Your questions/comments

IMPORTANT ANNOUNCEMENTS:
Review Tuesday 6-8pm, in 141 Loomis – will cover selected questions from SP13 EX2 and 3 (see website)

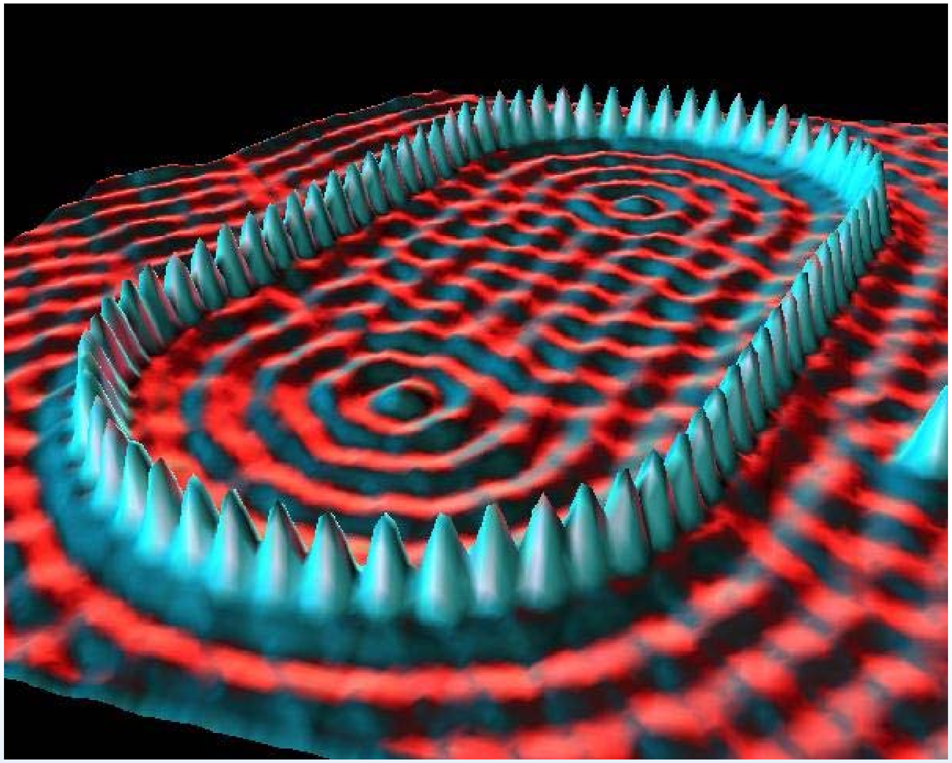
"I found all of this really confusing, especially the Uncertainty Principle equations. I'm hoping lecture is able to makes these topics more clear."

"Did not expect quantum in the scope of phys 102, but it's extremely welcome. Glad we get to explore some contemporary physics issues rather than just throwing objects in the air"

"Chemla, you got some 'splainin' to do!"

"plz go over de Broccoli wavelength. & eat your vegetables."

"I am Heisenberg."



Phys 102 – Lecture 24

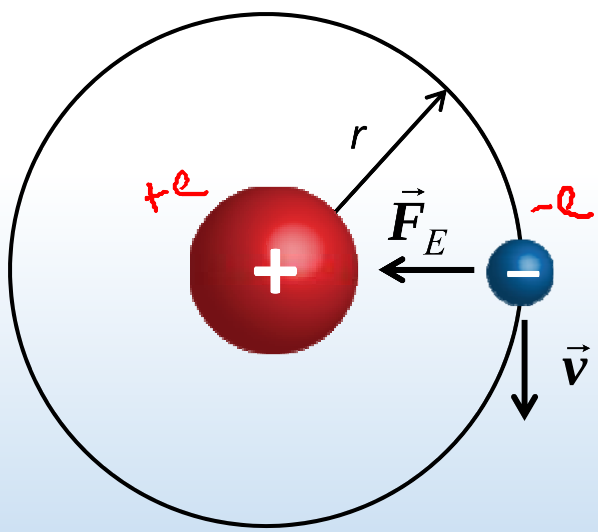
The classical and Bohr atom

State of late 19th Century Physics

- Two great theories *"Classical physics"*
  - Newton's laws of mechanics & gravity Phys. 101
  - Maxwell's theory of electricity & magnetism, including EM waves Phys. 102 Lect. 1–23
- But... some unsettling problems
  - Stability of atom & atomic spectra
  - Photoelectric effect – next lecture
  - ...and others
- New theory required *Quantum mechanics*

The "classical" atom

Negatively charged electron orbits around positively charged nucleus



Hydrogen atom

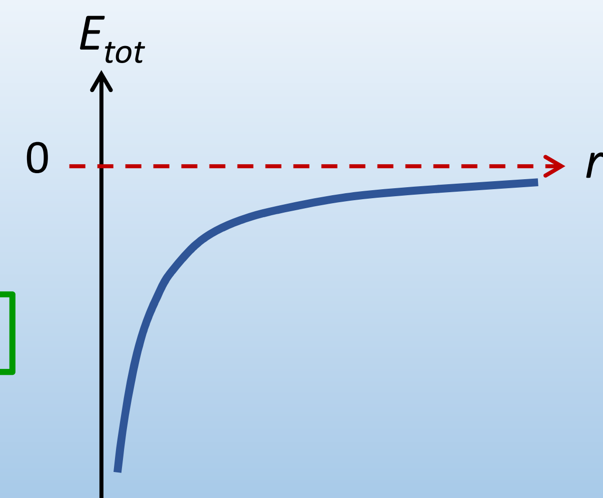
Orbiting e⁻ has centripetal acceleration:

$$F_E = k\frac{e^2}{r^2} = \cancel{ma_c}\,\frac{mv^2}{r} \quad \text{so} \quad \frac{ke^2}{r} = mv^2$$

