Your questions/comments

IMPORTANT ANNOUNCEMENTS:
I will be out of town for Lect. 6-7. Prof. Dahmen will give the lectures
Exam 1 is **Thursday, Feb. 19**. Start reviewing! Lect 1 – 5

"…I was wondering if you could post extra problems to work on from the text book."

"WORK WITH CHARGES. I DON'T GET IT"

signs matter!

"Could you please discuss the concept of negative vs. positive work in relation to the electric field. Also, how do you determine the amount of work done when you have multiple charges?"

"I am very confused about potential energy. I thought $U=kq1q2/r$. Thus the smaller the radius the more potential energy. Why then in the first question did the two charges FURTHER apart (increase r) have more potential energy? I also am really confused about the work it takes to 'assemble' that triangle."

Phys 102 – Lecture 4

Electric potential energy & work

Today we will...

- Learn about the *electric potential energy*
- Relate it to *work*

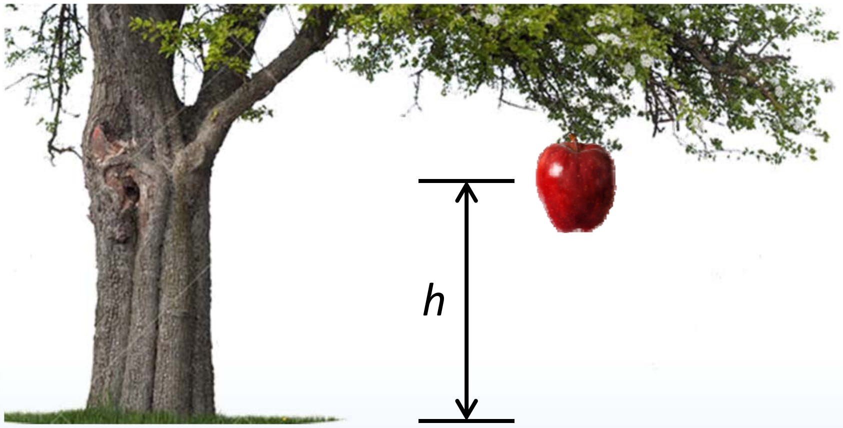
  Ex: charge in uniform electric field, point charges
- Apply these concepts

  Ex: electron microscope, assembly of point charges, dipole energy & hydration shells

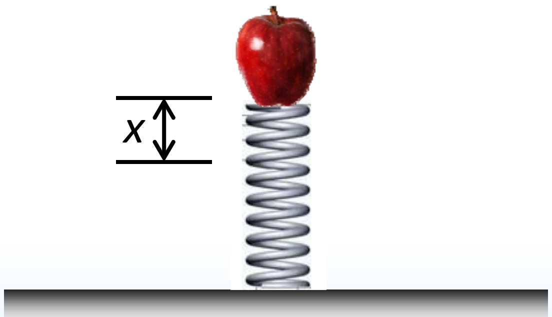
Potential energy

Potential energy U – stored energy, can convert to kinetic energy *K*

Review Phys. 101



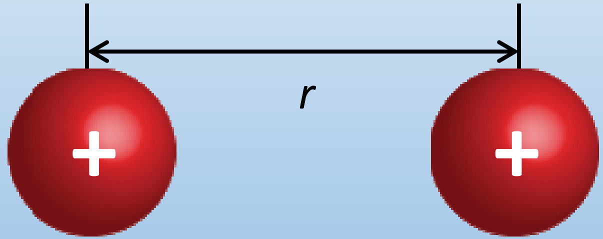
Gravitational potential energy (ex: falling object)



Elastic potential energy (ex: spring)

Total energy *K* + *U* is *conserved*

Same ideas apply to electricity



Electric potential energy (ex: repelling charges)

Work

Review Phys. 101

Ex: electrical force

Work – transfer of energy when a force acts on a moving object

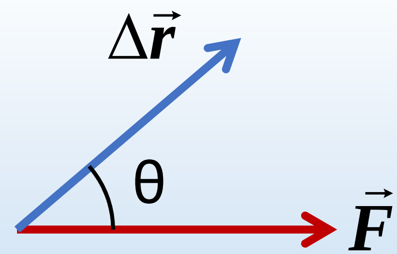
$$\boxed{W_F = F \Delta r \cos\theta} = -W_{you} = -\Delta U$$

