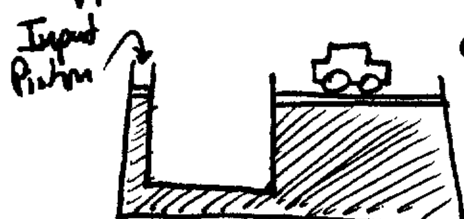
Chapter 9 - Fluids

A useful quantity to describe the forces a fluid exerts on the walls of its container is PRESSURE:

$$P = F/A$$

<u>Pascal's Principle</u> states that any pressure applied to a fluid is equally transmitted everywhere.

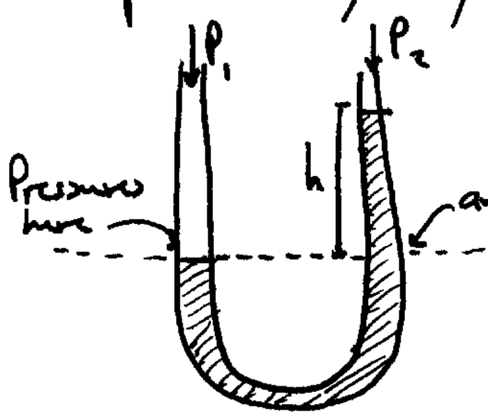
Important applications of this are, for example, pneumatic lifts:



Pascal's Principle says that any pressure applied to the input piston will be transmitted to the output piston:

$$P_{input} = P_{output}$$

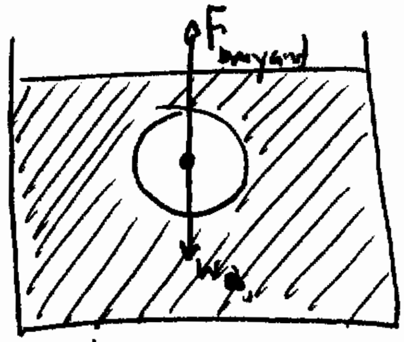
Another important consequence of this is that the pressure exerted by a fluid depends only only the depth within the fluid:



are equal hence $P_1 = P_2 + \rho g h$

Another type of fluid problem you'll encounter is Archimedes's Principle: the magnitude of the buoyant force a fluid exerts on a submerged object is equal to the weight of the displaced fluid, i.e.:

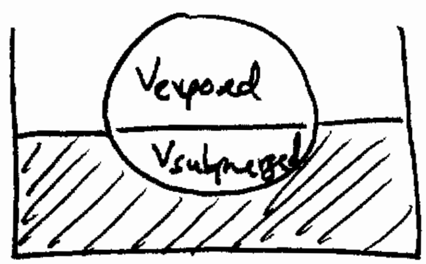
$$F_{buoyant} = W_{displaced\,fluid} = \rho_{fluid}\, g\, V_{displaced}$$



A consequence of this is that objects whose densities are greater than water will sink, whereas objects whose densities are less than water will float!

In particular, the submerged volume of a floating object is determined by its density:

$$\frac{\rho_{object}}{\rho_{H_2O}} = \frac{V_{submerged}}{V_{object}}$$

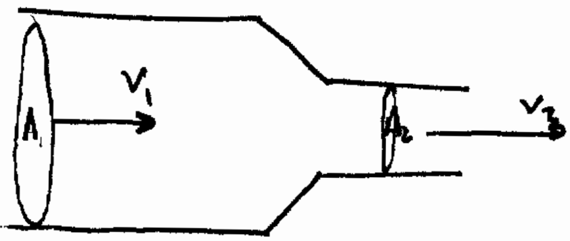


$V_{object} = V_{submerged} + V_{exposed}$

e.g. if an object's density is 50% of water's density, 50% of the object's volume will be submerged!

Finally, you've got to worry about flowing fluids.

The Volume flow rate is



$$VFR = Av$$

where A is the cross-sectional area of the pipe, and v is the speed of the fluid.

