

Your questions/comments

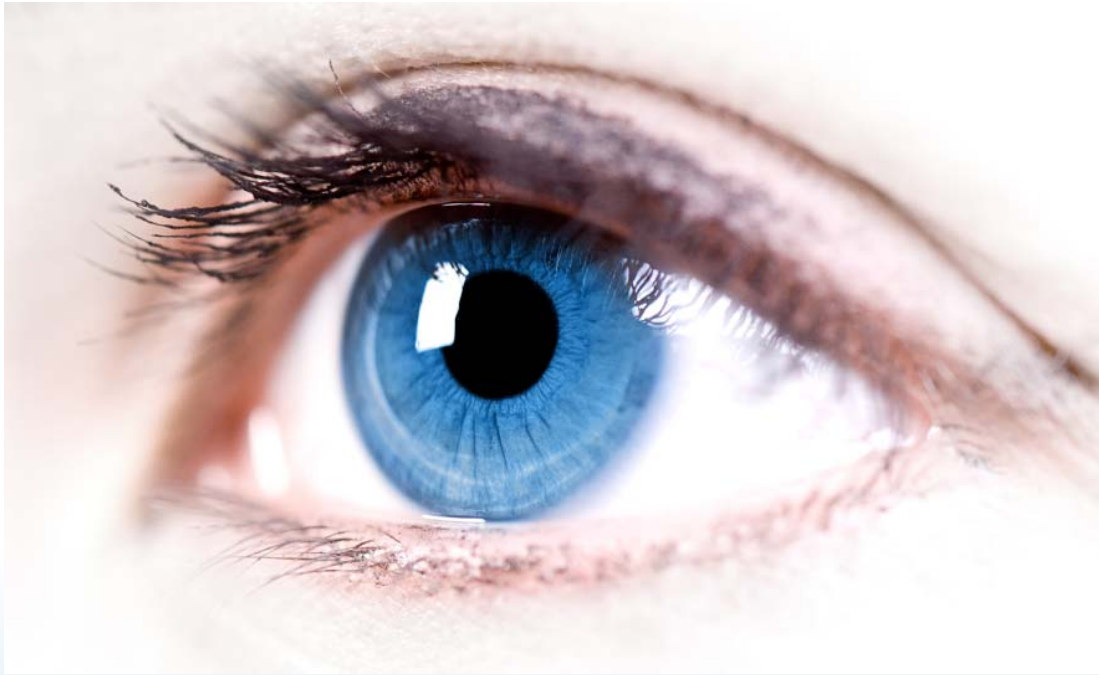
“I just wanted to say hello to everyone. Happy monday, let's all have a productive week <3”

“I just really want to be put up on the powerpoint. Please put me on the powerpoint!”

“Angular size was difficult to understand.”

“this stuff is tricky. do we need to know how to draw the rays going through the eye lens? because even without eye lenses, those rays still confuse me...”

“I don't really understand how to think about the image on the back of the retina versus the image that the eye 'sees'. I also don't intuitively understand how to think about light rays in general: are they supposed to converge on the retina (in which case, every object would be interpreted by the brain as a dot), or are they supposed to converge before the retina and produce an image on the retina? And would that not mean that the focal length of an eye is always less than the length of the eye itself?”



Phys 102 – Lecture 20

The eye & corrective lenses

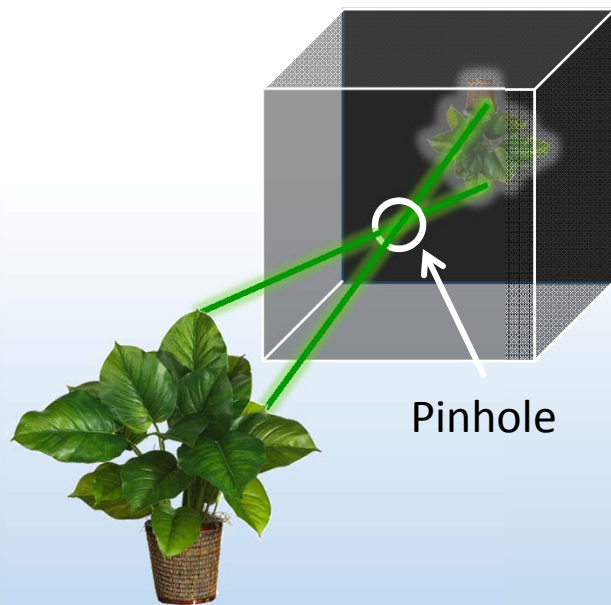
Today we will...

