

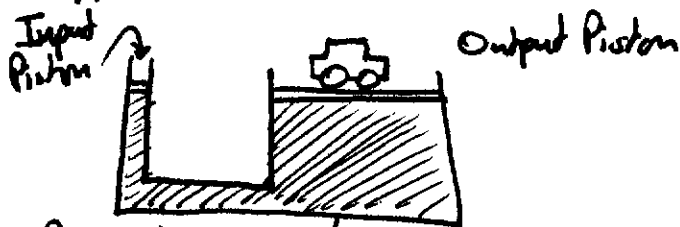
Chapter 9 - Fluids

A useful quantity to describe the force a fluid exerts on the walls of its container is **PRESSURE**:

$$P = F/A$$

Pascal's Principle states that any pressure applied to a fluid is equally transmitted everywhere.

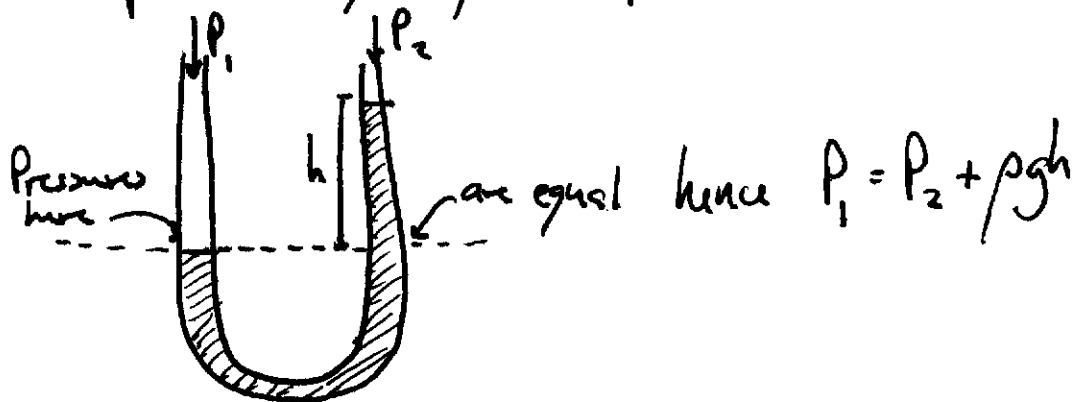
Important applications of this are, for example, pneumatic lifts:



Pascal's Principle says that any pressure applied to the input piston will be transmitted to the output piston:

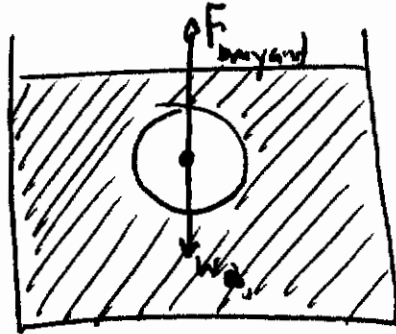
$$P_{\text{input}} = P_{\text{output}}$$

Another important consequence of this is that the pressure exerted by a fluid depends only on the depth within the fluid:



Another type of fluid problem you'll encounter is Archimede's Principle:
the magnitude of the buoyant force a fluid exerts on a submerged object is equal to the weight of the displaced fluid, i.e.:

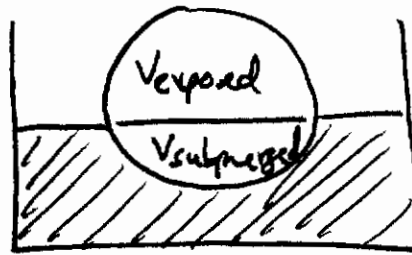
$$F_{\text{buoyant}} = W_{\text{displaced fluid}} = \rho_{\text{fluid}} g V_{\text{displaced}}$$



A consequence of this is that objects whose densities are greater than water will sink, whereas objects whose densities are less than water will float!

