# *Your questions/comments*

**END-OF-SEMESTER ANNOUNCEMENTS:**
Check your gradebook NOW, ICES course evaluation
James Scholar Credit projects due May 2
Final exam study resources: extra practice exam, *optional* homework -> see website

"Could you give us more info about the final exam: number of questions/ how many questions per topic/what to study/etc"

"Do we have homework due next week? There is one problem listed as NOT optional."

"Hmm this is very interesting, I remember learning some of this in General Chemistry, but it's nice to get a sort of refresher on all of this."

"Much more straightforward than quantum from last lecture! Can we review the rate law, and how to derive the half-life expression from it?"

# Phys 102 – Lecture 27

**The strong & weak nuclear forces**

# *4 Fundamental forces of Nature*

Gravitational force (solar system, galaxies)

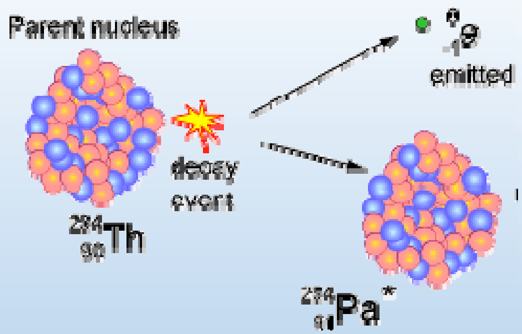
Electromagnetic force (atoms, molecules)

Today { Strong force (atomic nuclei)

Weak force (radioactive decay)

Gravitational

<




Weak

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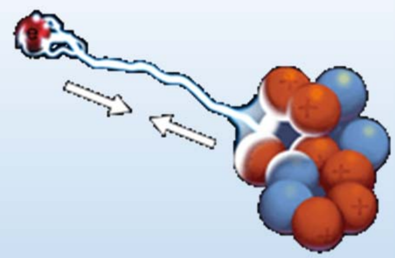

Electromagnetic

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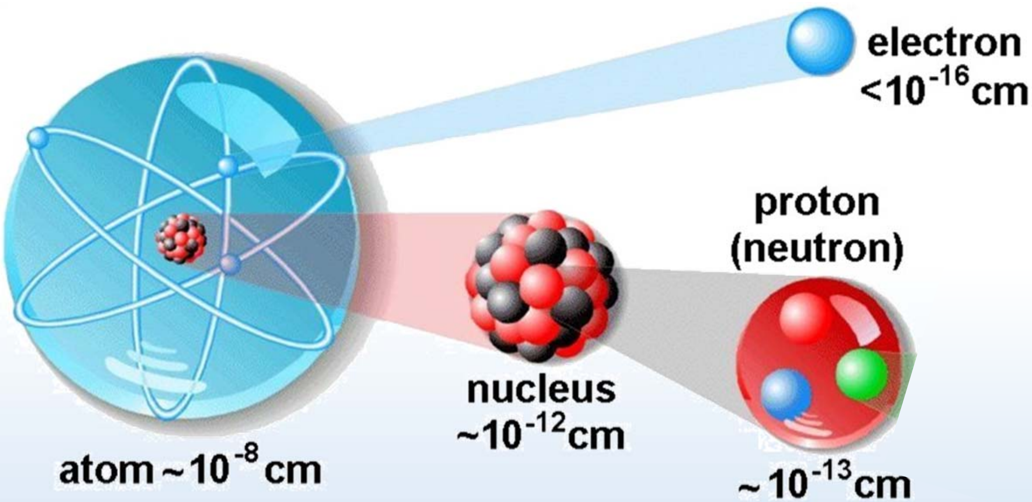
Strong

weakest strongest

# *The nucleus*

All + charge of the atom is inside a small *nucleus*, which is made up of *protons* and *neutrons* ("nucleons")




$E = mc^2$

| Particle | Mass (MeV/$c^2$) | Charge |
|---|---|---|
| electron | 0.511 | –e |
| proton | 938.3 | +e |
| neutron | 939.5 | 0 |

# *Nuclear nomenclature*

A nucleus is composed of $Z$ *protons* and $N$ *neutrons* ("nucleons")

*A* = "atomic mass number"

*Z* = "atomic number"

$$\boxed{{}^{A}_{Z}\mathrm{E}}$$

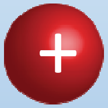
Element name

$$A = N + Z$$

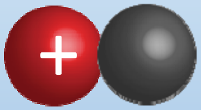
*A* = nucleon number (gives mass of nucleus since $m_{prot} \approx m_{neut} \gg m_{elec}$ )

*Z* = proton number = electron number (gives element name & chemical properties)