Total energy of electron:

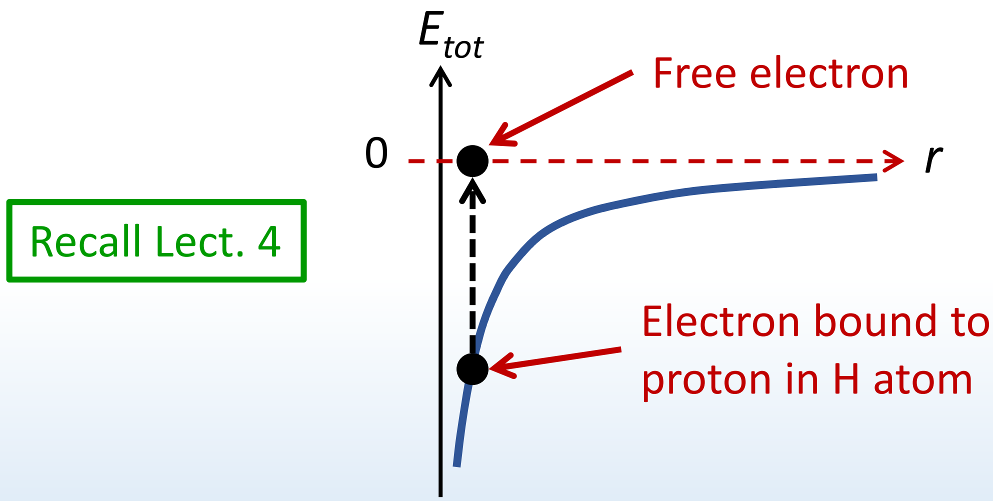
$$E_{tot} = K + U = \frac{1}{2}mv^2 - k\frac{e^2}{r} = -\frac{1}{2}\frac{ke^2}{r}$$

Recall Lect. 4



ACT: Quick review

What does the negative sign of the energy of the e^- mean?



A. Total energy is always positive

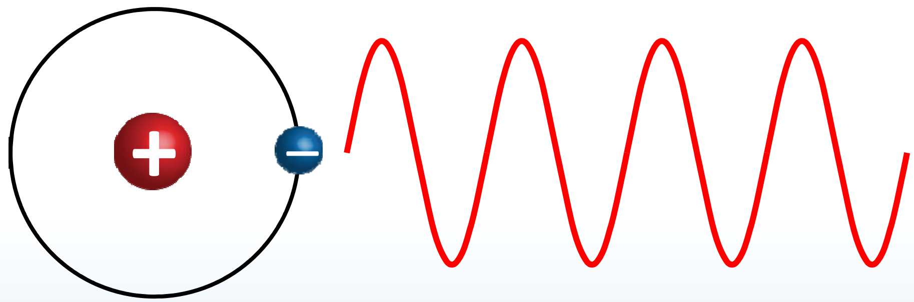
B. It is just a sign convention

C. It is negative relative to a free electron

Stability of classical atom

Prediction – orbiting e^- is an oscillating charge & should emit EM waves in every direction

Recall Lect. 15 & 16



$$\bar{u}_{tot} = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}B_{rms}^2$$

EM waves carry energy, so e^- should lose energy & fall into nucleus!

Classical atom is NOT stable!

