Exam I

Physics 101: Lecture 06
Two Dimensional Dynamics

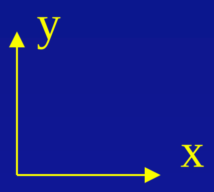
Today's lecture will cover Chapter 4

Brief Review Thus Far

- Newton's Laws of motion: $\Sigma F=ma$
- Kinematics: $x = x_0 + v_0 t + ½ a t^2$
- Dynamics

- Today we work in 2 Dimensions!

2-Dimensions



- X and Y are INDEPENDENT!
- Break 2-D problem into two 1-D problems

Position, Velocity and Acceleration

- Position, Velocity and Acceleration are Vectors!

$$\vec{v}_{av} = \frac{\vec{r}_f - \vec{r}_0}{t_f - t_0}$$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_0}{t_f - t_0}$$

x direction

$$v_x = \frac{x_f - x_0}{t_f - t_0}$$

$$a_x = \frac{v_{xf} - v_{x0}}{t_f - t_0}$$

y direction

$$v_y = \frac{y_f - y_0}{t_f - t_0}$$

$$a_y = \frac{v_{yf} - v_{y0}}{t_f - t_0}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

- x and y directions are INDEPENDENT!

Velocity in Two Dimensions

A ball is rolling on a horizontal surface at 5 m/s. It then rolls up a ramp at a 25 degree angle. After 0.5 seconds, the ball has slowed to 3 m/s.

What is the magnitude of the change in velocity?

A) 0 m/s B) 2 m/s C) 2.6 m/s D) 3 m/s E) 5 m/s

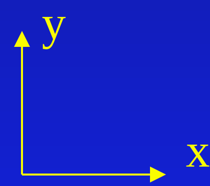
x-direction

$v_{ix} = 5$ m/s

$v_{fx} = 3$ m/s $\cos(25)$

$\Delta v_x = 3\cos(25) - 5 = -2.28$ m/s

y-direction

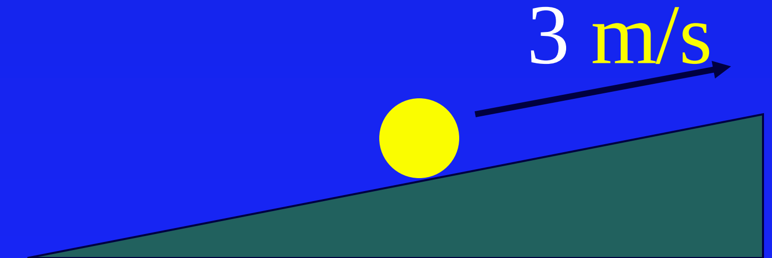


$v_{iy} = 0$ m/s

$v_{fy} = 3$ m/s $\sin(25)$

$\Delta v_y = 3\sin(25) = +1.27$ m/s

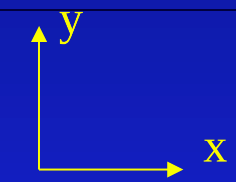
$$|\Delta v| = \sqrt{\Delta v_x^2 + \Delta v_y^2} = 2.6\text{ m/s}$$



Acceleration in Two Dimensions

A ball is rolling on a horizontal surface at 5 m/s. It then rolls up a ramp at a 25 degree angle. After 0.5 seconds, the ball has slowed to 3 m/s. What is the average acceleration?

x-direction	y-direction
$a_x = \frac{-2.28\text{m/s}}{0.5\,\text{s}} = -4.56\,\text{m/s}^2$	$a_y = \frac{1.27\text{m/s}}{0.5\,\text{s}} = 2.54\,\text{m/s}^2$



$$|a| = \sqrt{a_x^2 + a_y^2} = 5.21\,\text{m/s}^2$$

Kinematics in Two Dimensions

- $x = x_0 + v_{0x}t + 1/2\, a_x t^2$
- $v_x = v_{0x} + a_x t$
- $v_x^2 = v_{0x}^2 + 2a_x\, \Delta x$

- $y = y_0 + v_{0y}t + 1/2\, a_y t^2$
- $v_y = v_{0y} + a_y t$
- $v_y^2 = v_{0y}^2 + 2a_y\, \Delta y$

Must be able to identify variables in these equations!

x and y motions are independent!
They share a common time t

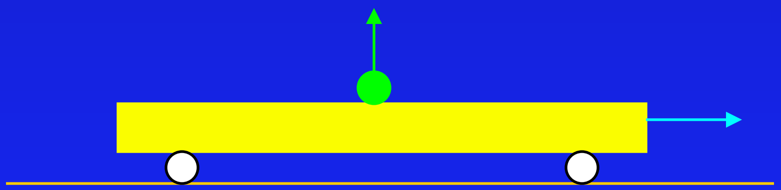
Train Act/Demo

A flatbed railroad car is moving along a track at constant velocity. A passenger at the center of the car throws a ball straight up. Neglecting air resistance, where will the ball land?