In particular, the submerged volume of a floating object is determined by its density:

$$\frac{\rho_{\text{object}}}{\rho_{\text{H}_2\text{O}}} = \frac{V_{\text{submerged}}}{V_{\text{object}}}$$



$$V_{\text{object}} = V_{\text{submerged}} + V_{\text{exposed}}$$

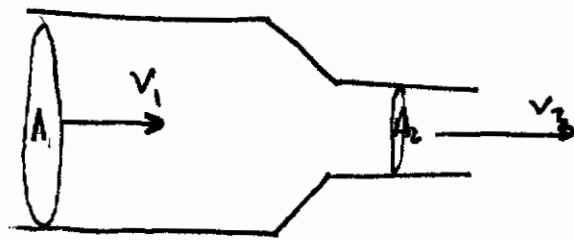
e.g. if an object's density is 50% of water's density, 50% of the object's volume will be submerged!

Finally, you've got to worry about Flowing Fluids.

The Volume Flow rate is

$$VFR = Av$$

where A is the cross-sectional area of the pipe, and v is the speed of the fluid.



The Mass Flow rate is

$$MFR = VFR \cdot \rho = \rho Av$$

The two primary equations that describe fluid flow are the Continuity Equation, which just says that everything that goes in has got to come out:

$$A_1 v_1 = A_2 v_2$$

The other is the Bernoulli Equation, which is a statement that energy is conserved:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

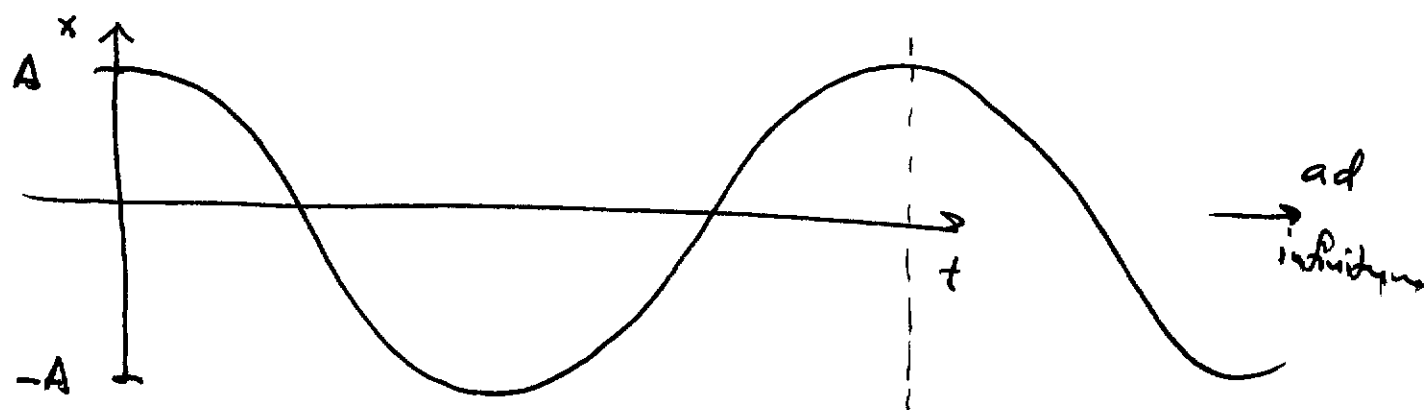
\nwarrow work done on fluid \nearrow kinetic energy of fluid \nearrow Potential energy of fluid

A consequence of the Bernoulli Eq. is that faster flowing fluids exert less pressure; this is how airplanes fly!

Chapter 10 - Simple Harmonic Motion

Simple Harmonic Motion ^(SHM) is any motion that repeats itself indefinitely, like that of a frictionless spring or a pendulum.

In general, it will look like this:



The time it takes to repeat itself is the PERIOD, T . The inverse of the period is the frequency, f :

$$f = \frac{1}{T}$$

Another useful variable is the angular frequency, ω :

$$\omega = 2\pi f = \frac{2\pi}{T}$$

The position, velocity, and acceleration of SHM is described by trig functions:

↙ Amplitude
 $x = A \cos(\omega t)$

$$v = -v_{\max} \sin(\omega t)$$

$$a = -a_{\max} \cos(\omega t)$$

$$v_{\max} = A\omega$$

$$a_{\max} = A\omega^2$$

The two types of Simple harmonic oscillators you will encounter are springs and pendula.

