

Physics 101: Lecture 24

Ideal Gas Law and Kinetic Theory

- Today's lecture will cover Textbook Chapter 13.5-13.7



Aside: The Periodic Table

Ia	IIa	IIIb	IVb	Vb	VIb	VIIb	VII					Ib	IIb	IIa	IVa	Va	VIa	VIIa	O
1 <u>H</u> 1.00797																		2 <u>He</u> 4.0026	
3 <u>Li</u> 6.941	4 <u>Be</u> 9.0122											5 <u>B</u> 10.81	6 <u>C</u> 12.011	7 <u>N</u> 14.0067	8 <u>O</u> 15.9995	9 <u>F</u> 18.9984	10 <u>Ne</u> 20.179		
11 <u>Na</u> 22.9898	12 <u>Mg</u> 24.305											13 <u>Al</u> 26.9815	14 <u>Si</u> 28.086	15 <u>P</u> 30.9738	16 <u>S</u> 32.06	17 <u>Cl</u> 35.453	18 <u>Ar</u> 39.948		
19 <u>K</u> 39.098	20 <u>Ca</u> 40.08	21 <u>Sc</u> 44.956	22 <u>Ti</u> 47.90	23 <u>V</u> 50.9414	24 <u>Cr</u> 51.996	25 <u>Mn</u> 54.9830	26 <u>Fe</u> 55.847	27 <u>Co</u> 58.9332	28 <u>Ni</u> 58.70	29 <u>Cu</u> 63.546	30 <u>Zn</u> 65.38	31 <u>Ga</u> 69.72	32 <u>Ge</u> 72.59	33 <u>As</u> 74.9216	34 <u>Se</u> 78.96	35 <u>Br</u> 79.904	36 <u>Kr</u> 83.80		
37 <u>Rb</u> 85.4678	38 <u>Sr</u> 87.62	39 <u>Y</u> 88.909	40 <u>Zr</u> 91.22	41 <u>Nb</u> 92.9064	42 <u>Mo</u> 95.94	43 <u>Tc</u> (97)	44 <u>Ru</u> 101.07	45 <u>Rh</u> 102.905	46 <u>Pd</u> 106.04	47 <u>Ag</u> 107.868	48 <u>Cd</u> 112.40	49 <u>In</u> 114.82	50 <u>Sn</u> 118.69	51 <u>Sb</u> 121.75	52 <u>Te</u> 127.60	53 <u>I</u> 126.9046	54 <u>Xe</u> 131.30		
55 <u>Cs</u> 132.905	56 <u>Ba</u> 137.34	57 * <u>La</u> 138.91	72 <u>Hf</u> 178.49	73 <u>Ta</u> 180.948	74 <u>W</u> 183.85	75 <u>Re</u> 186.207	76 <u>Os</u> 190.2	77 <u>Ir</u> 192.22	78 <u>Pt</u> 195.09	79 <u>Au</u> 196.967	80 <u>Hg</u> 200.59	81 <u>Tl</u> 204.37	82 <u>Pb</u> 207.2	83 <u>Bi</u> 208.980	84 <u>Po</u> (209)	85 <u>At</u> (210)	86 <u>Rn</u> (222)		
87 <u>Fr</u> (223)	88 <u>Ra</u> 226.03	89 * <u>Ac</u> (227)																	

The Periodic Table Explained ?

Look carefully

1 H 1.00797						2 He 4.0026						
3 Li 6.941	4 Be 9.0122						5 B 10.81	6 C 12.011	7 N 14.0067	8 O 15.9995	9 F 18.9984	10 Ne 20.179

● proton
● neutron
● electron

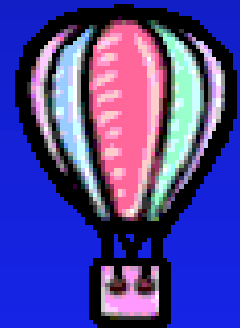
Diagram illustrating the periodic table and the corresponding number of protons, neutrons, and electrons for the first ten elements (H to Ne). The elements are arranged in two rows. The first row contains H (1), He (2). The second row contains Li (3), Be (4), B (5), C (6), N (7), O (8), F (9), and Ne (10). Below the table, the number of protons (red dots), neutrons (yellow dots), and electrons (green dots) are shown for each element. A legend indicates: red dot = proton, yellow dot = neutron, green dot = electron. A detailed atomic model of Helium is shown at the bottom, illustrating the nucleus (2 protons, 2 neutrons) and the electron cloud (2 electrons). A blue arrow points from the Helium entry in the periodic table to the atomic model.

