Your questions/comments

"What's on the exam?"

IMPORTANT ANNOUNCEMENTS:

Exam I Thursday!, on Lect. 1–5

Review Tuesday 6-8pm in 124 Burrill – will review selections from EX1 FA14 – COME PREPARED having attempted these problems

"It is hard to determine the direction of the flow when there are multiple batteries."

"I'm confused, does the voltage in the first battery change when a second battery is added or does it stay the same?"

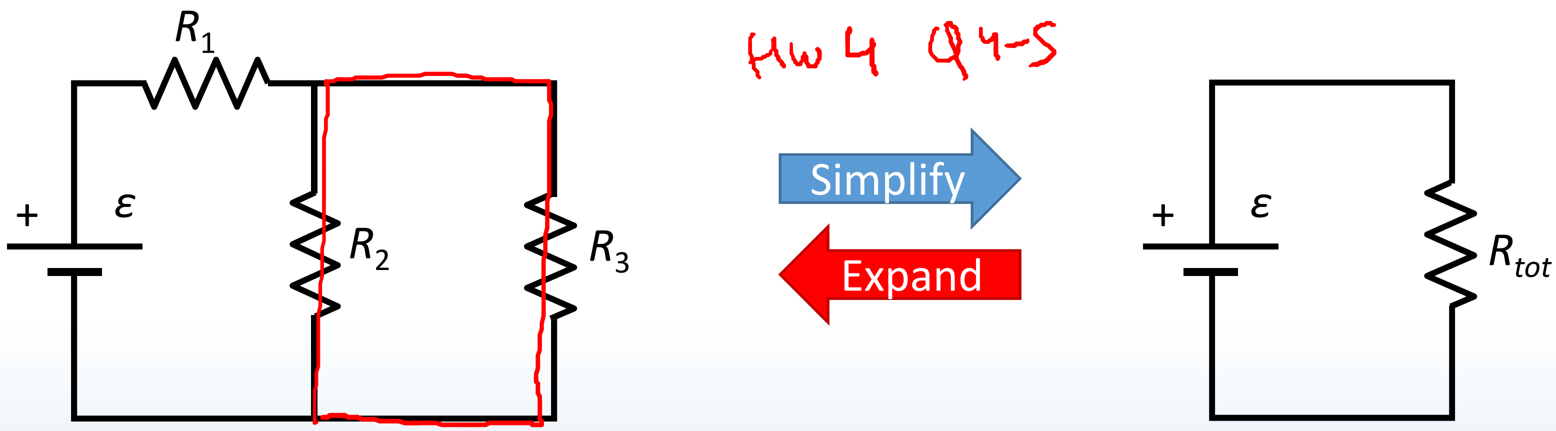
"I do however have trouble understanding which direction current is supposed to flow conventionally because I always went starting from the positive end of the battery(or power source) but in a few of the hw problems its seems though it is being followed starting on the negative end."

Phys 102 – Lecture 8

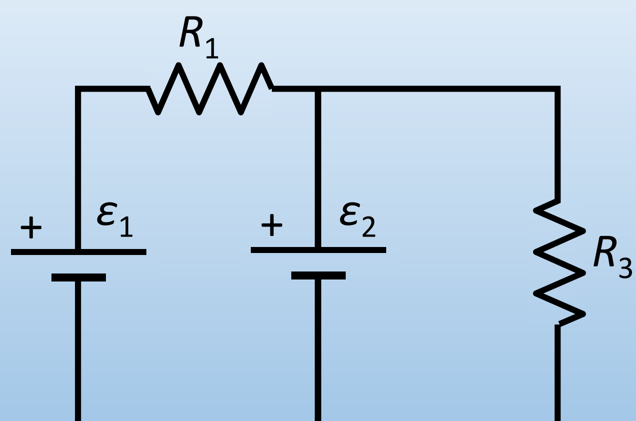
Circuit analysis and Kirchhoff's rules

Recall from last time...

We solved circuits like... by combining series & parallel components



What about a circuit like...

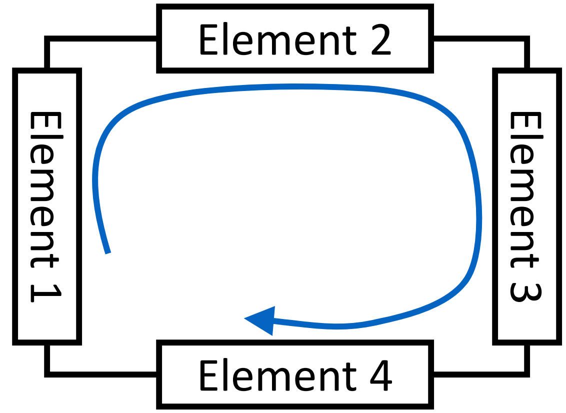


Kirchhoff's loop rule

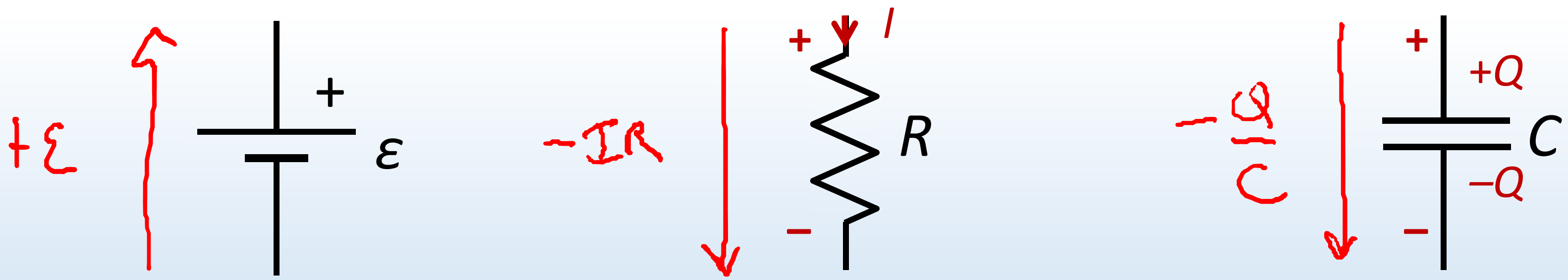
diff. in electric potential

Voltages around a loop sum to zero

$$\sum \Delta V = 0$$



Is voltage positive or negative?



