

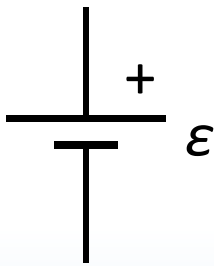
# Phys 102 – Lecture 7

Series and parallel circuits

# Recall from last time...

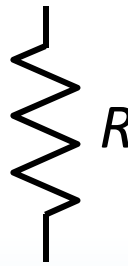
Electric potential difference across circuit element is its “voltage”

$V_{\text{element}}$  Should be “ $\Delta V$ ”, but we’ll usually drop the “ $\Delta$ ”



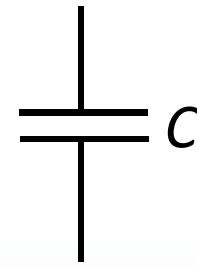
Batteries – pump charges  
Provide emf for charges

$$V_{\text{battery}} = \varepsilon$$



Resistors – regulate current  
Dissipate power

$$V_R = IR$$



Capacitors – store charge  
Store energy

$$V_C = \frac{Q}{C}$$

Wires are ideal conductors, with no resistance.  $E = 0$ , the electric potential is constant and the voltage across a wire is 0.

# ***Today we will...***

- Learn about electric circuits

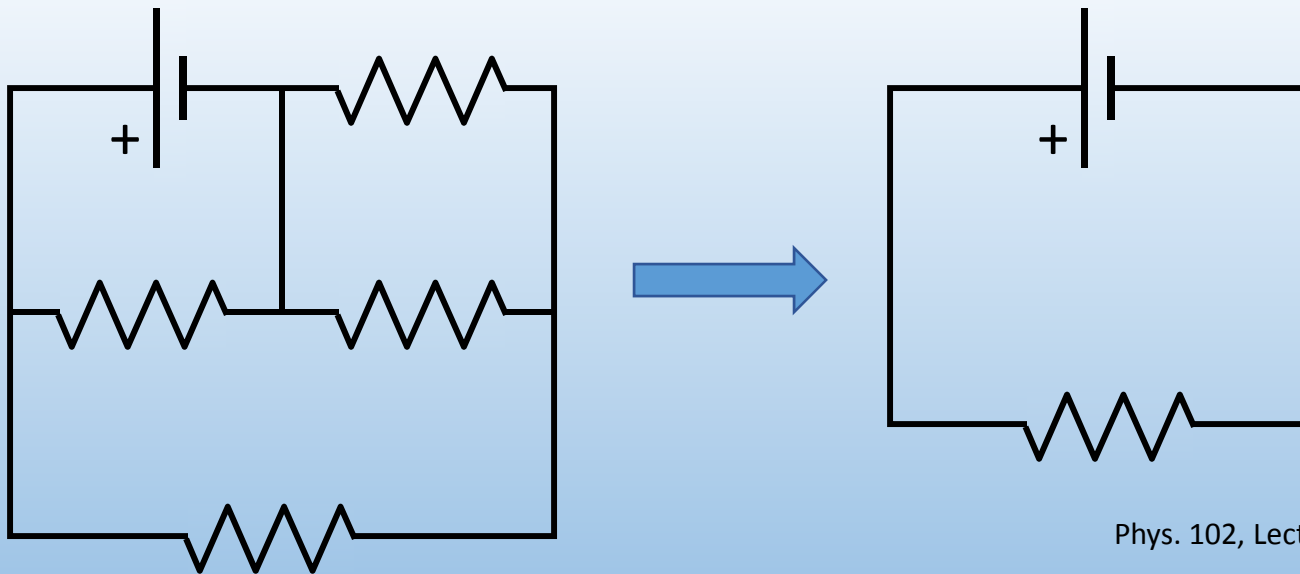
Circuits with a battery, wires, and resistors

Circuits with a battery, wires, and capacitors

- Analyze circuits

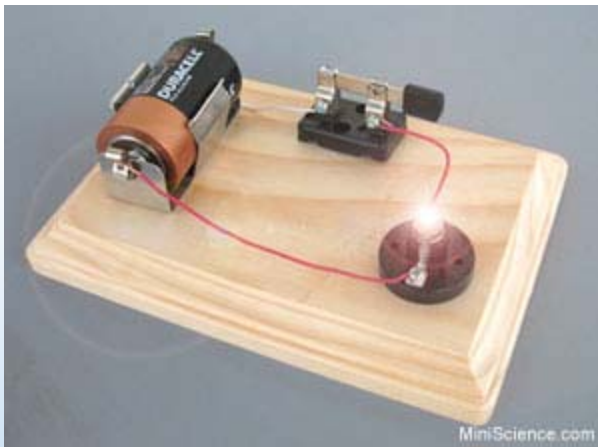
Take a complex-looking circuit like...

