Your questions/comments

IMPORTANT ANNOUNCEMENTS:

Exam I tomorrow on Lect. 1–5 material. Extra office hours today.

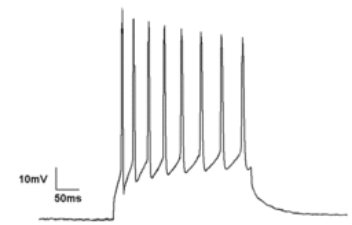
“Sorry went through it fast I am studying”

“I'm having trouble understanding the concept of discharging a capacitor. What are real world examples of discharging capacitors?”

“What exactly is a capacitor? What does it do? Why is it important?”

“I still don't completely understand why current stops when the capacitor is fully charged.”

“The biggest challenge for this prelecture was understanding when to use the exponential equations that were given to us. There were no examples in questions or checkpoints of when we have to use them, so some clarification would help!”

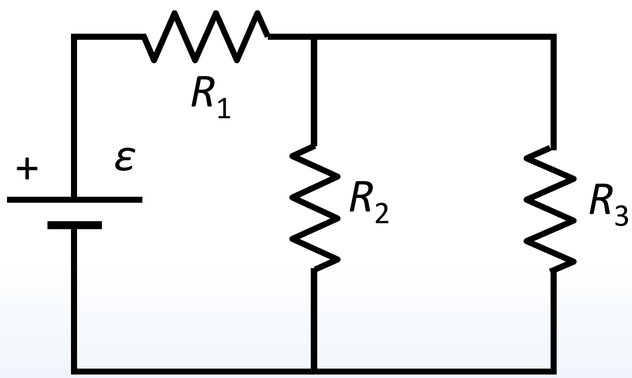


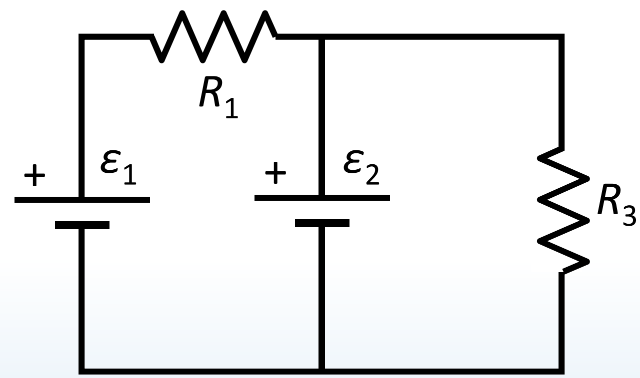
Phys 102 – Lecture 9

RC circuits

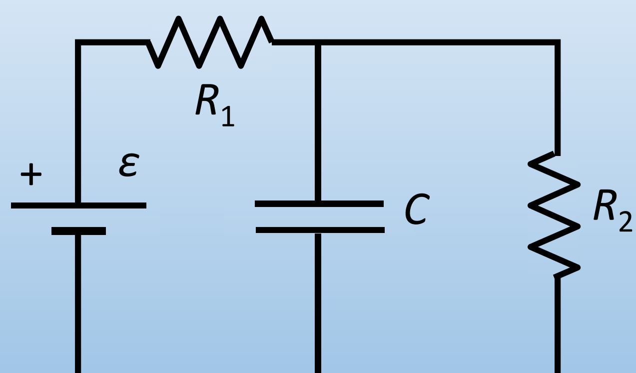
Recall from last time...

We solved various circuits with resistors and batteries (also capacitors and batteries)





What about circuits that combine all three... ...RC circuits



RC circuits

Circuits that store and release energy controllably...

Camera flash

Defibrillator

Nerve cells

Today we will...

- Learn about *RC circuits*

  Charge on capacitors cannot change instantly,

  so behavior of RC circuit *depends on time*

- Analyze RC circuits under different situations

  Charging capacitors at short/long times

  Discharging capacitors at short/long times

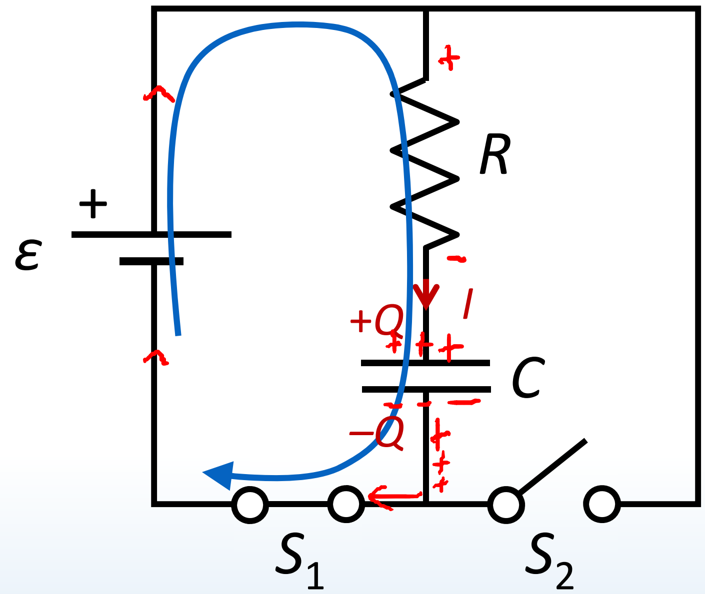
  Time dependence

- Apply these concepts

  Nerve cells and nerve impulses (action potential)

Charging capacitor

Initially the capacitor is uncharged ($Q_0 = 0$)
At $t = 0$ we close switch S_1.



Immediately after:
Current I_0 flows around loop, through C
No charge on C ($Q_0 = 0$)

$$\varepsilon - I_0 R - V_C = 0 \qquad V_C = \frac{Q_0}{C} = 0$$

$$I_0 = \frac{\varepsilon}{R}$$ C acts like a wire!

Does a current through C mean charges "hop" over C plates?

