

Your questions/comments

“The hardest part of this checkpoint was determining whether the work done on the particle was negative or positive.”

“Does a negative sign potential energy mean the energy decreases?”

“When does the electric field do positive work and when does it do negative work?”

“I didn't understand the concept of electrical potential energy, more specifically why the potential energy decreases as r decreases.”

“I am confused on how to find how much work it would take make the charges assemble because it is different from just moving the charges from point A to point B.”



Phys 102 – Lecture 4

Electric potential energy & work

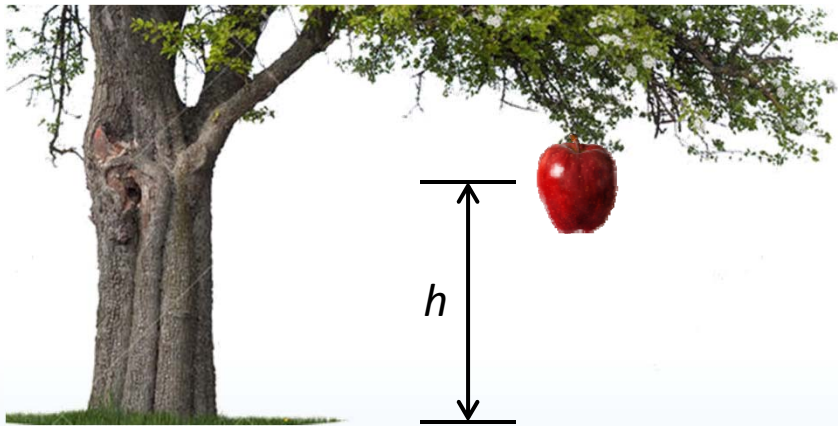
Today we will...

- Learn about the electric potential energy
- Relate it to work
 - Ex: charge in uniform electric field, point charges
- Apply these concepts
 - Ex: electron microscope, assembly of point charges, dipole energy & hydration shells

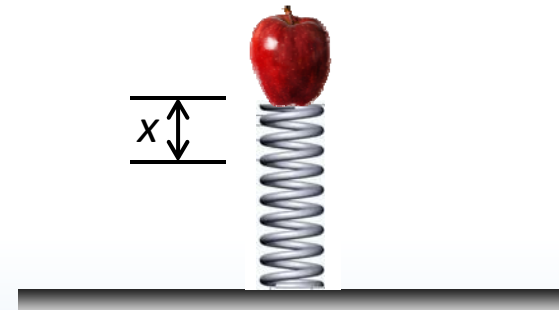
Potential energy

Potential energy U – stored energy, can convert to kinetic energy K

Review Phys. 101



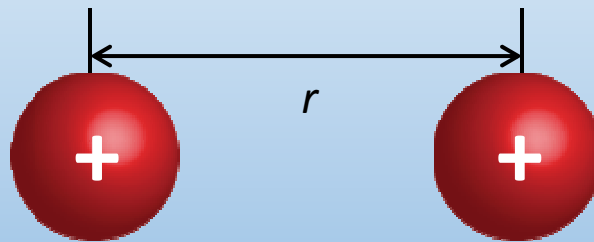
Gravitational potential energy (ex: falling object)



Elastic potential energy (ex: spring)

Total energy $K + U$ is conserved

Same ideas apply to electricity



Electric potential energy (ex: repelling charges)

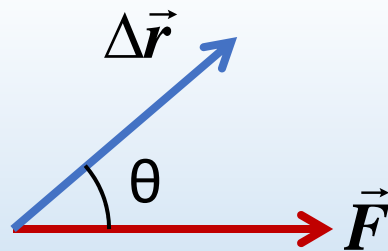
Work

Review Phys. 101

Work – transfer of energy when a force acts on a moving object

Work done by force F →
$$W_F = F \Delta r \cos \theta = -W_{you} = -\Delta U$$

Displacement Angle between force and displacement Change in potential energy



Units: J (“Joules”)

