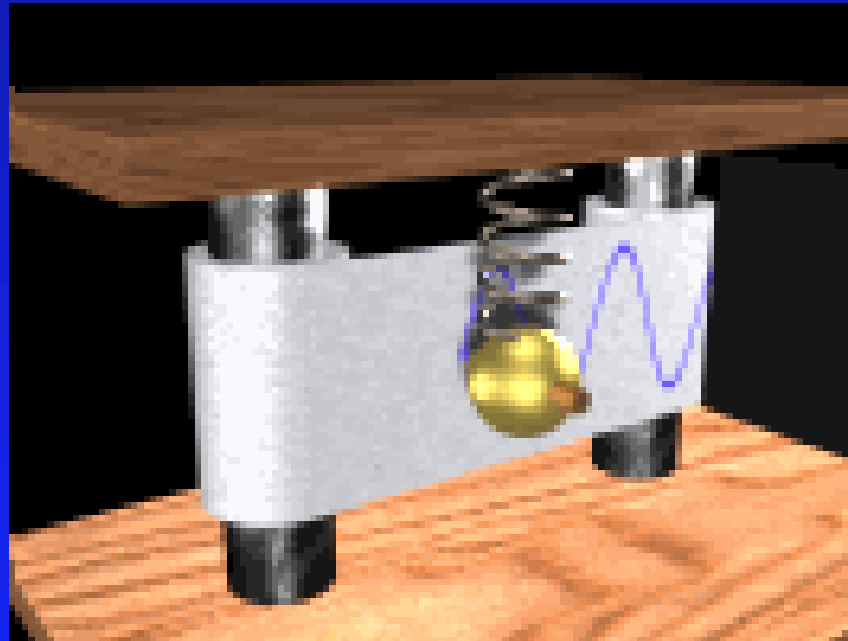# Physics 101: Lecture 19
Elasticity and Oscillations

Overview

- Springs (review)
  - ➔ Restoring force proportional to displacement
  - ➔ F = -k x (often a good approximation)
  - ➔ U = ½ k x^2

- Today
  - ➔ Simple Harmonic Motion
  - ➔ Springs Revisited
  - ➔ Young's Modulus (where does k come from?)

Springs

● **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

➜ $F_X = -k\,x$

Where x is the displacement from the relaxed position and k is the constant of proportionality.

