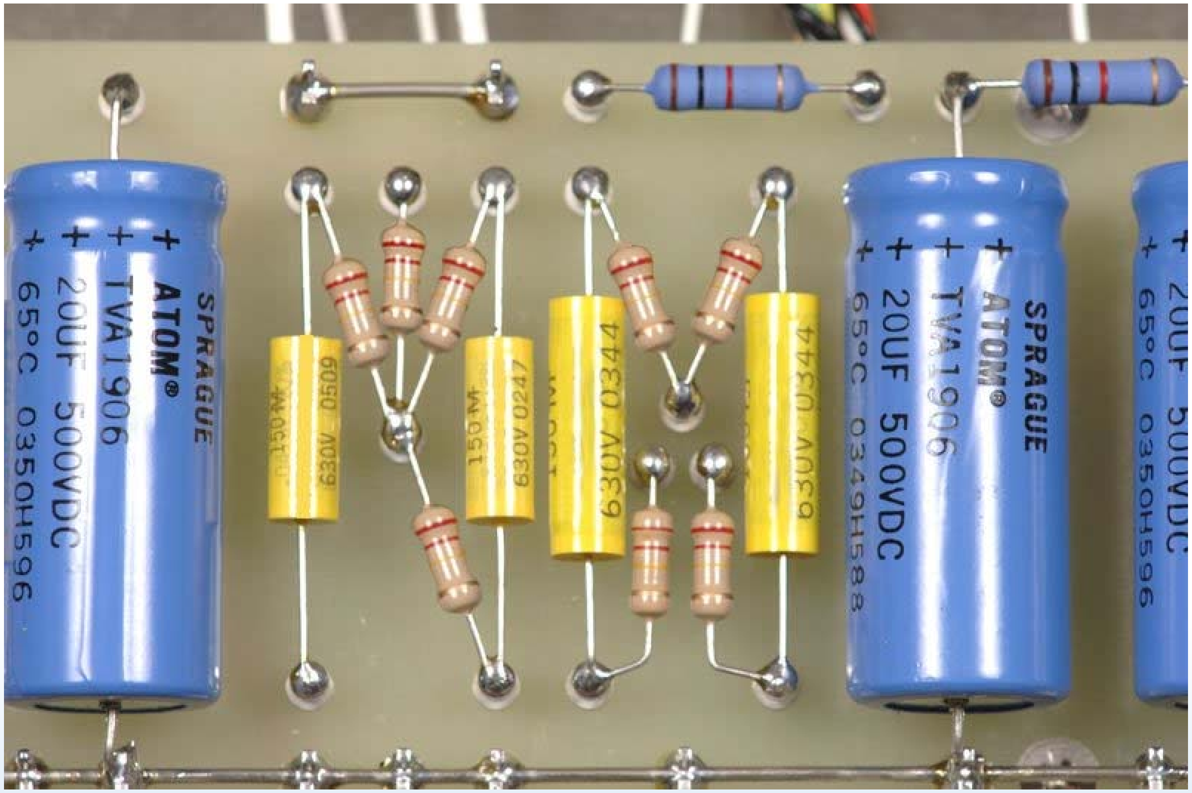
Physics 102 so far

Basic principles of electricity

- Lecture 1 – electric charge
- Lecture 2 – Coulomb's law
- Lecture 3 – the electric field
- Lecture 4 – electric potential energy and work
- Lecture 5 – electric potential

Applications of electricity – circuits

- Lecture 6 – capacitance & resistance
- Lecture 7 – simple circuits
- Lecture 8 – Kirchhoff's rules
- Lecture 9 – RC circuits