It matters who does the work

For *conservative* forces, work is related to potential energy

Electric potential energy & work

$$W_F = -W_{you} = -\Delta U = F \Delta r \cos \theta$$

Gravity



Electricity

Mass raised $y_i \rightarrow y_f$

Charge moved $x_i \rightarrow x_f$
(in uniform E field to left)

$$F_G = mg \quad \text{down}$$

$$F_E = qE \quad \text{left}$$

$$W_G = -mgh$$

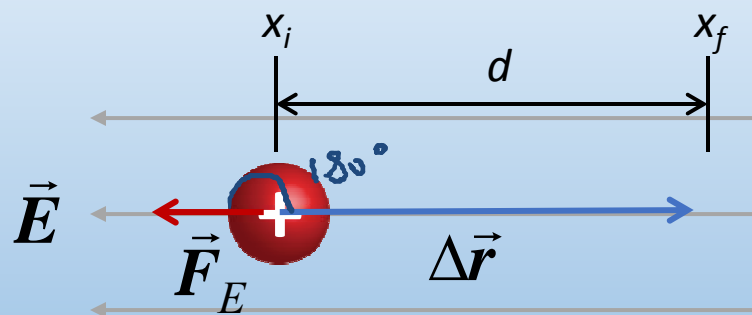
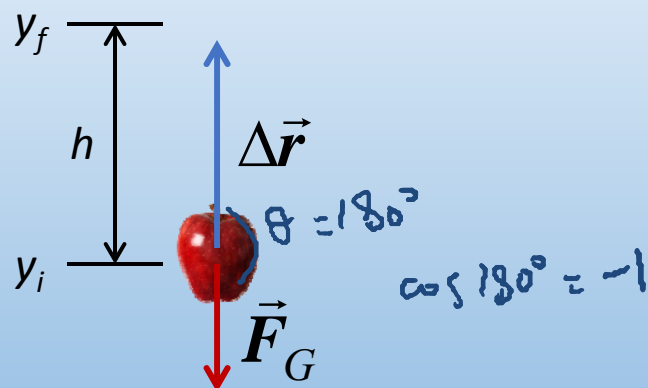
$$W_E = -qEd$$

$$W_{you} = +mgh$$

$$W_{you} = +qEd$$

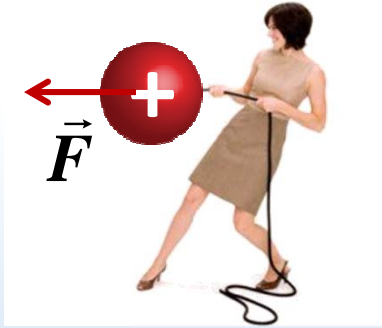
$$\Delta U_G = +mgh$$

$$\Delta U_E = +qEd$$



Positive and negative work

If you moved object against external force (gravitational, electric, etc.), you did positive work, force did negative work



$$W_{you} > 0 \quad W_F < 0$$



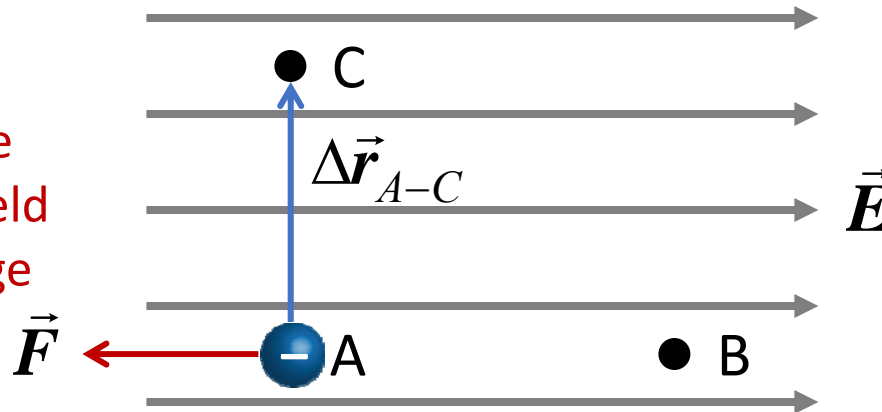
$$W_{you} < 0 \quad W_F > 0$$

If you moved object along external force (gravitational, electric, etc.), you did negative work, force did positive work

