Your questions/comments

IMPORTANT ANNOUNCEMENTS:

Homework solutions online in byteshelf

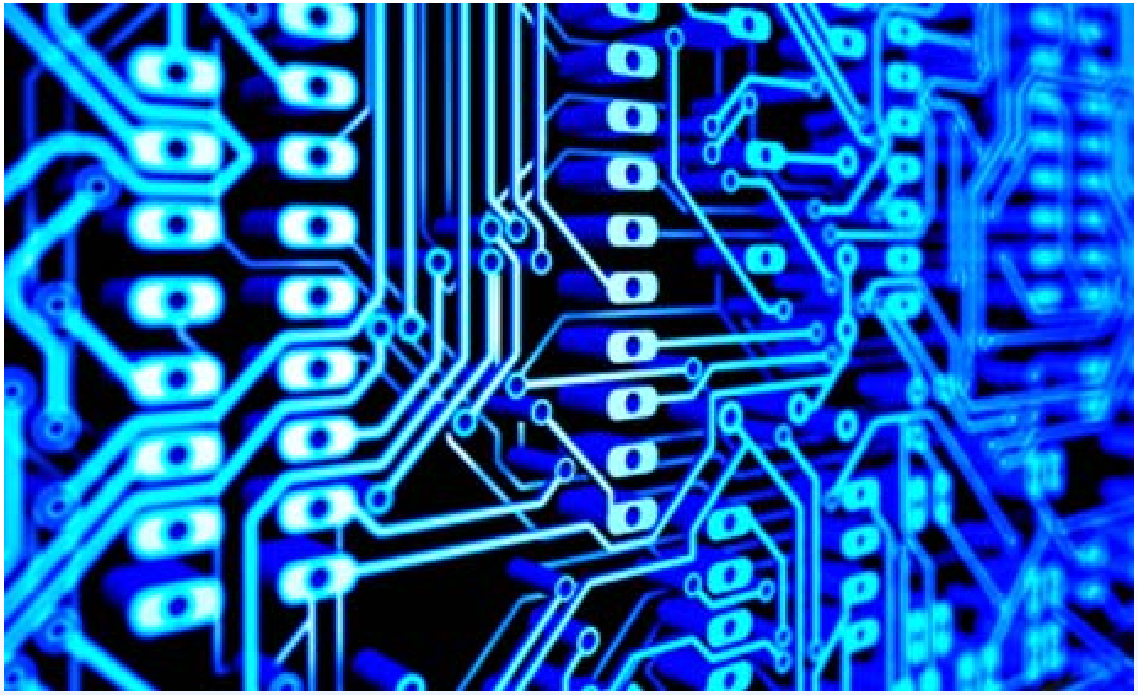
Review session Tuesday 6-8pm in 1092 Lincoln Hall – will review EX1 SP14 & SP13

Solutions to EX1 FA13 online in byteshelf

“Can you explain what the phrase "voltage across a resistor" means? And if this is the change in voltage, should it be written as a negative value?”

“More confused than before. When do currents add. Why would currents across two unlike resistors be the same: they have different resistances. What about the voltage in between, not across, the resistors.”

“There are a lot of relationships between voltage, current, and resistance that change with the addition of more elements to a circuit, as well as the way they are arranged in the circuit (series vs parallel). I would appreciate it if these relationships could be once again highlighted and clarified.”



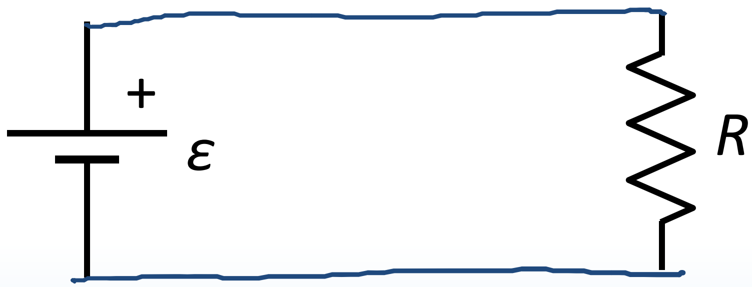
Phys 102 – Lecture 7

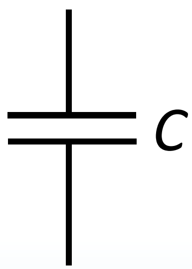
Series and parallel circuits

Recall from last time...

Electric potential difference across circuit element is its "voltage"

$V_{element}$ Should be "ΔV", but we'll usually drop the "Δ"





Batteries – pump charges
Provide emf for charges

$$V_{battery} = \varepsilon$$

Resistors – regulate current
Dissipate power

$$V_R = IR$$

Capacitors – store charge
Store energy

$$V_C = \frac{Q}{C}$$

Wires are ideal conductors, with no resistance. *E* = 0, the electric potential is constant and the voltage across a wire is 0.

Today we will...

- Learn about *electric circuits*

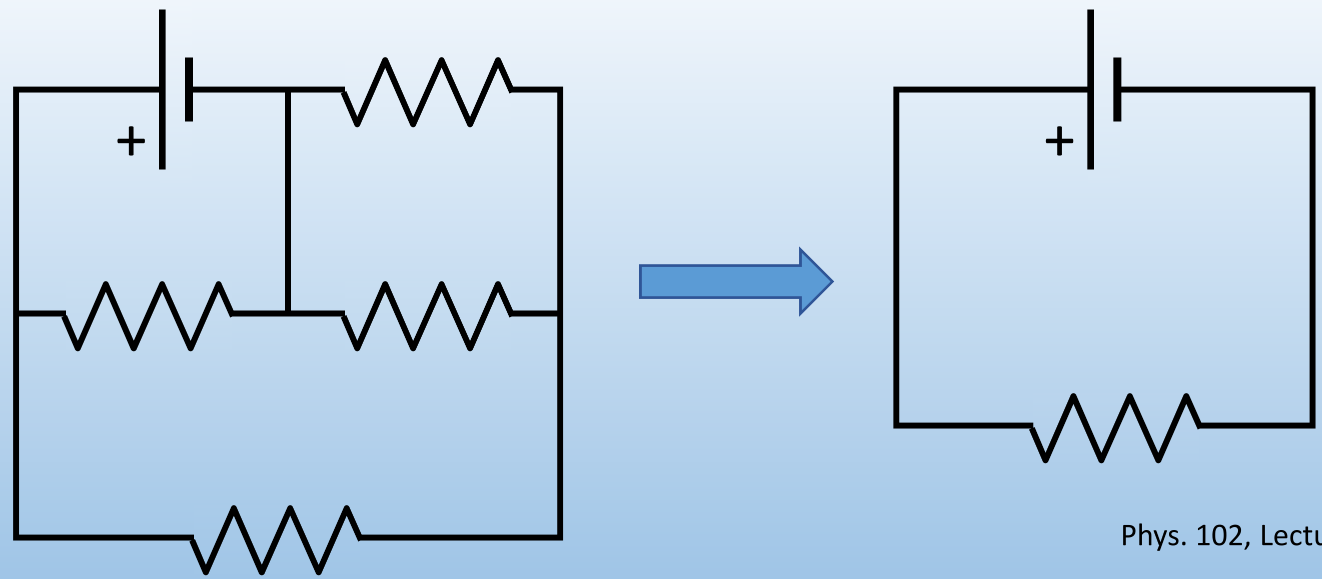
  Circuits with a battery, wires, and resistors

  Circuits with a battery, wires, and capacitors

- Analyze circuits

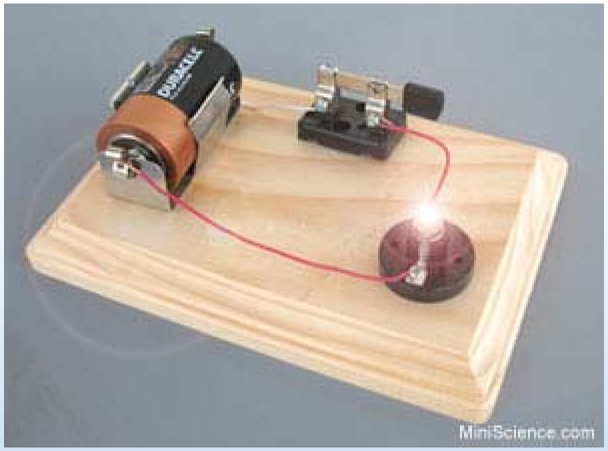
  Take a complex-looking circuit like...