...and turn into a simple-looking circuit like

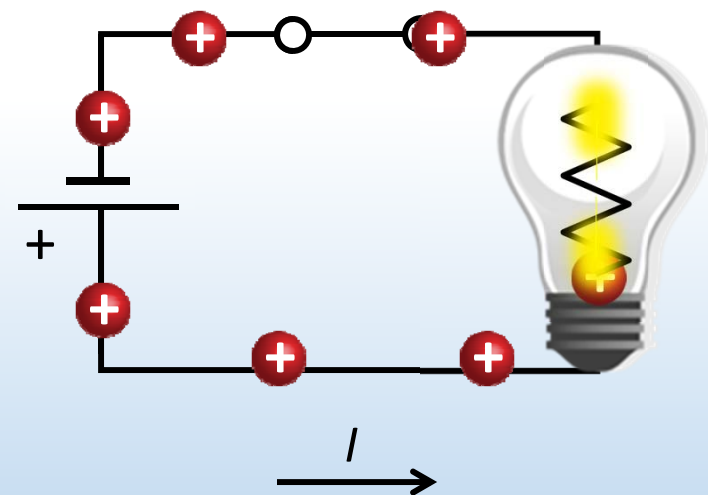


# *Electric circuits*

Electric circuits consist of one or more closed loops around which charges flow

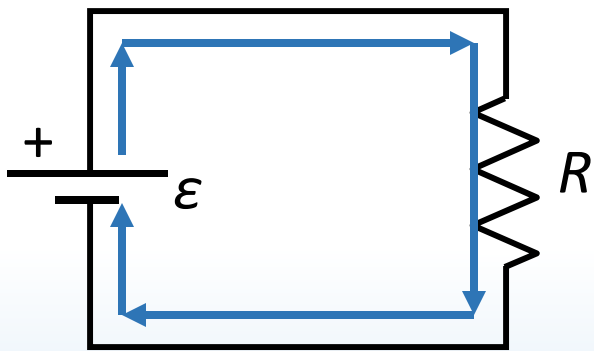


Closed circuit

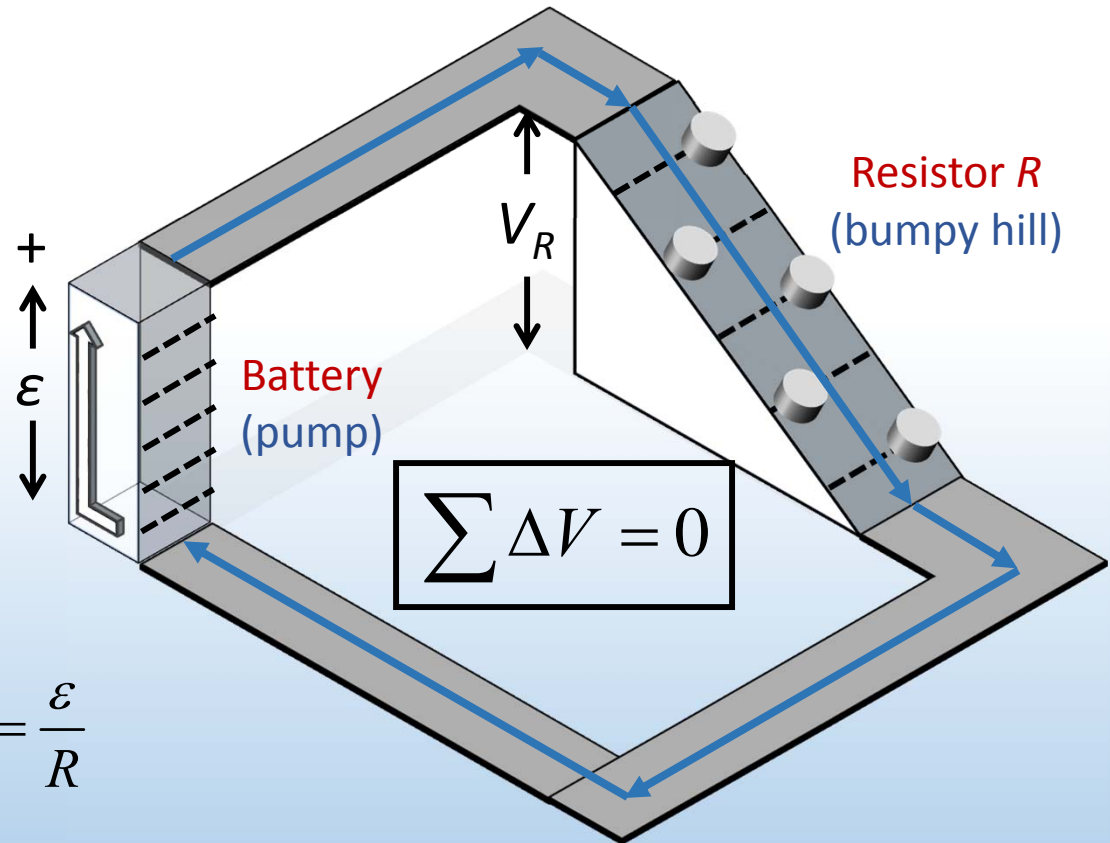


# Kirchhoff loop rule

A charge making a complete loop around a circuit must return to the same electric potential (“height”) at which it started



Sum of electric potential differences (voltages) around circuit loop is zero



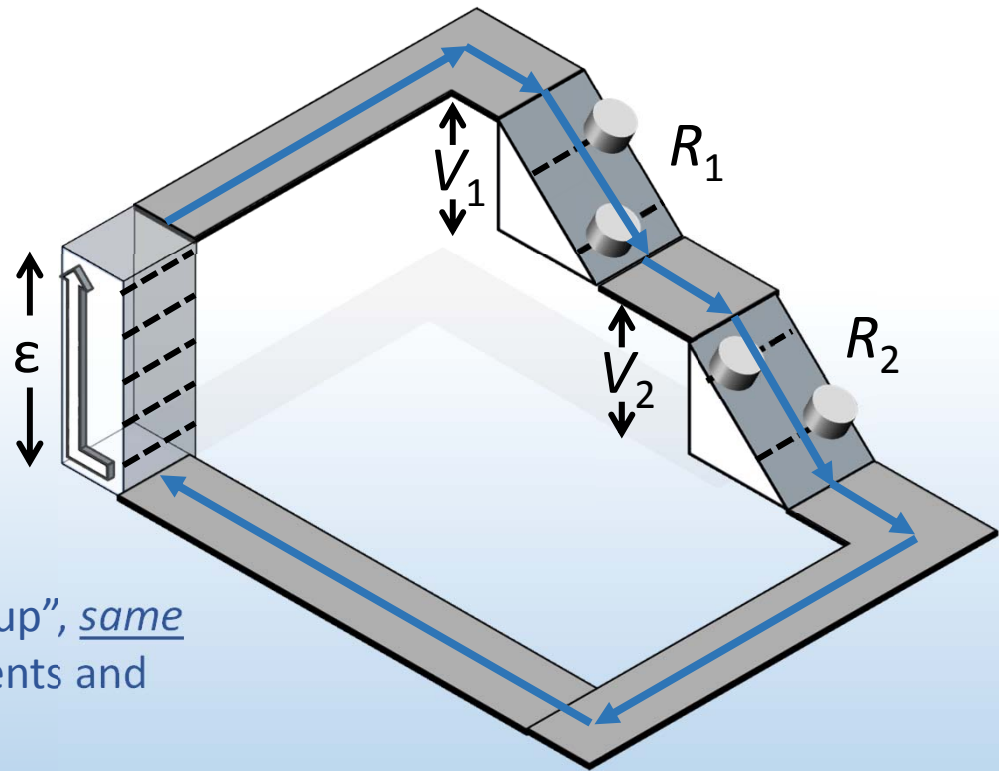
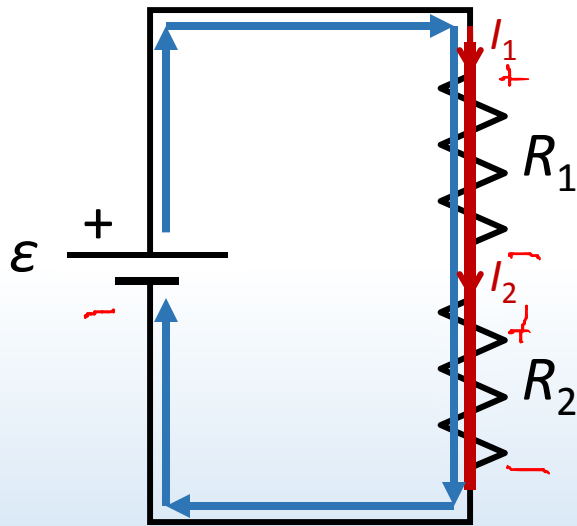
$$\varepsilon - IR_R = 0$$

Potential *increase* across battery      Potential *drop* across resistor

$$\text{So, } I = \frac{\varepsilon}{R}$$

# Series components

Two components are said to be in series when they are connected end-to-end by a single wire



Recall that charges do NOT get “used up”, same current  $I$  flows through both components and around circuit

