

Your questions/comments

“This pre-lecture involved many different equations and aspects of circuits, and I find keeping it all straight a little difficult. I need more clarification on how and when to use the different equations.”

“Capacitance is a bit confusing in what happens to the magnitude of it when the dielectric is removed or introduced.”

“Focus the lecture on capacitors, I don't get it!!! What are they used for, why does it show an example of charges flowing between plates? What situations are they applicable in? Especially the lectures on capacitor energy and dielectric constant. Finally please explain the different types of Power from the resistor video”

Physics 102 so far

Basic principles of electricity

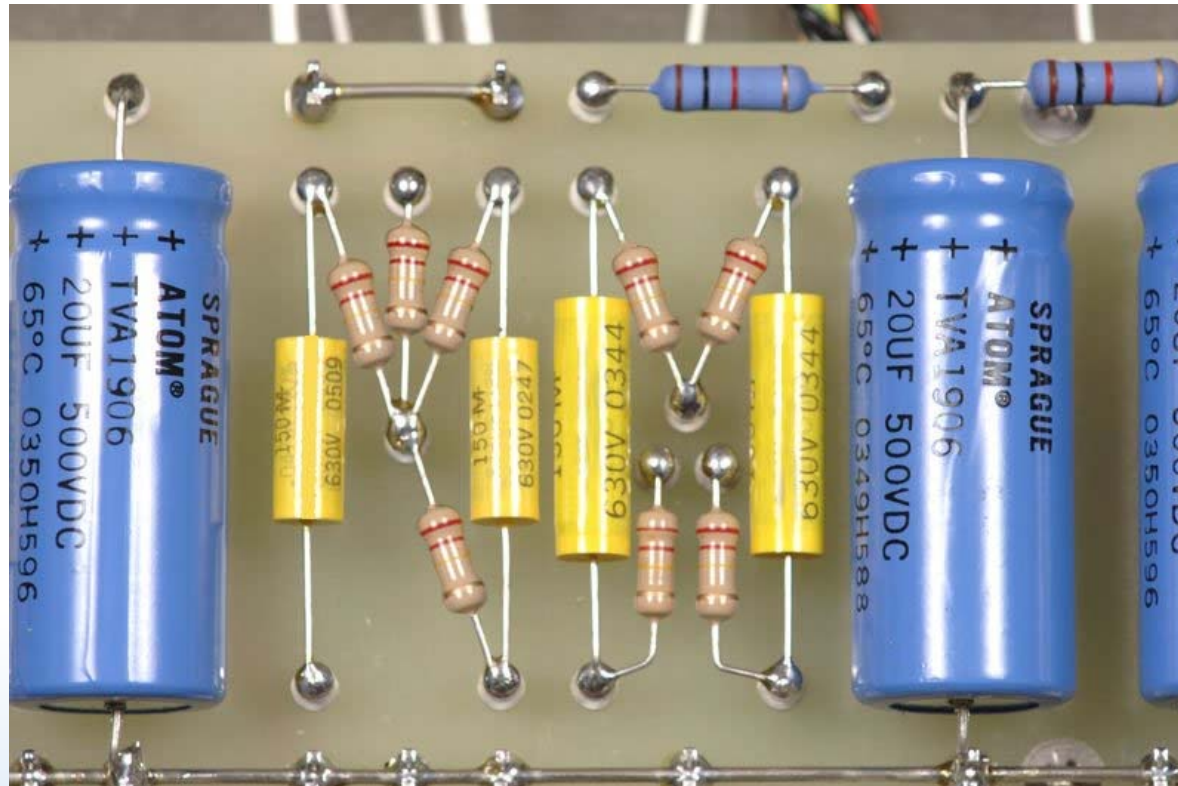
EXAM-1

- Lecture 1 – electric charge
- Lecture 2 – Coulomb's law
- Lecture 3 – the electric field
- Lecture 4 – electric potential energy and work
- Lecture 5 – electric potential

HW 1-3, Disc 1-4
LAB 1

Applications of electricity – circuits

- Lecture 6 – capacitance & resistance
- Lecture 7 – simple circuits
- Lecture 8 – Kirchhoff's rules
- Lecture 9 – RC circuits