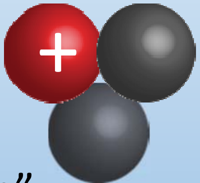
Ex: Elements with different nuclei are known as *isotopes*

${}^{1}_{1}\mathrm{H}$ 


"protium"

${}^{2}_{1}\mathrm{H}$ 


"deuterium"

${}^{3}_{1}\mathrm{H}$ 


"tritium"

# ACT: CheckPoint 1.1

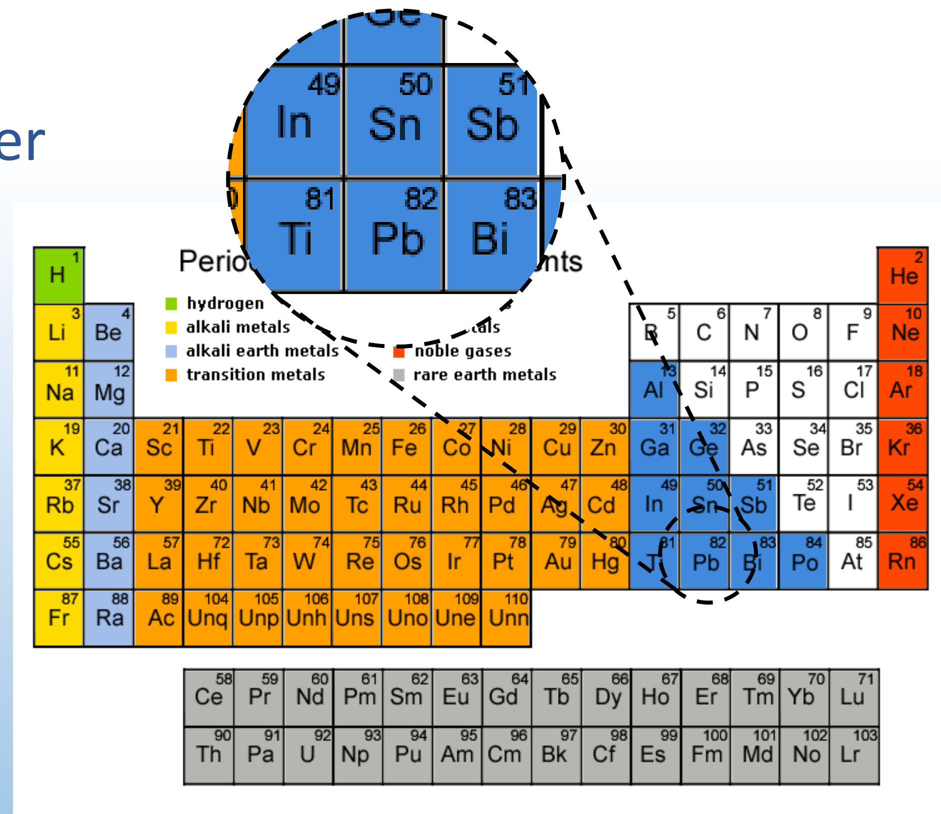
A material is known to be made from an isotope of lead (Pb).

Based on this information which of the following can you specify?

18% A. The atomic mass number

25% B. The neutron number

57% C. The proton number




Chemical properties (and name) given by number of protons (Z)

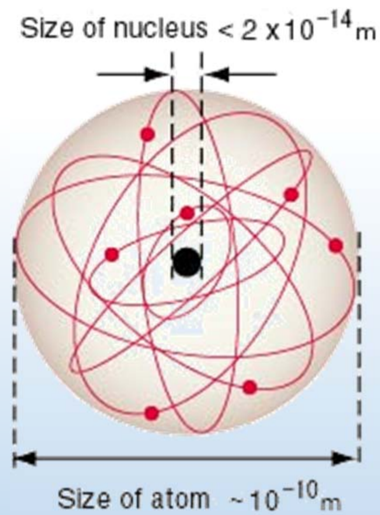
For lead Z = 82

# Nucleus size

Mass densities of nuclei are approximately the same

$$\rho_{nucleus} = \frac{M_{nucleus}}{V_{nucleus}} \quad \text{and} \quad M_{nucleus} \propto A \quad \text{so} \quad V_{nucleus} = \frac{4\pi}{3} r^3 \propto A$$

$$r \approx A^{1/3} \cdot 1.2 \times 10^{-15}\,\mathrm{m}$$ fm "femtometer"




A "

Ex: $^{27}_{13}\mathrm{Al}$

"

Z

$$\left.\begin{aligned} r_{nucleus} &\approx 3.6 \times 10^{-15}\,\mathrm{m} \\ r_{atom} &\approx 1.4 \times 10^{-10}\,\mathrm{m} \end{aligned}\right\} 10^4\text{–}10^5$$