Batteries: + end is always at higher potential

independent of current

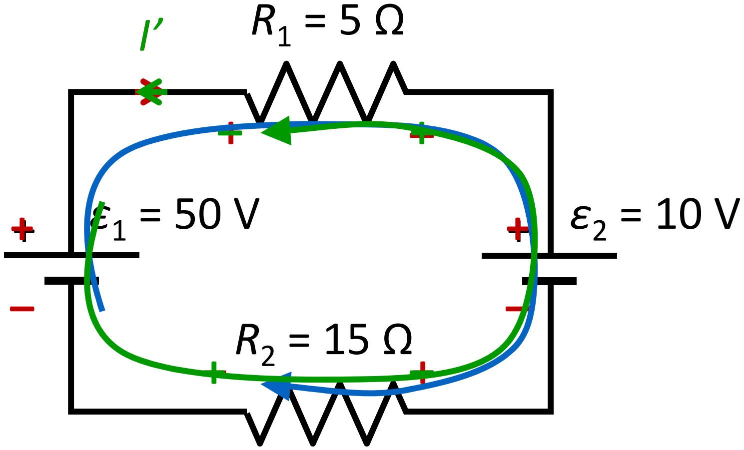
Resistors: higher/lower potential depends on current direction

Capacitors: higher/lower potential depends on which plate has $+Q/-Q$

Label +/– for higher/lower electric potential

Go around loop and write $+V_{element}$ if electric potential increases $-V_{element}$ if it decreases

Calculation: single loop practice



Calculate the current *I* in the circuit

$$+\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{50-10}{5+15} = 2\text{A}$$

What if we go around the loop the "wrong" way?

$$-\varepsilon_1 + IR_1 + \varepsilon_2 + IR_2 = 0$$

Get same equation multiplied by –1.
There is no "wrong" way.

What if we're not given the current direction? Pick one!

What if we pick the "wrong" direction?

$$\varepsilon_1 + I'R_1 - \varepsilon_2 + I'R_2 = 0$$

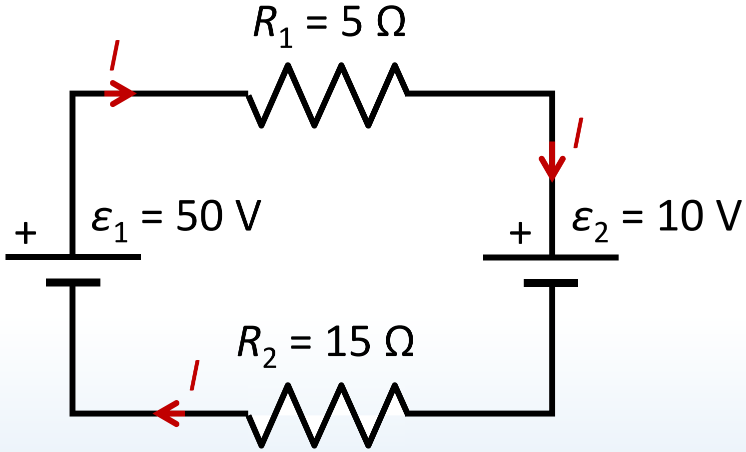
$$I' = \frac{\varepsilon_2 - \varepsilon_1}{R_1 + R_2} = -I$$

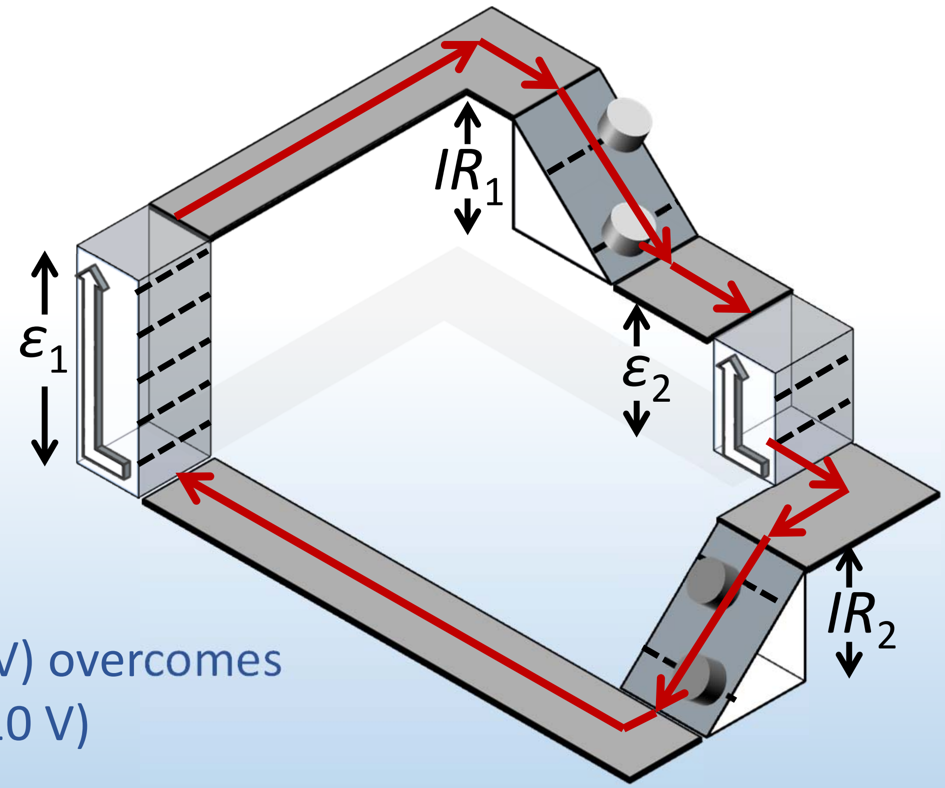
Get –2 A, so current goes in opposite direction. There is no "wrong" choice.

Calculation: single loop practice

Battery = voltage source ≠ current source

How can the current be driven opposite battery 2?





$$+\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2}$$

Battery 1 emf (ε_1 = 50 V) overcomes emf of battery 2 (ε_2 = 10 V)

You wouldn't design a circuit like this, but this occurs in nature

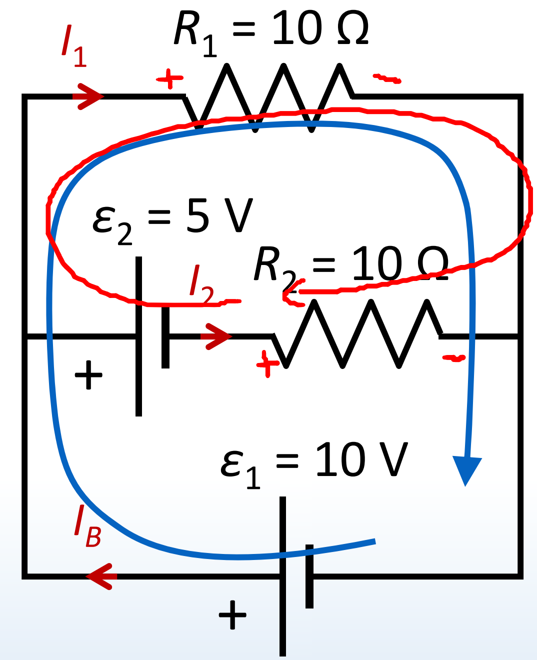
ACT: Checkpoint 1.1

Calculate the current through R_1.

