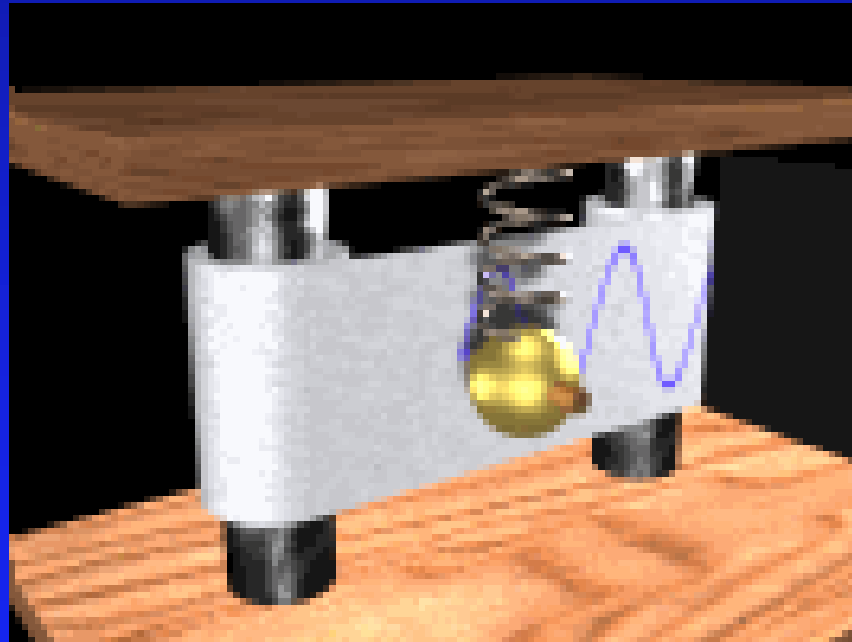


Physics 101: Lecture 19 Elasticity and Oscillations



Overview

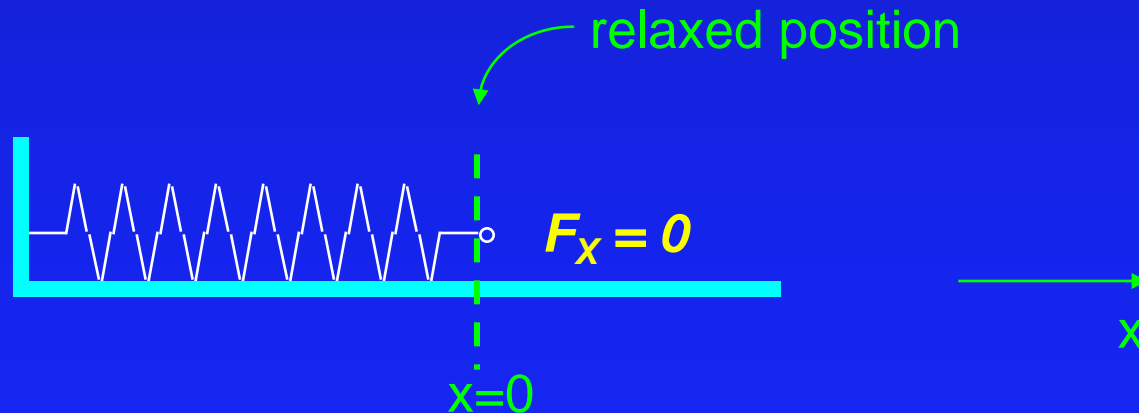
- Springs (review)
 - Restoring force proportional to displacement
 - $F = -k x$ (often a good approximation)
 - $U = \frac{1}{2} k x^2$
- Today
 - Simple Harmonic Motion
 - Springs Revisited
 - Young's Modulus (where does k come from?)

Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

$$\rightarrow F_x = -k x$$

Where x is the displacement from the relaxed position and k is the constant of proportionality.