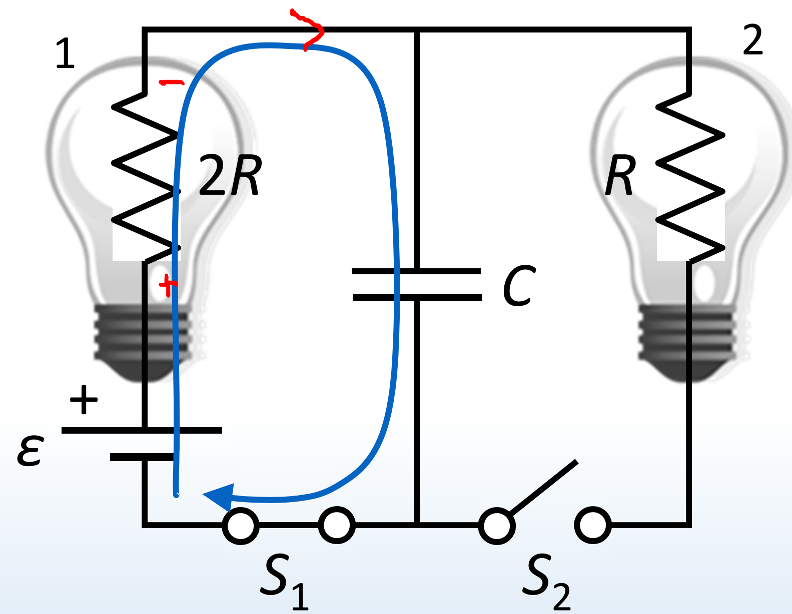
After a long time ($t = \infty$):
Charge on C builds until $V_C = \varepsilon$. Current decreases to zero ($I_\infty = 0$)

$$\varepsilon - I_\infty R - V_C = 0 \qquad I_\infty = 0$$

$$Q_\infty = C\varepsilon$$ C acts like an open circuit!

ACT: CheckPoint 1.1

Both switches are initially open and the capacitor is uncharged. What is the current through light bulb 1 ***right after*** switch S_1 is closed?



A. $I_b = 0$ 17%

B. $I_b = \varepsilon/3R$ 10%

C. $I_b = \varepsilon/2R$ 58%

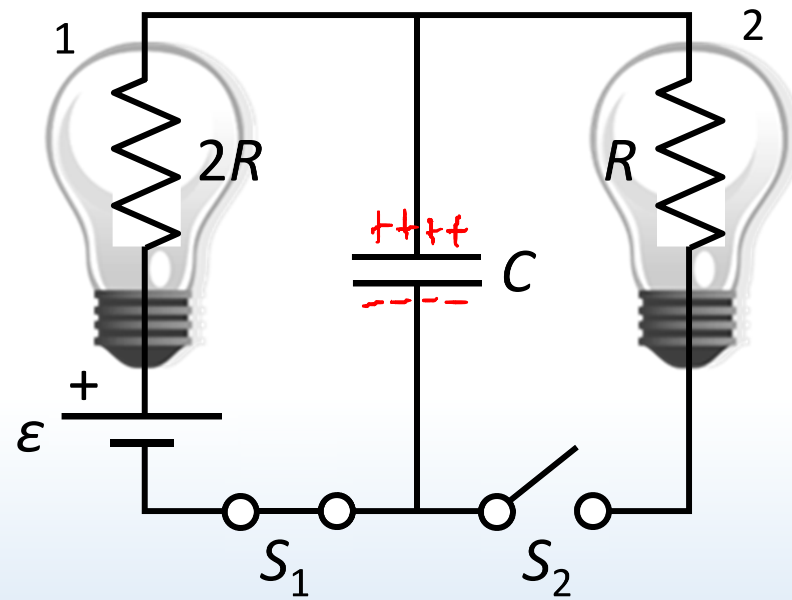
D. $I_b = \varepsilon/R$ 15%

DEMO

Capacitor is still uncharged so $V_C = Q/C = 0$ $\quad \varepsilon - I_b(2R) = 0$

ACT: charging

Both switches are initially open and the capacitor is uncharged. What is the voltage across the capacitor <u>*a long time after*</u> switch S_1 is closed?



A. $V_C = 0$

B. $V_C = \varepsilon/2$

C. $V_C = \varepsilon$

D. $V_C = 2\varepsilon$

DEMO

55%

Capacitor is fully charged so $I_b = 0$ (***CheckPoint 1.3***)

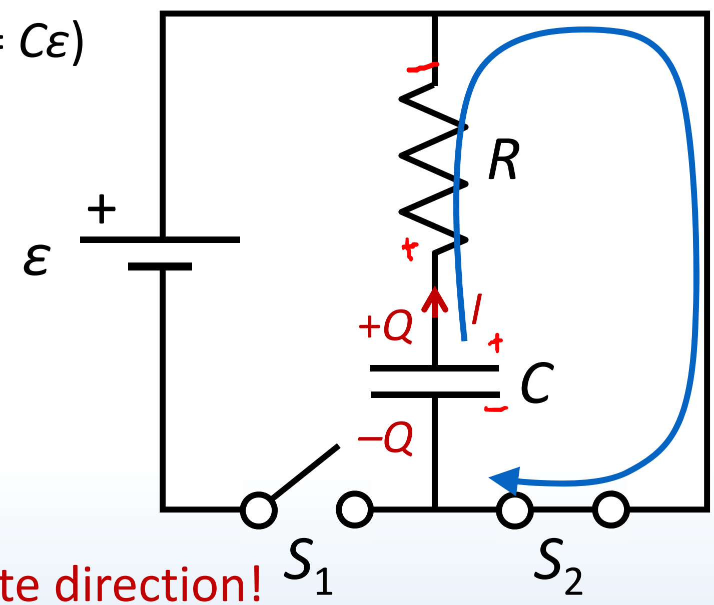
$V_R = I_b(2R) = 0$

$$\varepsilon - V_C = 0$$

$$Q_\infty = C\varepsilon$$

Discharging capacitor

Initially the capacitor is fully charged ($Q_0 = C\varepsilon$)
At $t = 0$ we close switch S_2.



Immediately after:
Current I_0 driven around loop, through C
Charge on C from before ($Q_0 = C\varepsilon$)

$$+V_C - I_0 R = 0 \qquad V_C = \frac{Q_0}{C}$$

$$I_0 = \frac{V_C}{R} = \frac{Q_0}{RC}$$

Note current in opposite direction!
C acts like a battery!