  ...and turn into a simple-looking circuit like

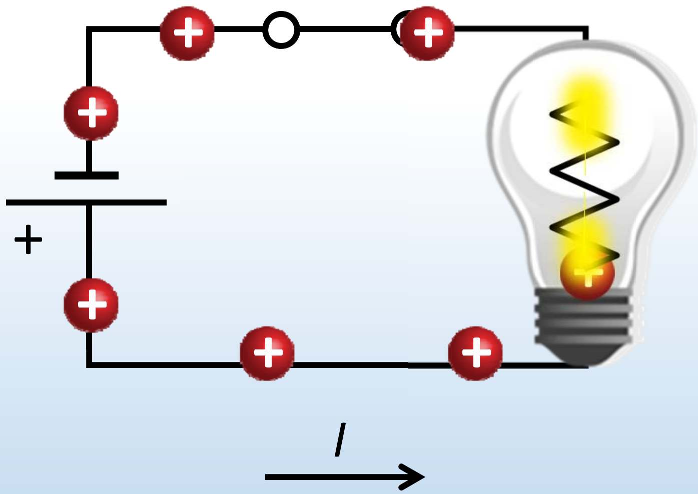


Electric circuits

Electric circuits consist of one or more closed loops around which charges flow

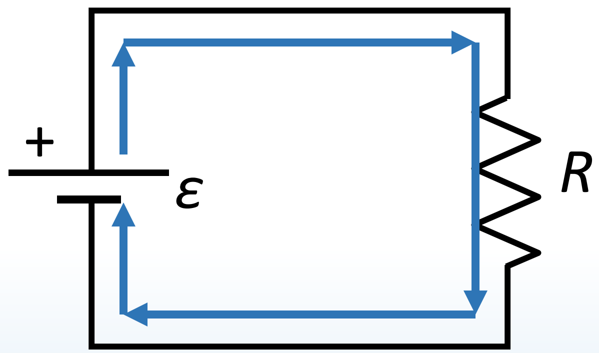


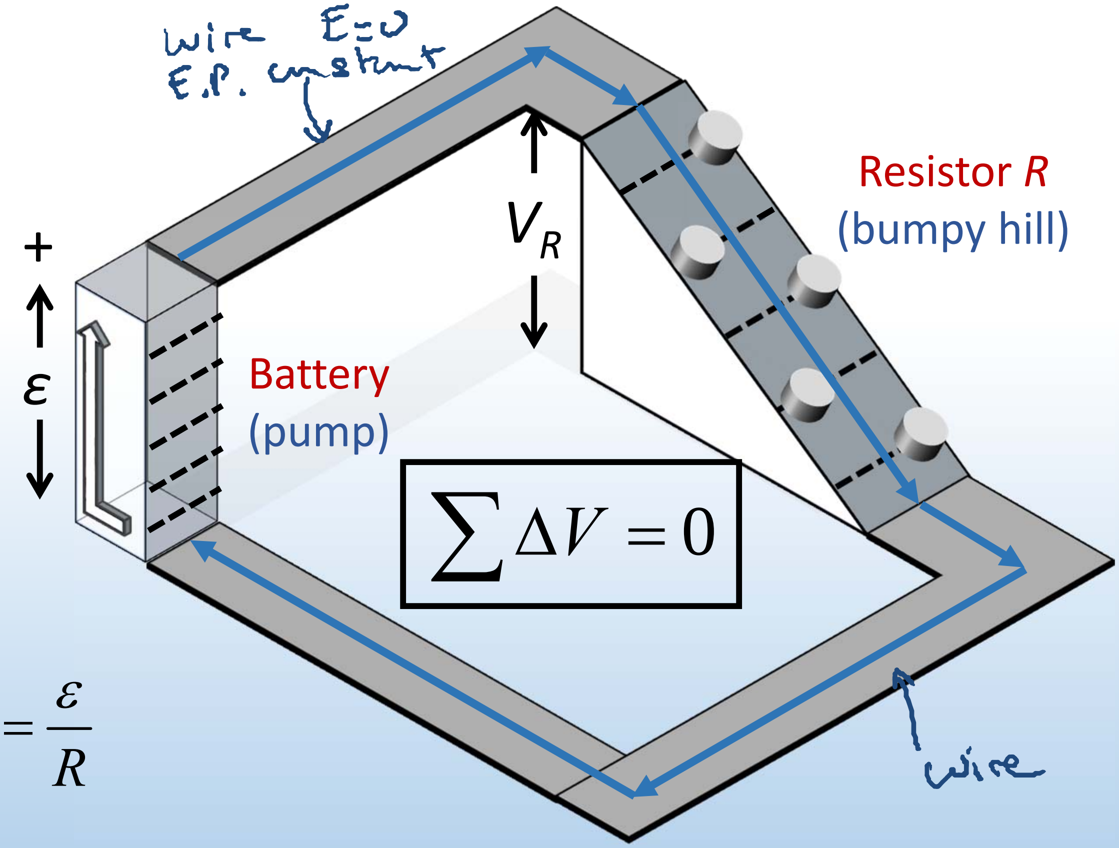
Closed circuit



Kirchhoff loop rule

A charge making a complete loop around a circuit must return to the same electric potential ("height") at which it started





Sum of electric potential differences (voltages) around circuit loop is zero

$$\varepsilon - IR = 0$$

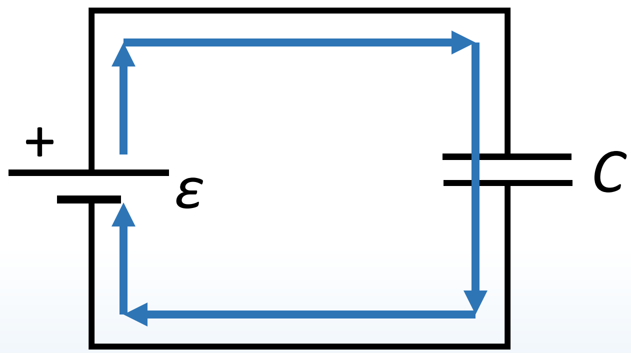
So, $I = \dfrac{\varepsilon}{R}$

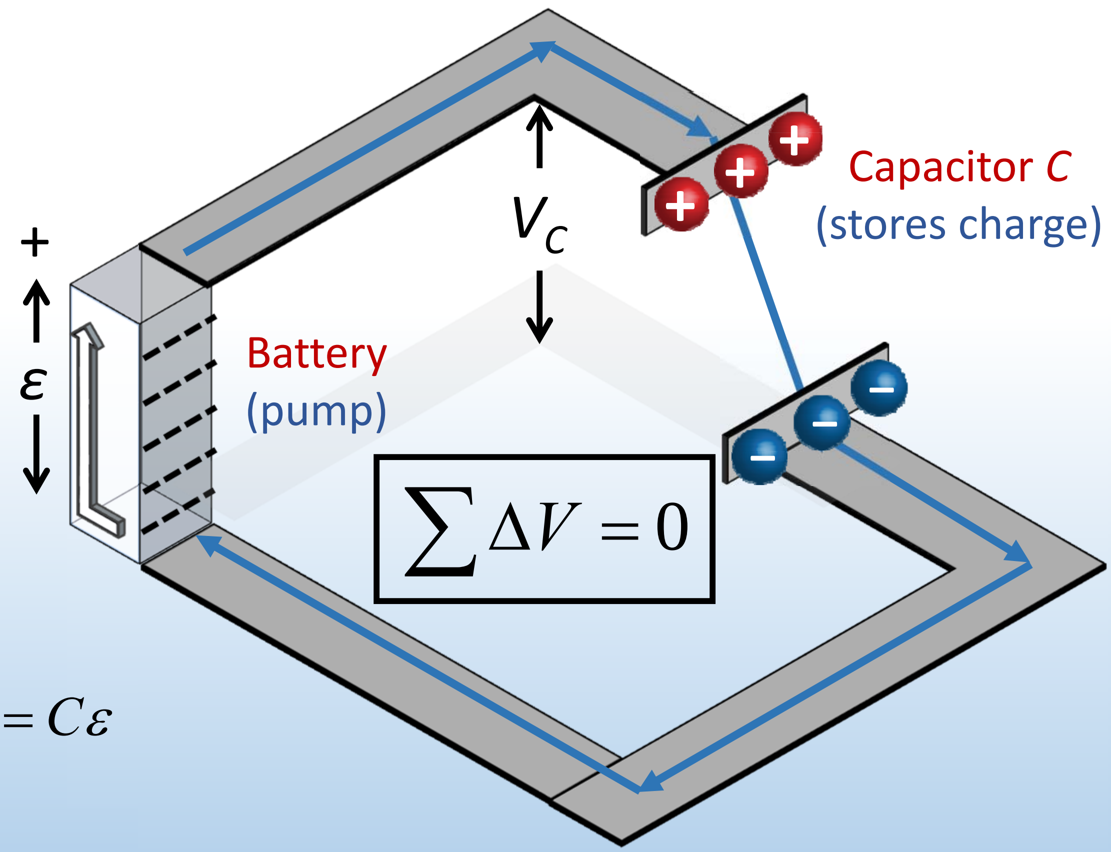
Potential *increase* across battery

Potential *drop* across resistor

Kirchhoff loop rule

A charge making a complete loop around a circuit must return to the same electric potential (“height”) at which it started





