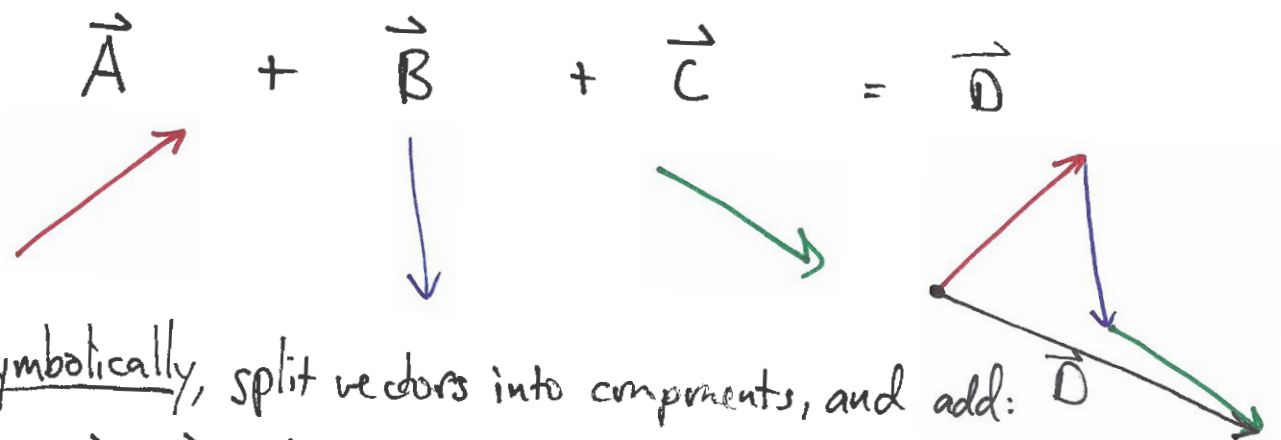


# MATH

## Vector Addition

Graphically, place vectors head to tail. The resultant is drawn from the tail of the first to the head of the last:



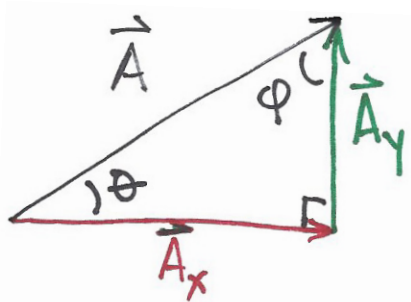
Symbolically, split vectors into components, and add:  $\vec{D}$

$$\vec{A} + \vec{B} + \vec{C} = \vec{D}$$

$$\Rightarrow D_x = A_x + B_x + C_x$$

$$D_y = A_y + B_y + C_y$$

Where the x and y components are found using trigonometry, e.g.



$$A_x = A \cos \theta = A \sin \phi$$

$$A_y = A \sin \theta = A \cos \phi$$

and  $A = \sqrt{A_x^2 + A_y^2}$

Remember

SOH  
sin = opposite / hypotenuse

CAH  
cos = adjacent / hypotenuse

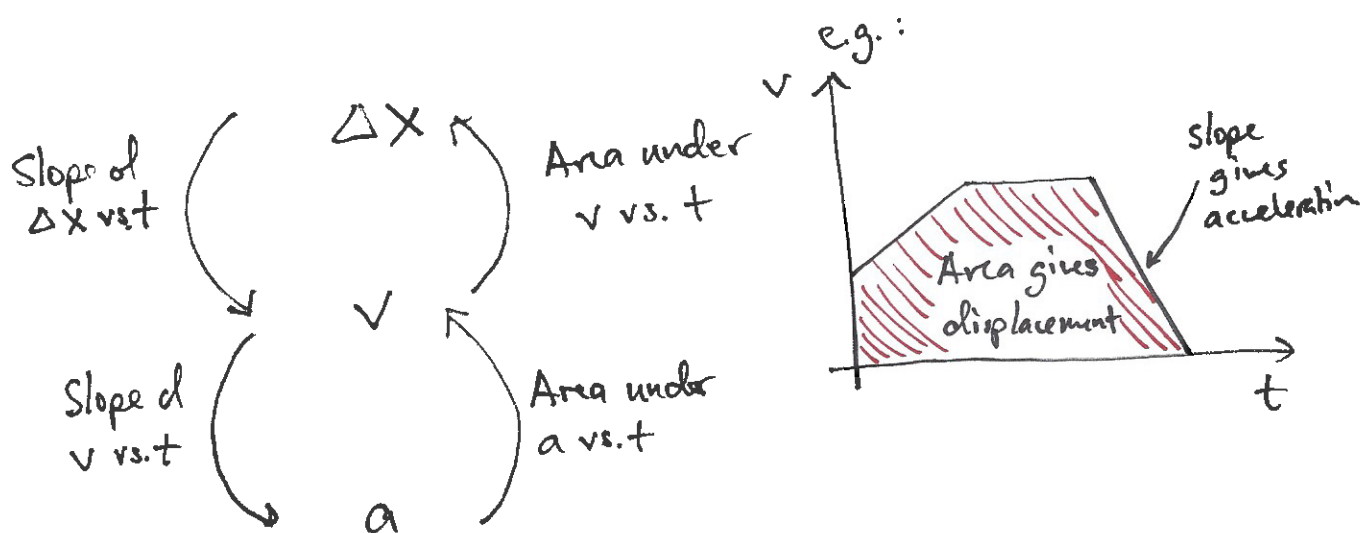
TOA  
tan = opposite / adjacent

Example type of Problem: Relative Velocities!

