

Physics 101: Lecture 02

Forces: Equilibrium Examples

- Today's lecture will cover **Textbook Sections 2.1-2.7**

Phys 101 URL:

<http://courses.physics.illinois.edu/phys101/>

Read the course web page!

Office hours start next week; see web page for locations & times.



About the Schedule

My Lecture Slides
(Posted after Lecture)

AY 2011-2012
Lecture Slides

Reading
Assignments

If you have any issues logging in to view any of the secured documents below, please try adding "uofI" (without the quotes) in front of your netid.

Week	Date	Checkpoint	Lecture	Lecture handouts	Lab	Discussion	Homework	Exam
1	Monday 8/26/2013	Checkpoint 1	Lecture 1 1-1.9 pdf ppt	Handout 1 pdf				
	Tuesday 8/27/2013							
	Wednesday 8/28/2013	Checkpoint 2	Lecture 2 2.1-2.7 pdf ppt	Handout 2 pdf		Discussion 1 Explanation 1		
	Thursday 8/29/2013							
	Friday 8/30/2013							
Monday 9/2/2013	Labor Day Holiday							
Tuesday 9/3/2013							Homework 1	
2	Wednesday 9/4/2013	Checkpoint 3	Lecture 3 Ch 3 pdf ppt	Handout 3 pdf	Lab 1	Discussion 2 Explanation 2		
	Thursday 9/5/2013				Lab 1			
	Friday 9/6/2013				Lab 1			

Overview

- Last Lecture

- **Newton's Laws of Motion**

- » FIRST LAW: Inertia

- » SECOND LAW: $F_{\text{net}} = ma$

- » THIRD LAW: Action/reaction pairs

- Gravity
$$W = G \frac{M_{\text{Earth}} m}{r_{\text{Earth}}^2} = m \left(G \frac{M_{\text{Earth}}}{r_{\text{Earth}}^2} \right)$$
$$\approx mg \quad (\text{near Earth's surface!})$$

- Today

- **Forces as Vectors**

- **Free Body Diagrams to Determine F_{net}**

- » Draw coordinate axes, each direction is independent.

- » Identify/draw all force vectors

- Friction: kinetic $\mathbf{f} = \mu_k \mathbf{N}$; static $\mathbf{f} \leq \mu_s \mathbf{N}$

- Contact Forces – Springs and Tension

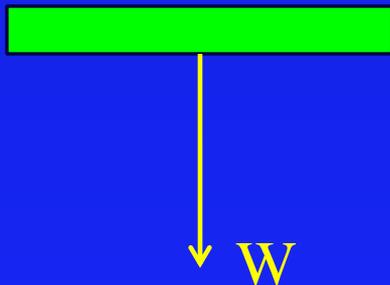
Forces as Vectors

- Last lecture we calculated the force of gravity on a book (i.e. its WEIGHT):

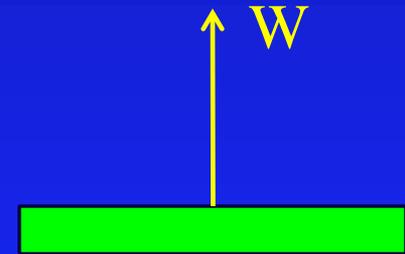
Calculate the gravitational force on a 3 kg book held 1 meter above the surface of the earth.

$$\begin{aligned}W &= G M_{\text{Earth}} m / r_{\text{Earth}}^2 \\&= (6.7 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2)) (6 \times 10^{24} \text{ kg}) (3 \text{ kg}) / (6.4 \times 10^6 + 1)^2 \text{ m}^2 \\&= 29.4 \text{ kg m/s}^2 = 29.4 \text{ N}\end{aligned}$$

- We missed something: The direction!



