Your questions/comments

END-OF-SEMESTER ANNOUNCEMENTS: Check your gradebook NOW
ICES course evaluation
James Scholar Credit projects due this Friday, Dec. 5

“Is there anyway we can get more practice problems for Lectures 22-28 that would reflect the types of questions that will be asked on the final? Personally, I learn best if I can apply what I studied to practice problems that I haven't seen before to see if I actually understand and apply the material. I know a few others that feel this way, too.”

“Could you give us more info about the final exam: number of questions/ how many questions per topic/what to study/etc”

“this lecture material seems better than the previous one. i hope. last time was cray cray.”

“Much more straightforward than quantum from last lecture! Can we review the rate law, and how to derive the half-life expression from it?“

Phys 102 – Lecture 27

The strong & weak nuclear forces

4 Fundamental forces of Nature

Gravitational force (solar system, galaxies) phys. 101

Electromagnetic force (atoms, molecules)

Today { Strong force (atomic nuclei)

Weak force (radioactive decay)

Gravitational

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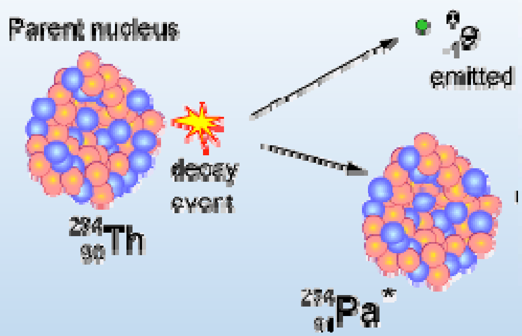

Weak

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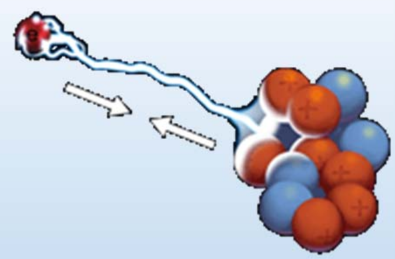

Electromagnetic

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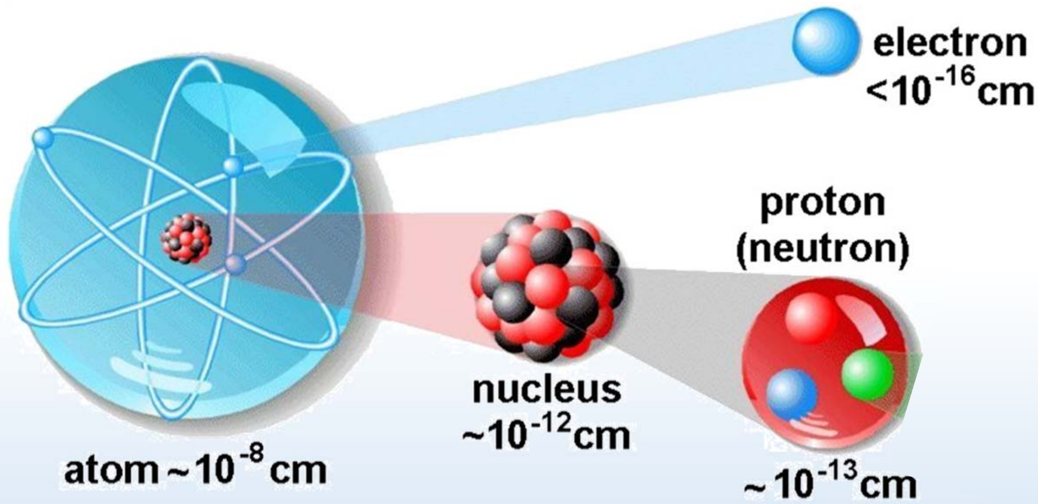
Strong

weakest — strongest

The nucleus

All + charge of the atom is inside a small *nucleus*, which is made up of *protons* and *neutrons* ("nucleons")



$E = mc^2$

Particle	Mass (MeV/c^2)	Charge
electron	0.511	–e
proton	938.3	+e
neutron	939.5	0

Nuclear nomenclature

A nucleus is composed of *Z protons* and *N neutrons* ("nucleons")

A = "atomic mass number"

Z = "atomic number"

$${}^{A}_{Z}\mathrm{E}$$

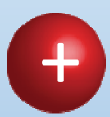
Element name

$$A = N + Z$$

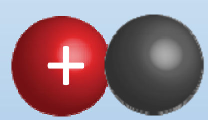
A = nucleon number (gives mass of nucleus since $m_{prot} \approx m_{neut} \gg m_{elec}$)

Z = proton number = electron number (gives element name & chemical properties)

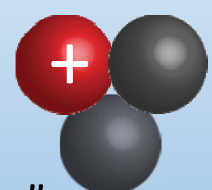
Ex: Elements with different nuclei are known as *isotopes* same Z, diff. A

${}^{1}_{1}\mathrm{H}$ 

"protium"

${}^{2}_{1}\mathrm{H}$ 

"deuterium"

${}^{3}_{1}\mathrm{H}$ 

"tritium"

ACT: CheckPoint 1.1

A material is known to be made from an isotope of lead (Pb).