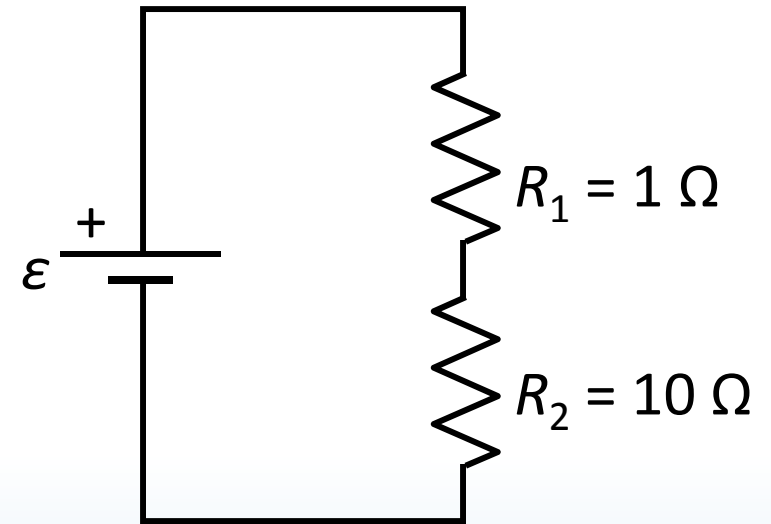
$$I_1 = I_2 = I = \frac{\varepsilon}{R_1 + R_2} \quad \text{Checkpoint 1.1}$$

From loop rule:  $\varepsilon - (V_1 + V_2) = 0$



## ACT: CheckPoint 1.2

Consider a circuit with two resistors  $R_1$  and  $R_2$  in series. Compare the voltages across the resistors:



A.  $V_1 > V_2$

B.  $V_1 = V_2$

C.  $V_1 < V_2$

Resistors are in series so  $I_1 = I_2 = I$  (**Checkpoint 1.1**)

$$V_1 = I_1 R_1 = I \times 1 \Omega$$

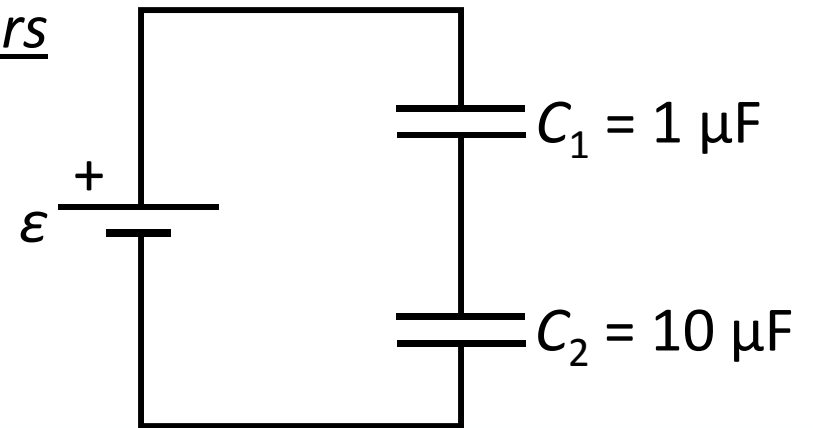
$$V_2 = I_2 R_2 = I \times 10 \Omega$$



# ACT: Capacitors in series

Consider a circuit with two capacitors  $C_1$  and  $C_2$  in series. Compare the voltages across the capacitors:

$$V_C = \frac{Q}{C}$$



A.  $V_1 > V_2$

B.  $V_1 = V_2$

C.  $V_1 < V_2$

Apply the same reasoning as for a circuit with resistors in series

Capacitors are in series so they have the same charge  $Q_1 = Q_2 = Q$

$$V_1 = Q_1 / C_1 = Q / 1 \mu\text{F}$$

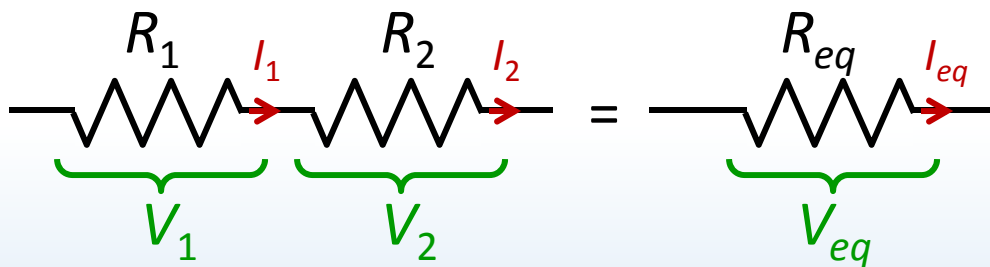
$$V_2 = Q_2 / C_2 = Q / 10 \mu\text{F}$$



# Equivalent resistance & capacitance

Circuit behaves the same as if *series* components were replaced by a *single, equivalent* component

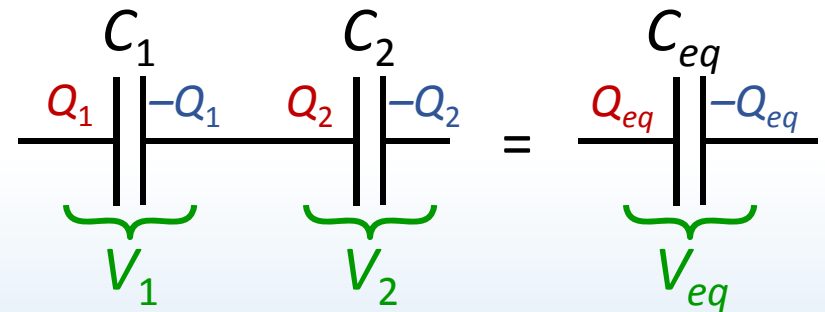
## Resistors



$$I_1 = I_2 = I_{eq} \quad V_1 + V_2 = V_{eq}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

## Capacitors



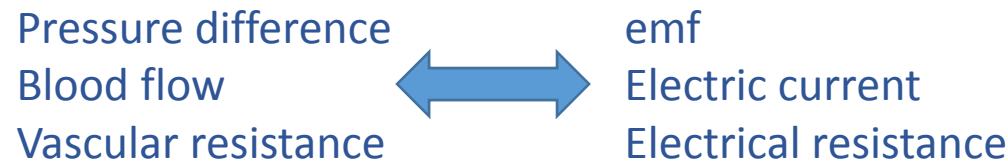
$$Q_{eq} = Q_1 = Q_2 \quad V_{eq} = V_1 + V_2$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

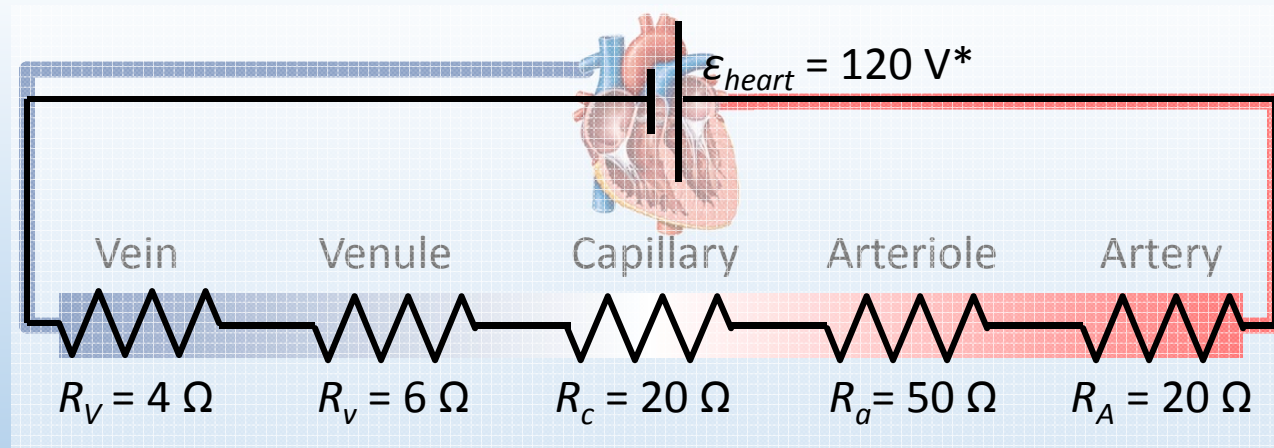
Note same reasoning for resistors and capacitors but final expression is different because  $V_R \propto R$  but  $V_C \propto 1/C$