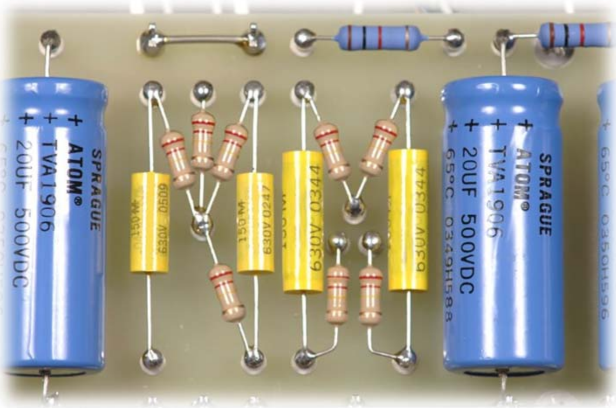
Phys 102 – Lecture 6

Circuit elements: resistors, capacitors, and batteries

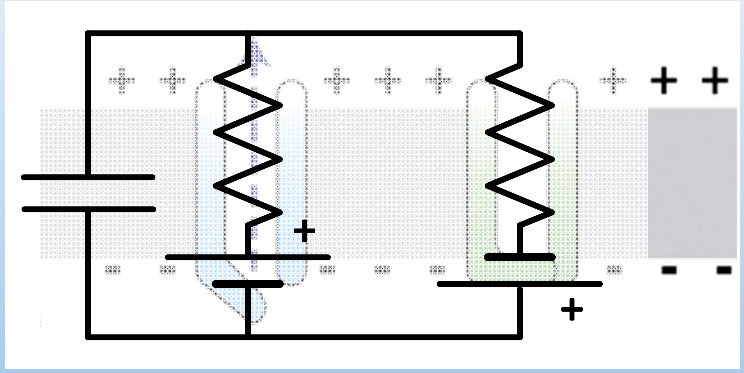
Today we will learn about...

Circuit elements that:

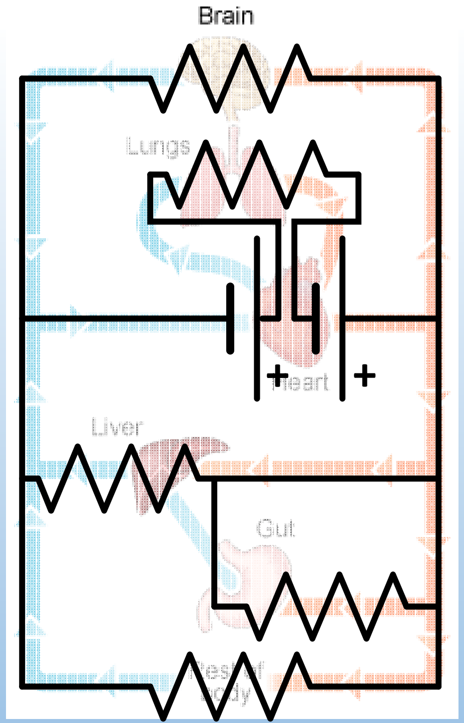
1) Serve as conduits for charge – *wires*
2) Pump charges around – *batteries*
3) Regulate flow of charge – *resistors*
4) Store and release charge – *capacitors*



These elements are idealizations of components in electronic circuits & in nature



Ex: neurons, circulatory system

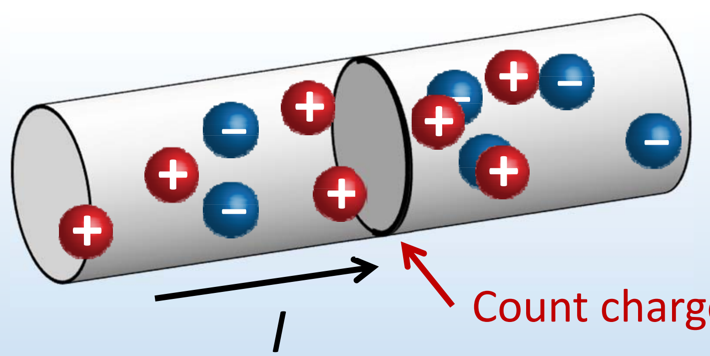


Electric current

<u>*Current*</u> – measure of flow of charge (+ charge, by convention)
Counts total charge ΔQ passing through area in a time interval Δt

$$\boxed{I \equiv \frac{\Delta Q}{\Delta t}}$$

Unit: A ("Amp" or "Ampere")
1A = 1C/s

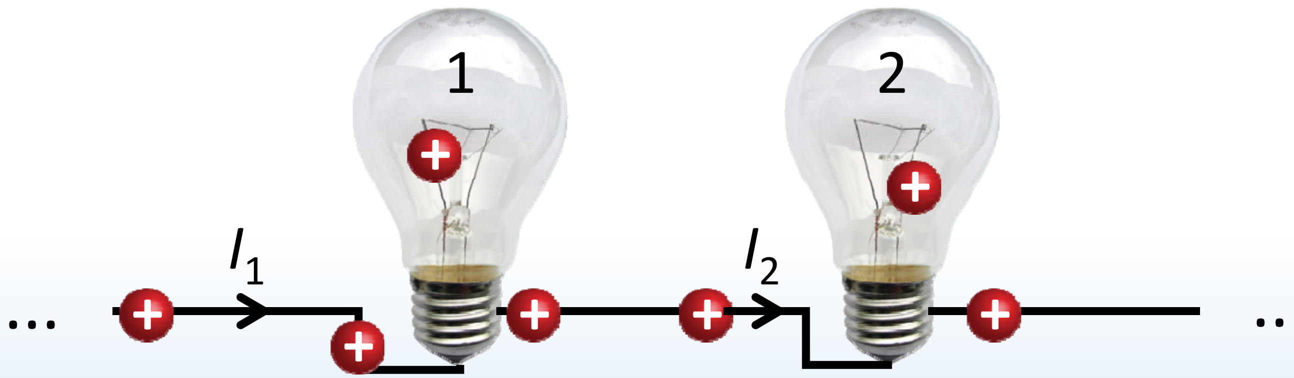


In electronic circuits, electrons (–e) carry current, flow <u>*opposite*</u> to current

In liquid or gas, both cations and anions can carry current

ACT: Two light bulbs

Two light bulbs 1 and 2 are connected end-to-end by conducting wire. If a current I_1 flows through bulb 1, what is the current I_2 in bulb 2?