- Apply concepts from ray optics & lenses
 - Simple optical instruments – the camera & the eye
- Learn about the human eye
 - Accommodation
 - Myopia, hyperopia, and corrective lenses
- Learn about perception of size
 - Angular size
 - Magnifying glass & angular magnification

The Camera

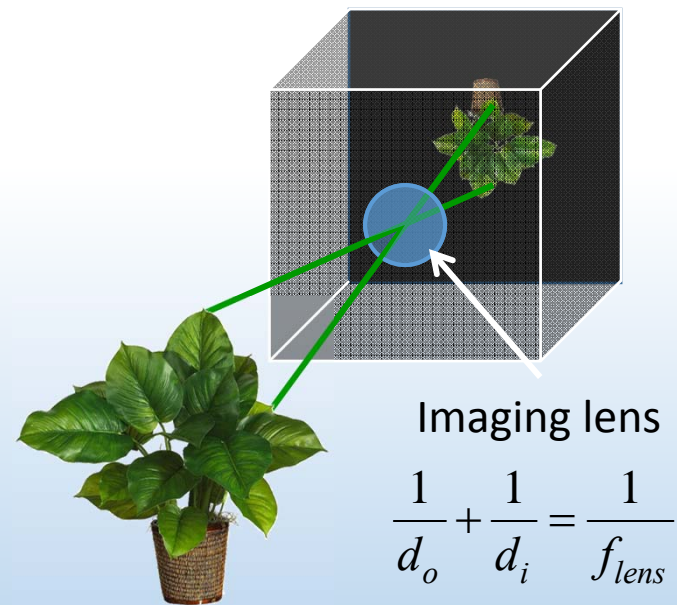
Cameras are one of simplest optical instruments, produce real image onto sensor

Pinhole camera ("*camera obscura*")



DEMO

Modern camera



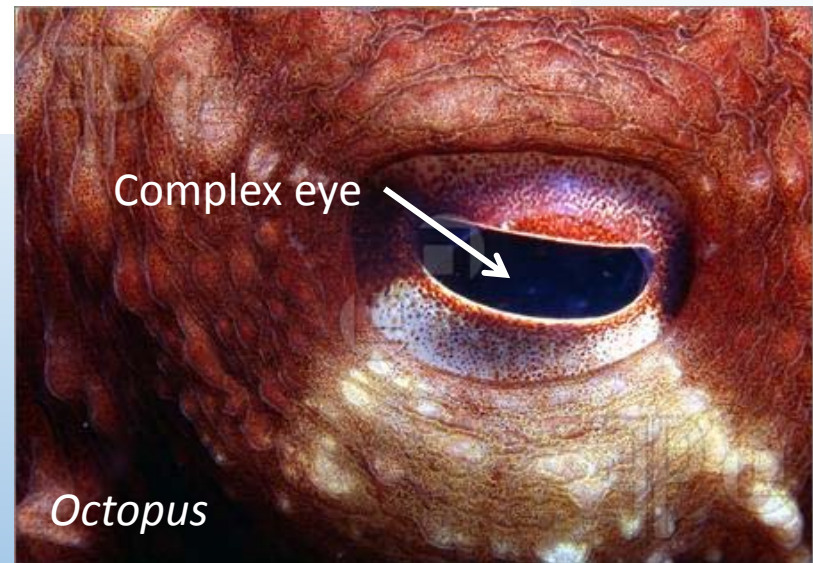
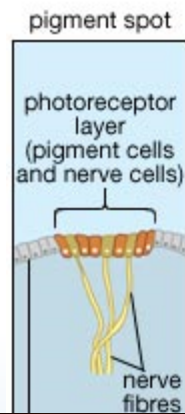
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_{lens}}$$

Not a true imaging system. Each point from object creates a circle of light on screen.

True imaging system. Each point from object has a corresponding point on screen.

