

Your questions/comments

“What's on the exam?”

IMPORTANT ANNOUNCEMENTS:

Exam I Thursday (!) on Lect. 1–5 material

Review Tuesday 6-8pm in 1092 Lincoln Hall – will review EX1 SP14 (#1-8) & SP13 (#1-2, 8-12, 17-18) – COME PREPARED having attempted these problems

“The two batteries is really confusing me. Does more current come out of the battery than goes in?”

“Can we go over how to do the current going through a battery? Not sure if I understand the concept of the way it works.”

“I do however have trouble understanding which direction current is supposed to flow conventionally because I always went starting from the positive end of the battery (or power source) but in a few of the hw problems its seems though it is being followed starting on the negative end.”

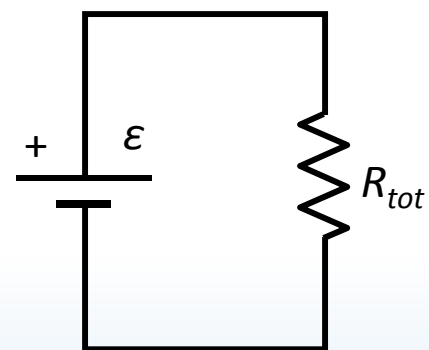
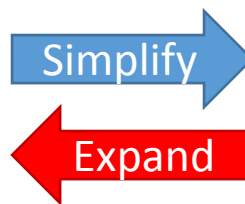
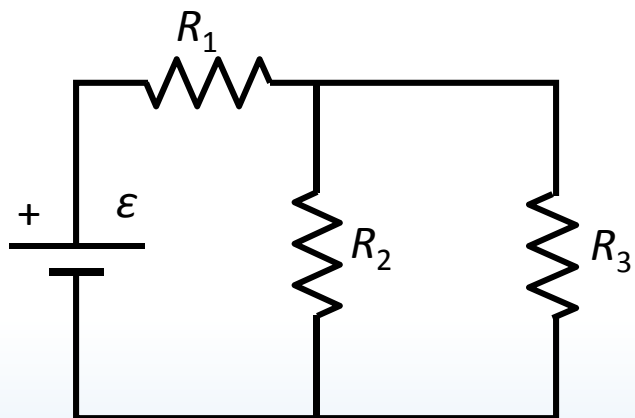


Phys 102 – Lecture 8

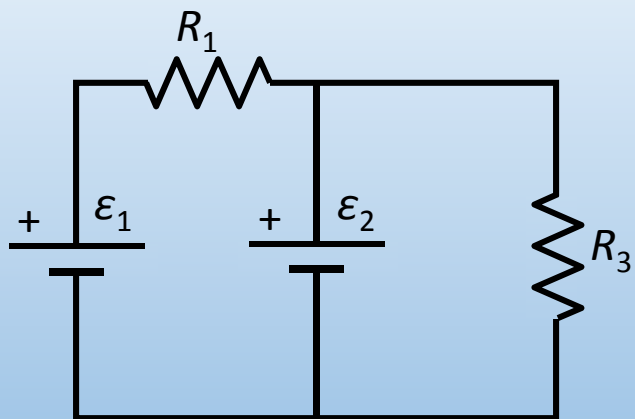
Circuit analysis and Kirchhoff's rules

Recall from last time...

We solved circuits like... by combining series & parallel components



What about a circuit like...

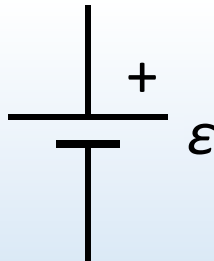


Kirchhoff's loop rule

Voltages around a loop sum to zero

$$\sum \Delta V = 0$$

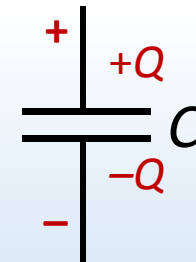
Is voltage positive or negative?



Batteries: + end is always at higher potential



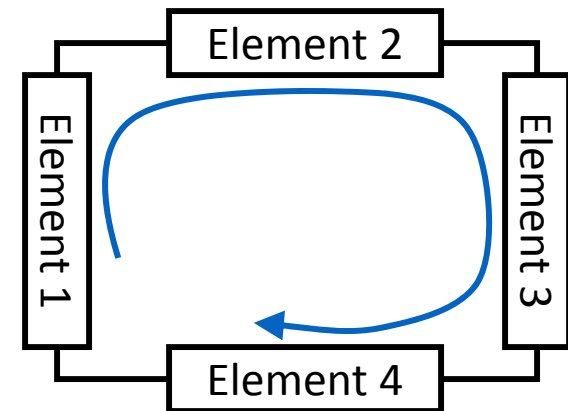
Resistors: higher/lower potential depends on current direction



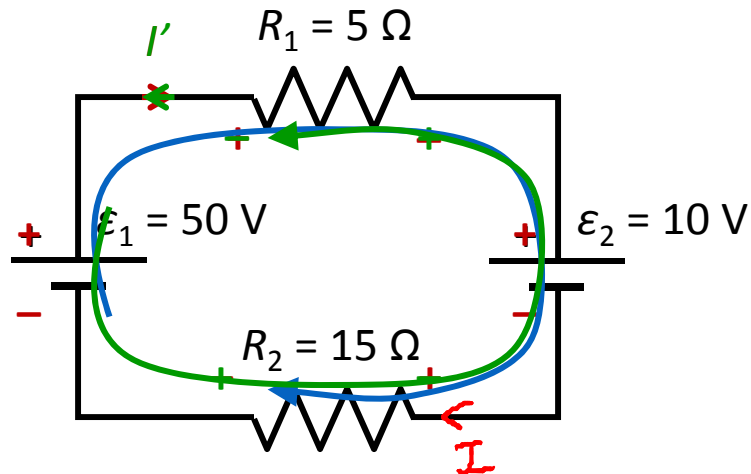
Capacitors: higher/lower potential depends on which plate has +Q/-Q

Label +/– for higher/lower electric potential

Go around loop and write $+V_{\text{element}}$ if electric potential increases $-V_{\text{element}}$ if it decreases



Calculation: single loop practice



Calculate the current I in the circuit

$$+\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{50 - 10}{5 + 15} = 2 \text{ A}$$

What if we go around the loop the “wrong” way?

$$-\varepsilon_1 + IR_1 + \varepsilon_2 + IR_2 = 0$$

Get same equation multiplied by -1 .
There is no “wrong” way.