Molecular Picture of Gas

- Gas is made up of many individual molecules
- Number density is number of molecules/volume:
 - $N/V = \rho/m$
 - ρ is the mass density
 - m is the mass for one molecule

- Number of moles: $n = N / N_A$
1 u = 1.66×10^{-27} kg = 1/12 of a mass of C^{12}
 - N_A = Avogadro's Number = 6.022×10^{23} mole⁻¹

- Mass of 1 mole of “stuff” in grams = molecular mass in μ
 - e.g., 1 mole of N_2 has mass of $2 \times 14 = 28$ grams



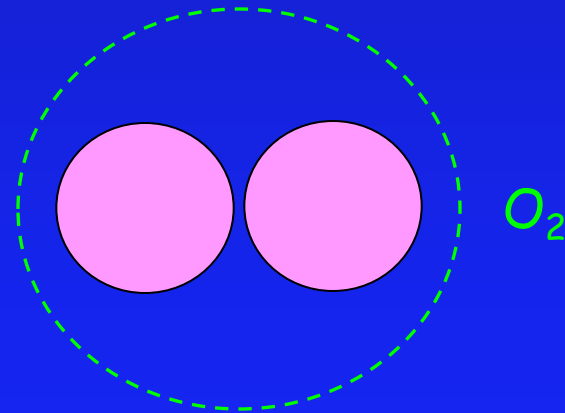
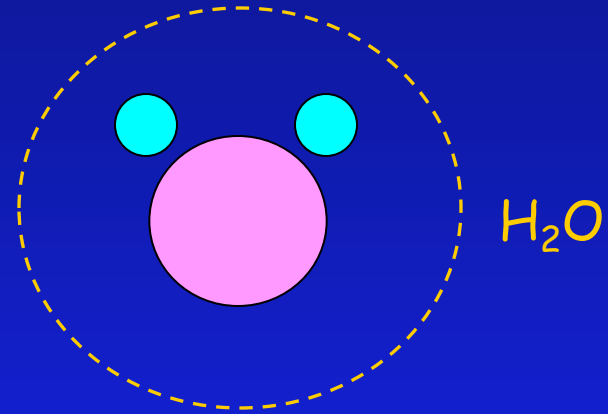
Atomic Act I

Which contains the most molecules ?