Work done by force *F* → W_F

Displacement → Δr

Angle between force and displacement → θ

Change in potential energy → ΔU



Units: J ("Joules") = N × m

It matters who does the work

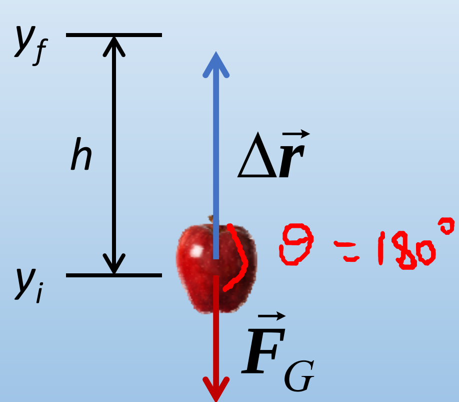
For *conservative* forces, work is related to potential energy

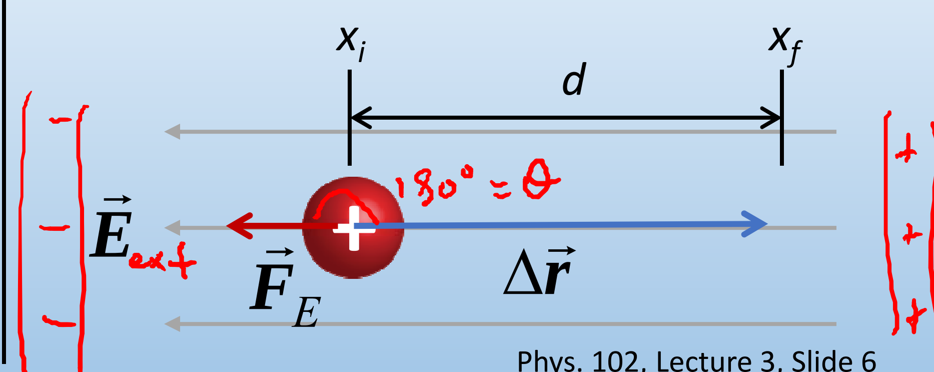
Electric potential energy & work

$$W_F = -W_{you} = -\Delta U = F\Delta r\cos\theta$$

Gravity ⟷ Electricity

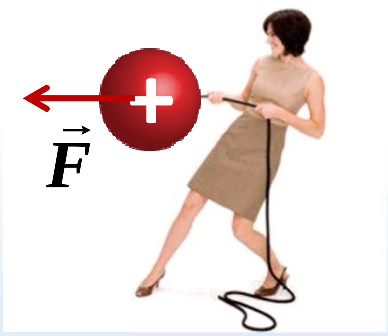
Mass raised $y_i \rightarrow y_f$	Charge moved $x_i \rightarrow x_f$ (in uniform E field to left)
$F_G = mg$ down	$F_E = qE$ left
$W_G = -mgh$	$W_E = -qEd$
$W_{you} = +mgh$	$W_{you} = +qEd$
$\Delta U_G = +mgh$	$\Delta U_E = +qEd$





Positive and negative work

If you moved object against external force (gravitational, electric, etc.), you did ***positive*** work, force did ***negative*** work



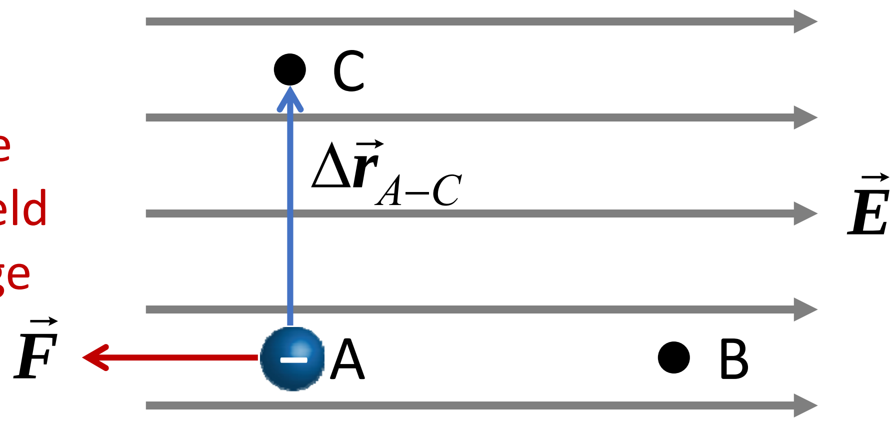
$W_{you} > 0 \quad W_F < 0$ $\qquad$ $W_{you} < 0 \quad W_F > 0$

If you moved object along external force (gravitational, electric, etc.), you did ***negative*** work, force did ***positive*** work