# Calculation: vascular resistance

The circulatory system is analogous to an electric circuit



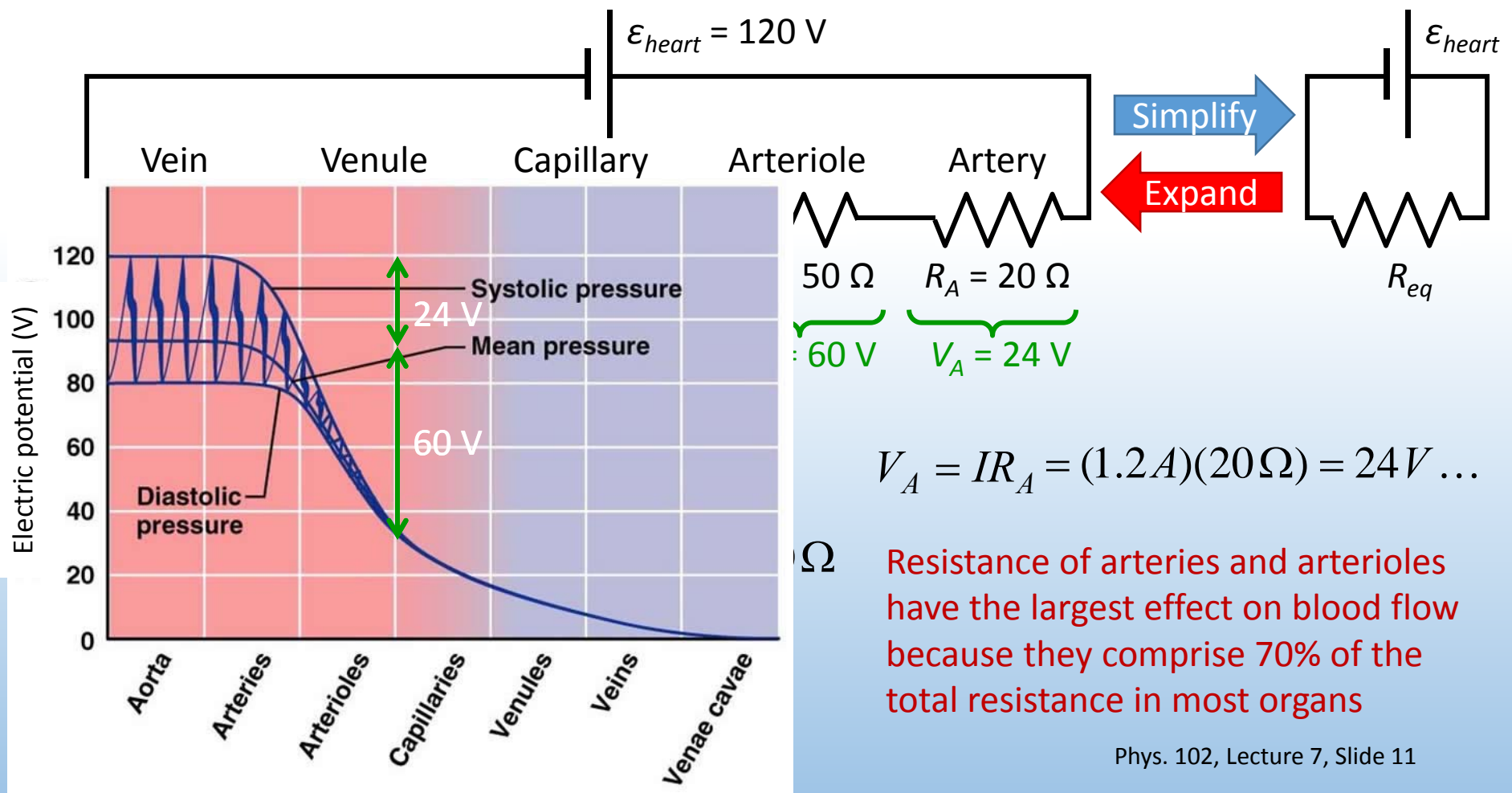
The circulatory system consists of different types of vessels in series with different resistances to flow



\*Numbers represent accurate relative values

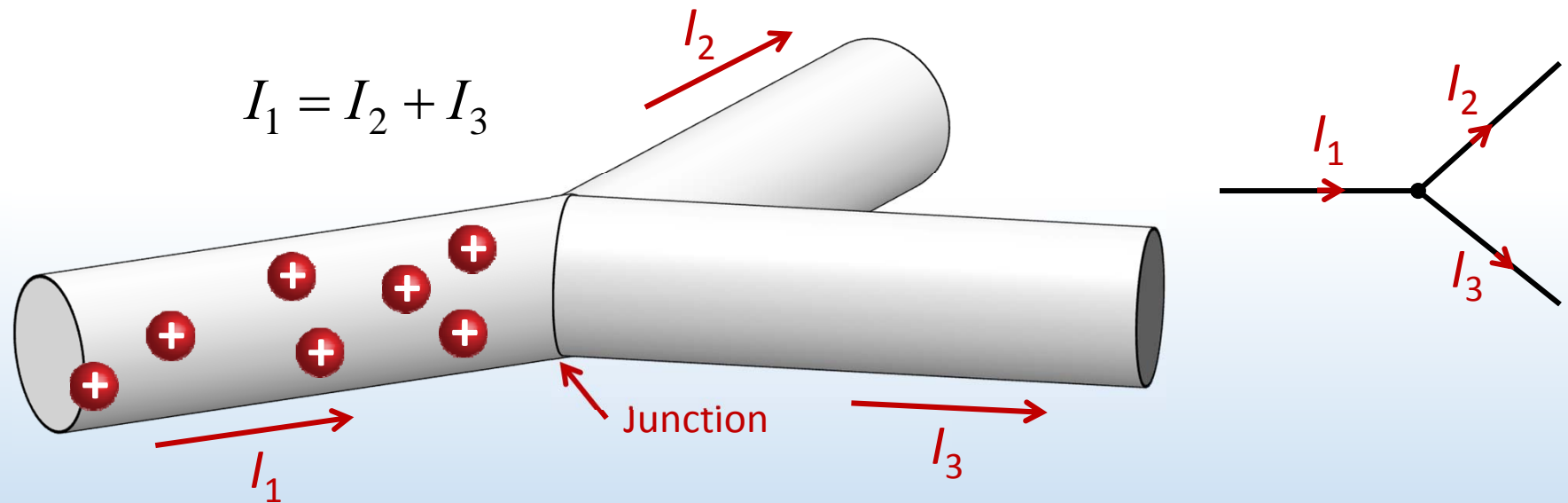
# Calculation: vascular resistance

Calculate the current  $I$  through the vascular circuit and the voltages across the different types of vessels