Based on this information which of the following can you specify?

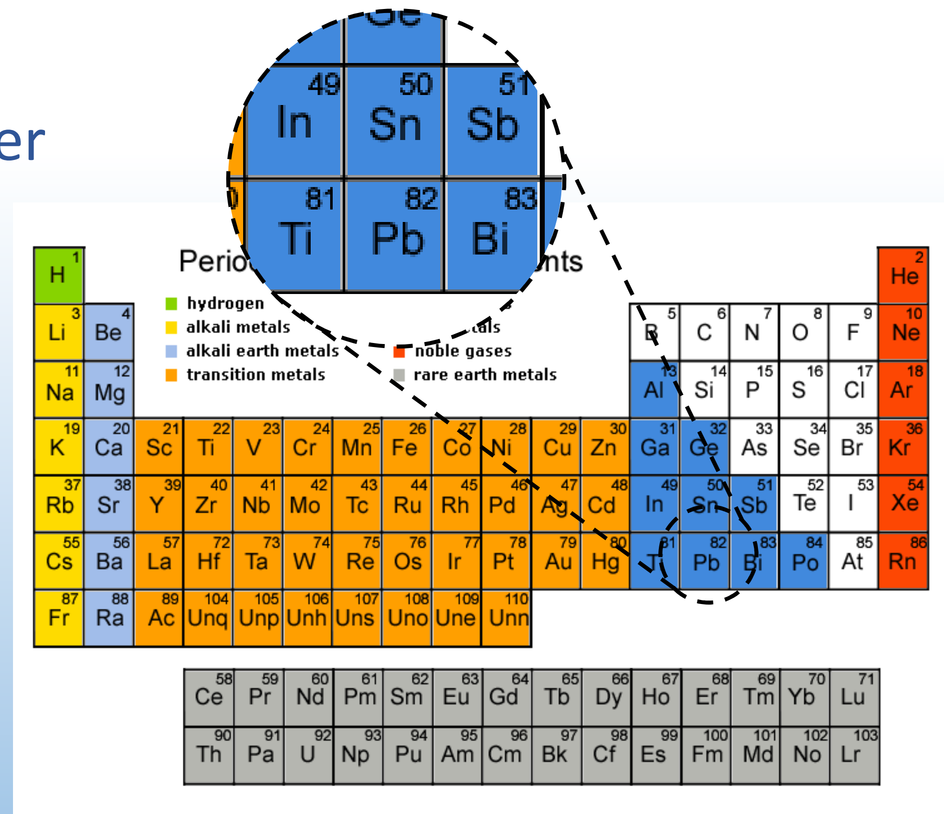
22% A. The atomic mass number

20% B. The neutron number

58% C. The proton number

Chemical properties (and name) given by number of protons (Z)

For lead Z = 82

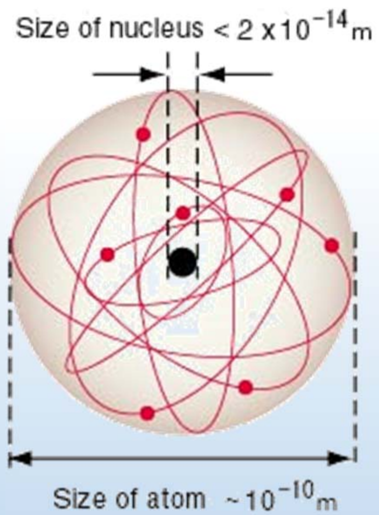


Nucleus size

Mass densities of nuclei are approximately the same

$$\rho_{nucleus} = \frac{M_{nucleus}}{V_{nucleus}} \quad \text{and} \quad M_{nucleus} \propto A \quad \text{so} \quad V_{nucleus} = \frac{4\pi}{3} r^3 \propto A$$

$$r \approx A^{1/3} \cdot 1.2 \times 10^{-15}\,\text{m}$$ fm "femtometer"



A =

Ex: ${}^{27}_{13}\text{Al}$

$$r_{nucleus} \approx 3.6 \times 10^{-15}\,\text{m}$$

$$r_{atom} \approx 1.4 \times 10^{-10}\,\text{m}$$

10^4–10^5

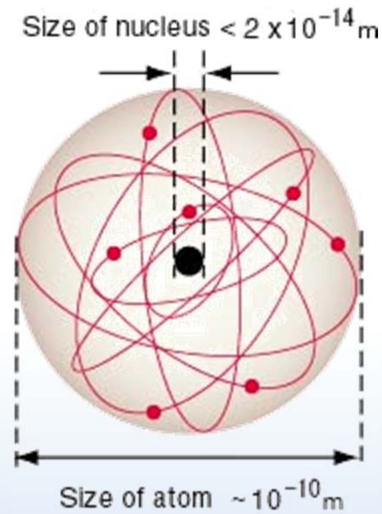
Compared to size of atom (outer e^- shell)

How do protons not repel each other inside nucleus?