After a long time ($t = \infty$):
Charge on C dissipates until $V_C = 0$. Current decreases to zero ($I_\infty = 0$)

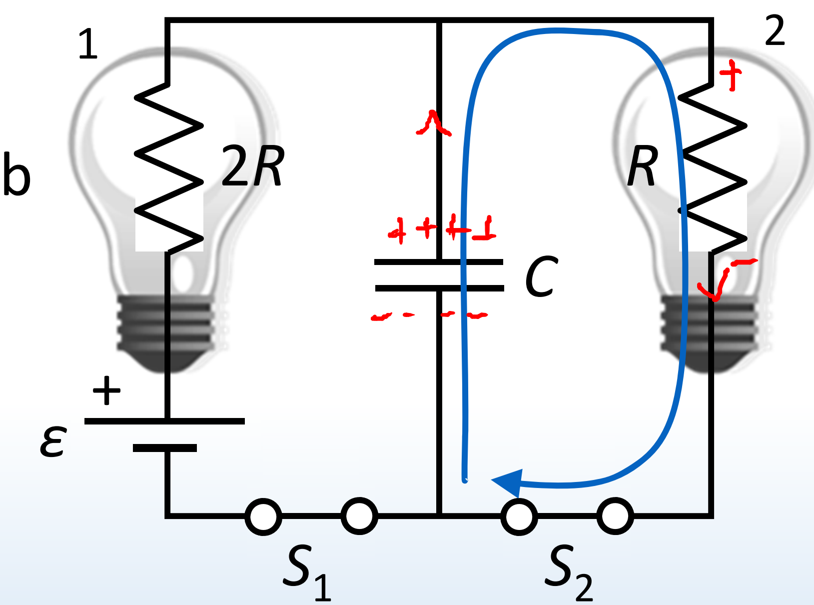
$$+V_C - I_\infty R = 0 \qquad I_\infty = 0$$

$$Q_\infty = 0$$

C acts like an open circuit!

ACT: CheckPoint 1.5

After S_1 has been closed for a long time, it is opened and S_2 is closed. What is the current through light bulb 2 *right after* S_2 is closed?



A. $I_b = 0$ 16%

B. $I_b = \varepsilon/3R$ 17%

C. $I_b = \varepsilon/2R$ 18%

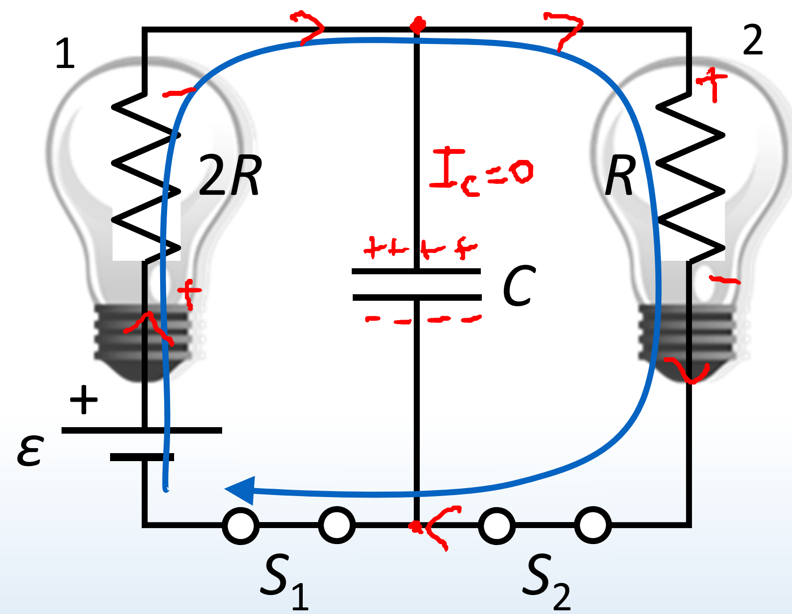
D. $I_b = \varepsilon/R$ 49%

DEMO

Capacitor is still fully charged from before, so $V_C = \varepsilon$ $V_C - I_b R = 0$

ACT: RC circuit practice

Now both S_1 *and* S_2 are closed. What is the current through light bulb 2 a *long time after* both switches are closed?



A. $I_b = 0$

B. $I_b = \varepsilon/3R$

C. $I_b = \varepsilon/2R$

D. $I_b = \varepsilon/R$

DEMO

Capacitor is fully charged so no current flows through it. BUT, current still flows around outer loop!

$$\varepsilon - I_b(2R) - I_b R = 0$$

Summary: charging & discharging

Study this slide

- Charge (and therefore voltage, since $V_C = Q/C$) on capacitors cannot change instantly
- *Short term* behavior of capacitor:

  If the capacitor is *charging*, current I drives charge onto it, and Q increases (acts like a wire)

  If the capacitor is *discharging*, current I drives charge off of it, and Q decreases (acts like a battery)

- *Long term* behavior of capacitor:

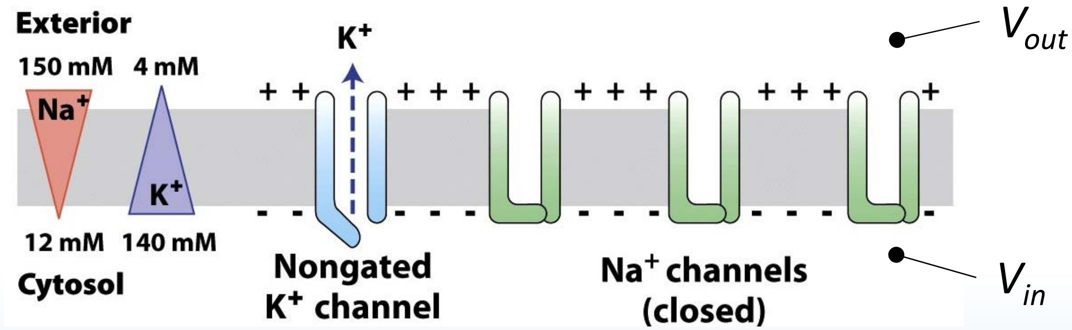
  If the capacitor is fully *charged*, I = 0 and Q is maximum (acts like an open circuit)