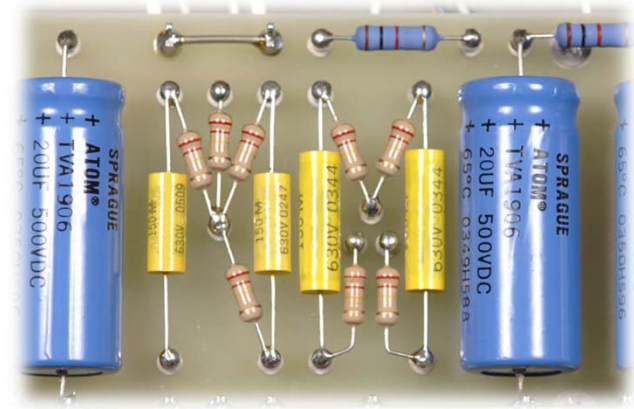
Phys 102 – Lecture 6

Circuit elements: resistors, capacitors, and batteries

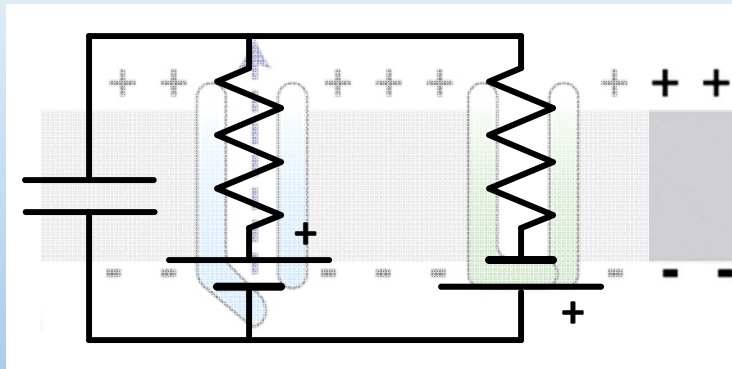
Today we will learn about...

Circuit elements that:

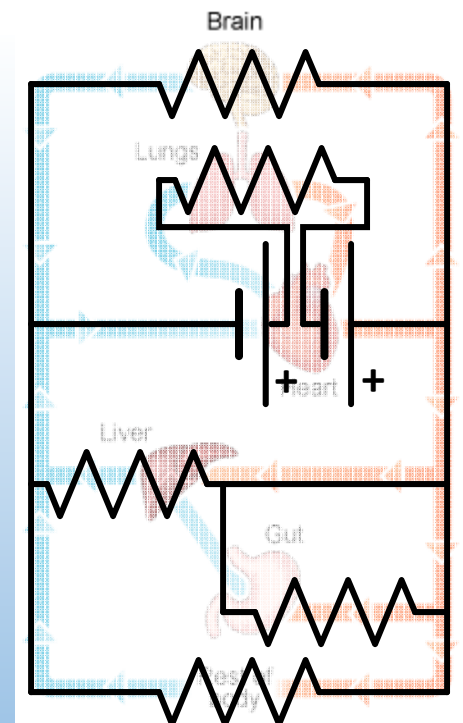
- 1) Serve as conduits for charge – wires
- 2) Pump charges around – batteries
- 3) Regulate flow of charge – resistors
- 4) Store and release charge – capacitors



These elements are idealizations of components in electronic circuits & in nature



Ex: neurons, circulatory system



Electric current

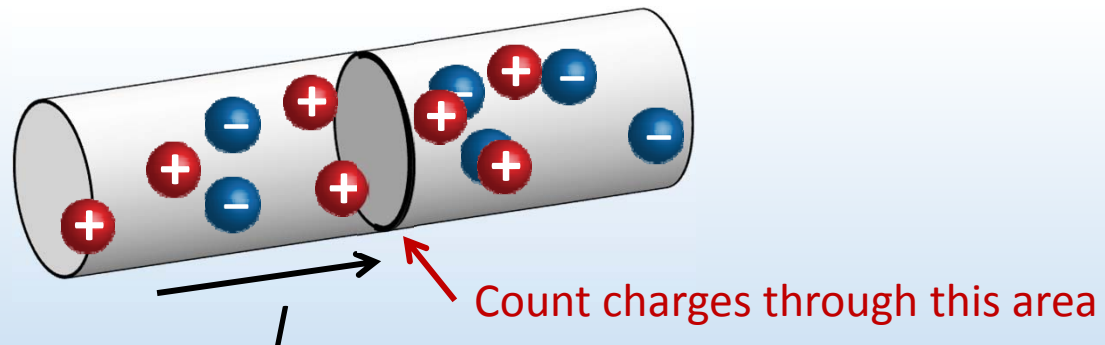
Current – measure of flow of charge (+ charge, by convention)

Counts total charge ΔQ passing through area in a time interval Δt

$$I \equiv \frac{\Delta Q}{\Delta t}$$

Unit: A (“Amp” or “Ampere”)