A. $I_2 < I_1$

B. $I_2 = I_1$

C. $I_2 > I_1$

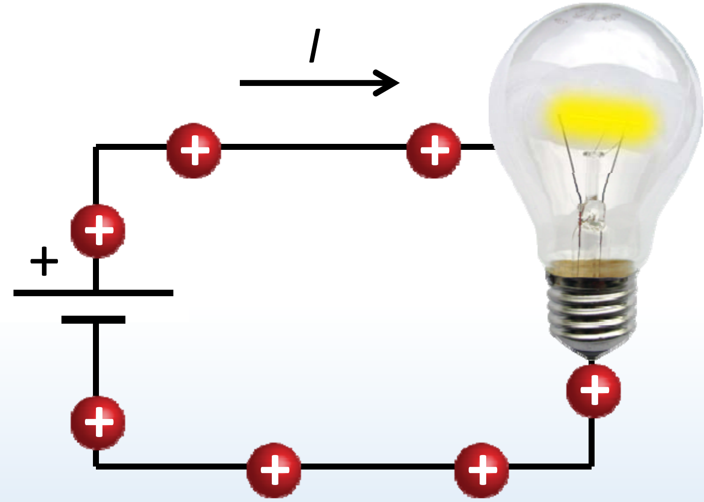
Charges are conserved! Current does NOT get "used up" by light bulb

Same current entering bulb 1, leaving bulb 1, entering bulb 2, leaving bulb 2 ...

Batteries & electromotive force

Battery – maintains a constant electric potential difference ("Electromotive force" – emf ε)

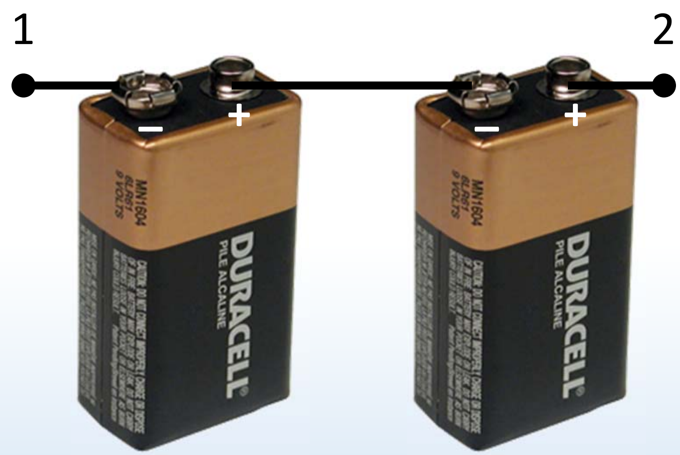
Electric potential is 9 V higher at + end relative to – end. Potential difference across a circuit element is its "voltage"



Electric potential difference drives current around circuit
Battery does NOT determine how much current flows
Battery does NOT generate new charges, it "pushes" charges, like a pump

ACT: Two batteries

Two 9 V batteries are connected end-to-end by conducting wire. What is the electric potential at point 2 relative to point 1?



A. +18 V

B. +9 V

C. –18 V

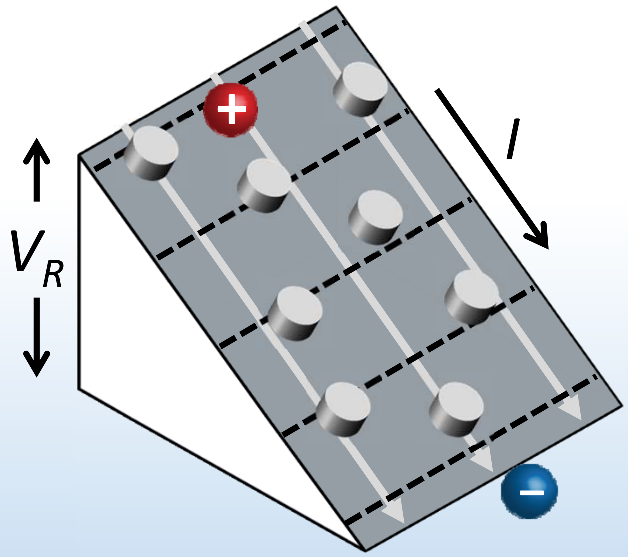
D. –9 V

Battery 1 increases electric potential by 9 V

Battery 2 increases electric potential by another 9 V

Resistance and Ohm's law

Moving charges collide with each other, ions, defects inside material
Flow rate depends on electric potential difference