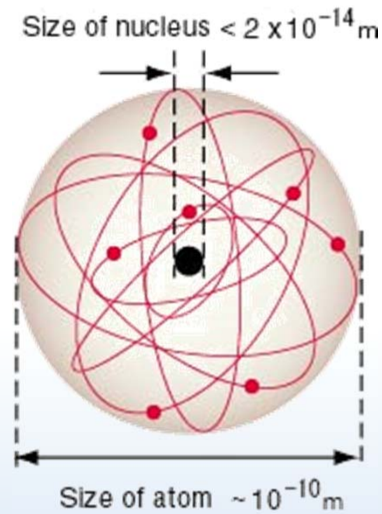
Compared to size of atom (outer $e^-$ shell)

How do protons not repel each other inside nucleus?

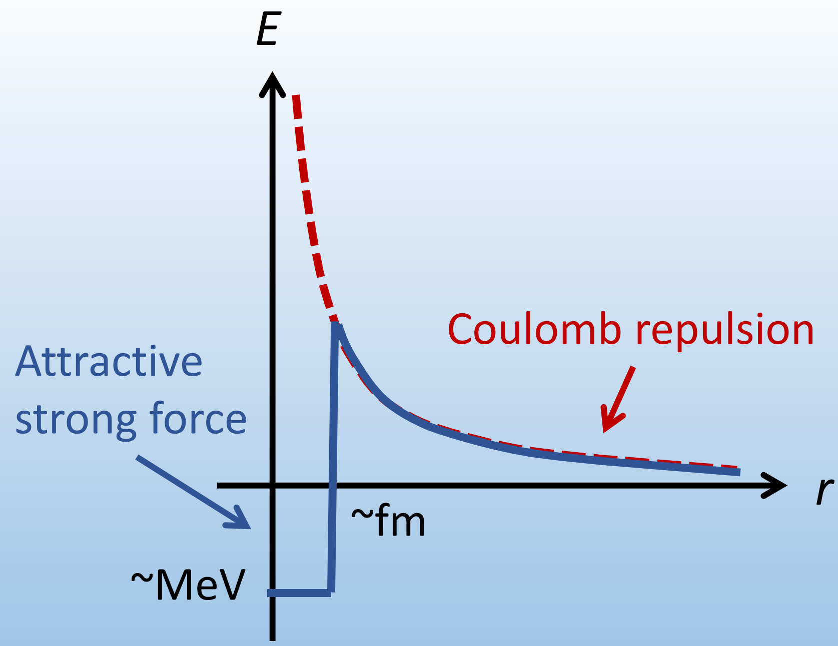
# *Strong nuclear force*

Electric potential energy of two protons in 1 fm nucleus:

$$U_{p-p} = +\frac{ke^2}{r} = \frac{1.44\,\text{eV}\cdot\text{nm}}{1\times10^{-6}\,\text{nm}} = 1.44\,\text{MeV}$$




*Strong nuclear force* binds nucleons together, overcomes Coulomb repulsion at fm distances




Ex: deuterium (1 proton + 1 neutron)

$$\left.\begin{aligned} E_{deuteron} &\approx 2.2\text{MeV} \\ E_{atom} &\approx 13.6\,\text{eV} \end{aligned}\right\} 10^5$$

DEMO

# *Binding energy & mass defect*

Nuclear binding energy decreases mass of nucleus

Einstein's equation: $$\boxed{E_0 = mc^2}$$ Equivalence of mass and energy

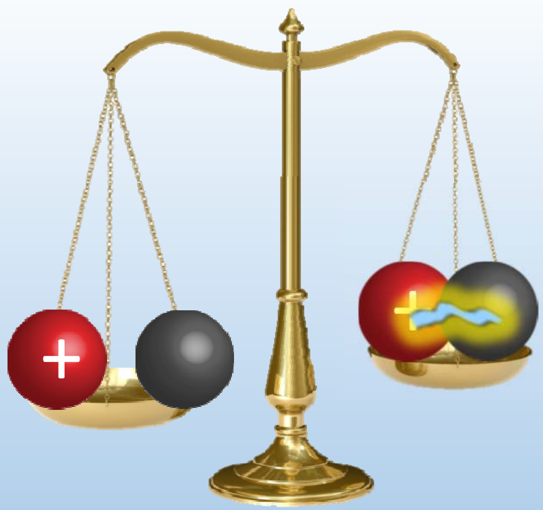
$$m_{nucleus} = Zm_{prot} + Nm_{neut} - \frac{|E_{bind}|}{c^2}$$ "Mass defect"

Ex: deuteron = 1 proton + 1 neutron

$$m_{prot} = 938.3\,\mathrm{MeV}/c^2$$

$$+ \quad m_{neut} = 939.5\,\mathrm{MeV}/c^2$$

$$= \quad m_{tot} = 1877.8\,\mathrm{MeV}/c^2$$



$$m_{deut} = 1875.6\,\mathrm{MeV}/c^2$$

Difference is binding energy!
For deuteron:

$$E_{bind} = 2.2\,\mathrm{MeV}$$

# *ACT: Binding Energy*

Which system "weighs" more?