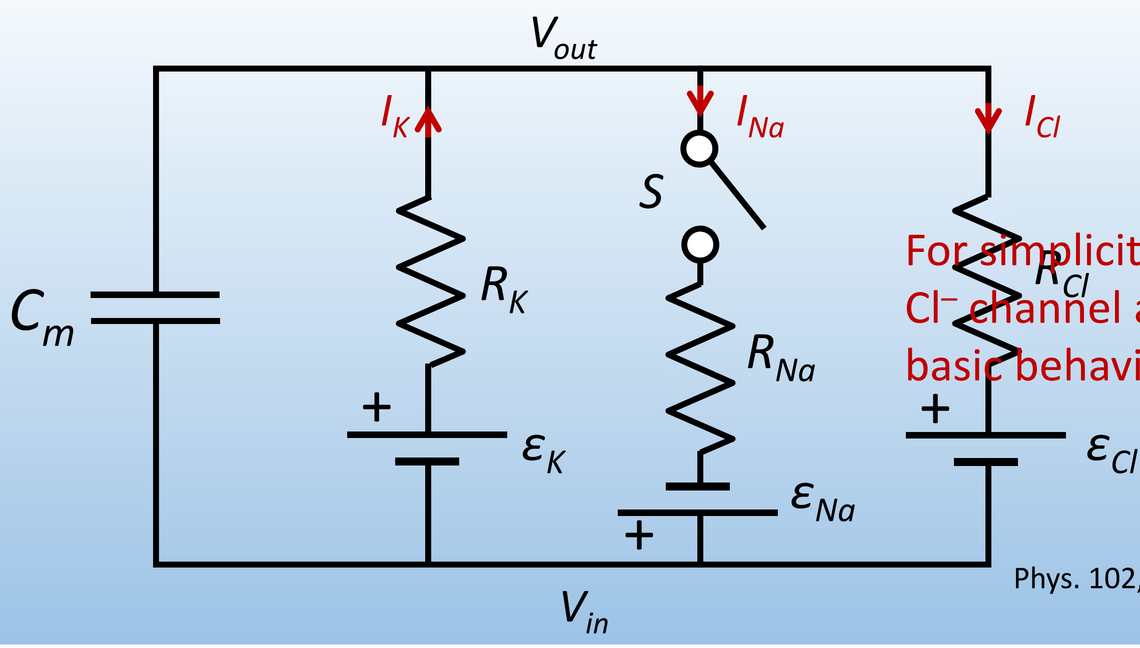
  If the capacitor is fully *discharged*, I = 0 and Q is minimum (acts like an open circuit)

KLR, KJR

Nerve cell equivalent circuit

Neurons have ion channels (K^+,Na^+, and Cl^-) that pump current into and out of cell (it is *polarized*). Cell membrane also has capacitance

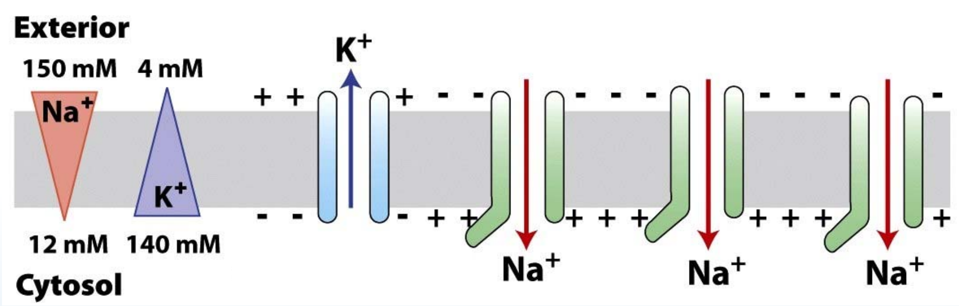


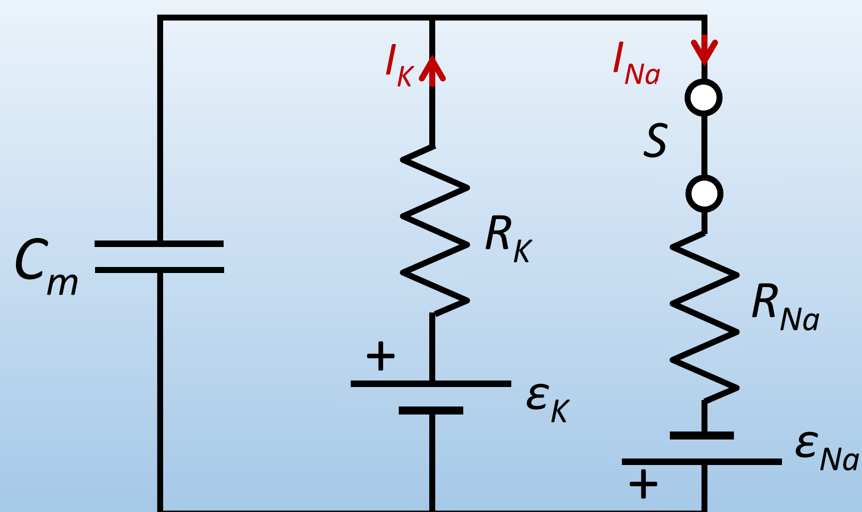


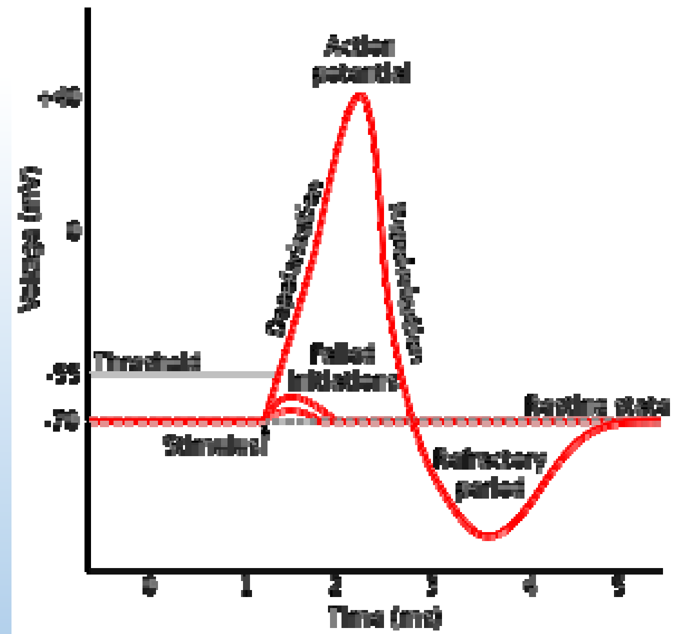
For simplicity we'll ignore the Cl^- channel and still get the basic behavior right