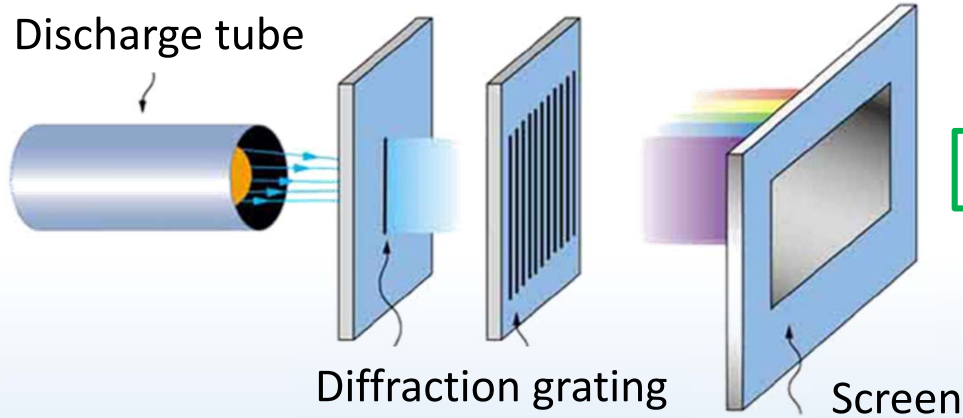
Lifetime of classical atom = 10^{-11} s

Reality – Atoms are stable

Atomic spectrum

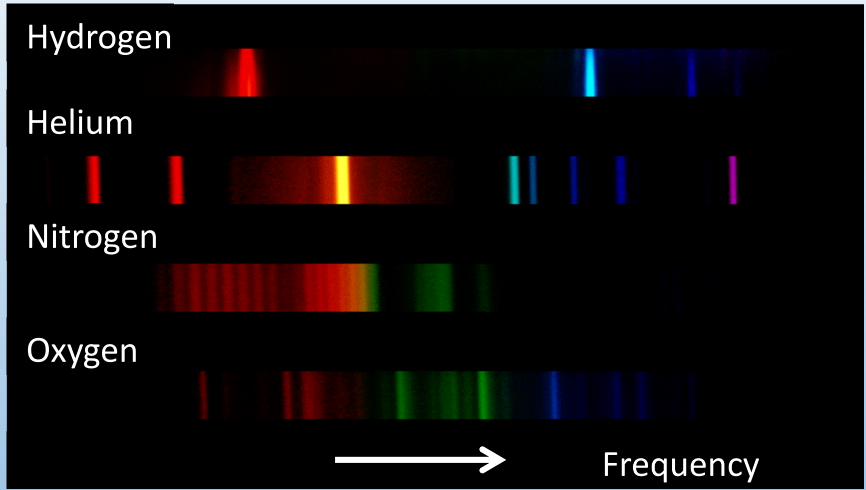
Prediction – e^- should emit light at whatever frequency f it orbits nucleus

$$d \sin\theta = m\lambda$$



Recall Lecture 22

Reality – Only *certain* frequencies of light are emitted & are different for different elements



Quantum mechanics

Quantum mechanics explains stability of atom & atomic spectra (and many other phenomena...)

QM is one of most successful and accurate scientific theories

Predicts measurements to <10^{-8} (ten parts per billion!)

Wave-particle duality – matter behaves as a wave

Particles can be in many places at the same time
Processes are probabilistic not deterministic
Measurement changes behavior

Certain quantities (ex: energy) are *quantized*

Matter waves

All Matter behaves as a wave with *de Broglie* wavelength

Wavelength of matter wave

$$\boxed{\lambda = \frac{h}{p}}$$

Planck's constant

$$h = 6.626\times10^{-34}\, J\cdot s$$

Momentum of particle

$p = mv$

Ex: a fastball (m = 0.5 kg, v = 100 mph ≈ 45 m/s)

$$\lambda_{fastball} = \frac{h}{p} = \frac{6.626\times10^{-34}}{0.5\cdot45} = 3\times10^{-35}\, m$$

20 orders of magnitude smaller than the proton!

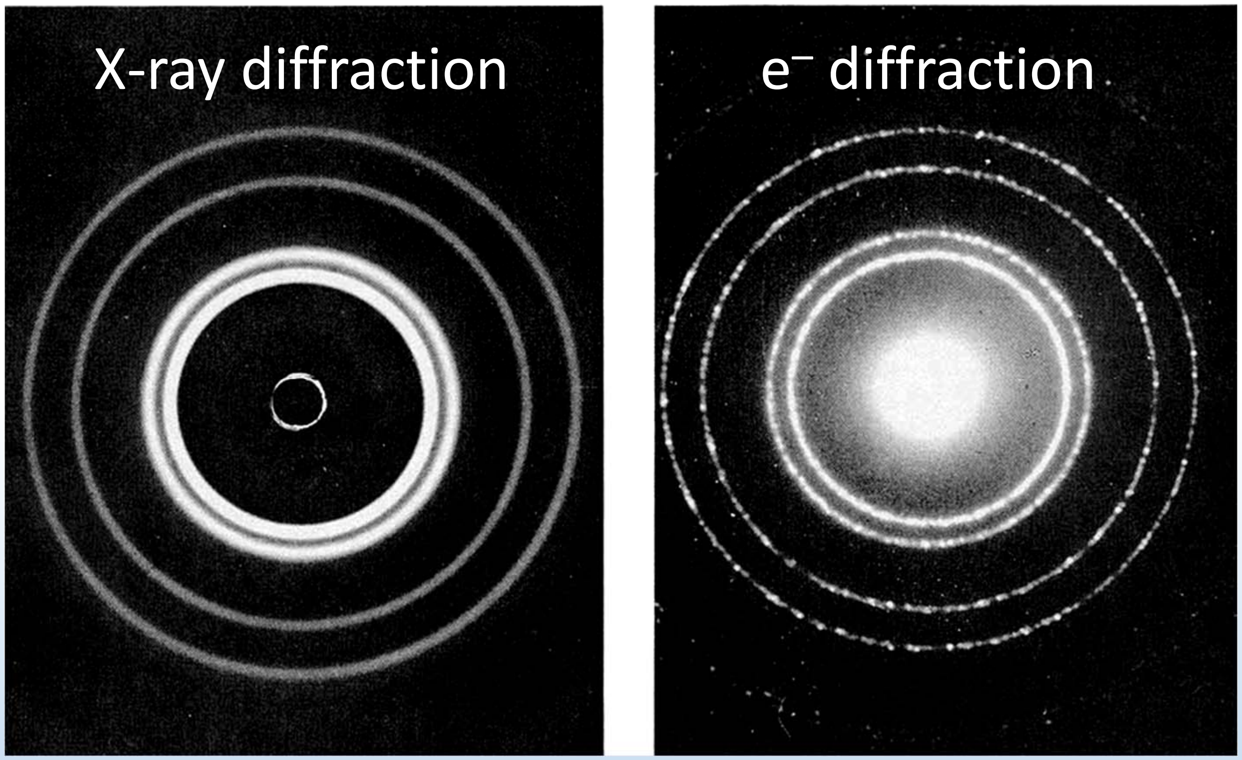
Ex: an electron ($m = 9.1\times10^{-31}$ kg, $v = 6\times10^{5}$ m/s)

