





# Phys 102 – Lecture 25

The quantum mechanical model of light

# Recall last time...

- Problems with classical physics

Stability of atoms

Atomic spectra

Photoelectric effect

} Today

- Quantum model of the atom

Bohr model – only orbits that fit  $n e^- \lambda$  allowed

Angular momentum, energy, radius quantized

$$L_n = n\hbar \quad E_n = -13.6 eV \frac{Z^2}{n^2} \quad r_n = 0.0529 nm \frac{n^2}{Z}$$

- Today: Quantum model of light

Einstein's photon model

$$\lambda = \frac{h}{p}$$



# ACT: Quick review

Consider an atom with a nuclear charge of  $+2e$  with a single electron orbiting, in its ground state ( $n = 1$ ), i.e. He<sup>+</sup>.

How much energy is required to ionize the atom totally?

- A. 13.6 eV
- B.  $2 \times 13.6$  eV
- C.  $4 \times 13.6$  eV

$$E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

Energy measured relative to free electron ( $E = 0$ )

# Atomic units

At atomic scales, Joules, meters, kg, etc. are not convenient units

“Electron Volt” – energy gained by charge  $+1e$  when accelerated

by 1 Volt:  $U = qV$        $1e = 1.6 \times 10^{-19} \text{ C}$ , so  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Planck constant:  $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$

Speed of light:  $c = 3 \times 10^8 \text{ m/s}$

$$hc \approx 2 \times 10^{25} \text{ J}\cdot\text{m} = \underline{\underline{1240 \text{ eV}\cdot\text{nm}}}$$

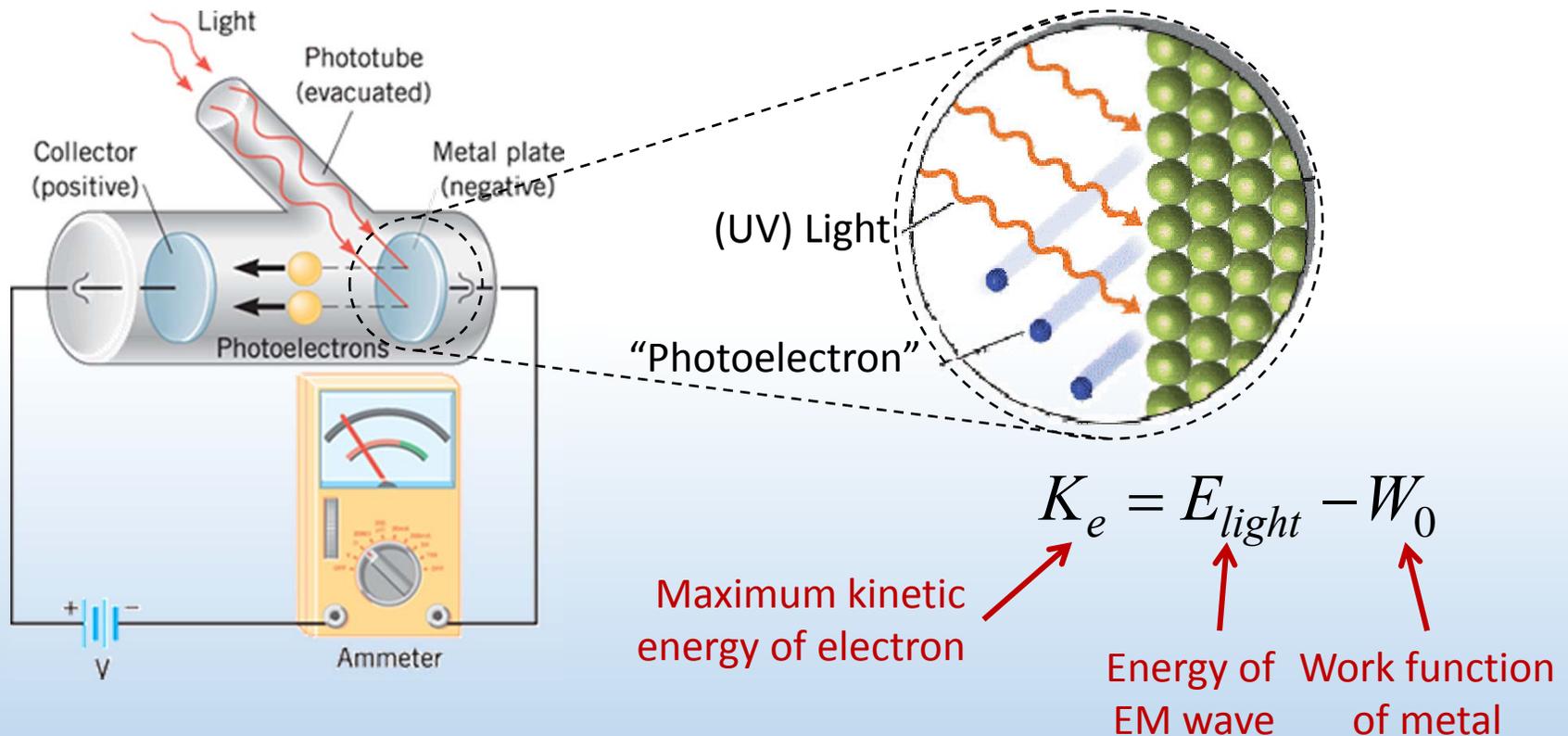
Electron mass:  $m = 9.1 \times 10^{-31} \text{ kg}$        $mc^2 = 8.2 \times 10^{-13} \text{ J} = \underline{\underline{511,000 \text{ eV}}}$