A. Forward of the center of the car

B. At the center of the car ← correct

C. Backward of the center of the car



- x- direction ball and car start with same position and velocity, a=0, so always have same position

Demo - train

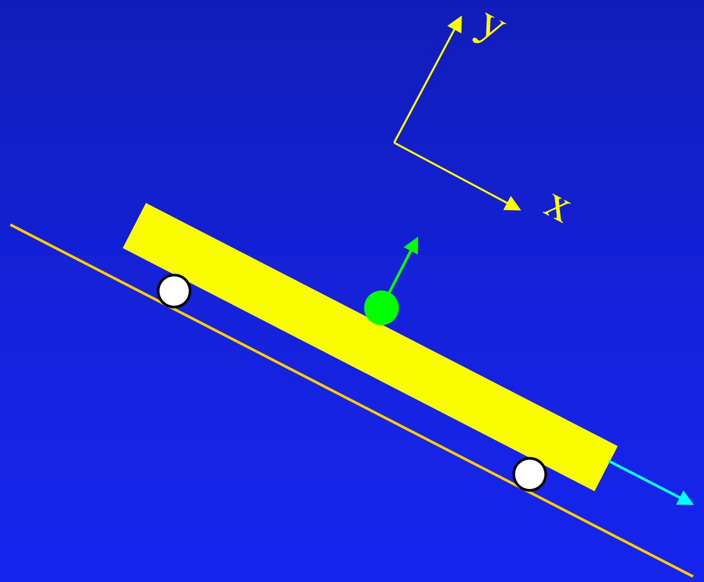
ACT

A flatbed railroad car is accelerating down a track due to gravity. The ball is shot perpendicular to the track. Where will it land?

A. Forward of the center of the car

B. At the center of the car ⟵ correct

C. Backward of the center of the car



x direction Ball

$$m_b g \sin(\theta) = m_b a_b$$

$$a_b = g \sin(\theta)$$

x direction Cart

$$m_c g \sin(\theta) = m_c a_c$$

$$a_c = g \sin(\theta)$$

Same acceleration gives same position

Projectile Motion ACT

One marble is given an initial horizontal velocity, the other simply dropped. Which marble hits the ground first?

A) dropped

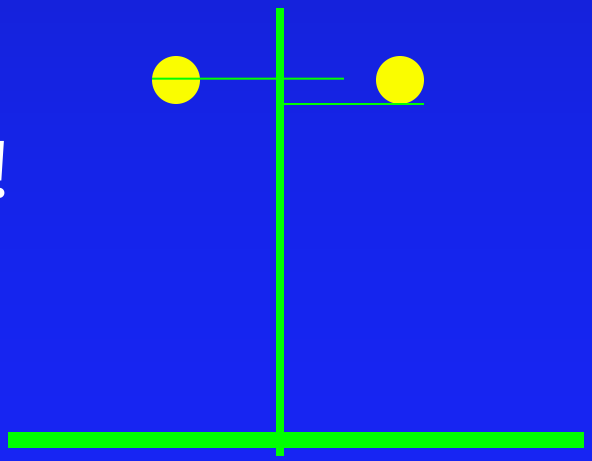
B) pushed

C) They both hit the ground at the same time

When ball hits depends on y only!

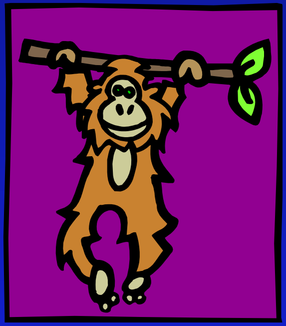
$y(t) = y_0 + v_{yo} + \frac{1}{2} a_y t$

Same for both balls!



