Exam II

Physics 101: Lecture 15
Rolling Objects

Today's lecture will cover Textbook Chapter 8.5-8.7

Overview

- Review
  - ➔ $K_{rotation} = \frac{1}{2} I \omega^2$
  - ➔ Torque = Force that causes rotation

    $\tau = F r \sin\theta$

  - ➔ Equilibrium

    $F_{Net} = 0$

    $\tau_{Net} = 0$
- Today
  - ➔ $\tau_{Net} = I \alpha$ (rotational $F = ma$)
  - ➔ Energy conservation revisited

Linear and Angular

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	½ m v2	$\frac{1}{2} I \omega^2$
N2L	F=ma	$\tau = I\alpha$
Momentum	p = mv	$L = I\omega$

Today

Rotational Form Newton's 2^{nd} Law

- $\tau_{Net} = I\,\alpha$
  - ➔Torque is amount of twist provide by a force
    - » Signs: positive = CCW

  - ➔Moment of Inertia like mass. Large I means hard to start or stop from spinning.
- Problems Solved Like Newton's 2nd
  - ➔Draw FBD
  - ➔Write Newton's 2^{nd} Law

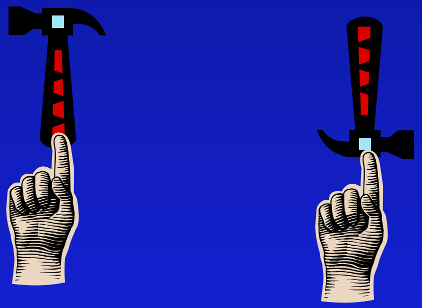
The Hammer!

You want to balance a hammer on the tip of your finger, which way is easier

32% A) Head up

62% B) Head down

6% C) Same



I just tried it in my home and I guess it is easier to balance the hammer with the head up.

Angular acceleration is smaller

the larger the radius the larger the moment of inertia.

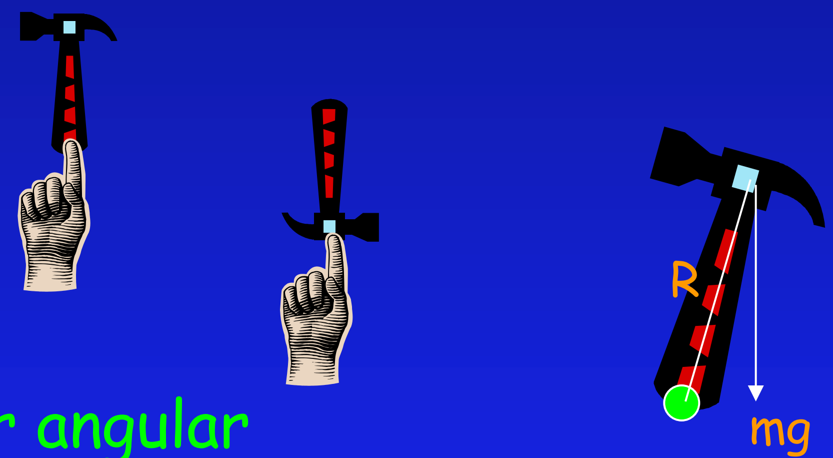
The Hammer!

You want to balance a hammer on the tip of your finger, which way is easier

29% A) Head up

63% B) Head down

8% C) Same



$\tau = I\,\alpha$

Key idea: higher angular acceleration means more difficult to balance.