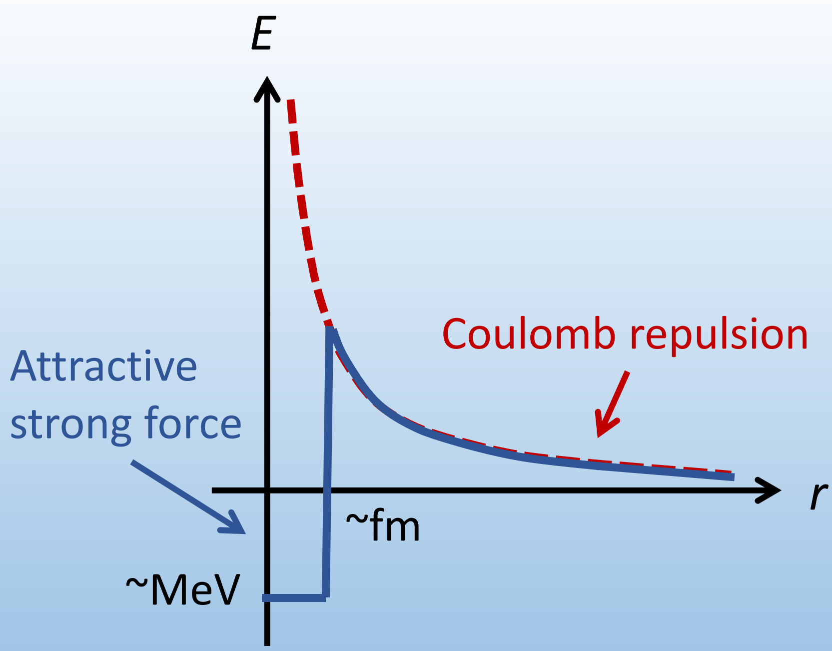
Strong nuclear force

Electric potential energy of two protons in 1 fm nucleus:

$$U_{p-p} = +\frac{ke^2}{r} = \frac{1.44\,\text{eV}\cdot\text{nm}}{1\times10^{-6}\,\text{nm}} = 1.44\,\text{MeV}$$



Strong nuclear force binds nucleons together, overcomes Coulomb repulsion at fm distances



Ex: deuterium (1 proton + 1 neutron)

$$\left.\begin{aligned} E_{deuteron} &\approx 2.2\text{MeV} \\ E_{atom} &\approx 13.6\,\text{eV} \end{aligned}\right\} 10^5$$

DEMO

Binding energy & mass defect

Nuclear binding energy decreases mass of nucleus

Einstein's equation: $$\boxed{E_0 = mc^2}$$ Equivalence of mass and energy

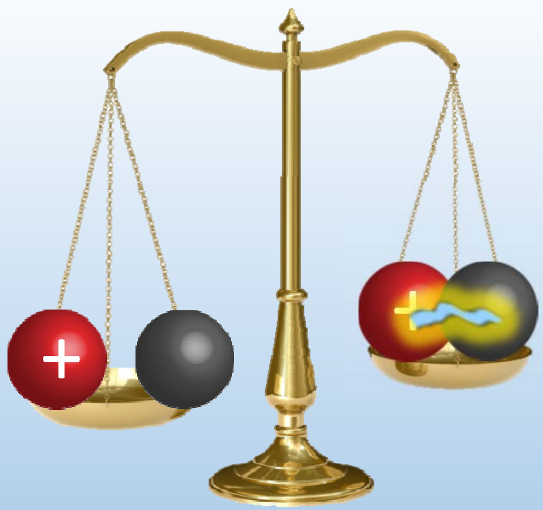
$$m_{nucleus} < Zm_{prot} + Nm_{neut} - \frac{|E_{bind}|}{c^2}$$ "Mass defect"

Ex: deuteron = 1 proton + 1 neutron

$$m_{prot} = 938.3\,\mathrm{MeV}/c^2$$

$$+ \quad m_{neut} = 939.5\,\mathrm{MeV}/c^2$$

$$= \quad m_{tot} = 1877.8\,\mathrm{MeV}/c^2$$



$$m_{deut} = 1875.6\,\mathrm{MeV}/c^2$$

Difference is binding energy!
For deuteron:

$$E_{bind} = 2.2\,\mathrm{MeV}$$

ACT: Binding Energy

Which system "weighs" more?