What if we’re not given the current direction? **Pick one!**

What if we pick the “wrong” direction?

$$\varepsilon_1 + I'R_1 - \varepsilon_2 + I'R_2 = 0$$

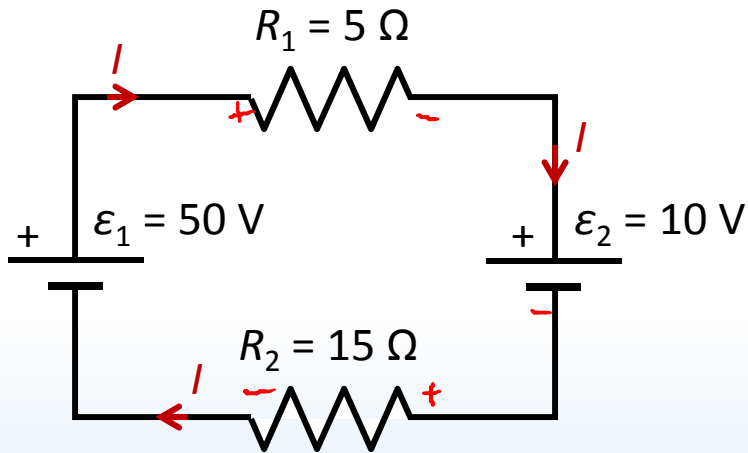
$$I' = \frac{\varepsilon_2 - \varepsilon_1}{R_1 + R_2} = -I$$

Get -2 A , so current goes in opposite direction. There is no “wrong” choice.

Calculation: single loop practice

Battery is a voltage source NOT a current source

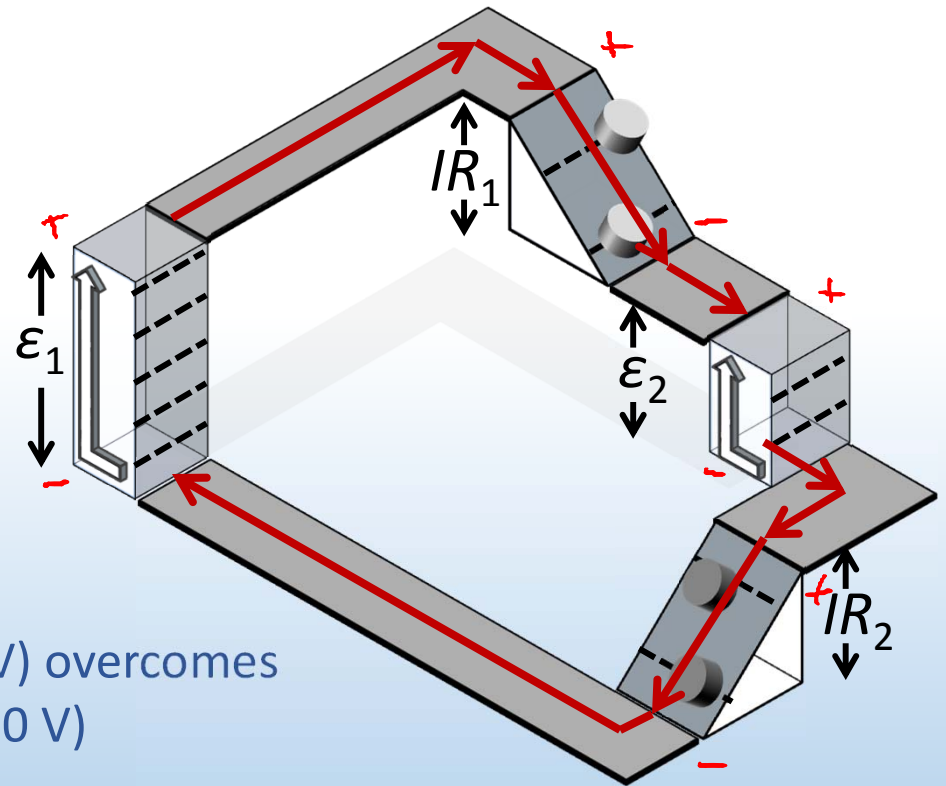
How can the current be driven opposite battery 2?



$$+\epsilon_1 - IR_1 - \epsilon_2 - IR_2 = 0$$

$$I = \frac{\epsilon_1 - \epsilon_2}{R_1 + R_2}$$

Battery 1 emf ($\epsilon_1 = 50 \text{ V}$) overcomes
emf of battery 2 ($\epsilon_2 = 10 \text{ V}$)



You wouldn't design a circuit like this, but this occurs in nature



ACT: Checkpoint 1.1

Calculate the current through R_1 .

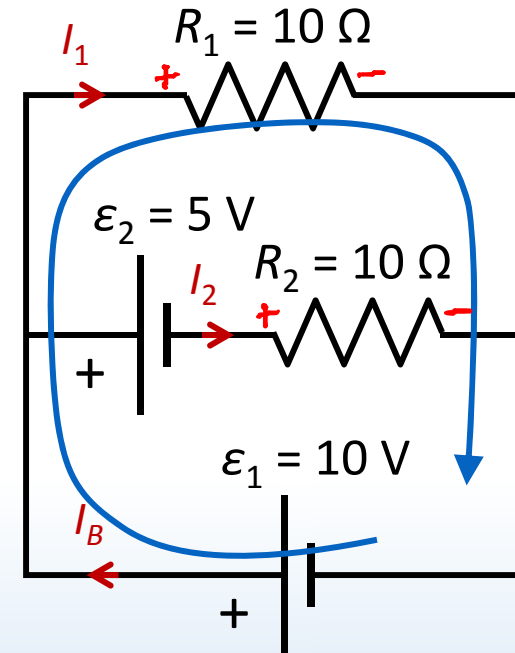
A. $I_1 = 0.5 \text{ A}$ 30%

B. $I_1 = 1.0 \text{ A}$ 43%

C. $I_1 = 1.5 \text{ A}$ 27%

$$+\varepsilon_1 - I_1 R_1 = 0$$

$$I_1 = \frac{\varepsilon_1}{R_1} = \frac{10}{10} = 1 \text{ A}$$

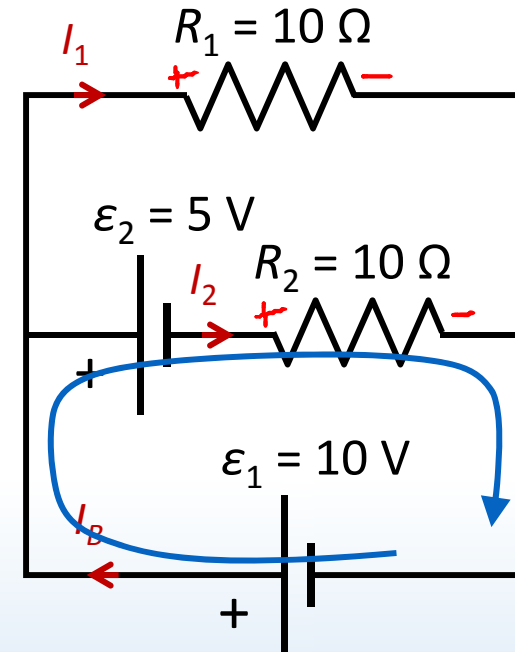




ACT: Checkpoint 1.2

Calculate the current through R_2 .