A. $I_1 = 0.5$ A 27%

B. $I_1 = 1.0$ A 49%

C. $I_1 = 1.5$ A 24%



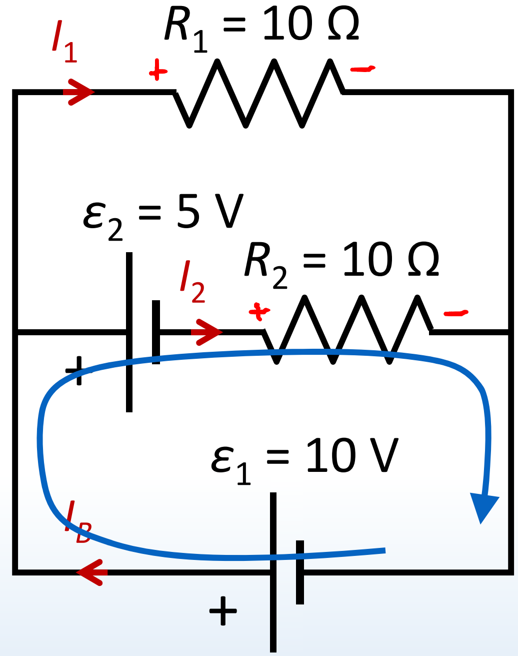
$$+\varepsilon_1 - I_1 R_1 = 0$$

$$I_1 = \frac{\varepsilon_1}{R_1} = \frac{10}{10} = 1\text{A}$$

$$+\mathcal{E}_2 - I_1 R_1 + I_2 R_2 = 0$$

ACT: Checkpoint 1.2

Calculate the current through R_2.



A. $I_2 = 0.5$ A 54%

B. $I_2 = 1.0$ A 28%

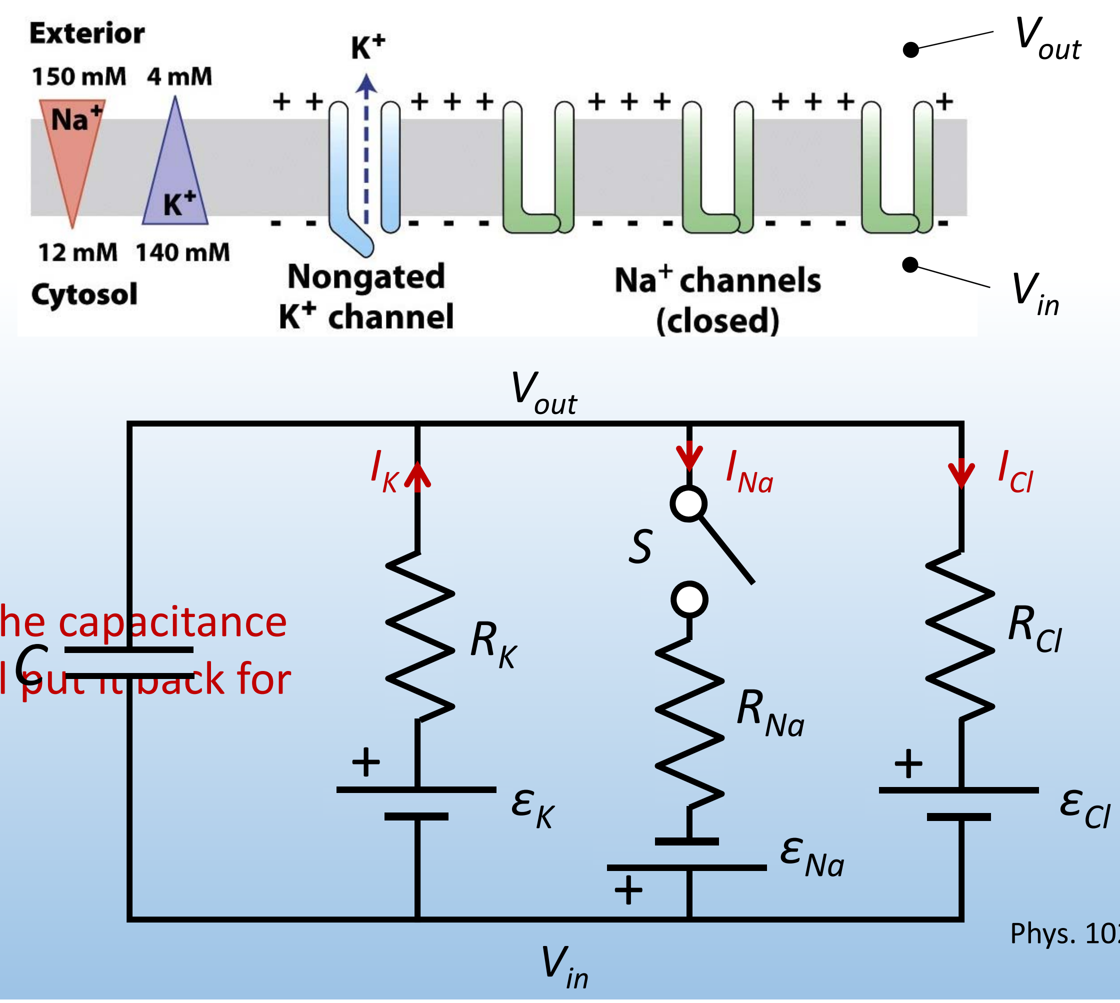
C. $I_2 = 1.5$ A 19%

$$+\varepsilon_1 - \varepsilon_2 - I_2 R_2 = 0$$

$$I_2 = \frac{\varepsilon_1 - \varepsilon_2}{R_2} = \frac{10-5}{10} = 0.5\text{A}$$

Nerve cell equivalent circuit

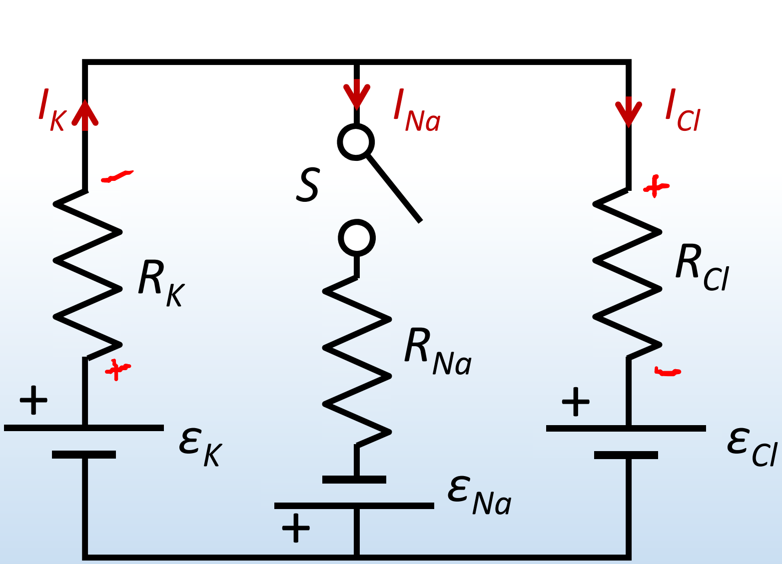
Neurons have different types of ion channels (K^+, Na^+, and Cl^-) that pump current into and out of cell – act like batteries!