is different than

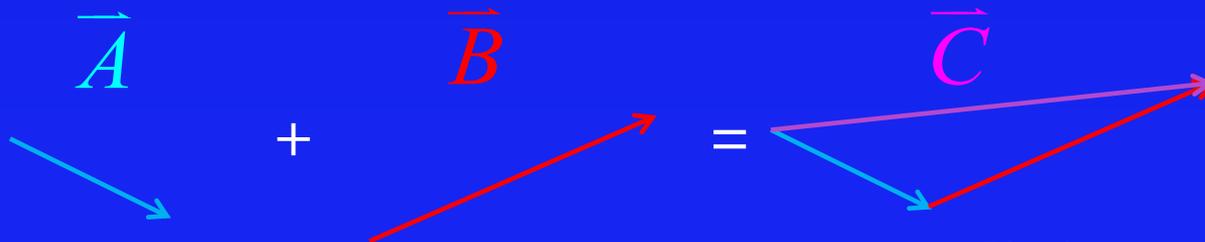


Forces as Vectors

- A quantity which has both magnitude and direction is called a VECTOR; FORCES are VECTORS
- Usually drawn as an arrow pointing in the proper direction, where the length indicates the magnitude

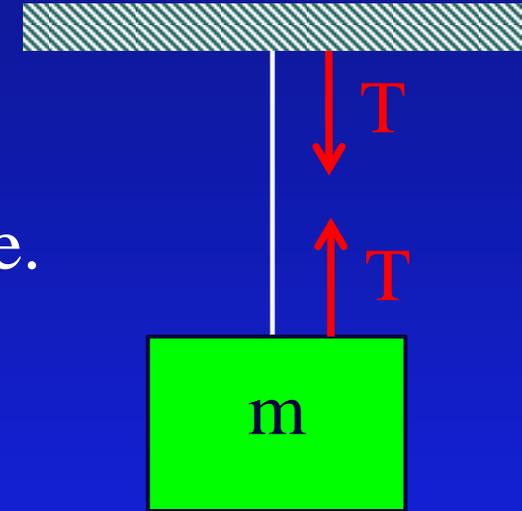


- This is an example of VECTOR ADDITION: to add vectors, you place them head to tail, and draw the RESULTANT from the start of the first to the end of the last



Another Example of a Force: Tension

- Tension in an Ideal String, T :
 - Direction is parallel to string (only pulls)
 - Magnitude of tension is equal everywhere.



- Now we are ready to do some physics!

QUESTION:

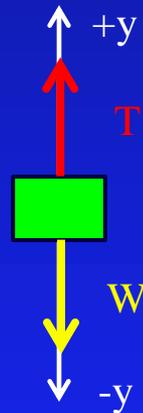
We suspend a mass $m = 5$ kg from the ceiling using a string. What is the tension in the string?

Newton's 2nd Law and Equilibrium Systems

We suspend a mass $m = 5 \text{ kg}$ from the ceiling using a string. What is the tension in the string?

● Every single one of these problems is done the same way!

→ Step 1: Draw a simple picture (called a Free Body Diagram), and label your axes!



→ Step 2: Identify and draw all force vectors Weight, W Tension, T

→ Step 3: Use your drawing to write down Newton's 2nd law

$$F_{\text{Net}} = ma \quad \text{In equilibrium, everything is balanced!} \quad a = 0$$