- A. $I_2 = 0.5 \text{ A}$ 47%
- B. $I_2 = 1.0 \text{ A}$ 27%
- C. $I_2 = 1.5 \text{ A}$ 26%

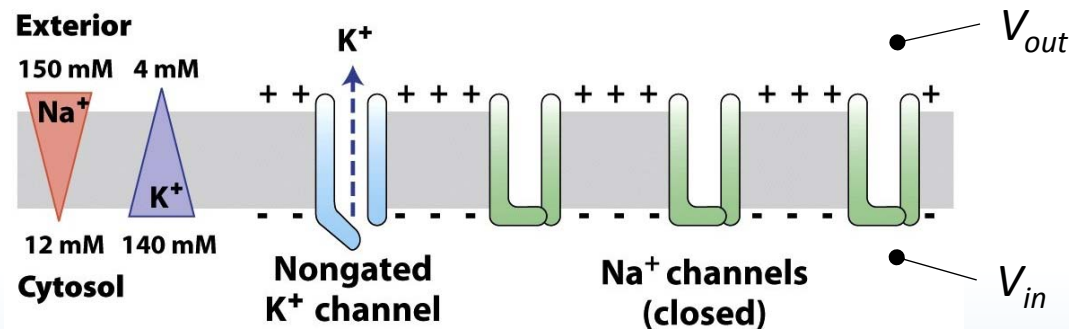


$$+\epsilon_1 - \epsilon_2 - I_2 R_2 = 0$$

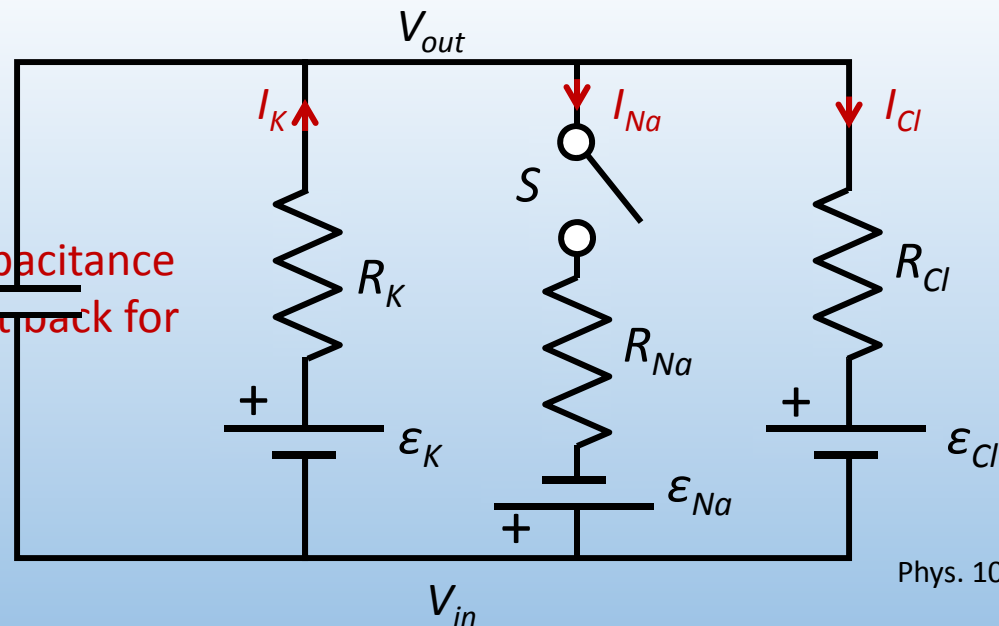
$$I_2 = \frac{\epsilon_1 - \epsilon_2}{R_2} = \frac{10 - 5}{10} = 0.5 \text{ A}$$

Nerve cell equivalent circuit

Neurons have different types of ion channels (K^+ , Na^+ , and Cl^-) that pump current into and out of cell – act like batteries!



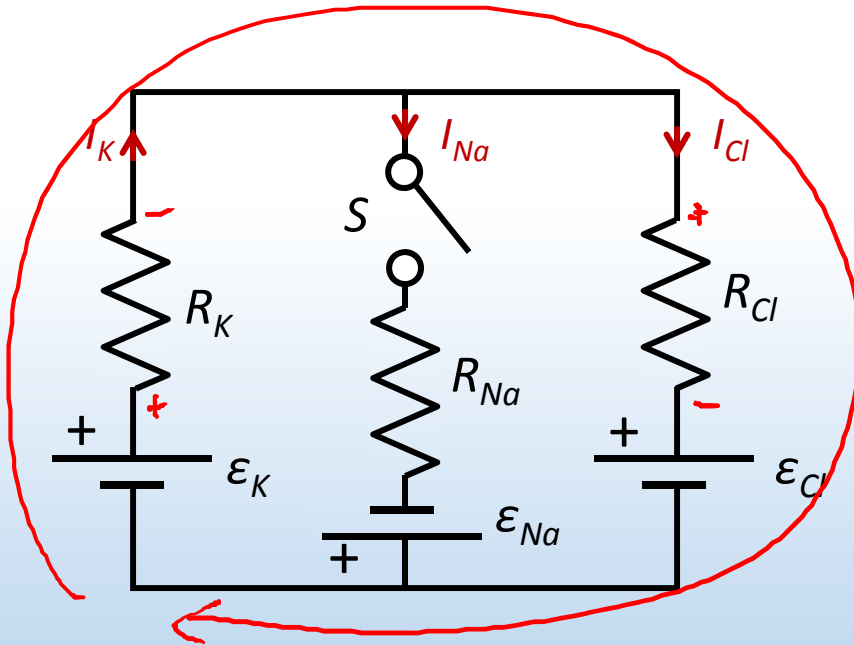
Today we ignore the capacitance of the cell (we will put it back for next lecture!)





ACT: loop

Na^+ channels have a “gate” (represented by the switch S) that allows or blocks ion flow. In its resting state, a Na^+ channel is shut (i.e. switch S is open). Which equation is correct?