The Mass flow rate is

$$MFR = VFR \cdot \rho = \rho A v$$

The two primary equations that describe fluid flow are the Continuity Equation, which just says that everything that goes in has got to come out:

$$A_1 v_1 = A_2 v_2$$

The other is the Bernoulli Equation, which is a statement that energy is conserved:

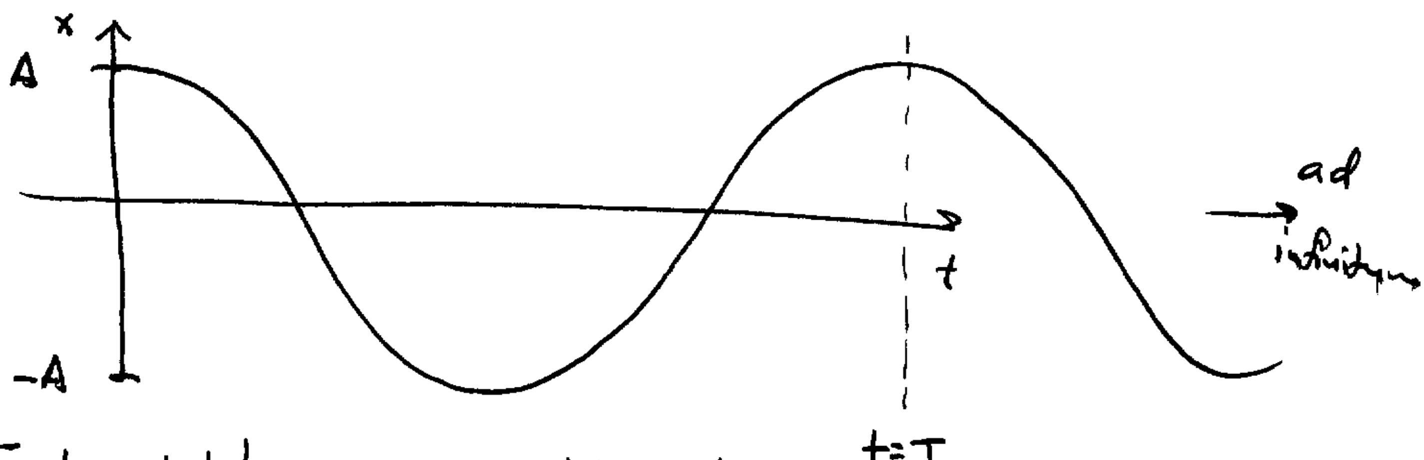
$$\underbrace{P_1}_{\text{work done on fluid}} + \underbrace{\tfrac{1}{2}\rho v_1^2}_{\text{kinetic energy of fluid}} + \underbrace{\rho g h_1}_{\text{Potential energy of fluid}} = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

A consequence of the Bernoulli Eq. is that faster flowing fluids exert less pressure; this is how airplanes fly!

Chapter 10 - Simple Harmonic Motion

Simple Harmonic Motion (SHM) is any motion that repeats itself indefinitely, like that of a frictionless spring or a pendulum.

In general, it will look like this:



The time it takes to repeat itself is the PERIOD, T. The inverse of the period is the frequency, f:

$$f = \frac{1}{T}$$

Another useful variable is the angular frequency, ω:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

The position, velocity, and acceleration of SHM is described by trig functions:

Amplitude

$$x = A\cos(\omega t)$$

$$v = -v_{max}\sin(\omega t) \qquad v_{max} = A\omega$$

$$a = -a_{max}\cos(\omega t) \qquad a_{max} = A\omega^2$$

The two types of simple harmonic oscillators you will encounter are springs and pendula.

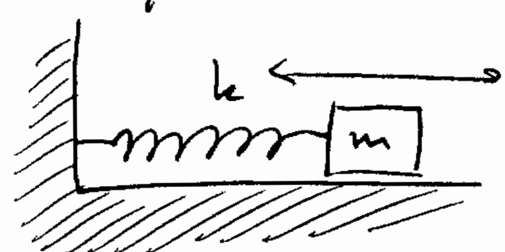
Springs

When a spring is stretched or compressed by an amount x, the spring will exert a restoring force in the direction opposite the displacement and with magnitude proportional to how far it is stretched (x) and how rigid the spring is (k, the spring constant):

$$\vec{F} = -k\vec{x} \qquad \text{Hooke's Law}$$

As a result, when you stretch and release a spring it will undergo simple harmonic motion with angular frequency