Springs ACT

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

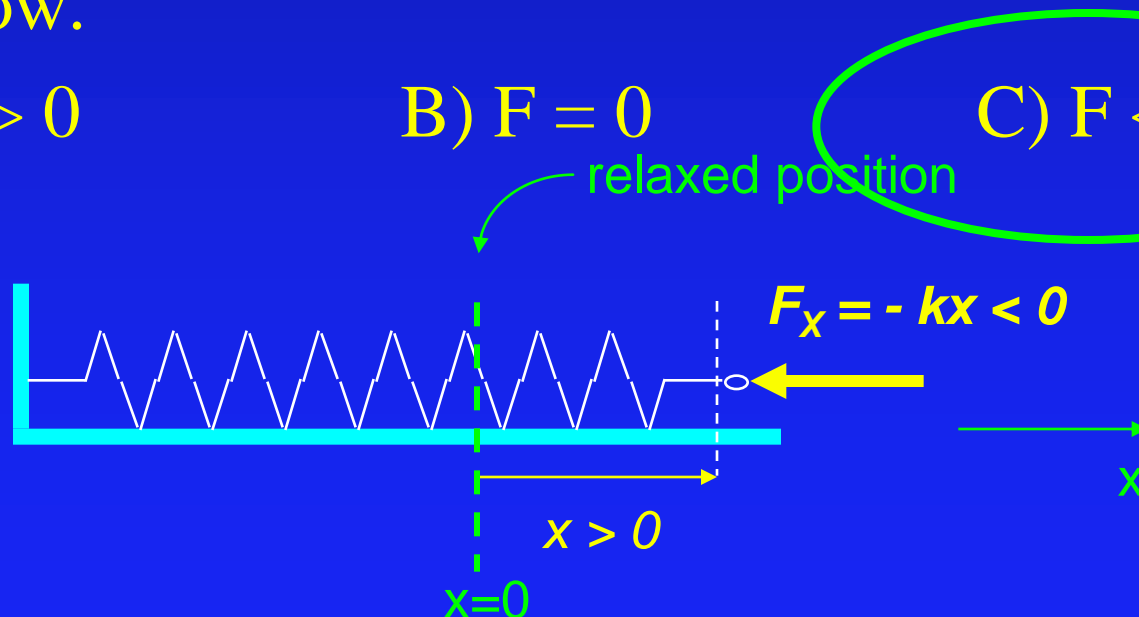
→ $F_x = -kx$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

What is force of spring when it is stretched as shown below.

A) $F > 0$

B) $F = 0$

C) $F < 0$

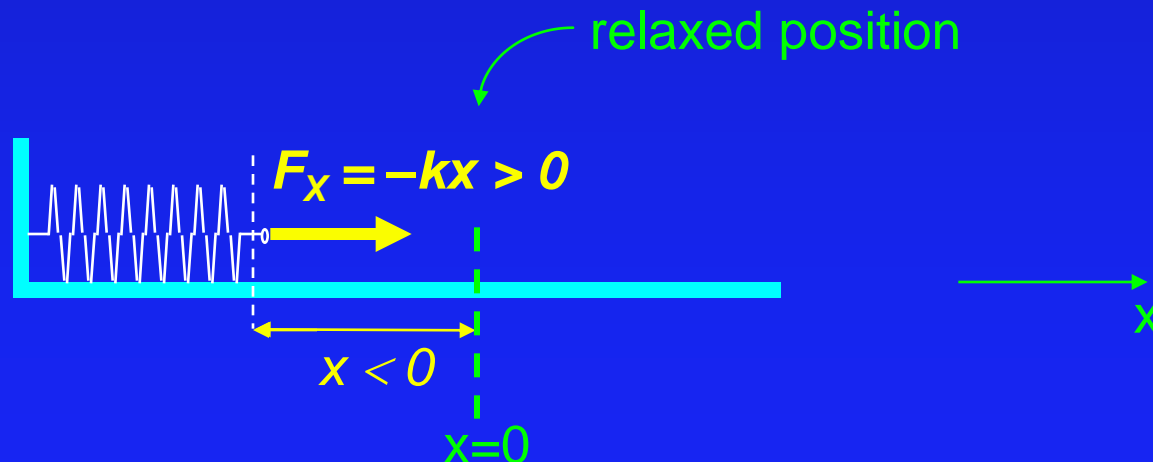


Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

$$\rightarrow F_x = -k x$$

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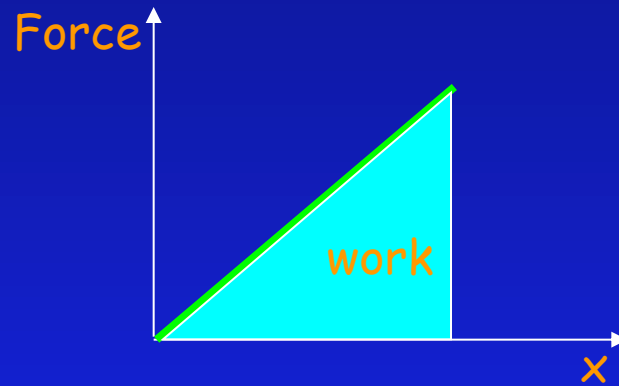


