Name: ______________________________ DISC: ____________ Score: _____ / 20

Instructions:

- Do your own work.
- Answer the questions below in the space provided.
- Make sure you show all your work and any equations that you use.
- Please place a box around your answers.
- Remember to give the correct units with all numerical answers.

Q1	Q2	Q3	Q4
10	10	5	5

1. A block of mass M is tied to a string fixed at the center of a frictionless circular disk as shown below. The block rotates with constant angular speed ω and radius R around the center.

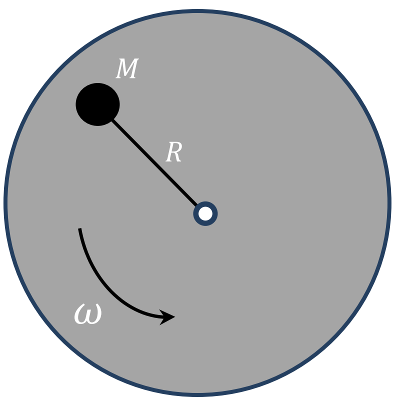


Figure 1: Top View of a Rotating Block

Table 1: Properties of the System

R	**M**	**I**	**ω**
$1.4\, m$	$2.5\, kg$	MR^2	$6\, rad/s$

External Torques (2 pts):

a. Are there external torques acting on this system around the axis of rotation?

i. No, since the disk is frictionless.

ii. Yes, since the string has tension pulling on the block.

b. Like linear momentum, angular momentum is a conserved quantity. In your own words, explain the conditions under which angular momentum is conserved.

Explanation of Conservation (3 pts):

Just like linear momentum is conserved in the absence of external forces, angular momentum is conserved in the absence of external torques.

Angular Momentum (2 pts):

c. Remember, angular momentum is $L = I\omega$. What is the angular momentum of the block?

$$L = I\omega = MR^2\omega = (2.5\, kg)(1.4\, m)^2\left(6\frac{rad}{s}\right) = 29.4\ {}^{kg\, m^2}/_{s}$$

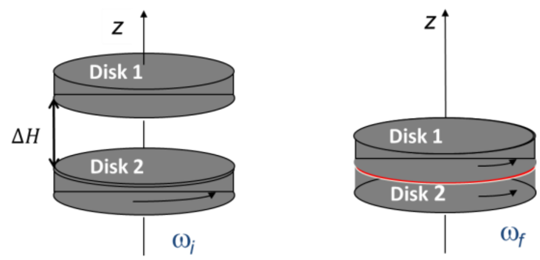
d. You pull on the string, reducing the radius of rotation to 1/4 of the original radius. Calculate the new angular speed ω_{new}.

New speed (3 pts):

Because there are no external torques, angular momentum must be conserved. Thus, $L_i = L_f$. We also remember that the mass of the object is not changing, but the radius does. Thus, I changes. The new moment of inertia, $I_{new} = \frac{1}{16}I$ because the radius is reduced to R/4. Since $L = I\omega$ and angular momentum is conserved, if I is smaller by 1/16, ω_{new} must be 16ω. Thus $\omega_{new} = 96\, rad/s$.

2. Consider a system consisting of two circular disks as shown below. Their masses, radii, and initial angular speeds are given as follows:

DISK	MASS	RADIUS	INITIAL ω
1	$30\ kg$	$0.6\ m$	$0\ rad/s$
2	$20\ kg$	$0.6\ m$	$40\ rad/s$



Initial (Start) Final (Finish)

Explanation (3 pts):

a. Initially, Disk 1 is stationary and Disk 2 is rotating with angular speed ω_i as shown in the figure above (left panel). Disk 1 suddenly falls onto Disk 2; then, they stick together and rotate with angular speed ω_f as depicted in the right panel. Explain in your own words what you expect to be conserved and why.

Since there are no external torques in this system, the total angular momentum must be conserved. Once the second disk is dropped on top of the first, the moment of inertia of the rotating system will increase because the mass will increase. This increase in moment of inertia will cause a change in the angular speed of the rotating disk system. Because angular momentum is conserved and the moment of inertia increases proportionally to the increase in the mass, the rotational speed must decrease by the same amount.

Angular Momentum (2 pts):

b. Calculate the angular momentum for the *Initial* system:

$$L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(20\ kg)(0.6\ m)^2\left(40\frac{rad}{s}\right) = 144\ kg\ m^2/s$$

Angular Momentum (2 pts):

c. What is the *final* angular momentum of the system?

Because of conservation of momentum $L_i = L_f = 144\ kg\ m^2/s$

Final speed (3 pts):

d. Find the *final* angular speed (ω_f) of the system.

To calculate the final angular speed, we need to figure out how the moment of inertia changes: $\frac{I_f}{I_i} = \frac{\frac{1}{2}(M_1+M_2)R^2}{\frac{1}{2}M_2R^2} = \frac{M_1+M_2}{M_2} = \frac{30+20}{20} = 2.5$. In other words, the final moment of inertia I_f is three times larger than the initial moment of inertia. Therefore, to conserve angular momentum, the angular speed must be reduced by a factor of 2.5. So, $\omega_f = \frac{\omega_i}{2.5} = \frac{40\frac{rad}{s}}{3} = 16\frac{rad}{s}$.