Checkpoint 1.2

Checkpoint 1.1

Force points in same direction as the E field for a POSITIVE charge



When a negative charge is moved from A to C the ELECTRIC force does

- A. positive work 6%
- B. zero work 90%**
- C. negative work 4%

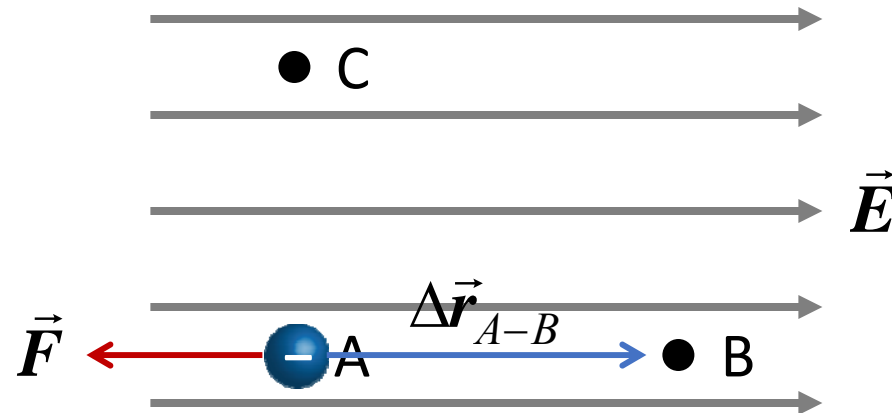
$$W_F = F \Delta r_{A-C} \cos \theta$$

$$\theta = 90^\circ \quad \cos(90^\circ) = 0$$

Think about moving the apple parallel to ground.
Does its gravitational potential energy change?



ACT: Checkpoint 1.3



When a negative charge is moved from A to B the ELECTRIC force does

A. positive work 54%

B. zero work 5%

C. negative work 41%

$$W_F = F \Delta r_{A-B} \cos \theta$$

$$\theta = 180^\circ \quad \cos(180^\circ) = -1$$

Electric force did *negative* work

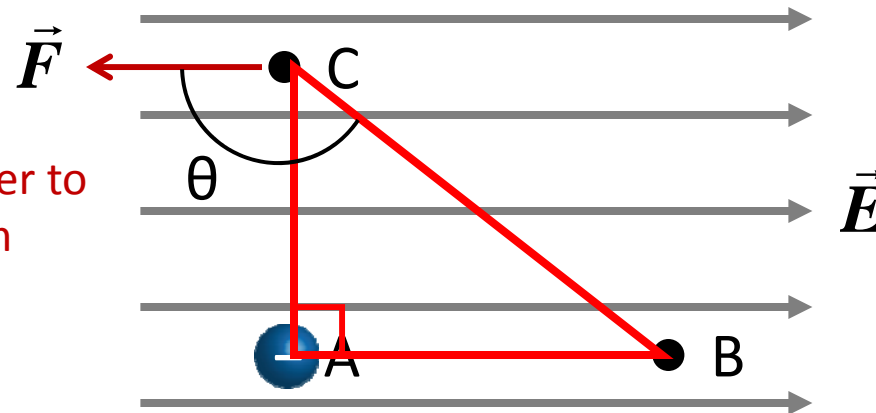
You did *positive* work

EPE increased (**CheckPoint 1.5**)



ACT: Work in a uniform E field

Let W_{A-B} be the answer to the previous problem



The negative charge is now moved from A to C to B. The work done by the electric force is

A. Greater than W_{A-B}

B. Same as W_{A-B}

C. Less than W_{A-B}

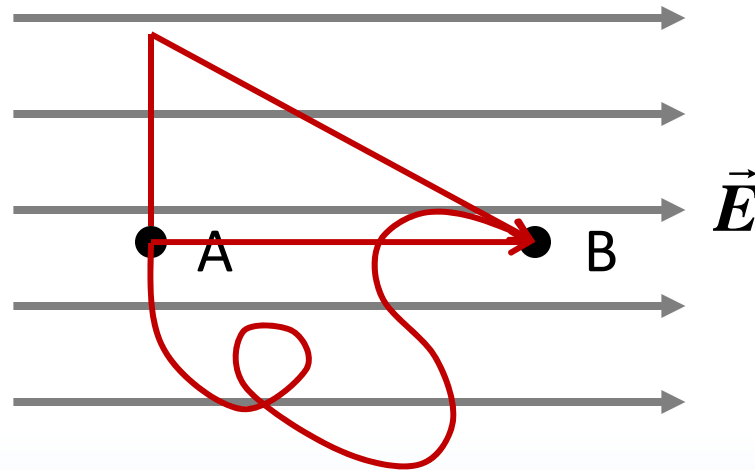
$$W_{A-C-B} = W_{A-C} + W_{C-B}$$

$$= 0 + F_E \Delta r_{C-B} \cos \theta$$

$$= 0 - F_E \Delta r_{A-B} = W_{A-B} \quad \checkmark$$

Path does not matter!

Path independence of work



For conservative forces (ex: gravitational, electric), work is independent of path. Work depends only on end points.