1A = 1C/s



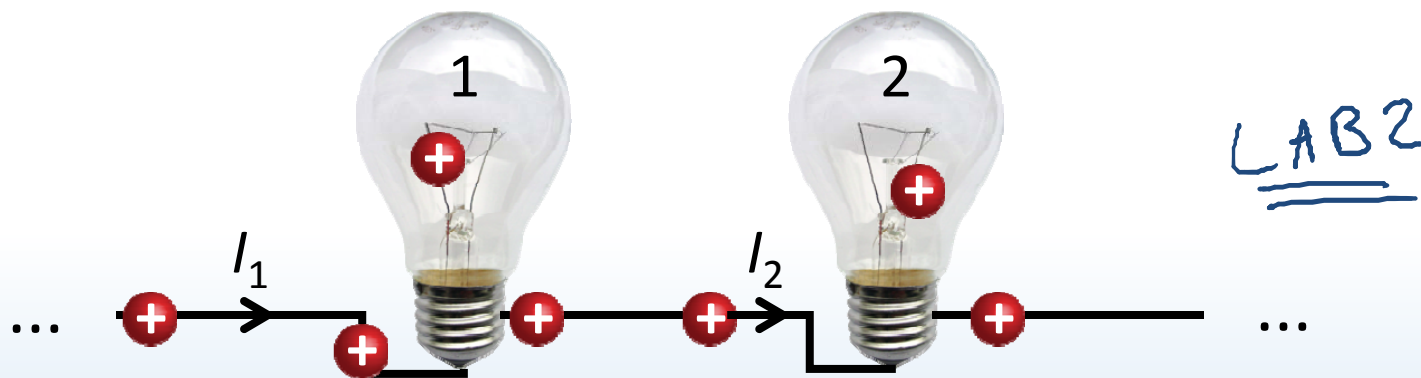
In electronic circuits, electrons ($-e$) carry current, flow opposite to current

In liquid or gas, both cations and anions can carry current



ACT: Two light bulbs

Two light bulbs 1 and 2 are connected end-to-end by conducting wire. If a current I_1 flows through bulb 1, what is the current I_2 in bulb 2?



A. $I_2 < I_1$

B. $I_2 = I_1$

C. $I_2 > I_1$

Charges are conserved! Current does NOT get “used up” by light bulb

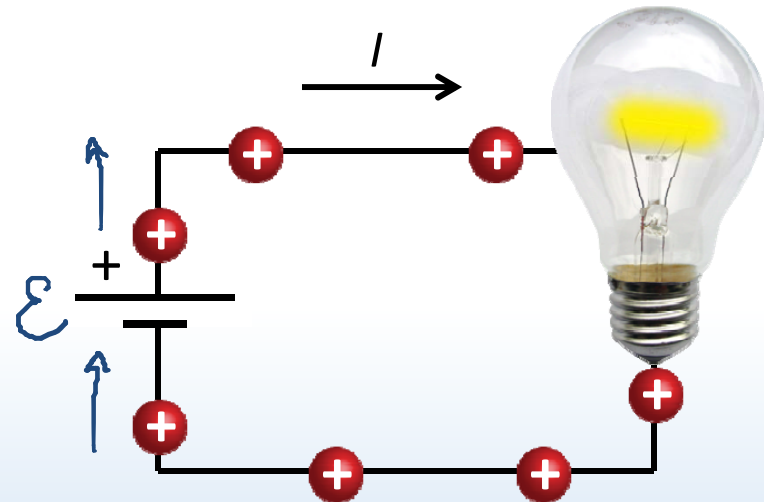
Same current entering bulb 1, leaving bulb 1, entering bulb 2, leaving bulb 2 ...

Batteries & electromotive force

Battery – maintains a constant electric potential difference
(“Electromotive force” – emf \mathcal{E})



Electric potential is 9 V higher at + end relative to – end. Potential difference across a circuit element is its “voltage”



Electric potential difference drives current around circuit

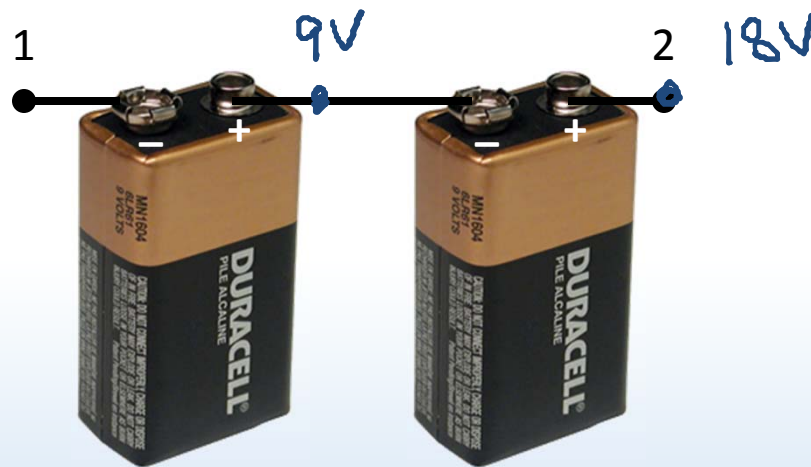
Battery does NOT determine how much current flows

Battery does NOT generate new charges, it “pushes” charges, like a pump



ACT: Two batteries

Two 9 V batteries are connected end-to-end by conducting wire. What is the electric potential at point 2 relative to point 1?