Potential Energy in Spring

- Hooke's Law force is Conservative

→ $F = -k x$

→ $W = -1/2 k x^2$

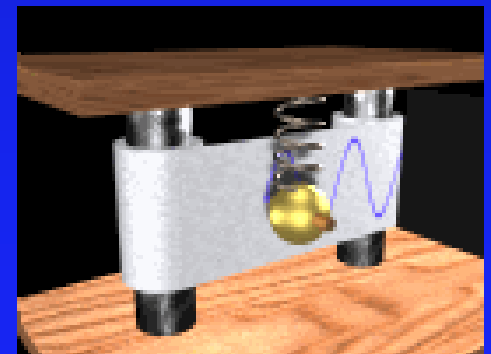


→ Work done only depends on initial and final position

→ Define Potential Energy $U_{\text{spring}} = 1/2 k x^2$

Simple Harmonic Motion

- Vibrations
 - ➔ Vocal cords when singing/speaking
 - ➔ String/rubber band
- Simple Harmonic Motion
 - ➔ Restoring force proportional to displacement
 - ➔ Springs $F = -kx$

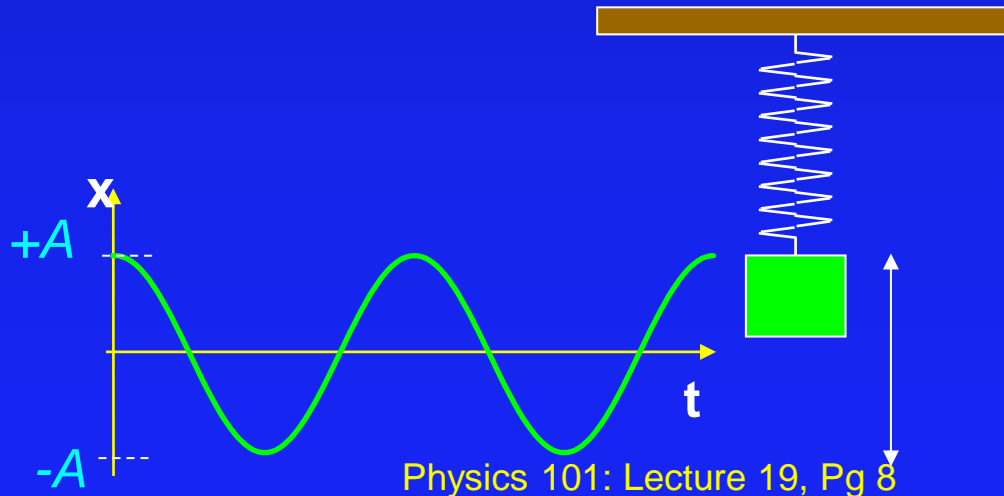


Spring ACT II

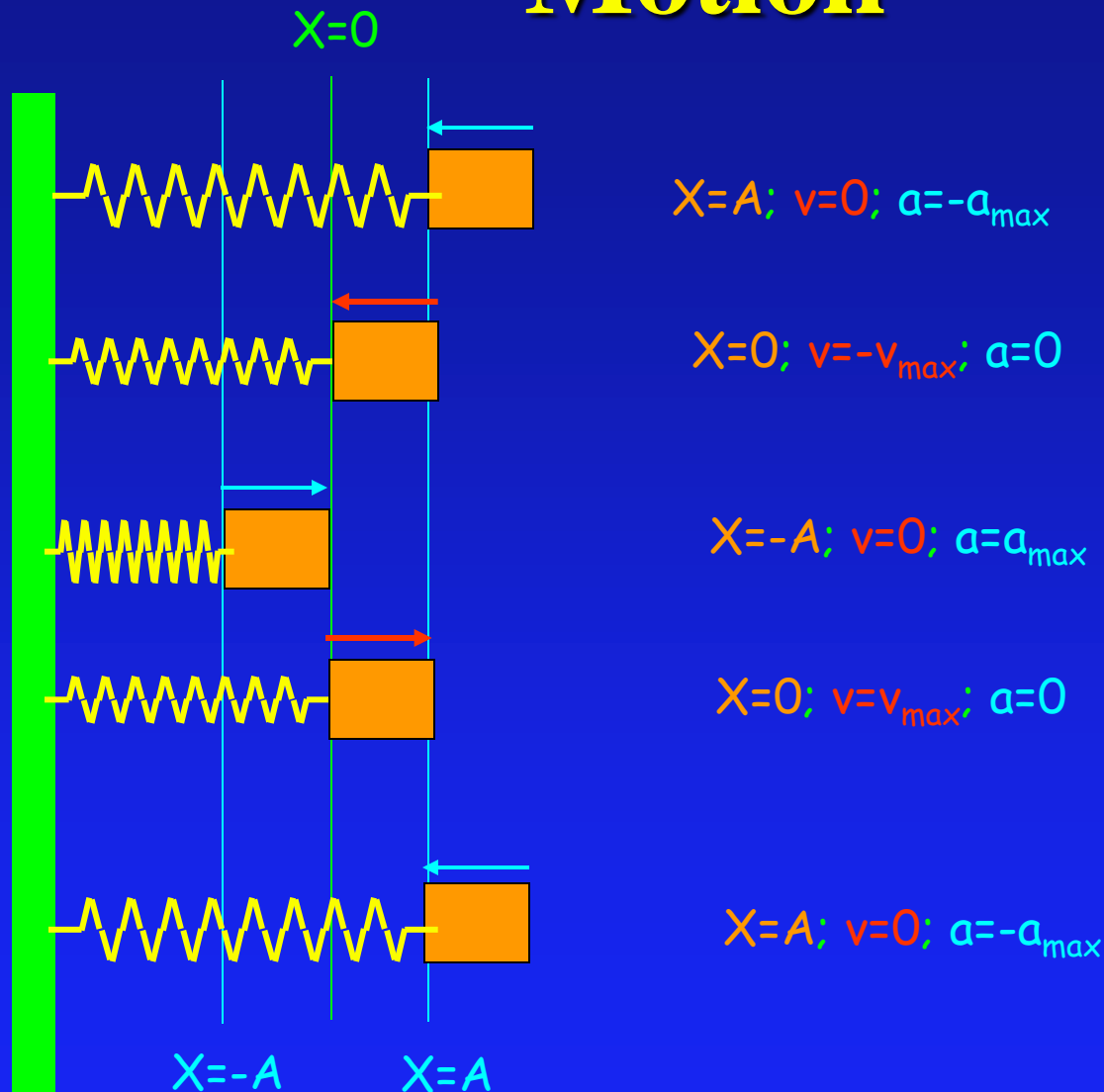
A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A . A plot of displacement (x) versus time (t) is shown below. **At what points during its oscillation is the magnitude of the acceleration of the block biggest?**

