

Physics 101: Lecture 08

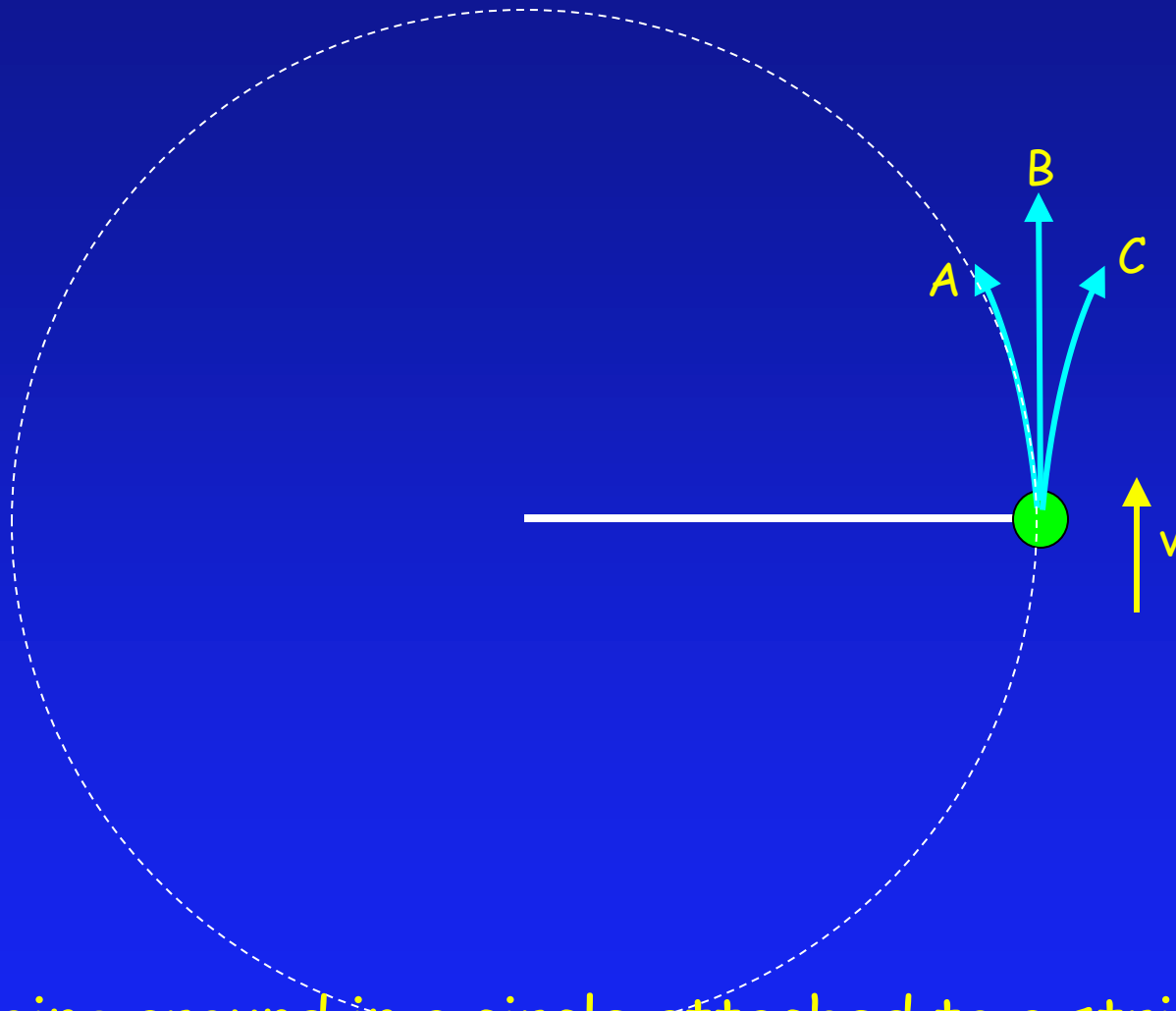
Centripetal Acceleration and Circular Motion