A 

B 

A. Two balls attached by a relaxed spring

B. Two balls attached by a stretched spring

C. They have the same weight

$$M_A = m_{balls} + m_{spring}$$

$$M_B = m_{balls} + m_{spring} + \frac{E_{spring}}{c^2}$$

Notice + sign because energy is *increased*

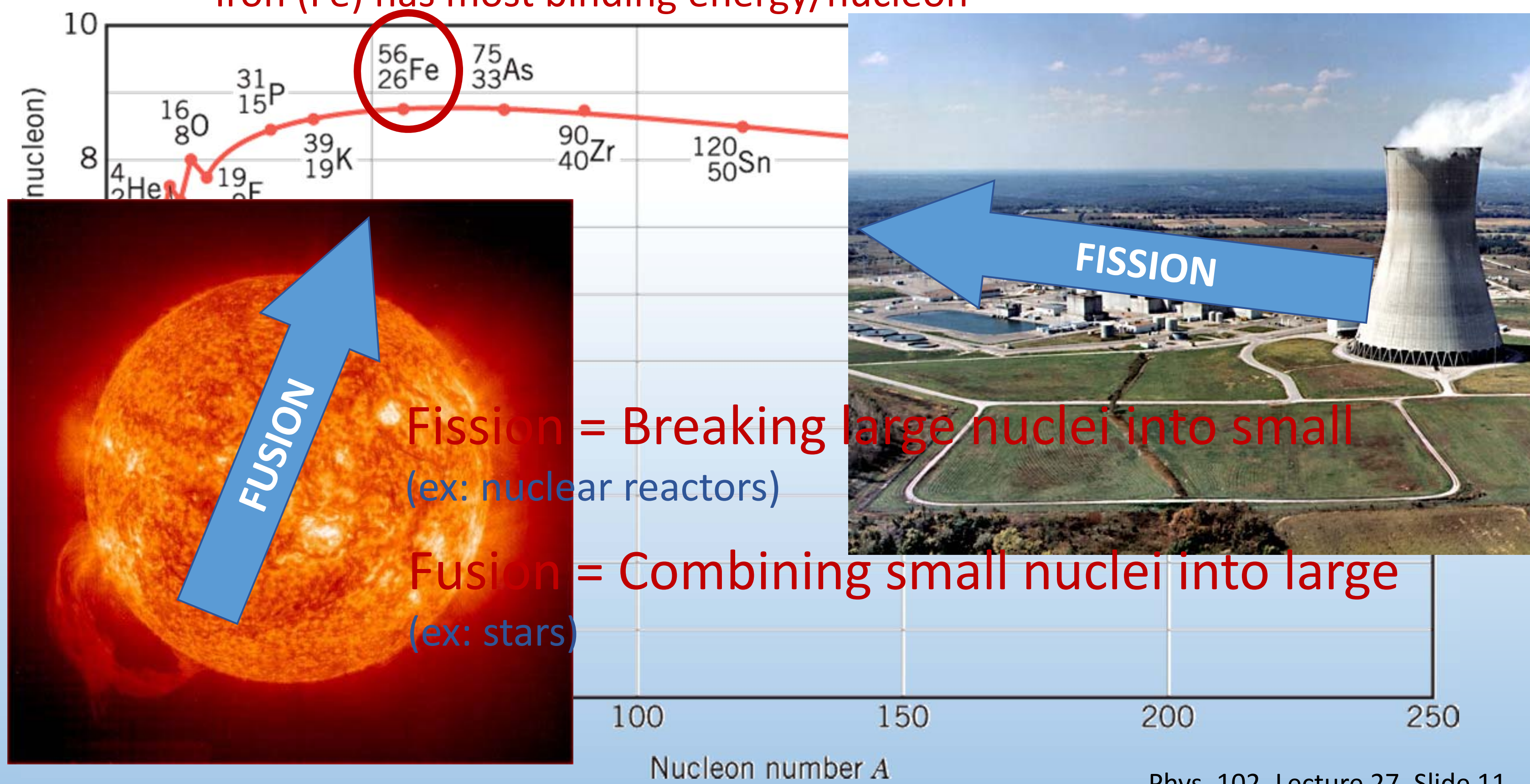
$$M_B - M_A = \frac{E_{spring}}{c^2} \sim \frac{1J}{(3\times10^8\ \mathrm{m/s})^2} \approx 10^{-17}\,kg$$

Binding energy plot

Binding energy per nucleon increases with *A* due to higher strong force, then decreases due to Coulomb repulsion

Iron (Fe) has most binding energy/nucleon

$$\frac{E_{bind}}{A}$$



Fission = Breaking large nuclei into small
(ex: nuclear reactors)

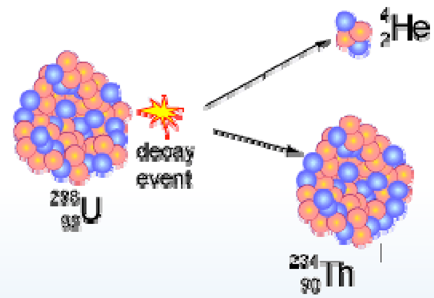
Fusion = Combining small nuclei into large
(ex: stars)

ACT: Uranium mass

^{238}U is long-lived but ultimately unstable. Eventually, it will spontaneously break into a ^{4}He and ^{234}Th nucleus, and release a tremendous amount of energy:

$$^{238}_{92}\text{U} \rightarrow \, ^{234}_{90}\text{Th} + \, ^{4}_{2}\text{He} + \text{energy}$$

$$m_U c^2 = m_{Th} c^2 + m_{He} c^2 + E$$



What must be true about the masses of the nuclei?