$m\,g\,R\,\sin(\theta) = mR^2\,\alpha$

Torque increases with R

Inertia increases as R^2

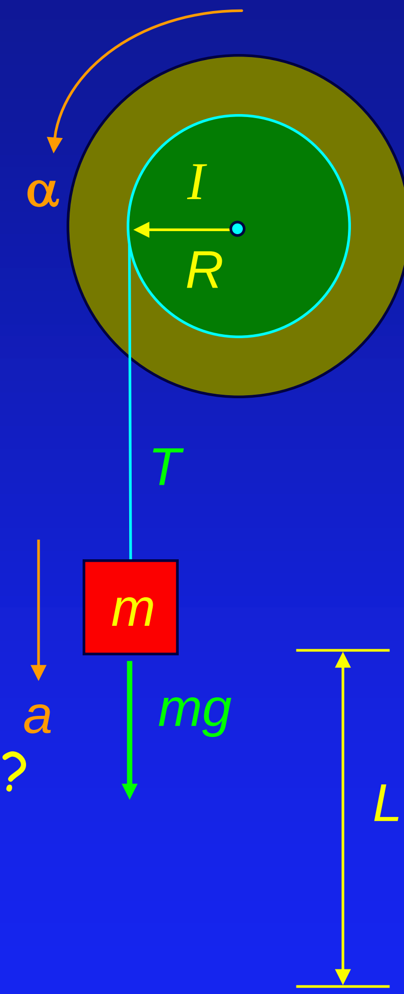
Angular acceleration decreases with R!
So large R is easier to balance.

$g\,\sin(\theta)\,/\,R = \alpha$

What is angular acceleration?

Falling weight & pulley

- A mass *m* is hung by a string that is wrapped around a pulley of radius *R* attached to a heavy flywheel. The moment of inertia of the pulley + flywheel is *I*. The string does not slip on the pulley.
  Starting at rest, how long does it take for the mass to fall a distance *L*.



What method should we use to solve this problem?

A) Conservation of Energy (including rotational)

B) $\tau_{Net} = I\alpha$ and then use kinematics

Either would work, but since it asks for time, we will use B.

Falling weight & pulley...

- For the hanging mass use $F_{Net} = ma$

$$\Rightarrow mg - T = ma$$

- For the flywheel use $\tau_{Net} = I\alpha$

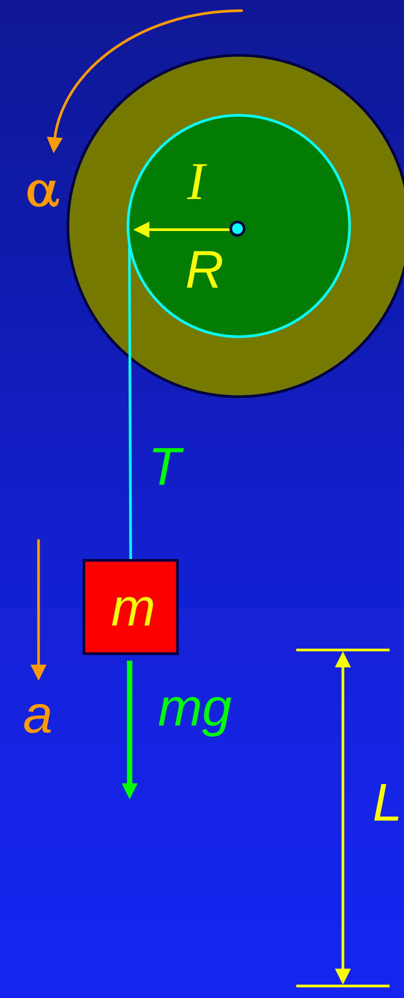
$$\Rightarrow TR \sin(90) = I\alpha$$

- Realize that $a = \alpha R$

$$\Rightarrow TR = I\frac{a}{R}$$

- Now solve for a, eliminate T:

$$a = \left(\frac{mR^2}{mR^2 + I}\right)g$$



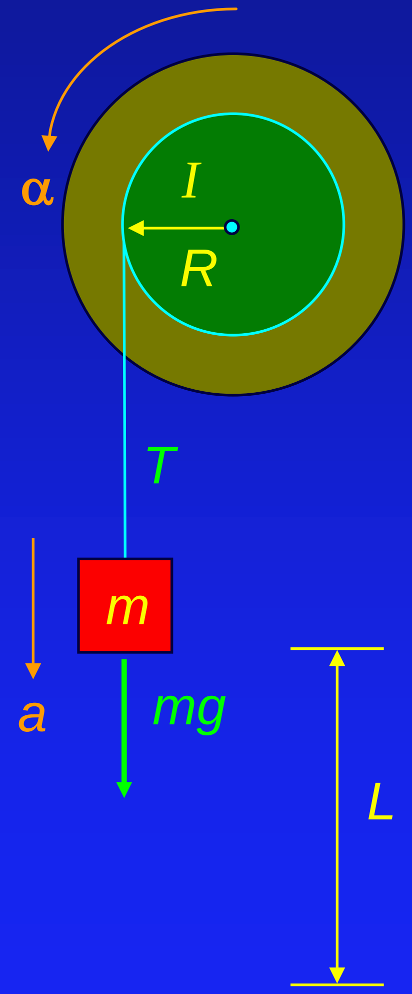
Falling weight & pulley...