Action potential

At rest, Na^+ channels in cell are closed. When stimulated, the cell's voltage increases (*depolarization*). If a threshold is exceeded, the Na^+ channels open & trigger a nerve impulse (*action potential*)

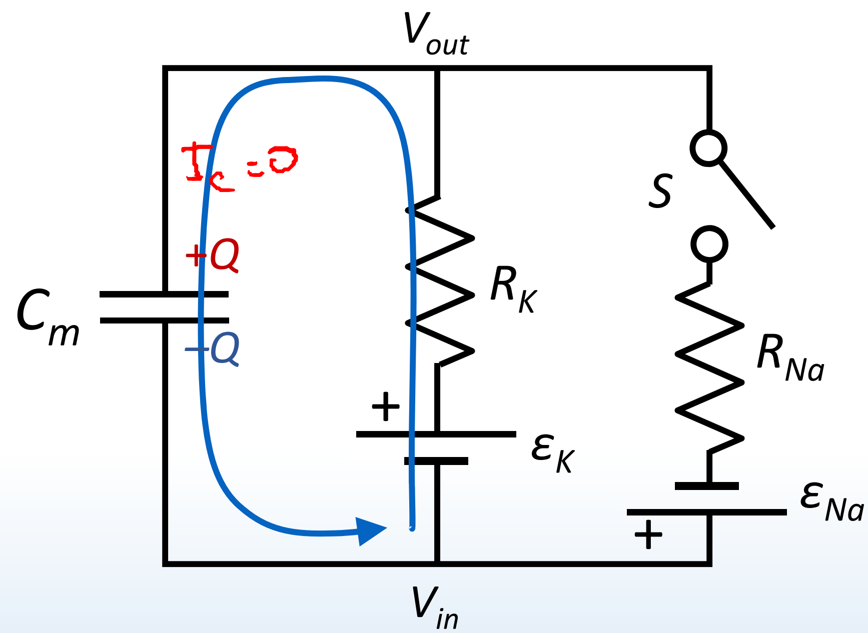






ACT: Resting state of neuron

The neuron has been in resting state for a long time. What is the voltage across the membrane capacitance?



A. $V_C > \varepsilon_K$

B. $V_C = \varepsilon_K$

C. $V_C < \varepsilon_K$

Na⁺ channel is closed (switch *S* is open)
At rest, no current flows through C_m
So, no current through K⁺ channel ($I_K = 0$)

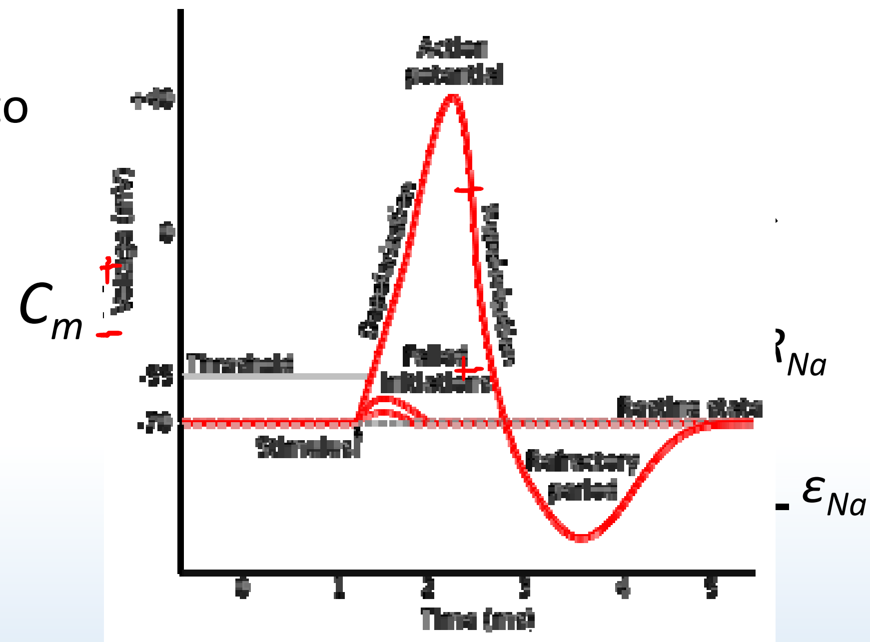
$\varepsilon_K = 70$ mV, $\varepsilon_{Na} = 60$ mV,
$R_K = 2$ MΩ, $R_{Na} = 0.4$ MΩ, $C_m = 300$ pF

$$+\varepsilon_K - V_C = 0 \quad V_C = \varepsilon_K = 70\,\text{mV}$$

Also, $V_{in} - V_{out} = -\varepsilon_K = -70\,\text{mV}$ ← Resting cell voltage

Calculation: action potential I

Some time ago, the cell was stimulated and depolarized to –60 mV, *less* than threshold to open Na^+ channels. What happens next?



<u>*Immediately after*</u>:

No current through Na^+ channel

Current I_K driven by K^+ channel

Charge Q_0 on C_m from V_C = 60 mV

$$+\varepsilon_K - I_K R_K - V_C = 0$$

$$I_K = \frac{\varepsilon_K - V_C}{R_K} = \frac{0.07 - 0.06}{2 \times 10^6} = 5\,\text{nA}$$

ε_K = 70 mV, ε_{Na} = 60 mV,
R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

<u>*After a long time*</u>:

Current I_K decays to 0

Charge on C_m returns to rest value

$$+\varepsilon_K - V_C = 0 \qquad V_C = 70\,\text{mV}$$

Current flows to repolarize the cell (*failed initiation*)

How long does this process take?