$I \propto V_R$ ← Double potential difference, double current

DEMO

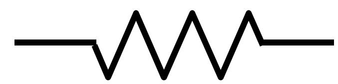
Resistance – proportionality constant between current and voltage

Ohm's law: $\boxed{R \equiv \frac{V_R}{I}}$ Units: Ω ("Ohms")

Potential difference causes current to flow ("downhill", by convention)
Resistance regulates the amount of flow

Physical resistance

Resistor – circuit element designed to have resistance



Resistance depends on material parameters and geometry

Resistivity – density of scatterers

$$\boxed{R = \rho \frac{L}{A}}$$

Length – the longer the resistor, the more scattering

Cross sectional area – the wider the resistor, the more charges flow

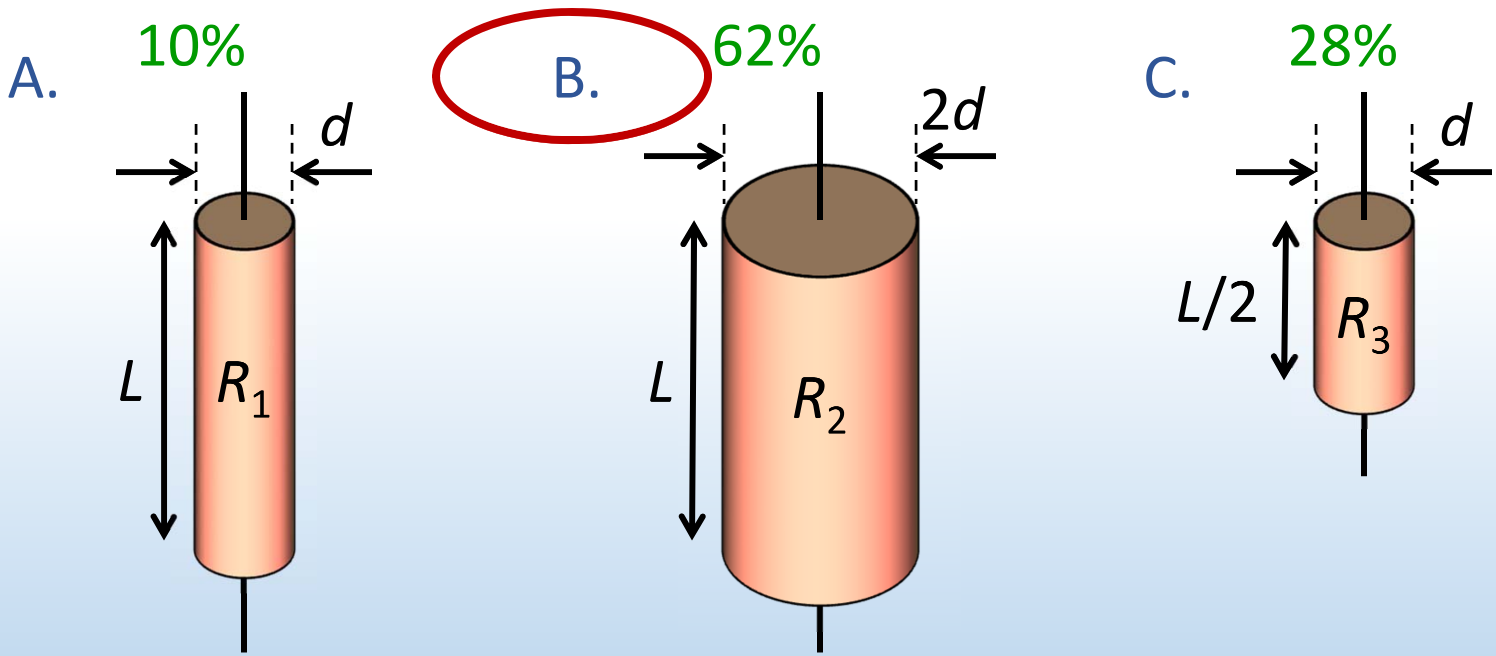
DEMO

Material	ρ ($\Omega \cdot$m)
Copper	1.7×10^{-8}
Iron	9.7×10^{-8}
Sea water	0.22
Muscle	13
Fat	25
Pure water	2.4×10^{5}

Basis for measuring body fat percentage

ACT: CheckPoint 1.1

Which of the following three copper resistors has the *lowest* resistance?



$$R_1 = \rho \frac{L}{A} \qquad R_2 = \rho \frac{L}{4A} \qquad R_3 = \rho \frac{L}{2A}$$