1. A mole of water (H_2O)

2. A mole of oxygen gas (O_2)

3. Same ← correct



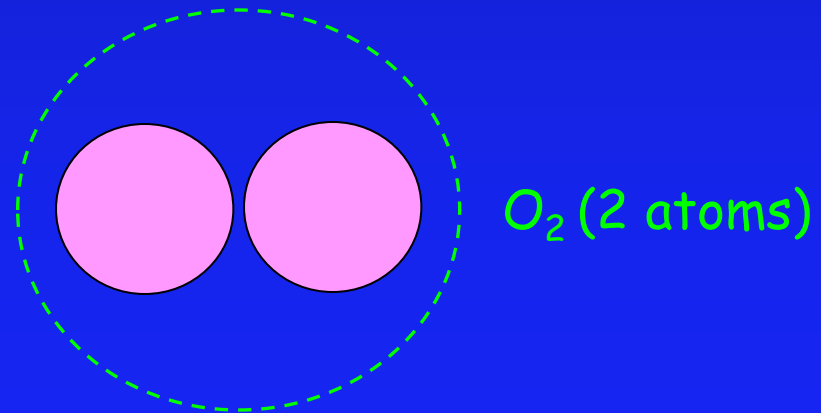
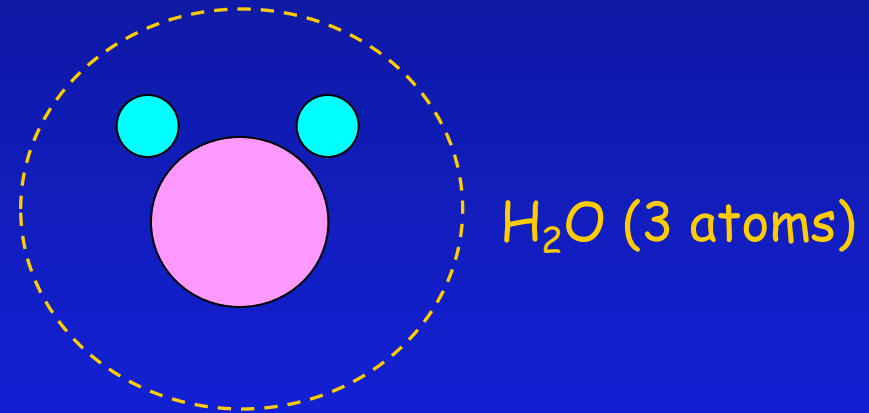
Atomic Act II

Which contains the most atoms ?

1. A mole of water (H_2O) ← correct

2. A mole of oxygen gas (O_2)

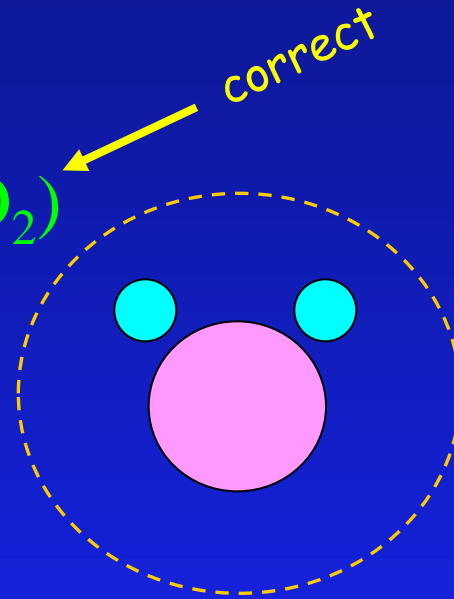
3. Same



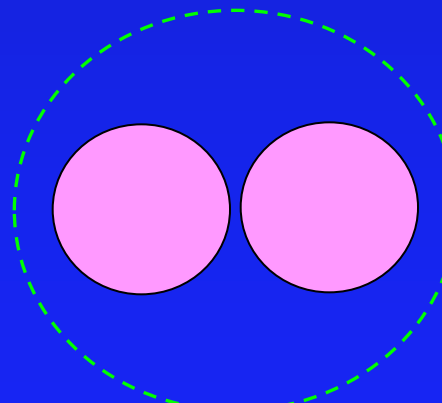
Atomic Act III

Which weighs the most ?

1. A mole of water (H_2O)
2. A mole of oxygen gas (O_2)
3. Same



H_2O ($M = 16 + 1 + 1$)



O_2 ($M = 16 + 16$)

The Ideal Gas Law

- $P V = N k_B T$

- P = pressure in N/m^2 (or Pascals)

- V = volume in m^3

- N = number of molecules

- T = absolute temperature in K

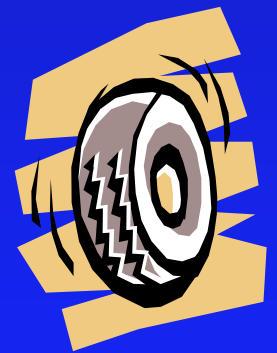
- k_B = Boltzmann's constant = $1.38 \times 10^{-23} \text{ J/K}$

- Note: $P V$ has units of N-m or J (energy!)