RC circuit time dependence

Charging:

$$\varepsilon - I(t)R - \frac{Q(t)}{C} = 0$$

Charge builds up:

$$Q(t) = Q_\infty (1 - e^{-t/RC})$$

Current decays:

$$I(t) = I_0 e^{-t/RC}$$

Discharging:

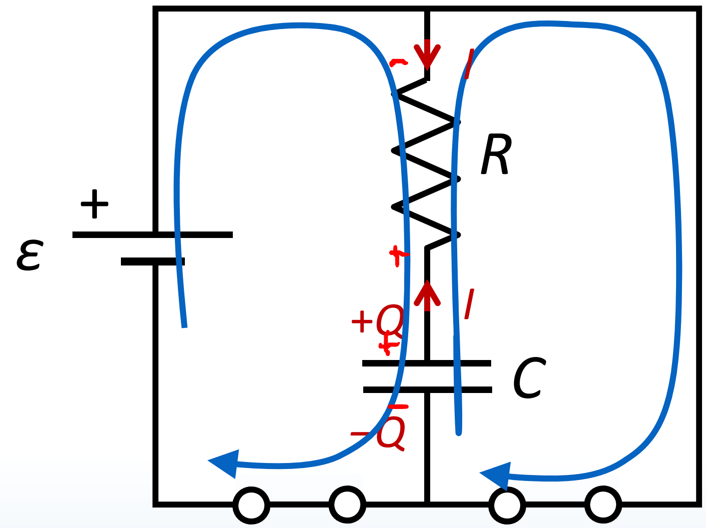
$$\frac{Q(t)}{C} - I(t)R = 0$$

Charge decays:

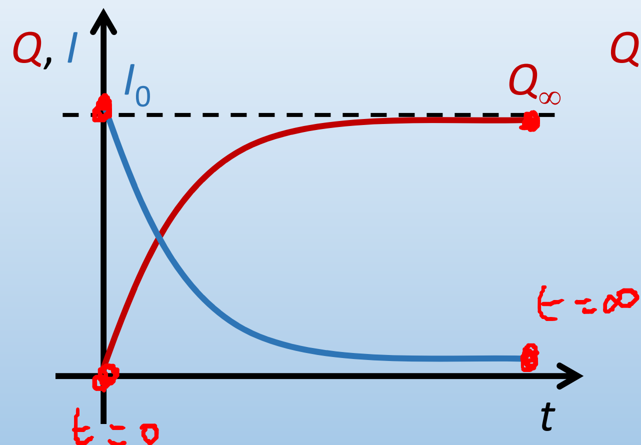
$$Q(t) = Q_0 e^{-t/RC}$$

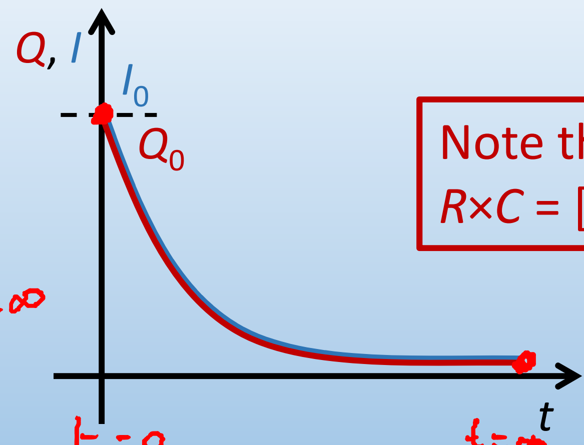
Current decays:

$$I(t) = I_0 e^{-t/RC}$$



DEMO

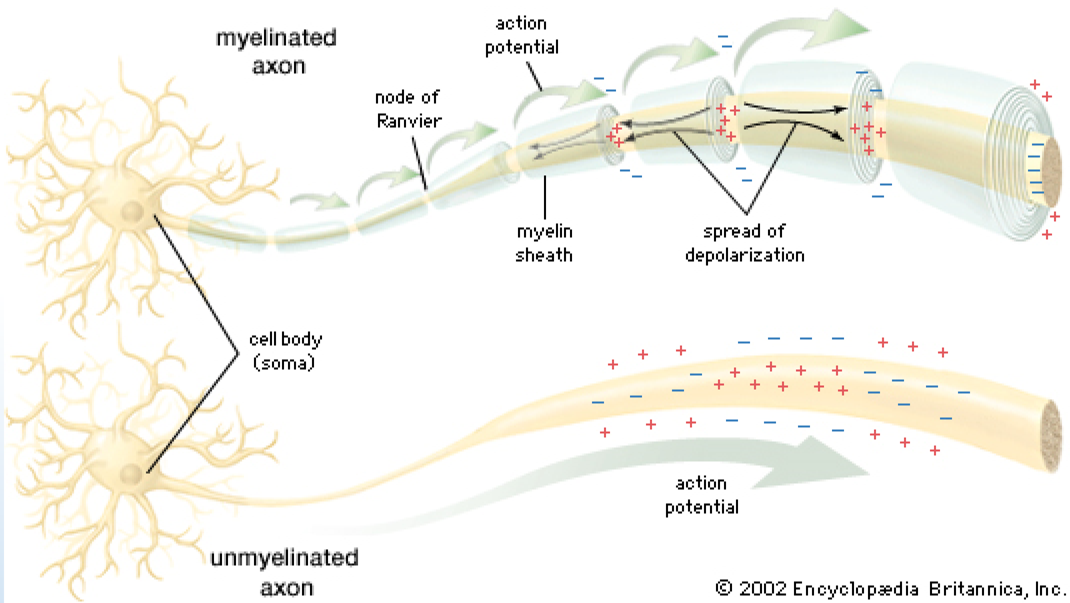




Note that *RC* has units of time!
R×*C* = [V]/[I] × [Q]/[V] = [Q]/[I] = [t]

Myelinated nerve cells

Action potentials propagate down nerve cell at rate determined by the cell's *RC* time constant.