# Kirchhoff junction rule

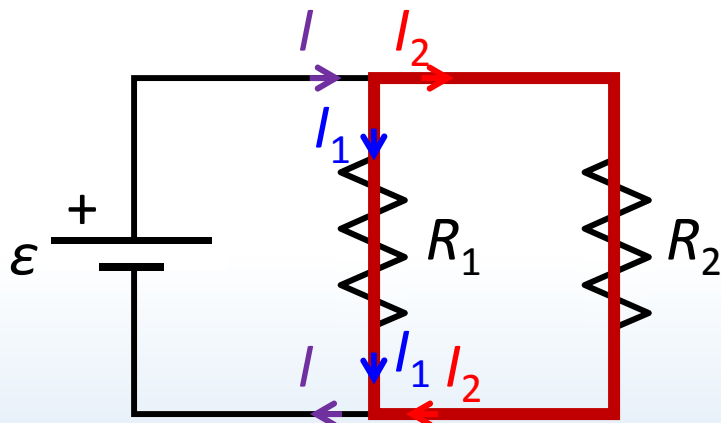
Charges flowing through a junction split. By conservation of charge, the sum of currents into a junction equals the sum of currents out of a junction



$$\sum I_{in} = \sum I_{out}$$

# Parallel components

Components are said to be in parallel when both ends are connected to each other, forming a loop containing only them



Junction rule:  $I$  splits into  $I_1$  and  $I_2$

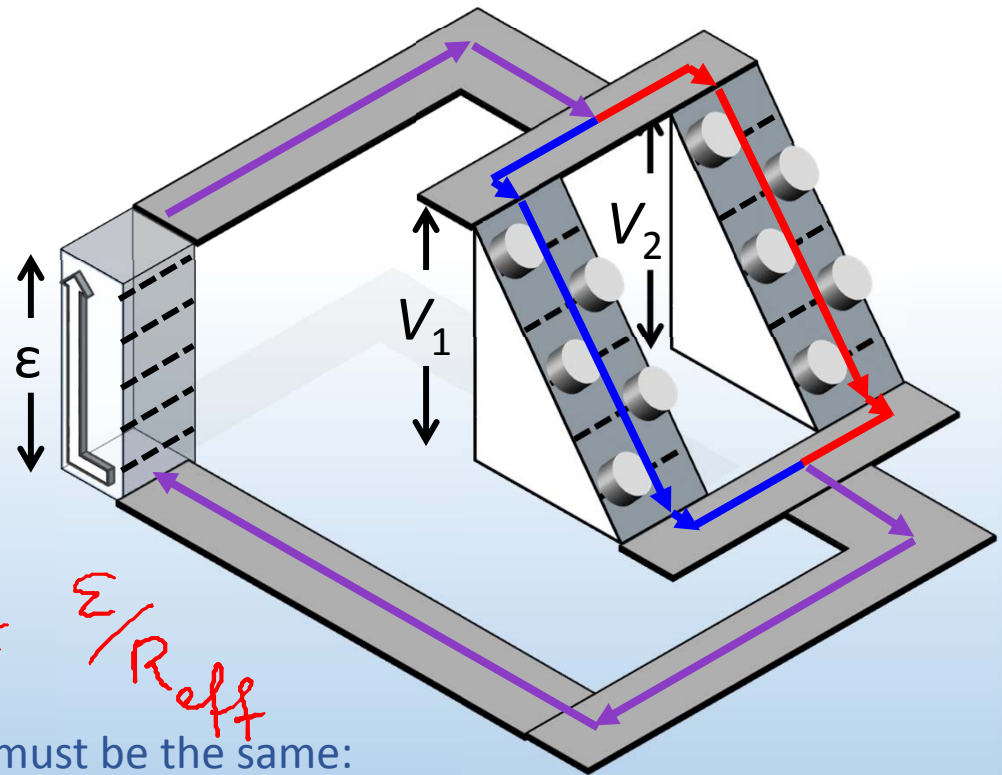
$I_1$  and  $I_2$  recombine into  $I$

$$I = I_1 + I_2 = \varepsilon \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \varepsilon / R_{\text{eff}}$$

From loop rule, voltage across resistors must be the same:

$$\varepsilon - V_1 = 0 \quad \varepsilon - V_2 = 0$$

$$V_1 R_1 = V_2 R_2 = \varepsilon$$

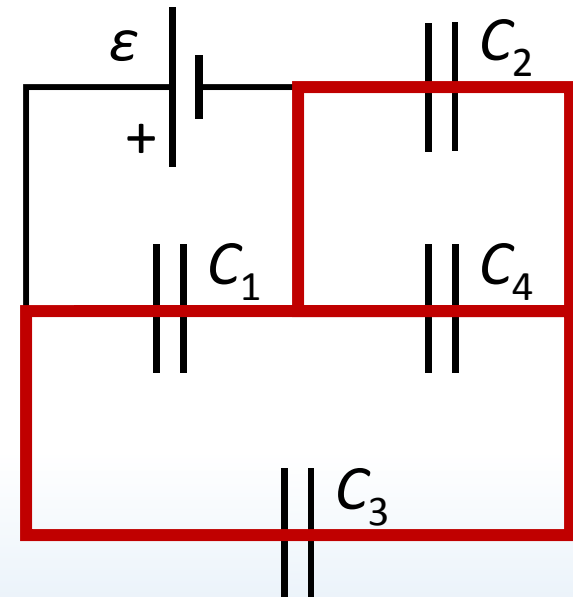


$$\frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2}$$



# ACT: Parallel or series?

Consider the circuit to the right. Which of the following statements is true?



A.  $C_1$  &  $C_4$  are in series

B.  $C_2$  &  $C_4$  are in parallel

C.  $C_1$  &  $C_3$  are in parallel

No, branch between  $C_1$  &  $C_4$

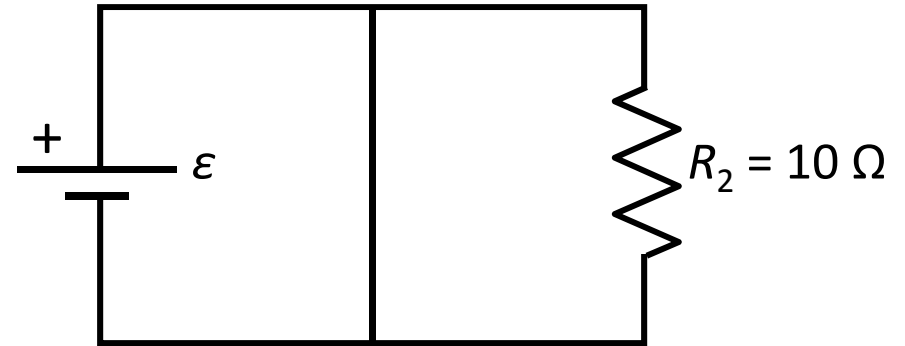
Yes, loop contains only  $C_2$  &  $C_4$

No, loop also contains  $C_3$



# ACT: Resistors in parallel

Consider a circuit with two resistors  $R_1$  and  $R_2$  in parallel. Compare  $I_1$ , the current through  $R_1$ , to  $I_2$ , the current through  $R_2$ :



A.  $I_1 > I_2$

B.  $I_1 = I_2$

C.  $I_1 < I_2$

Resistors & battery are in parallel so  $V_1 = V_2 = \epsilon$

$$I_1 = V_1 / R_1 = \epsilon / 1 \Omega$$

$$I_2 = V_2 / R_2 = \epsilon / 10 \Omega$$

$$I_2 = \frac{R_1}{R_2} I_1$$

Most current flows along “path of least resistance”!