1. When $x = +A$ or $-A$ (i.e. maximum displacement) ← **CORRECT**
2. When $x = 0$ (i.e. zero displacement)
3. The acceleration of the mass is constant

$$F=ma$$



Springs and Simple Harmonic Motion



Energy

- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$

$$\rightarrow \Sigma W_{nc} = \Delta K + \Delta U$$

$$\rightarrow 0 = \Delta K + \Delta U \text{ or Energy } U+K \text{ is constant!}$$

$$\text{Energy} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

$$\rightarrow \text{At maximum displacement } x=A, v=0$$

$$\text{Energy} = \frac{1}{2} k A^2 + 0$$

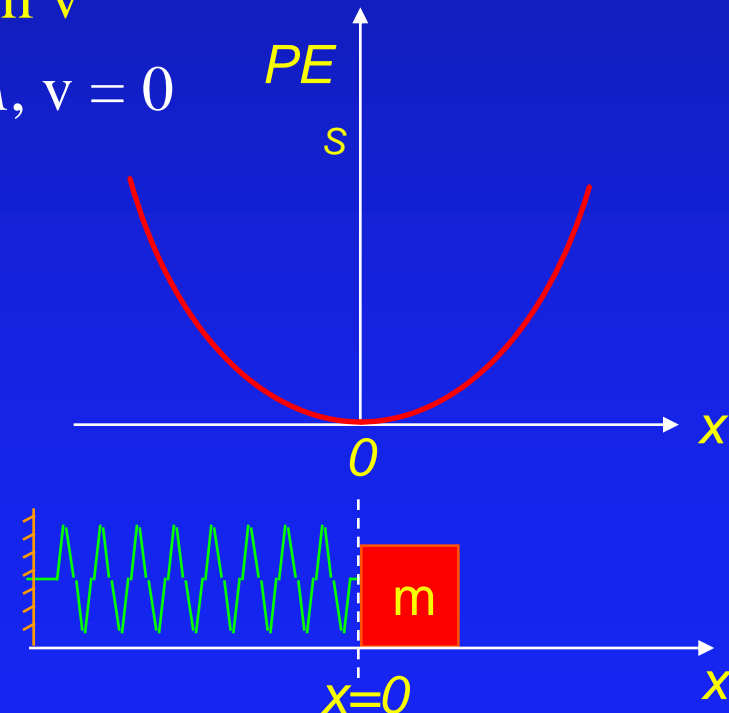
$$\rightarrow \text{At zero displacement } x=0$$