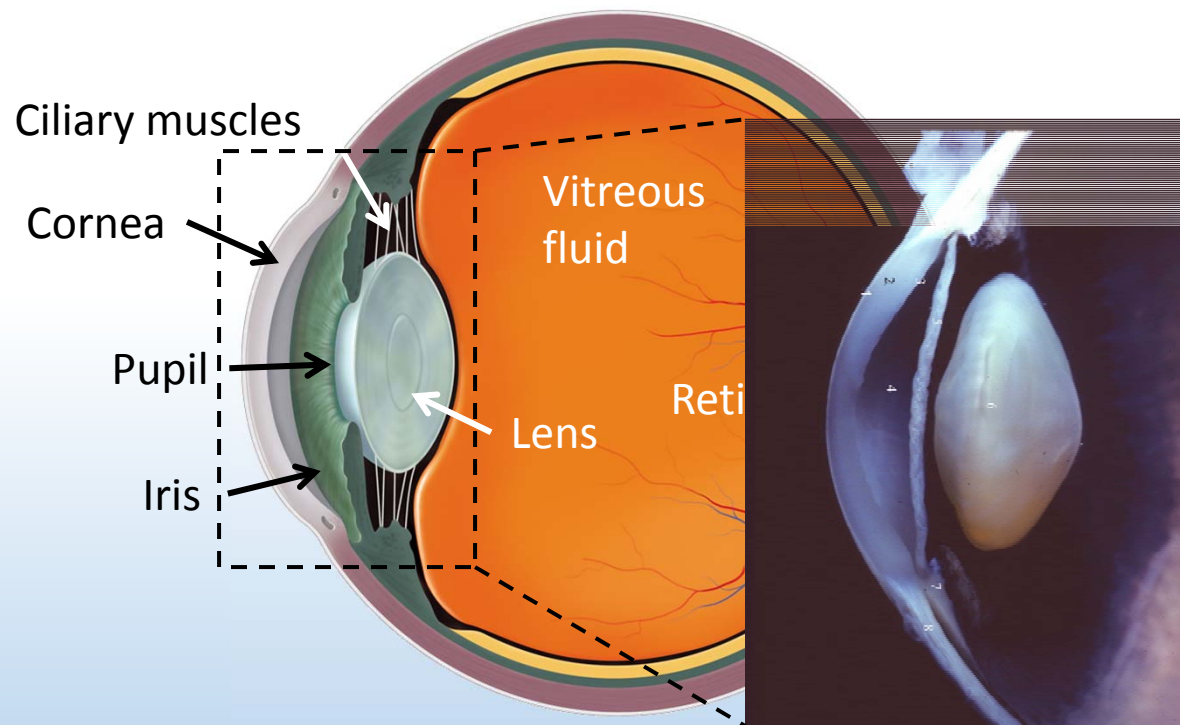
Evolution of the eye

The eye is like a camera



Anatomy of the human eye

As in a camera, eye lens creates image of object onto retina



| Part of eye | n |
|----------------|-------|
| Cornea | 1.351 |
| Lens | 1.437 |
| Vitreous fluid | 1.333 |

Pupil controls amount of light – diameter typically 2-8 mm

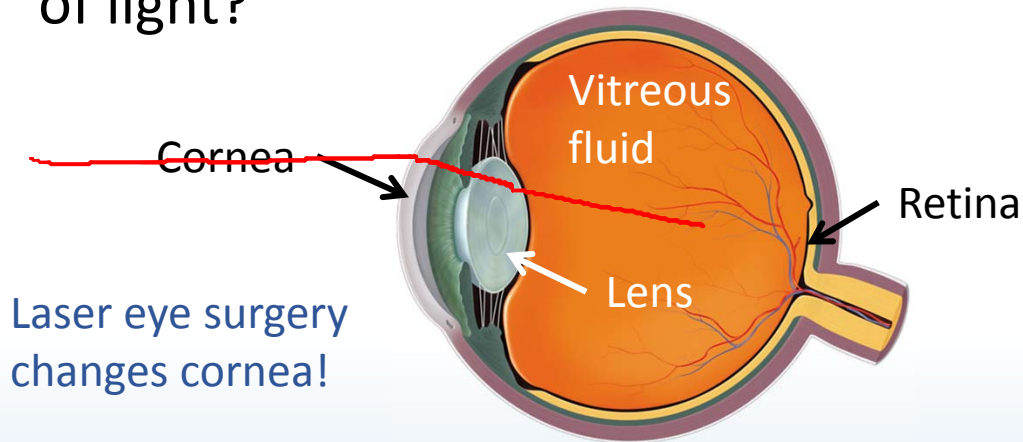
Retina has ~125 million photoreceptor cells (rods & cones)

DEMO



ACT: Anatomy of the Eye

Which part of the eye is responsible for most of the bending of light?



| Part of eye | n |
|----------------|-------|
| Cornea | 1.351 |
| Lens | 1.437 |
| Vitreous fluid | 1.333 |

A. Lens

B. Cornea

C. Retina

D. Vitreous fluid

Shape and index of refraction mismatch determine how much light bends:
Lens and cornea have similar shape and n

$$\frac{n_{\text{lens}}}{n_{\text{cornea}}} = \frac{1.44}{1.35} \approx 1.06$$

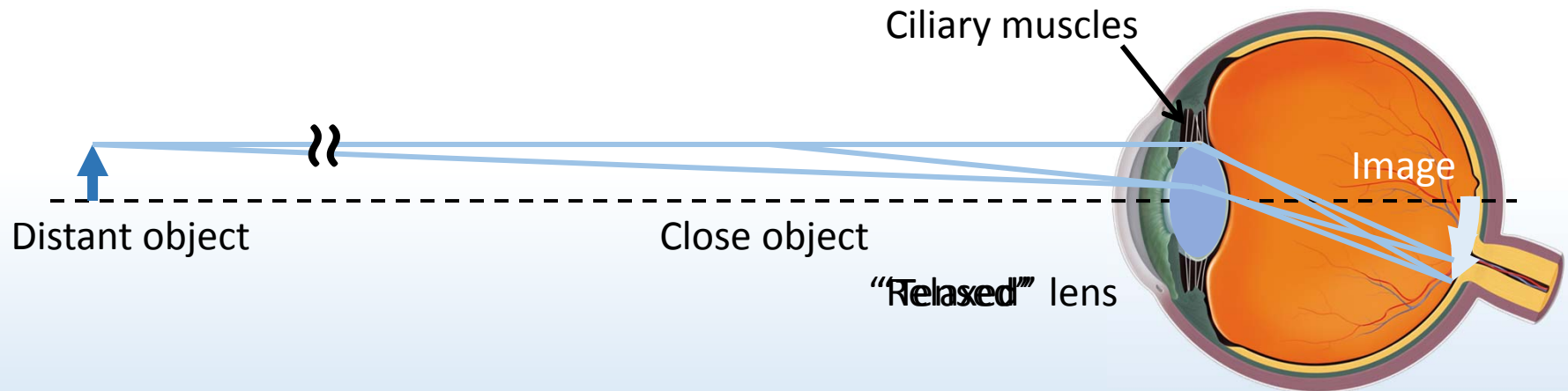
$$\frac{n_{\text{cornea}}}{n_{\text{air}}} = \frac{1.35}{1} \approx 1.35$$

$$\frac{n_{\text{lens}}}{n_{\text{vitreous}}} = \frac{1.44}{1.33} \approx 1.08$$

Most of bending occurs at air-cornea interface

Accommodation

Ciliary muscles around lens change its shape and focal length
The eye can focus on objects both close and far