What if we replace  $R_1$  with a wire?

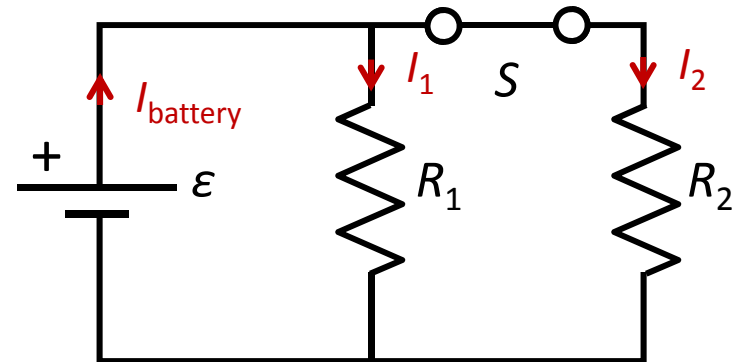
An ideal wire has no resistance, so all current flows through it and  $I_2 = 0$

This is called a short circuit



## ACT: CheckPoint 2.3

Now we add a switch  $S$ . What happens to the current out of the battery when the switch is closed?



A.  $I_{\text{battery}}$  increases

B.  $I_{\text{battery}}$  remains the same

C.  $I_{\text{battery}}$  decreases

$I_1$  stays the same,  $I_2$  increases  
(**Checkpoint 2.1-2.2**)

Before closing switch:

$$I_1 = \epsilon/R_1, I_2 = 0 \text{ and } I_{\text{battery}} = I_1$$

After closing switch:

$$I_1 = \epsilon/R_1, I_2 = \epsilon/R_2 \text{ and } I_{\text{battery}} = I_1 + I_2$$

This is how your home is wired. The more appliances you turn on, the more current you draw.

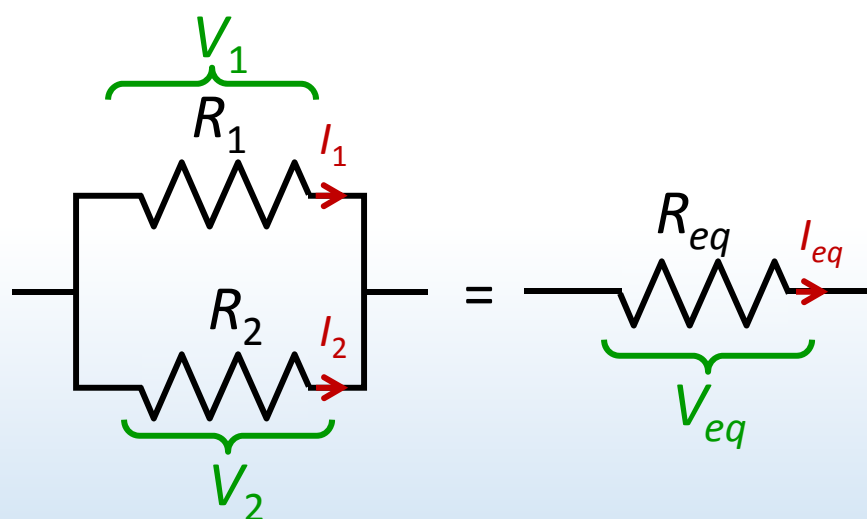
DEMO



# Equivalent resistance & capacitance

Circuit behaves the same as if *parallel* components were replaced by a *single, equivalent* component

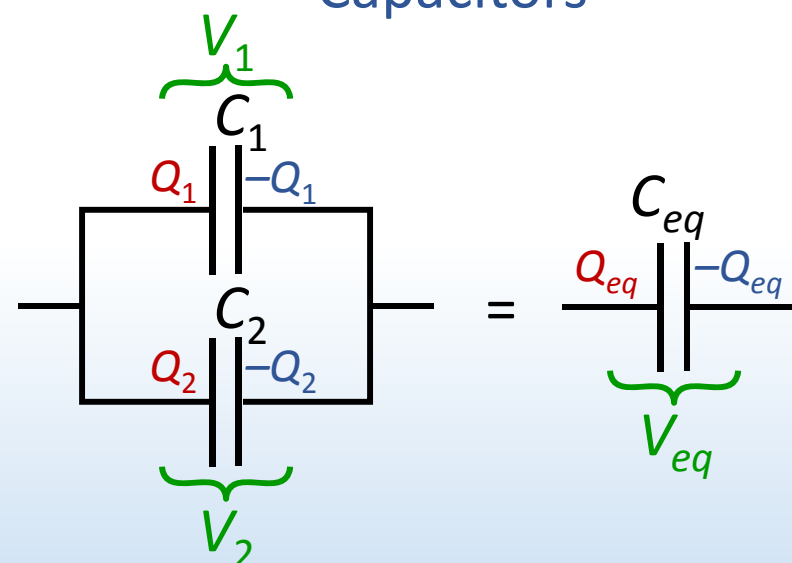
## Resistors



$$I_1 + I_2 = I_{eq} \quad V_1 = V_2 = V_{eq}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

## Capacitors

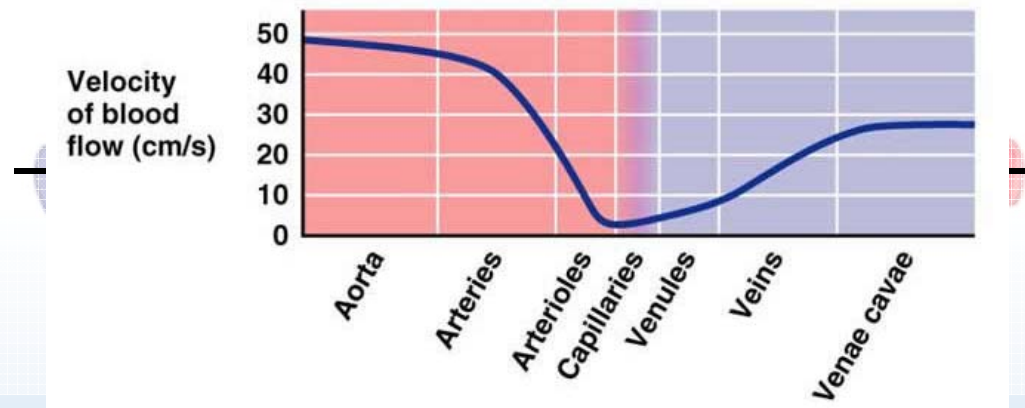


$$Q_1 + Q_2 = Q_{eq} \quad V_1 = V_2 = V_{eq}$$

$$C_{eq} = C_1 + C_2$$

# Calculation: vascular resistance

In previous calculation, capillarie resistance accounts for ~20% of total vascular resistance, yet capillaries are the thinnest blood vessels, and should have the *highest* resistance. Why?