Sum of electric potential differences (voltages) around circuit loop is zero

$$\varepsilon - \frac{Q}{C} = 0$$

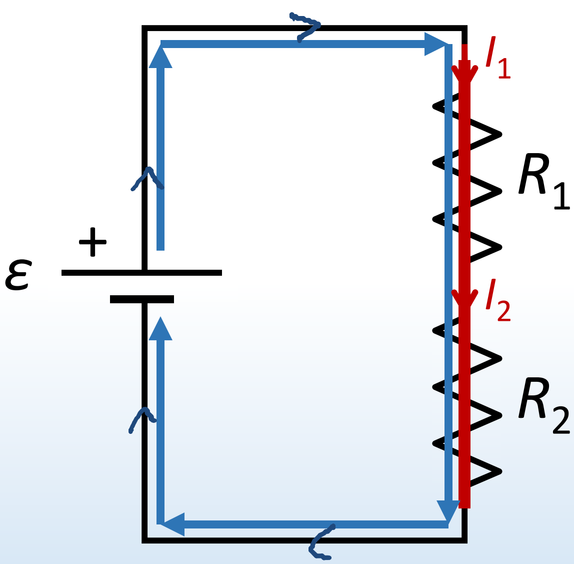
So, $Q = C\varepsilon$

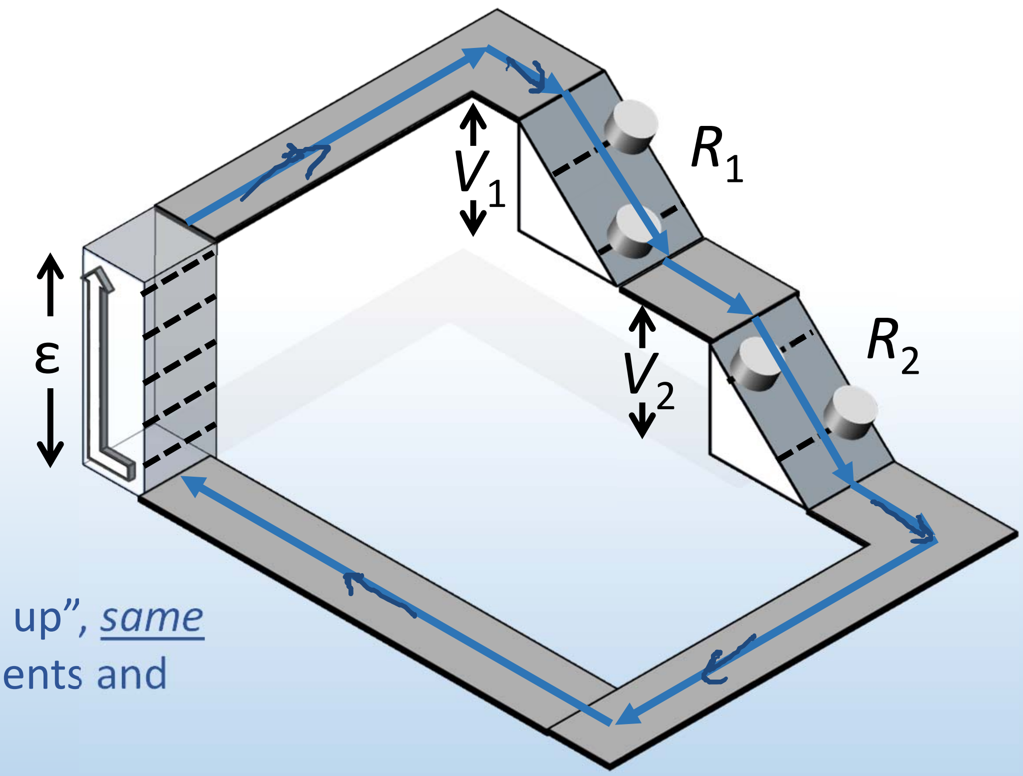
Potential *increase* across battery

Potential *drop* across capacitor

Series components

Two components are said to be *in series* when they are connected end-to-end by a *single* wire





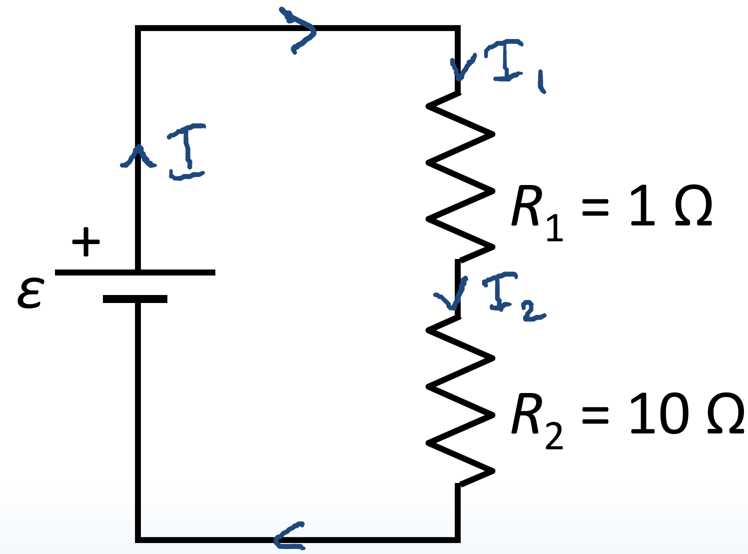
Recall that charges do NOT get "used up", same current I flows through both components and around circuit

$$I_1 = I_2 = I = \frac{\varepsilon}{R_1 + R_2}$$ ***Checkpoint 1.1*** 70%

From loop rule: $\varepsilon - (V_1 + V_2) = 0$

ACT: CheckPoint 1.2

Consider a circuit with two resistors R_1 and R_2 in series. Compare the voltages across the resistors:



A. $V_1 > V_2$ 28%

B. $V_1 = V_2$ 25%

C. $V_1 < V_2$ 47%

Resistors are in series so $I_1 = I_2 = I$ (***CheckPoint 1.1***)

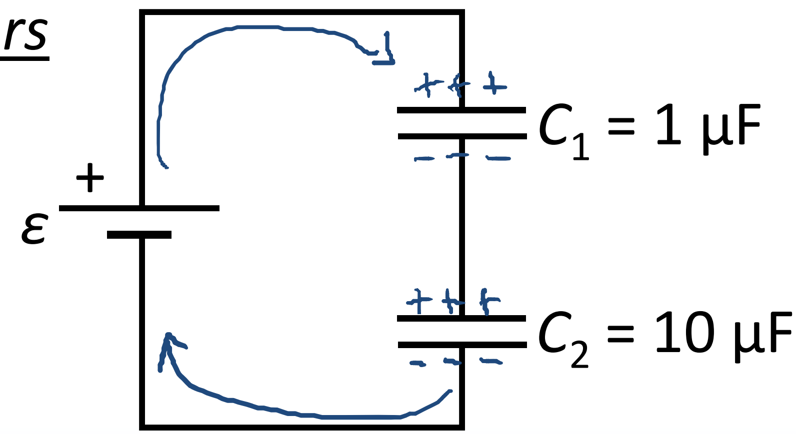
$V_1 = I_1 R_1 = I \times 1\ \Omega$

$V_2 = I_2 R_2 = I \times 10\ \Omega$

ACT: Capacitors in series

Consider a circuit with two *capacitors* C_1 and C_2 in series. Compare the voltages across the capacitors:

$$V_C = \frac{Q}{C}$$



A. $V_1 > V_2$

B. $V_1 = V_2$

C. $V_1 < V_2$

Apply the same reasoning as for a circuit with resistors in series

Capacitors are in series so they have the same charge $Q_1 = Q_2 = Q$

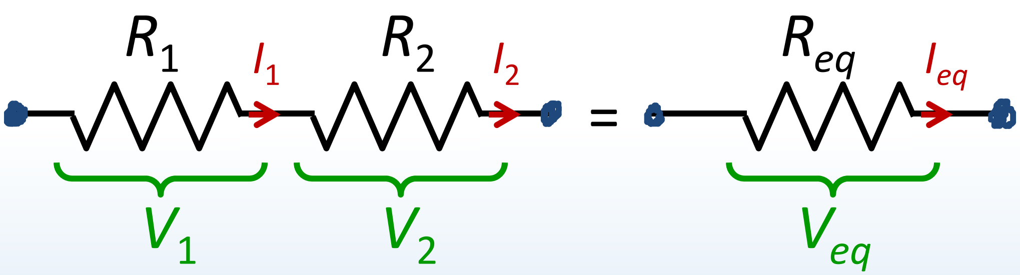
$V_1 = Q_1 / C_1 = Q / 1\ \mu\text{F}$

$V_2 = Q_2 / C_2 = Q / 10\ \mu\text{F}$

Equivalent resistance & capacitance

Circuit behaves the same as if *series* components were replaced by a *single, equivalent* component

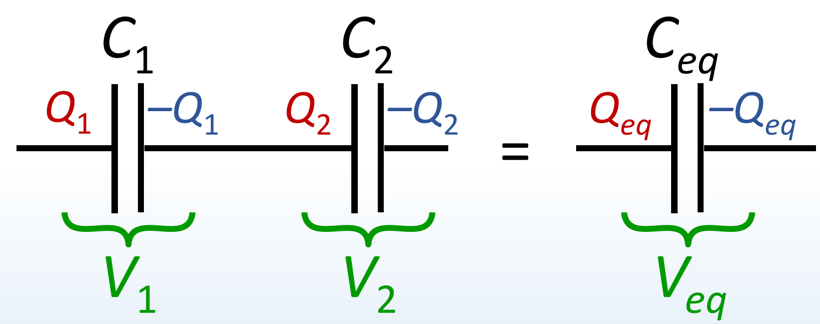
Resistors $V_R = IR$



$$I_1 = I_2 = I_{eq} \qquad V_1 + V_2 = V_{eq}$$

$$\boxed{I_1 R_1 + I_2 R_2 = I_{eq} R_{eq} \quad I_{eq}(R_1 + R_2) = I_{eq} R_{eq} \quad R_{eq} = R_1 + R_2}$$

Capacitors $V_c = Q/C$



$$Q_{eq} = Q_1 = Q_2 \qquad V_{eq} = V_1 + V_2$$

$$\boxed{\frac{Q_{eq}}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \quad \frac{Q_{eq}}{C_{eq}} = Q_{eq}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

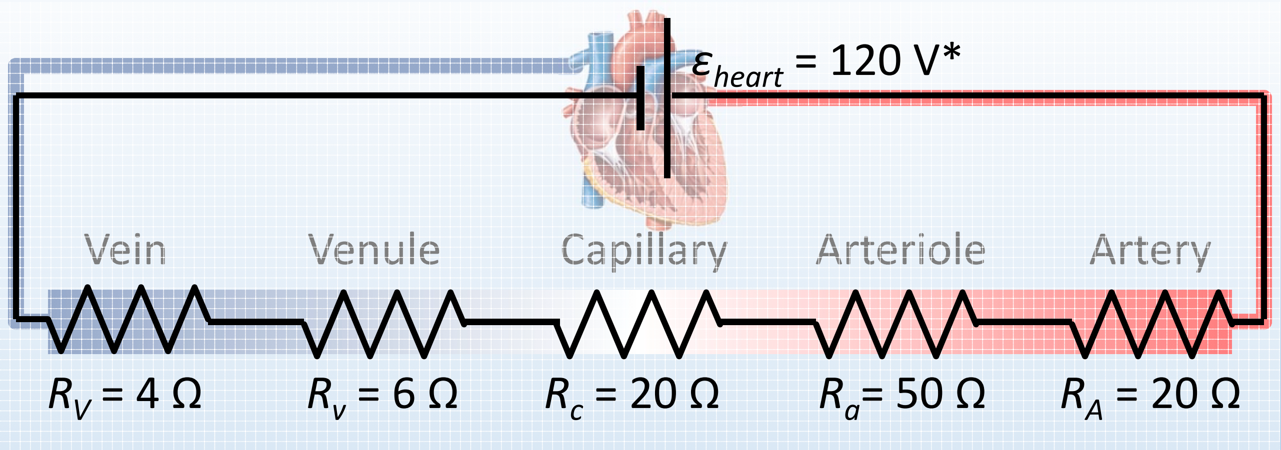
Note same reasoning for resistors and capacitors but final expression is different because $V_R \propto R$ but $V_C \propto 1/C$

Calculation: vascular resistance

The circulatory system is analogous to an electric circuit

Pressure difference	⟷	emf
Blood flow		Electric current
Vascular resistance		Electrical resistance

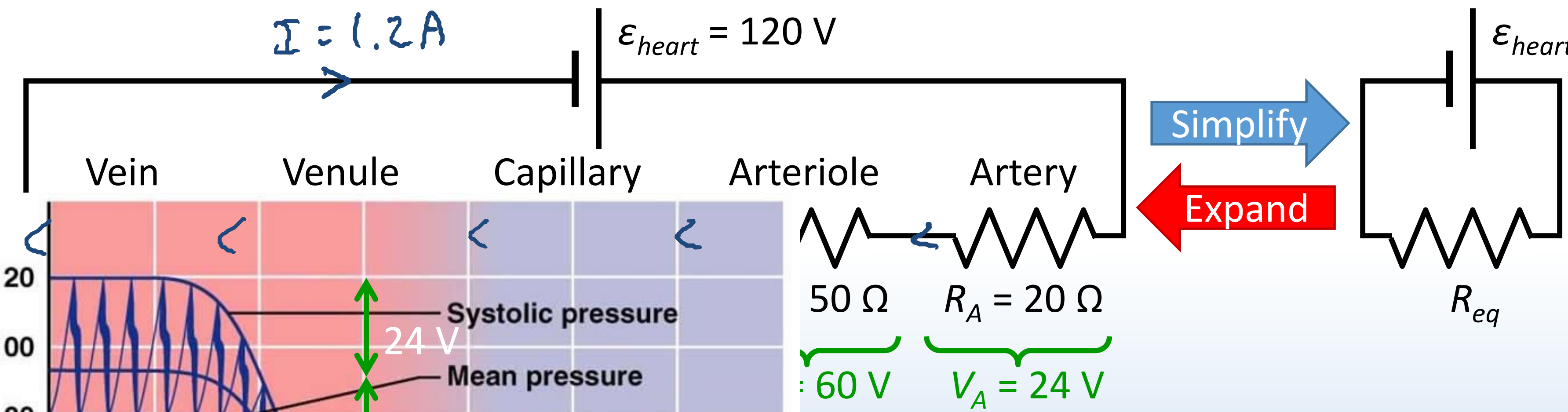
The circulatory system consists of different types of vessels in series with different resistances to flow