- Using 1-D kinematics we can solve for the time required for the weight to fall a distance *L*:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$L = \frac{1}{2} a t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2L}{a}}$$

where $a = \left(\dfrac{mR^2}{mR^2 + I} \right) g$



Torque ACT

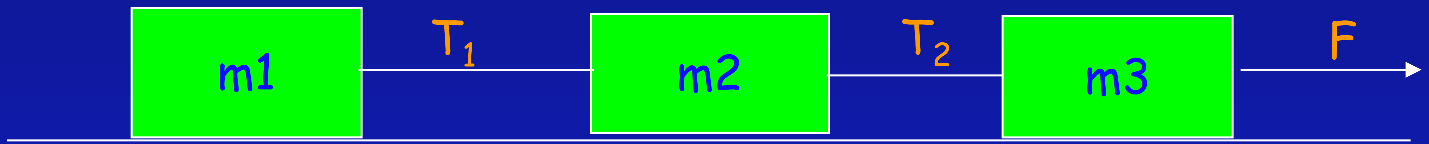
- Which pulley will make it drop fastest?

1) Small pulley
2) Large pulley
3) Same

$$a = \left(\frac{mR^2}{mR^2 + I} \right) g$$

Larger R, gives larger acceleration.

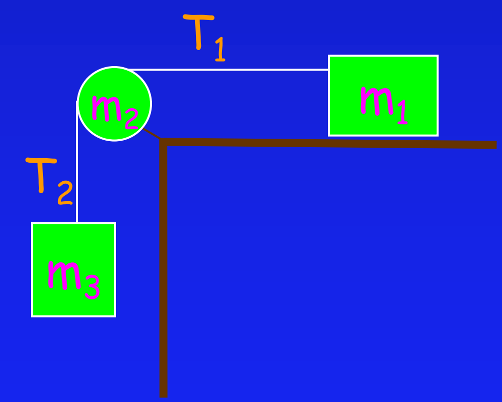
Tension...



Compare the tensions T_1 and T_2 as the blocks are accelerated to the right by the force F.

A) $T_1 < T_2$ B) $T_1 = T_2$ C) $T_1 > T_2$

$T_1 < T_2$ since $T_2 - T_1 = m_2 a$. It takes force to accelerate block 2.

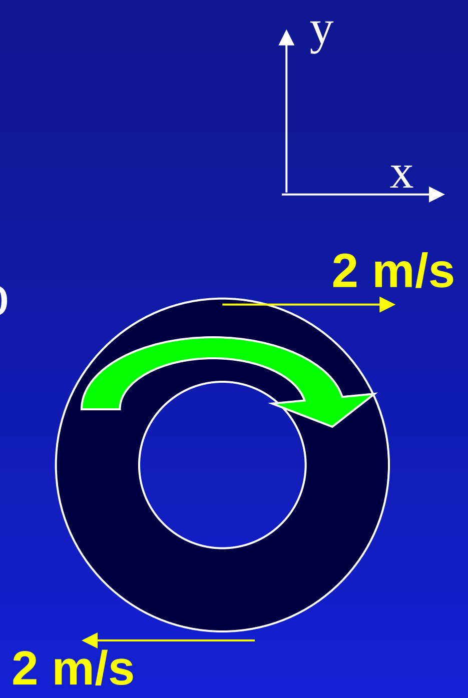


Compare the tensions T_1 and T_2 as block 3 falls

A) $T_1 < T_2$ B) $T_1 = T_2$ C) $T_1 > T_2$

$T_2 > T_1$ since $RT_2 - RT_1 = I_2\,\alpha$. It takes force (torque) to accelerate the pulley.