relaxed position

$F_X = 0$

x

x=0
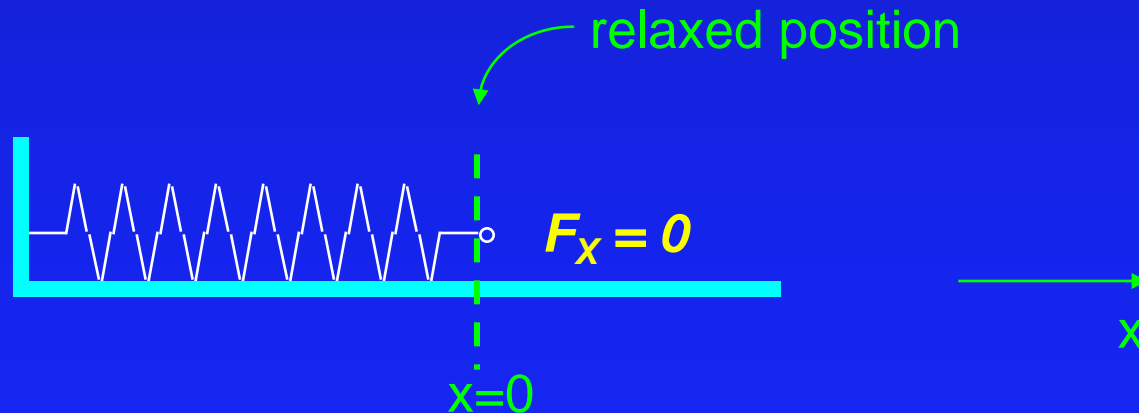
Springs ACT

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.
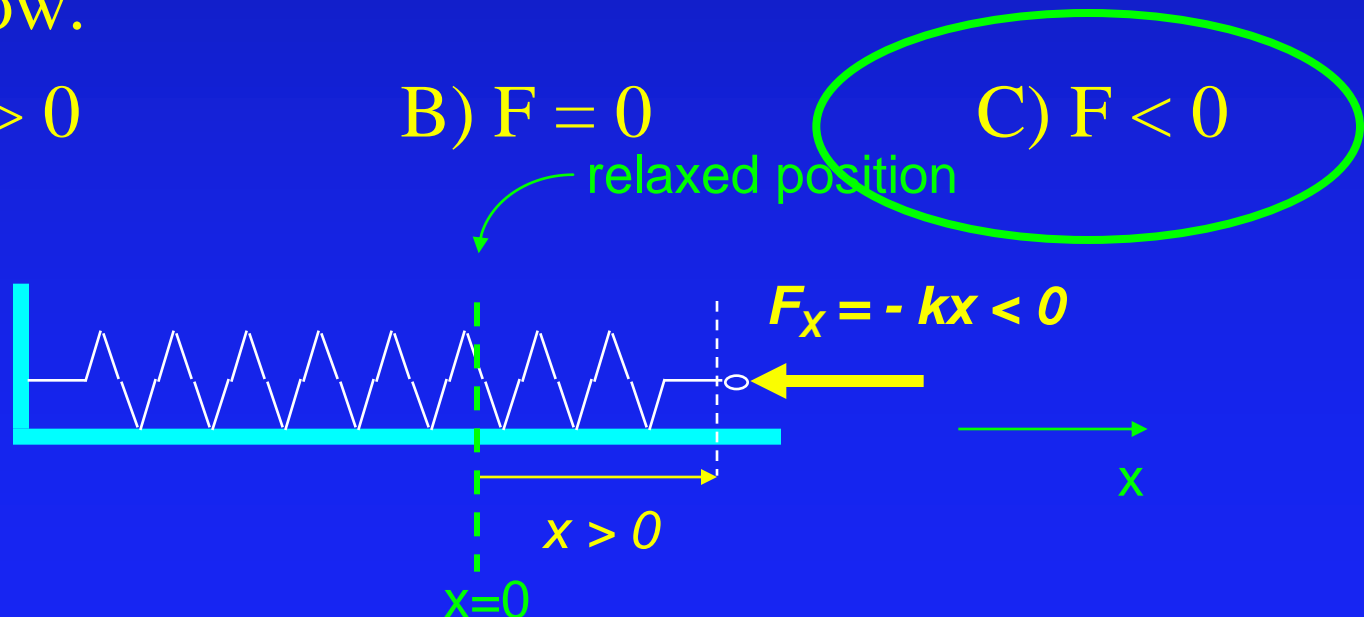
➔ $F_X = -k\,x$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

What is force of spring when it is stretched as shown below.

A) F > 0   B) F = 0   C) F < 0
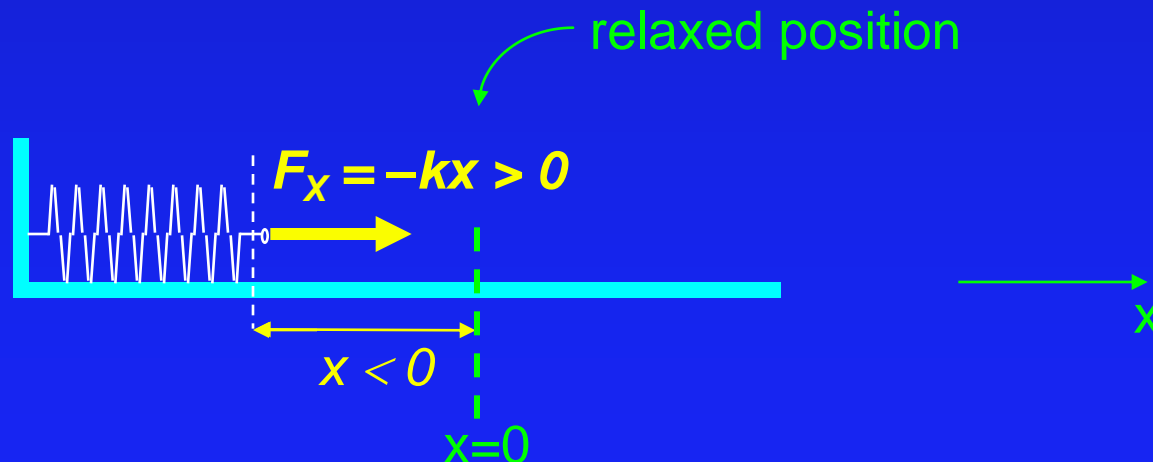
relaxed position

$F_X = -kx < 0$

x

x > 0

x=0

Springs

● **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

➔ $F_X = -kx$

Where x is the displacement from the relaxed position and k is the constant of proportionality.