A. $m_U > m_{Th} + m_{He}$

B. $m_U = m_{Th} + m_{He}$

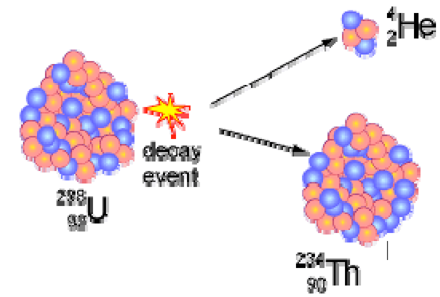
C. $m_U < m_{Th} + m_{He}$

Th + He is more stable (i.e. has a higher binding energy) than U alone

$$m_{nucleus} = Zm_{prot} + Nm_{neut} - \frac{|E_{bind}|}{c^2}$$

Calculation: Uranium decay

Calculate the energy released in this fission reaction:



$$ {}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} + \text{energy} $$

Initial energy:

$$ m_U c^2 = 92 m_p c^2 + 146 m_n c^2 - |E_U| $$

Final energy:

$$ m_{Th} c^2 = 90 m_p c^2 + 144 m_n c^2 - |E_{Th}| $$

$$ + m_{He} c^2 = 2 m_p c^2 + 2 m_n c^2 - |E_{He}| $$

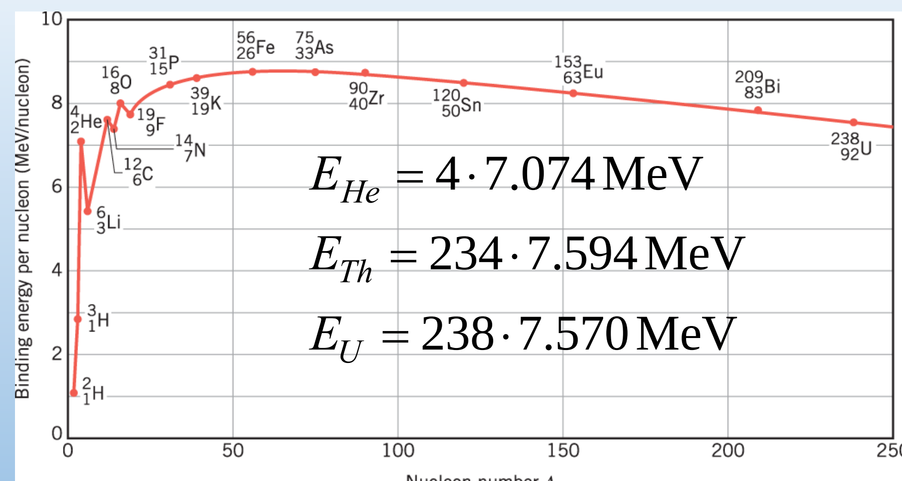
$$ + E_{released} $$

Released energy is difference in binding energies:

$$ E_{released} = E_{Th} + E_{He} - E_U $$

$$ = 4.3\,\mathrm{MeV} $$

In nuclear reactors, energy released as heat drives turbine (Recall Lect. 14)



$$ E_{He} = 4 \cdot 7.074\,\mathrm{MeV} $$

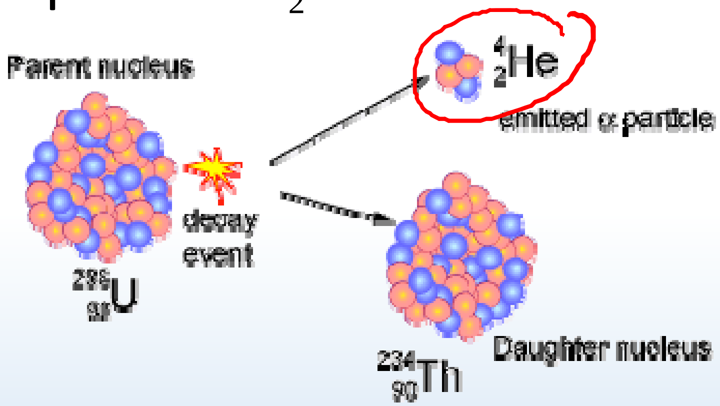
$$ E_{Th} = 234 \cdot 7.594\,\mathrm{MeV} $$

$$ E_U = 238 \cdot 7.570\,\mathrm{MeV} $$

Radioactive decay

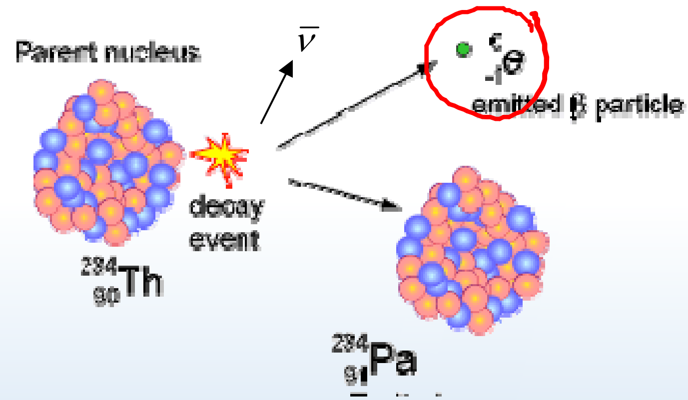
There are 3 types of radioactive decay:

α particle: $^{4}_{2}\mathrm{He}$ nucleus



Easily stopped

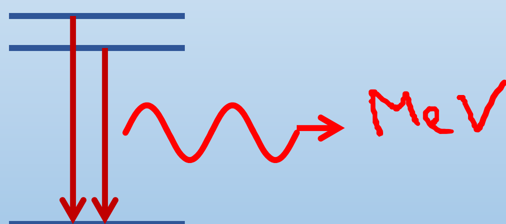
β⁻ particle: electron



Stopped by metal

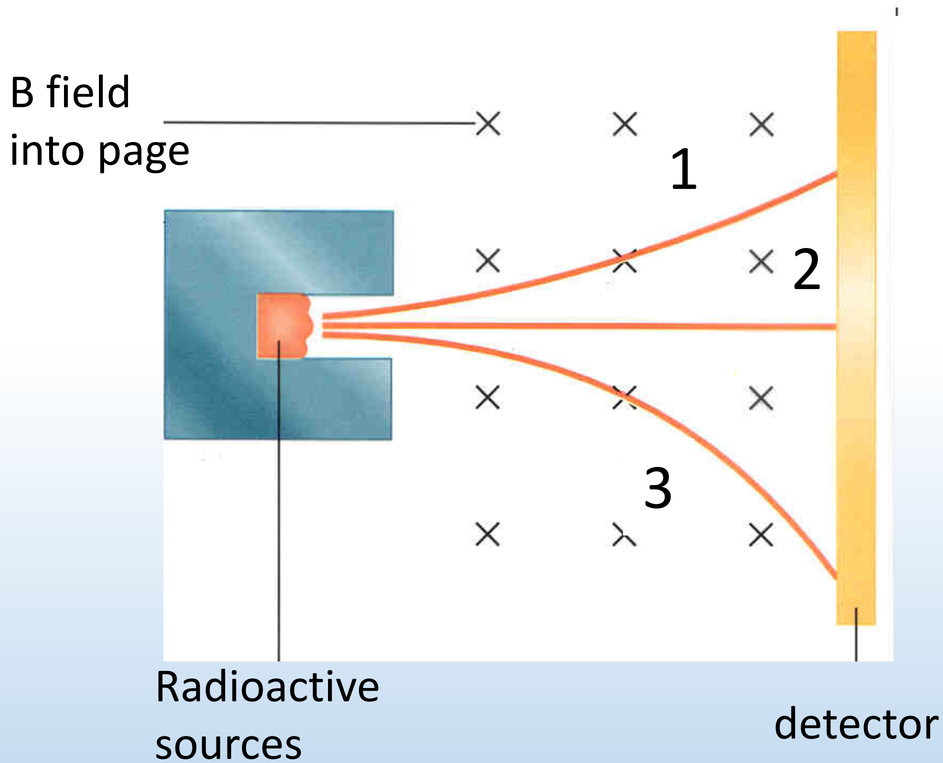
γ radiation: photon (more energetic than x-rays)

Penetrate



ACT: Types of radioactivity

Consider the following trajectories from α, β⁻, and γ sources



RHR! Recall Lect. 10

Which of the trajectories must belong to an α particle?

A. 1   B. 2   C. 3

Radioactive decay rules

1) Nucleon Number (A) is conserved.
2) Atomic Number (Z) is conserved.
3) Energy and momentum are conserved.

α decay: $^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^{4}_{2}\mathrm{He}$

α particle has 2 protons, 2 neutrons: A = 4
Charge is +2e: Z = +2

β⁻ decay: $^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z+1}\mathrm{D} + {}^{0}_{-1}e$

Electron is not a nucleon: A = 0
Charge is –1e: Z = –1

γ decay: $^{A}_{Z}\mathrm{P}^{*} \rightarrow {}^{A}_{Z}\mathrm{P} + {}^{0}_{0}\gamma$

Photon is not a nucleon: A = 0
Charge is 0: Z = 0

"nuclear isomer" excited state

ACT: Checkpoint 2.1

A nucleus undergoes α decay. Which of the following is FALSE?

25% A. Nucleon number decreases by 4

36% B. Neutron number decreases by 2

39% C. Charge on nucleus increases by 2

α decay is the emission of ${}^{4}_{2}\mathrm{He}$:

A decreases by 4

Ex: $${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

Z decreases by 2
(charge decreases!)

ACT: Checkpoint 2.2

The nucleus $^{234}_{90}\mathrm{Th}$ undergoes β^- decay. Which of the following is true?

61% A. The number of protons in the daughter nucleus increases by one.

