* be quantized?

1. Energy is quantized $E_n = -\frac{E_1}{n^2}$
2. Angular momentum is quantized $L_n = p_n r_n = n\hbar$
3. Radius is quantized $r_n = n^2 a_0$
4. Linear momentum & velocity are quantized $p_n = \frac{\hbar}{na_0}$

A. All of the above

B. #1 & 2

C. #3 & 4

If r and p are quantized, we would know both exactly, inconsistent with Uncertainty Principle

Summary of today's lecture

- Classical model of atom

Predicts unstable atom & cannot explain atomic spectra

- Quantum mechanics

Matter behaves as waves

Heisenberg Uncertainty Principle

- Bohr model

Only orbits that fit n electron wavelengths are allowed

Explains the stability of the atom

Energy quantization correct for single e^- atoms (H, He^+ , Li^{++})

However, it is *fundamentally* incorrect

Need complete quantum theory (Lect. 26)