A. +18 V

B. +9 V

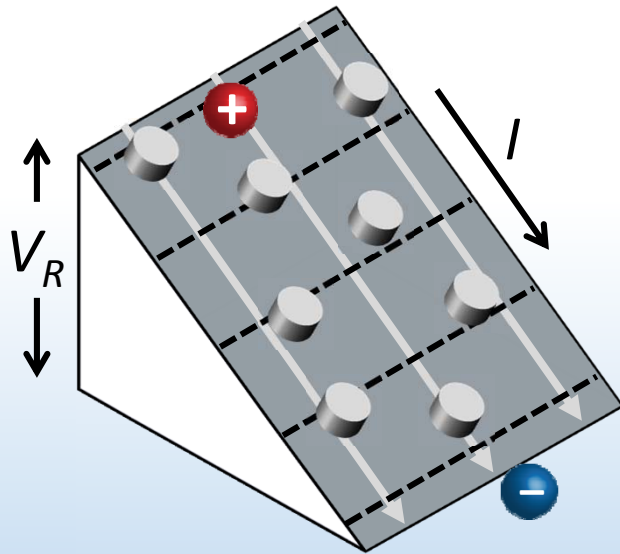
C. -18 V

D. -9 V

Battery 1 increases electric potential by 9 V
Battery 2 increases electric potential by another 9 V

Resistance and Ohm's law

Moving charges collide with each other, ions, defects inside material
Flow rate depends on electric potential difference



$$I \propto V_R \quad \leftarrow \text{Double potential difference, double current}$$

DEMO

Resistance – proportionality constant between current and voltage

Ohm's law:
$$R \equiv \frac{V_R}{I}$$
 Units: Ω ("Ohms")

Potential difference causes current to flow ("downhill", by convention)
Resistance regulates the amount of flow

Physical resistance

Resistor – circuit element designed to have resistance



Resistance depends on material parameters and geometry

Resistivity – density of scatterers

$$R = \rho \frac{L}{A}$$

Length – the longer the resistor, the more scattering

Cross sectional area – the wider the resistor, the more charges flow

DEMO

Material	ρ ($\Omega \cdot \text{m}$)
Copper	1.7×10^{-8}
Iron	9.7×10^{-8}
Sea water	0.22
Muscle	13
Fat	25
Pure water	2.4×10^5

Basis for measuring body fat percentage



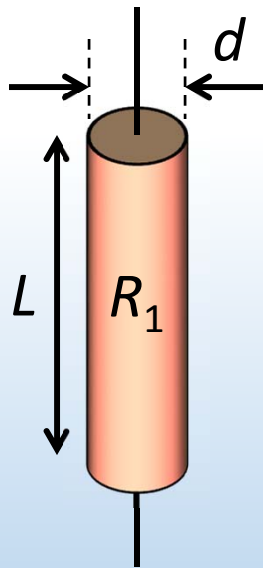


ACT: CheckPoint 1.1

Which of the following three copper resistors has the *lowest* resistance?

A.

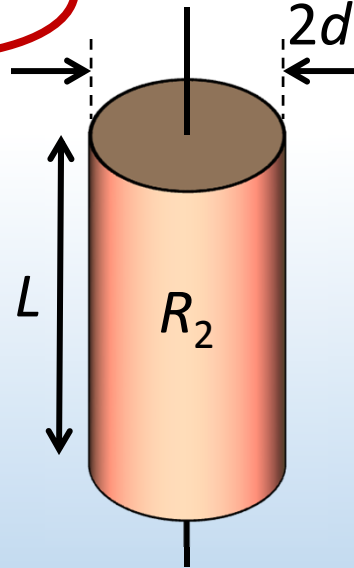
10%



$$R_1 = \rho \frac{L}{A}$$

B.

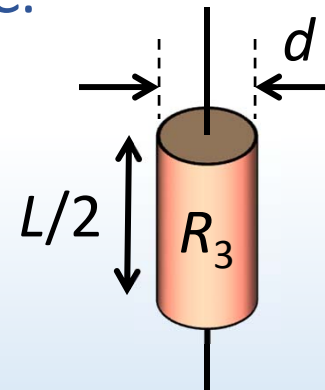
62%



$$R_2 = \rho \frac{L}{4A}$$

C.