The “far point” and “near point” are the maximum and minimum object distances where the image remains in focus

Far point: $d_{o, \text{far}} = \infty$

Near point: $d_{o, \text{near}} = 25 \text{ cm}$

} Normal adult

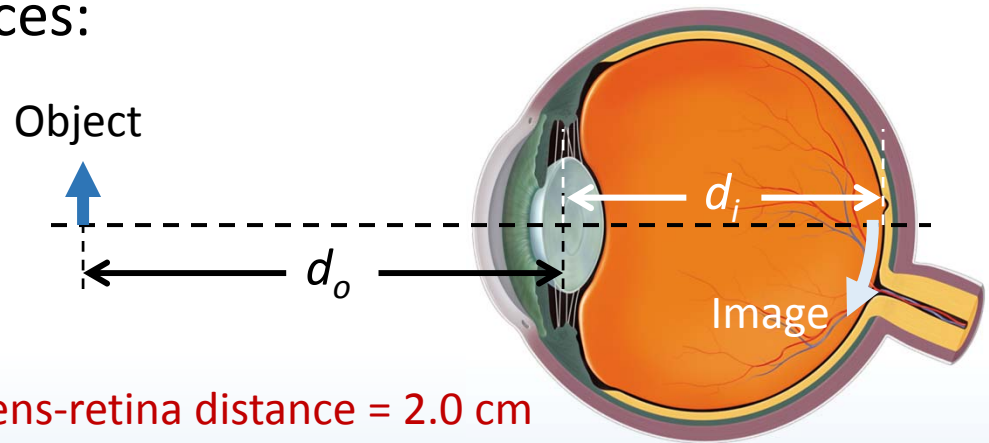
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Calculation: focal length of the eye

An adult with normal eyesight will see a focused image over a wide range of object distances:

“Far” point: $d_{o, \text{far}} = \infty$

“Near” point: $d_{o, \text{near}} = 25 \text{ cm}$



Typical lens-retina distance = 2.0 cm

What are the focal lengths of the relaxed and tensed eye?

$$\frac{1}{f_{\text{relaxed}}} = \frac{1}{d_{o, \text{far}}} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{2.0 \text{ cm}}$$

$$f_{\text{relaxed}} = 2.00 \text{ cm}$$

$$\frac{1}{f_{\text{tensed}}} = \frac{1}{d_{o, \text{near}}} + \frac{1}{d_i} = \frac{1}{25 \text{ cm}} + \frac{1}{2.0 \text{ cm}}$$

$$f_{\text{tensed}} = 1.85 \text{ cm}$$

Small change in f yields large change in d_o !



ACT: CheckPoint 1

A person with almost normal vision (near point at 26 cm) is standing in front of a plane mirror.

What is the closest distance to the mirror where the person can stand and still see himself in focus?

53 % A. 13 cm

42 % B. 26 cm

5 % C. 52 cm



Image from mirror
becomes object for eye!

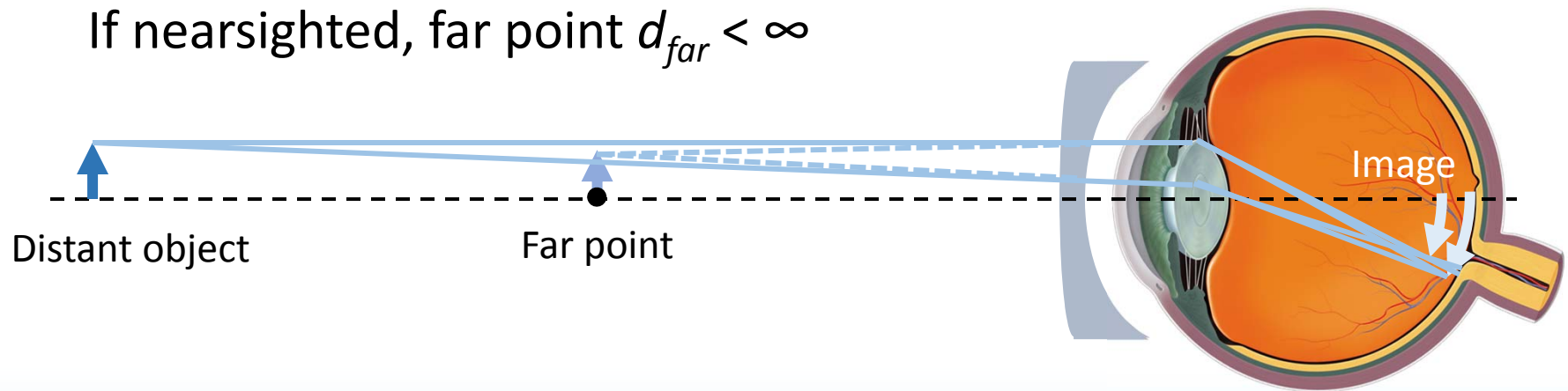
Near Point, Far Point

- Eye's lens changes shape (changes f)
Object at any d_o should produce image at retina ($d_i \approx 2.0$ cm)
Lens can only change shape so much
- “Far Point”
Furthest d_o where image can be at retina
Normally, $d_{far} = \infty$ (if nearsighted then closer)
- “Near Point”
Closest d_o where image can be at retina
Normally, $d_{near} \approx 25$ cm (if farsighted then further)

