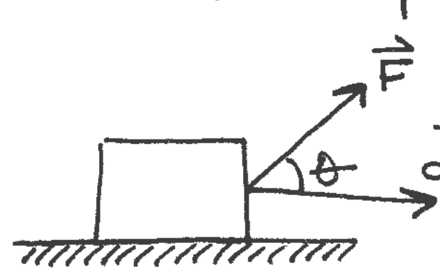
Work and Energy

"Work" is the concept that makes the connection between doing physics with forces and vectors and energy and scalars.

If you apply a force $\vec{F}$ to an object and cause a displacement $\vec{d}$, the amount of work done is



$$\boxed{W = |\vec{F}||\vec{d}|\cos\theta}$$

By doing work on an object, you cause it to change speed (accelerate!), i.e. you change its KINETIC ENERGY, K:

$$\boxed{W = \Delta K} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \qquad (1)$$

Similarly, if a conservative force (i.e. not friction!) does work on an object, you can define a Potential Energy, U, that describes that conservative force:

$$W = -\Delta U \qquad \text{For example } \boxed{U_{grav} = mgh} \qquad (2)$$

Combining (1) and (2), you see that:

$$\Delta K = -\Delta U$$

i.e. $\boxed{\underset{\text{Total Energy!}}{\Delta E} = \Delta K + \Delta U = 0}$ Energy is conserved!

If friction is present, energy is not conserved, and the change in Energy is the work done by friction:

$$\boxed{W_{friction} = \Delta E}$$

Work and Energy (continued)

You can use energy conservation to solve problems more easily with scalars!

In general, your problems will look like this:

① Start — Calculate initial total Energy

$E_0 = K_0 + U_0$

→ ② Something Happens! (roll down an incline, fall, or similar)

→ ③ End — Calculate Final total Energy

$E_f = K_f + U_f$

Energy conservation means that

$$E_0 = E_f$$

$$\frac{1}{2}mv_0^2 + U_0 = \frac{1}{2}mv_f^2 + U_f$$

Example type of problem: Dropping a ball, pendulum, sliding/rolling down inclined plane.

Momentum

Another useful conservation law is <u>Conservation of Momentum</u>.

Momentum is defined as

$$\boxed{\vec{p} = m\vec{v}}$$

Newton's 2^{nd} Law is more accurately written in terms of momentum:

$$\boxed{\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}}$$

i.e. application of a force $\vec{F}$ to an object for a time Δt will change that object's momentum:

$$\boxed{\Delta\vec{p} = \vec{F}_{net}\Delta t = \text{Impulse}}$$

If objects only interact with each other and there is no net external force applied to the system:

$$\Delta\vec{p} = F_{net}\Delta t = 0$$

no net force!

i.e. momentum does not change; it is conserved!

$$\boxed{\vec{p}_0 = \vec{p}_f} \qquad \text{Momentum Conservation}$$

You can use this in collisions and explosions, where kinetic energy is not necessarily conserved due to energy lost to sound, heat, etc.

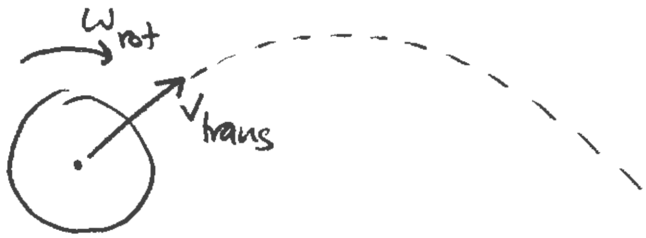
When energy is not conserved, it is an <u>inelastic collision</u>

If energy is conserved, it is an <u>elastic collision</u>

You can only treat a collision as elastic if you are <u>explicitly told it is elastic</u>!

Torque and Rotation

When analyzing the motion of solid objects, they can undergo two types of motion: translational and rotational



There is energy associated with rotation, since it is a kind of motion; this is Rotational Kinetic Energy:

$$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$$

where I is the "moment of inertia" or "rotational inertia"

$$I = \sum_i m_i r_i^2$$

which is a measure of how hard it is to make something rotate (just like mass is a measure of how hard it is to make something translate!)

In general, an object will be both translating and rotating, and so its kinetic energy will be:

$$\boxed{K = K_{trans} + K_{rot} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2}$$

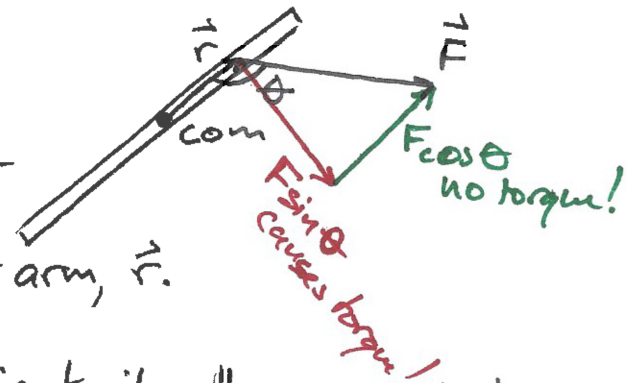
If it is rolling without slipping, v and ω are related!

$$\boxed{\begin{aligned} v &= r\omega \\ a &= r\alpha \end{aligned}}$$

Torque and Rotation (continued)

Just as Forces cause translational acceleration, rotational acceleration is caused by Torques, τ:

$$\tau = |\vec{F}||\vec{r}|\sin\theta$$



Torque is only caused by the component of the force perpendicular to the lever arm, $\vec{r}$.

When a net torque is applied to an object, it will cause it to rotate according to Newton's 2nd Law:

$$\boxed{\tau = I\alpha}$$

torque — moment of inertia — angular acceleration $\alpha = \frac{a}{R}$

Note that this looks exactly like Newton's 2nd Law for translation, just with angular variables! This is true in general: you can get the rotational equations from the translational ones by making the substitutions:

Translation		Rotation	
m	$\longrightarrow$	I	Inertia
x	$\longrightarrow$	θ	Displacement
v	$\longrightarrow$	ω	velocity
α	$\longrightarrow$	a	acceleration
F	$\longrightarrow$	τ	Force / torque
p	$\longrightarrow$	L	Momentum

Torque and Rotation (continued)

Everything we learned about translation motion is true about rotational motion!

Torques do work!

$$\boxed{W = \tau \Delta\theta} \qquad \text{(compare to } W = F\Delta x\text{)}$$

$$= \tfrac{1}{2} I \omega_f^2 - \tfrac{1}{2} I \omega_0^2 = \Delta K_{rot}$$

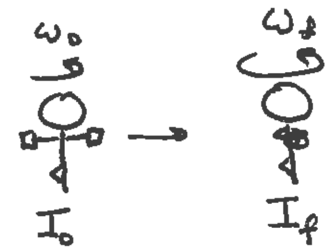
Angular Momentum is Conserved!

$$\tau_{net} = \frac{\Delta L}{\Delta t} = 0 \text{ if } \tau_{net} = 0$$

Hence

$$L_0 = L_f \quad \text{where} \quad L = I\omega$$

E.g.

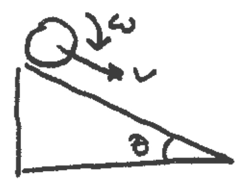


Rotational Energy is Conserved!

$$E_f = E_0$$

$$\tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega_f^2 + U_f = \tfrac{1}{2} m v_0^2 + \tfrac{1}{2} I \omega_0^2 + U_0$$

E.g.



If objects are in Equilibrium they must be in BOTH translational and rotational Equilibrium:

$$F_{net} = 0$$

$$\tau_{net} = 0$$

E.g.