Name: ______________________________ DISC: ____________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
10	10	5	5

1. Consider the two batons shown below. Each is made from a hollow, massless rod of length L and two small equal masses of mass $m/2$. They start rotating from rest when a force F is applied as shown. **(Lecture 13)**

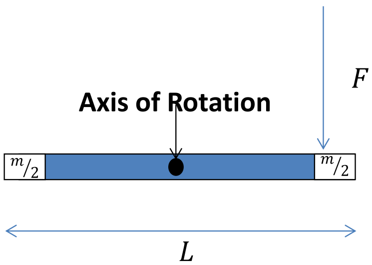


Figure (a)—Top View

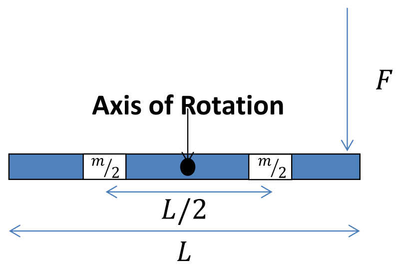


Figure (b)—Top View

Table 1: Useful Information

MASS (m)	**Length (L)**	**F**
$1\ kg$	$0.5\ m$	$2\ N$
I	K	$\omega(t)$
$I = \sum mR^2$	$\frac{1}{2} I\, \omega^2$	$\omega(t) = \omega_o + \alpha t$

$$R_{cm} = \frac{(r_1 M_1 + r_2 M_2)}{M_1 + M_2}$$

Fill in the table below:

Table (10 pts):

QUANTITY	FIGURE (a) VALUE	FIGURE (b) VALUE
I	$I = \frac{m}{2}\left(\frac{L}{2}\right)^2 + \frac{m}{2}\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4} = \frac{(1\ kg)(0.5\ m)^2}{4} = \frac{1}{16} kg\ m^2$	$I = \frac{m}{2}\left(\frac{L}{4}\right)^2 + \frac{m}{2}\left(\frac{L}{4}\right)^2 = \frac{mL^2}{16} = \frac{(1\ kg)(0.25\ m)^2}{4} = \frac{1}{64} kg\ m^2$
r_{cm}	See Diagrams Below	See Diagrams Below
$\tau = RF\ sin\theta = I\alpha$	$\tau = RF = \frac{L}{2} F = \frac{0.5}{2} \times 2\ (N\ m) = 0.5\ N\ m$	$\tau = RF = \frac{L}{2} F = \frac{0.5}{2} \times 2\ (N\ m) = 0.5\ N\ m$
α	$\alpha = \frac{\tau}{I} = \frac{(0.5\ N\ m)}{\left(\frac{1}{16} kg\ m^2\right)} = 8 \frac{rad}{s^2}$	$\alpha = \frac{\tau}{I} = \frac{(0.5\ N\ m)}{\left(\frac{1}{64} kg\ m^2\right)} = 32 \frac{rad}{s^2}$
K at $t = 2\ s$	$K = \frac{1}{2} I\, \omega^2 = \frac{1}{2} \frac{mL^2}{4} (\alpha t)^2 = \frac{1}{2} \times \frac{1}{16} \times 64 \times 4\ J = 8\ J$	$K = \frac{1}{2} I\, \omega^2 = \frac{1}{2} \frac{mL^2}{16} (\alpha t)^2 = \frac{1}{2} \times \frac{1}{64} \times 32 \times 4\ J = 1\ J$

Center-of-mass: Each of the following are equivalent for finding the center-of-mass (or any equivalent):

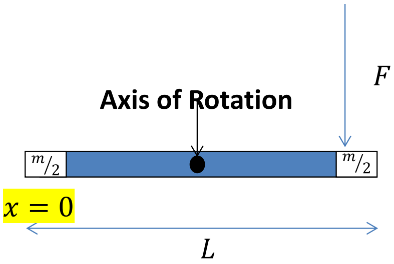


Figure (a)—Top View

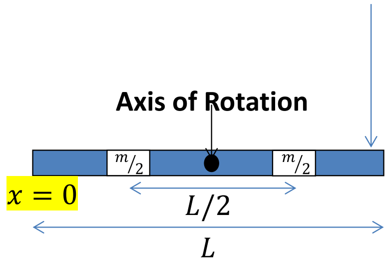


Figure (b)—Top View

Figure (a) With the origin at the left-hand mass we can use the expression $r_{cm} = \frac{\left(\frac{m}{2}\times 0+\frac{m}{2}\times L\right)}{(m)} = \frac{L}{2} = \frac{0.5}{2}m = 0.25\ m.$

Figure (b) With the origin at the left-hand edge we can use the expression $r_{cm} = \frac{\left(\frac{m}{2}\times\frac{L}{4}+\frac{m}{2}\times\frac{3L}{4}\right)}{(m)} = \frac{4L}{8} = \frac{0.5}{2}m = 0.25\ m.$