Checkpoint 1.2

Checkpoint 1.1

Force points in same direction as the *E* field for a POSITIVE charge



When a negative charge is moved from A to C the ELECTRIC force does

A. positive work 10%

B. zero work 85%

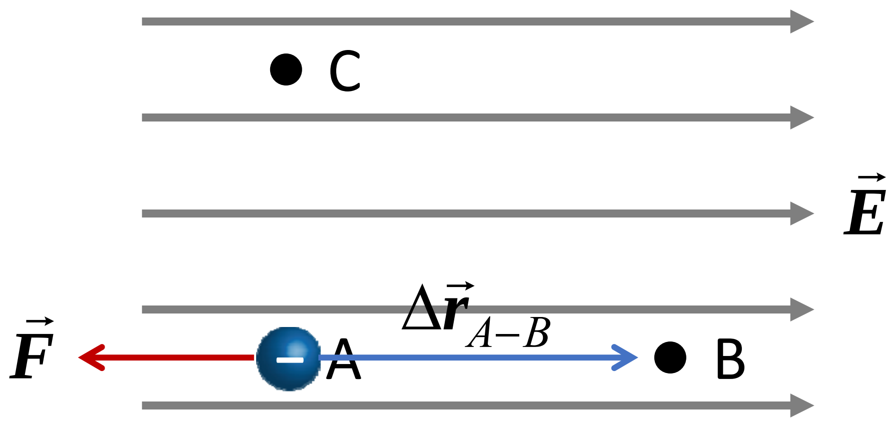
C. negative work 5%

$$W_F = F\Delta r_{A-C}\cos\theta$$

$\theta = 90°$ $\cos(90°) = 0$

Think about moving the apple parallel to ground.
Does its gravitational potential energy change?

ACT: Checkpoint 1.3



When a negative charge is moved from A to B the ELECTRIC force does

A. positive work 51%

B. zero work 8%

C. negative work 41%

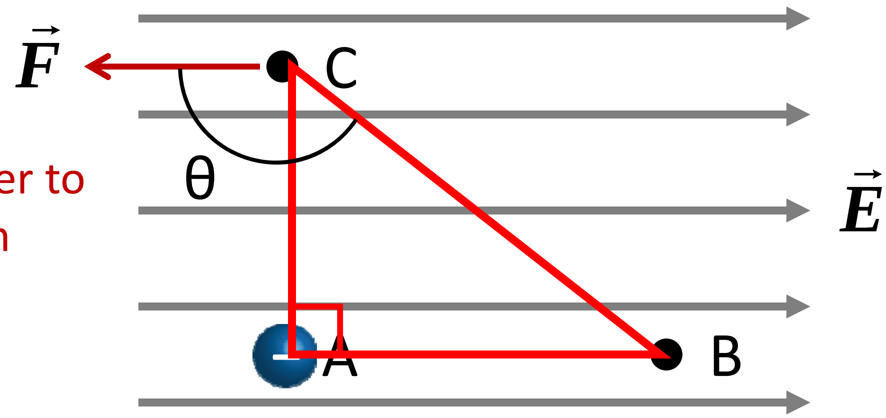
$$W_F = F\Delta r_{A-B}\cos\theta$$

θ = 180° cos(180°) = –1

Electric force did *negative* work
You did *positive* work
EPE increased (***CheckPoint 1.5***)

ACT: Work in a uniform E field

Let $W_{A\text{-}B}$ be the answer to the previous problem



The negative charge is now moved from A to C to B. The work done by the electric force is

A. Greater than $W_{A\text{-}B}$

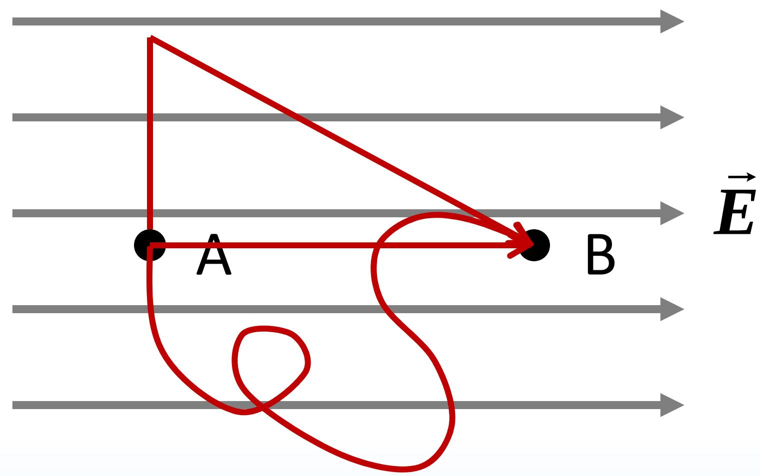
B. Same as $W_{A\text{-}B}$

C. Less than $W_{A\text{-}B}$

$$W_{A-C-B} = W_{A-C} + W_{C-B}$$
$$= 0 + F_E \Delta r_{C-B} \cos\theta$$
$$= 0 - F_E \Delta r_{A-B} = W_{A-B} \checkmark$$

Path <u>does not</u> matter!