Power generated and dissipated

Battery does work pumping charges through circuit

Ex: a 9 V battery does 9 J of work per 1 C of charge pumped

Where does the energy come from? Chemical energy in battery

Power – rate of energy conversion

$$P_{batt} = \frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\varepsilon = I\varepsilon$$

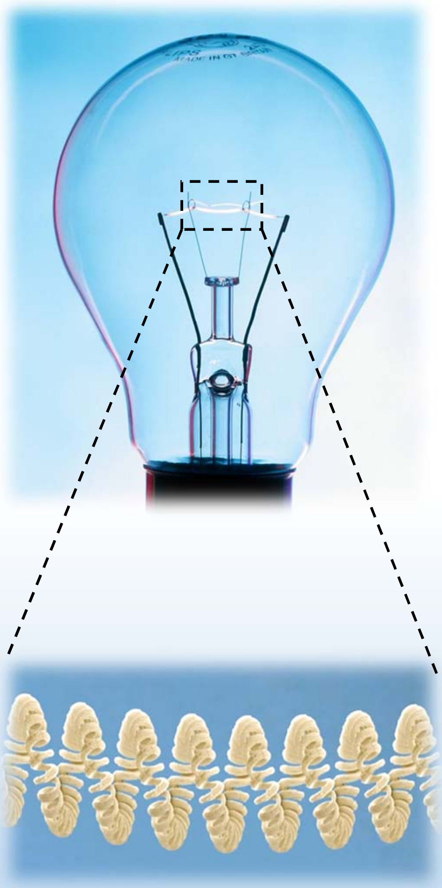
Units: W ("Watts")
1 W = 1 J/s = 1 V A

Resistor *dissipates* electric potential energy
Charges lose electric potential energy in collisions inside resistor

$$P_{diss} = IV_R = I^2 R = \frac{V_R^2}{R}$$

Where does the energy go? Heat and/or light

Calculation: light bulb filament



An incandescent light bulb is essentially a resistor that dissipates energy as heat and light. A typical light bulb dissipates 60 W with 120 V from an outlet.

The resistive element is a thin (40-µm diameter) *filament* of tungsten. How long must the filament be?

Approach: find resistance of filament, relate it to length

$$P = IV = I^2R = \frac{V^2}{R} \qquad R = \rho\frac{L}{A}$$

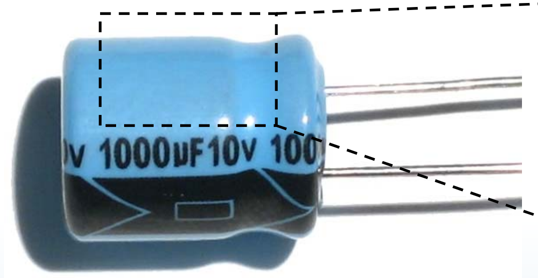
$$L = \frac{AR}{\rho} = \frac{AV^2}{\rho P} = \frac{\pi(20\times10^{-6}\,\mathrm{m})^2(120\,\mathrm{V})^2}{(5\times10^{-7}\,\Omega\cdot\mathrm{m})(60\,\mathrm{W})} = 0.6\,\mathrm{m}$$

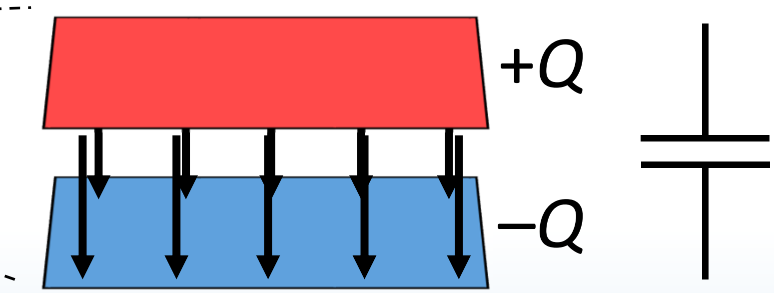
Resistivity of tungsten (at temp. of filament): $5.0 \times 10^{-7}\ \Omega\cdot\mathrm{m}$

$\boxed{\text{About 2 ft !}}$

Capacitance

<u>Capacitor</u> – circuit element that stores separated charge
Consists of two conductors separated by a small gap





<u>Capacitance</u> – measures the ability to store charge Q given a voltage V_C applied between the conductors