Myopia (nearsightedness)

HW

If nearsighted, far point $d_{far} < \infty$



Object at $d_o > d_{far}$ creates image in front of retina

Corrective lens creates image of distant object at the far point of the nearsighted eye

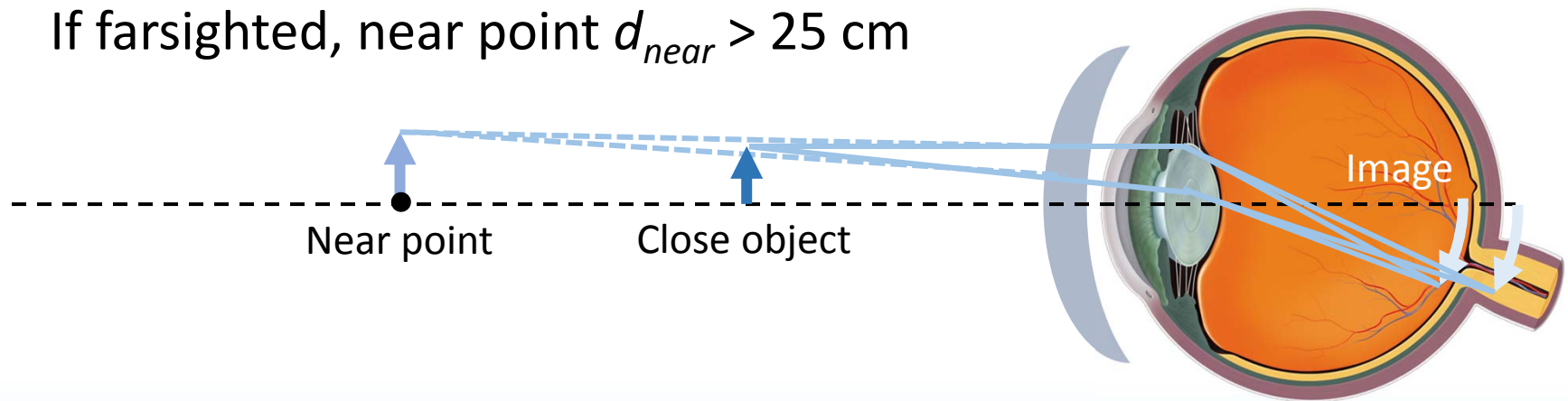
$$\frac{1}{d_o} + \frac{1}{-d_{far}} = \frac{1}{f_{lens}} \quad f_{lens} = -d_{far} \quad \text{Diverging lens!}$$

f_{lens} such that distant object at ∞ ("normal" far point) is in focus

DEMO

Hyperopia (farsightedness)

If farsighted, near point $d_{near} > 25$ cm



Object at $d_o < d_{near}$ creates image behind retina

Corrective lens creates image of close object at the near point of the farsighted eye

$$\frac{1}{25 \text{ cm}} + \frac{1}{-d_{near}} = \frac{1}{f_{lens}}$$

$$d_{near} > 25 \text{ cm} \quad \text{so} \quad f_{lens} > 0$$

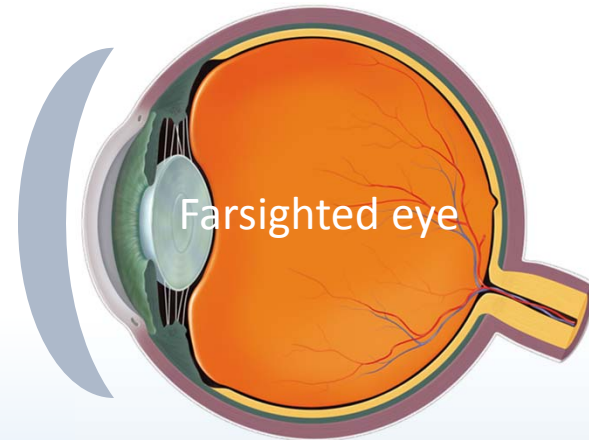
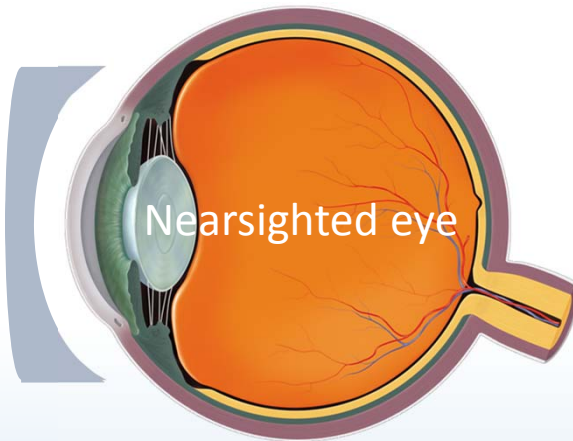
f_{lens} such that object at 25 cm ("normal" near point) is in focus **Converging lens!**

DEMO



ACT: Corrective lenses

For which type of eye correction is the image always virtual?