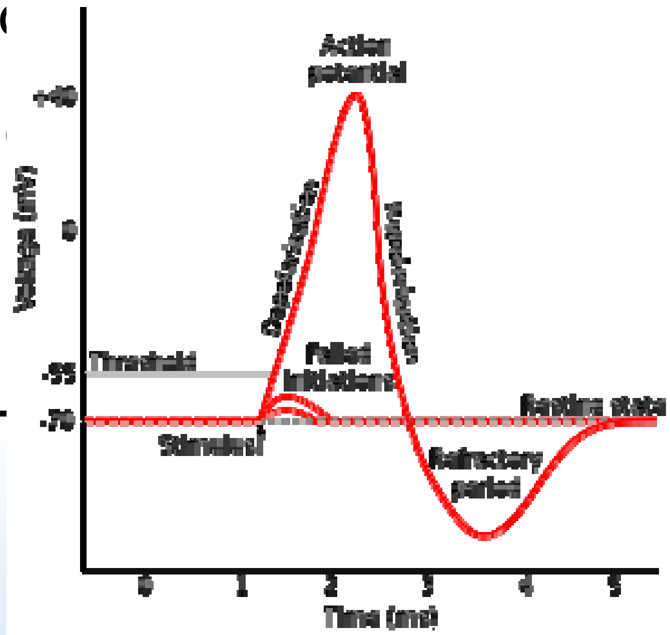


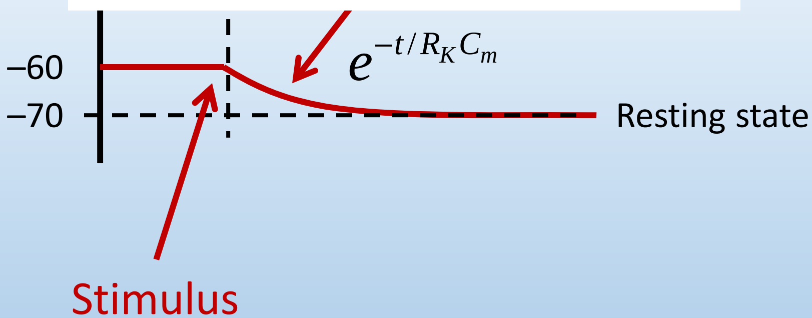
With very few exceptions (ex: C fibres) human neuron fibres are *myelinated.* Myelin reduces *C*, decreasing time constant & increasing propagation speed.

Many neurodegenerative diseases (ex: MS) cause progressive de-myelination.

Calculation: action potential I

How l eturn to 90% of its resting voltage?





Cell voltage $V_{in} - V_{out} = -V_C = -Q/C$:

$$\Delta V(t) = 10e^{-t/R_K C_m}$$

$$\Delta V(t_{90}) = 10(1-0.9) = 10e^{-t_{90}/R_K C_m}$$

Take natural log of both sides:

$$\frac{t_{90}}{R_K C_m} = \ln(0.1)$$

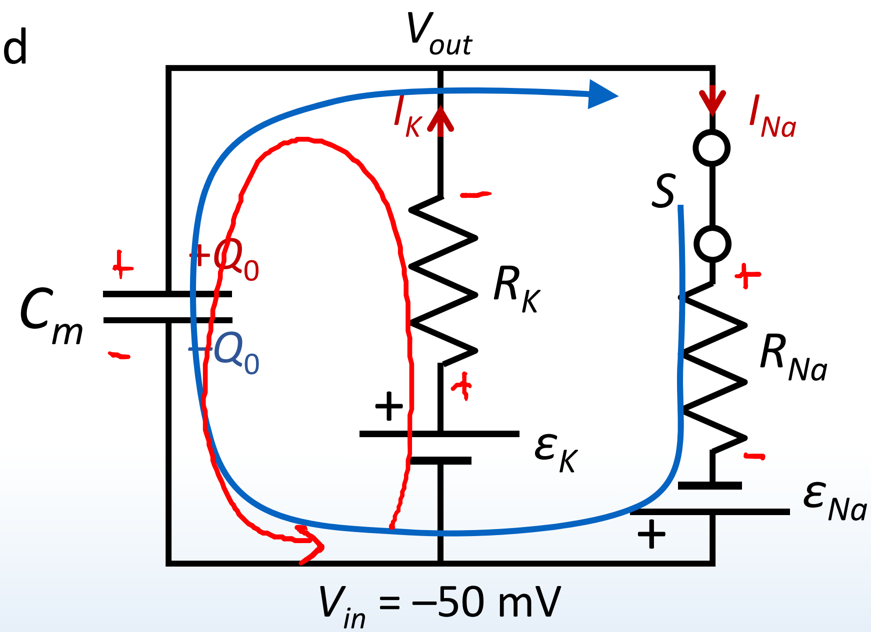
$$t_{90} = \ln(0.1) R_K C_m$$

$$= -(2\times10^{9})(300\times10^{-12})(-2.3)$$

$$\approx 1.4\,\text{ms}$$

Calculation: action potential II

Now, the cell was stimulated and depolarized to –50 mV, *over* the threshold to open Na^+ channels. What happens next?



<u>*Immediately after*</u>:

Current I_{Na} through Na^+ channel

Current I_K driven by K^+ channel

Charge Q_0 on C_m from before

ε_K = 70 mV, ε_{Na} = 60 mV,
R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

$$+\varepsilon_K - I_K R_K - V_C = 0$$

$$I_K = \frac{\varepsilon_K - V_C}{R_K} = \frac{0.07 - 0.05}{2\times 10^6} = 10\,\text{nA}$$