$$\text{Energy} = 0 + \frac{1}{2} m v_m^2$$

Since Total Energy is same

$$\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$$

$$v_m = \sqrt{k/m} A$$

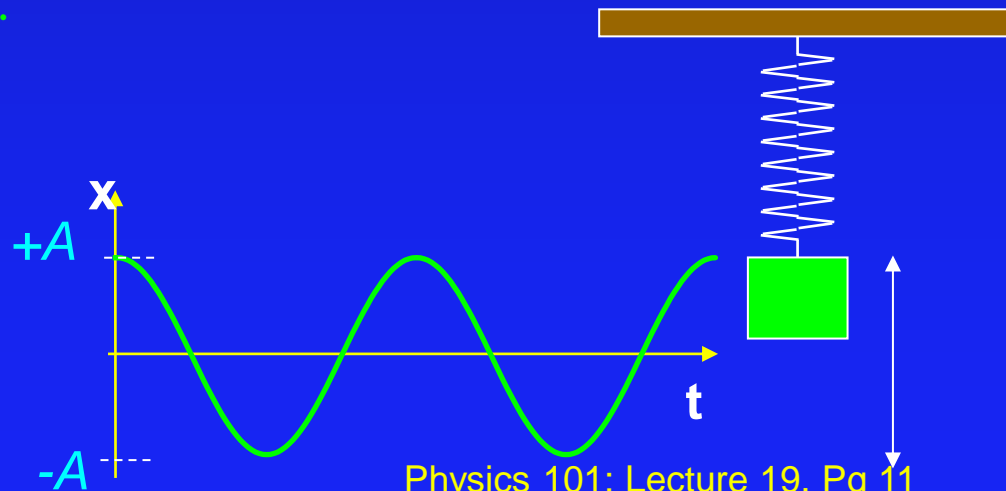
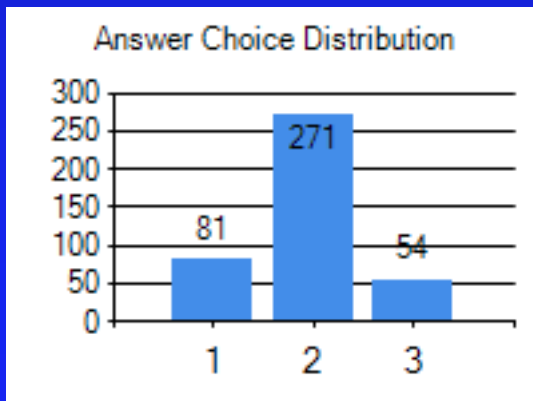


Prelecture 1+2

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A . A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

1. When $x = +A$ or $-A$ (i.e. maximum displacement)
2. When $x = 0$ (i.e. zero displacement) ← CORRECT
3. The speed of the mass is constant

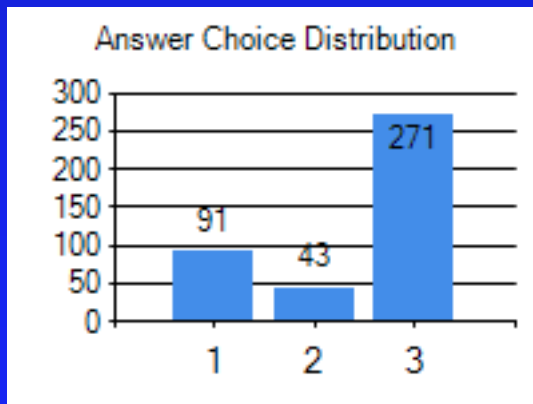
"At $x=0$ all spring potential energy is converted into kinetic energy and so the velocity will be greatest at this point."