Today we ignore the capacitance of the cell (we will put it back for next lecture!)

ACT: loop

Na$^+$ channels have a “gate” (represented by the switch *S*) that allows or blocks ion flow. In its resting state, a Na$^+$ channel is shut (i.e. switch *S* is open). Which equation is correct?



I_{Cl}

A. $+\varepsilon_K - I_K R_K - I_K R_{Cl} - \varepsilon_{Cl} = 0$

B. $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} - \varepsilon_{Na} = 0$

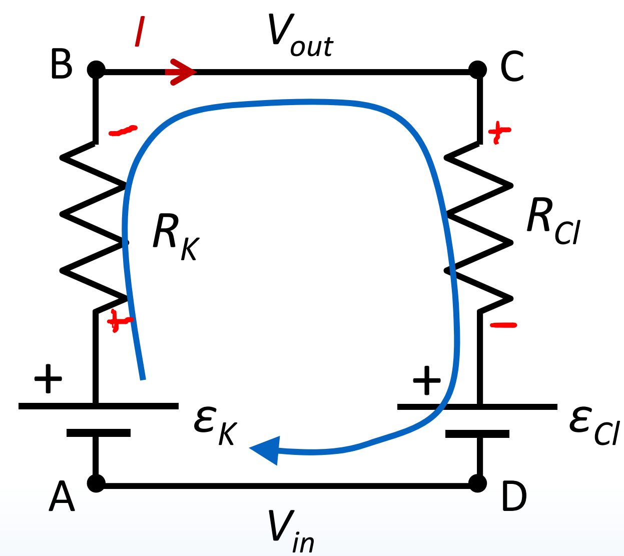
C. ~~$+\varepsilon_K + I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$~~

The switch is open, so all the current flows through outer loop and $I_K = I_{Cl}$

Calculation: electric potential

Find the electric potential difference across the cell $V_{in} - V_{out}$
(Assume $V_{out} = 0$ for reference)

$\varepsilon_K = 80$ mV, $\varepsilon_{Na} = 60$ mV, $\varepsilon_{Cl} = 50$ mV
$R_K = 2$ MΩ, $R_{Na} = 0.2$ MΩ, $R_{Cl} = 5$ MΩ



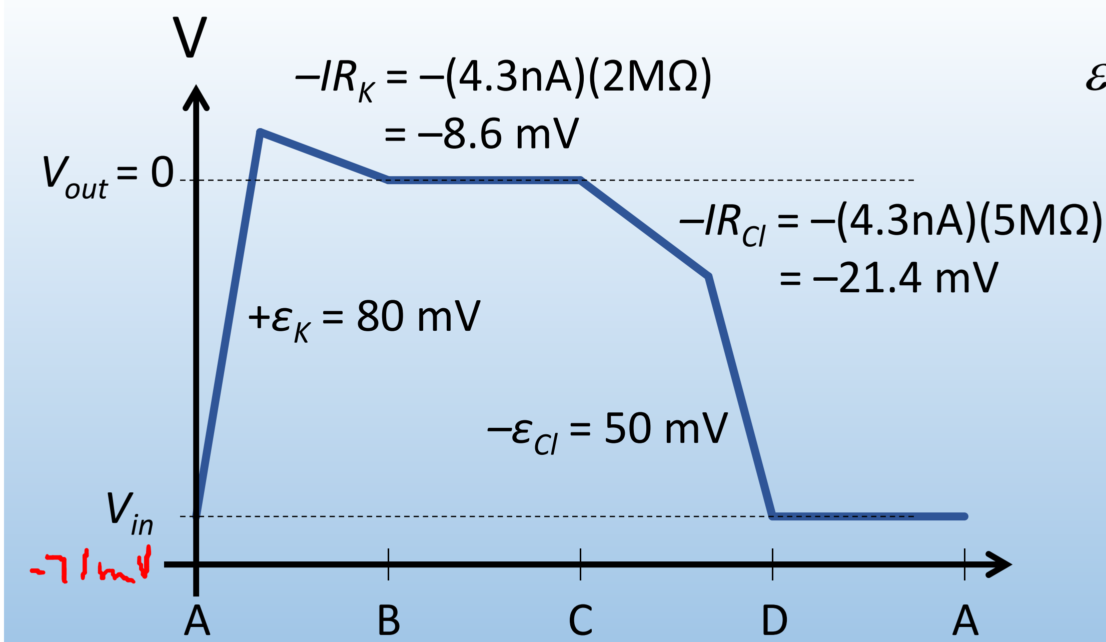
$$\varepsilon_K - IR_K - IR_{Cl} - \varepsilon_{Cl} = 0 \qquad I = 4.3\text{nA}$$

$$V_{in} + \varepsilon_K - IR_K = V_{out}$$

$$V_{in} = -80 + 8.6 + 0 \approx -71\text{mV}$$

or: $$V_{out} - IR_{Cl} - \varepsilon_{Cl} = V_{in}$$

$$0 - 21.4 - 50 \approx -71\text{mV}$$

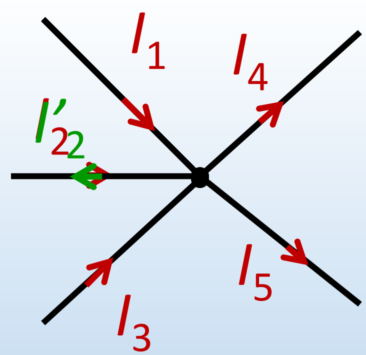


Kirchhoff's junction rule

The sum of currents into a junction equals the sum of currents out of a junction

$$\boxed{\sum I_{in} = \sum I_{out}}$$

Example:



$$I_1 + I_2 + I_3 = I_4 + I_5$$

$$I_1 + I_3 = I_4 + I_5 + I'_2$$

Again, there is no "wrong" choice for the current direction. If current is negative, it goes in a direction opposite that shown.