Monkey Pre-Flight

You are a vet trying to shoot a tranquilizer dart into a monkey hanging from a branch in a distant tree. You know that the monkey is very nervous, and will let go of the branch and start to fall as soon as your gun goes off. In order to hit the monkey with the dart, where should you point the gun before shooting?



74% 1 Right at the monkey ← correct

14% 2 Below the monkey

12% 3 Above the monkey

Demo - monkey

"Poor monkey...I hope there's something soft for the little guy to land on."

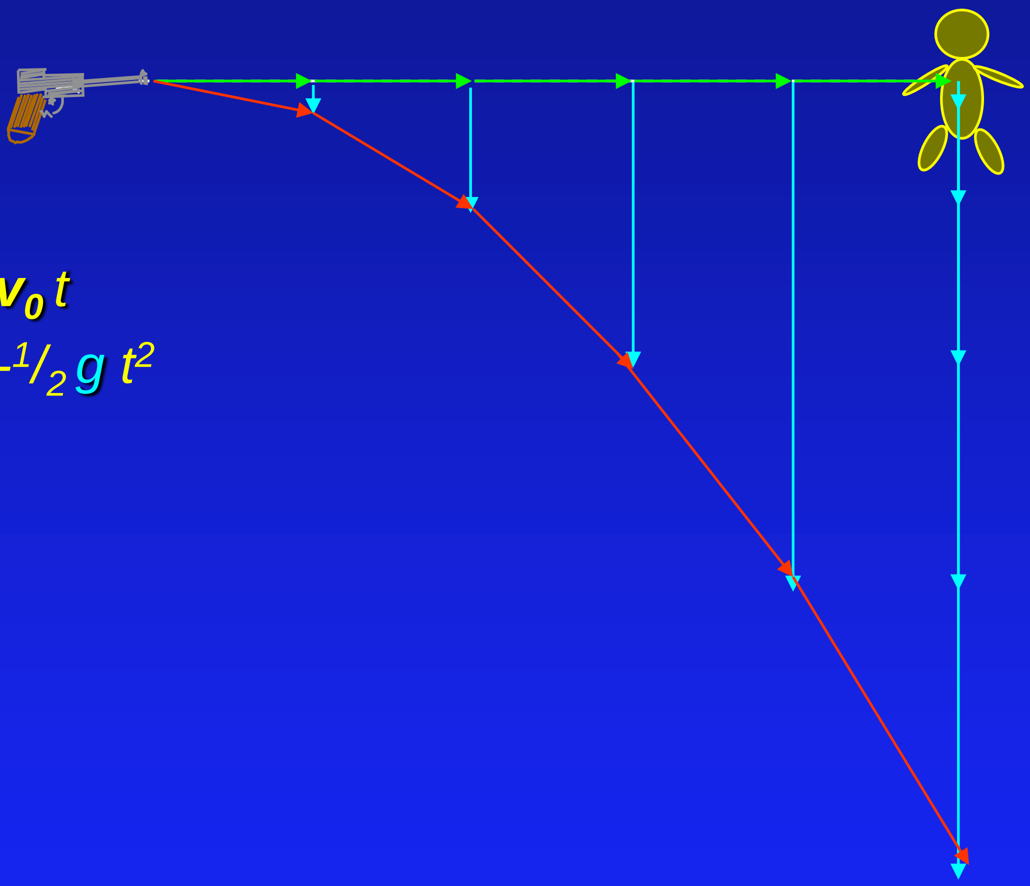
NO ANIMALS WERE HARMED IN THE MAKING OF THIS DEMO

Most intriguing answer

"The jungle night is hot, but I have waited too long to miss an opportunity like this one. The rare Bare-eared Squirrel Monkey, indigenous only to Brazil, was in my sights. This monkey could lead to mankind's survival and it was all up to me to capture it. This simple creature, hanging from a branch, was the key, the key to unlock the cure. Being the well-trained vet I am, I quickly realized how skittish these monkeys are...meaning I'd only have one shot...one shot to save the world. I lined up my dart gun and, thanks to my p.h.D in physics, did a quick calculation to judge how quickly the dart would fall and how quickly the monkey will fall. Based on the weight and reaction time of the monkey and the power of my gun, I knew that the dart and monkey will drop at the same rate. I thus aimed my weapon right at the animal and fired! At the sound of the shot the little guy dropped from his branch. I watched as the dart met its target dead on. I had done it. With this creature in hand, I had the tools to save the world. The only question that remained was if I was already to late..."