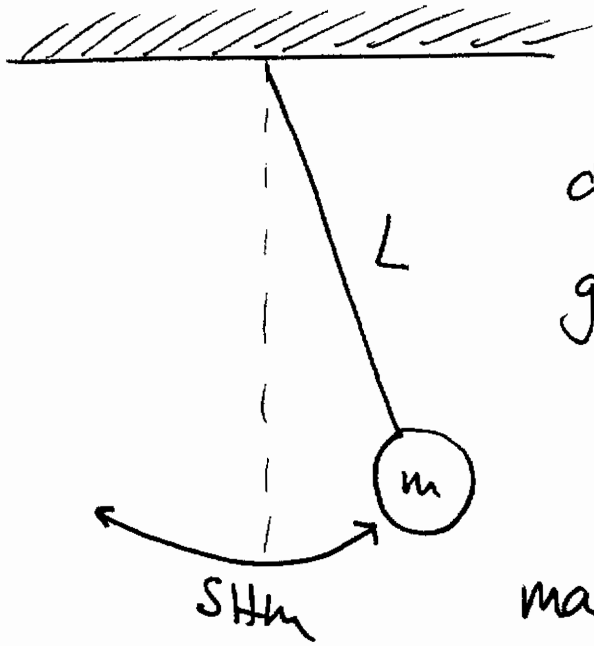
$$\omega = \sqrt{\frac{k}{m}}$$



When stretched, a spring stores potential energy; the amount of energy stored in a spring is

$$U = \frac{1}{2}kx^2$$

You'll also encounter pendula:



The angular frequency of a pendulum depends only upon the length, L, and gravitational acceleration, g:

$$\omega = \sqrt{\frac{g}{L}}$$

mass doesn't matter at all!

Don't forget: you'll calculate ω using either $\sqrt{\frac{k}{m}}$ (spring) or $\sqrt{\frac{g}{L}}$ (pendulum) ... this allows you to get the period as well!

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \quad \text{pendulum}$$

$$= 2\pi\sqrt{\frac{m}{k}} \quad \text{spring}$$

Chapter 11 - Waves

We'll mainly talk about two kinds of waves: waves in strings and sound.

The speed of a wave is related to its frequency and wavelength:

$$\boxed{v = \lambda f}$$ REMBER ME!

For a wave in a string, this is also determined by the tension in the string, T, and the string's mass density, $\mu = m/L$:

$$\boxed{v_{string} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}}$$

When you send waves down a string, they will hit the other end and reflect back. What happens depends on how the other end is attached.

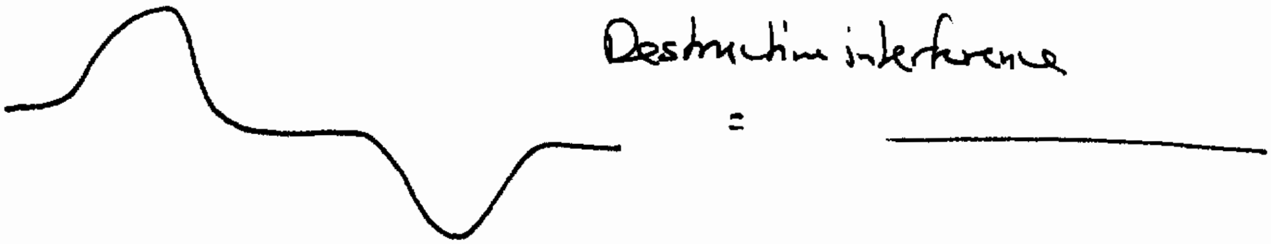
If the other end is <u>FIXED</u>, the reflected wave will be <u>INVERTED</u>

If the other end is <u>FREE</u>, the reflected wave will be <u>UPRIGHT</u>.

