Name: ______________________________ DISC: ____________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers

Q1	Q2	Q3	Q4
10	10	5	5

1. Consider the two batons shown below. Each is made from a hollow, massless rod of length L and two small equal masses of mass $m/2$. They start rotating from rest when a force F is applied as shown. Let the origin pass through the axis of rotation. **(Lecture 13)**

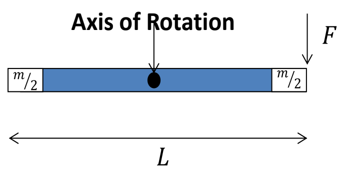


Figure (a)—Top View

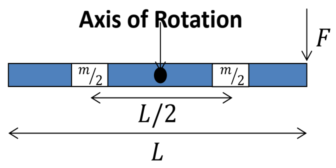


Figure (b)—Top View

Table 1: Useful Information

MASS (m)	**Length (L)**	**F**
$1\ kg$	$0.5\ m$	$2\ N$
I	K	$\omega(t)$
$I = \sum mR^2$	$\frac{1}{2} I\ \omega^2$	$\omega(t) = \omega_o + \alpha t$

$$r_{cm} = \frac{(r_1 M_1 + r_2 M_2)}{M_1 + M_2}$$

Fill in the table below:

Make sure you propagate errors through the table. For example, if the student got the wrong moment of inertia, but used it correctly later, don't take off extra points for using the wrong moment of inertia correctly.

Only take off once for exactly the same error. For example, if they didn't use the moment of inertia expression correctly in figure (a) and used it incorrectly the same way in figure (b), don't take off twice.

Table (10 pts):

QUANTITY	FIGURE (a) VALUE	FIGURE (b) VALUE
I ***Most common error: Failure to take the sum of the two masses (-1 pt—OK for figure (b) if same error)***	$I = \frac{m}{2}\left(\frac{L}{2}\right)^2 + \frac{m}{2}\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4}$ $I = \frac{(1\ kg)(0.5\ m)^2}{4} = \frac{1}{16} kg\ m^2$	$I = \frac{m}{2}\left(\frac{L}{4}\right)^2 + \frac{m}{2}\left(\frac{L}{4}\right)^2 = \frac{mL^2}{16}$ $I = \frac{(1\ kg)(0.25\ m)^2}{4} = \frac{1}{64} kg\ m^2$
r_{cm} ***Most common error: Wrong sign (-0.5 pt—OK for figure (b) if same error)***	**See Diagram Below**	**See Diagram Below**
$\tau = RF\ sin\theta = I\alpha$ ***Most common error: incorrect length (-0.5 pt ea.)***	$\tau = RF = \frac{L}{2}F = \frac{0.5}{2} \times 2\ (N\ m)$ $\tau = 0.5\ N\ m$	$\tau = RF = \frac{L}{2}F = \frac{0.5}{2} \times 2\ (N\ m)$ $\tau = 0.5\ N\ m$

α ***Most common error: arithmetic errors (-1 pt ea.)***	$\boldsymbol{\alpha = \frac{\tau}{I} = \frac{(0.5\ N\ m)}{\left(\frac{1}{16} kg\ m^2\right)} = 8\frac{rad}{s^2}}$	$\boldsymbol{\alpha = \frac{\tau}{I} = \frac{(0.5\ N\ m)}{\left(\frac{1}{64} kg\ m^2\right)} = 32\frac{rad}{s^2}}$
K at $t = 2\ s$ ***Most common error: arithmetic errors (-0.5 pt ea.)***	$\boldsymbol{K = \frac{1}{2} I\ \omega^2 = \frac{1}{2}\frac{mL^2}{4}(\alpha t)^2}$ $\boldsymbol{K = \frac{1}{2} \times \frac{1}{16} \times 64 \times 4\ J = 8\ J}$	$\boldsymbol{K = \frac{1}{2} I\ \omega^2 = \frac{1}{2}\frac{mL^2}{16}(\alpha t)^2}$ $\boldsymbol{K = \frac{1}{2} \times \frac{1}{64} \times 1024 \times 4\ J = 32\ J}$

Center-of-mass: Each of the following are equivalent for finding the center-of-mass (or any equivalent):