Path independence of work



For conservative forces (ex: gravitational, electric), work is *independent* of path. Work depends *only* on end points.

$$W_{A-B} = -\Delta U = -(U_B - U_A)$$

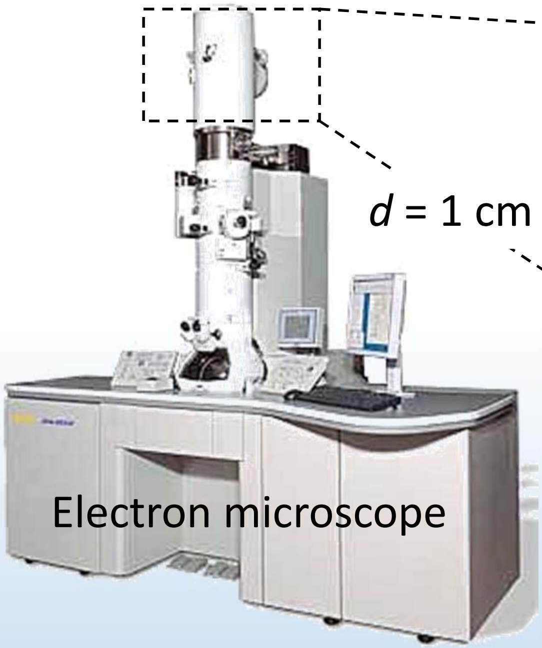
Potential energy of charge at position A

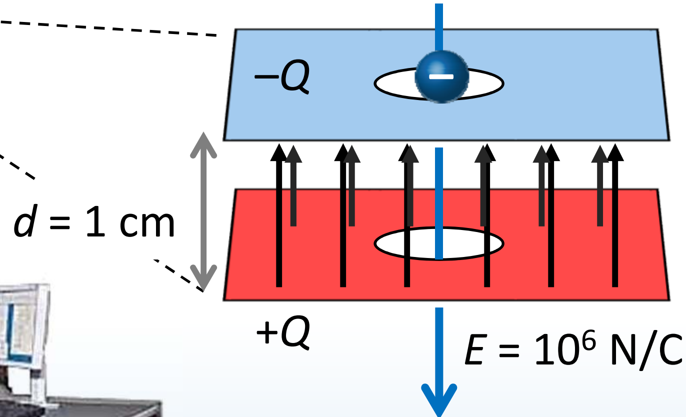
Potential energy of charge at position B

Calculation: Electron microscope (*revisited*)

Hw3 Q1

A uniform E field generated by parallel plates accelerates electrons in an electron microscope. If an electron starts from rest at the top plate what is its final velocity?





Approach: *conservation of energy*

Initial energy = Final energy

$$K_i + U_i = K_f + U_f$$

$$0 + U_i = \frac{1}{2}mv^2 + U_f$$

$U_f - U_i$

$$v = \sqrt{2\frac{(-\Delta U)}{m}} = \sqrt{2\frac{qEd}{m}}$$

← Review slide 5

$$= \sqrt{2\frac{1.6\times10^{-19}\cdot10^{6}\cdot0.01}{9.11\times10^{-31}}} = 5.9\times10^{7}\ \mathrm{m/s}$$

Same answer as in Lect. 3 using kinematic equations!