Since  $U = \frac{ke^2}{r}$ ,  $ke^2$  has units of  $\text{eV}\cdot\text{nm}$  like  $hc$        $ke^2 \approx 1.44 \text{ eV}\cdot\text{nm}$

$$\frac{ke^2}{\hbar c} = 2\pi \frac{ke^2}{hc} \approx \frac{1}{137} \quad \text{“Fine structure constant” (dimensionless)}$$

# Photoelectric effect

Light shining on a metal can eject electrons out of atoms



Light must provide enough energy to overcome Coulomb attraction of electron to nuclei:  $W_0$  ("Work function")

*Binding energy*

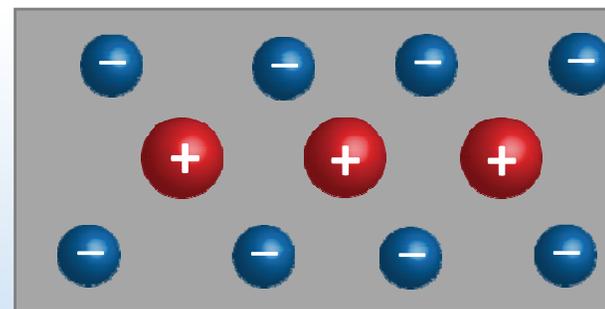
# Classical model vs. experiment

$$K_e = E_{light} - W_0$$

## Classical prediction

1. Increasing intensity should increase  $E_{light}$ ,  $K_e$
2. Changing  $f$  (or  $\lambda$ ) of light should change nothing

$$E_{light} \propto I_{light} = c\bar{u} \propto E_0^2$$



## Experimental result

1. Increasing intensity results in more  $e^-$ , at *same*  $K_e$
2. Decreasing  $f$  (or increasing  $\lambda$ ) *decreases*  $K_e$ , and below critical value  $f_0$ ,  $e^-$  emission stops

DEMO

# Photon Model of Light

Einstein proposed that light comes in discrete packets called *photons*, with energy:

$$E_{\text{photon}} = hf$$

Photon energy →      ← Frequency of EM wave

Planck's constant

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

Ex: energy of a single green photon ( $\lambda = 530 \text{ nm}$ , in vacuum)

$$f = \frac{c}{\lambda} \quad E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{530 \text{ nm}} = 2.3 \text{ eV}$$

Recall Lect. 24

$$\underline{hc = 1240 \text{ eV} \cdot \text{nm}}$$

Energy in a beam of green light (ex: laser pointer)

$$* E_{\text{light}} = N_{\text{photon}} E_{\text{photon}} *$$

**Checkpoint 2.1:** Higher/lower  $\lambda$   
= lower/higher  $E$



## ACT: CheckPoint 2.2

A **red** and **blue** light emitting diode (LEDs) both output 2.5 mW of light power.

Red = 650 nm & blue = 490nm

ROYGBIV

Which one emits more photons/second?

- 32% **A. Red**
- 49% B. Blue
- 19% C. The same

$$P = \frac{\Delta E_{light}}{\Delta t} = \frac{\Delta N}{\Delta t} E_{photon}$$

Number of photons per second

Energy per photon

Red light has less energy/photon so if they both have the same total power, red has to have more photons/time!

