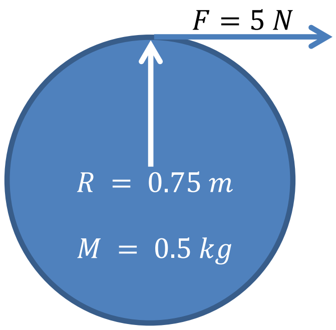
Name: ______________________________ DISC: ____________ Score: _____ / 20

Instructions:
- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
10	10	5	5

1. A solid, horizontal disk is free to rotate about its center. A force of $F = 5\ N$ acts tangentially at the edge of the disk.



MASS	RADIUS	I
0.5 kg	0.75 m	$I = \frac{1}{2}MR^2$

Table 1: Properties of the Disk

a. What is the torque on the disk ($\tau = RF\ sin\theta$)? **(Lecture 14, p. 4)**

Torque (3 pts):

From the diagram we notice that R and F are at 90^o to each other. Therefore, we know that $\sin\theta = 1$. We can use this to find the torque: $\tau = RF = (0.75\ m)(5\ N) = 3.75\ N\ m$. Now we need to check the sign: Remember that counter clockwise is positive (+). Observing the direction of the force (right) the disk will rotate clockwise, thus in the negative direction.

b. The disk starts rotating from rest. Using $\tau = I\ \alpha$, what is the angular acceleration of the disk?

Angular Acceleration (2 pts):

Letting $RF = I\alpha = \frac{1}{2}MR^2\alpha$ we can solve as follows: **(Homework—Disk with Weight)**

$$\alpha = \frac{2RF}{MR^2} = \frac{2(0.75\ m)(5\ N)}{(0.5\ kg)(0.75^2\ m^2)} = 26.7\ rad/s^2$$

c. Recall: $\omega(t) = \omega_0 + \alpha t$. Calculate the angular speed of the disk at $t = 5\ s$. **(Lecture 8, p. 14)**

Angular speed (2 pts):

Apply the kinematic formula: $\omega(5\ s) = 0\frac{rad}{s} + 26.7\frac{rad}{s^2}\ 5s = 133.5\ radians/s$

d. What is the kinetic energy of the disk at $t = 5\ s$ ($K = \frac{1}{2}I\omega^2$)? **(Lecture 13, p. 8)**

Kinetic Energy (3 pts):

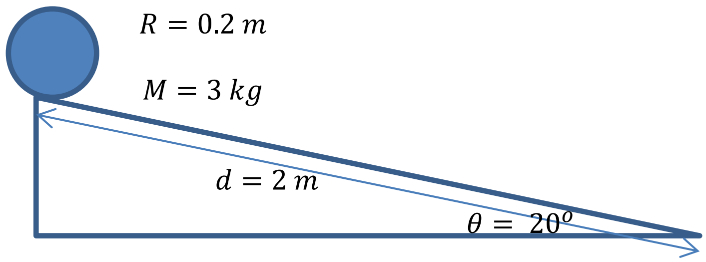
$$K_{rot} = \frac{1}{2}\ I\ \omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}MR^2\omega^2 = \frac{1}{4}(0.5\ kg)(0.75^2\ m^2)\left(133.5^2\ \left(\frac{rad}{s}\right)^2\right) = 1255\ J$$

2. A solid cylinder rolls without slipping down an inclined plane. The cylinder starts from rest at the top of the incline. The following are true about the cylinder-inclined plane system:

ANGLE OF INCLINE	LENGTH OF INCLINE	CYLINDER MASS	CYLINDER RADIUS
20^o	2 m	3 kg	0.2 m

a. Draw a figure which describes the cylinder and inclined plane *before* the cylinder starts to roll. Remember to label all parts of the diagram. **(Lecture 13, pp. 13-14; Homework—Sphere on Incline)**

Figure (2 pts):



b. As the cylinder rolls down the incline which of the following occur (select all correct responses):

Selections (2 pts):

i. Momentum is conserved.
ii. Potential energy is converted into kinetic energy.
iii. The cylinder will have both rotational and translational kinetic energy.

c. What is the potential energy of the cylinder at the top of the incline ($U = mgh$)? **(Lecture 10, p. 6)**

Potential Energy (2 pts):

$$U = (3\ kg)\left(9.81\ {}^{m}/_{s^2}\right)(2\ m)(\sin 20^o) = 20.13\ J$$

d. What is the total kinetic energy ($K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$) at the bottom of the incline (hint: total energy is conserved). **(Lecture 10, p. 7)**

Total Kinetic Energy (2 pts):

Since total energy is conserved, $\Delta U = \Delta K = 20.13\ J$

e. Using $\omega = v_{cm}/R$, find the rotational speed of the cylinder at the bottom of the ramp. For a solid cylinder $I = \frac{1}{2}MR^2$. **(Synthesis)**

Rotational Speed (2 pts):

First use conservation of energy to find the speed of the center of the cylinder:

$$K = \frac{1}{2}M\ v_{cm}^2 + \frac{1}{2}I\ \omega^2 = \frac{1}{2}M\ v_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\ (v_{cm}/R)^2\right)$$

$$K = \frac{1}{2}Mv_{cm}^2\left(1 + \frac{1}{2}\right) = \frac{1}{2}Mv_{cm}^2\left(\frac{3}{2}\right) = \frac{3}{4}Mv_{cm}^2 = \frac{3}{4}(3\ kg)v_{cm}^2 = 20.13\ J$$

$$20.13\ J\left(\frac{4}{9}kg\right) = v_{cm}^2$$

$$v_{cm} = \frac{2}{3}\sqrt{20.13} = 2.99\ m/s$$

Now we find the rotational speed: $\omega = \frac{v_{cm}}{R} = {}^{2.99\frac{m}{s}}\!/_{0.2\ m} = 14.95\ rad/s$