relaxed position
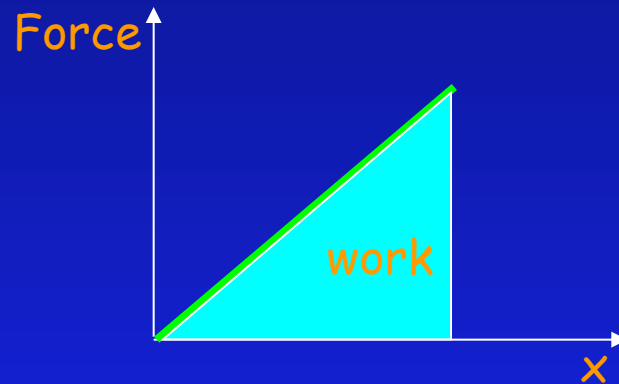
$F_X = -kx > 0$

$x < 0$

x=0

x

Potential Energy in Spring

- Hooke's Law force is Conservative

  ➔ $F = -k\,x$

  ➔ $W = -1/2\,k\,x^2$

  ➔ Work done only depends on initial and final position

  ➔ Define Potential Energy $U_{spring} = \frac{1}{2}\,k\,x^2$
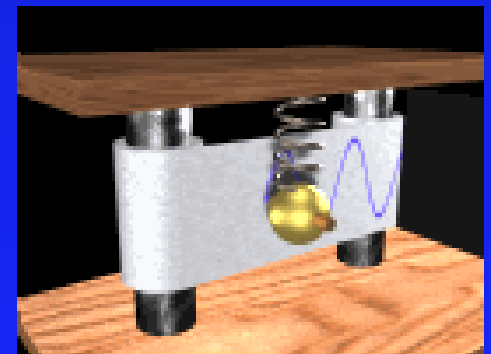
Simple Harmonic Motion

- Vibrations
  - ➔ Vocal cords when singing/speaking
  - ➔ String/rubber band

- Simple Harmonic Motion
  - ➔ Restoring force proportional to displacement
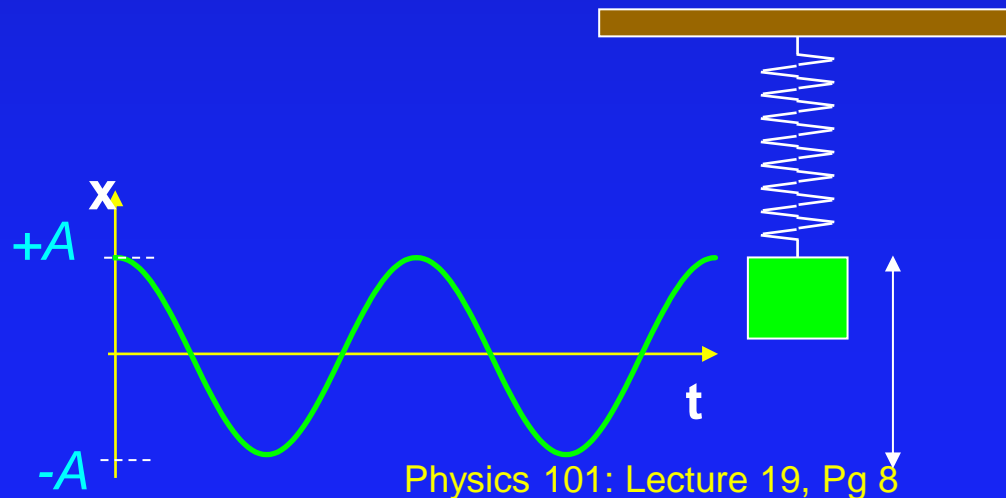 - ➔ Springs $F = -kx$