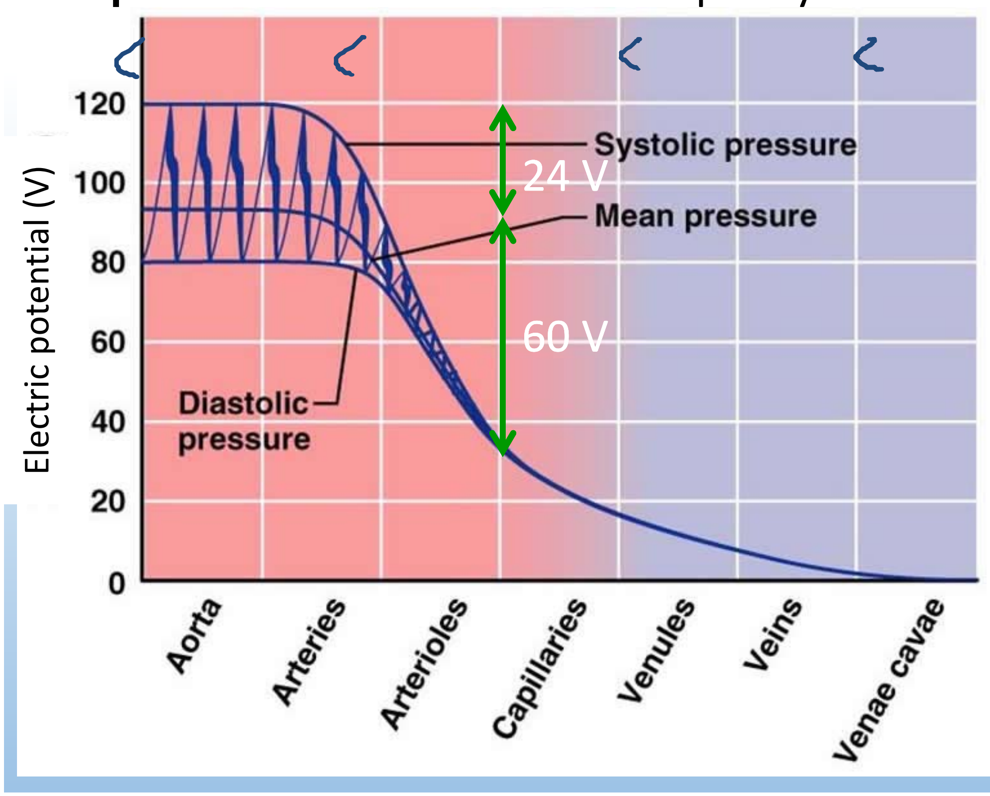


*Numbers represent accurate relative values

Calculation: vascular resistance

Calculate the current *I* through the vascular circuit and the voltages across the different types of vessels





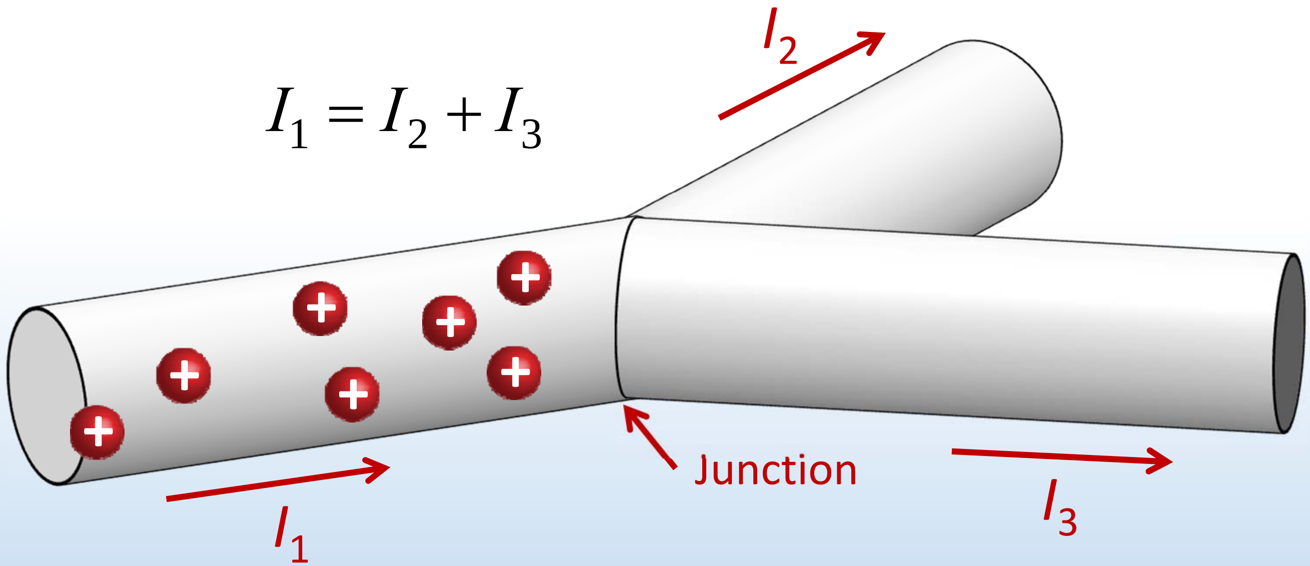
$$V_A = IR_A = (1.2A)(20\Omega) = 24V \ldots$$

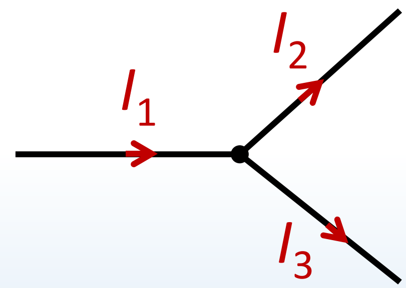
Resistance of arteries and arterioles have the largest effect on blood flow because they comprise 70% of the total resistance in most organs

Kirchhoff junction rule

Charges flowing through a junction split. By conservation of charge, the sum of currents into a junction equals the sum of currents out of a junction

$$I_1 = I_2 + I_3$$

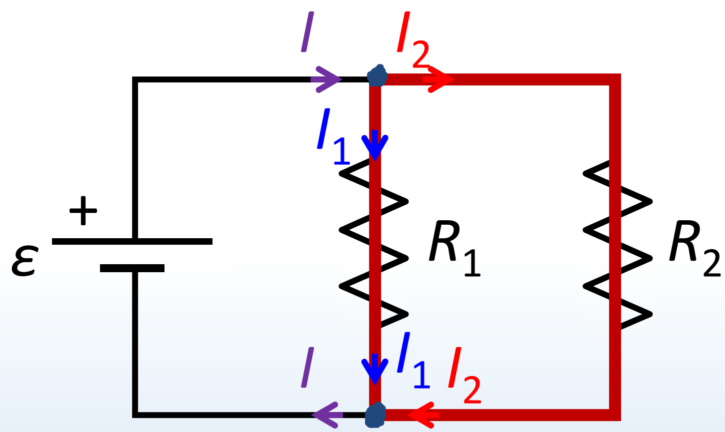


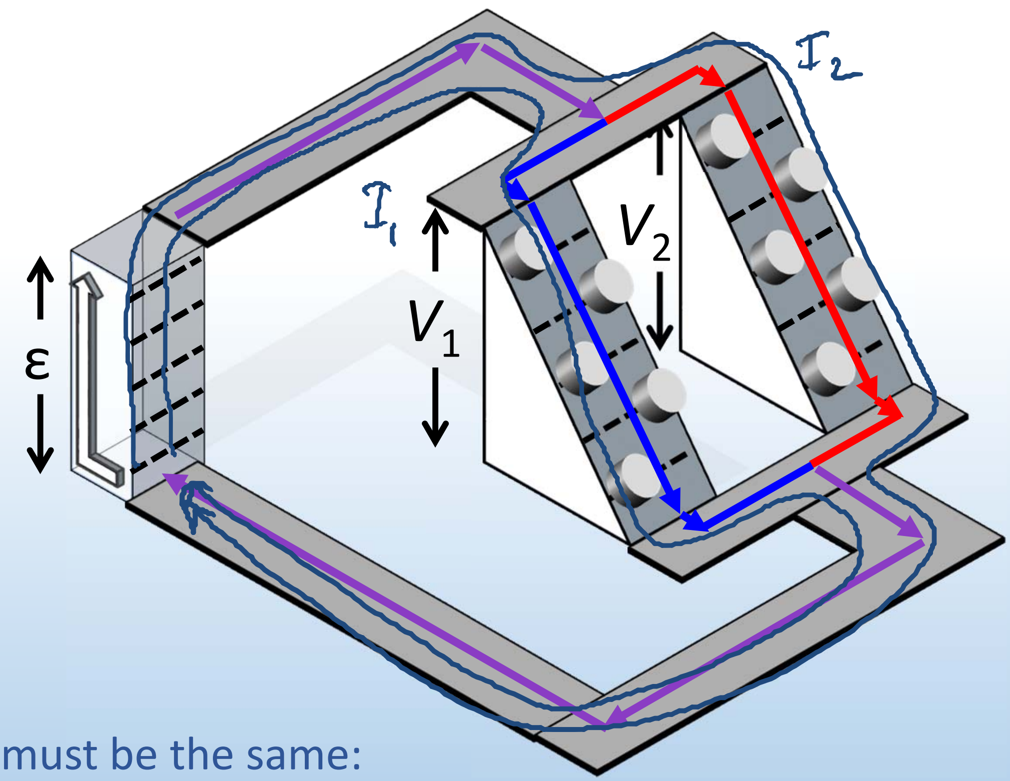


$$\boxed{\sum I_{in} = \sum I_{out}}$$

Parallel components

Components are said to be *in parallel* when both ends are connected to each other, forming a loop containing only them





