

## Physics 101: Lecture 24

# Ideal Gas Law and Kinetic Theory

- Today's lecture will cover Textbook Chapter 13.5-13.7



- Extra Office Hours this Friday!
  - I WILL NOT have extra office hours... I have to go give a talk
  
- Exam Review
  - Saturday, Nov. 23 7-9PM Loomis 141
  - I will try to record it and post videos!

# Aside: The Periodic Table

Ia	IIa	IIIb	IVb	Vb	VIb	VIIb	VII							Ib	IIb	IIIa	IVa	Va	VIa	VIIa	O
1 <u>H</u> 1.00797																					2 <u>He</u> 4.0026
3 <u>Li</u> 6.941	4 <u>Be</u> 9.0122											5 <u>B</u> 10.81	6 <u>C</u> 12.011	7 <u>N</u> 14.0067	8 <u>O</u> 15.9995	9 <u>F</u> 18.9984	10 <u>Ne</u> 20.179				
11 <u>Na</u> 22.9898	12 <u>Mg</u> 24.305											13 <u>Al</u> 26.9815	14 <u>Si</u> 28.086	15 <u>P</u> 30.9738	16 <u>S</u> 32.06	17 <u>Cl</u> 35.453	18 <u>Ar</u> 39.948				
19 <u>K</u> 39.098	20 <u>Ca</u> 40.08	21 <u>Sc</u> 44.956	22 <u>Ti</u> 47.90	23 <u>V</u> 50.9414	24 <u>Cr</u> 51.996	25 <u>Mn</u> 54.9830	26 <u>Fe</u> 55.847	27 <u>Co</u> 58.9332	28 <u>Ni</u> 58.70	29 <u>Cu</u> 63.546	30 <u>Zn</u> 65.38	31 <u>Ga</u> 69.72	32 <u>Ge</u> 72.59	33 <u>As</u> 74.9216	34 <u>Se</u> 78.96	35 <u>Br</u> 79.904	36 <u>Kr</u> 83.80				
37 <u>Rb</u> 85.4678	38 <u>Sr</u> 87.62	39 <u>Y</u> 88.909	40 <u>Zr</u> 91.22	41 <u>Nb</u> 92.9064	42 <u>Mo</u> 95.94	43 <u>Tc</u> (97)	44 <u>Ru</u> 101.07	45 <u>Rh</u> 102.905	46 <u>Pd</u> 106.04	47 <u>Ag</u> 107.868	48 <u>Cd</u> 112.40	49 <u>In</u> 114.82	50 <u>Sn</u> 118.69	51 <u>Sb</u> 121.75	52 <u>Te</u> 127.60	53 <u>I</u> 126.9046	54 <u>Xe</u> 131.30				
55 <u>Cs</u> 132.905	56 <u>Ba</u> 137.34	57 * <u>La</u> 138.91	72 <u>Hf</u> 178.49	73 <u>Ta</u> 180.948	74 <u>W</u> 183.85	75 <u>Re</u> 186.207	76 <u>Os</u> 190.2	77 <u>Ir</u> 192.22	78 <u>Pt</u> 195.09	79 <u>Au</u> 196.967	80 <u>Hg</u> 200.59	81 <u>Tl</u> 204.37	82 <u>Pb</u> 207.2	83 <u>Bi</u> 208.980	84 <u>Po</u> (209)	85 <u>At</u> (210)	86 <u>Rn</u> (222)				
87 <u>Fr</u> (223)	88 <u>Ra</u> 226.03	89 * <u>Ac</u> (227)																			

# The Periodic Table Explained ?

Look carefully

1 H 1.00797					2 He 4.0026		
3 Li 6.941	4 Be 9.0122	5 B 10.81	6 C 12.011	7 N 14.0067	8 O 15.9995	9 F 18.9984	10 Ne 20.179

● proton  
● neutron  
● electron

The diagram illustrates the periodic table with particle counts for the first row of elements. Below the table, vertical columns of colored dots represent the number of protons (red), neutrons (yellow), and electrons (green) for each element. A legend defines the colors: red for proton, yellow for neutron, and green for electron. A Bohr-style atomic model is shown at the bottom, with a blue arrow pointing to the electron column of the element it represents.