Individual capillaries are connected in *parallel*

Assuming each capillary has the same resistance:

$$\frac{1}{R_{c,tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} = \frac{N}{R_i} \quad R_{c,tot} = \frac{R_i}{N} \quad \text{Total capillarie resistance} \ll \text{resistance of single capillary}$$

$$I_{tot} = I_1 + I_2 + \dots + I_N = NI_i \quad \text{Current in single capillary} \ll \text{total current}$$

# Calculation: cardiovascular system

The human cardiovascular system consists of two circuits: pulmonary circulation which carries blood through the lungs, and systemic circulation which carries blood to the organs

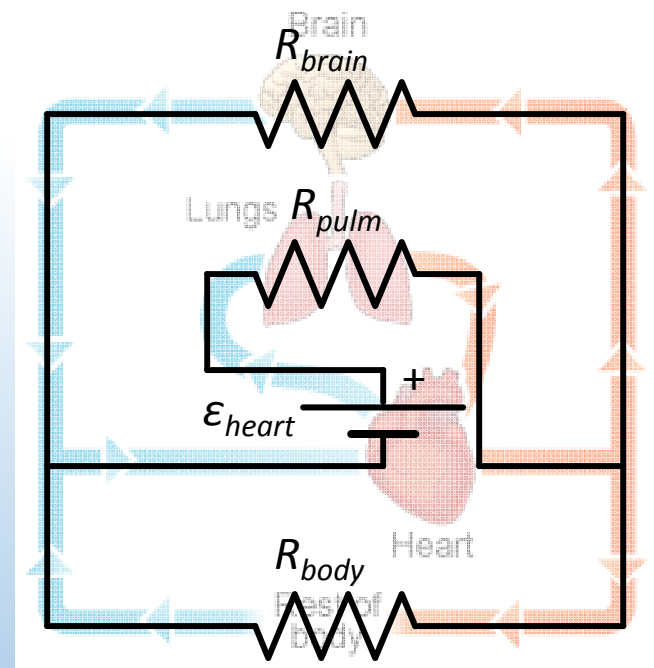
The organs of the body are connected in parallel in the systemic circuit

Simple circuit model\*:

$$R_{pulm} = 12 \, \Omega, R_{brain} = 1 \, \text{k}\Omega, \\ R_{body} = 160 \, \Omega, \epsilon_{heart} = 120 \, \text{V}$$

\*Numbers represent accurate relative values

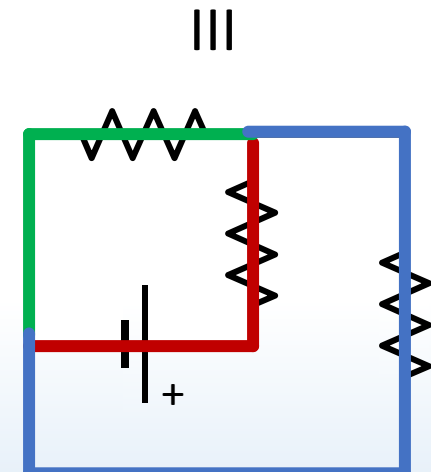
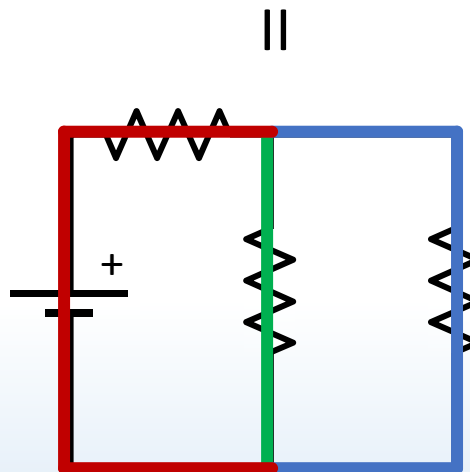
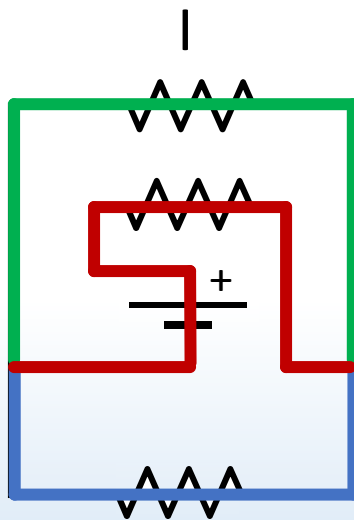
Calculate current through each component of circulatory system





# ***ACT: analyzing circuits***

Which of the following circuit is different than the others?



- A. Circuit I
- B. Circuit II
- C. Circuit III
- D. All three are equivalent**
- E. All three are different

Approach: identify parallel branches

Battery in series with resistor

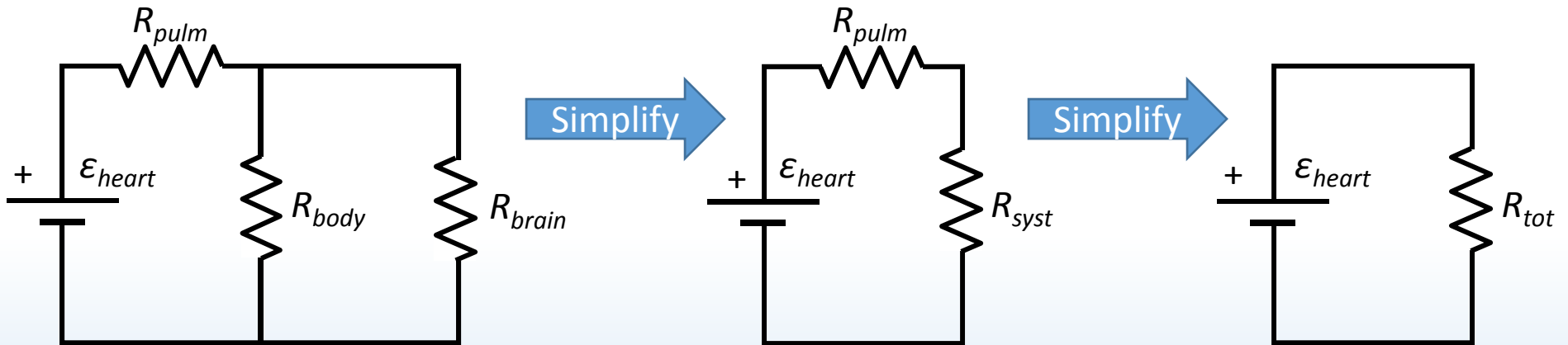
Resistor

Resistor

# Calculation: circulatory system

Calculate current through each component of circulatory system

$$R_{pulm} = 12 \, \Omega, R_{brain} = 1 \, \text{k}\Omega, R_{body} = 160 \, \Omega, \varepsilon_{heart} = 120 \, \text{V}$$