E.P.E of two point charges

Electric potential energy of two charges q_1 and q_2 separated by a distance r

$$U_E = k\frac{q_1 q_2}{r}$$

Note: NOT r^2

$+e =$ $+1.6 \times 10^{-19}$ C = q_1

$q_2 = -1.6 \times 10^{-19}$ C $= -e$

$r = 0.53 \times 10^{-10}$ m

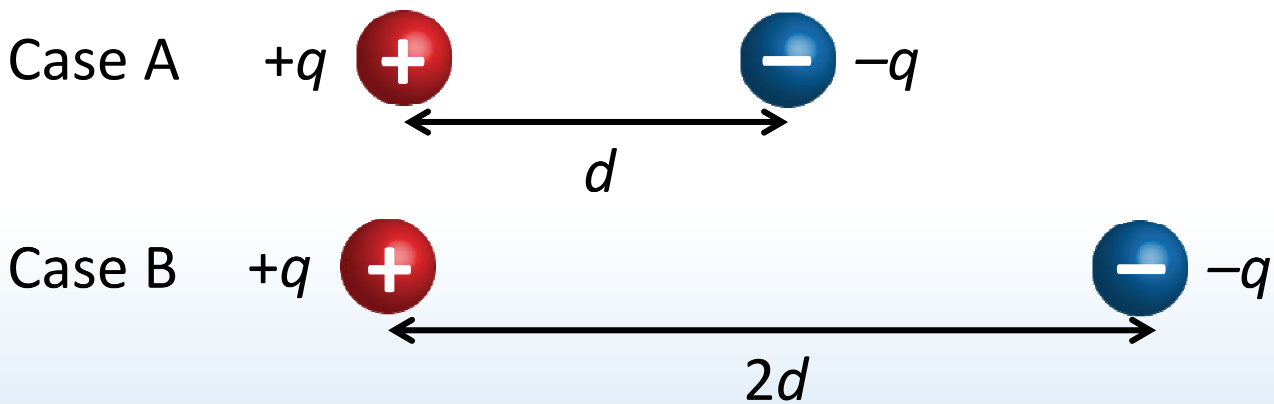
Ex: What is the electric potential energy of the proton and the electron in H?

Negative

$$U_E = k\frac{q_e q_p}{r} = \frac{9\times10^9\,\mathrm{N\cdot m^2/C^2}(-1.6\times10^{-19}\,\mathrm{C})(1.6\times10^{-19}\,\mathrm{C})}{0.53\times10^{-10}\,\mathrm{m}} = -4.3\times10^{-18}\ \mathrm{J}$$

ACT: E.P.E. of 2 charges

In case A, two charges of equal magnitude but opposite sign are separated by a distance *d*. In case B, they are separated by 2*d*.



$$U_E = -k\frac{q^2}{d}$$

$$U_E = -k\frac{q^2}{2d}$$

Which configuration has a higher electric potential energy?

A. Case A has a higher E.P.E.
B. Case B has a higher E.P.E.
C. Both have the same E.P.E.

$U_E < 0$ in case A and B.
Case B is ***less*** negative

Sign of potential energy

What does it mean to have a ***negative*** electric potential energy?

Ex: H atom

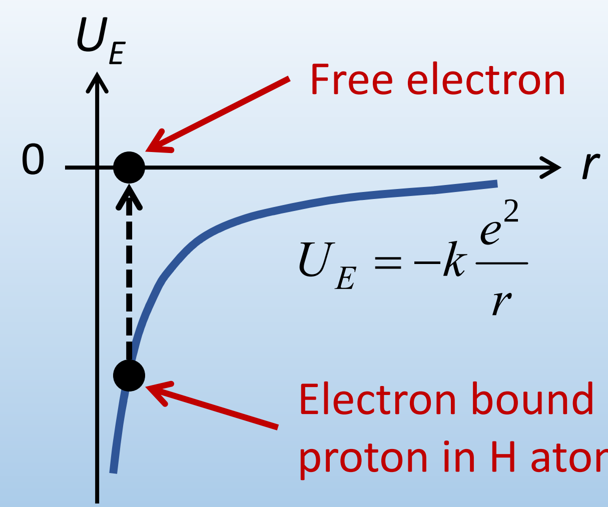
Proton

Electron

$U_E < 0$ relative to energy of an electron very far away ($r \rightarrow \infty$), away from E field of proton, i.e. a "free" electron

Energy must be added in order to free electron bound to proton

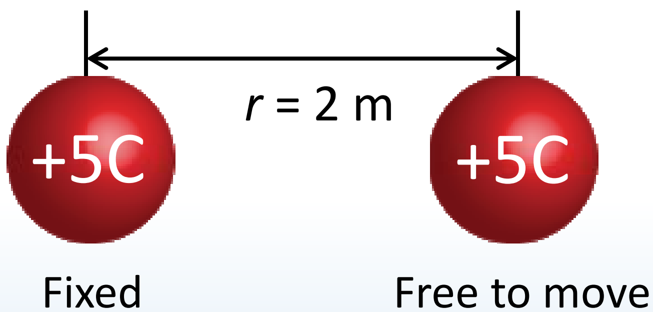
We will revisit this when discussing atomic physics later in the semester



Calculation: two charges

Two +5 C, 1 kg charges are separated by a distance of 2 m. At t = 0 the charge on the right is released from rest (the left charge is fixed). What is the speed of the right charge after a long time ($t \rightarrow \infty$)?