# Molecular Picture of Gas

- Gas is made up of many **individual molecules**
- **Number density** is number of molecules/volume:
  - $N/V = \rho/m$
  - $\rho$  is the mass density
  - $m$  is the mass for one molecule

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 1/12 \text{ of a mass of } C^{12}$$

- **Number of moles:**  $n = N / N_A$ 
  - $N_A = \text{Avogadro's Number} = 6.022 \times 10^{23} \text{ mole}^{-1}$
- Mass of 1 mole of “stuff” in grams = molecular mass in u
  - e.g., 1 mole of  $N_2$  has mass of  $2 \times 14 = 28$  grams



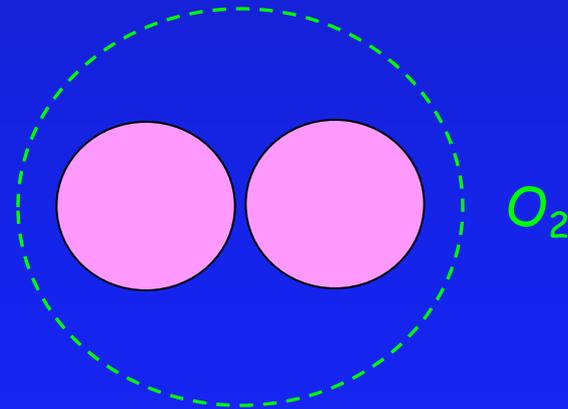
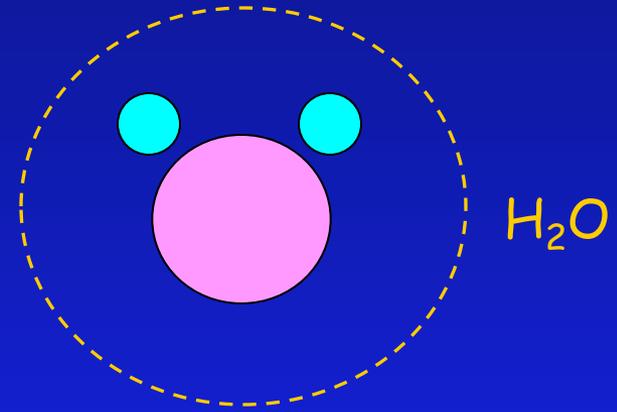
# Atomic Act I

Which contains the most molecules ?

1. A mole of water ( $\text{H}_2\text{O}$ )

2. A mole of oxygen gas ( $\text{O}_2$ )

3. Same ← correct



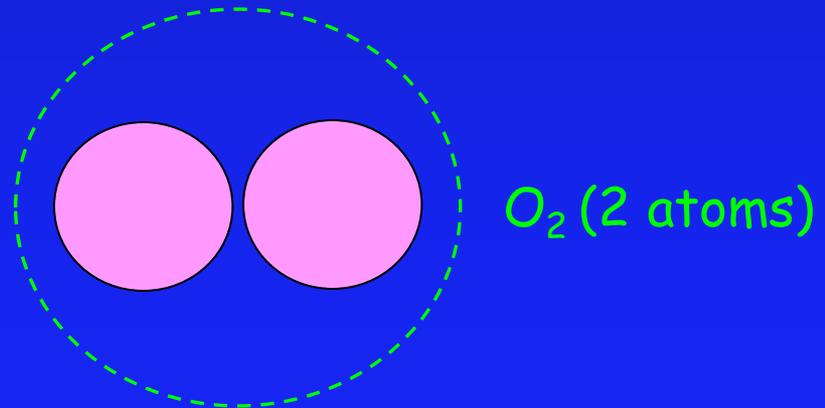
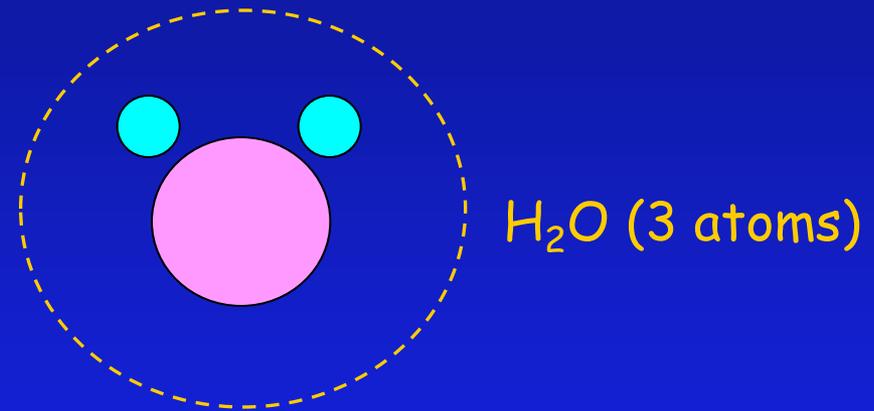
# Atomic Act II

Which contains the most atoms ?