Junction rule: I splits into I_1 and I_2
I_1 and I_2 recombine into I

$$I = I_1 + I_2 \;= \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

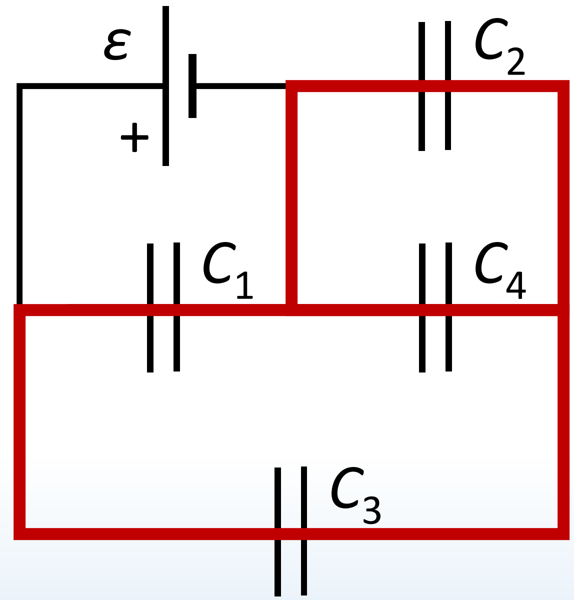
From loop rule, voltage across resistors must be the same:

$$\varepsilon - V_1 = 0 \qquad \varepsilon - V_2 = 0$$

$$V_1 = V_2 = \varepsilon$$
$$I_1 R_1 = I_2 R_2 = \varepsilon$$

ACT: Parallel or series?

Consider the circuit to the right. Which of the following statements is true?



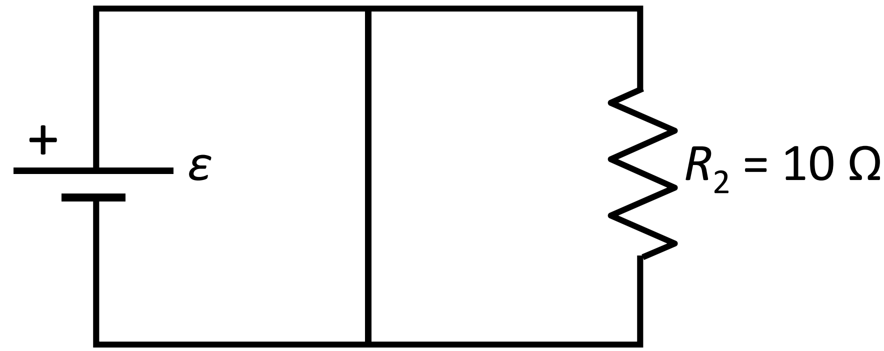
A. C_1 & C_4 are in series — No, branch between C_1 & C_4

B. C_2 & C_4 are in parallel — Yes, loop contains only C_2 & C_4

C. C_1 & C_3 are in parallel — No, loop also contains C_4

ACT: Resistors in parallel

Consider a circuit with two resistors R_1 and R_2 in parallel. Compare I_1, the current through R_1, to I_2, the current through R_2:



A. $I_1 > I_2$

B. $I_1 = I_2$

C. $I_1 < I_2$

Resistors are in parallel so $V_1 = V_2$ (= ε also)

$I_1 = V_1 / R_1 = \varepsilon / 1\ \Omega$

$I_2 = V_2 / R_2 = \varepsilon / 10\ \Omega$

$$I_2 = \frac{R_1}{R_2} I_1$$

Most current flows along "path of least resistance"!

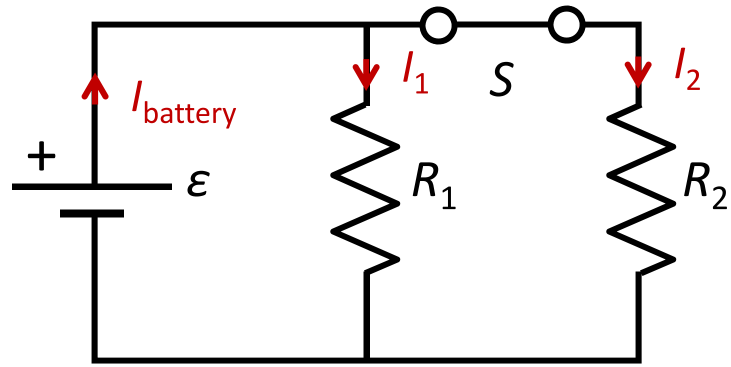
What if we replace R_1 with a wire?

An ideal wire has no resistance, so all current flows through it and $I_2 = 0$

This is called a ***short circuit***

ACT: CheckPoint 2.3

Now we add a switch S. What happens to the current out of the <u>*battery*</u> when the switch is closed?



24% A. $I_{battery}$ increases

71% B. $I_{battery}$ remains the same

6% C. $I_{battery}$ decreases

I_1 stays the same, I_2 increases
(***CheckPoint 2.1-2.2***) 45% 75%

Before closing switch:
$I_1 = \varepsilon/R_1$, $I_2 = 0$ and $I_{battery} = I_1$

After closing switch:
$I_1 = \varepsilon/R_1$, $I_2 = \varepsilon/R_2$ and $I_{battery} = I_1 + I_2$

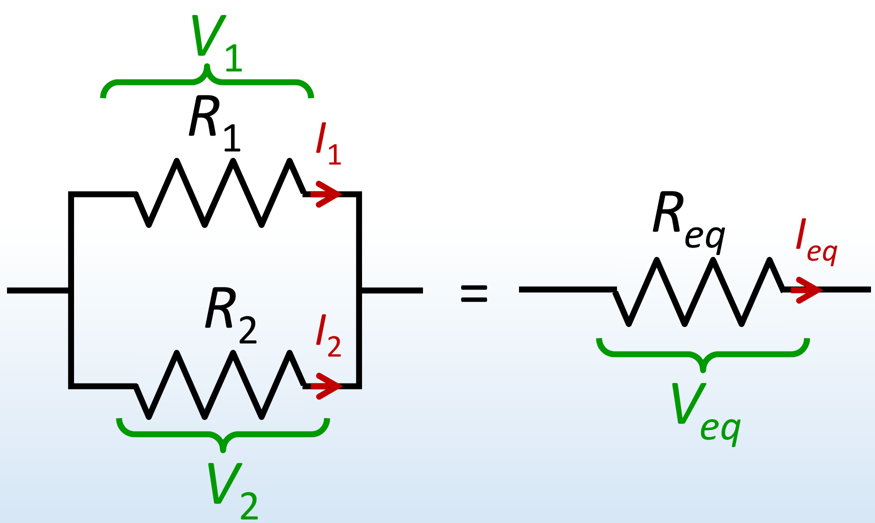
This is how your home is wired. The more appliances you turn on, the more current you draw.

