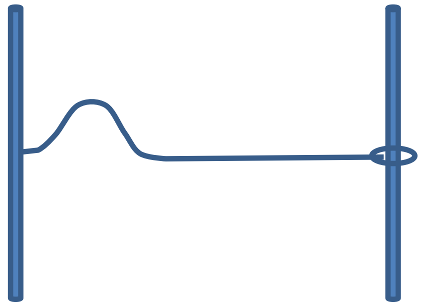
Name: ______________________________ DISC: ____________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
10	10	5	5

1. A pulse travels down a string fixed at one end and free at the other as shown in the diagram (the ring on the end of the string allows the string end to be free).



LENGTH OF STRING	M (STRING)	v_{pulse}
$0.5\ m$	$0.1\ kg$	$0.5\ m/s$

Table 1: Properties of the System

a. The reflected pulse will be

Selection (2 pts):

i. Inverted.

ii. Upright.

b. Given the parameters in the table above, what is the tension T in the string (remember $v = \sqrt{\frac{T}{M/L}}$)?

Tension (2 pts):

Using $v = \sqrt{\frac{T}{M/L}}$ we can solve for T: $T = v^2\left(\frac{M}{L}\right) = (0.5\frac{m}{s})^2\frac{0.1kg}{0.5\ m} = 0.05\ kg\ m/s^2$

c. If you double the string tension what is the speed of the pulse?

New speed (3 pts):

Again using $v = \sqrt{\frac{T}{\frac{M}{L}}}$ we can directly solve for v: $v = \sqrt{\frac{T}{\frac{M}{L}}} = \sqrt{\frac{2\,T}{\frac{M}{L}}} = \sqrt{2}\sqrt{\frac{T}{\frac{M}{L}}} = \sqrt{2}v$

$$v = 1.414 * 0.5\frac{m}{s} = 0.717\ m/s$$

d. At the same time the original pulse is reflected the string is plucked again. This produces a second pulse of the same amplitude. What will happen when the pulses meet?

Meeting of Pulses (3 pts):

Because the string is not fixed at the end, as illustrated by the wring on the right-hand end of the string, the reflected pulse will be upright. When the second upright pulse and the reflected pulse meet, they will constructively interfere with each other. This will result in a pulse which is twice the amplitude of either pulse separately.

2. Consider organ pipes with the parameters given in the following table. The pipes are open at both ends.

PIPE	LENGTH	FIRST HARMONIC	WAVELENGTH	FREQUENCY	ANGULAR FREQUENCY
1	$2\ m$	$150\ Hz$	$\lambda_1 = 2L/n$	$f_1 = v/\lambda_1$	$\omega_1 = 2\pi f_1$
2	$0.25\ m$	$1200\ Hz$	$\lambda_2 = 2L/n$	$f_2 = v/\lambda_2$	$\omega_2 = 2\pi f_2$

a. For the first harmonic, what is the value of n? Explain your reasoning.

First Harmonic (2 pts):

The first harmonic, whether on strings or anywhere else, is the mode *one more* than the fundamental harmonic. So the first harmonic has $n = 2$ for our organ, which behaves like a wave on a string with fixed ends. Remember, the first harmonic has 1 node in the middle!

b. Find the speed of sound in each pipe. Make sure your answers are clear.

Speeds (3 pts):

We can use $f = \frac{v}{\lambda}$ to give us $v = f\lambda$

PIPE	SPEED
1	$v = \frac{(150\ Hz)(2)(2\ m)}{2} = 300\frac{m}{s}$
2	$v = \frac{(1200\ Hz)(2)(0.25\ m)}{2} = 300\frac{m}{s}$

c. If both pipes are played simultaneously, when the waves meet will the sound waves interfere?

Interference (2 pts):

Yes, the waves will interfere with each other. Because of their different frequencies the waves will sometimes interfere constructively and sometimes destructively producing beats.

d. Remembering that a wave can be described as $A(t) = A\cos(\omega t)$,

i. Write down a wave function for pipe 1 and pipe 2. Use A as the amplitude for both waves.