A  B 

A. Two balls attached by a relaxed spring

B. Two balls attached by a stretched spring

C. They have the same weight

$$M_A = m_{balls} + m_{spring}$$

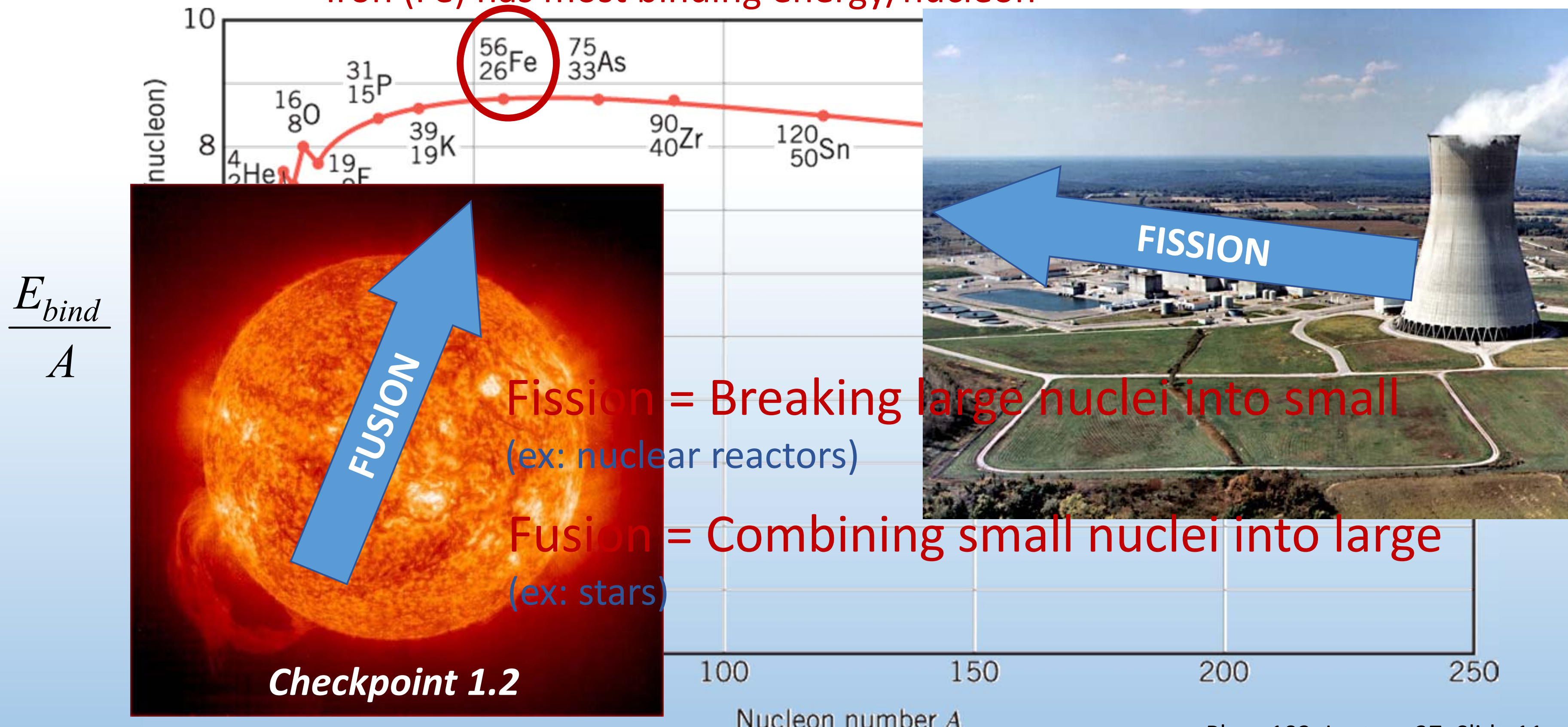
$$M_B = m_{balls} + m_{spring} + \frac{E_{spring}}{c^2}$$ Notice + sign because energy is *increased*

$$M_B - M_A = \frac{E_{spring}}{c^2} \sim \frac{1\,\mathrm{J}}{(3\times 10^8\,\mathrm{m/s})^2} \approx 10^{-17}\,\mathrm{kg}$$