A. $+\epsilon_K - I_K R_K - I_K R_{Cl} - \epsilon_{Cl} = 0$

B. $+\epsilon_K - I_K R_K - I_{Na} R_{Na} - \epsilon_{Na} = 0$

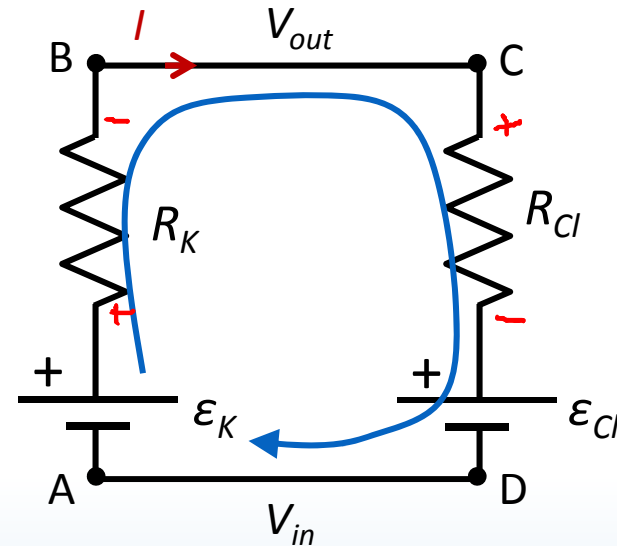
C. $+\epsilon_K + I_K R_K - I_{Cl} R_{Cl} - \epsilon_{Cl} = 0$

The switch is open, so all the current flows through outer loop and $I_K = I_{Cl}$

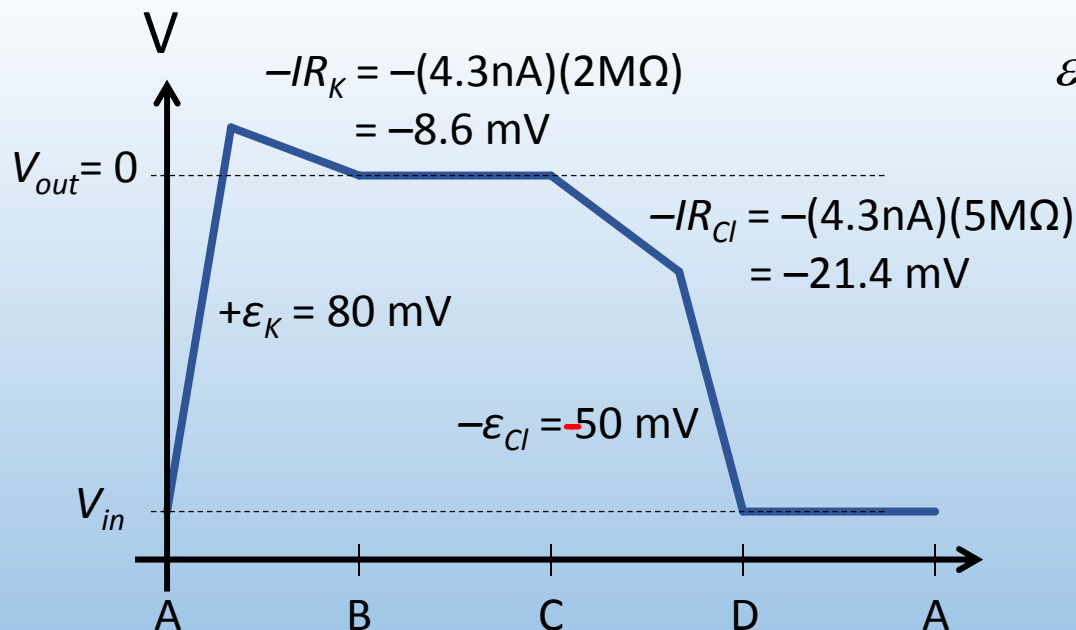
Calculation: electric potential

Find the electric potential difference across the cell $V_{in} - V_{out}$
(Assume $V_{out} = 0$ for reference)

$$\begin{aligned}\varepsilon_K &= 80 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV}, \varepsilon_{Cl} = 50 \text{ mV} \\ R_K &= 2 \text{ M}\Omega, R_{Na} = 0.2 \text{ M}\Omega, R_{Cl} = 5 \text{ M}\Omega\end{aligned}$$



$$\varepsilon_K - IR_K - IR_{Cl} - \varepsilon_{Cl} = 0 \quad I = 4.3 \text{ nA}$$



$$\begin{aligned}V_{in} + \varepsilon_K - IR_K &= V_{out} \\ V_{in} &= -80 + 8.6 + 0 \approx -71 \text{ mV}\end{aligned}$$

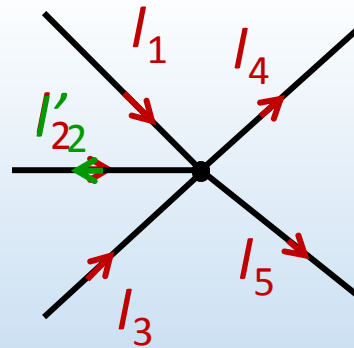
$$\begin{aligned}\text{or: } V_{out} - IR_{Cl} - \varepsilon_{Cl} &= V_{in} \\ 0 - 21.4 - 50 &\approx -71 \text{ mV}\end{aligned}$$

Kirchhoff's junction rule

The sum of currents into a junction equals the sum of currents out of a junction

$$\sum I_{in} = \sum I_{out}$$

Example:



$$I_1 + I_2 \neq I_3 \neq I_4 + I_5$$

Again, there is no “wrong” choice for the current direction. If current is negative, it goes in a direction opposite that shown.



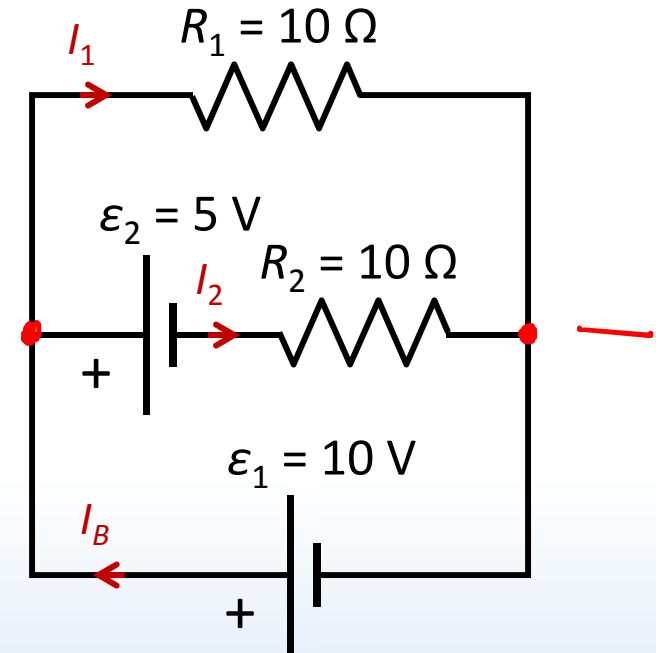
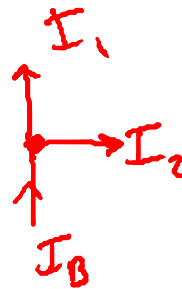
ACT: Checkpoint 1.3

Calculate the current through the battery I_B .