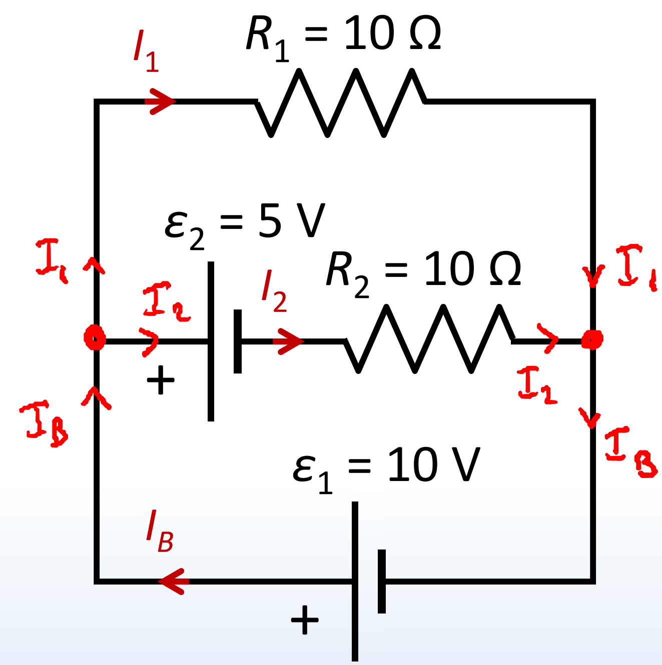
ACT: Checkpoint 1.3

Calculate the current through the battery I_B.

A. $I_B = 0.5$ A 21%

B. $I_B = 1.0$ A 44%

C. $I_B = 1.5$ A 35%

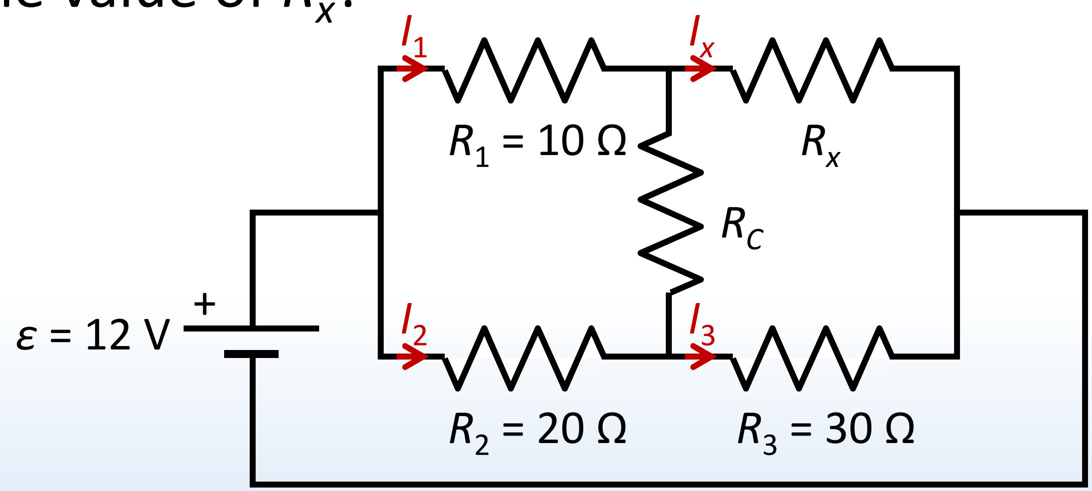


$$I_B = I_1 + I_2 = 1 + 0.5 = 1.5\text{A}$$

Calculation: Kirchhoff's laws

In the circuit, the current through R_C is 0. What is the current through R_3 and the value of R_x?

From EX1 FA13



No current flows through R_C so $I_2 = I_3$ and $I_1 = I_x$

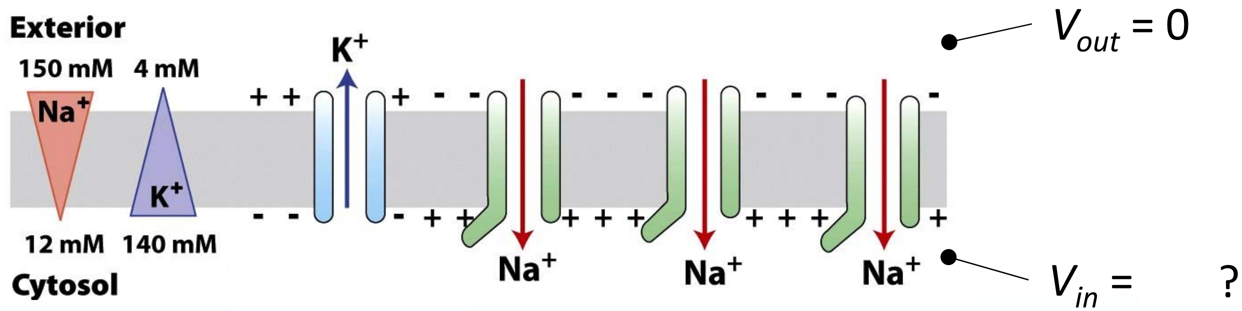
$$\varepsilon - I_2R_2 - I_3R_3 = 0 \qquad I_3 = I_2 = \frac{\varepsilon}{R_2 + R_3} = \frac{12}{20+30} = 0.24\,\text{A} \qquad \varepsilon - I_1R_1 - I_1R_x = 0$$

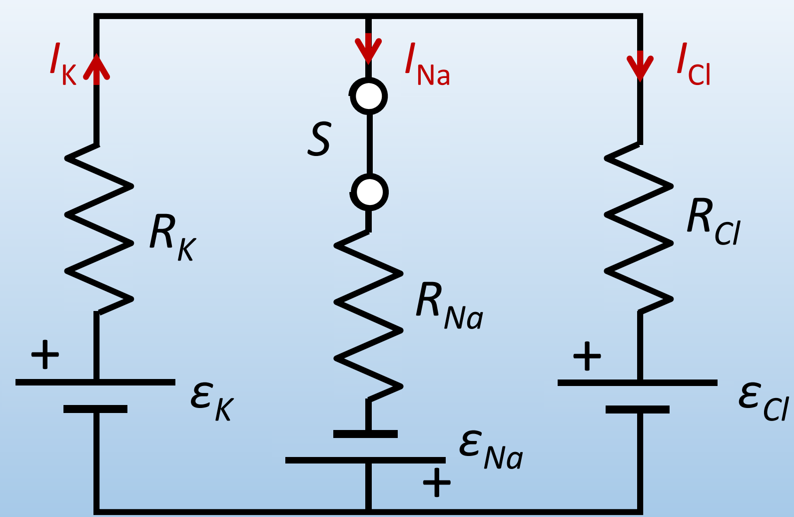
No current flows through R_C so $V_C = 0$

$$I_1R_1 = I_2R_2 \Rightarrow I_1 = I_2\frac{R_2}{R_1} = 0.24\frac{20}{10} = 0.48\,\text{A} \qquad R_x = \frac{\varepsilon}{I_1} - R_1 = \frac{12}{0.48} - 10 = 15\,\Omega$$