$$\lambda_{electron} = \frac{h}{p} = \frac{6.626\times10^{-34}}{9.11\times10^{-31}\cdot6\times10^{5}} = 1.2\, nm$$

X-ray wavelength

How could we detect matter wave? Interference!

X-ray vs. electron diffraction

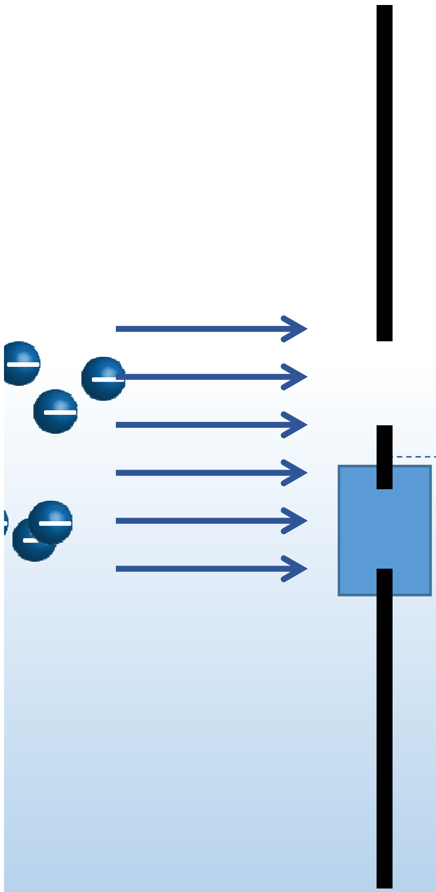


DEMO

Identical pattern emerges if de Broglie wavelength of e^- equals the X-ray wavelength!

Electron diffraction

Beam of monoenergetic e⁻ passes through double slit



Interference pattern = probability

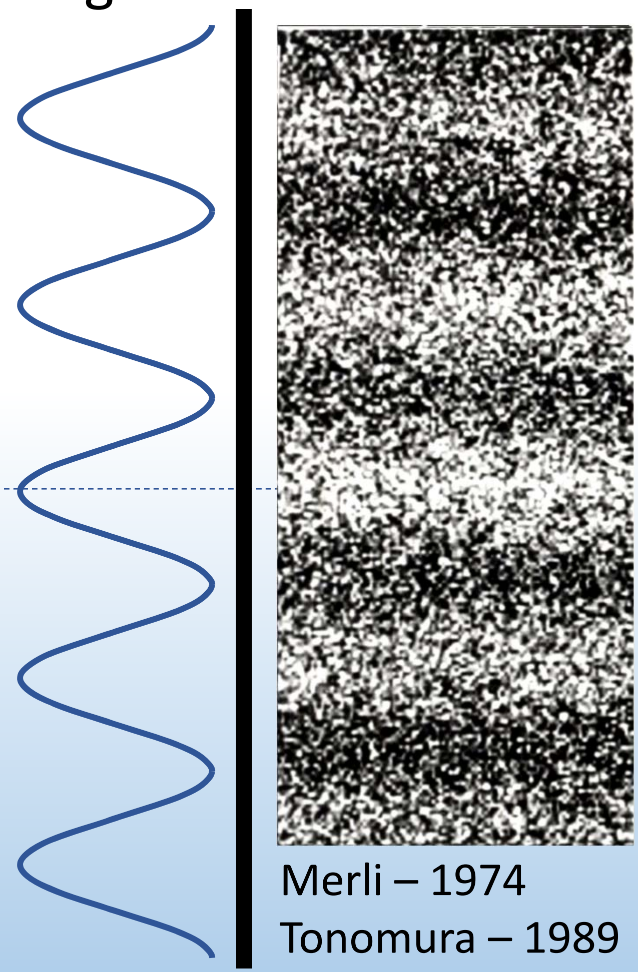
$$d \sin\theta = m\lambda \qquad \lambda = \frac{h}{p}$$

Wait! Does this mean e⁻ passes through both slits?

Yes! e⁻ is in both places at once

What if we measure which slit the e⁻ passes through?

Interference disappears!