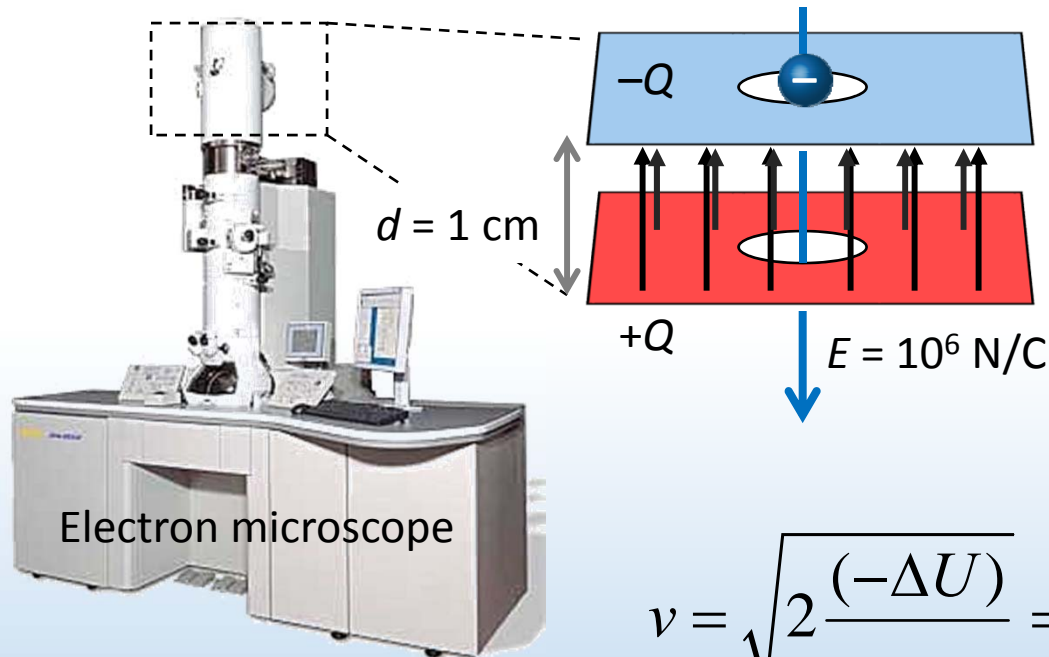
$$W_{A \rightarrow B} = -\Delta U = -(U_B - U_A)$$

Potential energy of
charge at position B

Potential energy of
charge at position A

Calculation: Electron microscope (revisited)

A uniform E field generated by parallel plates accelerates electrons in an electron microscope. If an electron starts from rest at the top plate what is its final velocity?



Approach: conservation of energy

Initial energy = Final energy

$$K_i + U_i = K_f + U_f$$

$$0 + U_i = \frac{1}{2} m \cancel{v^2} + U_f$$

$$v = \sqrt{2 \frac{(-\Delta U)}{m}} = \sqrt{2 \frac{qEd}{m}} \leftarrow \text{Review slide 5}$$

Same answer as in Lect. 3
using kinematic equations!

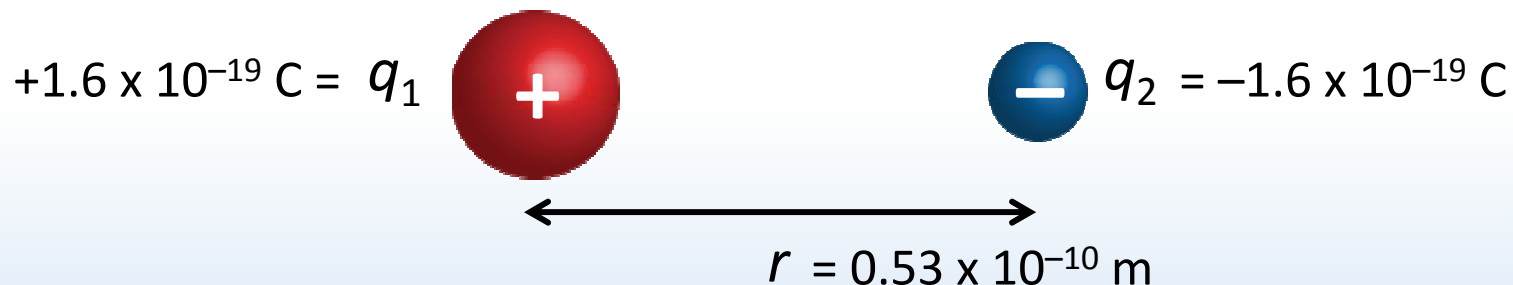
$$= \sqrt{2 \frac{1.6 \times 10^{-19} \cdot 10^6 \cdot 0.01}{9.11 \times 10^{-31}}} = 5.9 \times 10^7 \text{ m/s}$$

E.P.E of two point charges

Electric potential energy of two charges q_1 and q_2 separated by a distance r

$$U_E = k \frac{q_1 q_2}{r}$$

Note: NOT r^2



Ex: What is the electric potential energy of the proton and the electron in H?