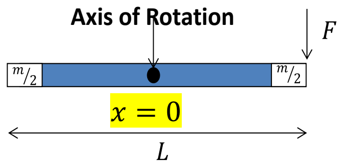


Figure (a)—Top View

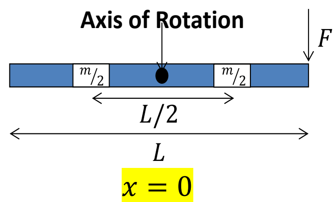


Figure (b)—Top View

Figure (a) With the origin at the center we use the expression $\boldsymbol{r_{cm} = \frac{\left(\frac{m}{2}\times\frac{L}{2} - \frac{m}{2}\times\frac{L}{2}\right)}{(m)} = 0\ m.}$

Figure (b) With the origin at the center we use the expression $\boldsymbol{r_{cm} = \frac{\left(\frac{m}{2}\times\frac{L}{4} - \frac{m}{2}\times\frac{L}{4}\right)}{(m)} = 0\ m.}$

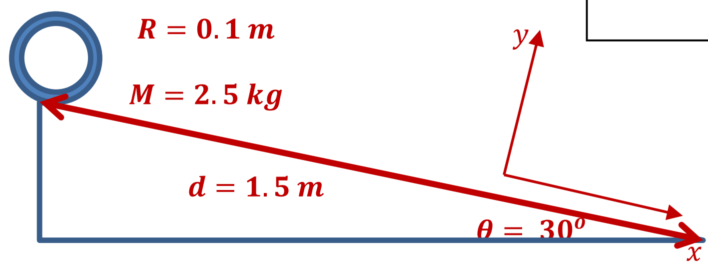
2. A thin, hollow cylinder rolls without slipping down an inclined plane. The cylinder starts from rest at the top of the incline. The following are true about the cylinder-inclined plane system: **(Lecture 13, pp. 11-12)**

ANGLE OF INCLINE	LENGTH OF INCLINE	CYLINDER MASS	CYLINDER RADIUS
$30°$	$1.5\ m$	$2.5\ kg$	$0.1\ m$

a. Draw a figure which describes the cylinder and inclined plane *before* the cylinder starts to roll. Remember to label all parts of the diagram.

This generally went well. It's kind of an all-or-nothing part.

Figure (2 pts):



b. As the cylinder rolls down the incline which of the following occur (select all correct responses):

Selections (2 pts):

i. Momentum is conserved.
ii. Potential energy is converted into kinetic energy.
iii. The cylinder will have both rotational and translational kinetic energy.

The most common errors:
1) including momentum as conserved (-1 pt)
2) including momentum as conserved & leaving off the cylinder with have both rotational and translational kinetic energy (-2 pts)

c. What is the potential energy of the cylinder at the top of the incline ($U = mgh$)?

Potential Energy (2 pts):

$$U = mgh = 2.5\ kg \times 9.81 \frac{m}{s^2} \times (1.5\ m) \sin 30^o\ J = 18.4\ J$$

Most common error: using incline length as the height rather than $L \sin\theta$. (-1 pt)

Total Kinetic Energy (2 pts):

d. What is the total kinetic energy ($K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$) at the bottom of the incline (hint: total energy is conserved).

$$\boldsymbol{K = U = 18.4\,J}$$

This is either right or wrong. Either they took the hint about conservation of energy, or they didn't know what conservation of energy meant.

e. Using $\omega = v_{cm}/R$, find the rotational speed of the cylinder at the bottom of the ramp. For a cylindrical shell $I = MR^2$.

Rotational Speed (2 pts):

$$\boldsymbol{K = \frac{1}{2}M\,v_{cm}^2 + \frac{1}{2}I\,\omega^2 = \frac{1}{2}M\,R^2\omega^2 + \frac{1}{2}(MR^2\,\omega^2) = MR^2\omega^2}$$

$$\boldsymbol{\omega^2 = \frac{K}{MR^2} = \frac{18.4\,J}{2.5\,kg \times 0.1^2\,m^2} = 736\frac{rad^2}{s^2}}$$

$$\boldsymbol{\omega = 27.1\frac{rad}{s}}$$

This generally went well. The most common error was failure to calculate $\boldsymbol{\omega}$ rather than $\boldsymbol{v}$. (-1 pt)