28%



$$R_3 = \rho \frac{L}{2A}$$

Power generated and dissipated

Battery does work pumping charges through circuit

Ex: a 9 V battery does 9 J of work per 1 C of charge pumped

Where does the energy come from? Chemical energy in battery

Power – rate of energy conversion

$$P_{batt} = \frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} \varepsilon = I \varepsilon$$

Units: W (“Watts”)
1 W = 1 J/s = 1 V A

Resistor dissipates electric potential energy

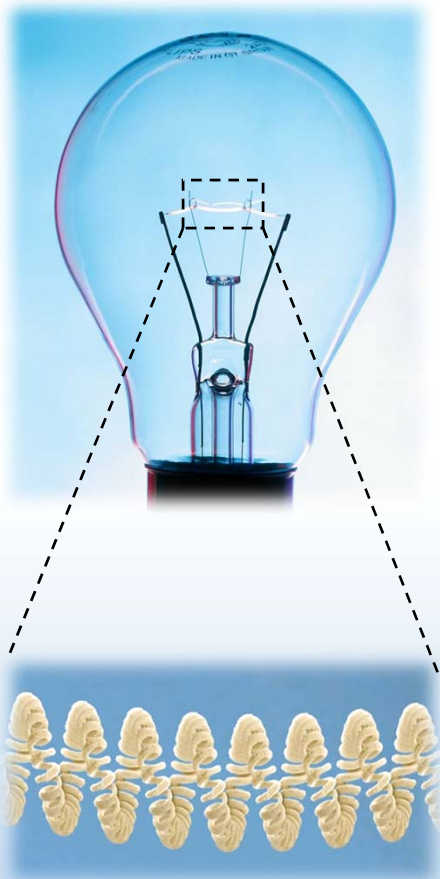
Charges lose electric potential energy in collisions inside resistor

$$P_{diss} = IV_R = I^2 R = \frac{V_R^2}{R}$$

$$V_R = IR$$

Where does the energy go? Heat and/or light

Calculation: light bulb filament



An incandescent light bulb is essentially a resistor that dissipates energy as heat and light. A typical light bulb dissipates 60 W with 120 V from an outlet.

The resistive element is a thin (40- μm diameter) *filament* of tungsten. How long must the filament be?

Approach: find resistance of filament, relate it to length

$$P = IV = I^2 R = \frac{V^2}{R} \quad R = \rho \frac{L}{A}$$

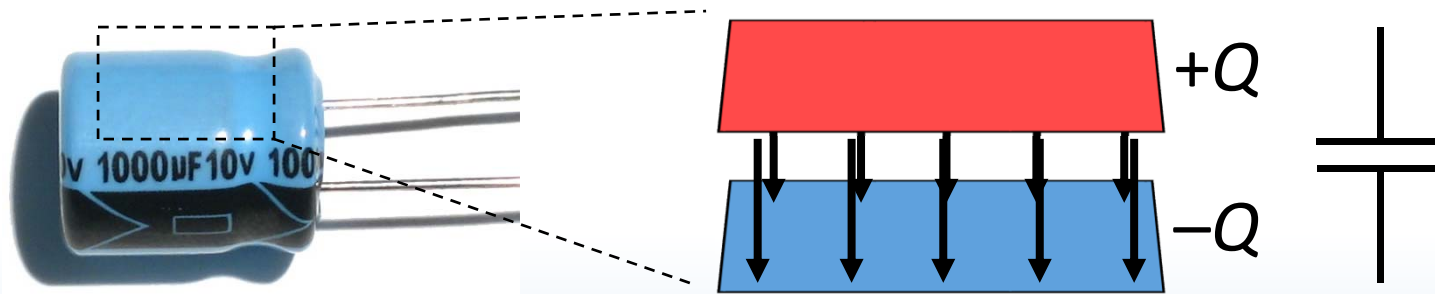
$$L = \frac{AR}{\rho} = \frac{AV^2}{\rho P} = \frac{\pi(20 \times 10^{-6} \text{ m})^2 (120 \text{ V})^2}{(5 \times 10^{-7} \Omega \cdot \text{m})(60 \text{ W})} = 0.6 \text{ m}$$

Resistivity of tungsten (at temp. of filament): $5.0 \times 10^{-7} \Omega \cdot \text{m}$

About 2 ft !

Capacitance

Capacitor – circuit element that stores separated charge
Consists of two conductors separated by a small gap



Capacitance – measures the ability to store charge Q given a voltage V_C applied between the conductors

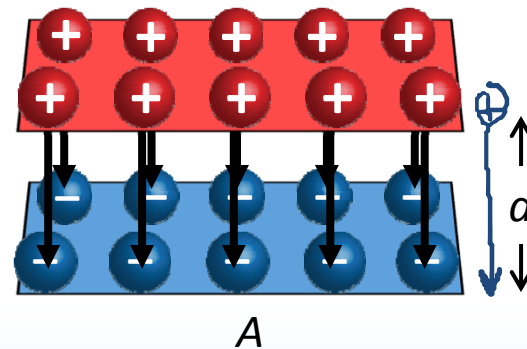
$$C \equiv \frac{Q}{V_C}$$

Units: F (“Farad”)
 $1 \text{ F} = 1 \text{ C/V}$

Parallel plate capacitor

Capacitor made up of two large conducting plates of area A separated by a small gap d