- $P V = n R T$

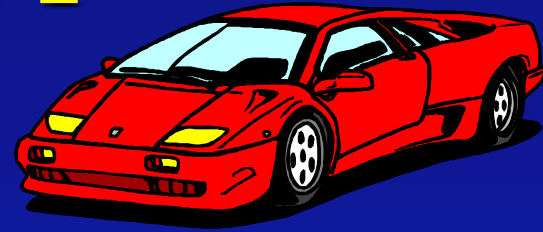
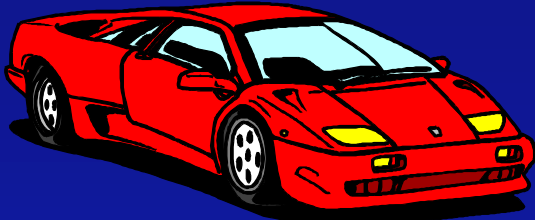
- n = number of moles

- R = ideal gas constant = $N_A k_B = 8.31 \text{ J/mol/K}$



Ideal Gas Law ACT I

$PV = nRT$



You inflate the tires of your car so the pressure is 30 psi, when the air inside the tires is at 20 degrees C. After driving on the highway for a while, the air inside the tires heats up to 38 C. Which number is closest to the new air pressure?

1) 16 psi

2) 32 psi

3) 57 psi

Careful, you need to use the temperature in K

$$P = P_0 (38+273)/(20+273)$$

Ideal Gas Law: ACT II

$$pV = nRT$$

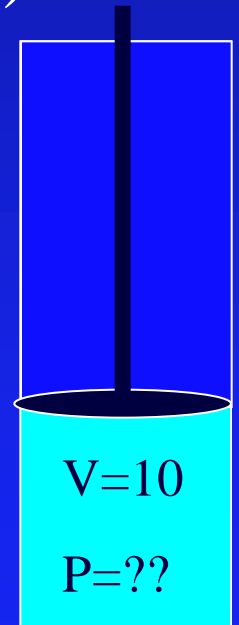
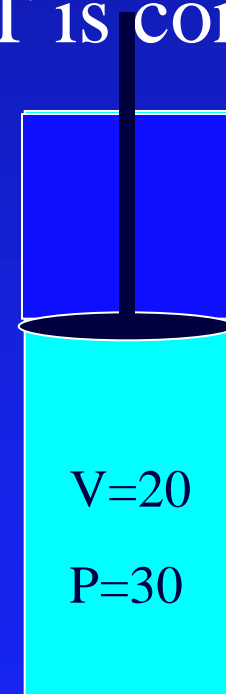
- A piston has volume 20 ml, and pressure of 30 psi. If the volume is decreased to 10 ml, what is the new pressure? (Assume T is constant.)

1) 60

2) 30

3) 15

- When n and T are constant, pV is constant (Boyle's Law)



Balloon ACT 1



- What happens to the pressure of the air inside a hot-air balloon when the air is heated? (Assume V is constant)

1) Increases 2) Same 3) Decreases

Balloon is still open to atmospheric pressure,
so it stays at 1 atm

Balloon ACT 2




- What happens to the buoyant force on the balloon when the air is heated?
(Assume V remains constant)

1) Increases 2) Same 3) Decreases

$$F_B = \rho V g$$

ρ is density of outside air!

Balloon ACT 3

- 
- What happens to the number of air molecules inside the balloon when the air is heated? (Assume V remains constant)

1) Increases 2) Same 3) Decreases

$$PV = NkT$$

P and V are constant. If T increases N decreases.

Lecture 24, Preflight 2

In terms of the ideal gas law, explain briefly how a hot air balloon works.

The flame lifts the balloon because it gets rid of the air inside, making it lighter inside.

The fire heats up the air inside the balloon. The air rises and pushes on the inside of the balloon which makes the pressure higher on the inside than the outside. Since there is more pressure pushing up than pushing down, the balloon rises.

hot air rises.

Dumbledore has cast levitation charms (which is pronounced leviosá not leviosa) on every hot air balloon.

Note! this is not a pressure effect, it is a density effect. As T increases, the density decreases the balloon then floats due to Archimedes principle. The pressure remains constant!