A. Nearsighted

B. Farsighted

C. Both

D. Neither

In both cases the image is formed *before* the lens, so it is virtual!

$$d_i < 0$$

Also, image is upright,
reduced (diverging lens) or
enlarged (converging lens)

$$m = -\frac{d_i}{d_o}$$

Calculation: Refractive Power

Optometrists use refractive power P instead of focal length f

$$P \equiv \frac{1}{f}$$

Units: “Diopters” (D) \equiv 1/meters

Your friend’s contact lens prescription is -3.3 diopters. What is the focal length? Is your friend near- or farsighted?



$$f_{\text{lens}} = \frac{1}{P} = \frac{1}{-3.3 \text{ D}} \approx -0.3 \text{ m} = -30 \text{ cm}$$

$$d_{\text{far}} = -f_{\text{lens}} \approx 30 \text{ cm}$$

A diverging lens!
Your friend is nearsighted



ACT: Refractive power

A relaxed, normal eye has a refractive power P_{norm} :

$$P_{norm} = \frac{1}{f_{norm}} = \frac{1}{0.02\text{ m}} = +50D$$

How does the refractive power P_{myopic} of a relaxed, nearsighted eye compare?

A. $P_{myopic} > +50\text{ D}$

B. $P_{myopic} = +50\text{ D}$

C. $P_{myopic} < +50\text{ D}$

Nearsighted eye forms an image of a distant object *in front* of retina so f must be smaller, P larger

Alternately,

$$\frac{1}{\infty} + \frac{1}{0.02\text{ m}} = \frac{1}{f_{norm}} = P_{norm}$$

$$\frac{1}{d_{far}} + \frac{1}{0.02\text{ m}} = \frac{1}{f_{myopic}} = P_{myopic} > P_{norm}$$



ACT: CheckPoint 2

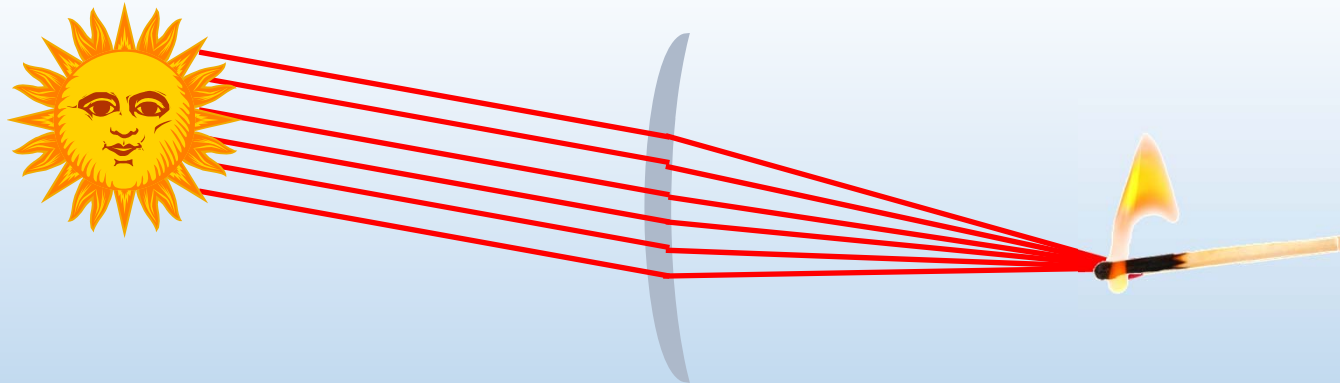
Two people who wear glasses are camping. One of them is nearsighted and the other is farsighted. Which person's glasses will be useful in starting a fire with the sun's rays?

A. Nearsighted

37 %

B. Farsighted

63 %

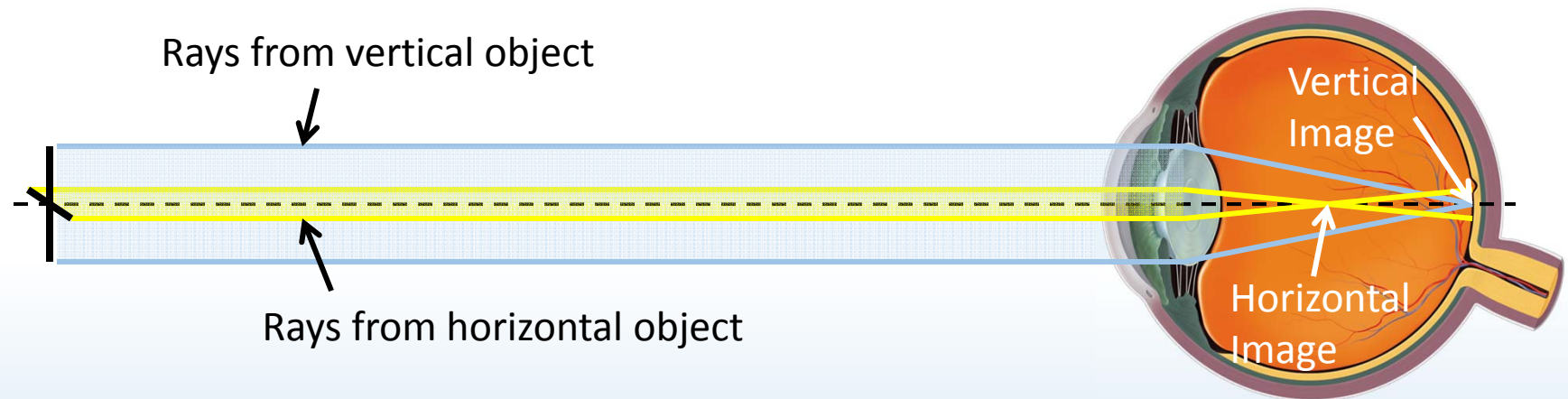


Farsighted person's glasses are converging – like magnifying glass!