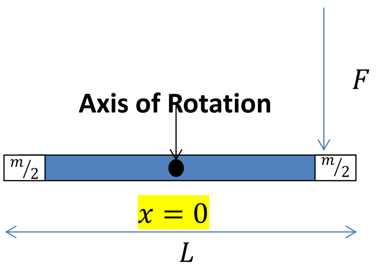


Figure (a)—Top View

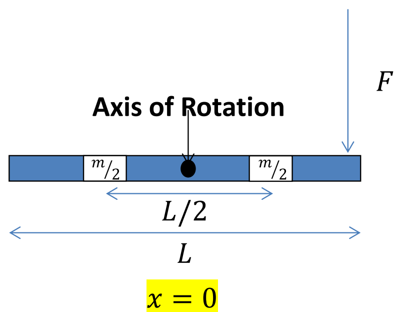


Figure (b)—Top View

a)

Figure (a) With the origin at the center we use the expression $r_{cm} = \frac{\left(\frac{m}{2}\times\frac{L}{2}-\frac{m}{2}\times\frac{L}{2}\right)}{(m)} = 0\ m.$

Figure (b) With the origin at the center we use the expression $r_{cm} = \frac{\left(\frac{m}{2}\times\frac{L}{4}-\frac{m}{2}\times\frac{L}{4}\right)}{(m)} = 0\ m.$

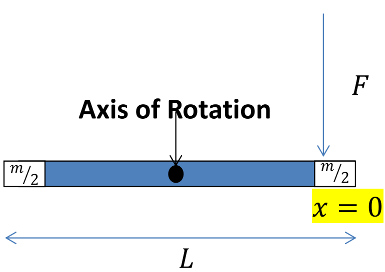


Figure (a)—Top View

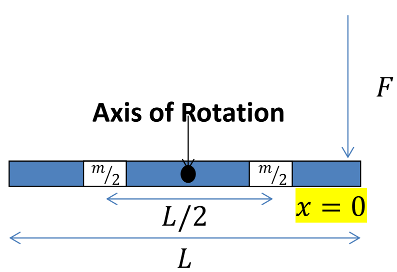


Figure (b)—Top View

b)

Figure (a) With the origin at the right-hand mass we use the expression $r_{cm} = \frac{\left(-\frac{m}{2}\times L+\frac{m}{2}\times 0\right)}{(m)} = -\frac{L}{2}m = -\frac{0.5}{2}m = -0.25\ m.$

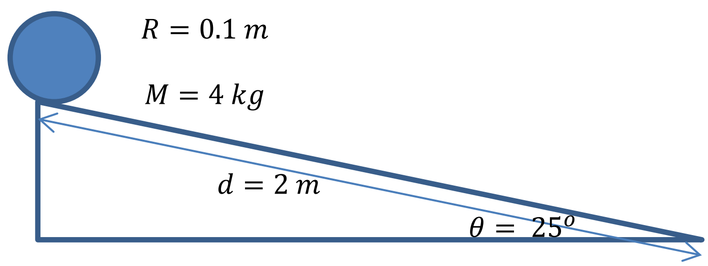
Figure (b) With the origin at the right-hand edge we use the expression $r_{cm} = \frac{\left(-\frac{m}{2}\times\frac{L}{4}-\frac{m}{2}\times\frac{3L}{4}\right)}{(m)} = -\frac{4L}{8} = -\frac{0.5}{2}m = -0.25\ m.$

2. A thin, hollow cylinder rolls without slipping down an inclined plane. The cylinder starts from rest at the top of the incline. The following are true about the cylinder-inclined plane system: **(Lecture 13, pp. 11-12)**

ANGLE OF INCLINE	LENGTH OF INCLINE	CYLINDER MASS	CYLINDER RADIUS
25^o	2 m	4 kg	0.1 m

a. Draw a figure which describes the cylinder and inclined plane *before* the cylinder starts to roll. Remember to label all parts of the diagram.

Figure (2 pts):



b. As the cylinder rolls down the incline which of the following occur (select all correct responses):

Selections (2 pts):

i. Momentum is conserved.
ii. Potential energy is converted into kinetic energy.
iii. The cylinder will have both rotational and translational kinetic energy.

c. What is the potential energy of the cylinder at the top of the incline ($U = mgh$)?

Potential Energy (2 pts):

$$U = mgh = 4 \times 9.81 \times 2 \sin 25^o\ J = 33.16\ J$$

Total Kinetic Energy (2 pts):

d. What is the total kinetic energy ($K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$) at the bottom of the incline (hint: total energy is conserved).

$$K = U = 33.16\ J$$

e. Using $\omega = v_{cm}/R$, find the rotational speed of the cylinder at the bottom of the ramp. For a cylindrical shell $I = MR^2$.

Rotational Speed (2 pts):

$$K = \frac{1}{2}M\,v_{cm}^2 + \frac{1}{2}I\,\omega^2 = \frac{1}{2}M\,R^2\omega^2 + \frac{1}{2}(MR^2\,\omega^2) = MR^2\omega^2$$

$$\omega^2 = \frac{K}{MR^2} = \frac{33.16\,J}{4\,kg \times 0.1^2\,m^2} = 829\frac{rad^2}{s^2} = 28.8\frac{rad}{s}$$