Name: ________________________________ DISC: _____________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
10	10	5	5

1. Consider a block tied to a string which rotates with constant speed on a frictionless surface as shown in the diagram.

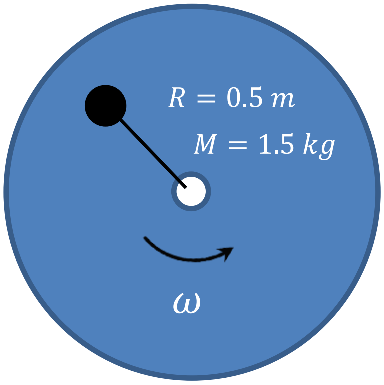


Figure 1: Top View of Rotating Block

R (disk-to-block)	M (block)	I	ω
$0.5\ m$	$1.5 kg$	MR^2	$12\ rad/s$

Table 1: Properties of the System

External Torques (2 pts):

a. There are external torques acting on this system?
   i. No, the table is frictionless.
   ii. Yes, the string has tension pulling on the block.

Explanation of Conservation (3 pts):

b. Like translational momentum, angular momentum is a conserved quantity. In your own words, explain the conditions under which angular momentum is conserved.

Just like linear momentum is conserved in the absence of external forces, angular momentum is conserved in the absence of external torques. **(Lecture 16, p. 4)**

Angular Momentum (2 pts):

c. Remember, angular momentum is $L = I\omega$. What is the angular momentum of the block?

$L = I\omega = MR^2\omega = (1.5\ kg)(0.5\ m)^2\left(12\frac{rad}{s}\right) = 4.5\ {kg\ m^2}/{s}$ **(Lecture 16, p. 18)**

New speed (3 pts):

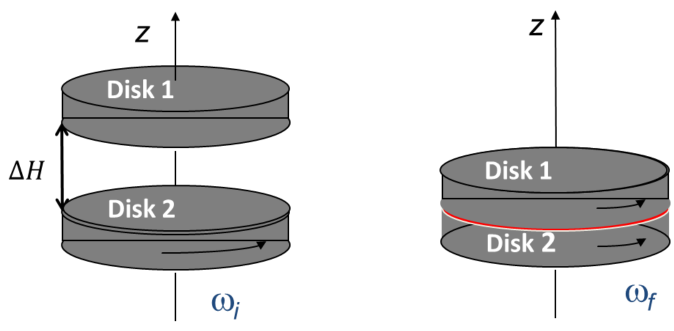
d. You pull on the string, reducing the radius of the rotation by $R/4$. Calculate the new rotational velocity ω_{new}.

Because there are no external torques, angular momentum must be conserved. Thus, $L_i = L_f$. We also remember that the mass of the object is not changing, but the radius does. Thus, I

changes. The new moment of inertia, $I_{new} = \frac{1}{16} I$ because the radius is reduced by 1/4. Since $L = I\omega$ and angular momentum is conserved, if I is smaller by $1/16$, ω_{new} must be 16ω. Thus $\omega_{new} = 192\ rad/s$. **(Lecture 16, p. 18)**

2. Consider the system of two disks as shown in the diagram. The important parameters are given in the table:

DISK	MASS	RADIUS	MOMENT OF	INITIAL ω
1	$40\ kg$	$0.5\ m$	$^1/_2\, MR^2$	$0\ rad/s$
2	$20\ kg$	$0.5\ m$	$^1/_2\, MR^2$	$50\ rad/s$



Initial (Start) Final (Finish)

Explaination (3 pts):

a. Disk 1 is initially stationary and Disk 2 is initially rotating as shown in the *Initial* diagram. Disk 1 suddenly falls resulting in the situation in the *Final* diagram. Explain in your own words what you expect to happen.

Because there are no external torques in this system angular momentum must be conserved. Once the second disk is dropped on top of the first, the moment of inertia of the rotating system will change. Because the mass will increase, the moment of inertia will increase. This increase in moment of inertia will cause a change in the angular frequency of the rotating disk system. Because angular momentum is conserved, since the moment of inertia went increased proportionally to the increase in the mass, the rotational speed must decrease by the same amount. **(Lecture 16, pp. 6-7)**

Angular Momentum (2 pts):

b. Calculate the angular momentum for the *Initial* system:

$$L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(20\ kg)(0.5\ m)^2\left(50\frac{rad}{s}\right) = 125\ kg\ m^2/s$$

Angular Momentum (2 pts):

c. What is the final angular momentum of the system?

Because of conservation of momentum $L_i = L_f = 125\ kg\ m^2/s$

Final speed (3 pts):

d. Find the final rotational speed (ω_f) of the system of disks in the *Final* diagram.

To calculate the final rotational speed we need to figure out how the moment of inertia changes: $\frac{I_f}{I_i} = \frac{\frac{1}{2}(M_1 + M_2)R^2}{\frac{1}{2}M_2 R^2} = \frac{M_1+M_2}{M_2} = \frac{40+20}{20} = 3$ In other words, the final moment of inertia I_f is three times larger than the initial moment of inertia. Therefore, to conserve angular momentum, the angular speed must be reduced by a factor of 3. So, $\omega_f = \frac{\omega_i}{3} = \frac{50\frac{rad}{s}}{3} = 16.7\,\frac{rad}{s}$.