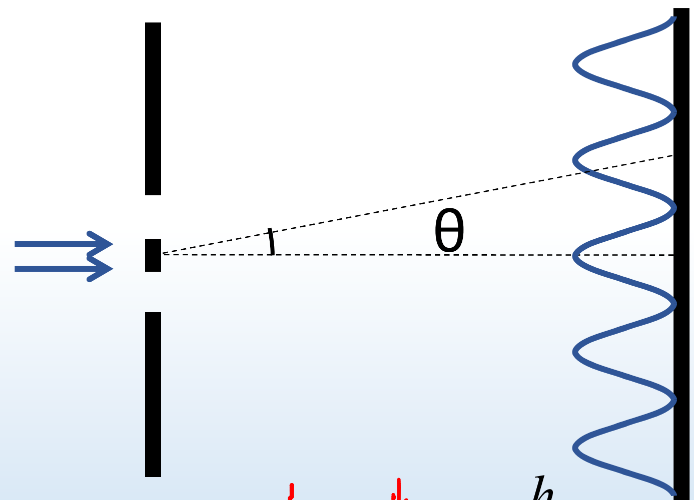


Merli – 1974
Tonomura – 1989

ACT: Double slit interference

Consider the interference pattern from a beam of mono-energetic electrons *A* passing through a double slit.

Now a beam of electrons *B* with 4x the energy of *A* enters the slits. What happens to the spacing θ between interference maxima?



A. $\theta_B = 4\,\theta_A$

B. $\theta_B = 2\,\theta_A$

C. $\theta_B = \theta_A$

D. $\theta_B = \theta_A / 2$

E. $\theta_B = \theta_A / 4$

$$d\sin\theta \approx d\theta = m\lambda \quad \lambda = \frac{h}{p}$$

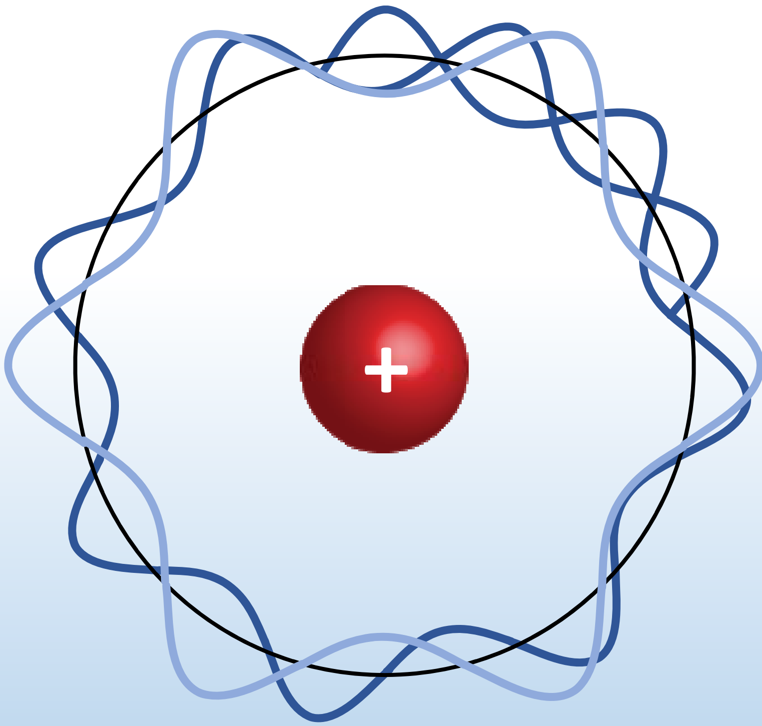
$$K.E. = \frac{1}{2}m_e v^2 = \boxed{\frac{p^2}{2m_e}}$$

$p = m_e v$

If $E_B = 4\,E_A$, $p_B = 2\,p_A$

The Bohr model

e^- behave as waves & only orbits that fit an integer number of wavelengths are allowed