From EX 1, SPRING '10



Approach: conservation of energy

$$K_i + U_i = K_f + U_f$$

$$0 + U_i = \tfrac{1}{2} m v^2 + 0$$

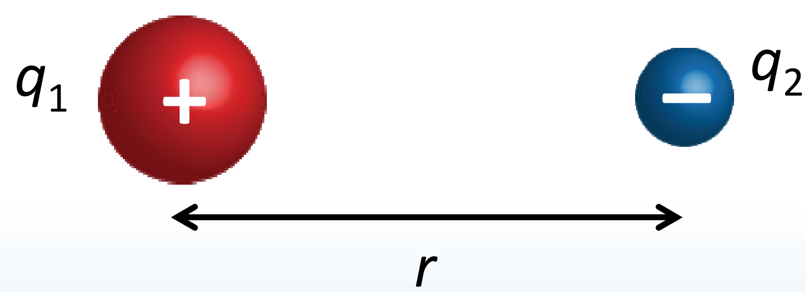
$$U = \frac{kq^2}{r}$$

$$v = \sqrt{2\frac{kq^2}{mr}} = \sqrt{2\frac{9\times 10^9 \cdot 5^2}{1\cdot 2}} = 4.7\times 10^5 \,\frac{\text{m}}{\text{s}}$$

Much harder to solve problem with kinematic equations! *F* and *a* are not constant!

Work done to assemble charges

How much work do *you* do assembling configuration of charges?



Imagine bringing charges from infinitely far away to a separation *r*

$$W_{you} = +\Delta U_E = k\frac{q_1 q_2}{r} - 0$$

Potential energy of charges infinitely far

Potential energy of charges in final configuration

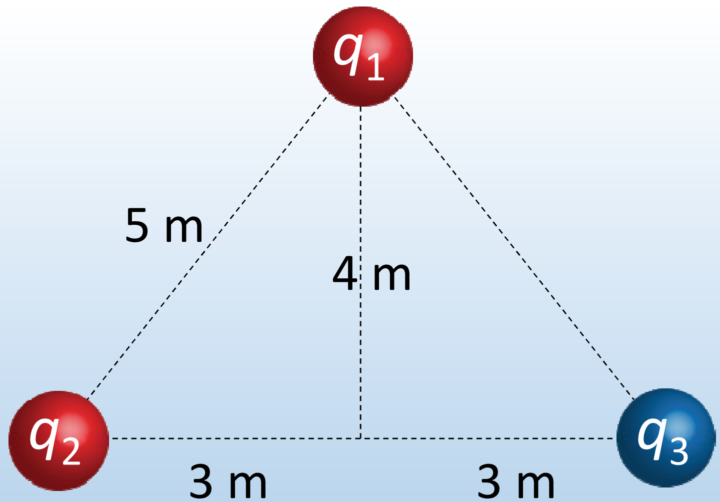
HW3 Q2-3

Calculation: assembling charges

How much work <u>do you do</u> to assemble the charges q_1 = +2 μC, q_2 = +7 μC, and q_3 = –3.5 μC into a triangle?

Approach:

- Bring each charge in from far away
- Add up work from each charge pair



Does the order matter?

$$W_1 = 0$$

No E fields from other charges!

$$W_2 = k\frac{q_1 q_2}{r_{12}}$$

$$= 9\times10^9 \frac{(2\times10^{-6})(7\times10^{-6})}{5} = +25.2\,\text{mJ}$$

$$W_3 = k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

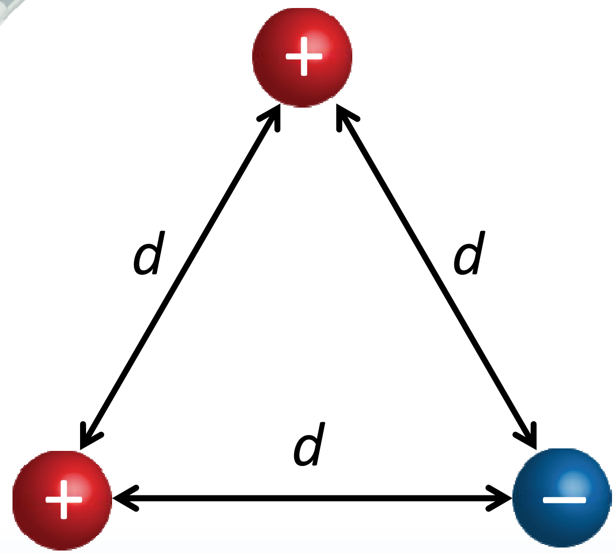
$$= 9\times10^9 \frac{(2\times10^{-6})(-3.5\times10^{-6})}{5}$$

$$+9\times10^9 \frac{(7\times10^{-6})(-3.5\times10^{-6})}{6} = -49.4\,\text{mJ}$$

$$W_{you} = -24.2\text{mJ} \qquad \Delta U = -24.2\text{mJ}$$

$$W_E = +24.2\text{mJ}$$

ACT: Checkpoint 2.1



Charges of equal magnitude are assembled into an equilateral triangle

The total work required *by you* to assemble this set of charges is:

A. positive 58%

B. zero 22%

C. negative 20%

Bring in (1): zero work $W_1 = 0$

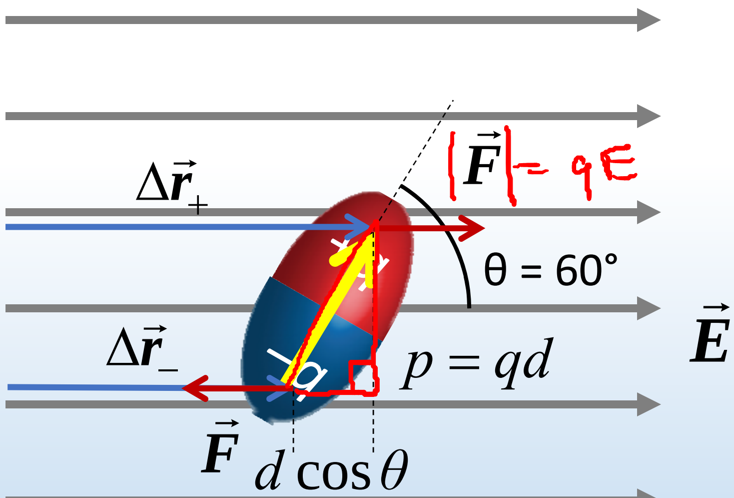
Bring in (2): positive work $W_2 = k\dfrac{q^2}{d}$

Bring in (3): negative work x 2 $W_3 = -k\dfrac{2q^2}{d}$

$$W_{you} = -k\frac{q^2}{d}$$

Calculation: dipole in E-field

An electric dipole with moment *p* = 6.2 x 10^{-30} C·m is placed in a uniform external electric field *E* = 10^6 N/C at an angle θ = 60°. Calculate the total *electric potential energy* of the dipole.



$$U_{dip.} = -pE\cos\theta$$

Approach: calculate work done by E field on the dipole

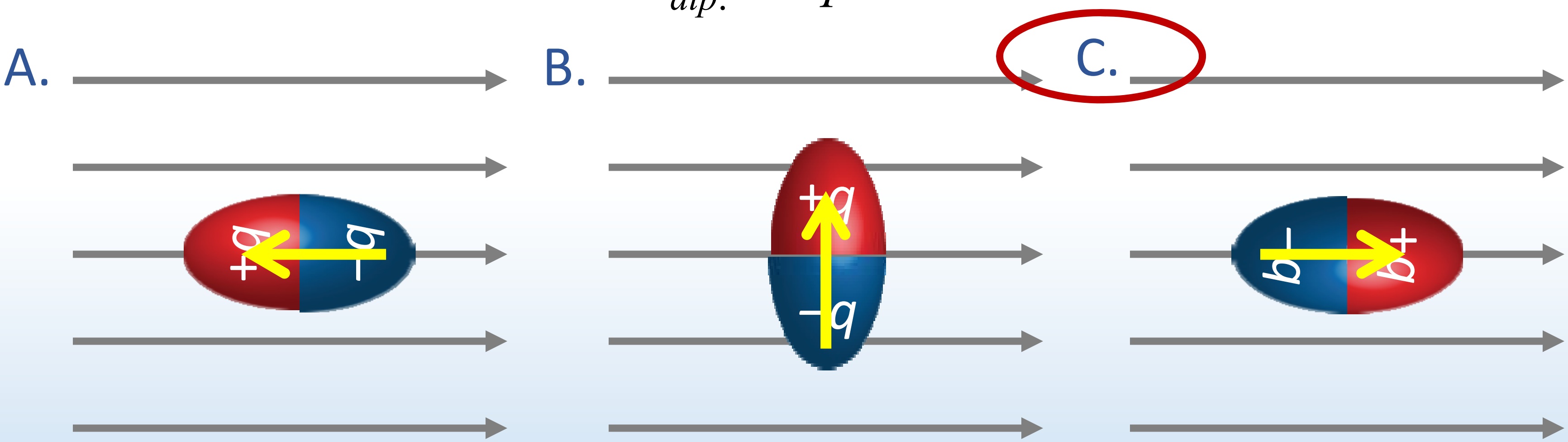
$$W_{E-dip.} = qE\Delta r_+ - qE\Delta r_-$$
$$= qE(\Delta r_+ - \Delta r_-)$$
$$= qEd\cos\theta$$
$$= pE\cos\theta \quad = -U_{dip.}$$

$$U_{dip} = -6.2\times10^{-30}\cdot10^{6}\cos(60^\circ)$$
$$= -3.1\times10^{-24}\,\text{J}$$