# *Binding energy plot*

Binding energy per nucleon increases with *A* due to higher strong force, then decreases due to Coulomb repulsion

Iron (Fe) has most binding energy/nucleon




Fission = Breaking large nuclei into small

(ex: nuclear reactors)

Fusion = Combining small nuclei into large

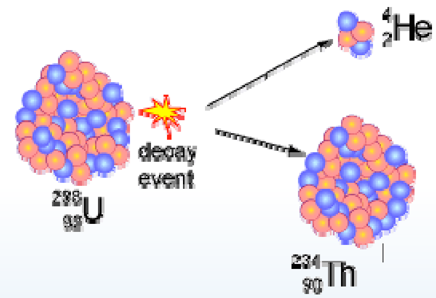
(ex: stars)

# ACT: Uranium mass

$^{238}$U is long-lived but ultimately unstable. Eventually, it will spontaneously break into a $^{4}$He and $^{234}$Th nucleus, and release a tremendous amount of energy:

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} + \text{energy}$$

$$m_u = m_{Th} + m_{He} + E/c^2$$




What must be true about the masses of the nuclei?

A. $m_U > m_{Th} + m_{He}$

B. $m_U = m_{Th} + m_{He}$

C. $m_U < m_{Th} + m_{He}$

Th + He is more stable (i.e. has a higher binding energy) than U alone

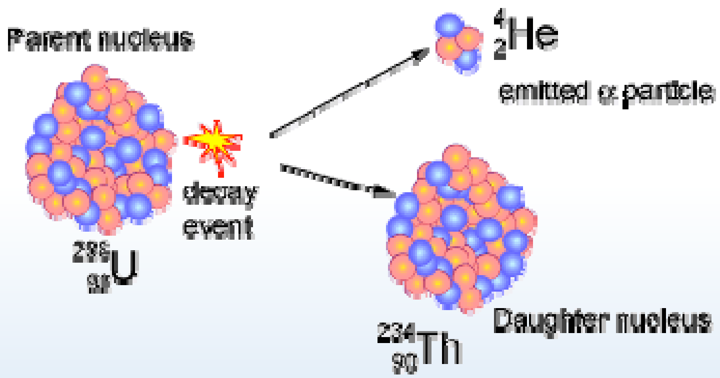
$$m_{nucleus} = Zm_{prot} + Nm_{neut} - \frac{|E_{bind}|}{c^2}$$

Extra HW

# *Radioactive decay*

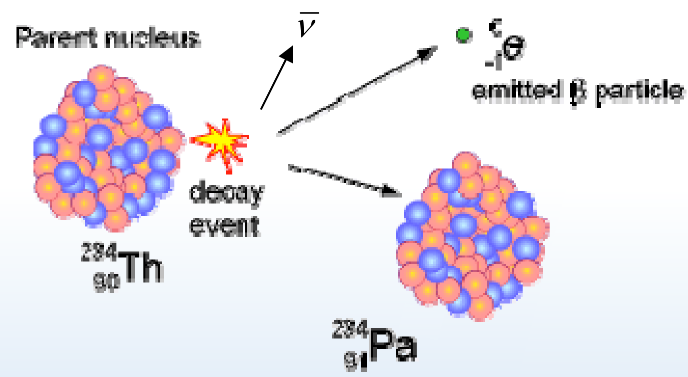
There are 3 types of radioactive decay:

α particle: $^{4}_{2}\mathrm{He}$ nucleus




Easily stopped

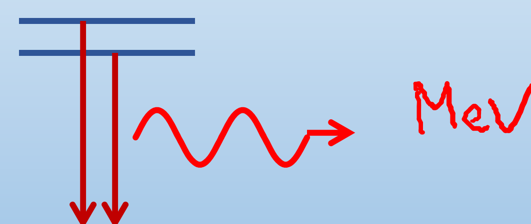
β⁻ particle: electron




Stopped by metal

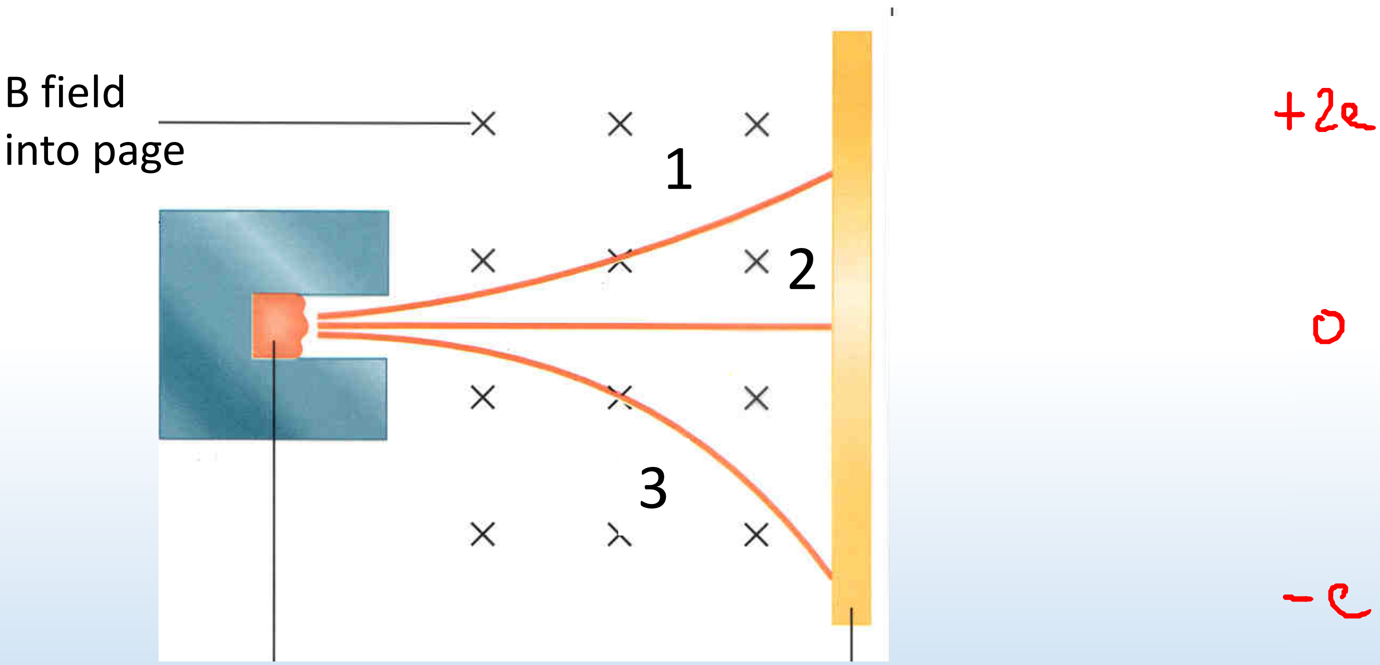
γ radiation: photon (more energetic than x-rays)

Penetrate

# ACT: Types of radioactivity

Consider the following trajectories from α, β⁻, and γ sources




RHR! Recall Lect. 10

Which of the trajectories must belong to an α particle?

A. 1    B. 2    C. 3

# *Radioactive decay rules*

1) Nucleon Number (A) is conserved.
2) Atomic Number (Z) is conserved.
3) Energy and momentum are conserved.

α decay: $^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^{4}_{2}\mathrm{He}$

α particle has 2 protons, 2 neutrons: A = 4
Charge is +2e: Z = +2

β⁻ decay: $^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z+1}\mathrm{D} + {}^{0}_{-1}e$

Electron is not a nucleon: A = 0
Charge is –1e: Z = –1

γ decay: $^{A}_{Z}\mathrm{P}^{*} \rightarrow {}^{A}_{Z}\mathrm{P} + {}^{0}_{0}\gamma$

Photon is not a nucleon: A = 0
Charge is 0: Z = 0

"nuclear isomer"
excited state

# ACT: Checkpoint 2.1

A nucleus undergoes α decay. Which of the following is FALSE?

28%   A. Nucleon number decreases by 4

35%   B. Neutron number decreases by 2

37%   C. Charge on nucleus increases by 2

α decay is the emission of ${}^{4}_{2}\mathrm{He}$ :

A decreases by 4

$A = N + Z$

Ex: $${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

Z decreases by 2
(charge decreases!)

# ACT: Checkpoint 2.2

The nucleus ${}^{234}_{90}\mathrm{Th}$ undergoes $\beta^-$ decay. Which of the following is true?

63% A. The number of protons in the daughter nucleus increases by one.

37% B. The number of neutrons in the daughter nucleus increases by one

$\beta^-$ decay emits $e^-$, increases nuclear charge by +1

Ex: $${}^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}e^- + \boxed{{}^{0}_{0}\bar{\nu}}$$

"anti-neutrino" needed to conserve momentum.

Z increases by 1
(charge increases!)

# ACT: Decay reactions

Which of the following decays is NOT allowed?