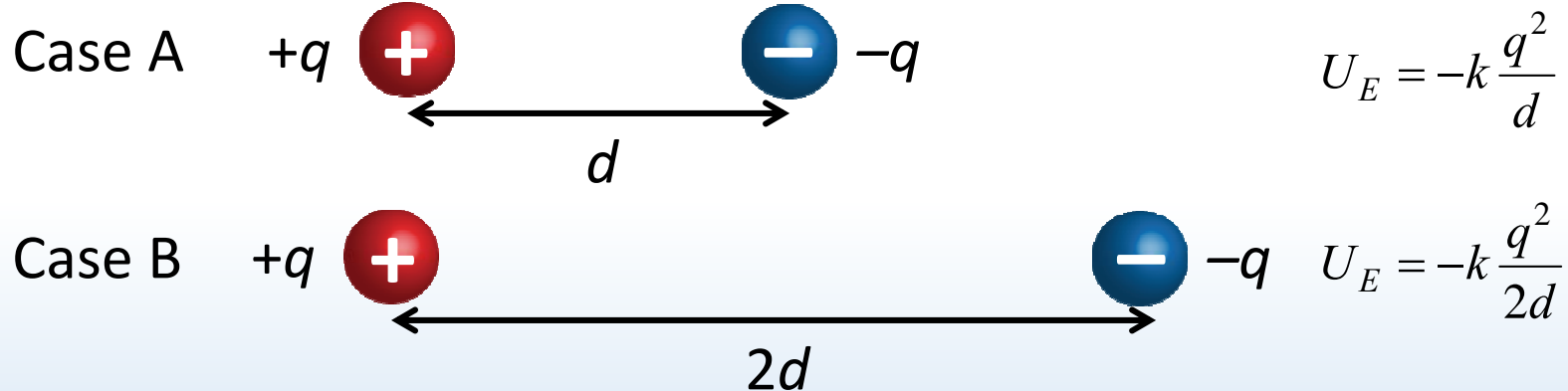
$$U_E = k \frac{q_e q_p}{r} = \frac{9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 (-1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})}{0.53 \times 10^{-10} \text{ m}} = -4.3 \times 10^{-18} \text{ J}$$

Negative



ACT: E.P.E. of 2 charges

In case A, two charges of equal magnitude but opposite sign are separated by a distance d . In case B, they are separated by $2d$.



Which configuration has a higher electric potential energy?

A. Case A has a higher E.P.E.

B. Case B has a higher E.P.E.

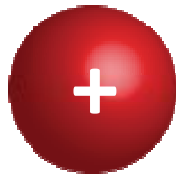
C. Both have the same E.P.E.

$U_E < 0$ in case A and B.
Case B is less negative

Sign of potential energy

What does it mean to have a negative electric potential energy?

Ex: H atom



Proton

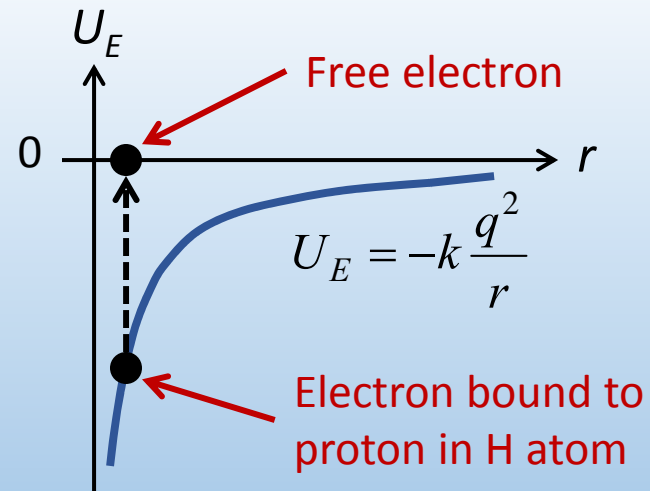


Electron

$U_E < 0$ relative to energy of an electron very far away ($r \rightarrow \infty$), away from E field of proton, i.e. a “free” electron

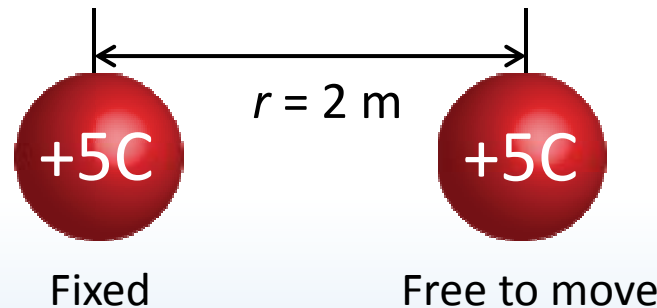
Energy must be added in order to free electron bound to proton

We will revisit this when discussing atomic physics later in the semester



Calculation: two charges

Two +5 C, 1 kg charges are separated by a distance of 2 m. At $t = 0$ the charge on the right is released from rest (the left charge is fixed). What is the speed of the right charge after a long time ($t \rightarrow \infty$)?