Electric field is uniform between plates (Recall Lect. 3)



PhET

$$k \equiv \frac{1}{4\pi\epsilon_0} \quad E \propto \frac{Q}{\epsilon_0 A}$$

Field strength \propto density of field lines \propto density of charges

Work to move $+q$ charge from $+$ to $-$ plate (Recall Lect. 4)

$$W_E = +qEd = -(U_- - U_+) \quad V_+ - V_- = \frac{U_+ - U_-}{q} = +Ed = V_C = \frac{Qd}{\epsilon_0 A} = \frac{Q}{C}$$

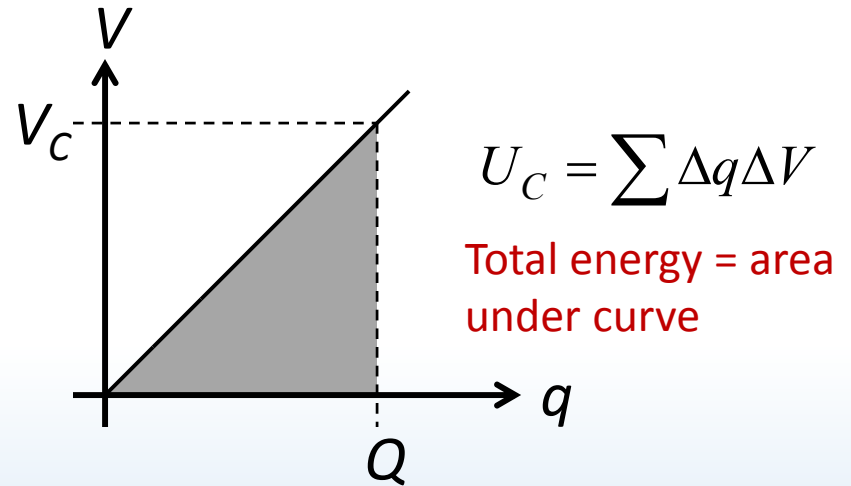
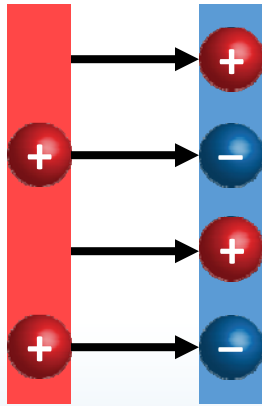
Capacitor voltage

For a parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$

Capacitance depends on geometry

Capacitor energy

Separated charges have potential energy (Recall Lect. 4)



Imagine transferring + charge from one plate to the other
Each time add Δq , electric field and voltage increase by ΔE and ΔV

$$U_C = \frac{1}{2} Q V_C = \frac{1}{2} C V_C^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$C = \frac{Q}{V}$$

Important factor of $\frac{1}{2}$! Don't confuse this equation with $U = qV$ for individual charge q

Why separate charge?

A way to store *and* release energy

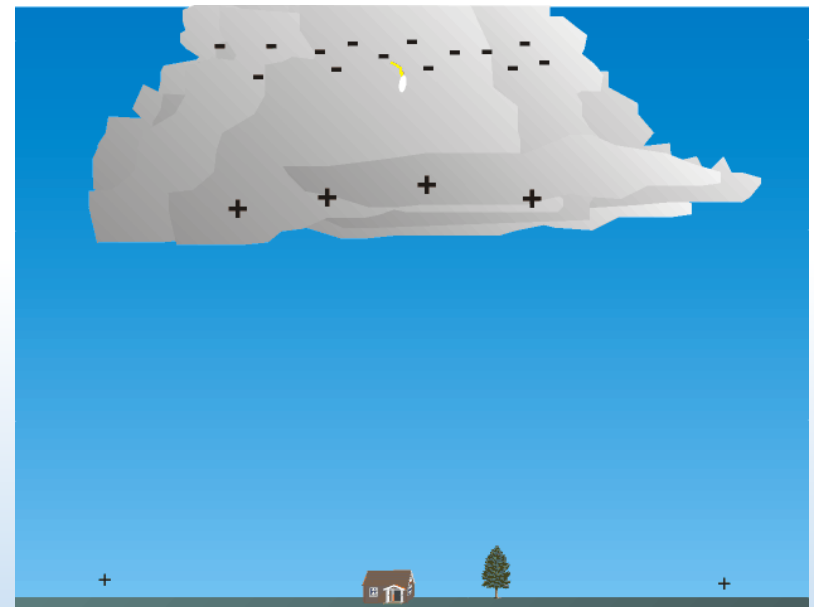
DEMO



Camera flash



Defibrillator



Lightning strike



ACT: Parallel plates

A parallel plate capacitor carries a charge Q . The plates are then pulled a small distance further apart.