$$T - W = 0$$

$$T = W = mg = (5 \text{ kg}) * (9.8 \text{ m/s}^2) = 49 \text{ N}$$

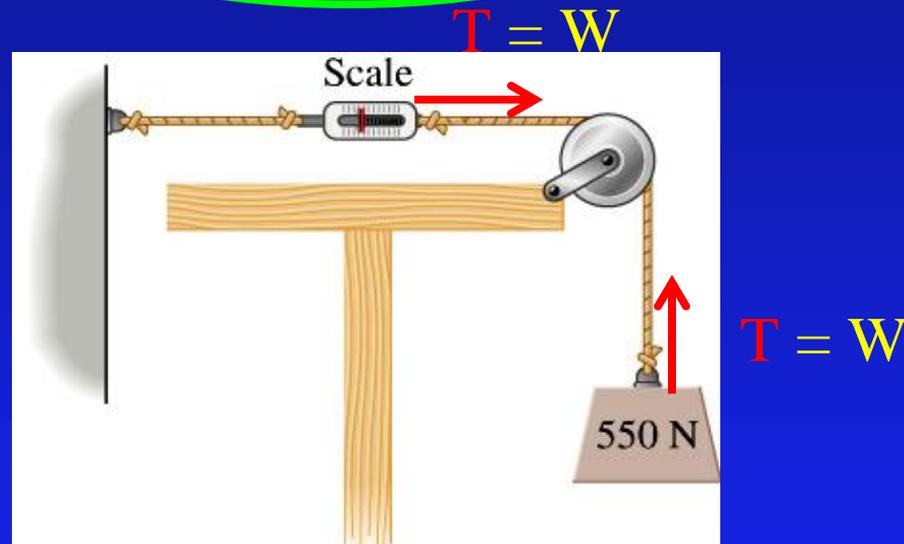
Prelecture!

• What does scale 1 read? (88% got correct!)

• A) 225 N

B) 550 N

C) 1100 N



The magnitude of tension in a ideal string is equal everywhere.

Tension ACT

- Two boxes are connected by a string over a frictionless pulley. **In equilibrium**, box 2 is lower than box 1. Compare the weight of the two boxes.

A) Box 1 is heavier

B) Box 2 is heavier

C) They have the same weight

Step 1 – Draw!

Step 2 – Forces!

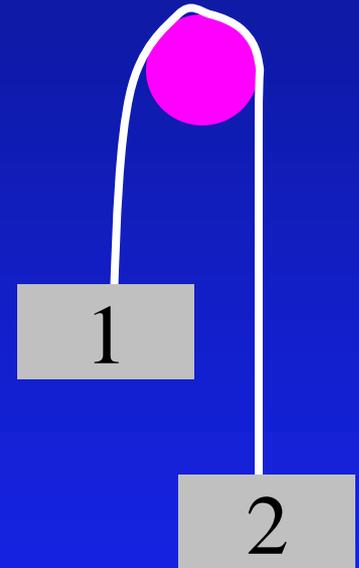
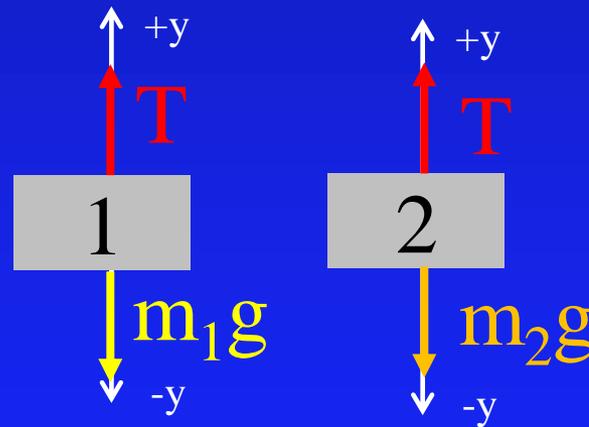
Step 3 – Newton's 2nd!

$$F_{\text{Net}} = m a$$

$$1) T - m_1 g = 0$$

$$2) T - m_2 g = 0$$

$$\Rightarrow m_1 = m_2$$



Another Force Example: Springs

- Force exerted by a spring is directly proportional to its displacement x (stretched or compressed).

$$F_{\text{spring}} = -k x$$

- Example:** When a 5 kg mass is suspended from a spring, the spring stretches $x_1 = 8$ cm. If it is hung by two identical springs, they will stretch $x_2 =$