A. $I_B = 0.5 \text{ A}$ 19%

B. $I_B = 1.0 \text{ A}$ 47%

C. $I_B = 1.5 \text{ A}$ 34%

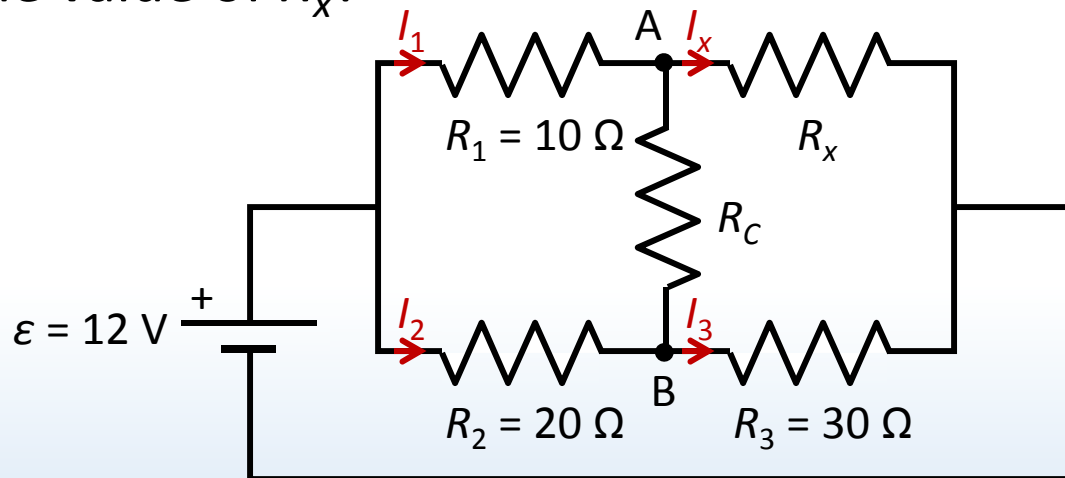


$$\begin{aligned} I_B &= I_1 + I_2 \\ &= 1 + 0.5 = 1.5 \text{ A} \end{aligned}$$

Calculation: Kirchhoff's laws

In the circuit, the current through R_C is 0. What is the current through R_3 and the value of R_x ?

From EX1 FA13



No current flows through R_C so $I_2 = I_3$ and $I_1 = I_x$

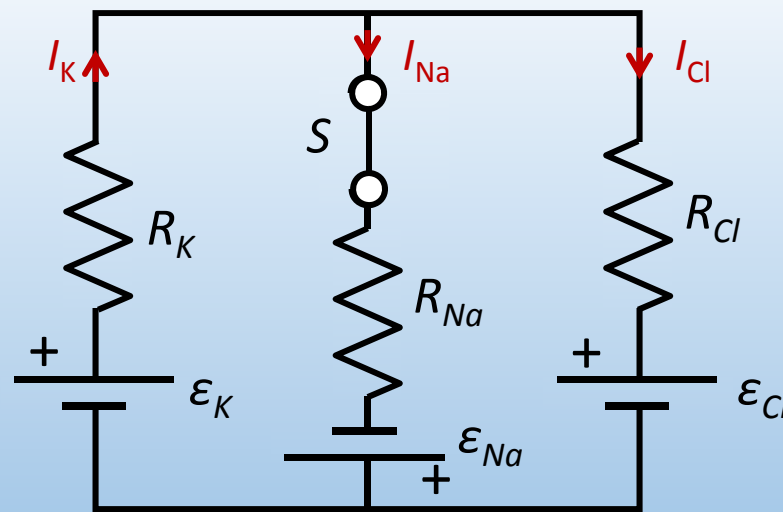
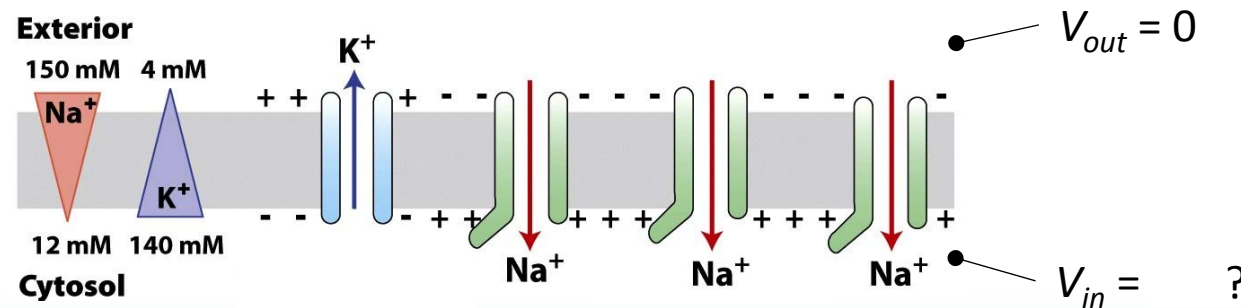
$$\varepsilon - I_3 R_2 - I_3 R_3 = 0 \quad I_3 = I_2 = \frac{\varepsilon}{R_2 + R_3} = \frac{12}{20 + 30} = 0.24 \text{ A} \quad \varepsilon - I_1 R_1 - I_1 R_x = 0$$

No current flows through R_C so $V_A = V_B$

$$I_1 R_1 = I_2 \frac{R_2}{R_1} = 0.24 \frac{20}{10} = 0.48 \text{ A} \quad R_x = \frac{\varepsilon}{I_1} - R_1 = \frac{12}{0.48} - 10 = 15 \Omega$$

Nerve cell equivalent circuit

During nerve impulse, Na^+ channels open (i.e. switch S closes) and allow Na^+ to enter the cell