A. $^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + \alpha$

238 = 234 + 4
92 = 90 + 2

B. $^{214}_{84}\mathrm{Po} \rightarrow {}^{210}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

214 = 210 + 4
84 = 82 + 2

C. $^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{0}\gamma$

14 = 14 + 0
6 ≠ 7 + 0

D. $^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca} + {}^{0}_{-1}\mathrm{e}^{-} + {}^{0}_{0}\overline{\nu}$

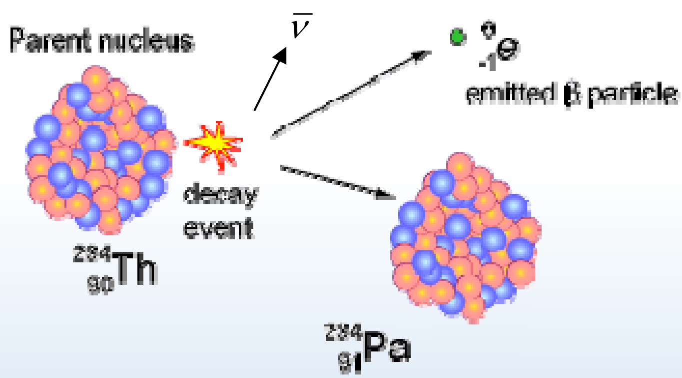
40 = 40 + 0 + 0
19 = 20 – 1 + 0

# *Weak nuclear interaction*

α decay is a fission reaction (strong force vs. Coulomb repulsion)

γ decay is transition between nuclear energy levels (like $e^-$ transition)

$\beta^-$ decay converts a neutron into a proton:

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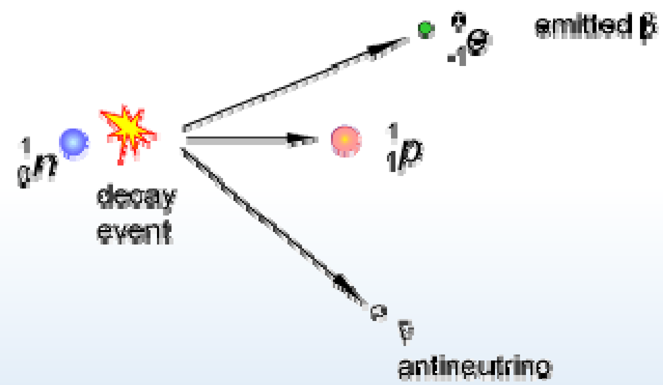



$$^{234}_{90}\mathrm{Th} \to {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}e^- + {}^{0}_{0}\overline{\nu} \qquad {}^{1}_{0}\mathrm{n} \to {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}e^- + {}^{0}_{0}\overline{\nu}$$

Neutron turns to proton inside nucleus.
Electron and anti-neutrino escape nucleus.

"Weak" interaction is mechanism behind this decay process

Range only $10^{-18}$ m and $10^{-6}\times$ weaker than strong interaction!

# *Radioactive decay rates*

Decay reactions are probabilistic

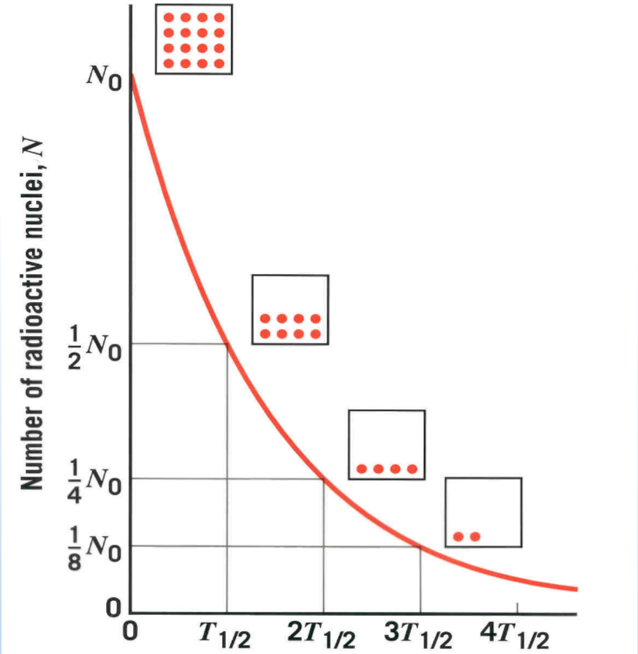
"Activity" or rate of decay

$$\boxed{\frac{\Delta N}{\Delta t} = -\lambda N}$$

Number of un-decayed nuclei

Decay constant

Units: "Becquerel"
(1 Bq ≡ 1 decay/s)




$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$$

$e^{\ln 2} = 2$

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

"Half-life" = time for ½ of the nuclei to decay

Ex: At $t = T_{1/2}$, ½ the nuclei survived & the activity decreased by ½
At $t = 2T_{1/2}$, ¼ the nuclei survived & the activity decreased by ¼

# Calculation: carbon dating

1 in ~$8 \times 10^{11}$ C atoms is $^{14}$C and $\beta^-$ decays with a $T_{1/2}$ of 5730 years. Determine how many decays/s per gram of carbon occur in a living organism.