# Variables for description of Motion

	<u>Translational</u>	<u>Rotational</u>
Displacement	$\Delta x$	$\Delta \theta$
velocity	$v = \frac{\Delta x}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$ (angular velocity)
acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$ (angular acceleration)

These variables are related to one another! By examining their plots versus time, you can extract the related variables:



Because of these relationships, you can derive the Kinematic Equations:

## Translational

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$v_f = v_0 + a t$$

$$v_f^2 = v_0^2 + 2 a \Delta x$$

## Rotational

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_0 + \alpha t$$

$$\omega_f^2 = \omega_0^2 + 2 \alpha \Delta \theta$$

When working in 2D, each dimension is described by a set of kinematic equations and you work them independently. Example: Projectile motion (2)

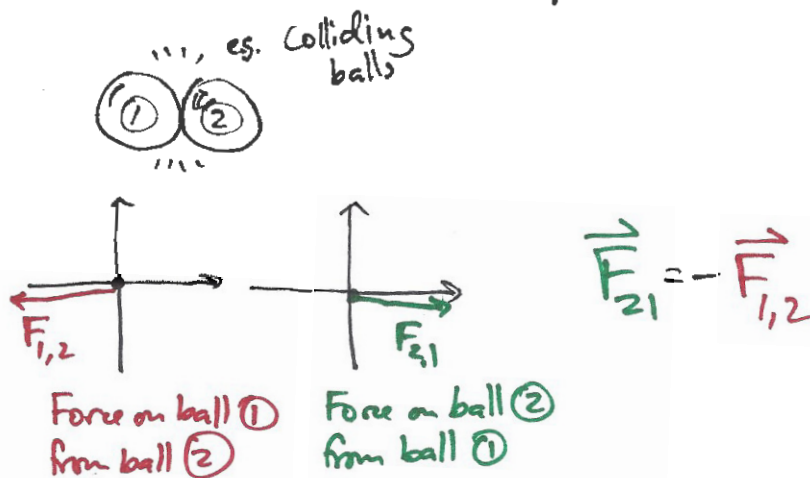
# Forces and Newton's Laws

## Newton's Laws:

- ① An object at rest will stay at rest, and an object in motion will remain in motion in a straight line unless acted upon by an external force.
- ② If an object experiences a net force, it will accelerate in the same direction as the force:

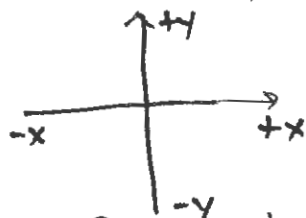
$$\vec{F}_{\text{net}} = m\vec{a}$$

- ③ For every action (force) there is an equal and opposite force

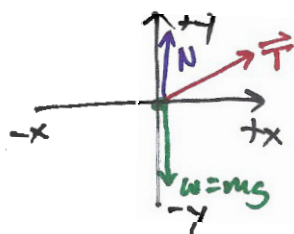


The law we use the most is Newton's 2<sup>nd</sup>:  $\vec{F}_{\text{net}} = m\vec{a}$ . To use it:

- ① Draw a picture (free body diagram), labelling coordinate axes:



- ② Identify and draw force vectors, e.g.



Example Problem:  
Block on inclined plane.

- ③ Use your drawing to split vectors into components and write Newton's 2<sup>nd</sup> law in each dimension

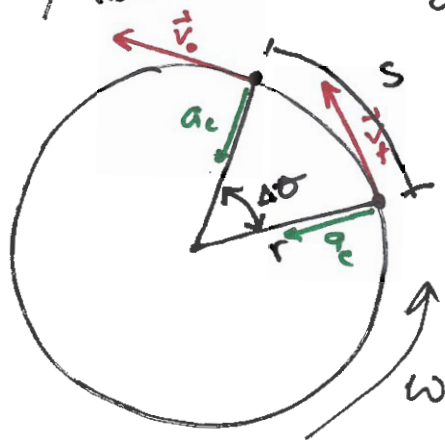
# Rotation and Circular Motion

You should familiarize yourself with the angular variables:

$$\Delta\theta = \frac{s}{r} \quad (\text{This is the definition of a radian!})$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$



Because these definitions are based on the definition of radians, you MUST use radians for the rotational kinematic equation to work!

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_f = \omega_0 + \alpha t$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$$

These are only true if your units are radians

The linear velocity  $\vec{v}$  of an object moving in a circle is everywhere tangent to its circular path, and is related to its angular velocity by

$$\vec{v} = r\omega$$

Similarly,

$$\Delta x = r\Delta\theta$$

$$a = r\alpha$$

Because the direction of its velocity vector is constantly changing, any object moving in a circle is constantly accelerated:

$$a_c = \frac{v^2}{r} = \omega^2 r \quad \text{centripetal acceleration points radially inward}$$

Example problems: A spinning disc, car driving in a circle

# Friction

There are two kinds of friction:

static friction: friction between stationary objects

The maximum possible static frictional force is

$$f_{\max} = \mu_s N$$

kinetic friction: friction between objects sliding relative to one another.

has magnitude

$$f_k = \mu_k N$$

In general, static friction is stronger than kinetic friction; that is

$$\mu_s > \mu_k$$

It is harder to get something moving than to keep it moving!

Examples: Sliding block on inclined plane, car driving in a circle