Orbit circumference

Phys.101

$$2\pi r = n\lambda \qquad pr = n\frac{h}{2\pi} \qquad n = 1, 2, 3\ldots$$

= mvr = L

de Broglie wavelength

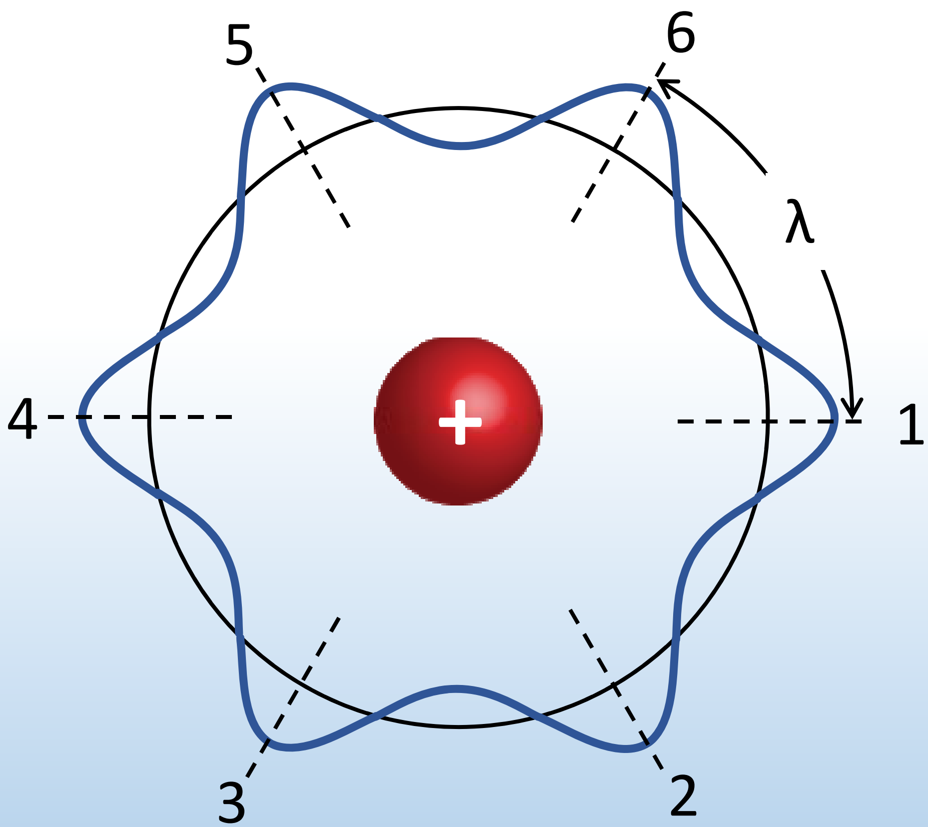
Angular momentum is *quantized*

$$L_n = n\hbar \qquad \hbar \equiv \frac{h}{2\pi}$$ "h bar"

"Quantum number"

ACT: Bohr model

What is the quantum number *n* of this hydrogen atom?



A. $n = 1$

B. $n = 3$

C. $n = 6$

D. $n = 12$

n is the number of e⁻ λ that fit around orbit circumference

Energy and orbit quantization

Angular momentum is *quantized*

$$L_n \equiv pr = mvr = n\hbar \qquad n = 1, 2, 3\ldots$$

From classical atom:

$$mr \cdot ke^2 = m^2v^2r^2$$

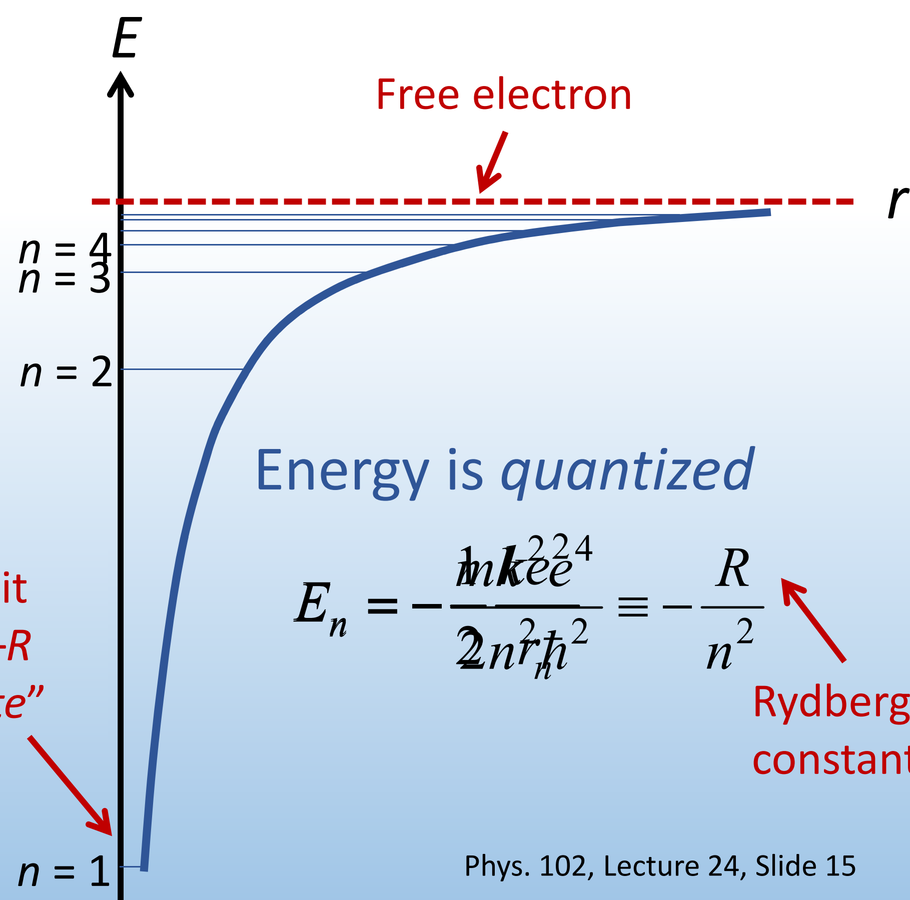
$n = 1$

$n = 2$

$n = 3$

Smallest
e^- canno

CheckPoint 3.1 radius is 4× that of n = 1 orbit



Radius of orbit is *quantized*

$$r_n = \frac{n^2\hbar^2}{mke^2} \equiv n^2 a_0$$

"Bohr radius"

Smallest orbit has energy –*R*
"ground state"

Energy is *quantized*

$$E_n = -\frac{mk^2e^4}{2n^2\hbar^2} \equiv -\frac{R}{n^2}$$

Rydberg constant

ACT: Bohr model momentum

According to the Bohr model, how is the electron *__linear momentum__* $p = mv$ quantized?