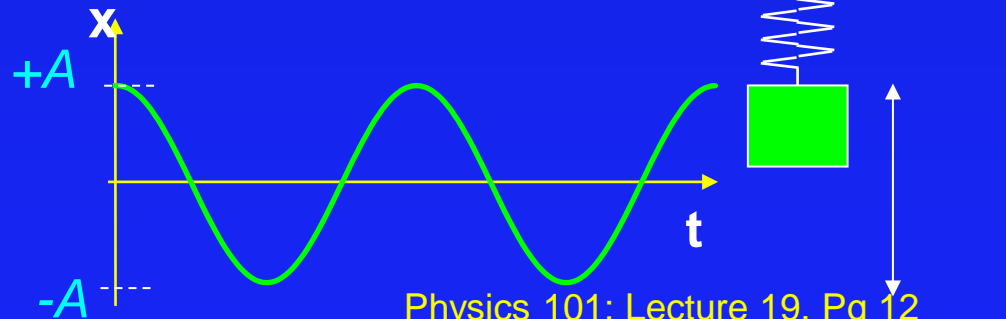
Prelecture 3+4

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A . A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total energy ($K+U$) of the mass and spring a maximum? (Ignore gravity).

1. When $x = +A$ or $-A$ (i.e. maximum displacement)
2. When $x = 0$ (i.e. zero displacement)
3. The energy of the system is constant ← CORRECT



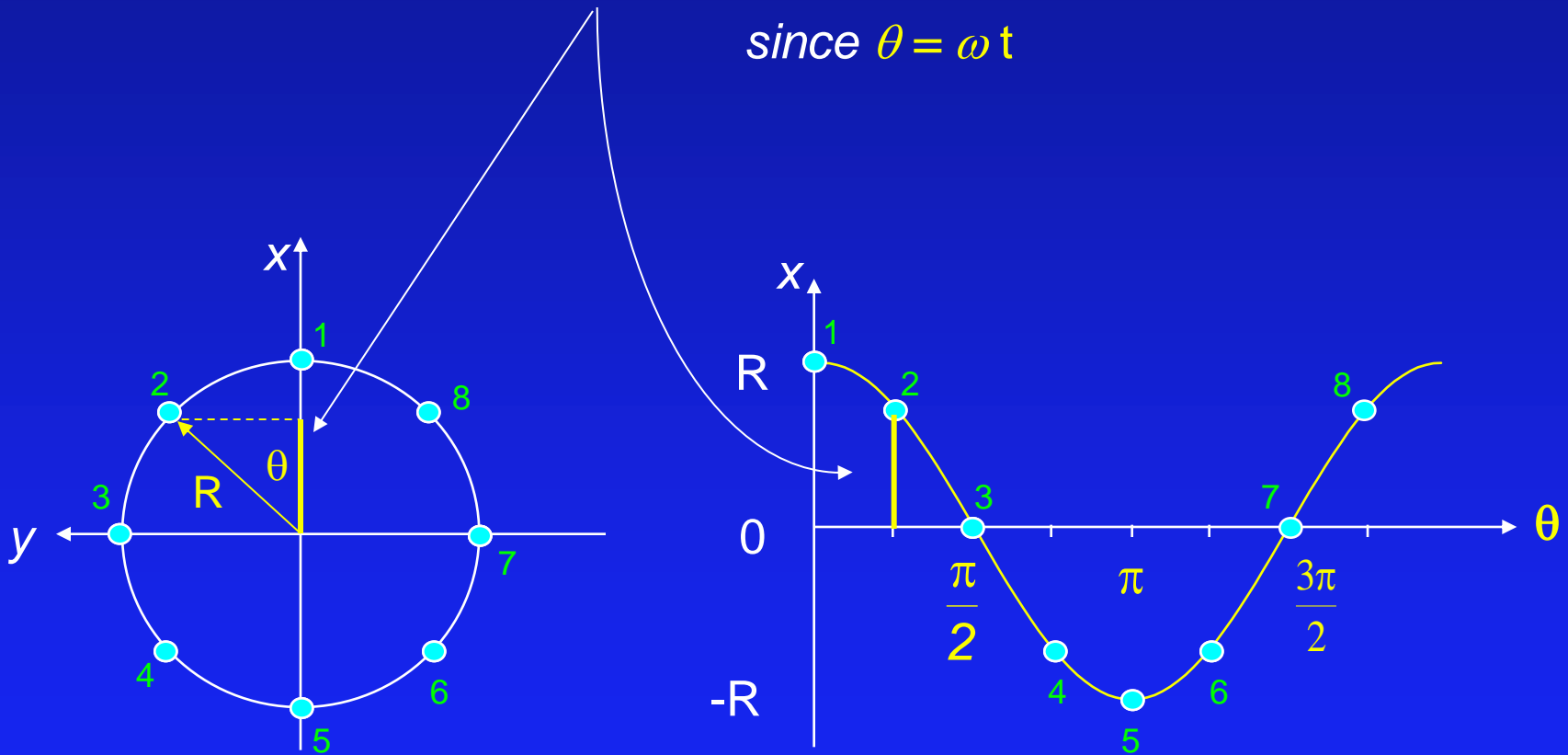
The energy changes from spring to kinetic but is not lost.