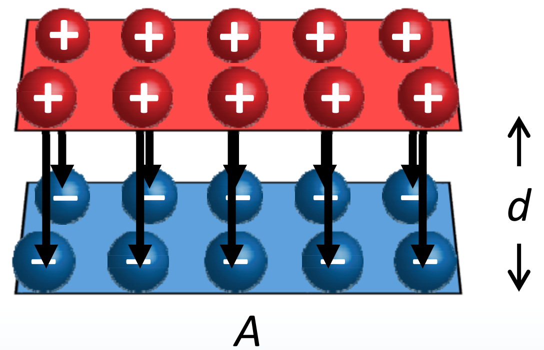
$$\boxed{C \equiv \frac{Q}{V_C}}$$

Units: F ("Farad")
1 F = 1 C/V

Parallel plate capacitor

Capacitor made up of two large conducting plates of area *A* separated by a small gap *d*

Electric field is uniform between plates (Recall Lect. 3)



$$E \propto \frac{Q}{\varepsilon_0 A}$$

Field strength ∝ density of field lines ∝ density of charges

Work to move +*q* charge from + to – plate (Recall Lect. 4)

$$W_E = +qEd = -(U_- - U_+) \qquad V_+ - V_- = \frac{U_+ - U_-}{q} = +Ed \quad = V_C$$

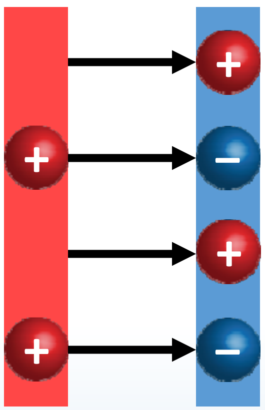
Capacitor voltage

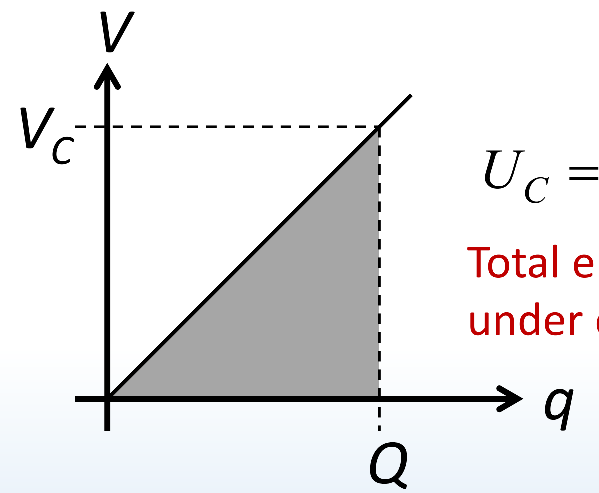
For a parallel plate capacitor: $C = \dfrac{\varepsilon_0 A}{d}$

Capacitance depends on geometry

Capacitor energy

Separated charges have potential energy (Recall Lect. 4)





$$U_C = \sum \Delta q \Delta V$$

Total energy = area under curve

Imagine transferring + charge from one plate to the other
Each time add Δq, electric field and voltage increase by ΔE and ΔV

$$U_C = \frac{1}{2} Q V_C = \frac{1}{2} C V_C^2 = \frac{1}{2} \frac{Q^2}{C}$$

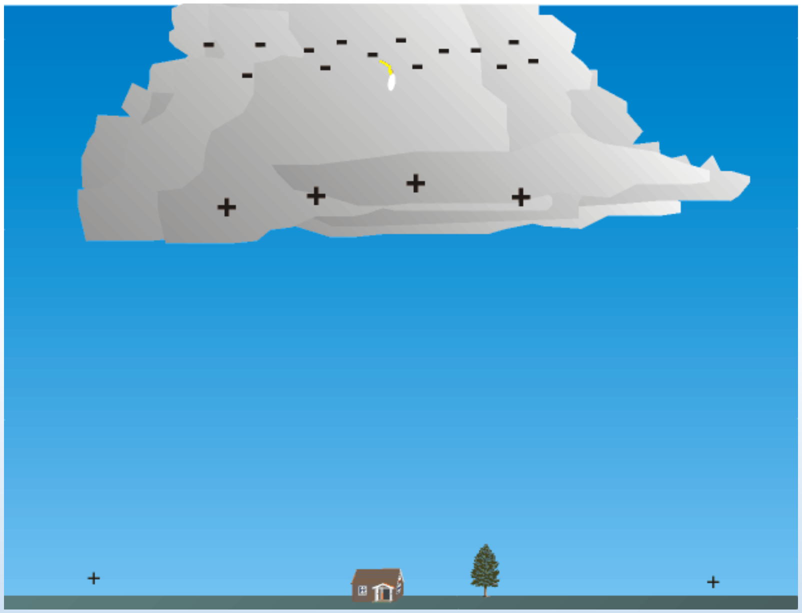
Important factor of ½! Don't confuse this equation with $U = qV$ for individual charge q

Why separate charge?