What happens to the charge Q on each plate?

A. Q increases

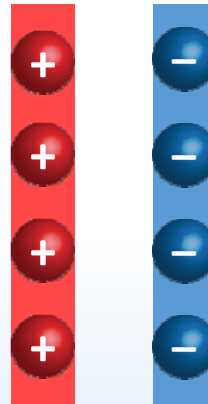
B. Q stays constant

C. Q decreases

Charge is conserved! There are no wires for charges to flow through, so they stay on the capacitor plates

Checkpoint 2

A parallel plate capacitor carries a charge Q . The plates are then pulled a small distance further apart.



DEMO

The capacitance increases $C = \epsilon_0 A / d$

True **False**

The electric field increases $E = Q / \epsilon_0 A$

True **False**

The voltage between the plates increases $V_C = Ed$

True False

The energy stored increases $U_C = \frac{1}{2} Q V_C$

True False

Dielectrics

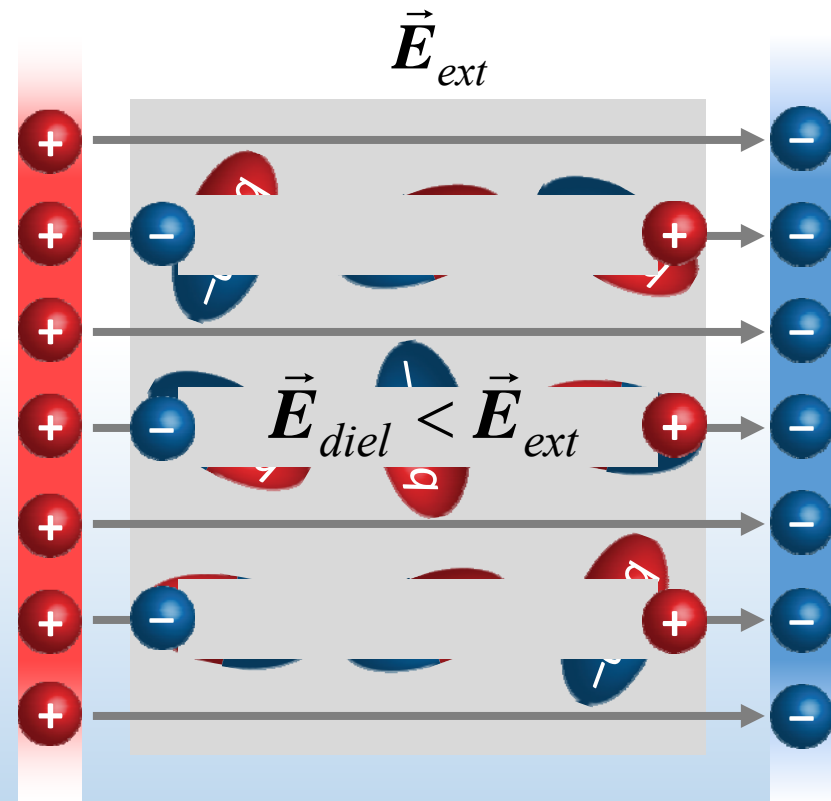
Imagine placing insulating material (dielectric) between plates

External field polarizes dielectric
Excess $+q$ and $-q$ charges build up on opposite planes

Parallel planes of $+q$ and $-q$
create own E field, cancel out
part of external E field

$$\vec{E}_{diel} = \frac{\vec{E}_{ext}}{\kappa}$$

Dielectric constant $\kappa > 1$



(Recall Lect. 3 – conductors)

Dielectric constant κ

Dielectric constant κ measures how much a material is polarized by electric field

Since $\vec{E} = \vec{E}_0 / \kappa$, need less E (or V) to store same Q , so $C = Q/V$ increases:

Capacitance with dielectric

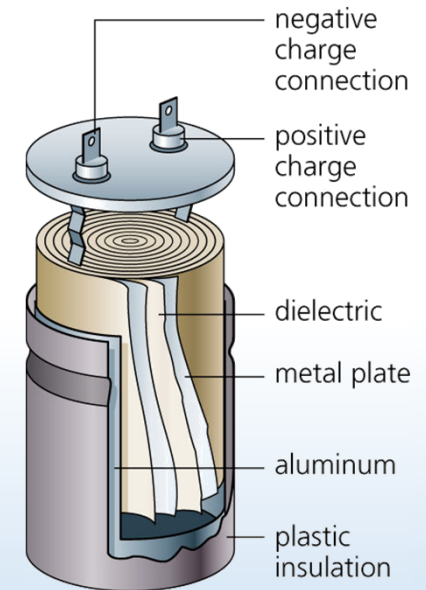
Dielectric constant

$$C = \kappa C_0$$

Capacitance without dielectric

Material	$\kappa (> 1)$
Vacuum	1 (exactly)
Air	1.00054
Rubber	3-4
Glass	5
Cell membrane	7-9
Pure water	80