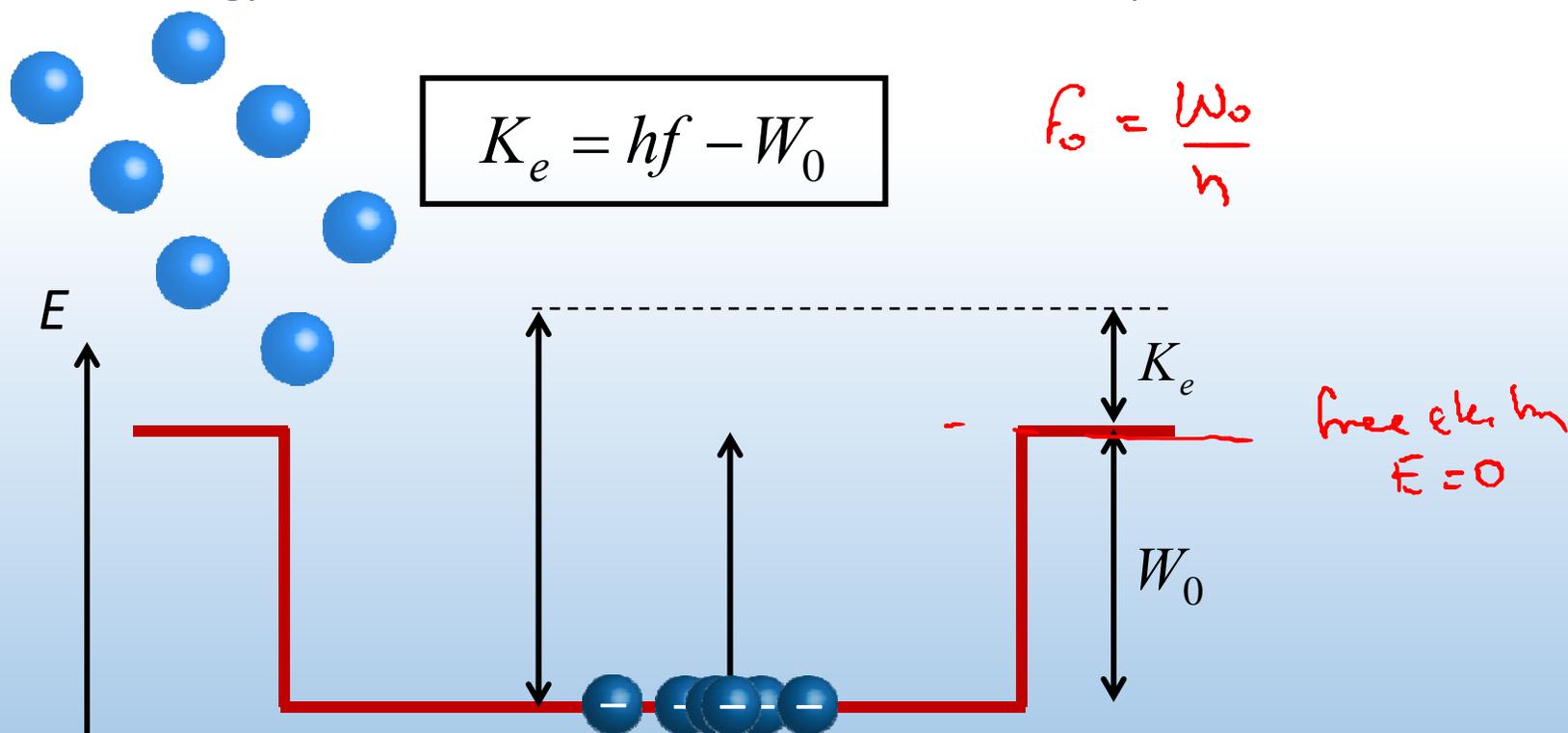
# Photoelectric effect explained

## Quantum model

1. Increasing intensity results in *more* photons of the same energy
2. Decreasing  $f$  (or increasing  $\lambda$ ) decreases photon energy

## Experimental result

1. More  $e^-$  emitted at *same*  $K_e$
2. Lower  $K_e$  and if  $hf_{\text{photon}} < hf_0 = W_0$   $e^-$  emission stops





# ACT: Photoelectric effect

You make a burglar alarm using infrared laser light ( $\lambda = 1000 \text{ nm}$ ) & the photoelectric effect. If the beam hits a metal detector, a current is generated; if blocked the current stops and the alarm is triggered.

Metal 1 –  $W_0 = 1 \text{ eV}$   
Metal 2 –  $W_0 = 1.5 \text{ eV}$   
Metal 3 –  $W_0 = 2 \text{ eV}$

You have a choice of 3 metals. Which will work?

- A. 1 and 2
- B. 2 and 3
- C. 1 only**
- D. 3 only

We need:

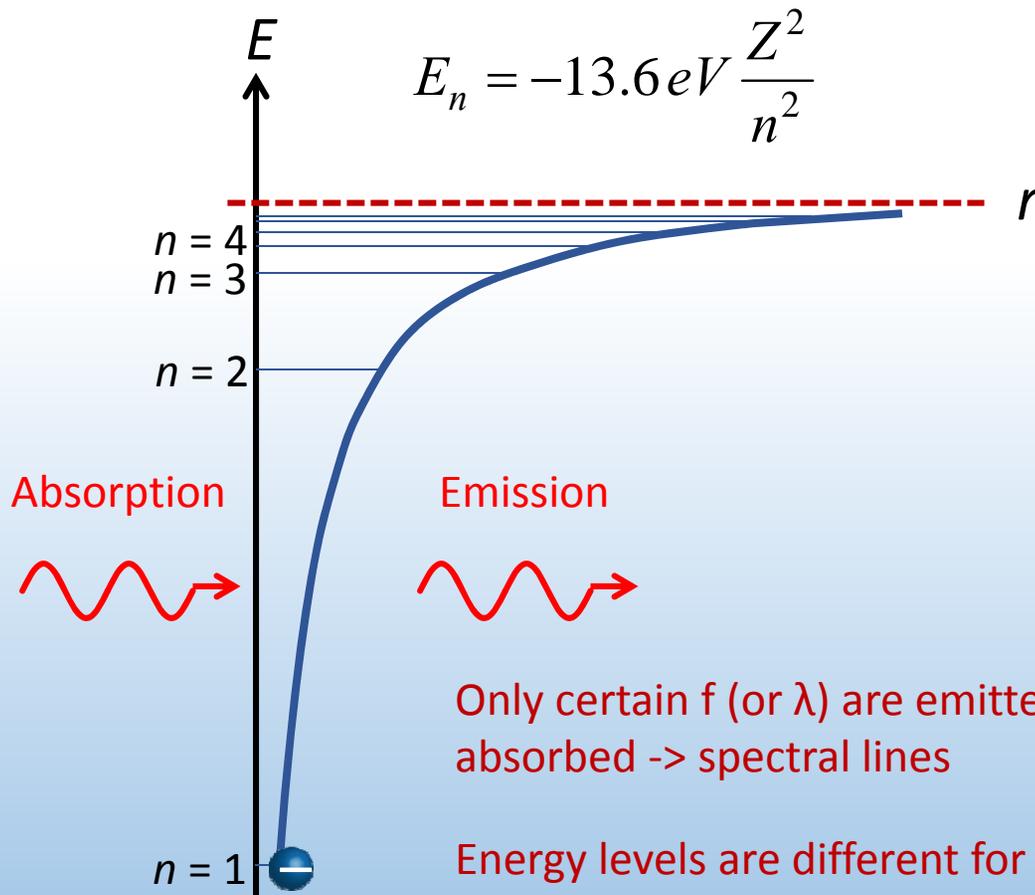
$$K_e = hf - W_0 > 0$$

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{1000 \text{ nm}} = 1.24 \text{ eV}$$



# Atomic spectra

Electrons in atom are in discrete energy levels



$$E_n = -13.6 eV \frac{Z^2}{n^2}$$

$e^-$  can jump from one level to another by absorbing or emitting a photon

Absorption ( $e^-$  jumps up in energy)

$$E_i + hf = E_f$$

Emission ( $e^-$  jumps down in energy)

$$E_i = E_f + hf$$

Energy is conserved

$$hf = E_n - E_{n'}$$

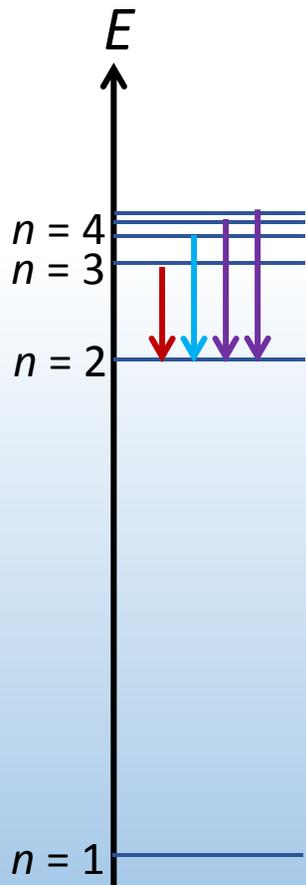
Only certain  $f$  (or  $\lambda$ ) are emitted or absorbed  $\rightarrow$  spectral lines

Energy levels are different for elements, so spectra are different

DEMO

# Calculation: H spectral lines

Calculate the wavelength of light emitted by hydrogen electrons as they transition from the  $n = 3$  to  $n = 2$  levels



Emission:

$$hf = E_i - E_f$$

$$\frac{hc}{\lambda} = 13.6eV Z^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^{-7} m^{-1} \left( Z^2 \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \right)$$

$$\lambda = 6.56 \times 10^{-7} m$$

Transition from  $n > 3$  to  $n = 2$  will generate higher energy/smaller  $\lambda$  photon