A) 4 cm B) 8 cm C) 16 cm

1 Spring

$$S_1 - W = 0$$

$$S_1 = W$$

$$kx_1 = mg$$

$$k = mg/x_1 = 612.5 \text{ N/m}$$

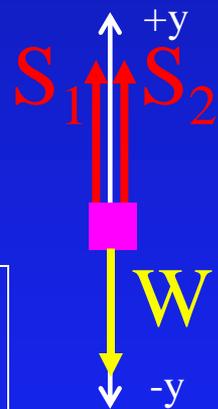
2 Springs

$$S_1 + S_2 - W = 0$$

$$kx_2 + kx_2 = 2kx_2 = W = mg$$

$$x_2 = mg/(2k) = (5\text{kg}) \cdot (9.8\text{m/s}^2) / (2 \cdot 612.5\text{N})$$

$$\text{So: } x_2 = 4 \text{ cm.}$$



2 Dimensional Equilibrium!



Calculate force of hand to keep a book sliding at *constant speed* (i.e. $a = 0$), if the mass of the book is 1 Kg, $\mu_s = .84$ and $\mu_k = .75$

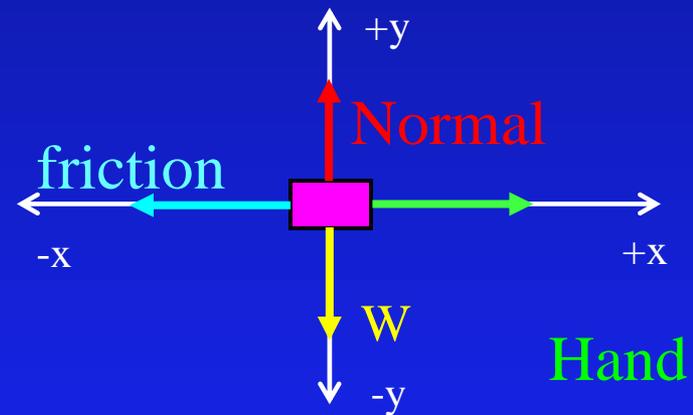
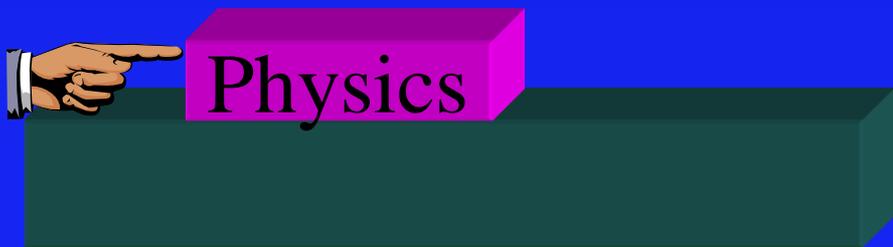
We do exactly the same thing as before, except in both x and y directions!

Step 1 – Draw!

Step 2 – Forces!

Step 3 – Newton's 2nd ($F_{\text{Net}} = ma$)!

Treat x and y independently!



$$F_{\text{Net}, y} = N - W = ma_y = 0$$

$$F_{\text{Net}, x} = H - f = ma_x = 0$$

This is what we want!

Calculate force of hand to keep the book sliding at a **constant speed** (*i.e.* $a = 0$), if the mass of the book is 1 Kg, $\mu_s = .84$ and $\mu_k = .75$.

$$F_{\text{Net}, y} = N - W = 0$$

$$F_{\text{Net}, x} = H - f = 0$$

$$N = W$$

$$H = f$$

- Magnitude of frictional force is proportional to the normal force and always opposes motion!