DEMO

Equivalent resistance & capacitance

Circuit behaves the same as if *parallel* components were replaced by a *single, equivalent* component

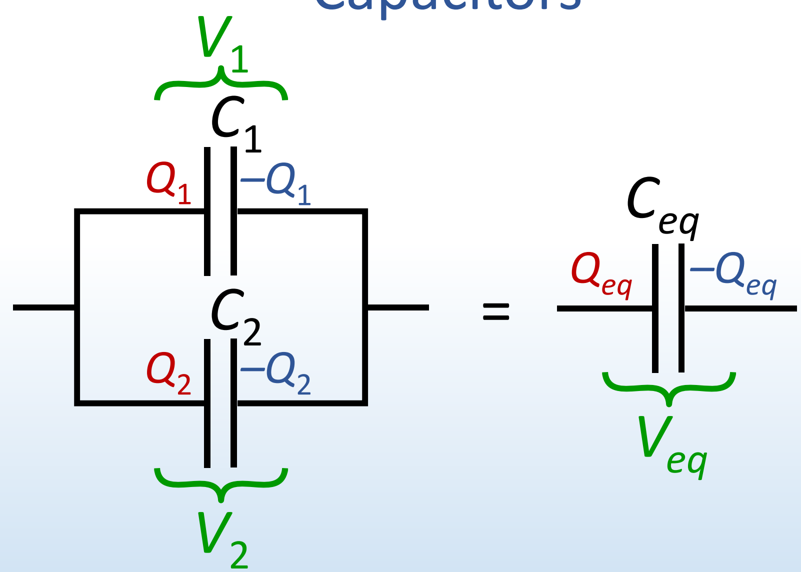
Resistors



$$I_1 + I_2 = I_{eq} \qquad V_1 = V_2 = V_{eq}$$

$$\boxed{V_{eq}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{V_{eq}}{R_{eq}} \quad \frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}}$$

Capacitors

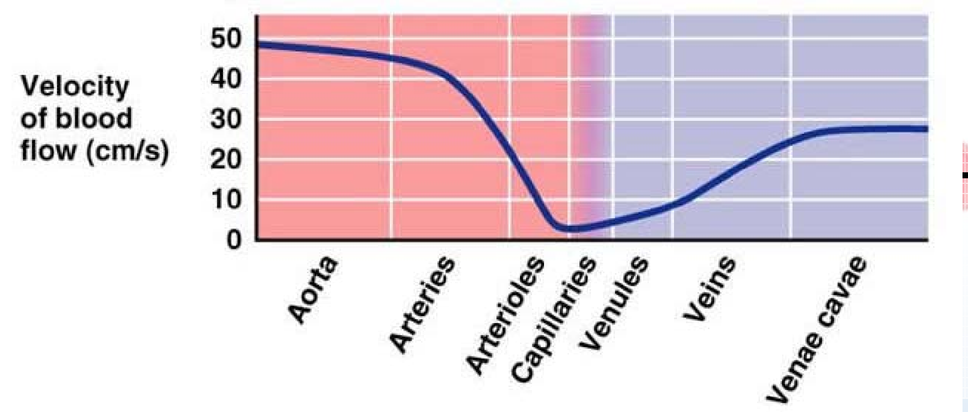


$$Q_1 + Q_2 = Q_{eq} \qquad V_1 = V_2 = V_{eq}$$

$$\boxed{(C_1 + C_2)V_{eq} = C_{eq}V_{eq} \quad C_{eq} = C_1 + C_2}$$

Calculation: vascular resistance

In previous calculation, capillaric resistance accounts for ~20% of total vascular resistance, yet capillaries are the thinnest blood vessels, and should have the *highest* resistance. Why?



Individual capillaries are connected in *parallel*

Assuming each capillary has the same resistance:

$$\frac{1}{R_{c,tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N} = \frac{N}{R_i} \qquad R_{c,tot} = \frac{R_i}{N}$$

Total capillaric resistance << resistance of single capillary

$$I_{tot} = I_1 + I_2 + \cdots I_N = NI_i$$

Current in single capillary << total current

Calculation: cardiovascular system

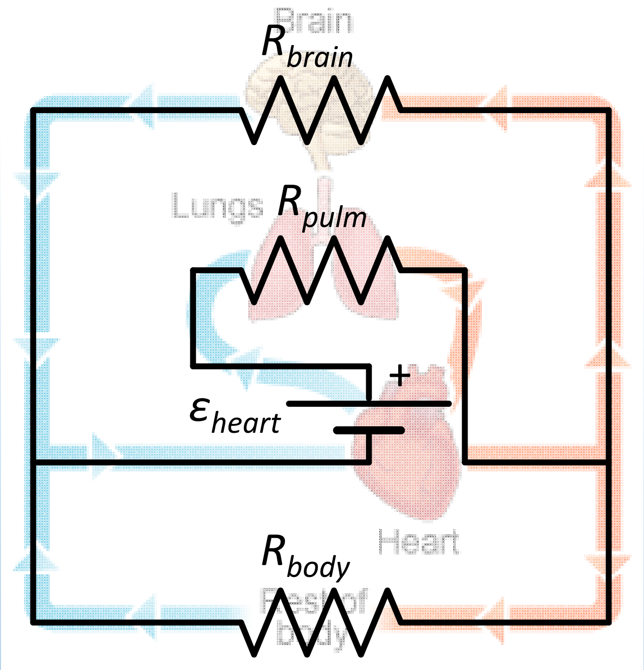
The human cardiovascular system consists of two circuits: *pulmonary* circulation which carries blood though the lungs, and *systemic* circulation which carries blood to the organs

The organs of the body are connected in parallel in the systemic circuit

Simple circuit model*:

R_{pulm} = 12 Ω, R_{brain} = 1 kΩ,
R_{body} = 160 Ω, ε_{heart} = 120 V

*Numbers represent accurate relative values

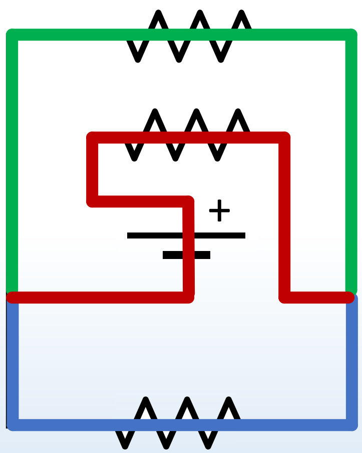


Calculate current through each component of circulatory system

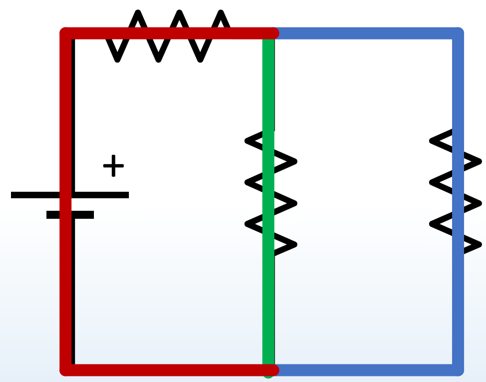
ACT: analyzing circuits

Which of the following circuit is different than the others?

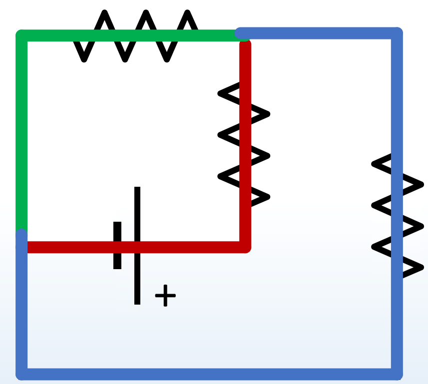
I



II



III



A. Circuit I
B. Circuit II
C. Circuit III
D. All three are equivalent
E. All three are different

Approach: identify parallel branches

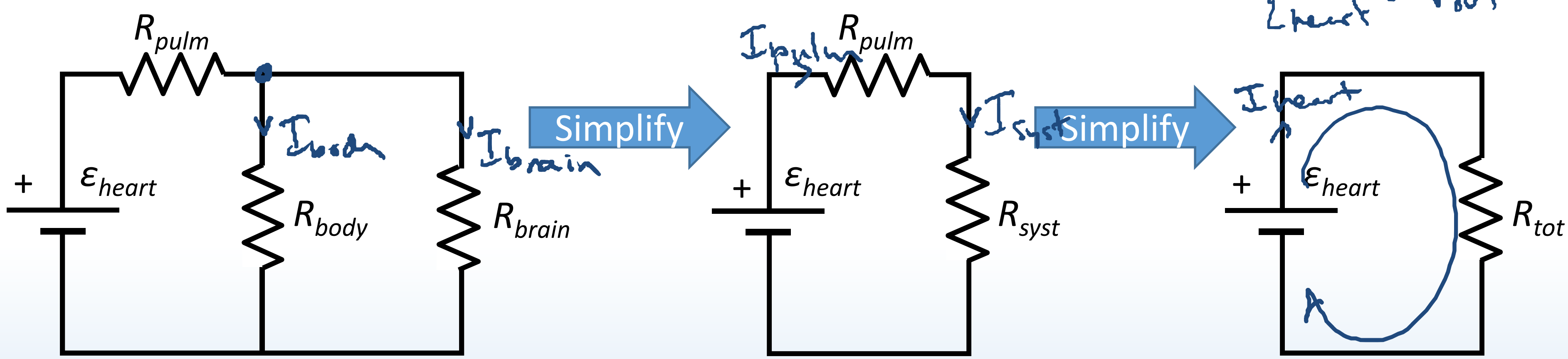
Battery in series with resistor
Resistor
Resistor