Spring ACT II

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?
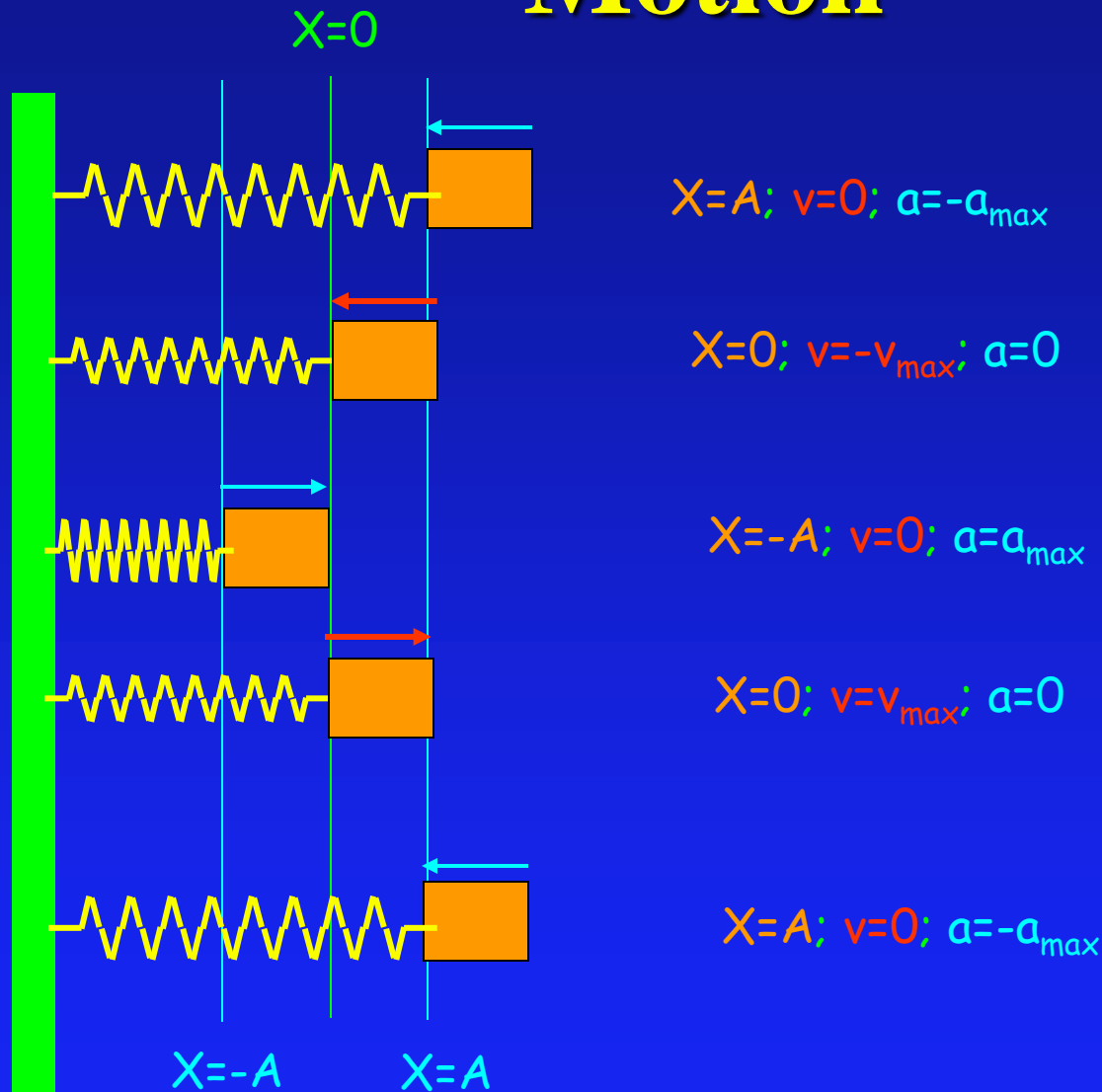
1. When x = +A or -A (i.e. maximum displacement) ⟵ CORRECT

2. When x = 0 (i.e. zero displacement)

3. The acceleration of the mass is constant

F=ma

Springs and Simple Harmonic Motion

X=0

$X=A$; $v=0$; $a=-a_{max}$

$X=0$; $v=-v_{max}$; $a=0$

$X=-A$; $v=0$; $a=a_{max}$

$X=0$; $v=v_{max}$; $a=0$

$X=A$; $v=0$; $a=-a_{max}$

X=-A   X=A

Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$ $x(t) = [A]\sin(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$ OR $v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$ $a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$

***Energy ***

- A mass is attached to a spring and set to motion. The maximum displacement is x=A

    ➔ $\Sigma W_{nc} = \Delta K + \Delta U$

    ➔ $0 = \Delta K + \Delta U$ or Energy U+K is constant!