Sum of 2 waves (1.5 pts):

From the table I notice that $\omega = 2\pi f$ so I can find the angular frequencies for each wave:

PIPE	ANGULAR FREQUENCY	WAVE FUNCTION
1	$\omega = 2\pi(150\ Hz) = 300\pi \frac{rad}{s} = 942 \frac{rad}{s}$	$A(t) = A\cos(942\ t)$
2	$\omega = 2\pi(\ 1200\ Hz) = 2400\pi \frac{rad}{s} = 7540 \frac{rad}{s}$	$A(t) = A\cos(7540\ t)$

And then we can use the expression given to find the *wave function* for the waves from the pipes!

ii. Find an expression for the superposition of the two waves. Can you sketch the result? Remember: $\omega_L = \frac{1}{2}(\omega_1 - \omega_2)$ and $\omega_H = \frac{1}{2}(\omega_1 + \omega_2)$

Result (1.5 pts):

To find the superposition we just add them up!

$A_{sup}(t) = [A\cos(942\ t) + \cos(7540\ t)]$ (full credit)

Now if you remember your trig identities you can use them here! We can make use of:

$$\cos(\theta_L + \theta_H) = \cos\theta_L \cos\theta_H + \sin\theta_L \sin\theta_H$$

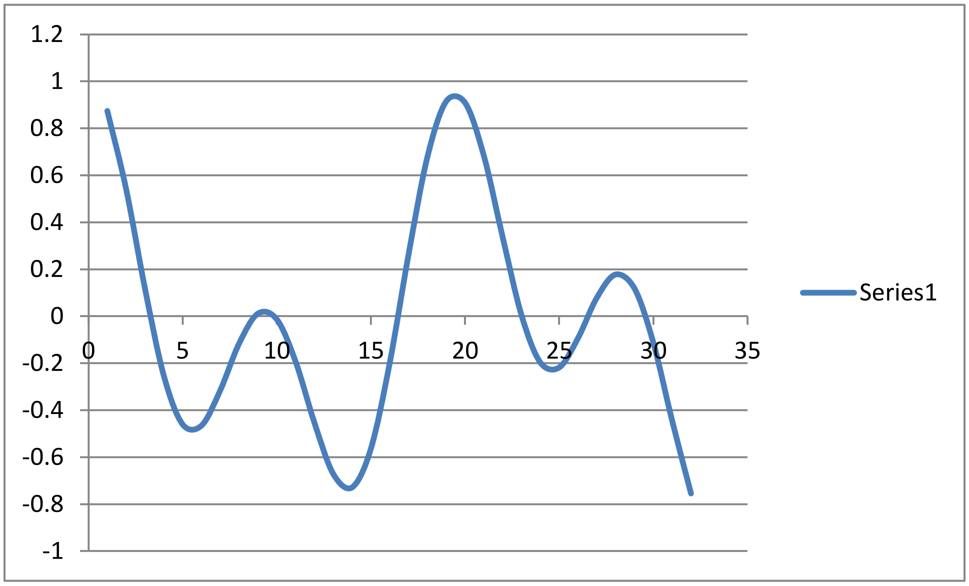
$$\cos(\theta_L - \theta_H) = \cos\theta_L \cos\theta_H - \sin\theta_L \sin\theta_H$$

And $\theta_L = \frac{1}{2}(942\ t + 7540\ t)$ and $\theta_H = \frac{1}{2}(942\ t - 7540\ t)$. Add the expressions for the sum and difference of the cosines of two angles to get:

$\cos(\theta_L + \theta_H) + \cos(\theta_L - \theta_H) = 2\cos\theta_L \cos\theta_H$

If you substitute for θ_L and θ_H and you will find you get an expression which looks just like your superposition above! So: $A_{sup}(t) = [A\cos(942\ t) + \cos(7540\ t)] = 2A\cos\theta_L \cos\theta_H = A\cos 8482t \cos 6598t$

A sketch would look something like



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Waves are neat!