Rolling



A wheel is spinning clockwise such that the speed of the outer rim is 2 m/s.

What is the velocity of the top of the wheel relative to the ground? + 2 m/s

What is the velocity of the bottom of the wheel relative to the ground? -2 m/s

You now carry the spinning wheel to the right at 2 m/s.

What is the velocity of the top of the wheel relative to the ground?

A) -4 m/s B) -2 m/s C) 0 m/s D) +2m/s E) +4 m/s

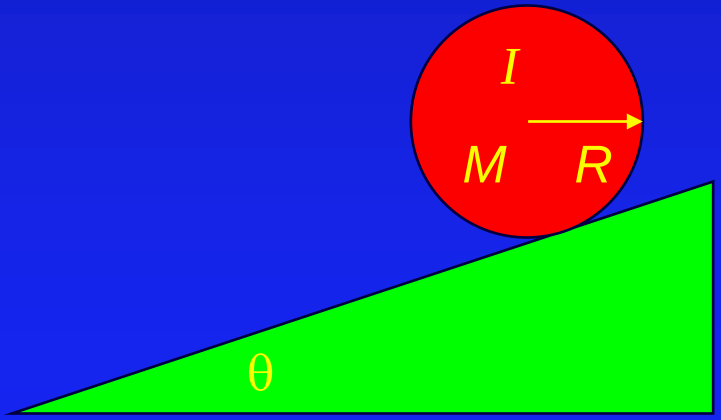
What is the velocity of the bottom of the wheel relative to the ground?

A) -4 m/s B) -2 m/s C) 0 m/s D) +2m/s E) +4 m/s

Rolling

- An object with mass M, radius R, and moment of inertia I rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

- Consider CM motion and rotation about the CM separately when solving this problem



Rolling...

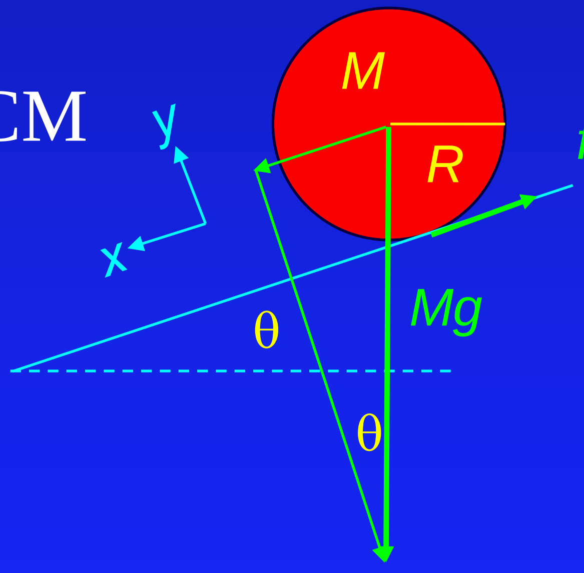
- Static friction f causes rolling. It is an unknown, so we must solve for it.
- First consider the free body diagram of the object and use $F_{NET} = Ma_{cm}$:

  In the x direction $Mg \sin\theta - f = Ma_{cm}$

- Now consider rotation about the CM and use $\tau_{N\varepsilon\tau} = I\alpha$ realizing that

  $\tau = Rf$ and $a = \alpha R$

$$Rf = I\frac{a}{R} \Rightarrow f = I\frac{a}{R^2}$$



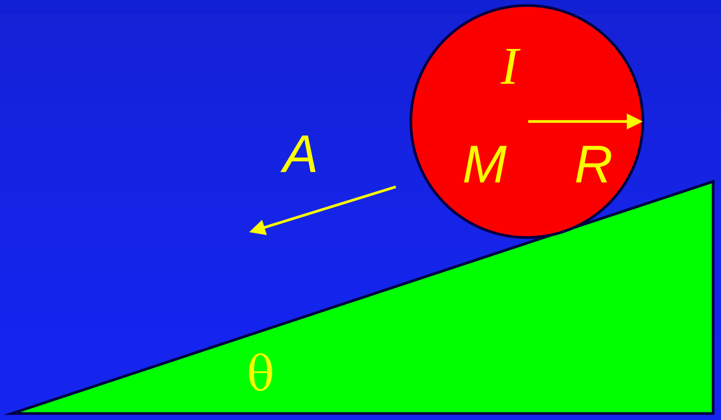
Rolling...