<http://www.youtube.com/watch?v=ZyF5WsmXRaI>

- Today's lecture will cover Chapter 5



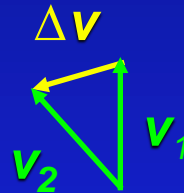
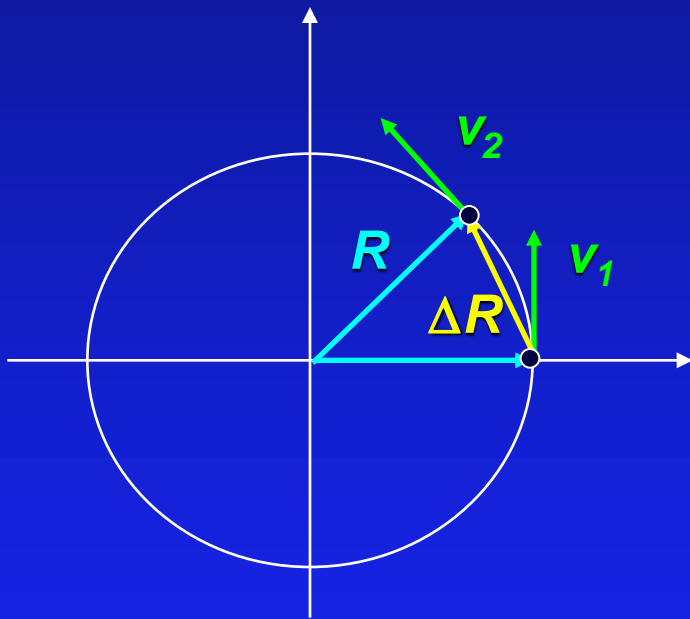
Circular Motion Act



Answer: B

A ball is going around in a circle attached to a string. If the string breaks at the instant shown, which path will the ball follow (demo)?

Acceleration in Uniform Circular Motion



$$\mathbf{a}_{\text{ave}} = \Delta \mathbf{v} / \Delta t$$

Acceleration inward

$$a = \frac{v^2}{R}$$

centripetal acceleration

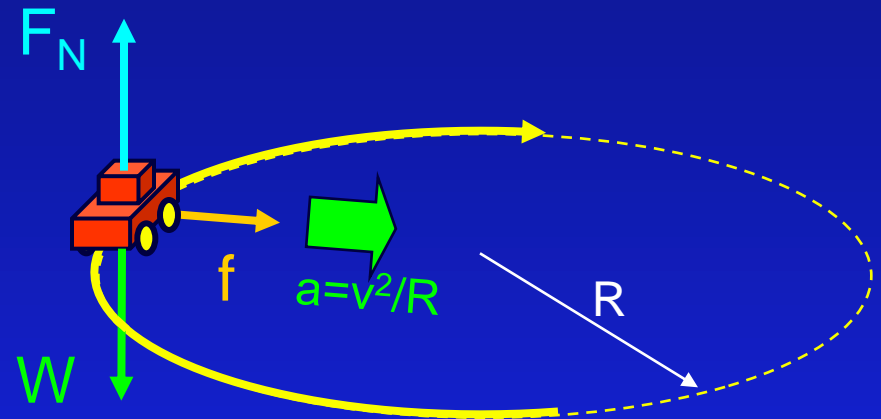
Acceleration is due to change in direction, not speed. Since the object turns “toward” center, there must be a force toward center.