Astigmatism

A normal eye is spherical, curved the same in every direction

An astigmatic eye is distorted (oval) along one direction



So, an astigmatic eye has a different f along different directions



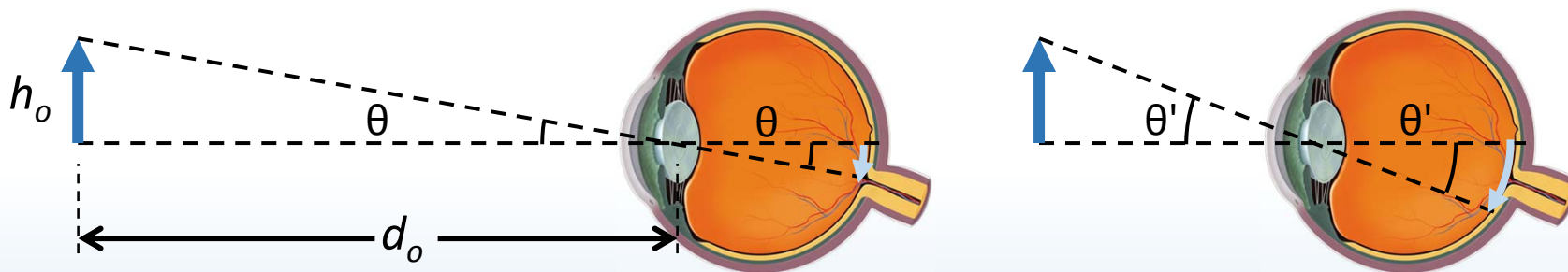
Images are blurry in one direction



Corrected with toric lens

Angular Size: CheckPoint 3.1-3.2

Angular size refers to how large the image is on your retina, and how big it *appears* to be.



Both objects are same size, but nearer one looks bigger.

$$\tan \theta \approx \frac{h_o}{d_o} \quad (\text{in radians}) \quad \text{if angle is small}$$

$$\theta \leq 0.3 \text{ (rad)}$$

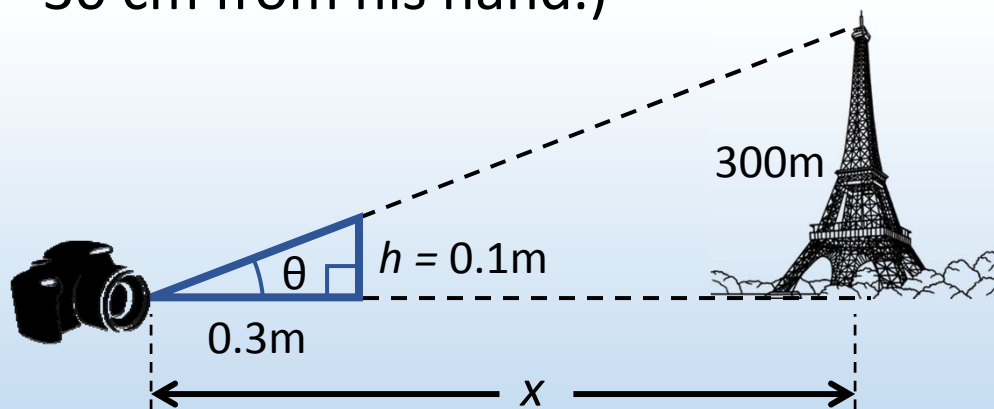
What is the maximum possible angular size?