Nerve cell equivalent circuit

During nerve impulse, Na^+ channels open (i.e. switch *S* closes) and allow Na^+ to enter the cell





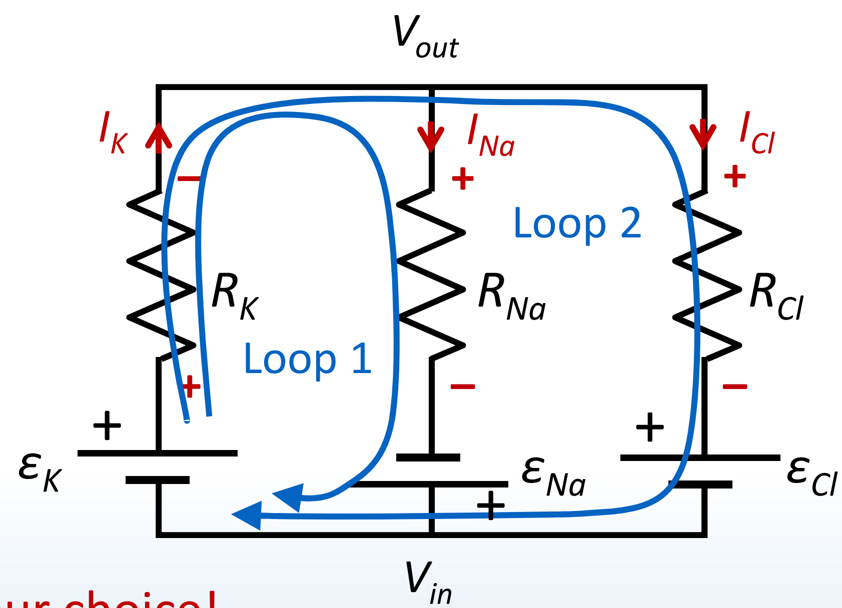
What happens to the currents through the channels and the potential in the cell?

Hws q1

Calculation: two loop circuit

Given the circuit to the right, find I_K, I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

ε_K = 80 mV, ε_{Na} = 60 mV, ε_{Cl} = 50 mV
R_K = 2 MΩ, R_{Na} = 0.2 MΩ, R_{Cl} = 5 MΩ



✓ 1. Label all currents — Your choice!

✓ 2. Label +/– for all elements — Current flows from + to – for resistors

✓ 3. Choose loop and direction — Choose any direction

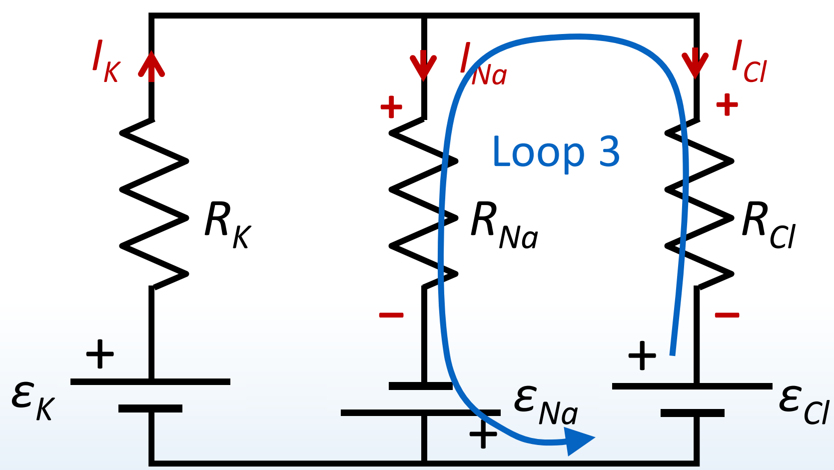
✓ 4. Write down voltage differences — Potential increases or decreases?

Loop 1: $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$

Loop 2: $+\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$

ACT: Kirchhoff loop rule

What is the correct expression for "Loop 3" in the circuit below?



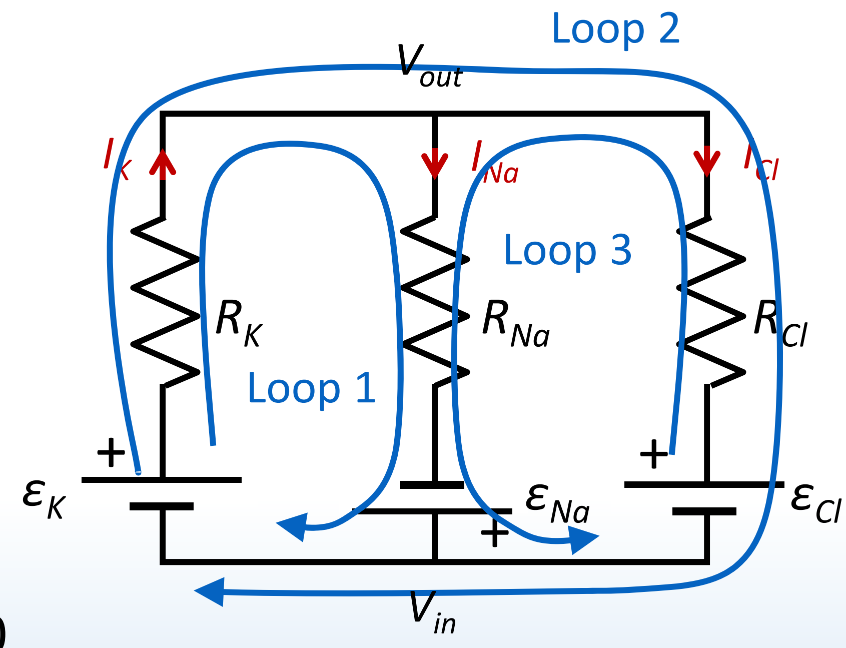
A. $+\varepsilon_{Cl} - I_{Cl}R_{Cl} - I_{Na}R_{Na} + \varepsilon_{Na} = 0$

B. $+\varepsilon_{Cl} - I_{Cl}R_{Cl} + I_{Na}R_{Na} + \varepsilon_{Na} = 0$

C. $+\varepsilon_{Cl} + I_{Cl}R_{Cl} - I_{Na}R_{Na} + \varepsilon_{Na} = 0$

Calculation: two loop circuit

Given the circuit to the right, find I_K, I_{Na} and I_{Cl} and $V_{in} - V_{out}$.