A way to store *and* release energy

Camera flash

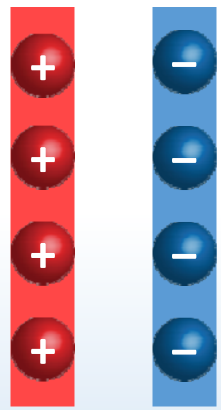
Defibrillator



Lightning strike

ACT: Parallel plates

A parallel plate capacitor carries a charge *Q*. The plates are then pulled a small distance further apart.



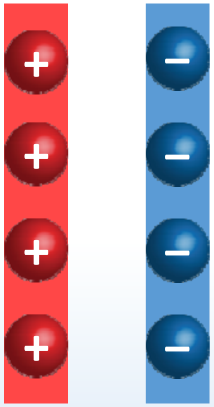
What happens to the charge *Q* on each plate?

A. *Q* increases
B. *Q* stays constant
C. *Q* decreases

Charge is conserved! There are no wires for charges to flow through, so they stay on the capacitor plates

Checkpoint 2

A parallel plate capacitor carries a charge *Q*. The plates are then pulled a small distance further apart.



DEMO

The capacitance increases	$C = \varepsilon_0 A/d$	True	False
The electric field increases	$E = Q/\varepsilon_0 A$	True	False
The voltage between the plates increases	$V_C = Ed$	True	False
The energy stored increases	$U_C = \frac{1}{2} C V_C^2$	True	False

Dielectrics

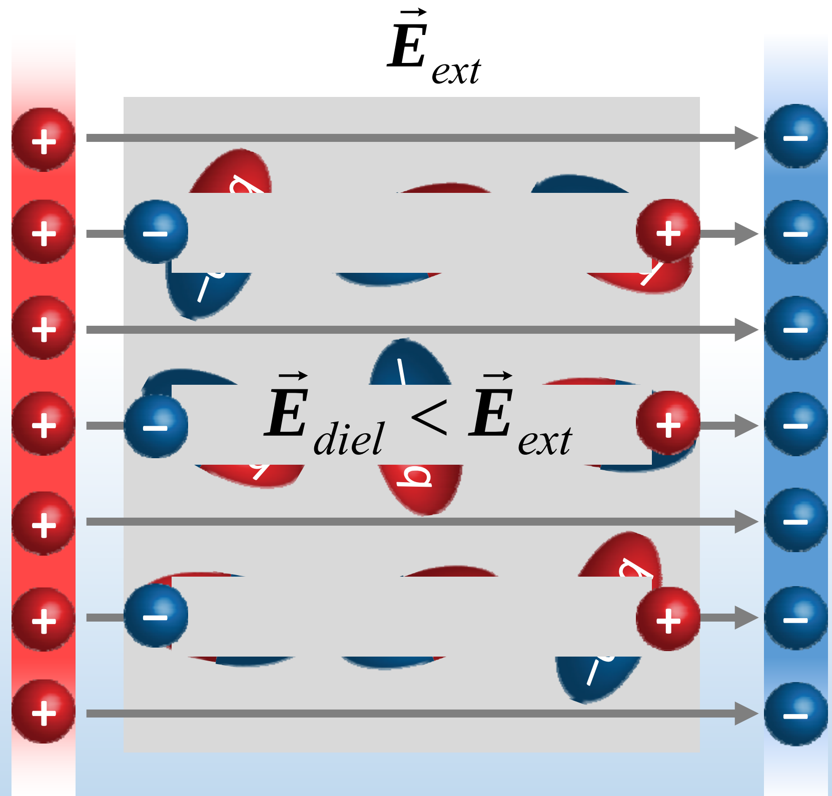
Imagine placing insulating material (dielectric) between plates

External field polarizes dielectric
Excess +*q* and –*q* charges build up on opposite planes

Parallel planes of +*q* and –*q* *create own E field*, cancel out part of external E field

$$\vec{E}_{diel} = \frac{\vec{E}_{ext}}{\kappa}$$

Dielectric constant κ > 1

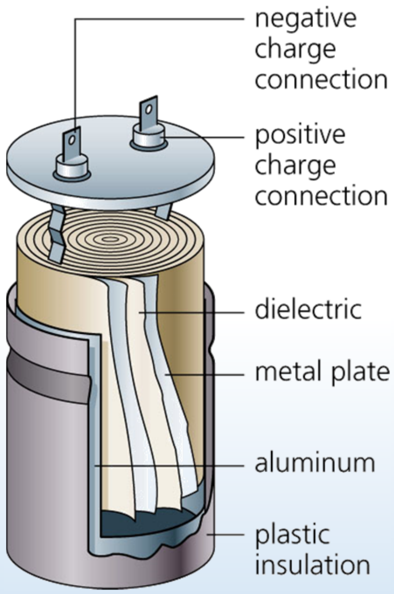


(Recall Lect. 3 – conductors)