$$\theta_{\max} \approx \frac{h_o}{d_{\text{near}}}$$

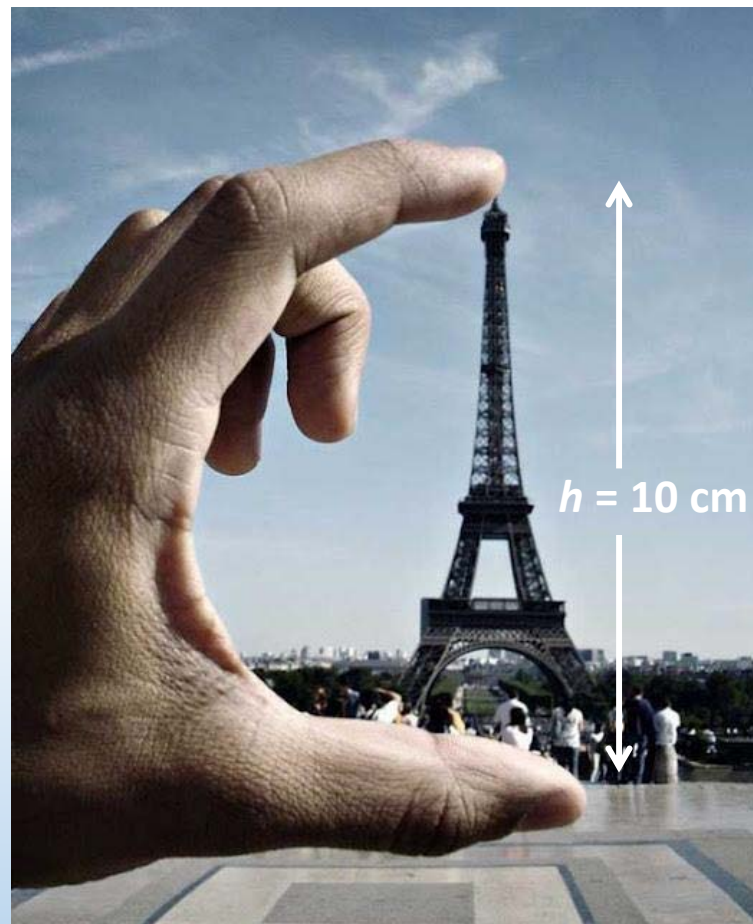
Calculation: Angular size

A cameraman takes a trick shot of the Eiffel tower, which is 300 m tall.

How far is the cameraman from the Eiffel tower? (Assume the camera is 30 cm from his hand.)

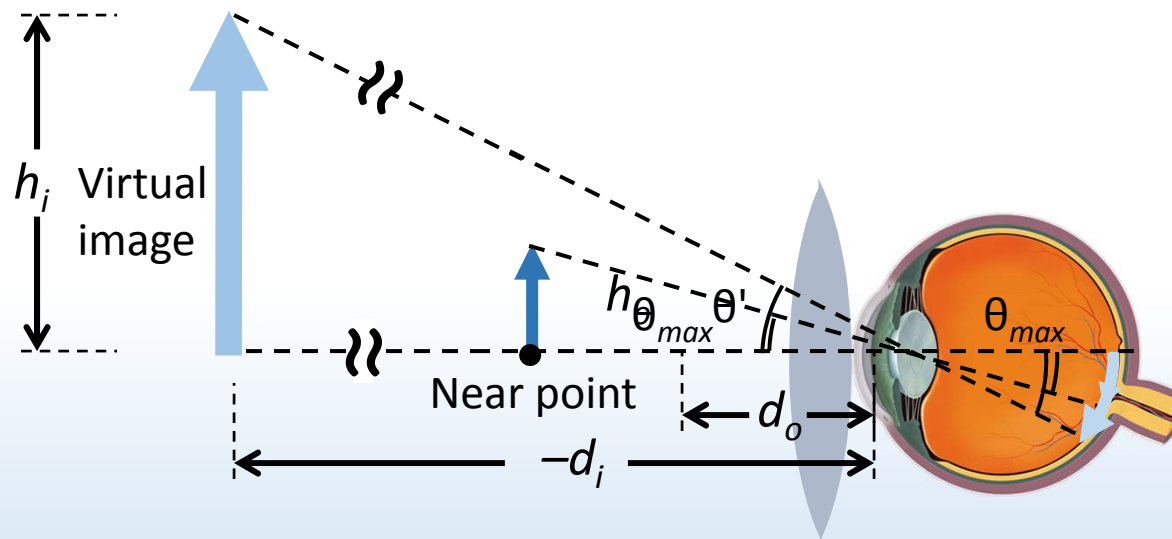


$$\tan \theta = \frac{0.1}{0.3} = \frac{300}{x} \quad x = 900 \text{ m}$$



Magnifying glass

A magnifying glass produces a virtual image behind object, allowing a closer object $d_o < d_{near}$ and a larger θ'



$$\theta' \approx \frac{h_i}{-d_i} = \frac{h_o}{d_o}$$

Angular magnification gives how much angular size increases:

$$M = \frac{\theta'}{\theta_{max}} \approx \frac{h_o/d_o}{h_o/d_{near}} = \frac{d_{near}}{d_o}$$

unaided eye θ_{max}

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Typically set image at $d_i = \infty$, for a relaxed eye (so $d_o = f$)



ACT: Magnifying glass

A person with normal vision ($d_{\text{near}} = 25 \text{ cm}$, $d_{\text{far}} = \infty$) has a set of lenses with different focal lengths. She wants to use one as a magnifying glass.



Which of the following focal lengths will magnify the image?

A. $f = 50 \text{ cm}$

B. $f = 2.5 \text{ cm}$

C. $f = -6 \text{ cm}$

D. $f = -40 \text{ cm}$

Magnifying glass is a converging lens ($f > 0$)

Want $f < d_{\text{near}}$ to magnify

$$M = \frac{d_{\text{near}}}{f}$$

DEMO

Summary of today's lecture

- Accommodation – eye lens changes shape
 - Near point – closest object (~ 25 cm, further if farsighted)
 - Far point – furthest object (∞ , closer if nearsighted)
- Corrective lenses
 - Nearsighted – diverging lens creates virtual image at far point
 - Farsighted – converging lens creates virtual image at near point
- Angular size & angular magnification
 - Magnifying glass creates virtual image of object placed closer than near point