See text: 4-3

Shooting the Monkey...



$x = \mathbf{v_0}\, t$

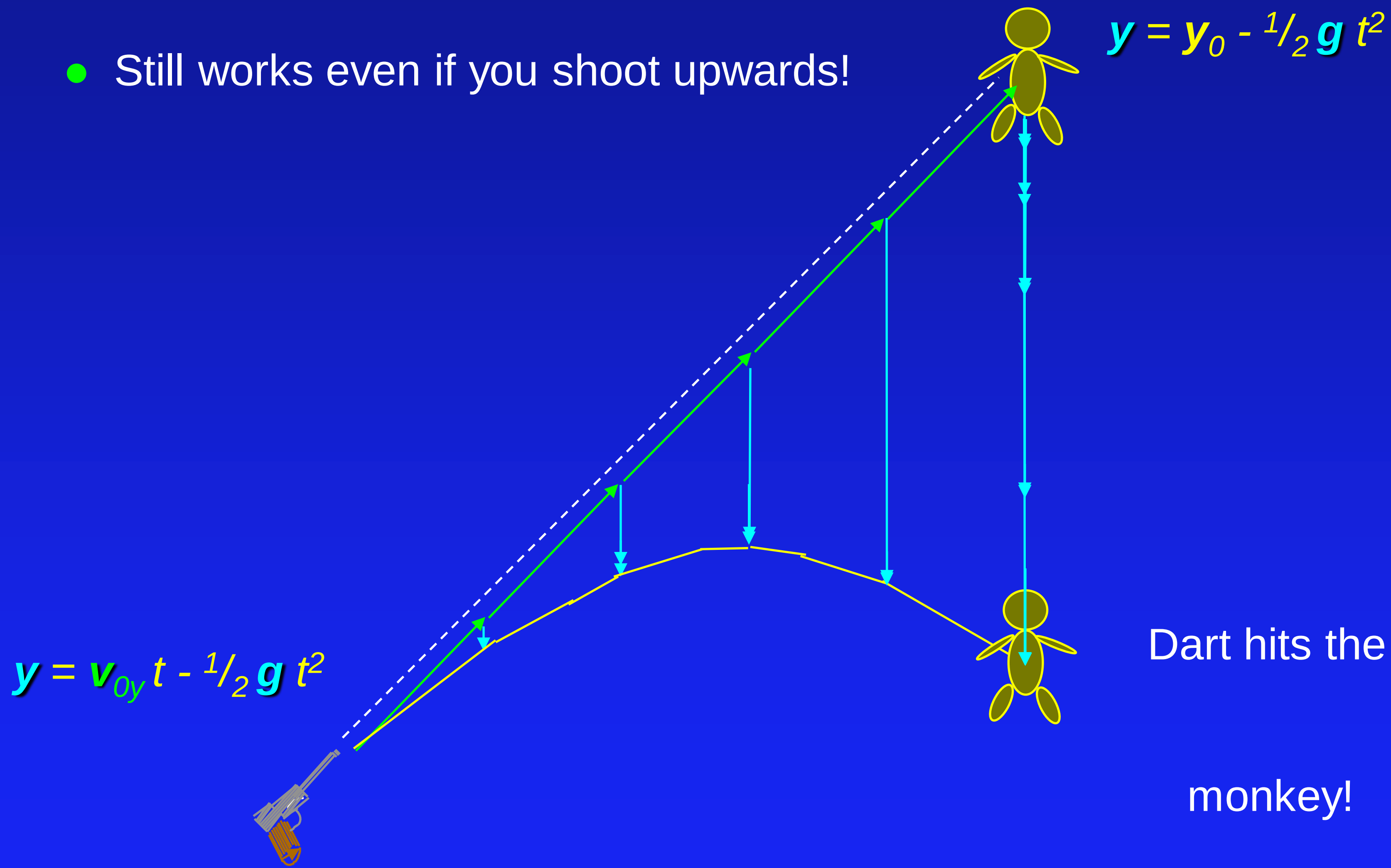
$y = -{}^1/_2\, g\, t^2$

$x = x_0$

$y = -{}^1/_2\, g\, t^2$

Shooting the Monkey...

- Still works even if you shoot upwards!