Dielectric constant κ

Dielectric constant κ measures how much a material is polarized by electric field

Since $\vec{E} = \vec{E}_0/\kappa$, need less E (or V) to store same Q, so $C = Q/V$ increases:



Dielectric constant

Capacitance with dielectric

$$\boxed{C = \kappa C_0}$$

Capacitance without dielectric

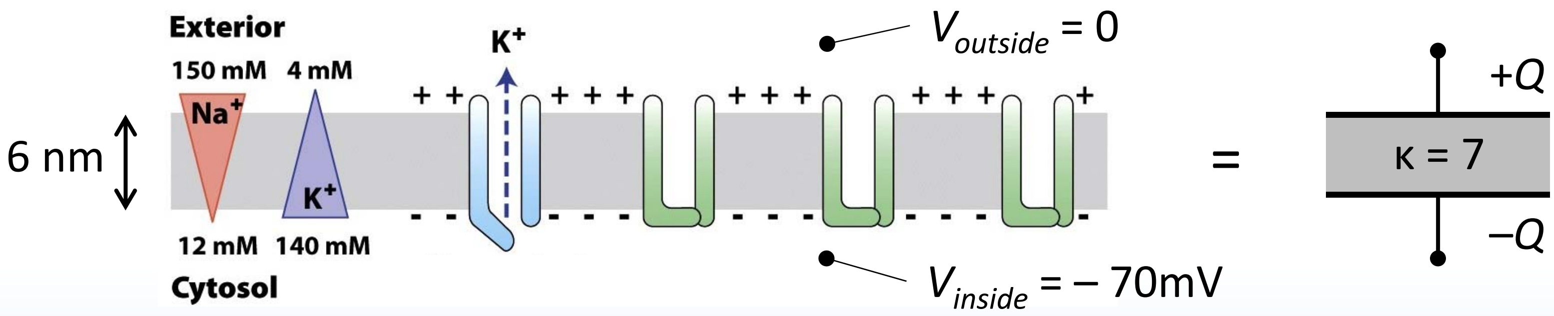
Material	κ (> 1)
Vacuum	1 (exactly)
Air	1.00054
Rubber	3-4
Glass	5
Cell membrane	7-9
Pure water	80

Capacitance depends on material parameters (dielectric) and geometry

Calculation: capacitance of a cell

Channels in a cell's membrane create a charge imbalance (recall Lect. 5), with + charge outside, – inside. The separated charge gives the cell *capacitance*, with the membrane acting as a dielectric ($\kappa = 7$).

Based on EXAM 1, FA09



What is the capacitance of a 1-μm² flat patch of cell?

$$C = \kappa\varepsilon_0 \frac{A}{d} = 7(8.85\times 10^{-12})\frac{(10^{-6}\,\text{m})^2}{6\times 10^{-9}\,\text{m}} = 0.01\text{pF}$$

At rest, a cell has a –70 mV voltage across it. How much charge is necessary to generate this voltage?

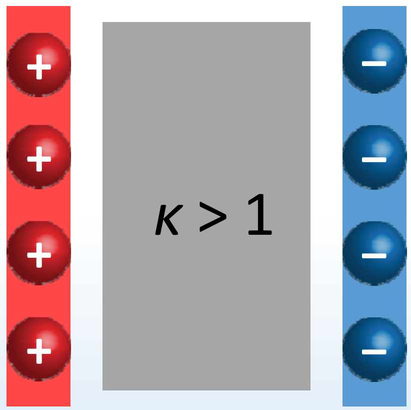
$$Q = CV = (10^{-14}\,\text{F})(0.07\,\text{V}) = 0.7\text{fC}$$

$$N = \frac{Q}{e} = \frac{7\times 10^{-16}\,C}{1.6\times 10^{-19}\,C} \approx 4,500$$

Not very much! <1 excess ion per 10 × 10 nm²

ACT: Capacitor dielectric

A parallel plate capacitor carries a charge *Q*. A dielectric with $\kappa > 1$ is inserted between the plates.



DEMO

What happens to energy U_C stored in the capacitor?

A. U_C increases

B. U_C stays constant

C. U_C decreases

Approach: what stays constant, what changes?

Q stays constant, *C* increases

$$U_C = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Summary of today's lecture

- Batteries generate emf ε, pump charges
- Resistors *dissipate* energy as power: $P = IV$

  Resistance: how difficult it is for charges to get through: $R = \rho L/A$

  Voltage determines *current*: $V = IR$

  Ideal wires have $R = 0$, $V = 0$
- Capacitors *store* energy as separated charge: $U = \frac{1}{2}QV$

  Capacitance: ability to store separated charge: $C = \kappa\varepsilon_0 A/d$

  Voltage determines *charge*: $V = Q/C$
- Don't mix capacitor and resistor equations!