Calculation: circulatory system

Calculate current through each component of circulatory system

R_{pulm} = 12 Ω, R_{brain} = 1 kΩ, R_{body} = 160 Ω, ε_{heart} = 120 V

$\varepsilon_{heart} - V_{tot} = 0$



R_{brain} & R_{body} are in parallel

$$\frac{1}{R_{syst}} = \frac{1}{R_{brain}} + \frac{1}{R_{body}} \qquad R_{syst} = 138\ \Omega$$

$$V_{syst} = V_{brain} = V_{body}$$

$$I_{syst} = I_{brain} + I_{body}$$

R_{pulm} & R_{syst} are in series

$$R_{tot} = R_{pulm} + R_{syst} = 150\ \Omega$$

$$V_{tot} = V_{pulm} + V_{syst} = \varepsilon_{heart}$$

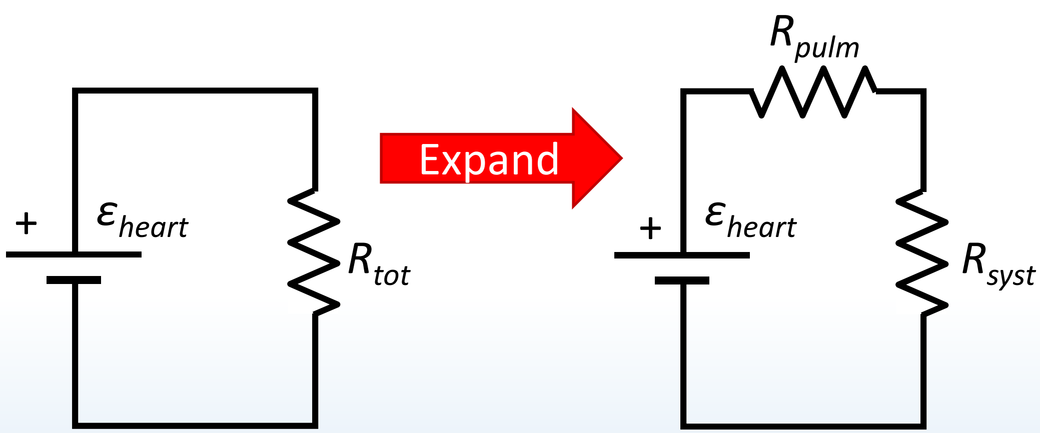
$$\frac{V_{tot}}{R_{tot}} = I_{tot} =$$

$$I_{heart} = I_{pulm} = I_{syst} = 120\ \text{V}/150\ \Omega = 0.8\ \text{A}$$

Calculation: circulatory system

Calculate current through each component of circulatory system

R_{pulm} = 12 Ω, R_{brain} = 1 kΩ, R_{body} = 160 Ω, ε_{heart} = 120 V



R_{pulm} & R_{syst} are in series

$$R_{tot} = R_{pulm} + R_{syst}$$

$$V_{tot} = V_{pulm} + V_{syst} = \varepsilon_{heart}$$

120 V = 10 + 110 V ✓

$$I_{pulm} = I_{syst} = I_{heart} = 0.8 \text{ A}$$

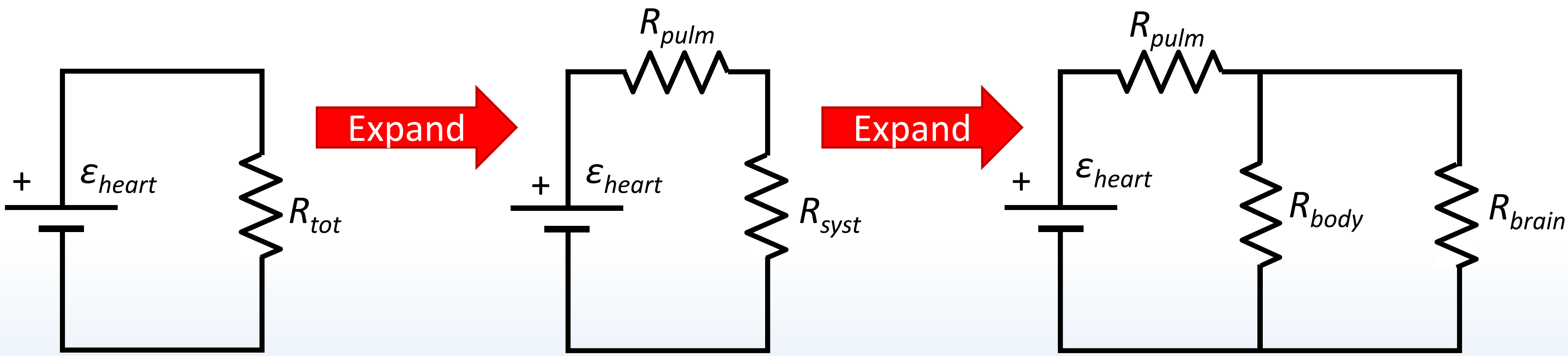
$V_{pulm} = I_{pulm} R_{pulm}$ = 10 V

$V_{syst} = I_{syst} R_{syst}$ = 110 V

Calculation: circulatory system

Calculate current through each component of circulatory system

R_{pulm} = 12 Ω, R_{brain} = 1 kΩ, R_{body} = 160 Ω, ε_{heart} = 120 V



R_{brain} & R_{body} are in parallel

$$\frac{1}{R_{syst}} = \frac{1}{R_{brain}} + \frac{1}{R_{body}}$$

About 14% of total blood flow goes to the brain

$$V_{syst} = V_{brain} = V_{body}$$

$I_{brain} = V_{brain}/R_{brain}$ = 110/1000 = 0.11 A

$$I_{syst} = I_{brain} + I_{body}$$

$I_{body} = V_{body}/R_{body}$ = 110/160 = 0.69 A

0.8 A = 0.11 + 0.69 A ✓

Summary of today's lecture

- Two basic principles:
- Kirchhoff loop rule

  Voltages around circuit loop sum to zero (based on conservation of energy)

$$\sum \Delta V = 0$$

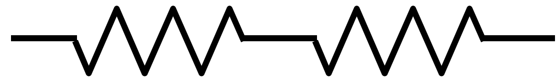
- Kirchhoff junction rule

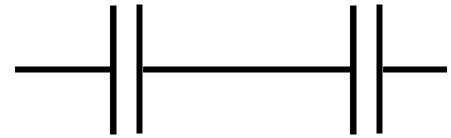
  Currents into a circuit branch equal currents out (based on conservation of charge)

$$\sum I_{in} = \sum I_{out}$$

Summary of today's lecture

- <u>Series</u> components



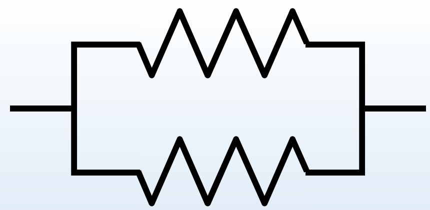


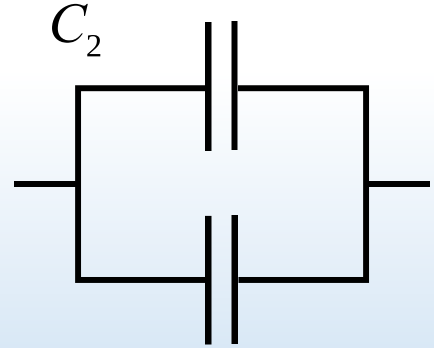
  Currents are the same $I_{eq} = I_1 = I_2$

  Voltages add $V_{eq} = V_1 + V_2$

  Resistors $R_{eq} = R_1 + R_2$ Capacitors $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

- <u>Parallel</u> components





  Voltages are the same $V_{eq} = V_1 = V_2$

  Currents add $I_{eq} = I_1 + I_2$

  Resistors $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ Capacitors $C_{eq} = C_1 + C_2$

- Don't mix equations!