Preflights

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. **How many forces are acting on the car?**

- 1% A) 1
- 18% B) 2
- 42% C) 3
- 25% D) 4
- 13% E) 5

← correct

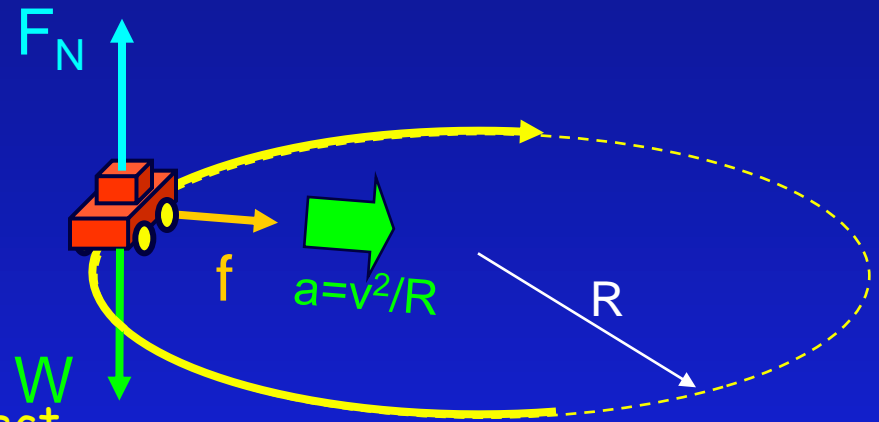


$$\Sigma \mathbf{F} = m\mathbf{a} = mv^2/R$$

"Friction, Gravity, & Normal"

Preflights

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. The net force on the car is



- 13% A. Zero
- 83% B. Pointing radially inward ← correct
- 4% C. Pointing radially outward

$$\Sigma \mathbf{F} = m\mathbf{a} = m\mathbf{v}^2/R$$

ACT

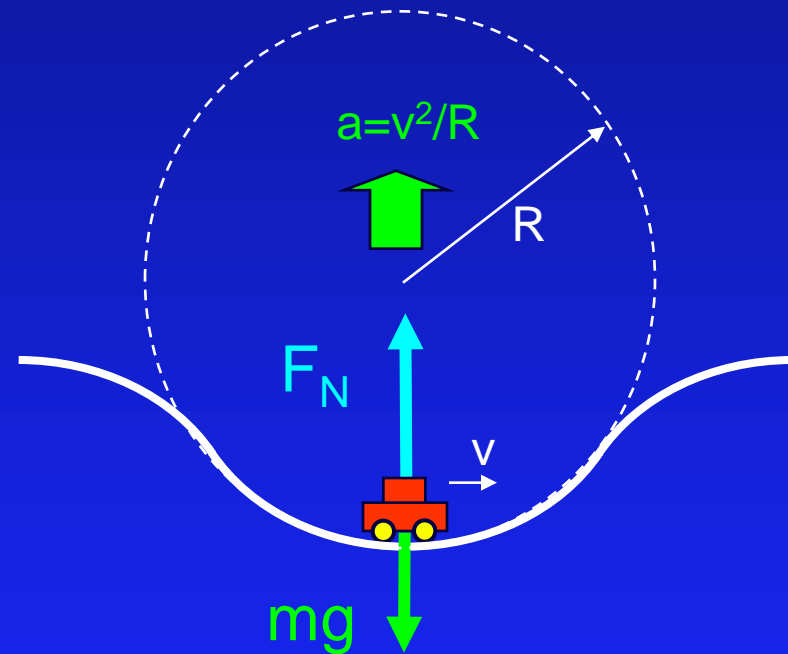
Suppose you are driving through a valley whose bottom has a circular shape. If your mass is m , what is the magnitude of the normal force F_N exerted on you by the car seat as you drive past the bottom of the hill

- A. $F_N < mg$
- B. $F_N = mg$
- C. $F_N > mg$ ← correct

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$F_N - mg = mv^2/R$$

$$F_N = mg + mv^2/R$$



Roller Coaster Example

What is the minimum speed you must have at the top of a 20 meter roller coaster loop, to keep the wheels on the track.