$$L_n \equiv pr = mvr = n\hbar$$

$$r_n = n^2 a_0$$

$$p_n = mv_n = \frac{n\hbar}{r_n} = \frac{\hbar}{n a_0}$$

A. *p* is not quantized
B. *p* is quantized as 1/*n*
C. *p* is quantized as *n*

ACT: CheckPoint 3.2

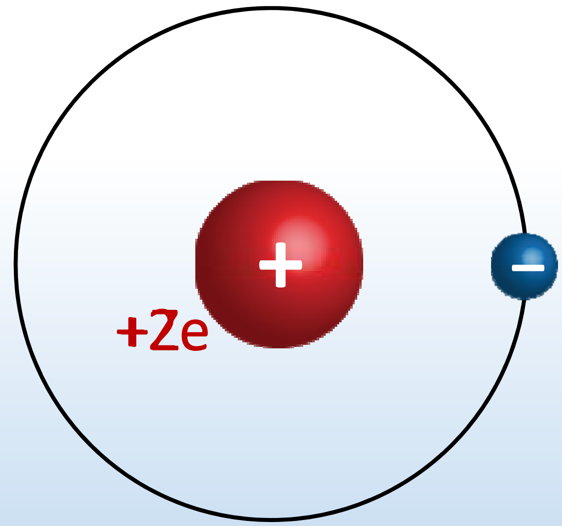
Suppose the charge of the nucleus is doubled (+2e), but the e⁻ charge remains the same (–e). How does *r* for the ground state (*n* = 1) orbit compare to that in hydrogen?

For hydrogen: $r_n = \dfrac{n^2\hbar^2}{mke^2}$

In general: $r_n = \dfrac{n^2\hbar^2}{mkZe^2} = \dfrac{n^2}{Z}a_0$

One factor of e comes e⁻ charge,
second factor from nuclear charge

$$E_n = -\frac{mk^2Z^2e^4}{2n^2\hbar^2} = -\frac{Z^2}{n^2}R$$



A. 1/2 as large 46%

B. 1/4 as large 36%

C. the same 18%

Z = 1, 2, 3

Works for any single e⁻ atom (ex: H, He⁺, Li⁺⁺)

ACT: CheckPoint 3.3

There is a particle in nature called a *muon*, which has the same charge as the electron but is 207 times heavier. A muon can form a hydrogen-like atom by binding to a proton.

$$r_n = \frac{n^2\hbar^2}{mke^2}$$

mass of electron

How does the radius of the ground state (n = 1) orbit for this hydrogen-like atom compare to that in hydrogen?

A. 207× larger
27%

B. The same
37%

C. 207× smaller
36%

Atomic units

At atomic scales, Joules, meters, kg, etc. are not convenient units

"Electron Volt" – energy gained by charge +1e when accelerated by 1 Volt: $U = qV$

1e = 1.6×10^{-19} C, so 1 eV = 1.6×10^{-19} J

Planck constant: $h = 6.626 \times 10^{-34}$ J·s
Speed of light: $c = 3 \times 10^{8}$ m/s

$$hc \approx 2\times10^{25}\,\mathrm{J\cdot m} = 1240\,\mathrm{eV\cdot nm}$$

Electron mass: $m = 9.1 \times 10^{-31}$ kg

$$mc^2 = 8.2\times10^{-13}\,J = 511{,}000\,\mathrm{eV}$$

Since $U = \dfrac{ke^2}{r}$, ke^2 has units of eV·nm like hc

$$ke^2 \approx 1.44\,\mathrm{eV\cdot nm}$$

$$\frac{ke^2}{\hbar c} = 2\pi\frac{ke^2}{hc} \approx \frac{1}{137}$$

"Fine structure constant" (dimensionless)

Rydberg constant & Bohr radius

Energy of electron in H-like atom (1 e⁻, nuclear charge +Ze):

$$E_n = -\frac{Z^2}{n^2}R = -\frac{mk^2Z^2e^4}{2n^2\hbar^2} = -\frac{Z^2}{2n^2}mc^2\left(\frac{ke^2}{\hbar c}\right)^2 = -\frac{511,000\,eV}{2\cdot 137^2}\frac{Z^2}{n^2}$$

$$= -13.6\,eV\frac{Z^2}{n^2}$$

Radius of electron orbit:

$$r_n = \frac{n^2}{Z}a_0 = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2}{Z}\frac{\hbar c}{mc^2}\left(\frac{\hbar c}{ke^2}\right) = \frac{n^2}{Z}\frac{1240\,eV\cdot nm}{2\pi\cdot 511,000\,eV}137$$

$$= 0.0529\,nm\frac{n^2}{Z}$$

ACT: Hydrogen-like atoms

Consider an atom with a nuclear charge of +2e with a single electron orbiting, in its ground state (*n* = 1), i.e. He^{+}.

How much energy is required to ionize the atom totally?