         Energy = ½ k x^2 + ½ m v^2

    ➔ At maximum displacement x=A, v = 0

         Energy = ½ k A^2 + 0

    ➔ At zero displacement x = 0

         Energy = 0 + ½ mv_m^2

    Since Total Energy is same

         ½ k A^2 = ½ m v_m^2

         v_m = sqrt(k/m) A
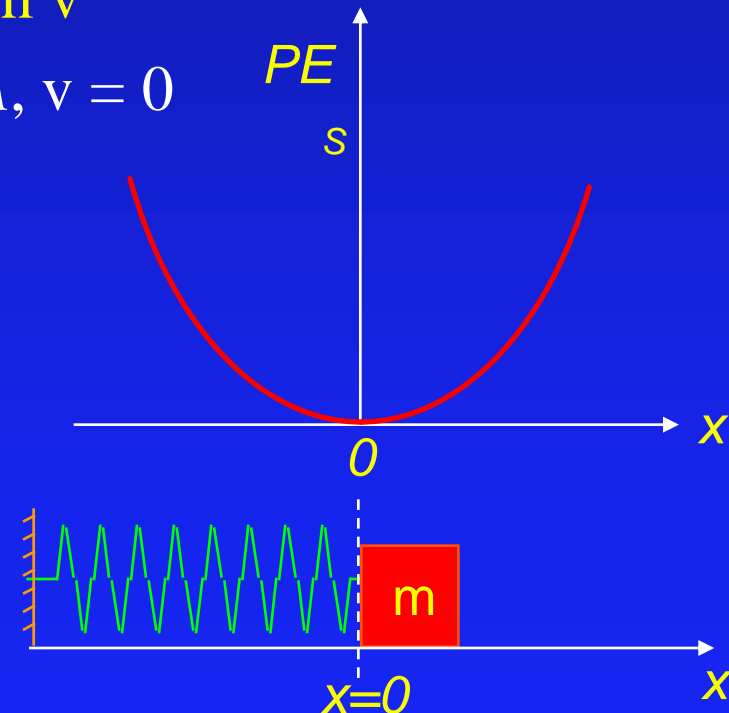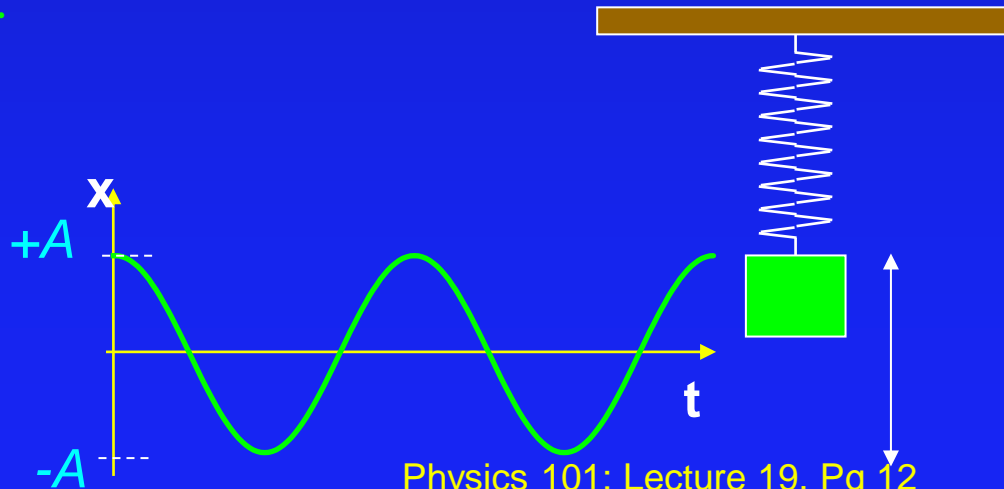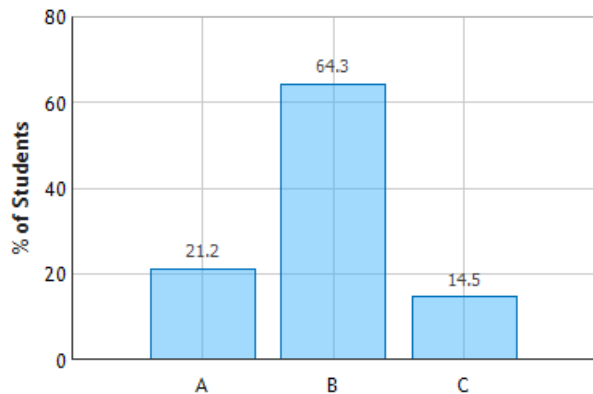
Prelecture 1+2

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

1. When x = +A or -A (i.e. maximum displacement)

2. When x = 0 (i.e. zero displacement)   ⟵ CORRECT

3. The speed of the mass is constant

"At x=0 all spring potential energy is converted into kinetic energy and so the velocity will be greatest at this point."
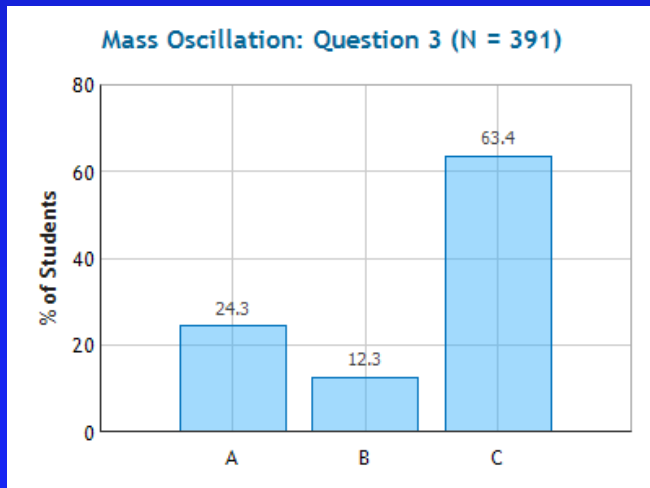
Mass Oscillation: Question 1 (N = 392)

80

64.3

60

40

% of Students

21.2

20

14.5

0

A   B   C

x

+A

t

-A

Prelecture 3+4

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total energy (K+U) of the mass and spring a maximum? (Ignore gravity).