1. A mole of water ( $\text{H}_2\text{O}$ ) ← correct

2. A mole of oxygen gas ( $\text{O}_2$ )

3. Same



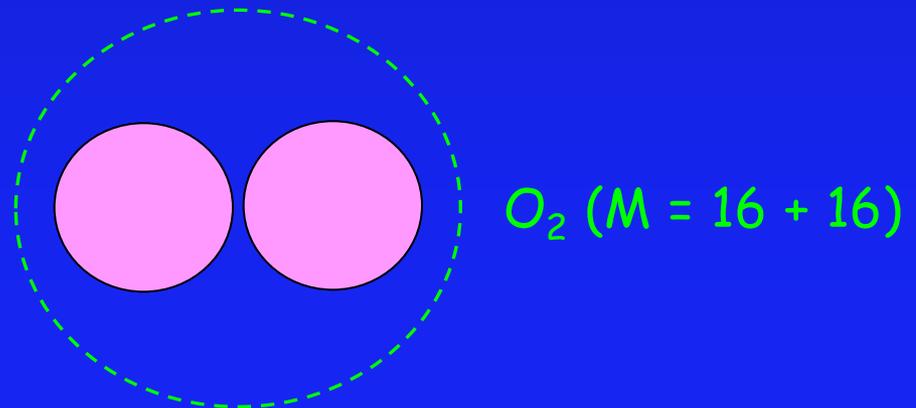
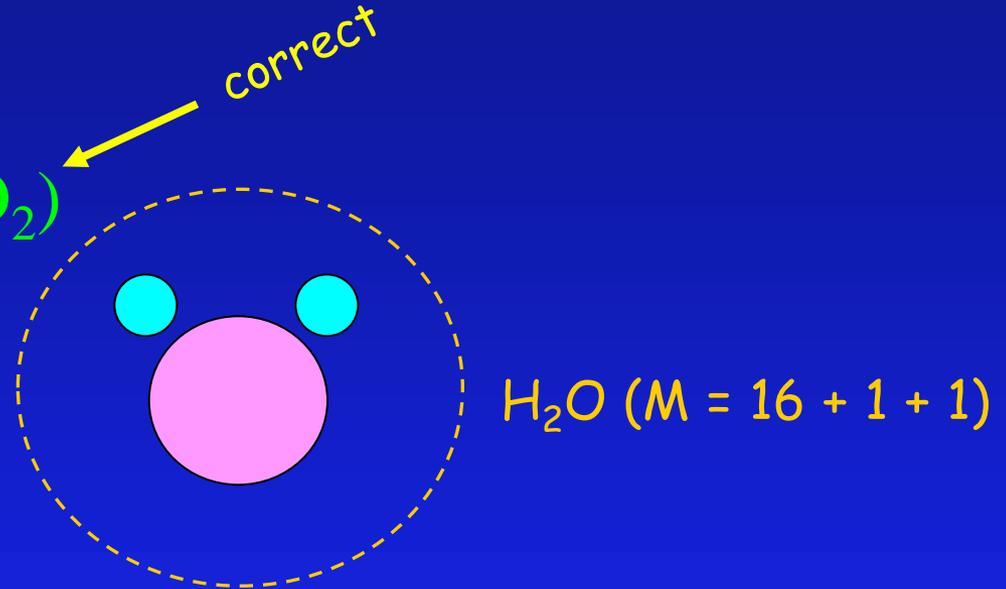
# Atomic Act III

Which weighs the most ?

1. A mole of water ( $\text{H}_2\text{O}$ )

2. A mole of oxygen gas ( $\text{O}_2$ )

3. Same



# The Ideal Gas Law

- $P V = N k_B T$

- $P$  = pressure in  $\text{N/m}^2$  (or Pascals)

- $V$  = volume in  $\text{m}^3$

- $N$  = number of molecules

- $T$  = absolute temperature in  $\text{K}$

- $k_B$  = Boltzmann's constant =  $1.38 \times 10^{-23} \text{ J/K}$

- Note:  $P V$  has units of  $\text{N}\cdot\text{m}$  or  $\text{J}$  (energy!)