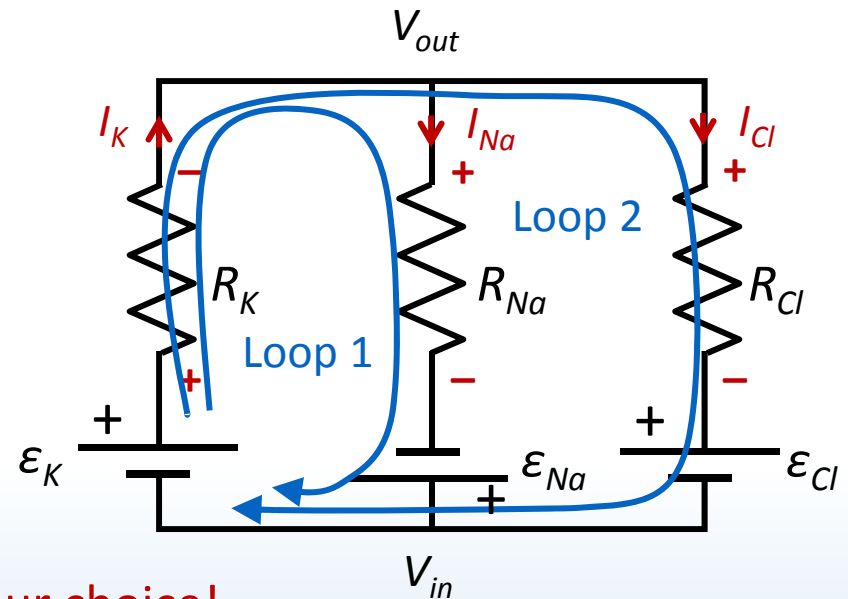


What happens to the currents through the channels and the potential in the cell?

Calculation: two loop circuit

Given the circuit to the right,
find I_K , I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

$$\begin{aligned}\varepsilon_K &= 80 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV}, \varepsilon_{Cl} = 50 \text{ mV} \\ R_K &= 2 \text{ M}\Omega, R_{Na} = 0.2 \text{ M}\Omega, R_{Cl} = 5 \text{ M}\Omega\end{aligned}$$



- ✓ 1. Label all currents
 - ✓ 2. Label +/– for all elements
 - ✓ 3. Choose loop and direction
 - ✓ 4. Write down voltage differences
- Your choice!
Current flows from + to – for resistors
Choose any direction
Potential increases or decreases?

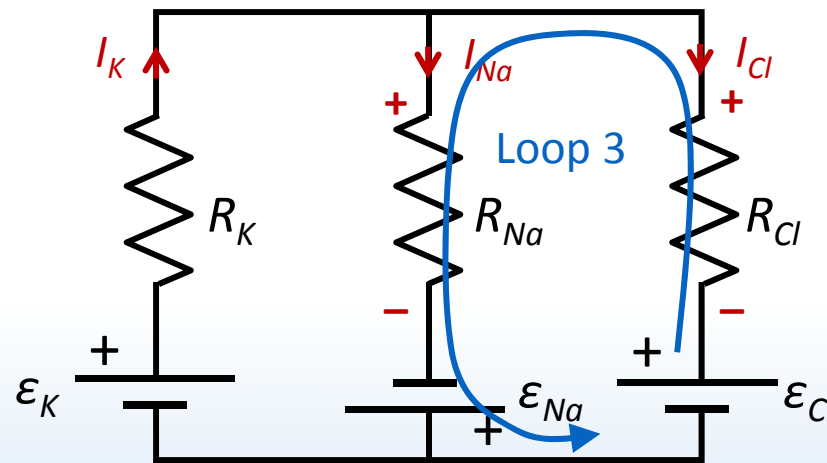
$$\text{Loop 1: } +\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$$

$$\text{Loop 2: } +\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$$



ACT: Kirchhoff loop rule

What is the correct expression for “Loop 3” in the circuit below?



A. $+\epsilon_{Cl} - I_{Cl}R_{Cl} - I_{Na}R_{Na} + \epsilon_{Na} = 0$

B. $+\epsilon_{Cl} - I_{Cl}R_{Cl} + I_{Na}R_{Na} + \epsilon_{Na} = 0$

C. $+\epsilon_{Cl} + I_{Cl}R_{Cl} - I_{Na}R_{Na} + \epsilon_{Na} = 0$

Calculation: two loop circuit

Given the circuit to the right,
find I_K , I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

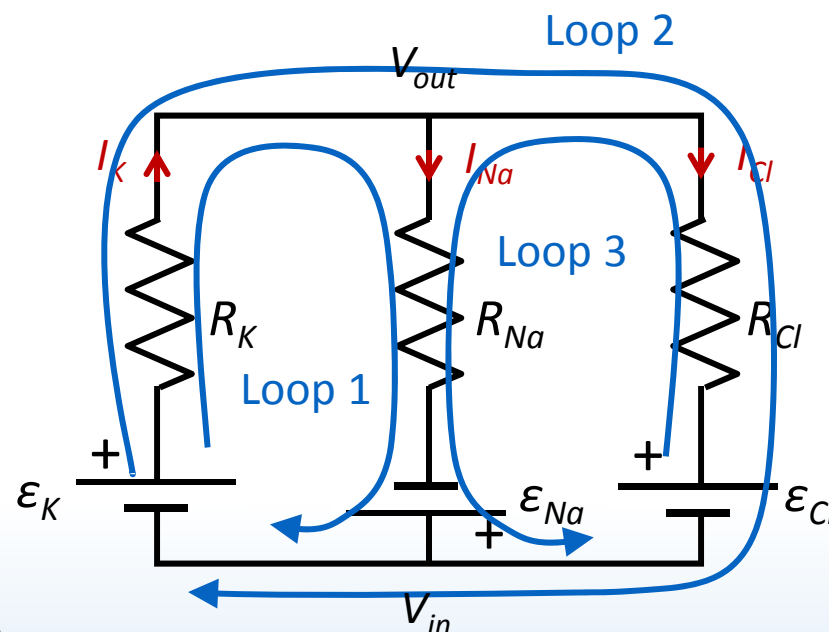
$$\begin{aligned}\varepsilon_K &= 80 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV}, \varepsilon_{Cl} = 50 \text{ mV} \\ R_K &= 2 \text{ M}\Omega, R_{Na} = 0.2 \text{ M}\Omega, R_{Cl} = 5 \text{ M}\Omega\end{aligned}$$

We have 3 unknowns, need 3 equations

$$\text{Loop 1: } +\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$$

$$\text{Loop 2: } +\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$$

$$\text{Loop 3: } +\varepsilon_{Cl} + I_{Cl} R_{Cl} - I_{Na} R_{Na} + \varepsilon_{Na} = 0$$