1. When x = +A or -A (i.e. maximum displacement)

2. When x = 0 (i.e. zero displacement)
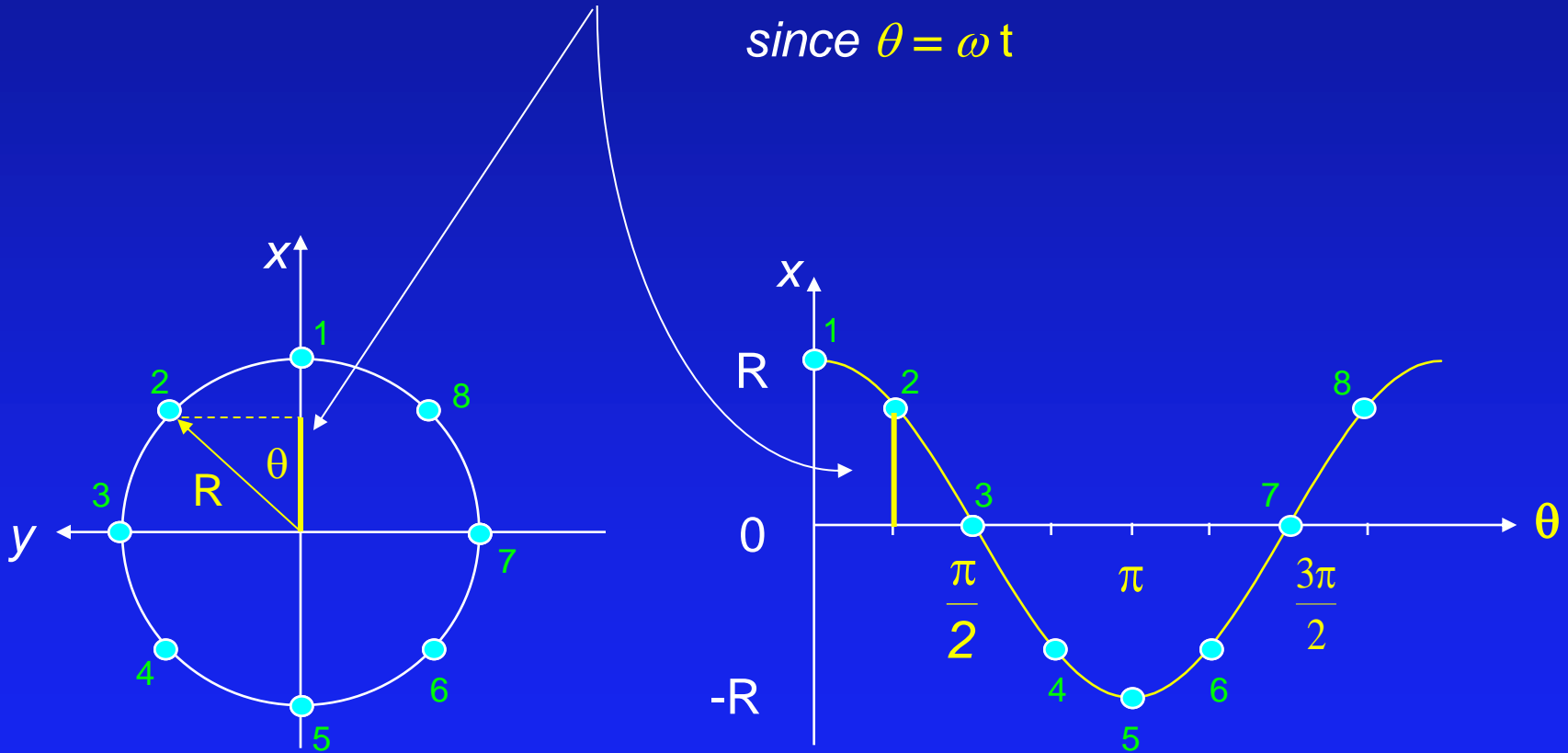
3. The energy of the system is constant ← CORRECT

The energy changes from spring to kinetic but is not lost.