$$\rightarrow f_{\text{kinetic}} = \mu_k N$$

μ_k coefficient of Kinetic (sliding) friction

$$\rightarrow f_{\text{static}} \leq \mu_s N$$

μ_s coefficient of Static (stationary) friction

$$H = f = \mu_k N = \mu_k W = \mu_k mg$$

$$= (0.75) * (1 \text{ kg}) * (9.8 \text{ m/s}^2)$$

$$H = 7.35 \text{ N}$$

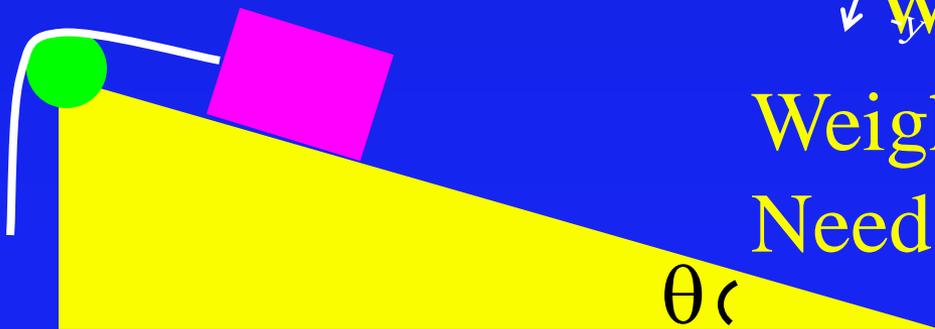
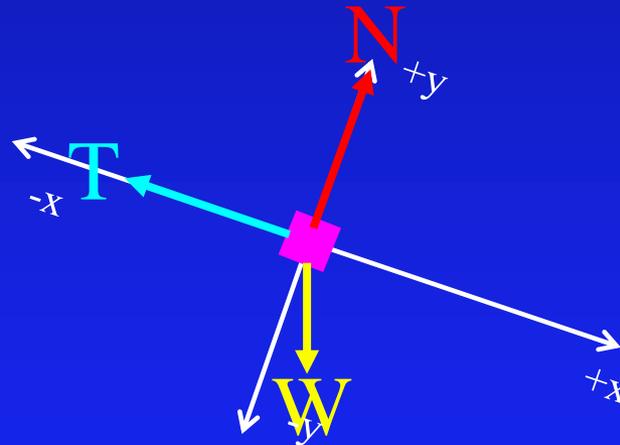
Forces in 2 Dimensions: Ramp

Calculate tension in the rope necessary to keep the 5 kg block from sliding down a **frictionless** incline of 20 degrees.

Step 1 - Draw!

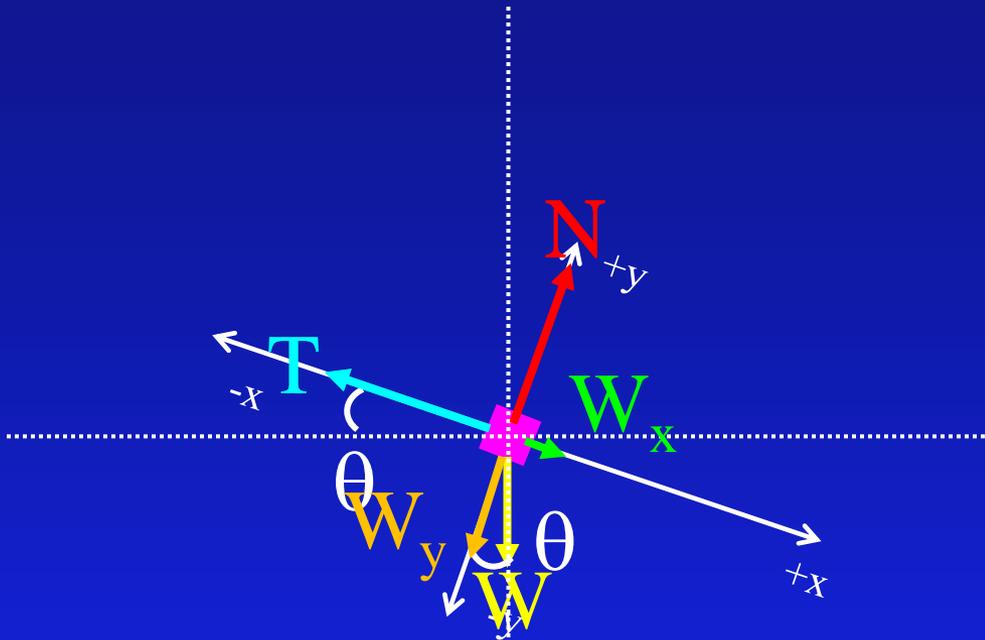
You should draw axes parallel and perpendicular to motion!