ACT: dipole energy

Which configuration of dipole in a uniform electric field has the lowest electric potential energy?

$$U_{dip.} = -pE\cos\theta$$



θ = 180° $U_{dip.} = +pE$

θ = 90° $U_{dip.} = 0$

θ = 0° $U_{dip.} = -pE$

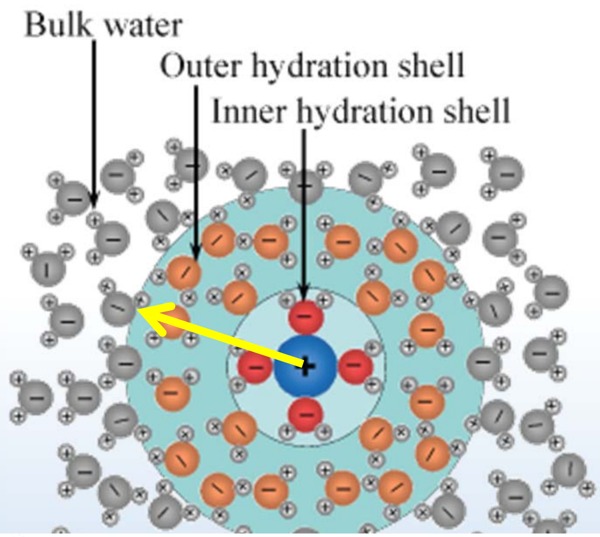
Recall Lect. 2: Dipoles in A & B will rotate to align to E field: *K.E.* converted from *E.P.E.* Dipole in C has minimum *E.P.E*: no rotation

Hydration shell radius

Recall that H_2O dipole aligns to electric field of ions. However, at room temperature, H_2O also has rotational kinetic energy that tends to randomize dipole orientation

$K_{dip.} + U_{dip.} \geq 0$ Dipoles are randomized (bulk water)

$K_{dip.} + U_{dip.} < 0$ Dipoles tend to be aligned (hydration shell)

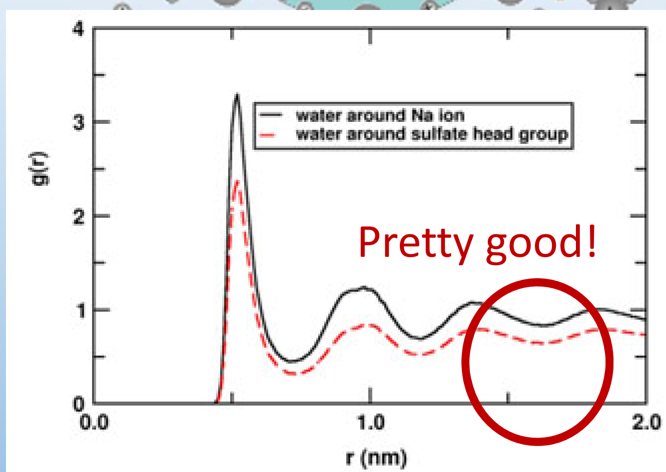


We can estimate distance to interface between bulk and ordered water for a monovalent ion (Ex: Na^+)

$$U_{dip.} = -pE = -p\frac{kq}{r^2} \qquad p_{H_2O} = 6.2\times10^{-30}\,\mathrm{C\cdot m}$$

$$K_{dip.} = 4\times10^{-21}\,\mathrm{J}$$

$$r = \sqrt{\frac{pkq}{K_{dip.}}} = \sqrt{\frac{6.2\times10^{-30}\cdot 9\times10^{9}\cdot 1.6\times10^{-19}}{4\times10^{-21}}} = 1.5\,\mathrm{nm}$$



Summary of today's lecture

- Electric potential energy & work

  $$W_F = -W_{you} = -\Delta U = F\Delta r\cos\theta$$

  Path independence

  Conservation of energy

- Electric potential energy for point charges $U_E = k\dfrac{q_1 q_2}{r}$