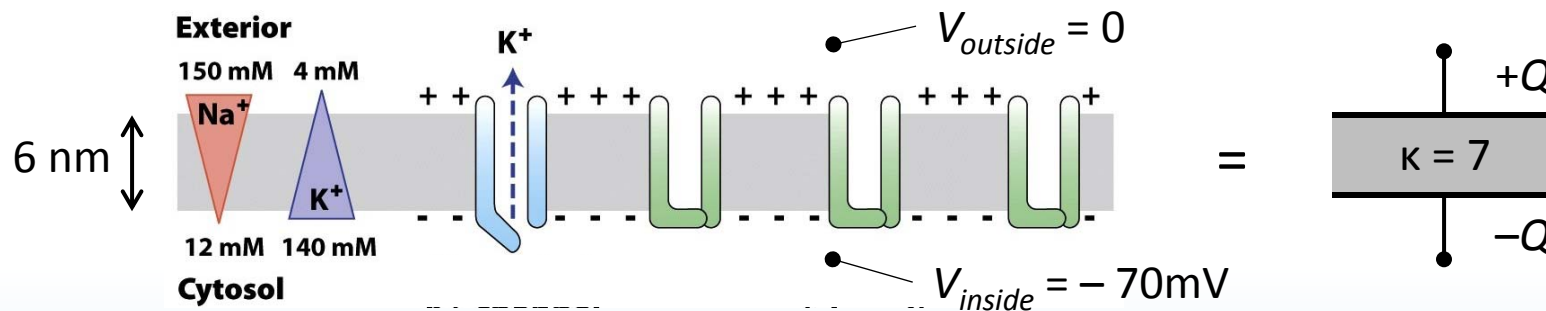
Capacitance depends on material parameters (dielectric) and geometry



Calculation: capacitance of a cell

Channels in a cell's membrane create a charge imbalance (recall Lect. 5), with + charge outside, – inside. The separated charge gives the cell *capacitance*, with the membrane acting as a dielectric ($\kappa = 7$).

Based on EXAM 1, FA09



What is the capacitance of a $1\text{-}\mu\text{m}^2$ flat patch of cell?

$$C = \kappa \epsilon_0 \frac{A}{d} = 7(8.85 \times 10^{-12}) \frac{(10^{-6} \text{ m})^2}{6 \times 10^{-9} \text{ m}} = 0.01 \text{ pF}$$

At rest, a cell has a -70 mV voltage across it. How much charge is necessary to generate this voltage?

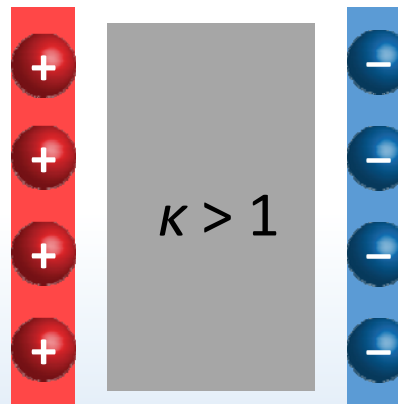
$$Q = CV = (10^{-14} \text{ F})(0.07 \text{ V}) = 0.7 \text{ fC} \quad N = \frac{Q}{e} = \frac{7 \times 10^{-16} \text{ C}}{1.6 \times 10^{-19} \text{ C}} \approx 4,500$$

Not very much! <1 excess ion per $10 \times 10 \text{ nm}^2$



ACT: Capacitor dielectric

A parallel plate capacitor carries a charge Q . A dielectric with $\kappa > 1$ is inserted between the plates.



DEMO

What happens to energy U_C stored in the capacitor?

- A. U_C increases
- B. U_C stays constant
- C. U_C decreases

Approach: what stays constant, what changes?

Q stays constant, C increases

$$U_C = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

Summary of today's lecture

- Batteries generate emf \mathcal{E} , pump charges
- Resistors *dissipate* energy as power: $P = IV$
Resistance: how difficult it is for charges to get through: $R = \rho L/A$
Voltage determines *current*: $V = IR$
Ideal wires have $R = 0$, $V = 0$
- Capacitors *store* energy as separated charge: $U = \frac{1}{2}QV$
Capacitance: ability to store separated charge: $C = \kappa\epsilon_0 A/d$
Voltage determines *charge*: $V = Q/C$
- Don't mix capacitor and resistor equations!