Step 2 - Forces!

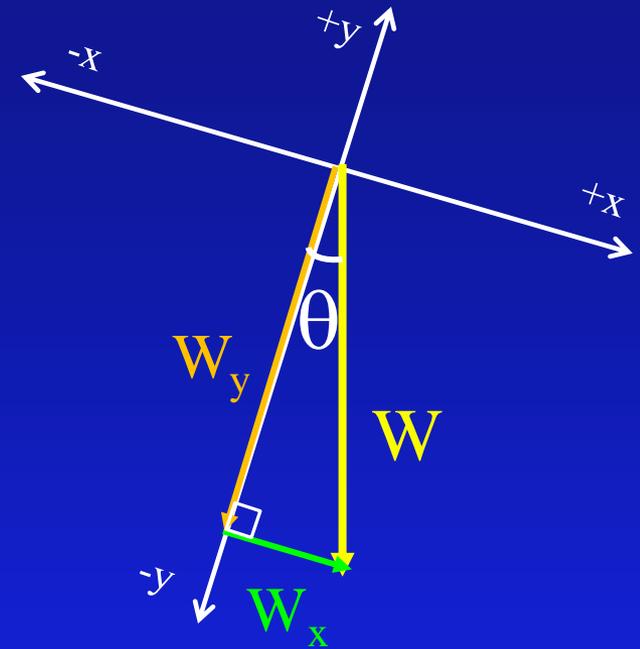


Weight is not in x or y direction!
Need to DECOMPOSE it!

Vector Decomposition



Now: Step 3 – Newton's 2nd!



Split W into COMPONENTS parallel to axes

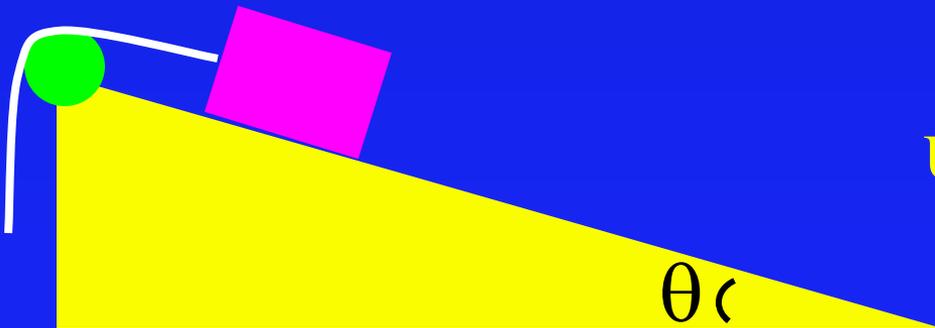
Note that

$$\vec{W} = \vec{W}_y + \vec{W}_x$$

$$W_x = W \sin \theta$$

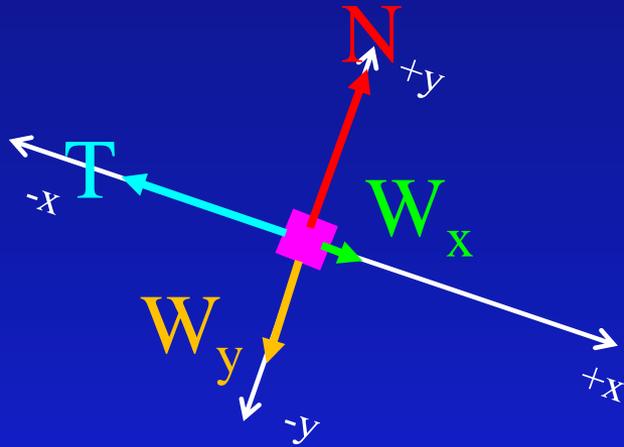
$$W_y = W \cos \theta$$

Using Trig:



Calculate force necessary to keep the 5 kg block from sliding down a frictionless incline of 20 degrees.