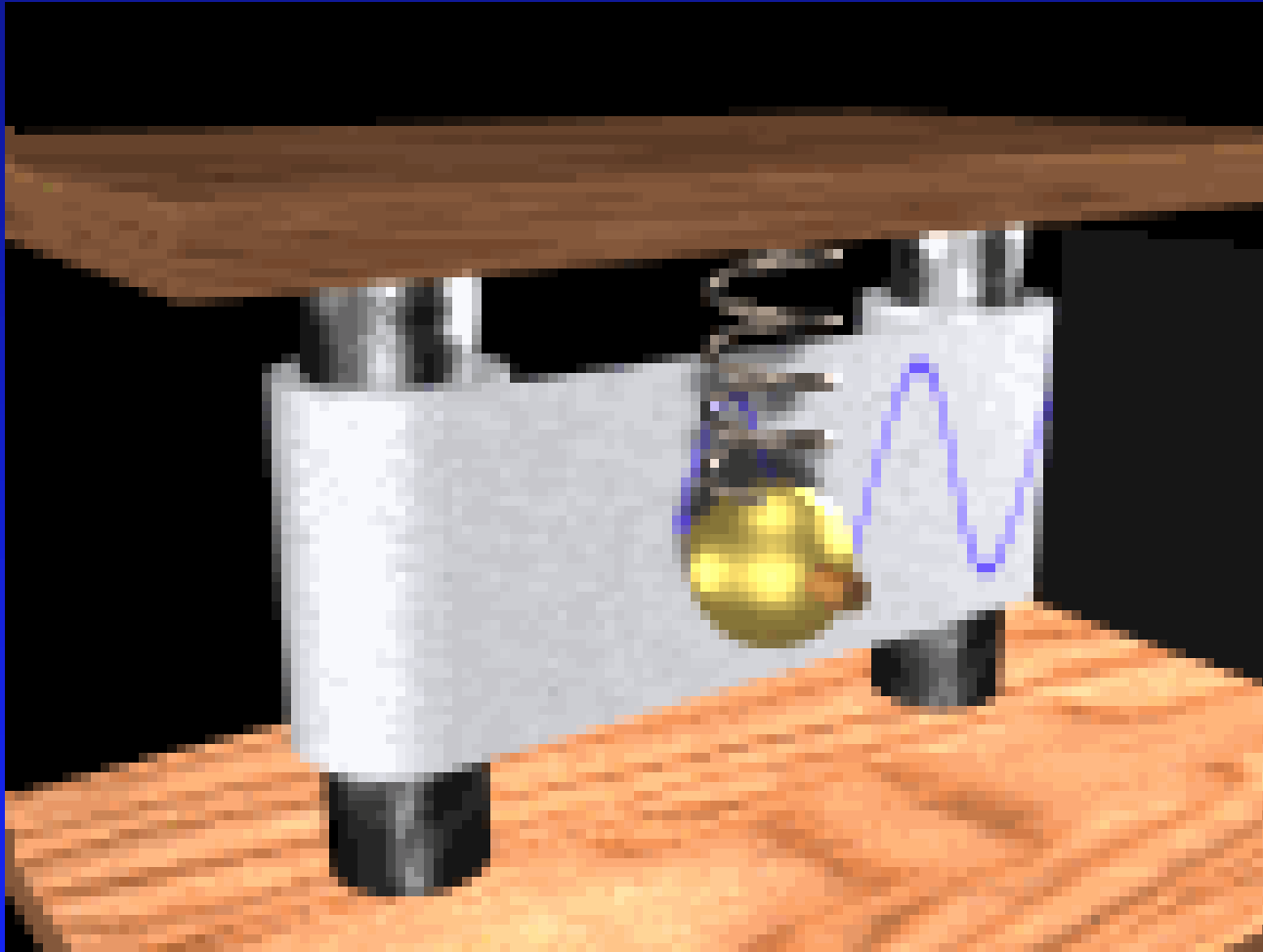
What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$$x = R \cos \theta = R \cos (\omega t)$$

since $\theta = \omega t$



SHM and Circles



Simple Harmonic Motion:

$$x(t) = [A]\cos(\omega t)$$

$$x(t) = [A]\sin(\omega t)$$

$$v(t) = -[A\omega]\sin(\omega t) \quad \text{OR} \quad v(t) = [A\omega]\cos(\omega t)$$