- $P V = n R T$

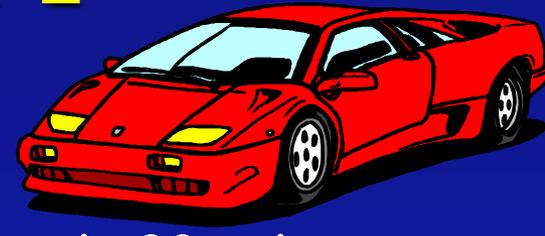
- $n$  = number of moles

- $R$  = ideal gas constant =  $N_A k_B = 8.31 \text{ J/mol/K}$



# Ideal Gas Law ACT I

$$PV = nRT$$



You inflate the tires of your car so the pressure is 30 psi, when the air inside the tires is at 20 degrees C. After driving on the highway for a while, the air inside the tires heats up to 38 C. Which number is closest to the new air pressure?

1) 16 psi

2) 32 psi

3) 57 psi

Careful, you need to use the temperature in K

$$P = P_0 (38+273)/(20+273)$$

# Ideal Gas Law: ACT II

$$pV = nRT$$

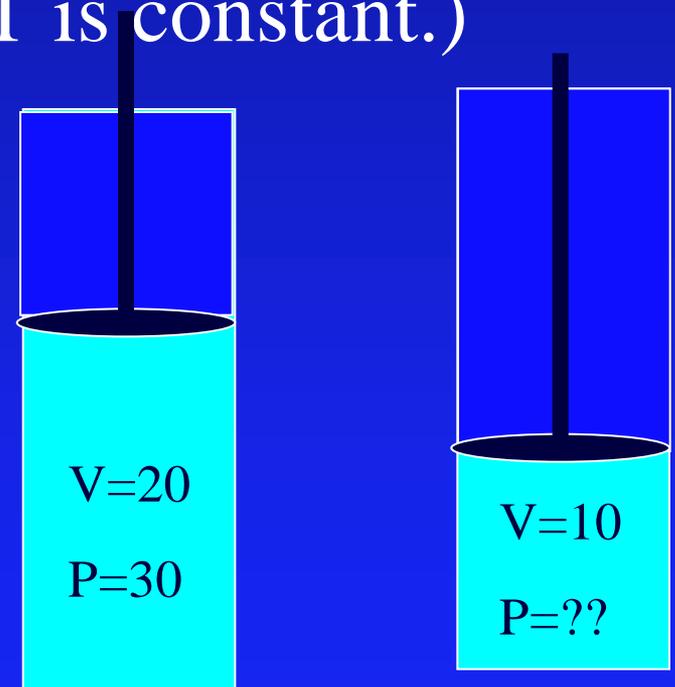
- A piston has volume 20 ml, and pressure of 30 psi. If the volume is decreased to 10 ml, what is the new pressure? (Assume T is constant.)

1) 60

2) 30

3) 15

- When n and T are constant, pV is constant (Boyle's Law)



# Balloon ACT 1



- What happens to the pressure of the air inside a hot-air balloon when the air is heated? (Assume  $V$  is constant)

1) Increases    2) Same    3) Decreases

Balloon is still open to atmospheric pressure,  
so it stays at 1 atm

# Balloon ACT 2

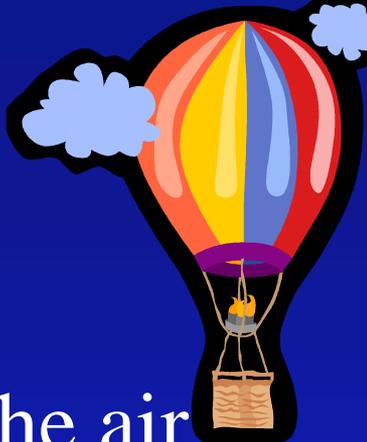
- What happens to the buoyant force on the balloon when the air is heated?  
(Assume  $V$  remains constant)

1) Increases   2) Same   3) Decreases

$$F_B = \rho V g$$

$\rho$  is density of outside air!

# Balloon ACT 3

- 
- 
- What happens to the number of air molecules inside the balloon when the air is heated? (Assume  $V$  remains constant)

1) Increases    2) Same    3) Decreases

$$PV = NkT$$

$P$  and  $V$  are constant. If  $T$  increases  $N$  decreases.

Note! this is not a pressure effect, it is a density effect. As  $T$  increases, the density decreases the balloon then floats due to Archimedes principle. The pressure remains constant!