Projectile Motion

$a_x = 0$ $a_y = -g$

- $x = x_0 + v_{0x} t$
- $v_x = v_{0x}$

- $y = y_0 + v_{0y} t - \frac{1}{2} g t^2$
- $v_y = v_{0y} - g t$
- $v_y^2 = v_{0y}^2 - 2 g \Delta y$

- Choose direction where you "know" information
- Solve kinematics in that direction.
- Use *t* from that direction as *t* in other direction

Throw ball to Monkey

You throw a ball to a monkey who is on a platform 12 meters above and 5 meters to the right of you. Determine the speed and angle you should throw it such that it "just reaches" the monkey.

Y-direction (v_{iy})

$v_{fy}^2 - v_{iy}^2 = 2\ a\ \Delta y$

$v_{iy} = \text{sqrt}(2\ 9.8\ 12)$

$= 15.3$ m/s

$v_{fy} = v_{iy} + a\ t$

$t = v_{iy}/g = 1.56$ s.

X-direction (v_{ix})

$v_{ix} = d/t$

$= 5$ m / 1.56 s

$= 3.2$ m/s

Throw ball to Monkey

You throw a ball to a monkey who is on a platform 12 meters above and 5 meters to the right of you. Determine the speed and angle you should throw it such that it "just reaches" the monkey.

Y-direction (v_{iy})

$v_{fy}^2 - v_{iy}^2 = 2\,a\,\Delta y$

$v_{iy} = \text{sqrt}(2*9.8*12)$

$= 15.3$ m/s

$v_{fy} = v_{iy} + a\,t$

$t = v_{iy}/g = 1.56$ s.

X-direction (v_{ix})

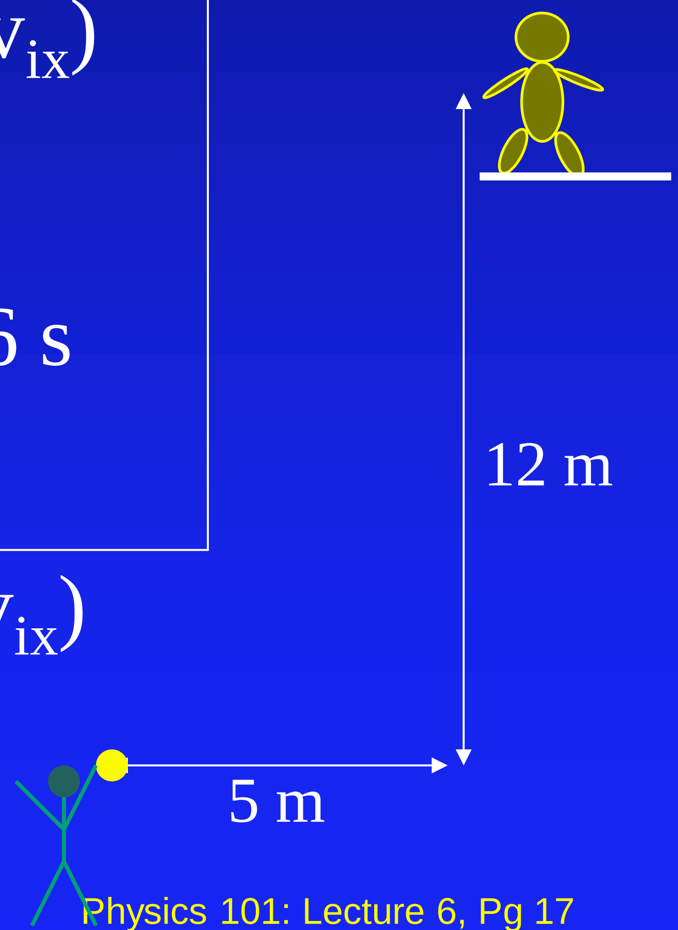
$v_{ix} = d/t$

$= 5$ m / 1.56 s

$= 3.2$ m/s

$\alpha = \arctan(v_{iy}/v_{ix})$

$|v| = (v_{ix}^2 + v_{iy}^2)^{1/2}$



Summary of Concepts

- X and Y directions are Independent!
  - ➔ Position, velocity and acceleration are vectors
  - ➔ "Share" t
- F = m a applies in both x and y direction
- Projectile Motion
  - ➔ $a_x = 0$ in horizontal direction
  - ➔ $a_y = g$ in vertical direction