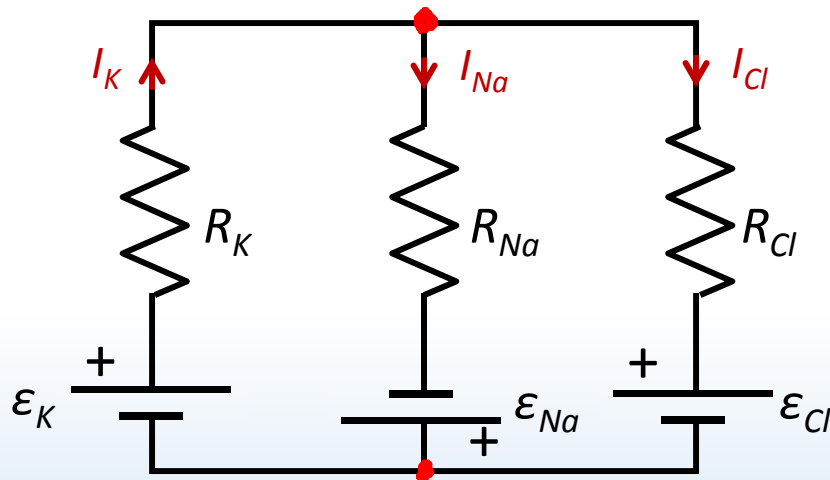
Careful! These 3 equations are not independent. We need 3 independent equations.

5. Write down junction rule



ACT: Kirchhoff junction rule

What is the correct expression for junction in the circuit?



A. $I_K + I_{Na} = I_{Cl}$

B. $I_{Na} + I_{Cl} = I_K$

C. $I_{Cl} + I_K = I_{Na}$

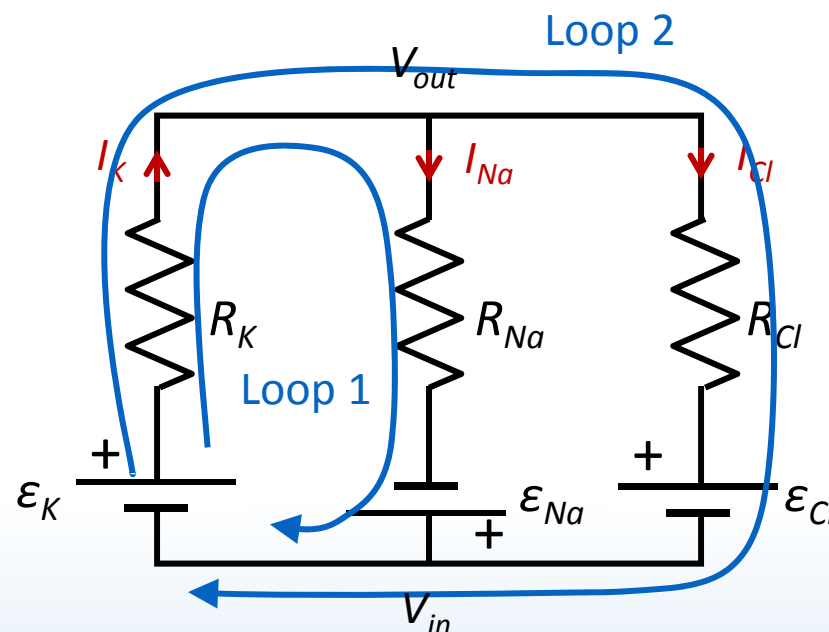
- ✓ 1. Label all currents
- ✓ 2. Label +/– for all elements
- ✓ 3. Choose loop and direction
- ✓ 4. Write down voltage differences
- ✓ 5. Write down junction rule

Calculation: two loop circuit

Given the circuit to the right,
find I_K , I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

$$\begin{aligned}\epsilon_K &= 80 \text{ mV}, \epsilon_{Na} = 60 \text{ mV}, \epsilon_{Cl} = 50 \text{ mV} \\ R_K &= 2 \text{ M}\Omega, R_{Na} = 0.2 \text{ M}\Omega, R_{Cl} = 5 \text{ M}\Omega\end{aligned}$$

3 equations, 3 unknowns,
the rest is algebra!



$$(1) \quad +\epsilon_K - I_K R_K - I_{Na} R_{Na} + \epsilon_{Na} = 0 \quad +80 - 2I_K - 0.2I_{Na} + 60 = 0$$

$$(2) \quad +\epsilon_K - I_K R_K - I_{Cl} R_{Cl} - \epsilon_{Cl} = 0 \quad +80 - 2I_K - 5I_{Cl} - 50 = 0$$

$$(3) \quad I_{Na} + I_{Cl} = I_K \quad +80 - 2I_K - 5(I_K - I_{Na}) - 50 = 0$$

Substitute Eq. (3) into Eq. (2) and rearrange

$$(2') \quad +30 - 7I_K + 5I_{Na} = 0$$

Calculation: two loop circuits

Now 2 equations (1 and 2'), 2 unknowns (I_K and I_{Na})

$$\begin{aligned} (1) \quad +70 - I_K - 0.1I_{Na} &= 0 & I_K &= 70 - 0.1I_{Na} \\ (2') \quad +30 - 7I_K + 5I_{Na} &= 0 & +30 - 7(70 - 0.1I_{Na}) + 5I_{Na} &= 0 \end{aligned}$$

Substitute I_K in Eq. (1) into Eq. (2') and rearrange

$$-460 + 5.7I_{Na} = 0 \quad I_{Na} = \frac{460 \text{ mV}}{5.7 \text{ M}\Omega} = 81 \text{ nA}$$

Plug solution into Eq. (2') to get I_K

$$+30 - 7I_K + 5 \cdot 81 = 0 \quad I_K = \frac{435 \text{ mV}}{7 \text{ M}\Omega} = 62 \text{ nA}$$

Use junction Eq. (3) to get I_{Cl}

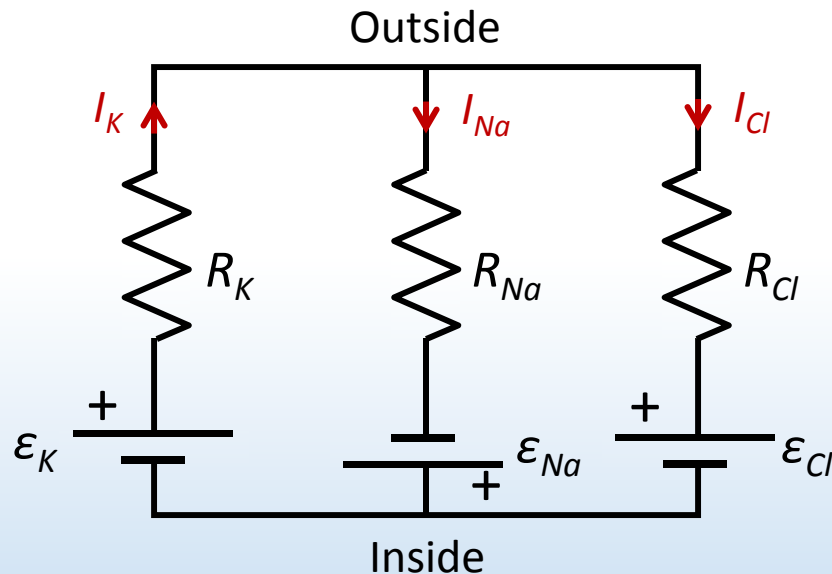
$$I_{Cl} = 62 - 81 = -19 \text{ nA}$$

ALMOST DONE!