Now: Step 3 – Newton's 2nd!



$$W_x = W \sin \theta$$

$$W_y = W \cos \theta$$

x direction:

$$F_{\text{net}, x} = ma_x$$

System is in equilibrium ($a = 0$)!

$$F_{\text{net}, x} = 0$$

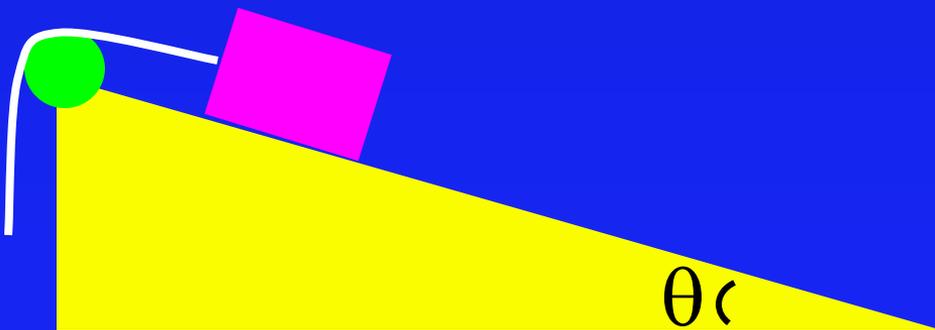
$$W_x - T = 0$$

$$T = W_x = W \sin \theta$$

$$= mg \sin \theta$$

$$= (5\text{kg})(9.8\text{m/s}^2) \sin(20^\circ)$$

$$T = 16.8 \text{ N}$$



Normal Force ACT

What is the normal force of the ramp on the block?

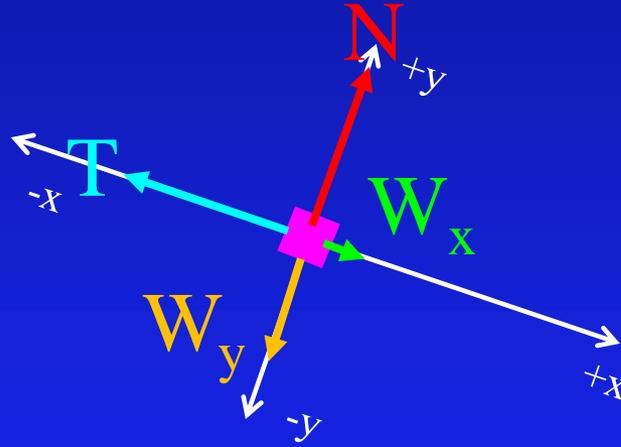
A) $N > mg$

B) $N = mg$

C) $N < mg$

$$W_x = W \sin \theta$$

$$W_y = W \cos \theta$$



y direction:

$$F_{\text{net}, y} = ma_y$$

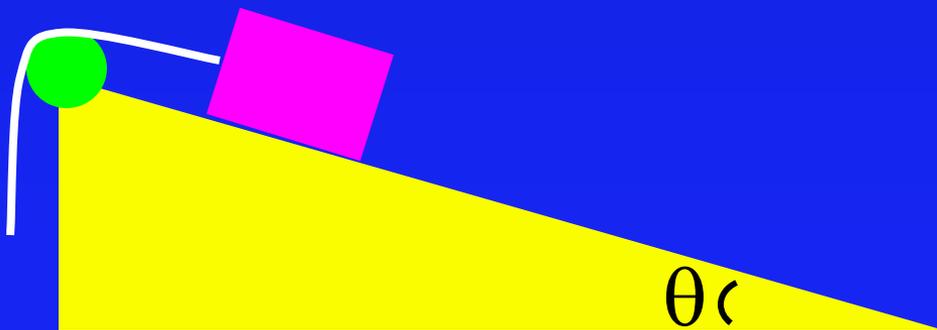
Equilibrium ($a = 0$)!