Y Direction: $F = ma$

$$-N - mg = m a$$

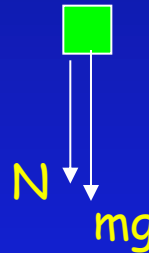
Let $N = 0$, just touching

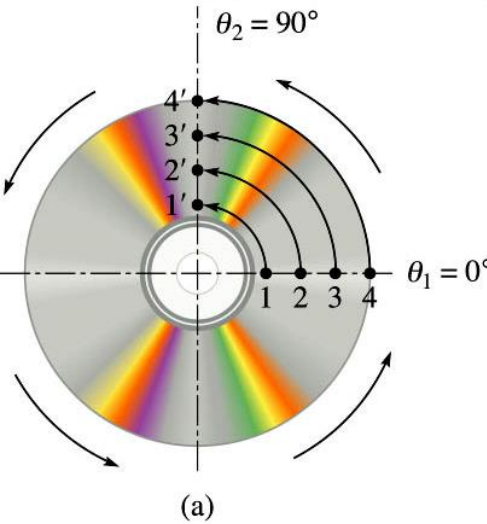
$$-mg = m a$$

$$-mg = -m v^2 / R$$

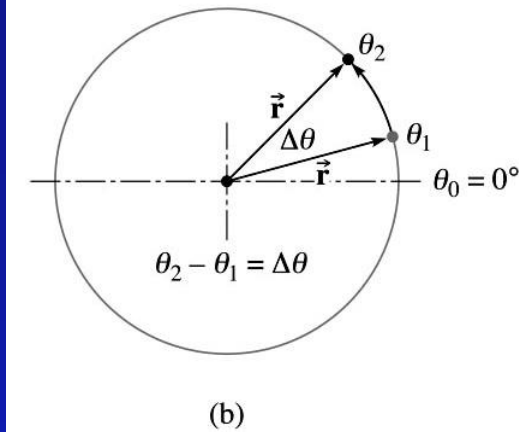
$$g = v^2 / R$$

$$v = \sqrt{g \cdot R} = 9.9 \text{ m/s}$$





Circular Motion



- Angular displacement $\Delta\theta = \theta_2 - \theta_1$
 - How far it has rotated
 - Units radians ($2\pi = 1$ revolution)
- Angular velocity $\omega = \Delta\theta / \Delta t$
 - How fast it is rotating
 - Units radians/second
- Period = 1/frequency $T = 1/f = 2\pi / \omega$
 - Time to complete 1 revolution

Circular to Linear

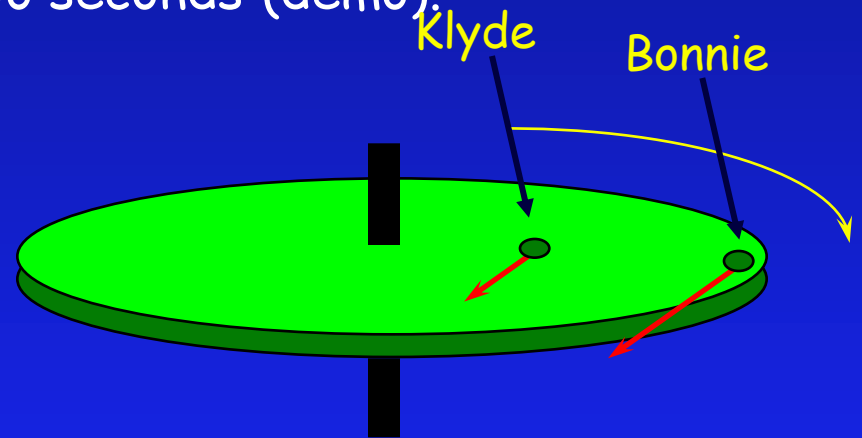
- Displacement $\Delta s = r \Delta\theta$ (θ in radians)
- Speed $|v| = \Delta s / \Delta t = r \Delta\theta / \Delta t = r\omega$
- Direction of v is tangent to circle

Merry-Go-Round ACT

- Bonnie sits on the outer rim of a merry-go-round with radius 3 meters, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds (demo).