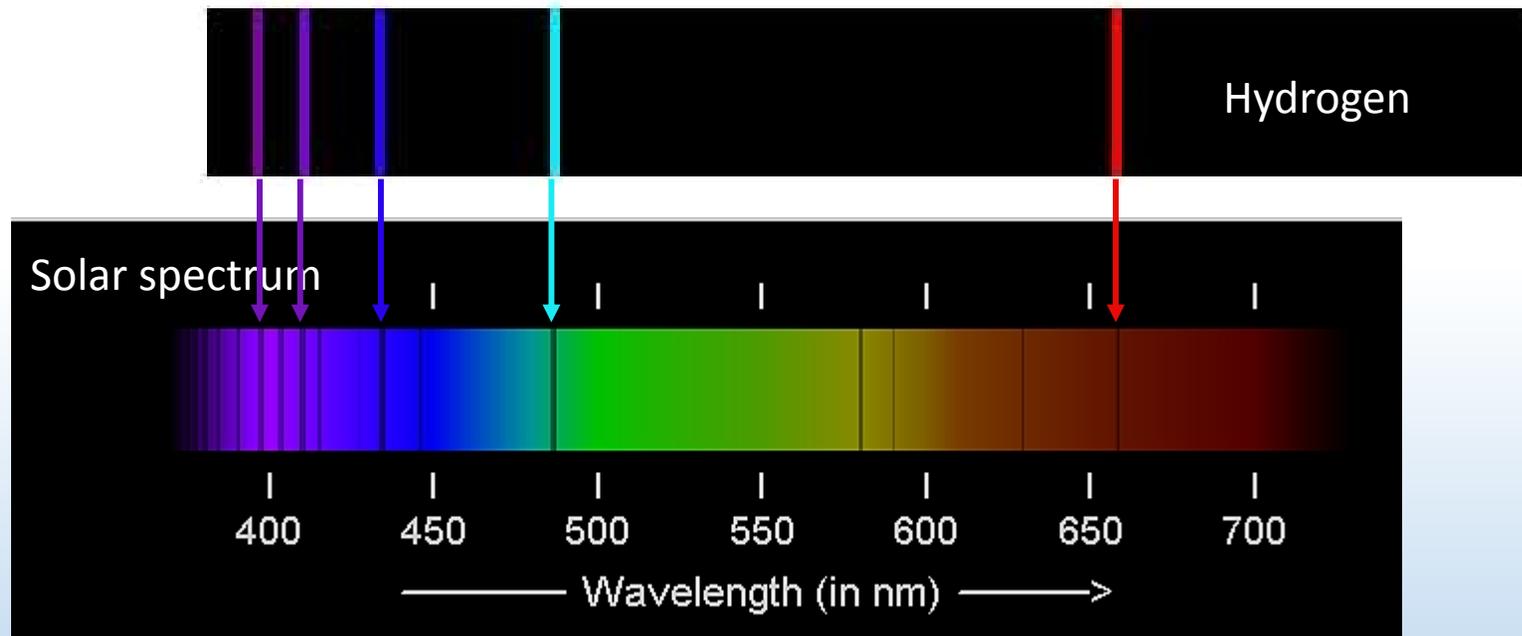
Hydrogen Emission Spectrum



Using  $hc = 1240 eV \cdot nm$

# *Solar spectrum*

Spectrum from celestial bodies can be used to identify its composition



Sun radiates over large range of  $\lambda$  because it is hot (5800K). Black spectral lines appear because elements inside sun absorb light at those  $\lambda$ .



# ACT: CheckPoint 3.1

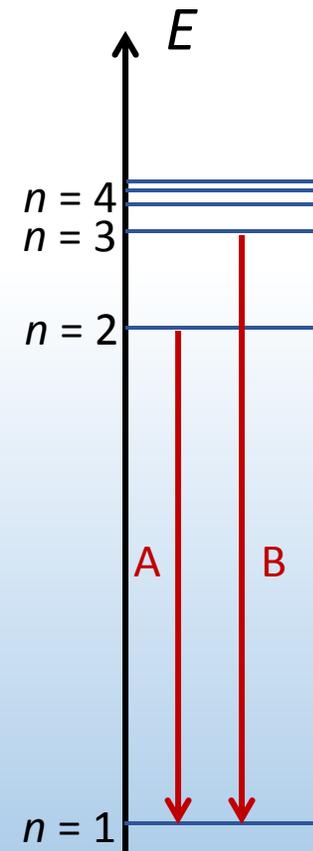
Electron A falls from energy level  $n = 2$  to  $n = 1$ . Electron B falls from energy level  $n = 3$  to energy level  $n = 1$ .

Which photon has a longer wavelength?

- 50% **A. Photon A**
- 36% B. Photon B
- 14% C. Both the same

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

So,  $E_A < E_B$  and  $\lambda_A > \lambda_B$





# ACT: CheckPoint 3.2

The electrons in a large group of hydrogen atoms are excited to the  $n = 3$  level.

How many spectral lines will be produced?