ε_K = 80 mV, ε_{Na} = 60 mV, ε_{Cl} = 50 mV
R_K = 2 MΩ, R_{Na} = 0.2 MΩ, R_{Cl} = 5 MΩ

We have 3 unknowns, need 3 equations

Loop 1: $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$

– Loop 2: $+\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$

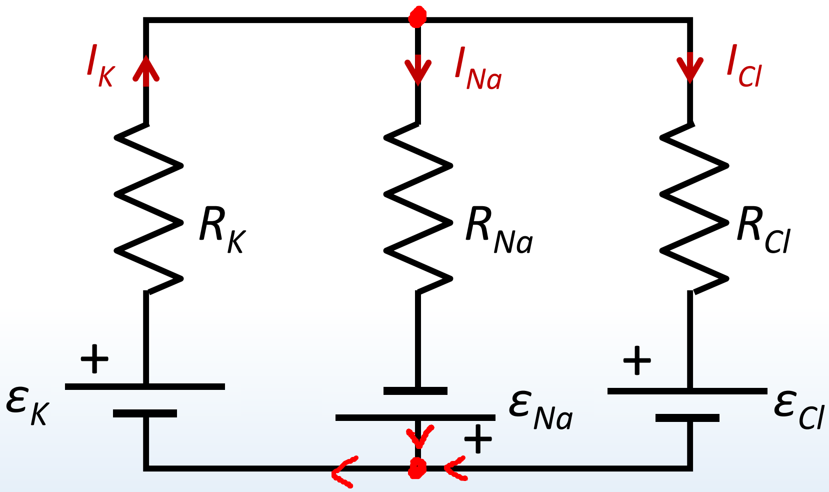
= Loop 3: $+\varepsilon_{Cl} + I_{Cl} R_{Cl} - I_{Na} R_{Na} + \varepsilon_{Na} = 0$

Careful! These 3 equations are not independent. We need 3 independent equations.

5. Write down junction rule

ACT: Kirchhoff junction rule

What is the correct expression for junction in the circuit?



A. $I_K + I_{Na} = I_{Cl}$

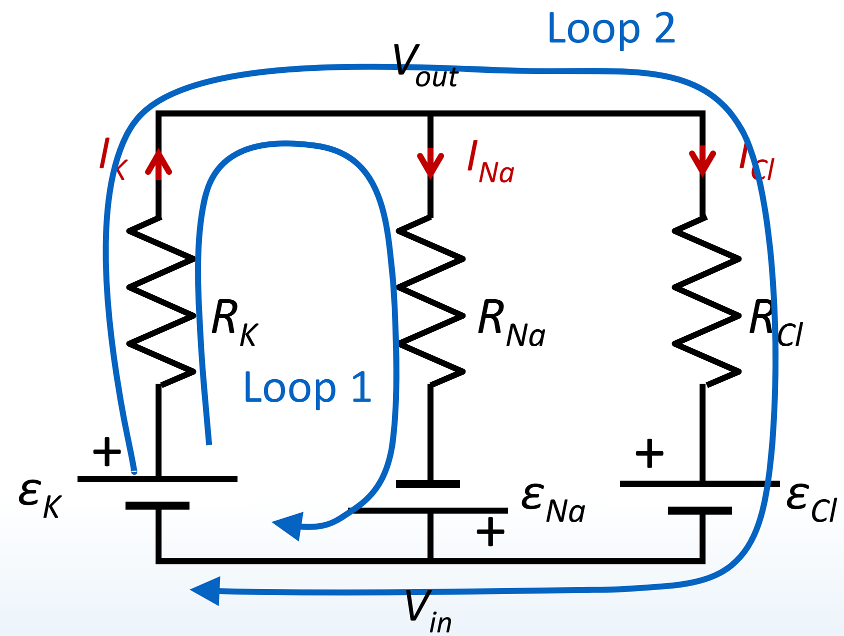
B. $I_{Na} + I_{Cl} = I_K$

C. $I_{Cl} + I_K = I_{Na}$

1. Label all currents
2. Label +/– for all elements
3. Choose loop and direction
4. Write down voltage differences
5. Write down junction rule

Calculation: two loop circuit

Given the circuit to the right, find I_K, I_{Na} and I_{Cl} and $V_{in} - V_{out}$.



ε_K = 80 mV, ε_{Na} = 60 mV, ε_{Cl} = 50 mV
R_K = 2 MΩ, R_{Na} = 0.2 MΩ, R_{Cl} = 5 MΩ

3 equations, 3 unknowns, the rest is algebra!

(1) $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$ $+80 - 2I_K - 0.2I_{Na} + 60 = 0$

(2) $+\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$ $+80 - 2I_K - 5I_{Cl} - 50 = 0$

(3) $I_{Na} + I_{Cl} = I_K$ $+80 - 2I_K - 5(I_K - I_{Na}) - 50 = 0$

Substitute Eq. (3) into Eq. (2) and rearrange

(2') $+30 - 7I_K + 5I_{Na} = 0$

Calculation: two loop circuits

Now 2 equations (1 and 2'), 2 unknowns (I_K and I_{Na})

(1) $+70 - I_K - 0.1 I_{Na} = 0$ $\qquad I_K = 70 - 0.1 I_{Na}$

(2') $+30 - 7 I_K + 5 I_{Na} = 0$ $\qquad +30 - 7(70 - 0.1 I_{Na}) + 5 I_{Na} = 0$

Substitute I_K in Eq. (1) into Eq. (2') and rearrange

$$-460 + 5.7 I_{Na} = 0 \qquad I_{Na} = \frac{460\,\text{mV}}{5.7\,\text{M}\Omega} = 81\,\text{nA}$$

Plug solution into Eq. (2') to get I_K

$$+30 - 7 I_K + 5 \cdot 81 = 0 \qquad I_K = \frac{435\,\text{mV}}{7\,\text{M}\Omega} = 62\,\text{nA}$$

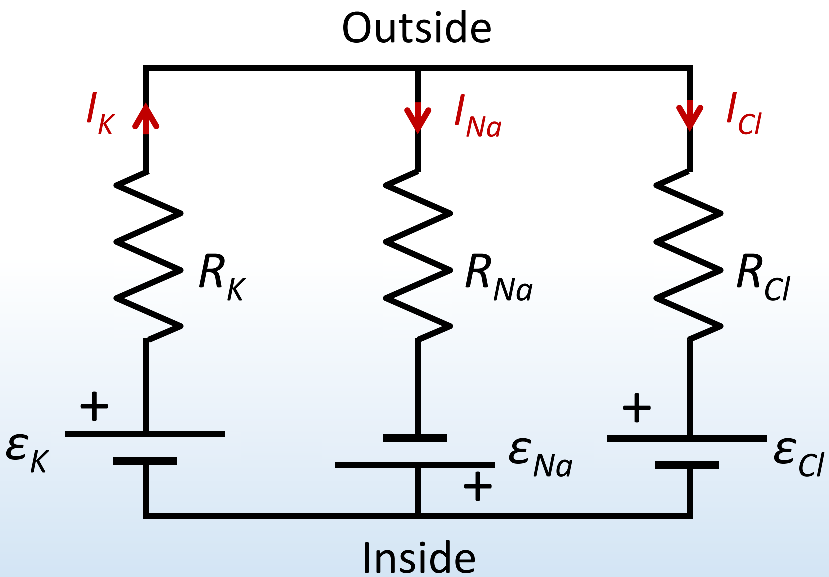
Use junction Eq. (3) to get I_{Cl}

$$I_{Cl} = 62 - 81 = -19\,\text{nA}$$

ALMOST DONE!