A. 13.6 eV

B. 2 × 13.6 eV

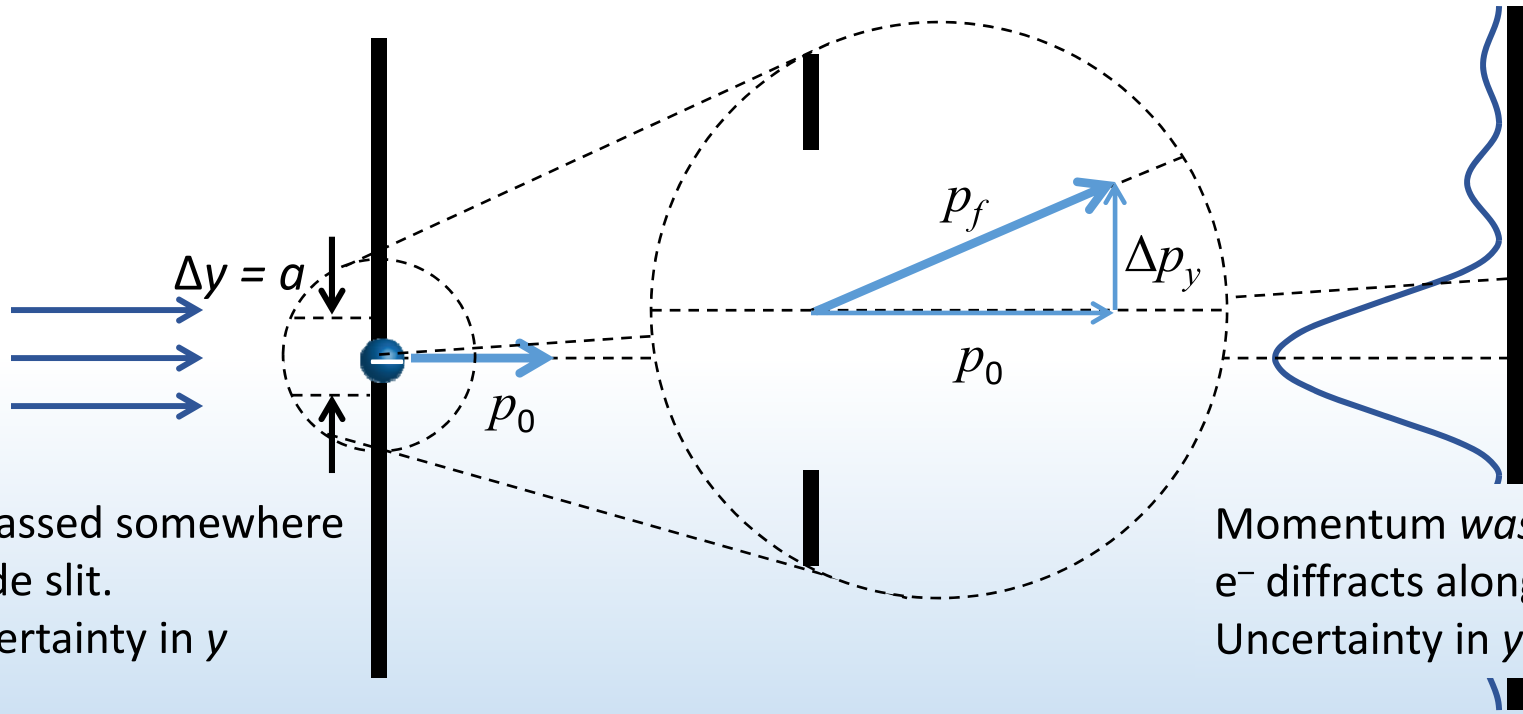
C. 4 × 13.6 eV

$$E_n = -13.6\,eV\,\frac{Z^2}{n^2}$$

Energy measured relative to free electron (*E* = 0)

Heisenberg Uncertainty Principle

e⁻ beam with momentum p_0 passes through slit will diffract



e⁻ passed somewhere inside slit.
Uncertainty in *y*

Momentum *was* along *x*, but e⁻ diffracts along *y*.
Uncertainty in *y*-momentum

If slit narrows, diffraction pattern spreads out

Uncertainty in *y*

$$\boxed{\Delta y \cdot \Delta p_y \gtrsim \frac{\hbar}{2}}$$

Uncertainty in *y*-momentum

CheckPoint 2.1

ACT: CheckPoint 4

The Bohr model cannot be correct! To be consistent with the Heisenberg Uncertainty Principle, which of the following properties *cannot* be quantized?

1. Energy is quantized $E_n = -\frac{R}{n^2}$
2. Angular momentum is quantized $L_n = n\hbar$
3. Radius is quantized $r_n = n^2 a_0$
4. Linear momentum & velocity are quantized $p_n = \frac{\hbar}{n a_0}$

A. All of the above

B. #1 & 2

C. #3 & 4

If *r* and *p* are quantized, we would know both *exactly*, inconsistent with Uncertainty Principle

Summary of today's lecture

- Classical model of atom

  Predicts unstable atom & cannot explain atomic spectra

- Quantum mechanics

  Matter behaves as waves

  Heisenberg Uncertainty Principle

- Bohr model

  Only orbits that fit *n* electron wavelengths are allowed

  $$\underbrace{\begin{array}{l}\text{Explains the stability of the atom}\\ \text{Energy quantization correct for single } e^{-} \text{ atoms (H, } He^{+}\text{, } Li^{++}\text{)}\\ \text{However, it is \textit{fundamentally} incorrect}\end{array}}$$

Need complete quantum theory (Lect. 26)