8% A. 1

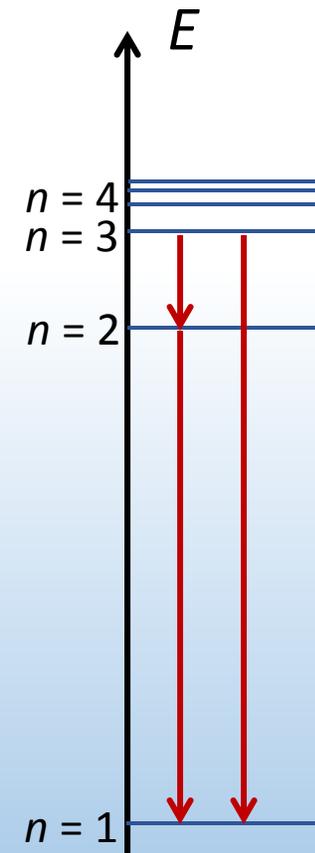
30% B. 2

47% C. 3

11% D. 4

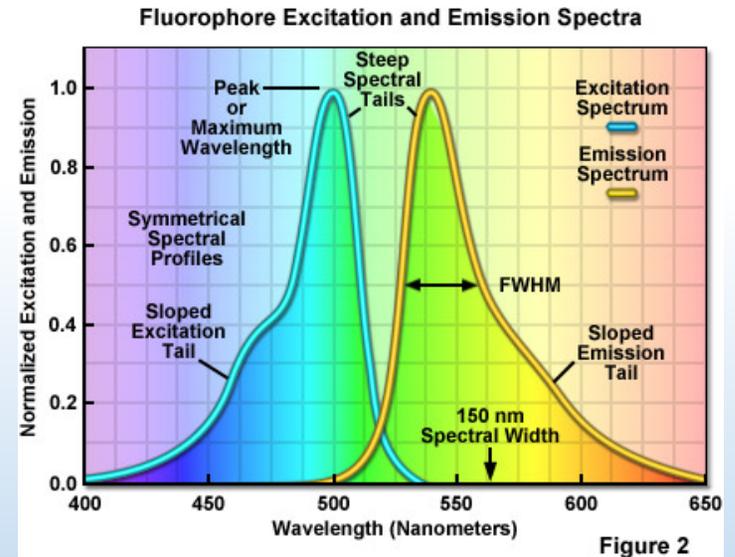
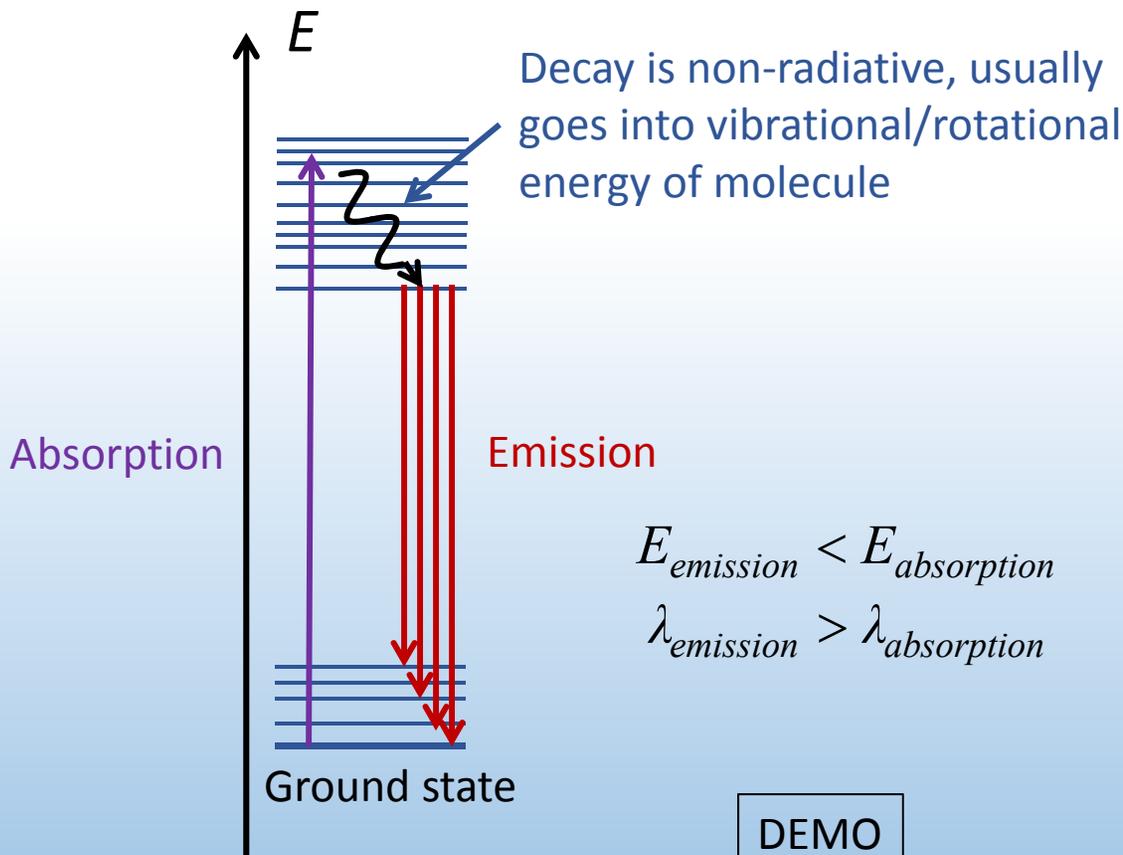
4% E. 5