From EX 1, SPRING '10

Approach: conservation of energy

$$K_i + U_i = K_f + U_f$$

$$0 + U_i = \frac{1}{2}mv^2 + 0$$

$$U = \frac{kq^2}{r}$$

Handwritten note: kq, q^2 over r

$$v = \sqrt{2 \frac{kq^2}{mr}} = \sqrt{2 \frac{9 \times 10^9 \cdot 5^2}{1 \cdot 2}} = 4.7 \times 10^5 \frac{\text{m}}{\text{s}}$$

Much harder to solve problem with kinematic equations! F and a are not constant!

Work done to assemble charges

How much work do you do assembling configuration of charges?



Imagine bringing charges from infinitely far away to a separation r

$$W_{you} = +\Delta U_E = k \frac{q_1 q_2}{r} - 0$$

Potential energy of charges in final configuration

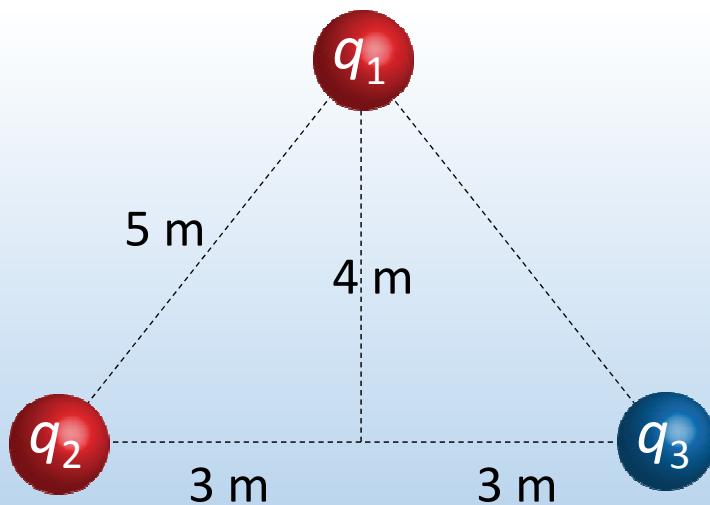
Potential energy of charges infinitely far

Calculation: assembling charges

How much work do you do to assemble the charges $q_1 = +2 \mu\text{C}$, $q_2 = +7 \mu\text{C}$, and $q_3 = -3.5 \mu\text{C}$ into a triangle?

Approach:

- Bring each charge in from far away
- Add up work from each charge pair



Does the order matter?

$$W_1 = 0 \quad \text{No E fields from other charges!}$$

$$W_2 = k \frac{q_1 q_2}{r_{12}} \\ = 9 \times 10^9 \frac{(2 \times 10^{-6})(7 \times 10^{-6})}{5} = 25.2 \text{ mJ}$$

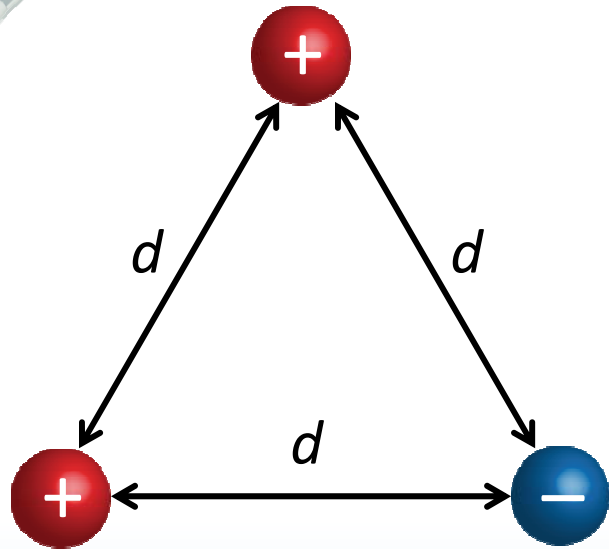
$$W_3 = k \frac{q_1 q_3}{r_{13}} + k \frac{q_2 q_3}{r_{23}} \\ = 9 \times 10^9 \frac{(2 \times 10^{-6})(-3.5 \times 10^{-6})}{5} \\ + 9 \times 10^9 \frac{(7 \times 10^{-6})(-3.5 \times 10^{-6})}{6} = -49.4 \text{ mJ}$$

$$W_{\text{you}} = -24.2 \text{ mJ} \quad \Delta U = -24.2 \text{ mJ}$$

$$W_E = +24.2 \text{ mJ}$$



ACT: Checkpoint 2.1



Charges of equal magnitude are assembled into an equilateral triangle

The total work required by you to assemble this set of charges is:

A. positive 58%

B. zero 17%

C. negative 25%

Bring in (1): zero work $W_1 = 0$

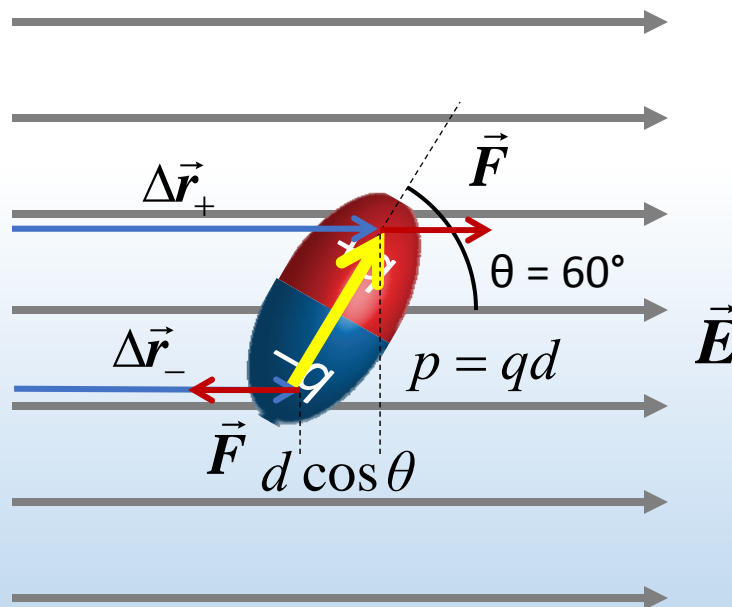
Bring in (2): positive work $W_2 = k \frac{q^2}{d}$

Bring in (3): negative work x 2 $W_3 = -k \frac{2q^2}{d}$

$$W_{you} = -k \frac{q^2}{d}$$

Calculation: dipole in E-field

An electric dipole with moment $p = 6.2 \times 10^{-30} \text{ C}\cdot\text{m}$ is placed in a uniform external electric field $E = 10^6 \text{ N/C}$ at an angle $\theta = 60^\circ$. Calculate the total *electric potential energy* of the dipole.



Approach: calculate work done by E field assembling dipole

$$\begin{aligned} W_{E-dip.} &= qE\Delta r_+ - qE\Delta r_- \\ &= qE(\Delta r_+ - \Delta r_-) \\ &= qEd \cos \theta \\ &= pE \cos \theta = -U_{dip.} \end{aligned}$$

$$\begin{aligned} U_{dip} &= -6.2 \times 10^{-30} \cdot 10^6 \cos(60^\circ) \\ &= -3.1 \times 10^{-24} \text{ J} \end{aligned}$$

$U_{dip.} = -pE \cos \theta$



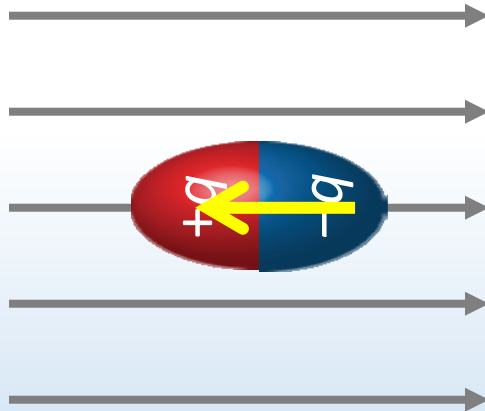


ACT: dipole energy

Which configuration of dipole in a uniform electric field has the lowest electric potential energy?

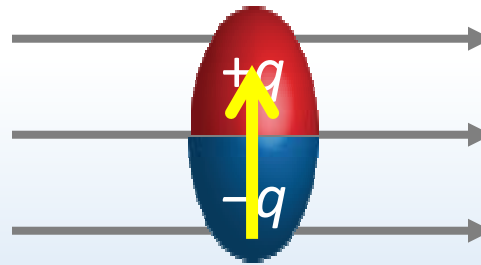
$$U_{dip.} = -pE \cos \theta$$

A.