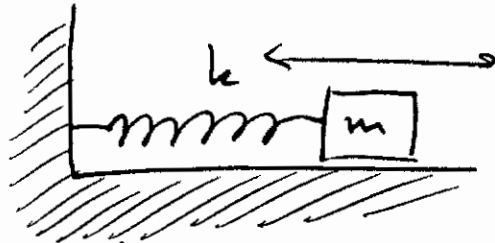
Springs

When a spring is stretched or compressed by an amount x , the spring will exert a restoring force in the direction opposite the displacement and with magnitude proportional to how far it is stretched (x) and how rigid the spring is (k , the spring constant):

$$\vec{F} = -k\vec{x} \quad \text{Hooke's Law}$$

As a result, when you stretch and release a spring it will undergo simple harmonic motion with angular frequency

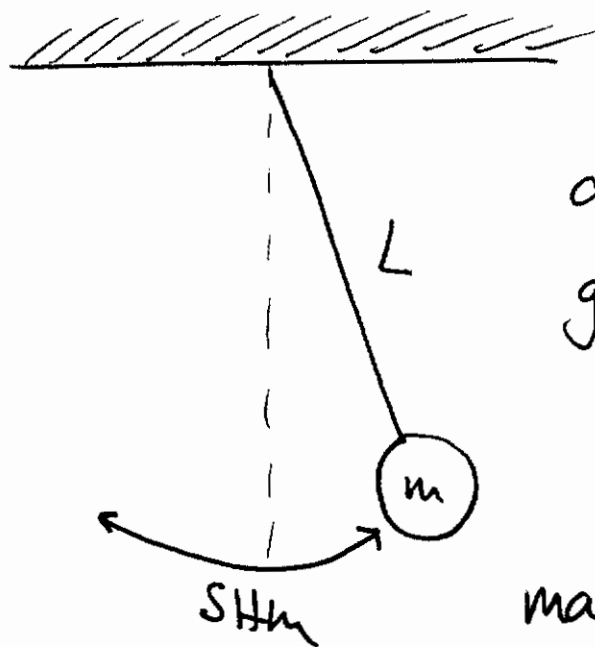
$$\omega = \sqrt{\frac{k}{m}}$$



When stretched, a spring stores potential energy; the amount of energy stored in a spring is

$$U = \frac{1}{2} k x^2$$

You'll also encounter pendula:



The angular frequency of a pendulum depends only upon the length, L , and gravitational acceleration, g :

$$\omega = \sqrt{\frac{g}{L}}$$

mass doesn't matter at all!

Don't forget: you'll calculate ω using either $\sqrt{\frac{k}{m}}$ (spring) or $\sqrt{\frac{g}{L}}$ (pendulum) ... this allows you to get the period as well!

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}} \quad \text{pendulum}$$

$$= 2\pi \sqrt{\frac{m}{k}} \quad \text{spring}$$

Chapter 11 - Waves

We'll mainly talk about two kinds of waves: waves in strings and sound.

The speed of a wave is related to its frequency and wavelength:

$$\boxed{v = \lambda f} \quad \text{REMEMBER ME!}$$

For a wave in a string, this is also determined by the tension in the string, T , and the string's mass density, $\mu = m/L$:

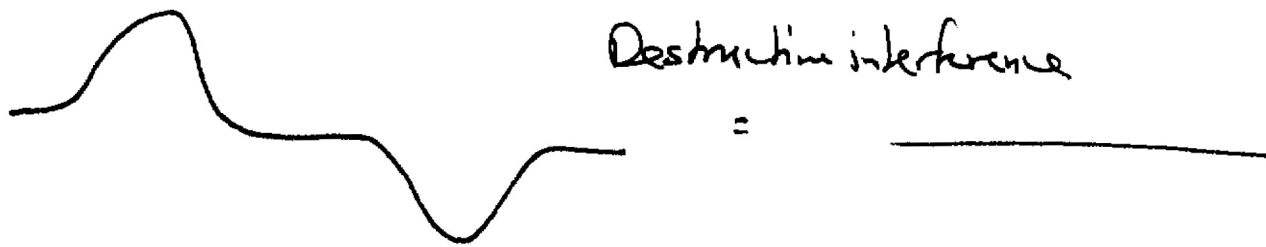
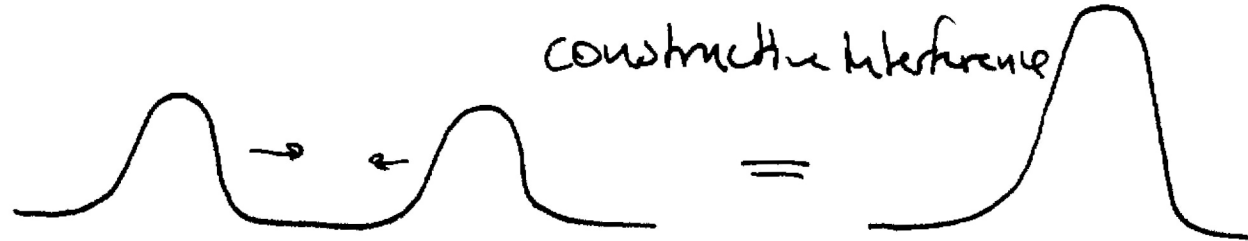
$$\boxed{v_{\text{string}} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}}$$

When you send waves down a string, they will hit the other end and reflect back. What happens depends on how the other end is attached.

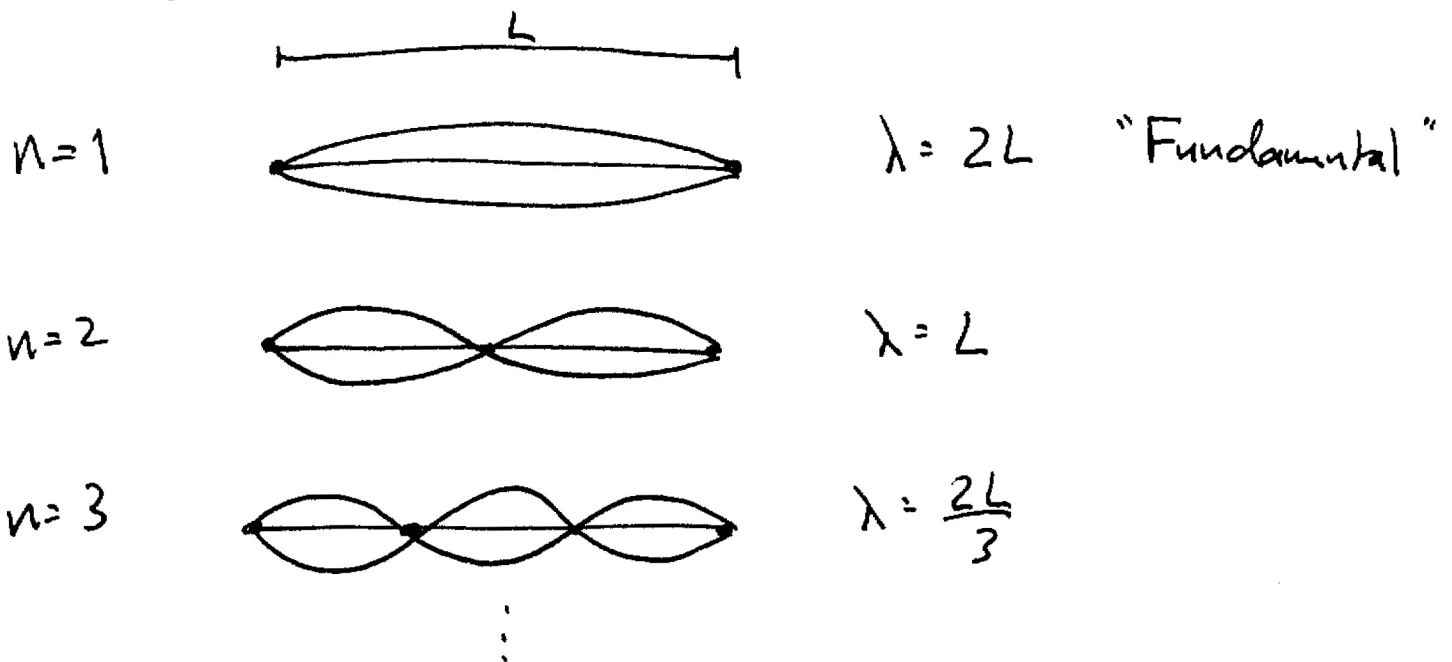
If the other end is FIXED, the reflected wave will be INVERTED.