39% B. The number of neutrons in the daughter nucleus increases by one

β^- decay emits e^-, increases nuclear charge by +1

Ex: $$^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}e^- + {}^{0}_{0}\overline{\nu}$$

Z increases by 1
(charge increases!)

"anti-neutrino" needed to conserve momentum.

ACT: Decay reactions

Which of the following decays is NOT allowed?

A. ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \alpha$

238 = 234 + 4
92 = 90 + 2

B. ${}^{214}_{84}\mathrm{Po} \rightarrow {}^{210}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

214 = 210 + 4
84 = 82 + 2

C. ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{0}\gamma$

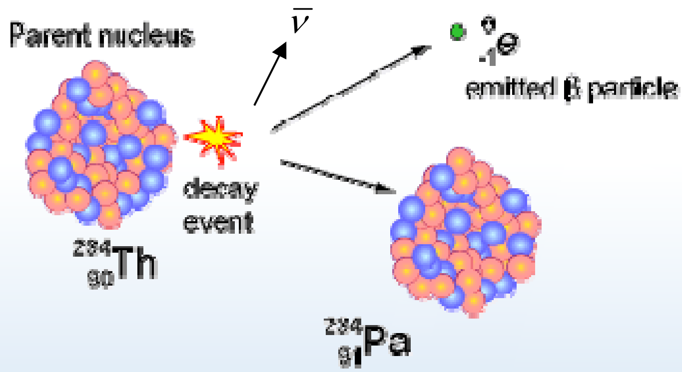
14 = 14 + 0
6 ≠ 7 + 0

D. ${}^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca} + {}^{0}_{-1}\mathrm{e}^{-} + {}^{0}_{0}\overline{\nu}$

40 = 40 + 0 + 0
19 = 20 – 1 + 0

Weak nuclear interaction

α decay is a fission reaction (strong force vs. Coulomb repulsion)
γ decay is transition between nuclear energy levels (like e⁻ transition)
β⁻ decay converts a neutron into a proton:



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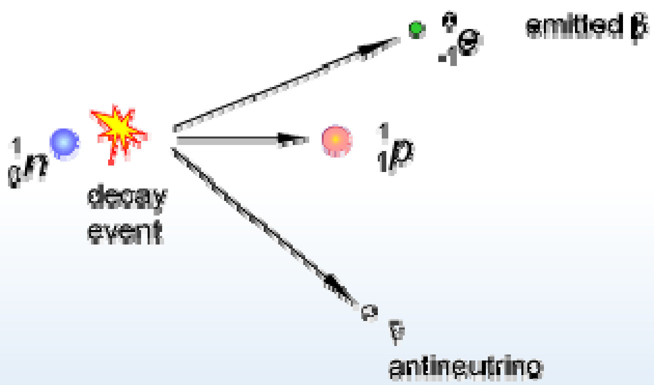


$$^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}e^{-} + {}^{0}_{0}\overline{\nu} \qquad {}^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}e^{-} + {}^{0}_{0}\overline{\nu}$$

Neutron turns to proton inside nucleus.
Electron and anti-neutrino escape nucleus.

"Weak" interaction is mechanism behind this decay process

Range only 10^{-18} m and $10^{-6}\times$ weaker than strong interaction!

Radioactive decay rates

Decay reactions are probabilistic

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

"Activity" or rate of decay → $\frac{\Delta N}{\Delta t}$

Number of un-decayed nuclei → N

Decay constant → λ

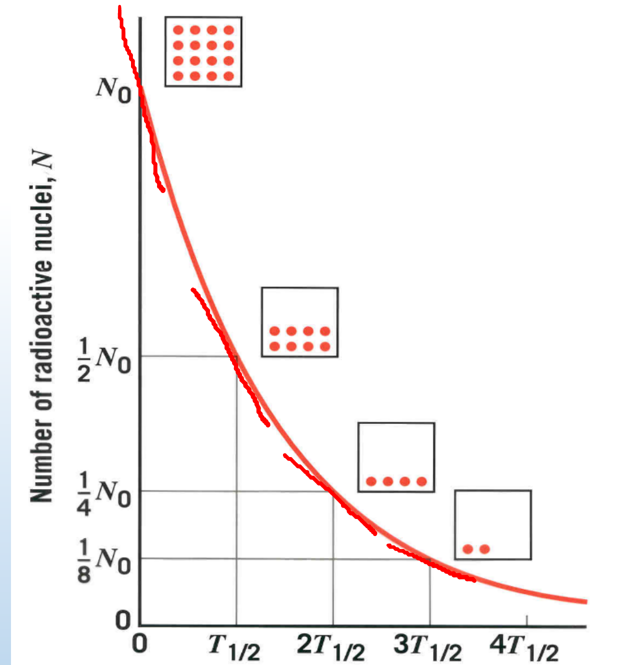
Units: "Becquerel"
(1 Bq ≡ 1 decay/s)