Ideal Gas Law: Demos

$$pV = nRT$$

- When T is constant, pV is constant (Boyle's Law)
 - Boyle's law demo
- When p is constant, V is proportional to T
 - Hot air balloon, helium and oxygen in LN_2
- When V is constant, p is proportional to T
 - Explosion!

Preflight 1

root-mean-square?

Suppose you want the rms (*root-mean-square*) speed of molecules in a sample of gas to double. By what factor should you increase the temperature of the gas?

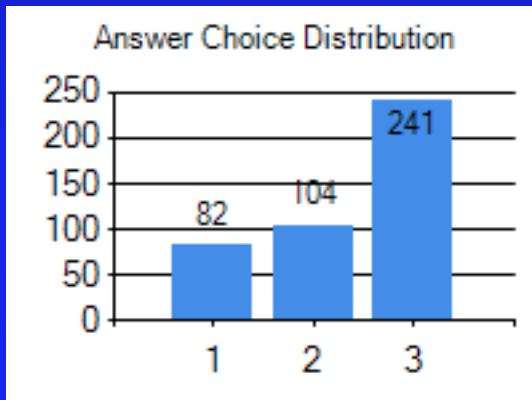
1. 2

2. $\sqrt{2}$

3. 4 ← correct

$$\langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

- If v doubles, v^2 quadruples
- Therefore, T quadruples



$$x_{\text{rms}} = \sqrt{\frac{1}{n} (x_1^2 + x_2^2 + \cdots + x_n^2)}$$

Example

- What is the rms speed of a nitrogen (N_2) molecule in this classroom?

$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3 k_B T}{m}$$

$$v = 510 \text{ m/s}$$

$$= 1150 \text{ mph!}$$

$$\langle v^2 \rangle = \frac{3(1.38 \times 10^{-23} \text{ J/K})(273 + 20) \text{ K}}{(28 \text{ u}) \times (1.66 \times 10^{-27} \text{ kg/u})}$$

Kinetic Theory:

The relationship between energy and temperature
(for monatomic ideal gas)

$$\Delta p_x = 2mv_x$$

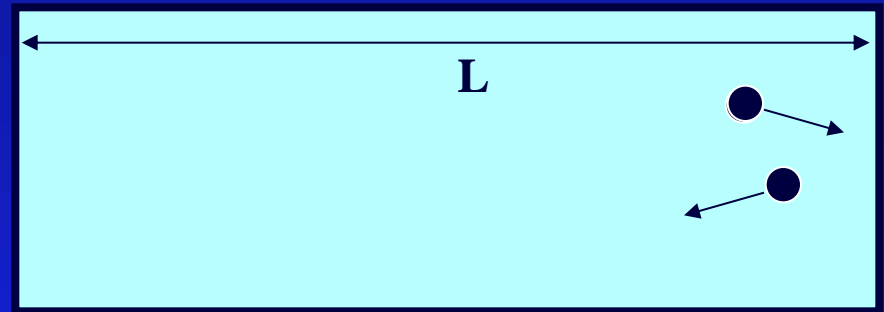
$$\Delta t = 2 \frac{L}{v_x}$$

$$F_{avg} = \frac{\Delta p_x}{\Delta t} = \frac{mv_x^2}{L}$$

For N molecules, multiply by N

$$P = \frac{F}{A} = \frac{Nmv_x^2}{V}$$

Note $KE = \frac{1}{2} m v^2 = \frac{3}{2} m v_x^2$



$$P = \frac{2}{3} \frac{N}{V} \langle K_{tr} \rangle$$

Using $PV = NkT$

$$\langle K_{tr} \rangle = \frac{3}{2} kT$$

$\langle \rangle$ means *average*.

$kT/2$ energy per
degree of
freedom =
equipartition
theorem

Summary

- Ideal Gas Law $PV = n R T$
 - P = pressure in N/m^2 (or Pascals)
 - V = volume in m^3
 - n = # moles
 - $R = 8.31 \text{ J/ (K mole)}$
 - T = Temperature (K)
- Kinetic Theory of Monatomic Ideal Gas
 - $\langle K_{\text{tr}} \rangle = \frac{3}{2} k_B T$