If the other end is FREE, the reflected wave will be UPRIGHT.

Multiple waves on a string will interfere with each other, and the result can be determined by superposition, i.e., you add up the individual waves to get the result:



If you keep sending wave pulses down a string at the correct frequencies, you will get STANDING WAVES.



In general,

$$\lambda_n = \frac{2L}{n}$$

Chapter 12 - Sound

Sound is just another kind of wave, but because we use it every day to hear, we spend a little more time on it!

We quantify the "strength" of sound using two variables:

Intensity, I : corresponds to the amount of Energy in the sound.

Remember that the intensity increases with the square of the amplitude and decreases with the square of the distance:

$$I \propto \frac{A^2}{D^2}$$

Loudness, β : corresponds to how "loud" a sound seems to our ears; units are decibels.

Loudness is related to Intensity by:

$$\beta = 10 \log_{10} \left(\frac{I}{I_0} \right) \quad ; \quad I_0 = 10^{-12} \text{ W/m}^2$$

Remember : Intensities Add, Loudness does NOT!

The total intensity of 3 objects emitting sound with intensities I_1 , I_2 and I_3 is

$$I_{\text{tot}} = I_1 + I_2 + I_3$$

But Loudness is $\beta_{\text{tot}} = 10 \log_{10} \left(\frac{I_1 + I_2 + I_3}{I_0} \right) \neq \beta_1 + \beta_2 + \beta_3$ (9)

The other kind of sound problem you gotta worry about is the Doppler EFFECT: The perceived frequency of sound depends upon the relative state of motion between source and observer:

$$f_{\text{obs}} = f_{\text{source}} \frac{v_{\text{wave}} \pm v_{\text{obs}}}{v_{\text{wave}} \mp v_{\text{source}}}$$

top sign for towards,
bottom sign for away

If source and observer are approaching each other, the perceived frequency, f_{obs} , should be greater than f_{source} .

If source and observer are separating, the perceived frequency f_{obs} , should be LESS than f_{source} .

Chapter 13 - Temperature and Ideal Gases

We use 3 temperature scales; Celsius, Fahrenheit and Kelvin:

$$K = C + 273.15$$
$$= \frac{5}{9}(F + 459.67)$$

ALWAYS USE KELVIN!!

~~The~~ An increase in temperature corresponds to increased molecular motion; consequently, objects will tend to expand:

Linear $\Delta L = \alpha L_0 (T_f - T_0)$ USE T in KELVIN!

Area $\Delta A = \gamma A_0 (T_f - T_0)$

Volume $\Delta V = \beta V_0 (T_f - T_0)$

The constants, α , β , and γ , depend upon the material, and are related

$$\gamma = 2\alpha$$

$$\beta = 3\alpha$$

Remember that if you have a hole in an object, the hole will expand as if it were also made out of the same material!

Gases will also expand when heated; this is described by the Ideal Gas Law:

$$PV = nRT$$
$$= Nk_B T$$

Where n = number of moles of gas, and N = number of molecules of gas. R is the ideal gas constant (8.314 J/mol K) whereas k_B is the Boltzmann constant ($k = R/N_A = 1.3806 \times 10^{-23} \text{ J/K}$).

Finally, we've got the kinetic molecular theory of gases, which states that the average kinetic energy of gas molecules is determined by the temperature:

$$\langle K \rangle = \frac{3}{2} k_B T$$

Now, kinetic energy is related to speed by:

$$K = \frac{1}{2} m v^2$$

Hence the mean-squared speed is

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\Rightarrow \langle v^2 \rangle = \frac{3 k_B T}{m}$$

or the root-mean-squared speed is

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$