$R_{brain}$  &  $R_{body}$  are in parallel

$$\frac{1}{R_{syst}} = \frac{1}{R_{brain}} + \frac{1}{R_{body}} \quad R_{syst} = 138 \, \Omega$$

$$V_{syst} = V_{brain} = V_{body}$$

$$I_{syst} = I_{brain} + I_{body}$$

$R_{pulm}$  &  $R_{syst}$  are in series

$$R_{tot} = R_{pulm} + R_{syst} = 150 \, \Omega$$

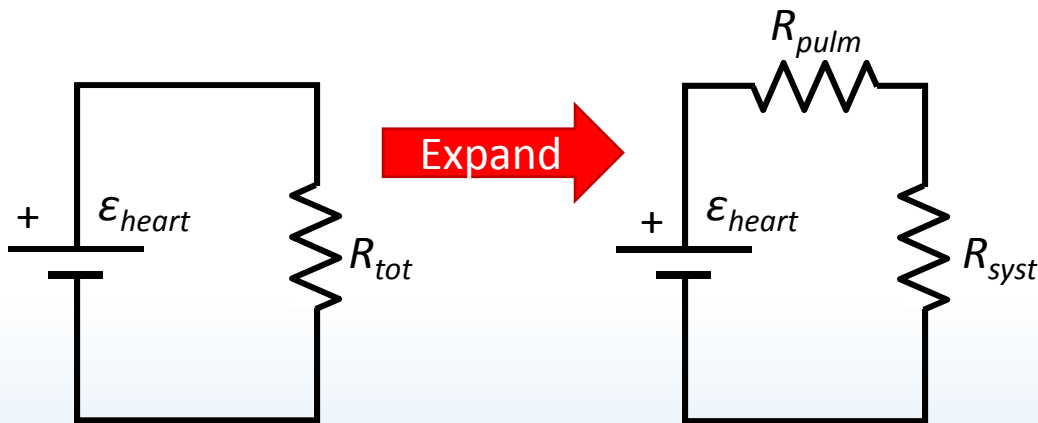
$$V_{tot} = V_{pulm} + V_{syst} = \varepsilon_{heart}$$

$$I_{heart} = I_{pulm} = I_{syst} = 120 \, \text{V} / 150 \, \Omega = 0.8 \, \text{A}$$

# Calculation: circulatory system

Calculate current through each component of circulatory system

$$R_{pulm} = 12 \, \Omega, R_{brain} = 1 \, \text{k}\Omega, R_{body} = 160 \, \Omega, \varepsilon_{heart} = 120 \, \text{V}$$



$R_{pulm}$  &  $R_{syst}$  are in series

$$R_{tot} = R_{pulm} + R_{syst}$$

$$V_{pulm} = I_{pulm} R_{pulm} = 10 \, \text{V}$$

$$V_{tot} = V_{pulm} + V_{syst} = \varepsilon_{heart}$$

$$V_{syst} = I_{syst} R_{syst} = 110 \, \text{V}$$

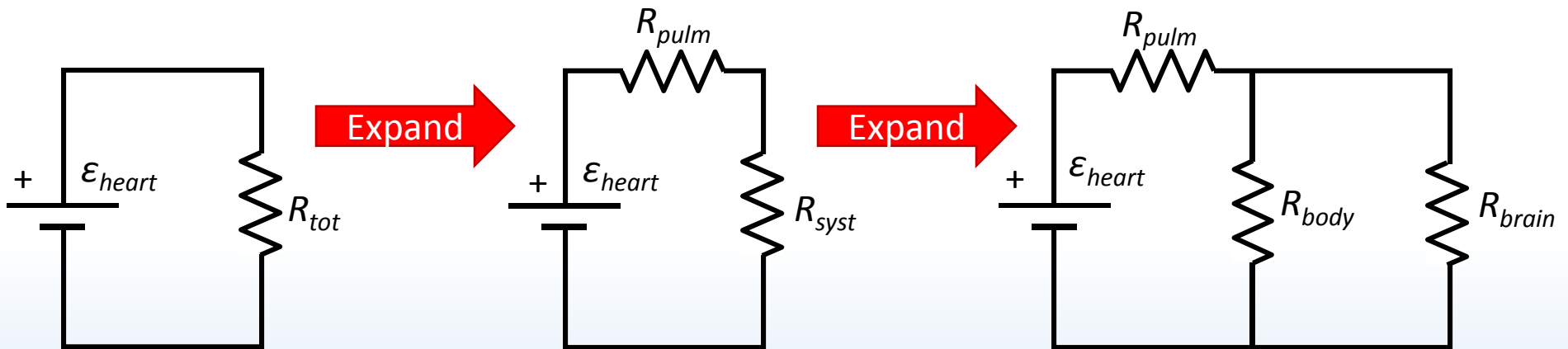
$$120 \, \text{V} = 10 + 110 \, \text{V} \quad \checkmark$$

$$I_{pulm} = I_{syst} = I_{heart} = 0.8 \, \text{A}$$

# Calculation: circulatory system

Calculate current through each component of circulatory system

$$R_{pulm} = 12 \, \Omega, R_{brain} = 1 \, \text{k}\Omega, R_{body} = 160 \, \Omega, \epsilon_{heart} = 120 \, \text{V}$$



$R_{brain}$  &  $R_{body}$  are in parallel

$$\frac{1}{R_{syst}} = \frac{1}{R_{brain}} + \frac{1}{R_{body}}$$

$$V_{syst} = V_{brain} = V_{body}$$

$$I_{syst} = I_{brain} + I_{body}$$

$$0.8 \, \text{A} = 0.11 + 0.69 \, \text{A} \quad \checkmark$$

About 14% of total blood flow goes to the brain

$$I_{brain} = V_{brain}/R_{brain} = 110/1000 = 0.11 \, \text{A}$$

$$I_{body} = V_{body}/R_{body} = 110/160 = 0.69 \, \text{A}$$

# *Summary of today's lecture*

- Two basic principles:
- Kirchhoff loop rule

Voltages around circuit loop sum to zero (based on conservation of energy)

$$\sum \Delta V = 0$$

- Kirchhoff junction rule

Currents into a circuit branch equal currents out (based on conservation of charge)

$$\sum I_{in} = \sum I_{out}$$



# Summary of today's lecture

- Series components



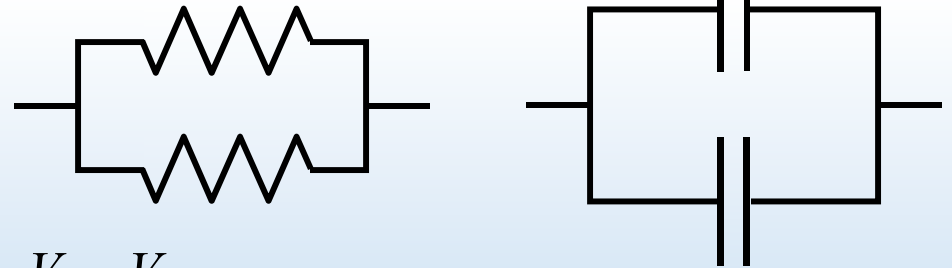
Currents are the same  $I_{eq} = I_1 = I_2$

Voltages add  $V_{eq} = V_1 + V_2$

Resistors  $R_{eq} = R_1 + R_2$

Capacitors  $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

- Parallel components



Voltages are the same  $V_{eq} = V_1 = V_2$

Currents add  $I_{eq} = I_1 + I_2$

Resistors  $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

Capacitors  $C_{eq} = C_1 + C_2$

- Don't mix equations!