Notice that  $n = 3$   $e^-$  could first decay to  $n = 2$ , then to  $n = 1$



# Fluorescence

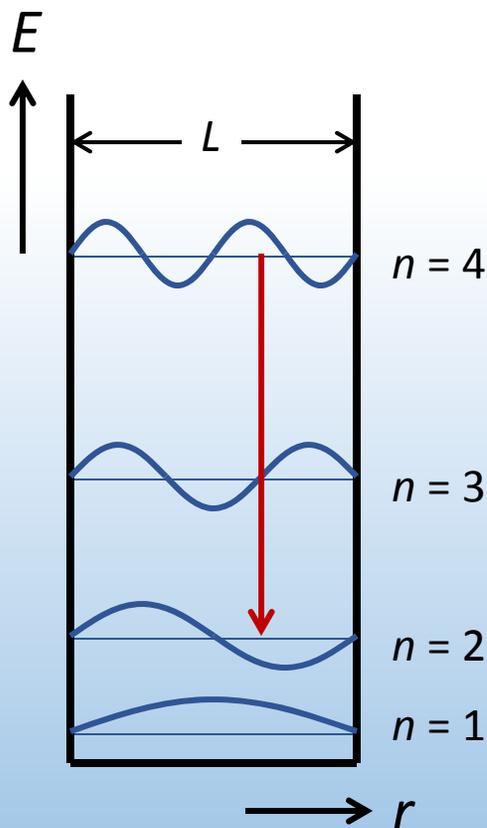
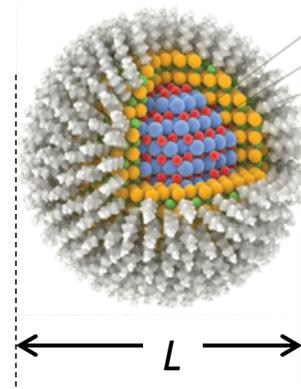
Molecules, like atoms, have discrete energy levels. Usually many more, and organized in *bands*



Fluorescent molecules that emit visible light absorb shorter  $\lambda$  (ex: UV)

# Quantum dots: “electron in a box”

Quantum dots (“Q-dots”) are nm-sized particles. Electrons are confined inside nm-sized “box”



Like Bohr model, only  $e^- \lambda$  that fit inside box are allowed:

$$n \frac{\lambda_e}{2} = L \quad n = 1, 2, 3, \dots$$

$$\lambda = h/p$$

$$E_{tot} = \frac{1}{2} m v^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda_e^2} = \frac{h^2 n^2}{8mL^2}$$

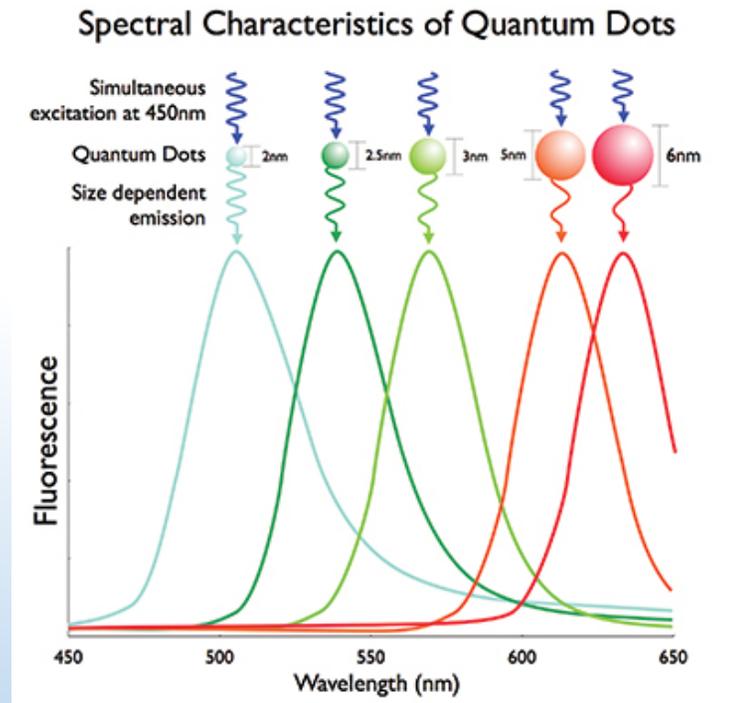
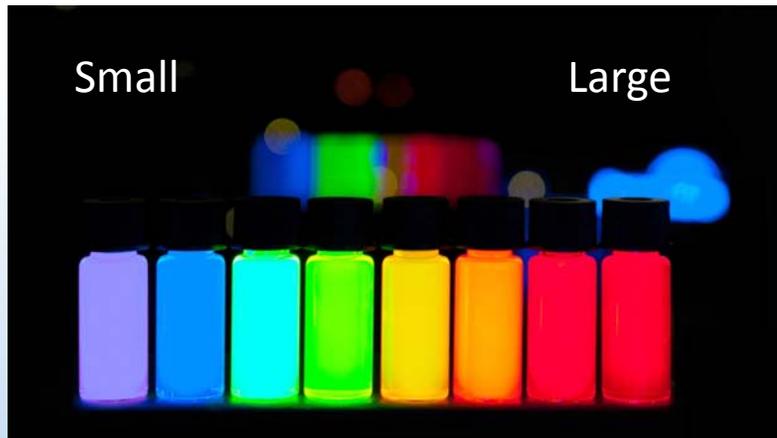
Emitted photon energy depends on Q-dot size:

$$hf = E_n - E_{n'} = \frac{h^2}{8mL^2} (n^2 - n'^2)$$

# Quantum dot emission

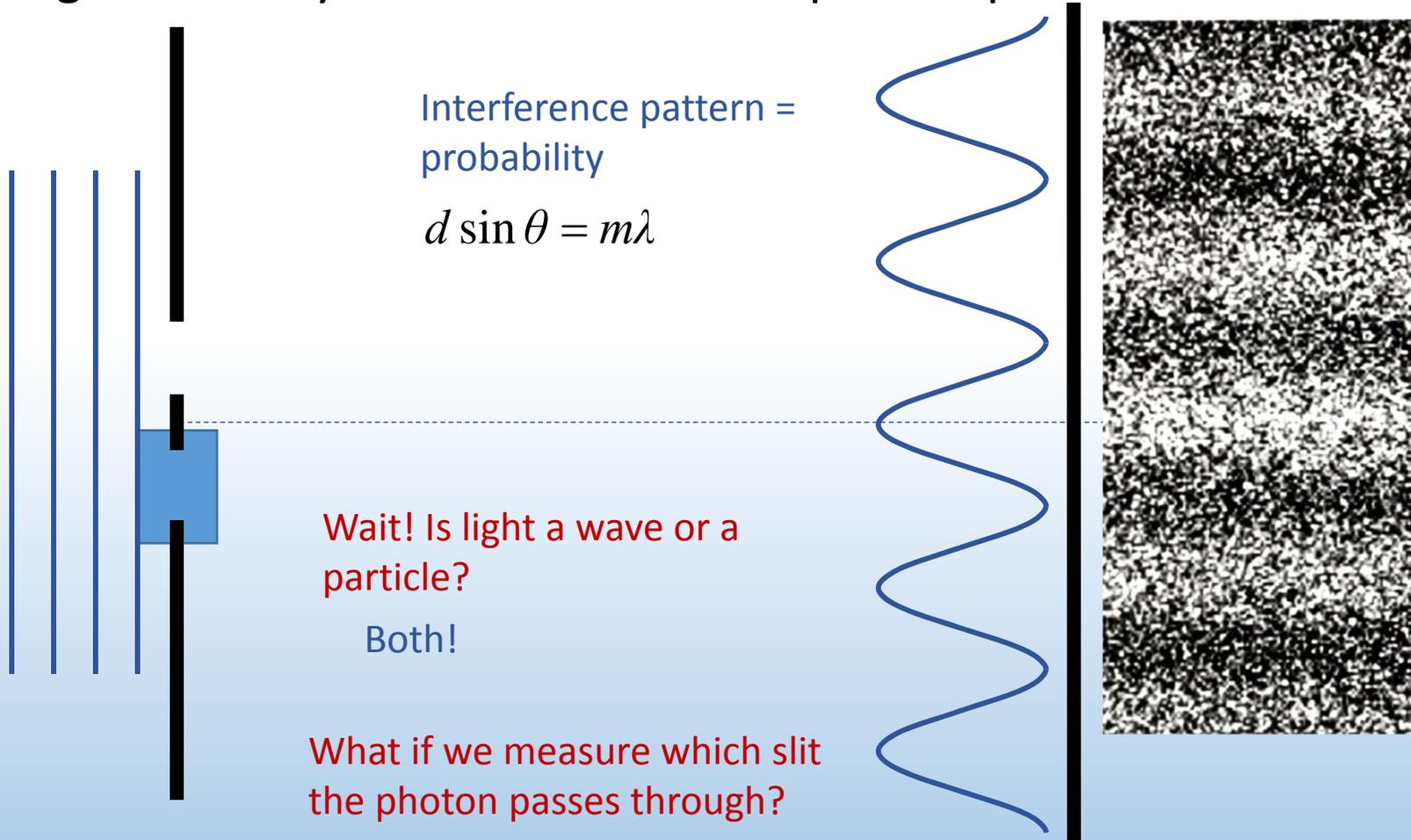
The larger the quantum dot, the longer the emitted photon wavelength

$$hf = \frac{hc}{\lambda} = E_n - E_{n'} = \frac{h^2}{8mL^2}(n^2 - n'^2)$$



# Young's double slit revisited

Light intensity is reduced until *one* photon passes at a time



Interference pattern =  
probability

$$d \sin \theta = m\lambda$$

Wait! Is light a wave or a  
particle?

Both!

What if we measure which slit  
the photon passes through?

Interference disappears!



# ACT: Photons & electrons

A free photon and an electron have the same energy of 1 eV.

Therefore they must have the same wavelength.

A. True

B. False

$$E_{\text{photon}} = hf = \frac{hc}{\lambda_{\text{photon}}} \quad \lambda_{\text{photon}} = \frac{hc}{E_{\text{photon}}}$$
$$= \frac{1240 \text{ eV} \cdot \text{nm}}{1 \text{ eV}} = 1240 \text{ nm}$$

$$E_{\text{elec}} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda_{\text{elec}}^2} \quad \lambda_{\text{elec}} = \frac{hc}{\sqrt{2mE_{\text{elec}}}}$$
$$= \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{511,000 \text{ eV} \cdot 1 \text{ eV}}} = 1.23 \text{ nm}$$

Notice BIG difference!

# *Summary of today's lecture*

- Quantum model of light

Light comes in discrete packets of energy  $E_{\text{photon}} = hf = \frac{hc}{\lambda}$

Light intensity is related to number of photons, not photon energy

- Spectral lines

Transitions between energy levels  $hf = E_n - E_{n'}$

- Wave-particle duality

Waves behave like particles (photons)

Particles behave like waves (electrons)