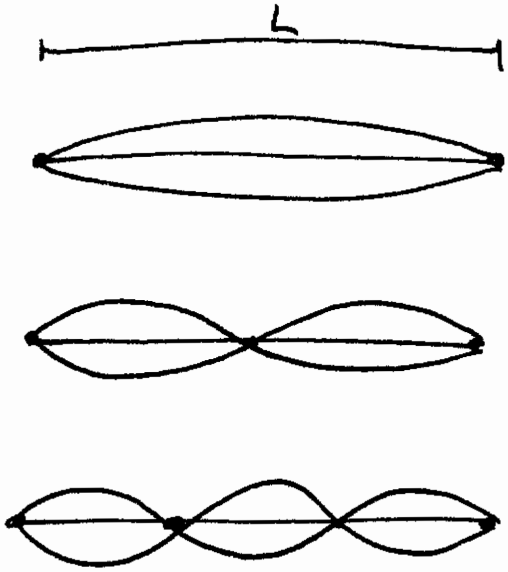
Multiple waves on a string will <u>interfere</u> with each other, and the result can be determined by <u>Superposition</u>, i.e., you add up the individual waves to get the result:

constructive interference

Destructive interference



If you keep sending wave pulses down a string at the correct frequencies, you will get STANDING WAVES:



$n=1$ $\lambda = 2L$ "Fundamental"

$n=2$ $\lambda = L$

$n=3$ $\lambda = \frac{2L}{3}$

$\vdots$

In general,

$$\lambda_n = \frac{2L}{n}$$

Don't worry about pipes for this class!

Chapter 12 - Sound

Sound is just another kind of wave, but because we use it every day to hear, we spend a little more time on it!

We quantify the "strength" of sound using two variables:

Intensity, I : corresponds to the amount of Energy in the sound.

Remember that the intensity increases with the square of the amplitude and decreases with the square of the distance:

$$I \propto \frac{A^2}{D^2}$$

Loudness, β : corresponds to how "loud" a sound seems to our ears; units are decibels.

Loudness is related to Intensity by:

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right) \quad ; \quad I_0 = 10^{-12}\, W/m^2$$

Remember: Intensities Add, Loudness does NOT!

The total intensity of 3 objects emitting sound with intensities I_1, I_2 and I_3 is

$$I_{tot} = I_1 + I_2 + I_3$$

But loudness is $\beta_{tot} = 10 \log_{10}\left(\frac{I_1 + I_2 + I_3}{I_0}\right) \neq \beta_1 + \beta_2 + \beta_3$

The other kind of sound problem you gotta worry about is the Doppler EFFECT: The perceived frequency of sound depends upon the relative state of motion between source and observer:

$$f_{obs} = f_{source} \frac{v_{wave} \pm v_{obs}}{v_{wave} \mp v_{source}}$$

top sign for towards, bottom sign for away

If source and observer are approaching each other, the perceived frequency, f_{obs}, should be greater than f_{source}.

If source and observer are separating, the perceived frequency f_{obs}, should be LESS than f_{source}.

Chapter 13 - Temperature and Ideal Gases

We use 3 temperature scales; Celsius, Fahrenheit and Kelvin:

$$K = C + 273.15$$
$$= \frac{5}{9}(F + 459.67)$$

ALWAYS USE KELVIN!!

An increase in temperature corresponds to increased molecular motion; consequently, objects will tend to expand:

Linear $\quad \Delta L = \alpha L_0 (T_f - T_0)$ $\quad$ use T in KELVIN!

Areal $\quad \Delta A = \gamma A_0 (T_f - T_0)$

Volume $\quad \Delta V = \beta V_0 (T_f - T_0)$

The constants, α, β, and γ, depend upon the material, and are related

$$\gamma = 2\alpha$$
$$\beta = 3\alpha$$

Remember that if you have a hole in an object, the hole will expand as if it were also made out of the same material!

Gases will also expand when heated; this is described by the <u>Ideal Gas Law</u>:

$$PV = nRT$$
$$= Nk_BT$$

where n = number of moles of gas, and N = number of molecules of gas. R is the ideal gas constant (8.314 J/mol K) whereas k_B is the Boltzmann constant ($k = R/N_A = 1.3806 \times 10^{-23}$ J/K).

Finally, we've got the <u>kinetic molecular theory of gases</u>, which states that the average kinetic energy of gas molecules is determined by the temperature:

$$\langle K \rangle = \frac{3}{2} k_B T$$

Now, kinetic energy is related to speed by:

$$K = \frac{1}{2} m v^2$$

Hence the <u>mean-squared speed</u> is

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\Rightarrow \langle v^2 \rangle = \frac{3 k_B T}{m}$$

or the root-mean-squared speed is

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$