$$a(t) = -[A\omega^2]\cos(\omega t) \quad a(t) = -[A\omega^2]\sin(\omega t)$$

$$x_{\max} = A$$

Period = T (seconds per cycle)

$$v_{\max} = A\omega$$

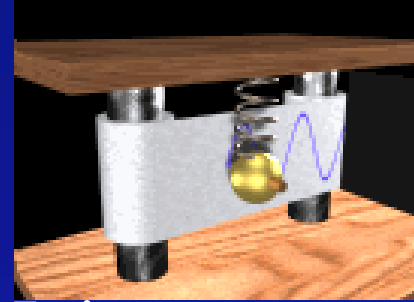
Frequency = $f = 1/T$ (cycles per second)

$$a_{\max} = A\omega^2$$

Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$

Example



A 3 kg mass is attached to a spring ($k=24$ N/m). It is stretched 5 cm. At time $t=0$ it is released and oscillates.

Which equation describes the position as a function of time $x(t) =$

- A) $5 \sin(\omega t)$ B) $5 \cos(\omega t)$ C) $24 \sin(\omega t)$
D) $24 \cos(\omega t)$ E) $-24 \cos(\omega t)$

We are told at $t=0$, $x = +5$ cm. $x(t) = 5 \cos(\omega t)$ only one that works.

Example



A 3 kg mass is attached to a spring ($k=24 \text{ N/m}$). It is stretched 5 cm. At time $t=0$ it is released and oscillates.

What is the total energy of the block spring system?

A) 0.03 J

B) .05 J

C) .08 J

$$E = U + K$$

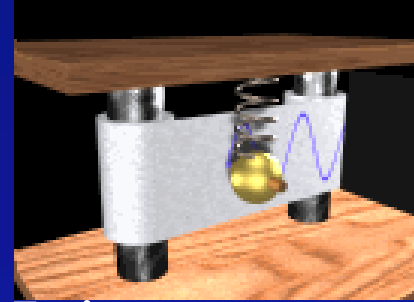
At $t=0$, $x = 5 \text{ cm}$ and $v=0$:

$$E = \frac{1}{2} k x^2 + 0$$

$$= \frac{1}{2} (24 \text{ N/m}) (5 \text{ cm})^2$$

$$= 0.03 \text{ J}$$

Example



A 3 kg mass is attached to a spring ($k=24 \text{ N/m}$). It is stretched 5 cm. At time $t=0$ it is released and oscillates.

What is the maximum speed of the block?

A) .45 m/s

B) .23 m/s

C) .14 m/s

$$E = U + K$$

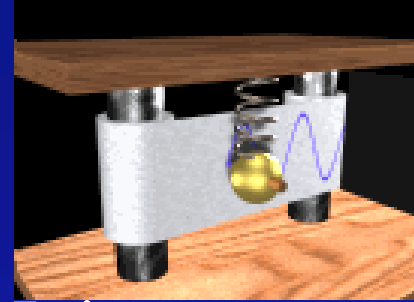
When $x = 0$, maximum speed:

$$E = \frac{1}{2} m v^2 + 0$$

$$.03 = \frac{1}{2} 3 \text{ kg } v^2$$

$$v = .14 \text{ m/s}$$

Example



A 3 kg mass is attached to a spring ($k=24 \text{ N/m}$). It is stretched 5 cm. At time $t=0$ it is released and oscillates.

How long does it take for the block to return to $x=+5\text{cm}$?

A) 1.4 s

B) 2.2 s

C) 3.5 s

$$\omega = \sqrt{k/m}$$

$$= \sqrt{24/3}$$

$$= 2.83 \text{ radians/sec}$$

Returns to original position after 2π radians

$$T = 2\pi / \omega = 6.28 / 2.83 = 2.2 \text{ seconds}$$

Summary

- Springs

- ➔ $F = -kx$

- ➔ $U = \frac{1}{2} k x^2$

- ➔ $\omega = \sqrt{k/m}$

- Simple Harmonic Motion

- ➔ Occurs when have linear restoring force $F = -kx$

- ➔ $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$

- ➔ $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$

- ➔ $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$

Young's Modulus

- Spring $F = -k x$ [demo]
 - What happens to “k” if cut spring in half?
 - A) decreases B) same C) increases
- k is inversely proportional to length!
- Define
 - Strain = $\Delta L / L$
 - Stress = F/A
- Now
 - Stress = Y Strain
 - $F/A = Y \Delta L/L$
 - $k = Y A/L$ from $F = k x$
- Y (Young's Modulus) independent of L