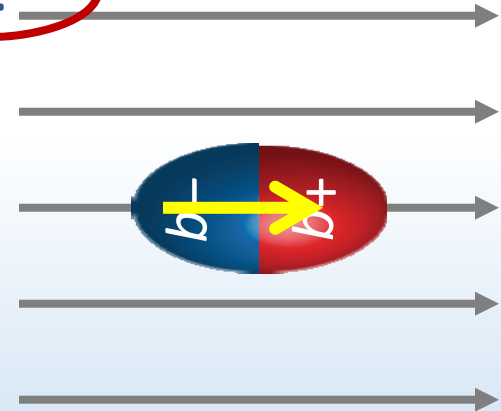
$$\theta = 180^\circ \quad U_{dip.} = +pE$$

B.



$$\theta = 90^\circ \quad U_{dip.} = 0$$

C.



$$\theta = 0^\circ \quad U_{dip.} = -pE$$

Recall Lect. 2: Dipoles in A & B will rotate to align to E field: *K.E.* converted from *E.P.E.* Dipole in C has minimum *E.P.E.*: no rotation

Hydration shell radius

Recall that H_2O dipole aligns to electric field of ions. However, at room temperature, H_2O also has rotational kinetic energy that tends to randomize dipole orientation

$K_{dip.} + U_{dip.} \geq 0$ Dipoles are randomized (bulk water)

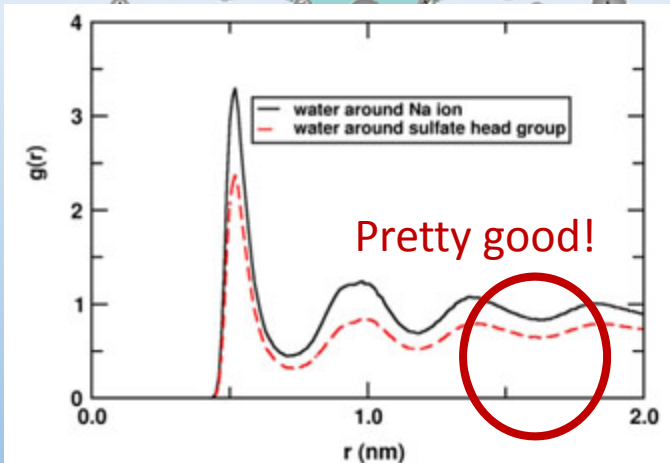
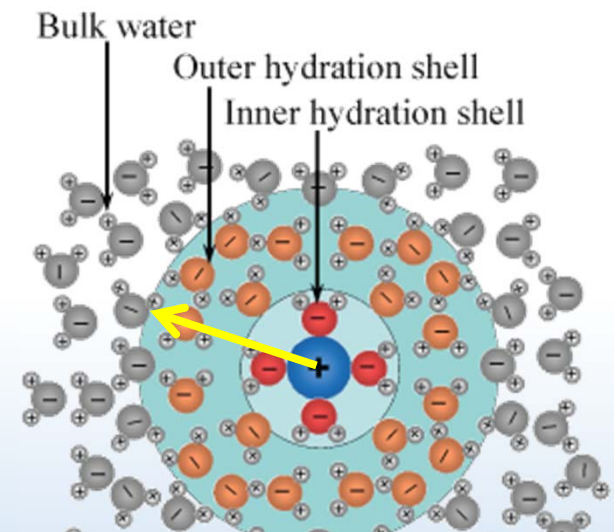
$K_{dip.} + U_{dip.} < 0$ Dipoles tend to be aligned (hydration shell)

We can estimate distance to interface between bulk and ordered water for a monovalent ion (Ex: Na^+)

$$U_{dip.} = -pE = -p \frac{kq}{r^2} \quad p_{\text{H}_2\text{O}} = 6.2 \times 10^{-30} \text{ C} \cdot \text{m}$$

$$K_{dip.} = 4 \times 10^{-21} \text{ J}$$

$$r = \sqrt{\frac{pkq}{K_{dip.}}} = \sqrt{\frac{6.2 \times 10^{-30} \cdot 9 \times 10^9 \cdot 1.6 \times 10^{-19}}{4 \times 10^{-21}}} = 1.5 \text{ nm}$$



Summary of today's lecture

- Electric potential energy & work

$$W_F = -W_{you} = -\Delta U = F \Delta r \cos \theta$$

Path independence

Conservation of energy

- Electric potential energy for point charges $U_E = k \frac{q_1 q_2}{r}$