$$\frac{\Delta N}{\Delta t} = -\lambda N = 3.83\times10^{-12}\cdot 6.2\times10^{10}$$

$$= \mathbf{0.24} \text{ Bq/gm decays/s/gm}$$

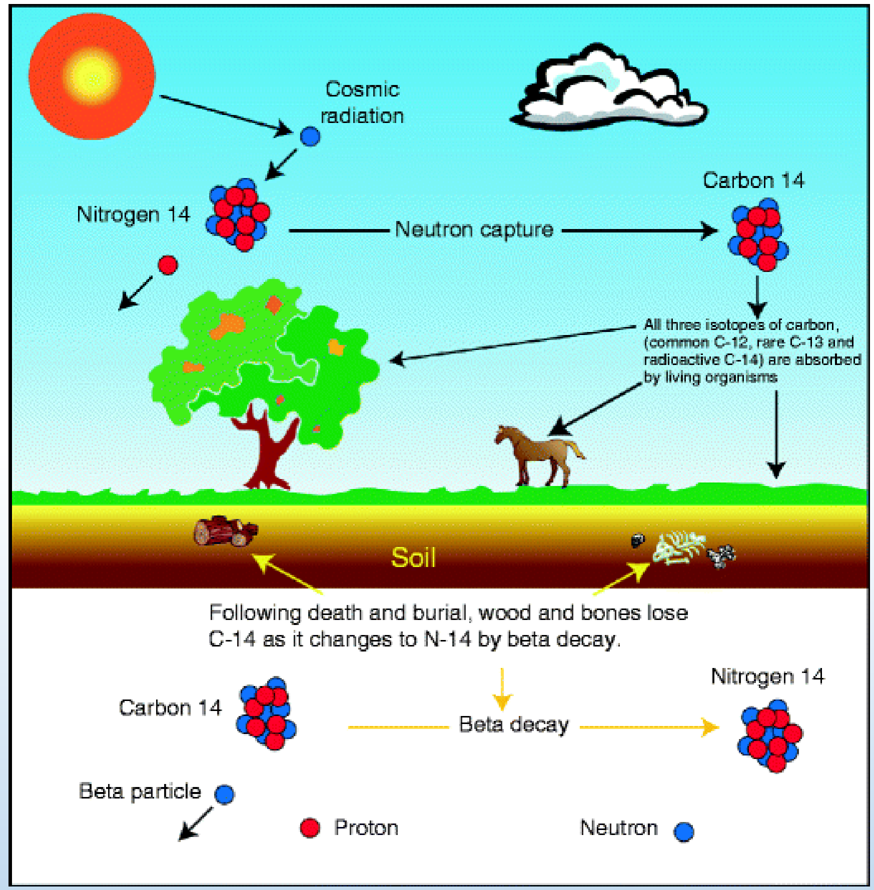
Decay constant:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5730\cdot365\cdot24\cdot60\cdot60}$$

$$= 3.83\times10^{-12}\,\text{s}^{-1}$$

Number $N$ of $^{14}$C atoms per gm:

$$N = \left(\frac{\#\text{ C atoms/gm}}{8\times10^{11}}\right) = \frac{1}{8\times10^{11}}\left(\frac{\text{mole}}{12\,\text{gm}}\right)\left(6.02\times10^{23}\frac{\text{atoms}}{\text{mole}}\right) = 6.2\times10^{10}\frac{^{14}\text{C atoms}}{\text{gm}}$$



# ACT: Carbon dating

In the previous example we found that the $^{14}C$ activity in living organisms is 0.24 Bq per gram of sample.

The half-life for $\beta^-$ decay of $^{14}C$ is ~6,000 years. You test a fossil and find that its activity is 0.06 Bq/gm. How old is the fossil?

A. 3,000 years

B. 6,000 years

C. 12,000 years

At 0 yrs: activity is 0.24 Bq/gm
100% of $^{14}C$ nuclei remain

At ~6,000 yrs: activity is 0.12 Bq/gm
50% of $^{14}C$ nuclei remain

At ~12,000 yrs: activity is 0.06 Bq/gm
25% of $^{14}C$ nuclei remains

# *Summary of today's lecture*

- Nuclear atom

    Nuclei composed of neutrons & protons (nucleons)

    Strong force binds nucleons together -> "mass defect"

- Radioactive decay

    Three types: α (He nucleus), $\beta^-$ (electron), γ (photon)

    Nucleon number, charge, energy/momentum conserved

- Decay rate

    $T_{1/2}$ is time for ½ of nuclei to decay & for activity to decrease by ½

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad N(t) = N_0 2^{-t/T_{1/2}}$$