ACT: Kirchhoff junction rule

We found that I_K = 62 nA, I_{Na} = 81 nA and I_{Cl} = –19 nA. Which of the following statements is FALSE?



A. I_K is out of the cell

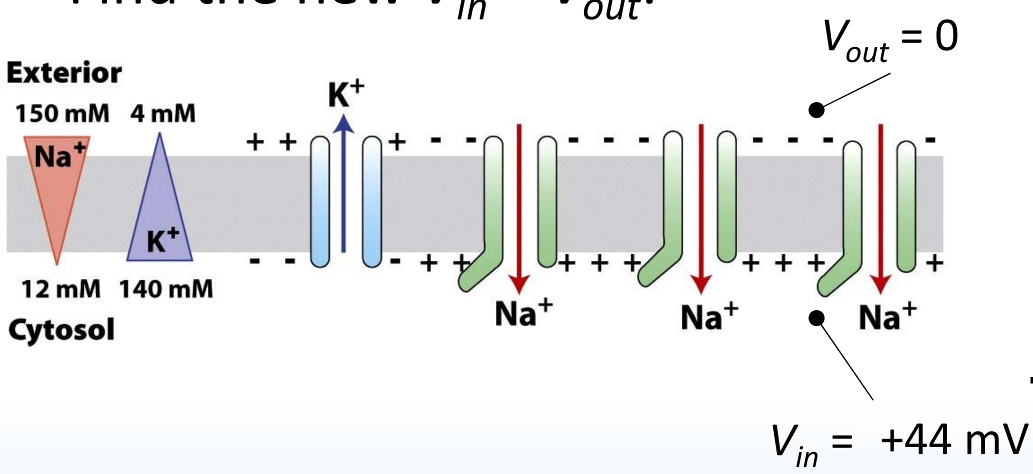
B. I_{Na} is into the cell

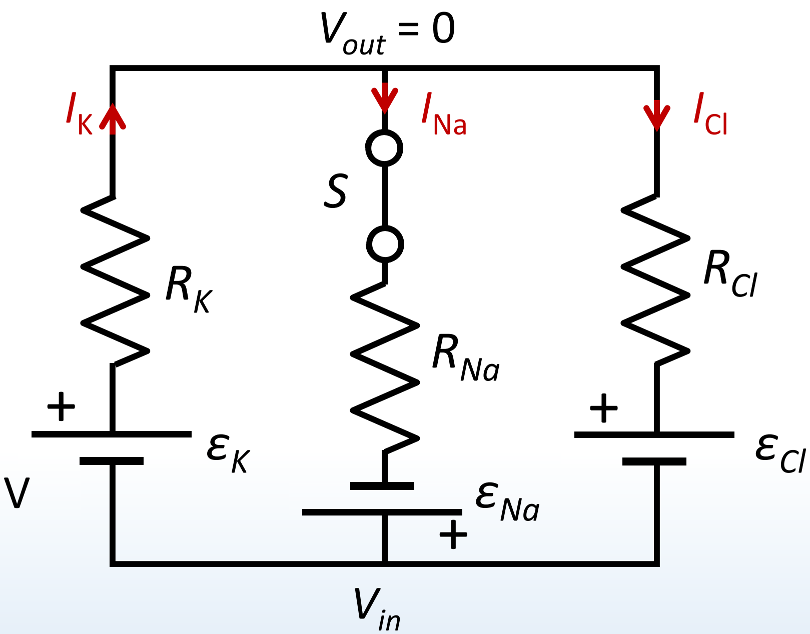
C. I_{Cl} is into the cell

I_{Cl} is negative, so current flows in direction opposite that drawn.
Note that Cl^- is an anion, so Cl^- ions actually flow into the cell.

Calculation: two loop circuit

Find the new $V_{in} - V_{out}$:

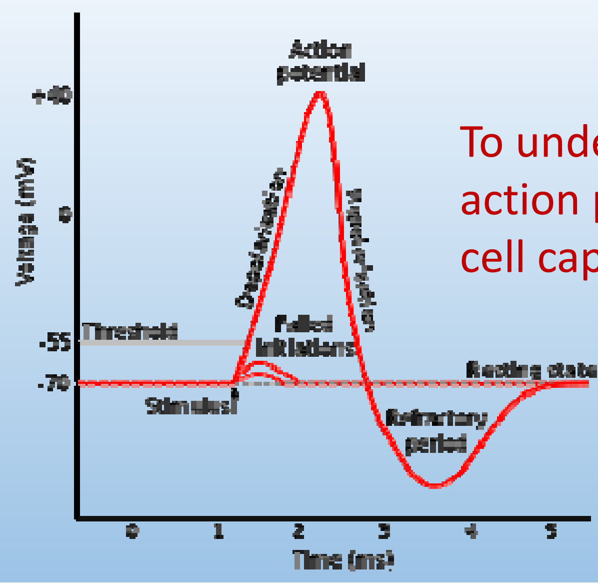




$$V_{in} + \varepsilon_K - I_K R_K = V_{out}$$

$$V_{in} + 80 - 62 \cdot 2 = 0$$

$$V_{in} = +44\,\mathrm{mV}$$



To understand the time scales for the action potential, we'll need to put the cell capacitance back in. Next lecture!

Summary of today's lecture

- Two basic principles:
- Kirchhoff loop rule

  Voltages around circuit loop sum to zero (based on conservation of energy)

$$\sum \Delta V = 0$$

- Kirchhoff junction rule

  Currents into a circuit branch equal currents out (based on conservation of charge)

$$\sum I_{in} = \sum I_{out}$$

Summary of today's lecture

- Basic approach to solving complex circuits:
  1. Label all currents
  2. Label +/– for all elements
  3. Choose loop(s) and direction(s)
  4. Write down voltage differences
  5. Write down junction rule

  The rest is algebra!