Name: ______________________ DISC: ____________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers.

Q1	Q2	Q3	Q4
10	10	5	5

1. Consider the two batons shown below. Each is made from a hollow, massless rod of length L and two small equal masses of mass $m/2$. Each starts rotating from rest around an axis at the center when a force F is applied on one end perpendicularly.

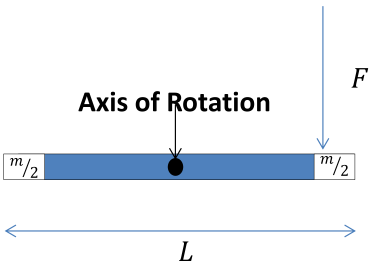


Figure (a)—Top View

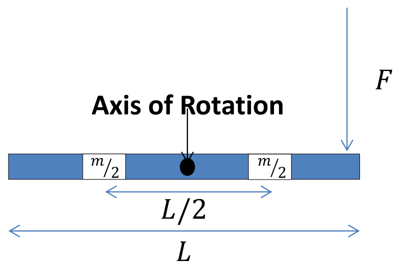


Figure (b)—Top View

Table 1: Useful Information

MASS (m)	Length (L)	F
$2.5\ kg$	$1.2\ m$	$1.5\ N$
I	K	$\omega(t)$
$I = \sum mr^2$	$\frac{1}{2} I\,\omega^2$	$\omega(t) = \omega_o + \alpha t$

$$r_{cm} = \frac{(r_1 m_1 + r_2 m_2)}{m_1 + m_2}$$

Fill in the table below. Make sure to include correct units.

Table (10 pts):

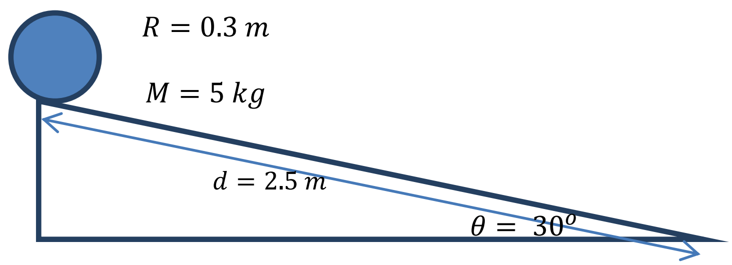
QUANTITY	FIGURE (a) VALUE	FIGURE (b) VALUE
I	$I = \frac{m}{2}\left(\frac{L}{2}\right)^2 + \frac{m}{2}\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4} = \frac{(2.5\ kg)(1.2\ m)^2}{4} = 0.9\ kg\ m^2$	$I = \frac{m}{2}\left(\frac{L}{4}\right)^2 + \frac{m}{2}\left(\frac{L}{4}\right)^2 = \frac{mL^2}{16} = \frac{(2.5\ kg)(1.2\ m)^2}{16} = 0.225\ kg\ m^2$
r_{cm} (measured from the left end)	$r_{cm} = \frac{\left(\frac{m}{2} \times 0 + \frac{m}{2} \times L\right)}{(m)} = \frac{L}{2} = \frac{1.2}{2} m = 0.6\ m.$	$r_{cm} = \frac{\left(\frac{m}{2} \times \frac{L}{4} + \frac{m}{2} \times \frac{3L}{4}\right)}{(m)} = \frac{L}{2} = \frac{1.2}{2} m = 0.6\ m.$
$\tau = RF\,sin\theta = I\alpha$	$\tau = RF = \frac{L}{2} F = \frac{1.2\ m}{2} \times 1.5\ N = 0.9\ N\ m$	$\tau = RF = \frac{L}{2} F = \frac{1.2\ m}{2} \times 1.5\ N = 0.9\ N\ m$
α	$\alpha = \frac{\tau}{I} = \frac{(0.9\ N\ m)}{(0.9\ kg\ m^2)} = 1 \frac{rad}{s^2}$	$\alpha = \frac{\tau}{I} = \frac{(0.9\ N\ m)}{(0.225\ kg\ m^2)} = 4 \frac{rad}{s^2}$
K at $t = 2\ s$	$K = \frac{1}{2} I\,\omega^2 = \frac{1}{2}\frac{mL^2}{4}(\alpha t)^2 = \frac{1}{2} \times 0.9 \times (1 \times 2)^2\ J = 1.8\ J$	$K = \frac{1}{2} I\,\omega^2 = \frac{1}{2}\frac{mL^2}{4}(\alpha t)^2 = \frac{1}{2} \times 0.225 \times (4 \times 2)^2\ J = 7.2\ J$

2. A thin, hollow cylinder rolls down an inclined plane without slipping. The cylinder starts from rest at the top of the incline. The following are true about the cylinder-inclined plane system:

ANGLE OF INCLINE	LENGTH OF INCLINE	CYLINDER MASS	CYLINDER RADIUS
30^o	2.5 m	5 kg	0.3 m

a. Draw a figure which describes the cylinder and inclined plane *before* the cylinder starts to roll. Remember to label all parts of the diagram.

Figure (2 pts):



Selections (2 pts):

b. As the cylinder rolls down the incline which of the following are true (select all correct responses):

i. Momentum is conserved (we consider the cylinder only).
ii. Potential energy is converted into kinetic energy.
iii. The cylinder will have both rotational and translational kinetic energy.

c. Calculate the potential energy of the cylinder at the top of the incline ($U = mgh$).

Potential Energy (2 pts):

$$U = mgh = 5 \times 9.8 \times 2.5 \sin 30^o\ J = 61.25\ J$$

d. Find the total kinetic energy ($K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$) at the bottom of the incline (hint: total energy is conserved).

Total Kinetic Energy (2 pts):

$$K = U = 61.25\ J$$

e. Using $\omega = v_{cm}/R$, find the angular speed of the cylinder at the bottom of the ramp. For a cylindrical shell $I = MR^2$.

Rotational Speed (2 pts):

$$K = \frac{1}{2}M\ v_{cm}^2 + \frac{1}{2}I\ \omega^2 = \frac{1}{2}M\ R^2\omega^2 + \frac{1}{2}(MR^2\ \omega^2) = MR^2\omega^2$$

$$\omega^2 = \frac{K}{MR^2} = \frac{61.25\ J}{5\ kg \times 0.3^2\ m^2} = 136.1\frac{rad^2}{s^2}$$

$$\omega = 11.7\frac{rad}{s}$$