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$$

$(e^{\ln 2} = 2)$

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

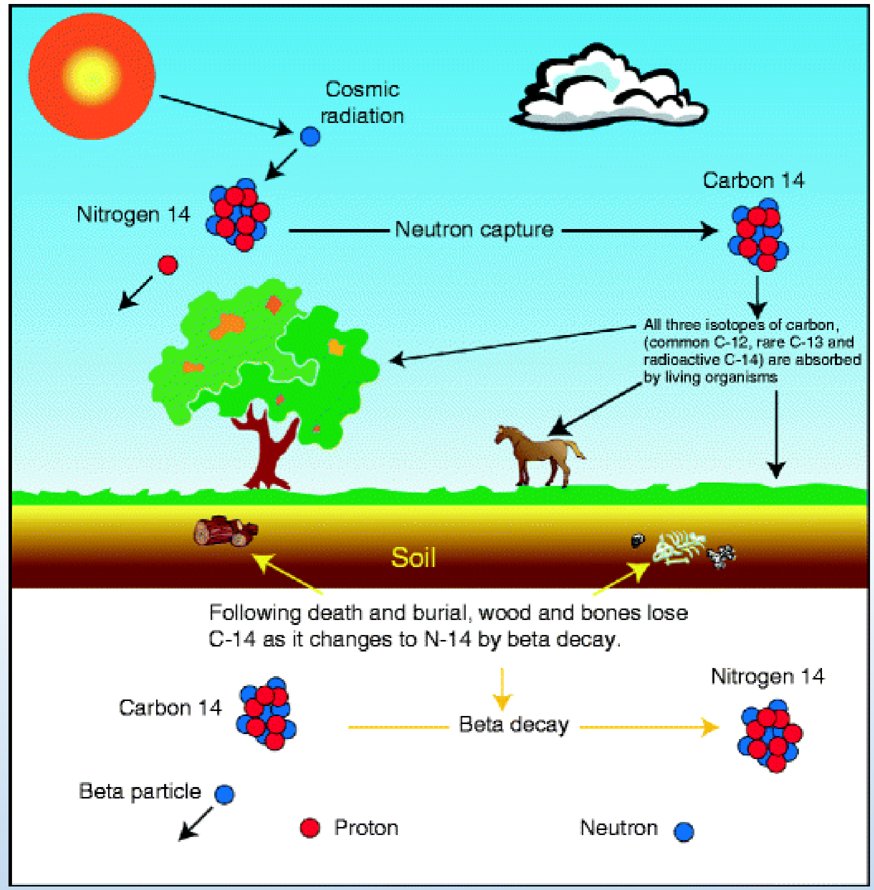
"Half-life" = time for ½ of the nuclei to decay



Ex: At $t = T_{1/2}$, ½ the nuclei survived & the activity decreased by ½
At $t = 2T_{1/2}$, ¼ the nuclei survived & the activity decreased by ¼

Calculation: carbon dating

1 in ~8 × 10^{11} C atoms is ^{14}C and β^- decays with a $T_{1/2}$ of 5730 years. Determine how many decays/s per gram of carbon occur in a living organism.



$$\frac{\Delta N}{\Delta t} = -\lambda N = 3.83\times10^{-12}\cdot 6.2\times10^{10}$$

$$= \mathbf{0.24}\ \text{Bq/gm decays/s/gm}$$

Decay constant:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5730\cdot365\cdot24\cdot60\cdot60}$$

$$= 3.83\times10^{-12}\,\text{s}^{-1}$$

Number *N* of ^{14}C atoms per gm:

$$N = \left(\frac{\#\ \text{C atoms/gm}}{8.3\times10^{11}}\right) = \frac{1}{8\times10^{11}}\left(\frac{\text{mole}}{12\,\text{gm}}\right)\left(6.02\times10^{23}\,\frac{\text{atoms}}{\text{mole}}\right) = 6.2\times10^{10}\,\frac{^{14}\text{C atoms}}{\text{gm}}$$

ACT: Carbon dating

In the previous example we found that the ^{14}C activity in living organisms is 0.24 Bq per gram of sample.

The half-life for β^- decay of ^{14}C is ~6,000 years. You test a fossil and find that its activity is 0.06 Bq/gm. How old is the fossil?

A. 3,000 years

B. 6,000 years

C. 12,000 years

At 0 yrs: activity is 0.24 Bq/gm
100% of ^{14}C nuclei remain

↓

At ~6,000 yrs: activity is 0.12 Bq/gm
50% of ^{14}C nuclei remain

↓

At ~12,000 yrs: activity is 0.06 Bq/gm
25% of ^{14}C nuclei remains

Summary of today's lecture

- Nuclear atom

  Nuclei composed of neutrons & protons (nucleons)

  Strong force binds nucleons together -> "mass defect"

- Radioactive decay

  Three types: α (He nucleus), β^- (electron), γ (photon)

  Nucleon number, charge, energy/momentum conserved

- Decay rate

  $T_{1/2}$ is time for ½ of nuclei to decay & for activity to decrease by ½

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad N(t) = N_0 2^{-t/T_{1/2}}$$