→ Klyde's speed is:

- (a) the same as Bonnie's
- (b) twice Bonnie's
- (c) half Bonnie's



$$V_{Klyde} = \frac{1}{2} V_{Bonnie}$$

Bonnie travels $2\pi R$ in 2 seconds $v_B = 2\pi R / 2 = 9.42 \text{ m/s}$

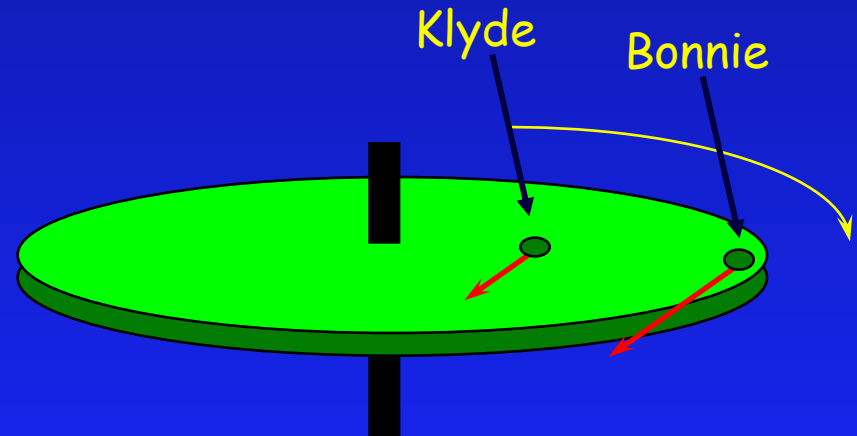
Klyde travels $2\pi (R/2)$ in 2 seconds $v_K = 2\pi (R/2) / 2 = 4.71 \text{ m/s}$

Merry-Go-Round ACT II

- Bonnie sits on the outer rim of a merry-go-round, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds.

→ Klyde's angular velocity is:

- (a) **the same as Bonnie's**
- (b) **twice Bonnie's**
- (c) **half Bonnie's**



- The angular velocity ω of any point on a solid object rotating about a fixed axis is the same.
 - Both Bonnie & Klyde go around once (2π radians) every two seconds.

Angular Acceleration

- Angular acceleration is the change in angular velocity ω divided by the change in time.

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_0}{\Delta t}$$

- If the speed of a roller coaster car is 15 m/s at the top of a 20 m loop, and 25 m/s at the bottom. What is the cars average angular acceleration if it takes 1.6 seconds to go from the top to the bottom?

$$\omega = \frac{V}{R}$$

$$\omega_f = \frac{25}{10} = 2.5$$

$$\omega_0 = \frac{15}{10} = 1.5$$

$$\bar{\alpha} \equiv \frac{2.5 - 1.5}{1.6} = 0.64 \text{ rad/s}^2$$

Summary

(with comparison to 1-D kinematics)

Angular	Linear
$\alpha = \text{constant}$	$a = \text{constant}$
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$
<p>And for a point at a distance R from the rotation axis:</p> $x = R\theta \quad v = \omega R \quad a = \alpha R$	

CD Player Example

- The CD in your disk player spins at about 20 radians/second. If it accelerates uniformly from rest with angular acceleration of 15 rad/s², how many revolutions does the disk make before it is at the proper speed?

$$\omega^2 - \omega_0^2 = 2\alpha\Delta\theta$$

$$\frac{\omega_f^2 - \omega_0^2}{2\alpha} = \Delta\theta$$

$$\frac{20^2 - 0^2}{2 \times 15} = \Delta\theta$$

$$\Delta\theta = 13.3 \text{ radians}$$

$$1 \text{ Revolution} = 2\pi \text{ radians}$$

$$\Delta\theta = 13.3 \text{ radians}$$

$$= 2.12 \text{ revolutions}$$

Summary of Concepts

- Uniform Circular Motion
 - Speed is constant
 - Direction is changing
 - Acceleration toward center $a = v^2 / r$
 - Newton's Second Law $F = ma$
- Circular Motion
 - θ = angular position radians
 - ω = angular velocity radians/second
 - α = angular acceleration radians/second²
 - Linear to Circular conversions $s = r \theta$
- Uniform Circular Acceleration Kinematics
 - Similar to linear!