# Ideal Gas Law: Demos

$$pV = nRT$$

- When  $T$  is constant,  $PV$  is constant (Boyle's Law)
  - Boyle's law demo
- When  $P$  is constant,  $V$  is proportional to  $T$ 
  - Hot air balloon, helium and oxygen in  $LN_2$
- When  $V$  is constant,  $P$  is proportional to  $T$ 
  - Explosion!

# Kinetic Theory:

The relationship between energy and temperature  
(for monatomic ideal gas)

$$\Delta p_x = 2mv_x$$

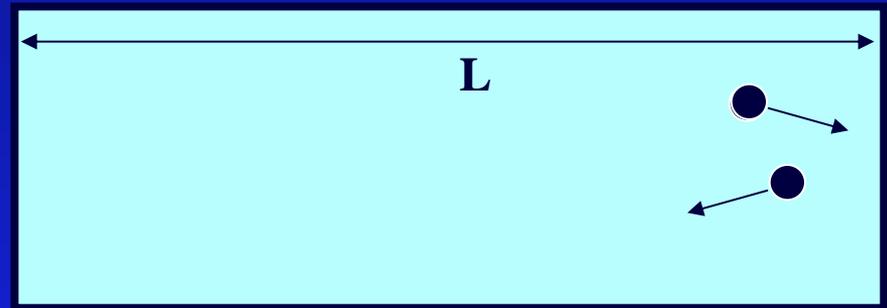
$$\Delta t = 2 \frac{L}{v_x}$$

$$F_{avg} = \frac{\Delta p_x}{\Delta t} = \frac{mv_x^2}{L}$$

For N molecules, multiply by N

$$P = \frac{F}{A} = \frac{Nmv_x^2}{V}$$

Note  $KE = \frac{1}{2} m v^2 = \frac{3}{2} m v_x^2$



$$P = \frac{2N}{3V} \langle K_{tr} \rangle$$

Using  $PV = NkT$

$$\langle K_{tr} \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$\langle \rangle$  means *average*.  
 $kT/2$  energy per  
degree of  
freedom =  
equipartition  
theorem

# Prelecture 1

root-mean-square?

Suppose you want the rms (*root-mean-square*) speed of molecules in a sample of gas to double. By what factor should you increase the temperature of the gas?

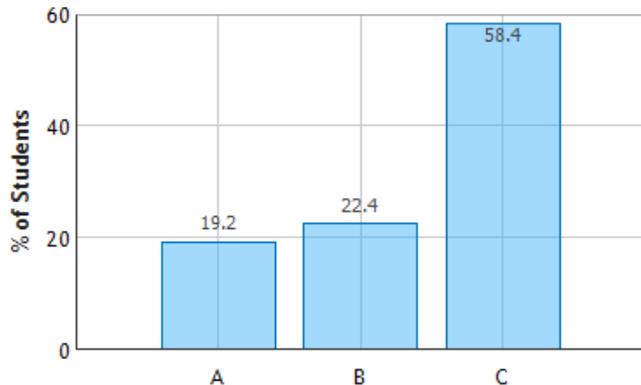
1. 2

2.  $\sqrt{2}$

3. 4 ← correct

$$\langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

Ideal Gas and Kinetic Theory: Question 1 (N = 411)



- If  $v$  doubles,  $v^2$  quadruples
- Therefore,  $T$  quadruples

$$x_{\text{rms}} = \sqrt{\frac{1}{n} (x_1^2 + x_2^2 + \dots + x_n^2)}$$

# Example

- What is the rms speed of a nitrogen ( $\text{N}_2$ ) molecule in this classroom?

$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3k_B T}{m}$$

$$v = 510 \text{ m/s}$$

$$= 1150 \text{ mph!}$$

$$\langle v^2 \rangle = \frac{3(1.38 \times 10^{-23} \text{ J/K})(273 + 20) \text{ K}}{(28 \text{ u}) \times (1.66 \times 10^{-27} \text{ kg/u})}$$

# Summary

- Ideal Gas Law  $PV = n R T$ 
  - $P$  = pressure in  $\text{N/m}^2$  (or Pascals)
  - $V$  = volume in  $\text{m}^3$
  - $n$  = # moles
  - $R$  =  $8.31 \text{ J/ (K mole)}$
  - $T$  = Temperature (K)
- Kinetic Theory of Monatomic Ideal Gas
  - $\langle K_{\text{tr}} \rangle = 3/2 k_B T$