- We have two equations: $Mg \sin\theta - f = Ma$

$$f = I\frac{a}{R^2}$$

- We can combine these to eliminate f:

$$\boxed{a = g\left(\frac{MR^2 \sin\theta}{MR^2 + I}\right)}$$



For a sphere:

$$a = g\left(\frac{MR^2 \sin\theta}{MR^2 + \frac{2}{5}MR^2}\right) = \frac{5}{7}g\sin\theta$$

Energy Conservation!

- Friction causes object to roll, but if it rolls w/o slipping friction does NO work!
  - ➔ $W = F\,d\cos\theta$ d is zero for point in contact
- No dissipated work, energy is conserved
- Need to include both translational and rotational kinetic energy.
  - ➔ $K = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,I\,\omega^2$

Translational + Rotational KE

- Consider a cylinder with radius R and mass M, rolling w/o slipping down a ramp. Determine the ratio of the translational to rotational KE.

Translational: $K_T = \frac{1}{2} M v^2$

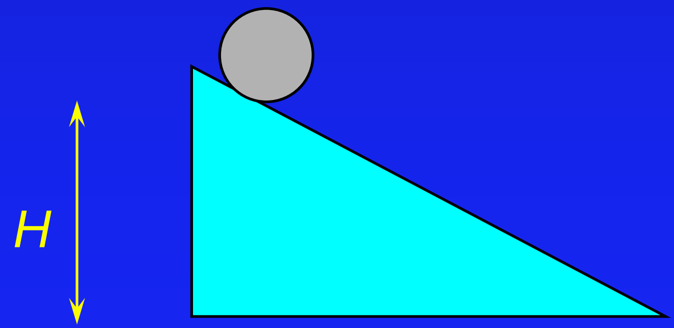
Rotational: $K_R = \frac{1}{2} I \omega^2$

use $I = \frac{1}{2} MR^2$ and $\omega = \frac{V}{R}$

Rotational: $K_R = \frac{1}{2} \left(\frac{1}{2} M R^2\right) (V/R)^2$

$= \frac{1}{4} M v^2$

$= \frac{1}{2} K_T$



Rolling Act

- Two uniform cylinders are machined out of solid aluminum. One has twice the radius of the other.
  - ➔ If both are placed at the top of the same ramp and released, which is moving faster at the bottom?

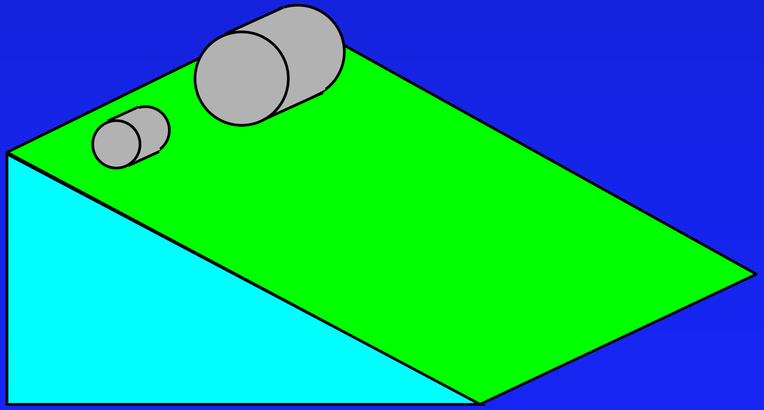
(a) bigger one (b) smaller one (c) same

$K_i + U_i = K_f + U_f$

$$MgH = \frac{1}{2} I \omega^2 + \frac{1}{2} MV^2$$

$$MgH = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{V^2}{R^2} + \frac{1}{2}MV^2$$

$$V = \sqrt{\frac{4}{3}gH}$$



Summary

- $\tau = I\,\alpha$
- Energy is Conserved
  - ➔ Need to include translational and rotational