$$+\varepsilon_{Na} - I_{Na} R_{Na} + V_C = 0$$

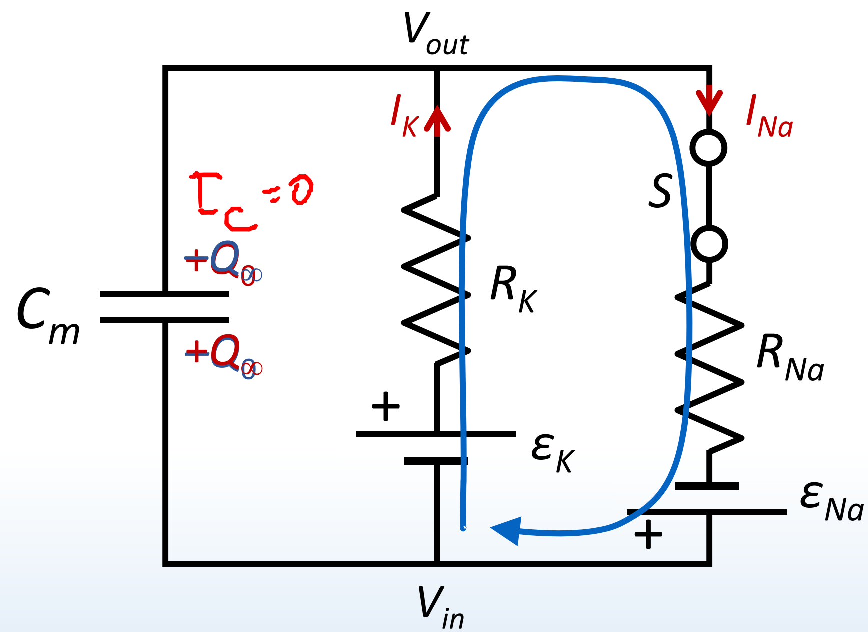
$$I_{Na} = \frac{\varepsilon_{Na} + V_C}{R_{Na}} = \frac{0.06 + 0.05}{0.4\times 10^6} = 275\,\text{nA}$$

Net current flows to *depolarize* the cell further (*action potential*)

$I_{Na} >> I_K$

ACT: action potential II

A long time after stimulating the cell, which statement below holds TRUE?



A. All currents are 0

B. The currents $I_K = I_{Na} \neq 0$

C. Voltage across C_m is 0

After a long time, current through C_m is 0, but current still flows through K^+ and Na^+ channels

ε_K = 70 mV, ε_{Na} = 60 mV,
R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

$$+\varepsilon_K - IR_K - IR_{Na} + \varepsilon_{Na} = 0$$

$$I = \frac{\varepsilon_K + \varepsilon_{Na}}{R_K + R_{Na}} = \frac{0.13}{2.4\times 10^6} \approx 54\,\text{nA}$$

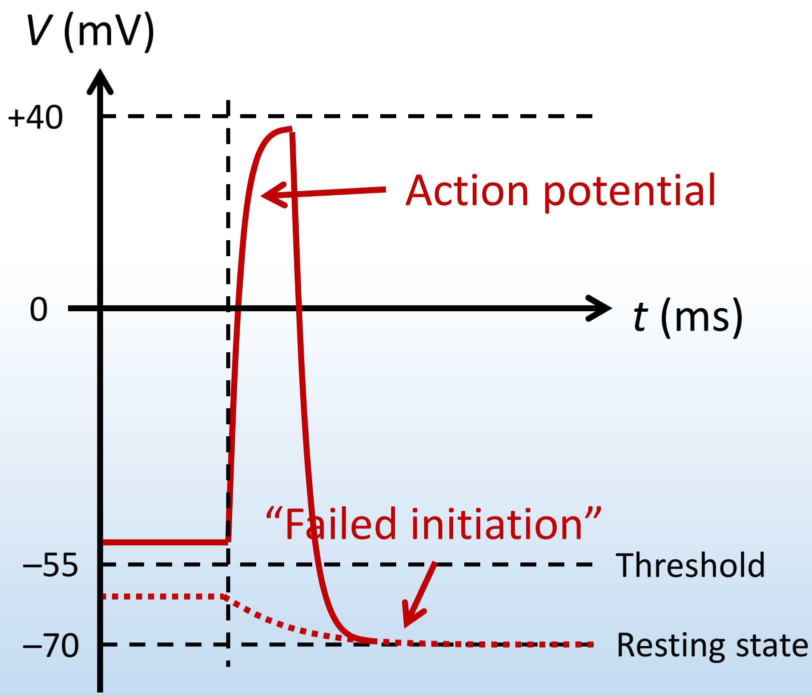
$$V_{in} - V_{out} = \varepsilon_{Na} - IR_{Na}$$

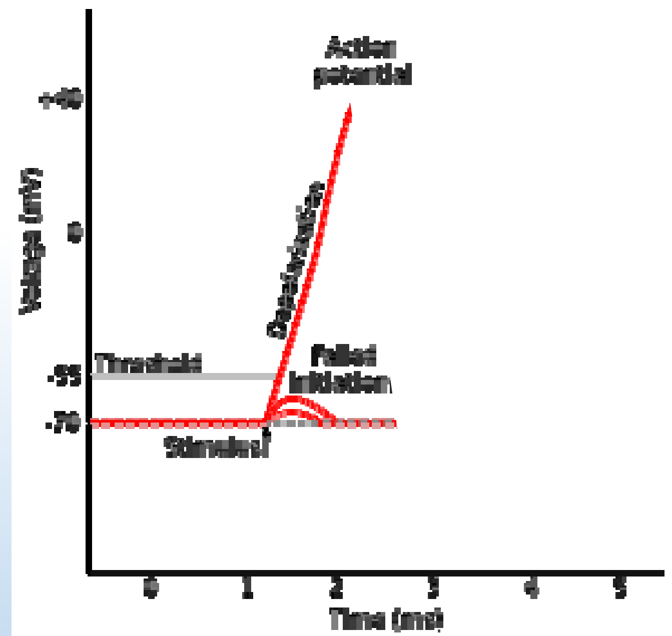
$$= 0.06 - 54\times 10^{-9}\cdot 0.4\times 10^6 \approx +38\,\text{mV}$$

Cell polarity reverses

Action potential summary

If the stimulus exceeds –55 mV, the Na^+ channels open, *depolarize* the cell & trigger an *action potential*.





Once a +40 mV potential is reached, the Na^+ channels close again & the cell *repolarizes* to its resting potential.

Summary of today's lecture

- RC circuits depend on time

  Charge on capacitors cannot change instantly

- Short/long times & charging/discharging

  t = 0: I flows Q onto/off of C, Q increases/decreases (charging/discharging)

  $t = \infty$: I through C decays to 0, Q reaches maximum/minimum (charging/discharging)

  $\tau = RC$: provides time to charge/discharge

Next week magnetism!