Mass Oscillation: Question 3 (N = 391)

% of Students

80

60 — 63.4

40

24.3

20 — 12.3
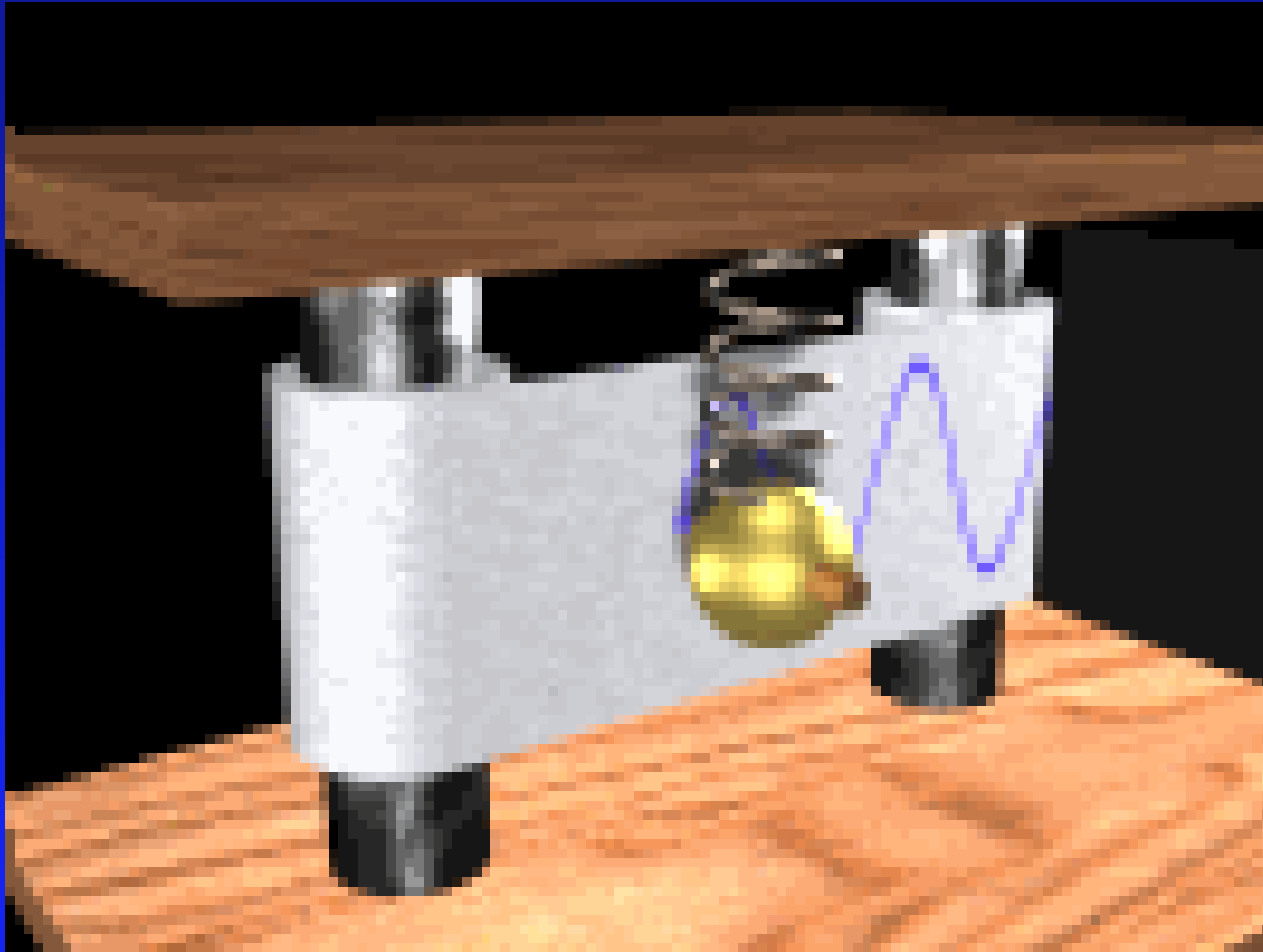
0

A   B   C

x

+A

t

-A
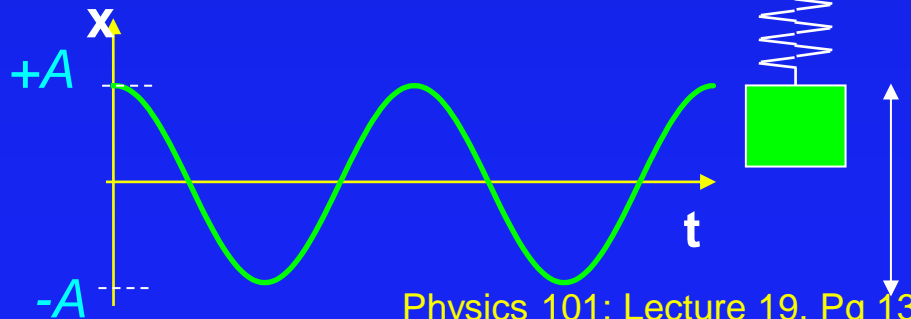
What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$$x = R \cos \theta = R \cos (\omega t)$$

since $\theta = \omega t$

SHM and Circles

Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$ $x(t) = [A]\sin(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$ **OR** $v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$ $a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$ Period = T (seconds per cycle)