ACT: Kirchhoff junction rule

We found that $I_K = 62 \text{ nA}$, $I_{Na} = 81 \text{ nA}$ and $I_{Cl} = -19 \text{ nA}$. Which of the following statements is FALSE?



A. I_K is out of the cell

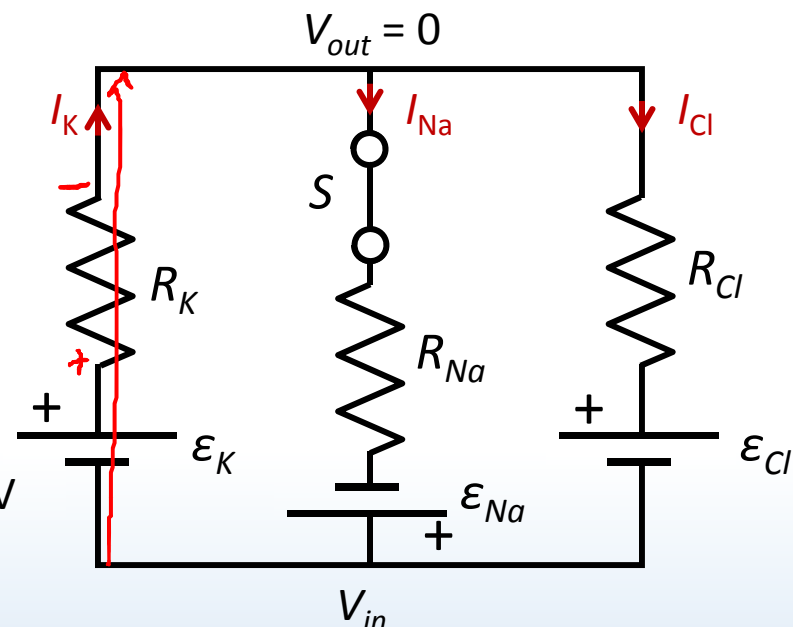
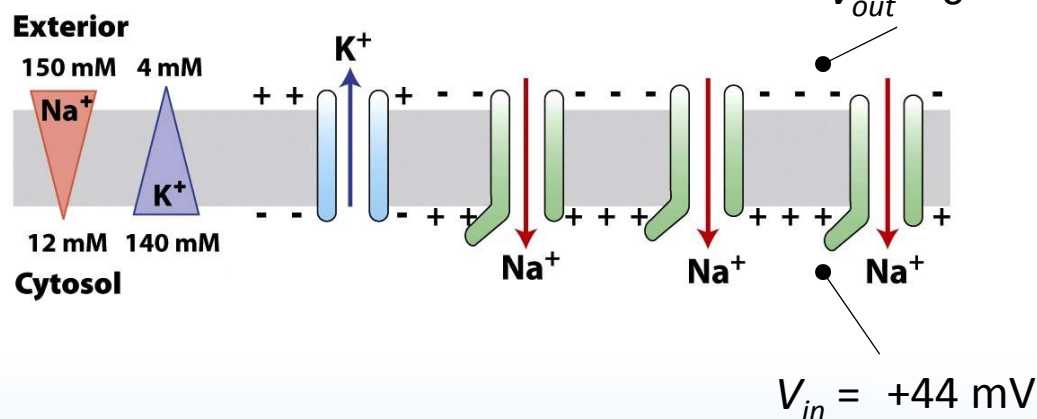
B. I_{Na} is into the cell

C. I_{Cl} is into the cell

I_{Cl} is negative, so current flows in direction opposite that drawn.
Note that Cl^- is an anion, so Cl^- ions actually flow into the cell.

Calculation: two loop circuit

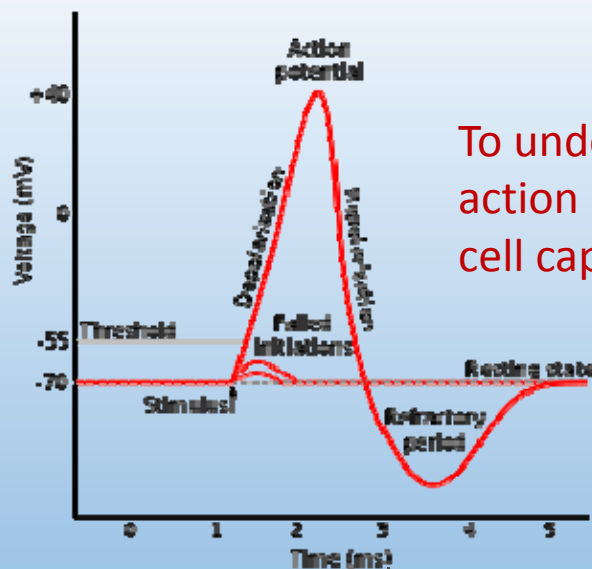
Find the new $V_{in} - V_{out}$:



$$V_{in} + \epsilon_K - I_K R_K = V_{out}$$

$$V_{in} + 80 - 62 \cdot 2 = 0$$

$$V_{in} = +44 \text{ mV}$$



To understand the time scales for the action potential, we'll need to put the cell capacitance back in. Next lecture!

Summary of today's lecture

- Two basic principles:
- Kirchhoff loop rule

Voltages around circuit loop sum to zero (based on conservation of energy)

$$\sum \Delta V = 0$$

- Kirchhoff junction rule

Currents into a circuit branch equal currents out (based on conservation of charge)

$$\sum I_{in} = \sum I_{out}$$

Summary of today's lecture

- Basic approach to solving complex circuits:
 1. Label all currents
 2. Label $+/-$ for all elements
 3. Choose loop(s) and direction(s)
 4. Write down voltage differences
 5. Write down junction ruleThe rest is algebra!