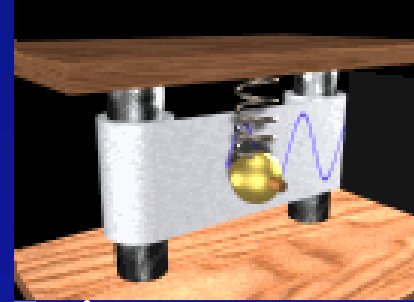$v_{max} = A\omega$ Frequency = f = 1/T (cycles per second)

$a_{max} = A\omega^2$ Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$

Example
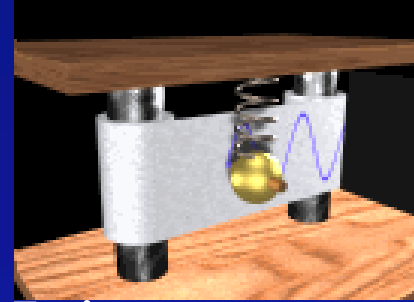

A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

Which equation describes the position as a function of time x(t) =

A) $5 \sin(\omega t)$ B) $5 \cos(\omega t)$ C) $24 \sin(\omega t)$

D) $24 \cos(\omega t)$ E) $-24 \cos(\omega t)$

We are told at t=0, x = +5 cm. $x(t) = 5 \cos(\omega t)$ only one that works.

Example

A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the total energy of the block spring system?
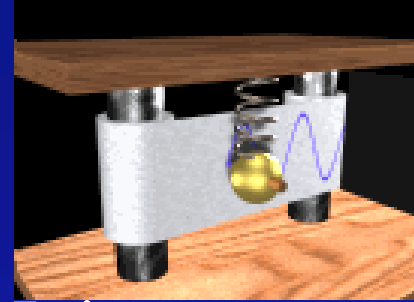
A) 0.03 J   B) .05 J   C) .08 J

E = U + K

At t=0, x = 5 cm and v=0:

E = ½ k x² + 0

$= ½ (24 \text{ N/m}) (5 \text{ cm})^2$

= 0.03 J

Example

A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the maximum speed of the block?

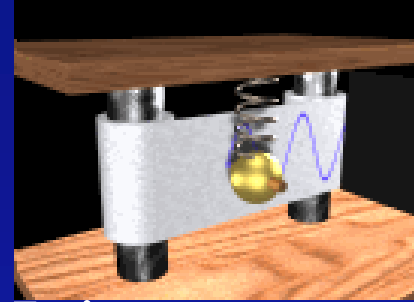A) .45 m/s   B) .23 m/s   C) .14 m/s

$E = U + K$

When $x = 0$, maximum speed:

$E = \frac{1}{2} m v^2 + 0$

$.03 = \frac{1}{2} \, 3 \text{ kg } v^2$

$v = .14 \text{ m/s}$

Example

A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

How long does it take for the block to return to x=+5cm?

A) 1.4 s   B) 2.2 s   C) 3.5 s

$\omega = \text{sqrt}(k/m)$

$\quad = \text{sqrt}(24/3)$

$\quad = 2.83$ radians/sec

Returns to original position after $2\,\pi$ radians

$T = 2\,\pi / \omega = 6.28 / 2.83 = 2.2$ seconds

Summary

● Springs

➔ F = -kx

➔ U = ½ k x^2

➔ ω = sqrt(k/m)

● Simple Harmonic Motion

➔ Occurs when have linear restoring force F= -kx

➔ x(t) = [A] cos(ωt)   or   [A] sin(ωt)

➔ v(t) = -[Aω] sin(ωt)   or   [Aω] cos(ωt)

➔ a(t) = -[Aω^2] cos(ωt) or   -[Aω^2] sin(ωt)

Young's Modulus

- Spring $F = -k\,x$ [demo]
  - ➔ What happens to "k" if cut spring in half?
  - ➔ A) decreases   B) same   C) increases

- k is inversely proportional to length!
- Define
  - ➔ Strain $= \Delta L / L$
  - ➔ Stress $= F/A$
- Now
  - ➔ Stress $= Y$ Strain
  - ➔ $F/A = Y\ \Delta L/L$
  - ➔ $k = Y\,A/L$ from $F = k\,x$
- Y (Young's Modules) independent of L