$$F_{\text{net}, y} = 0$$

$$N - W_y = 0$$

$$N = W_y = W \cos \theta$$

$$N = mg \cos \theta$$



Summary

● Contact Force: Tension

- Force parallel to string
- Always Pulls, tension equal everywhere

● Contact Force: Spring

- Can push or pull, force proportional to displacement
- $F = k x$

● Contact Force: Friction

- Static and kinetic
- Magnitude of frictional force is proportional to N

● Two Dimensional Examples

- Choose coordinate system; choose wisely!
- Analyze each direction independently

Force at Angle Example

- A person is pushing a 15 kg block across a floor with $\mu_k = 0.4$ at a constant speed. If she is pushing down at an angle of 25 degrees, what is the magnitude of her force on the block?

x- direction: $F_{\text{Net}, x} = ma_x$

$$P_x - f = P \cos(\theta) - f = 0$$

$$P \cos(\theta) - \mu N = 0$$

$$N = P \cos(\theta) / \mu$$

y- direction: $F_{\text{Net}, y} = ma_y$

$$N - W - P_y = N - W - P \sin(\theta) = 0$$

$$N - mg - P \sin(\theta) = 0$$

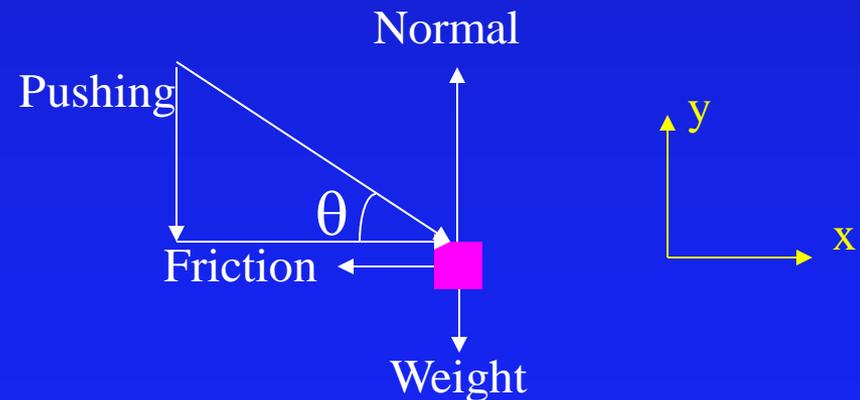
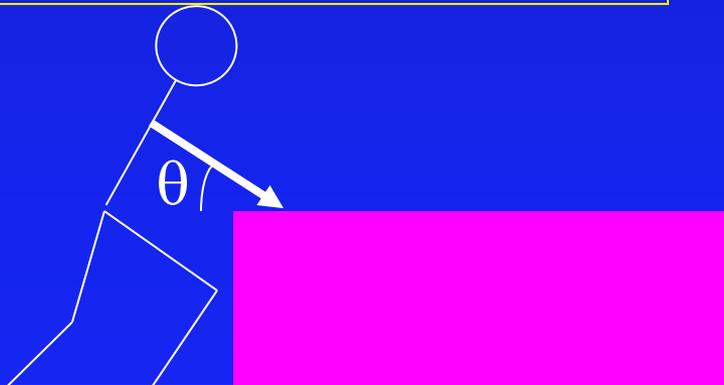
Combine:

$$(P \cos(\theta) / \mu) - mg - P \sin(\theta) = 0$$

$$P (\cos(\theta) / \mu - \sin(\theta)) = mg$$

$$P = mg / (\cos(\theta) / \mu - \sin(\theta))$$

$$P = 80 \text{ N}$$



Tension Example:

- Determine the force exerted by the hand to suspend the 45 kg mass as shown in the picture.

A) 220 N

B) 440 N

C) 660 N

D) 880 N

E) 1100 N

$$F_{\text{Net}} = m a$$

$$T + T - W = 0$$

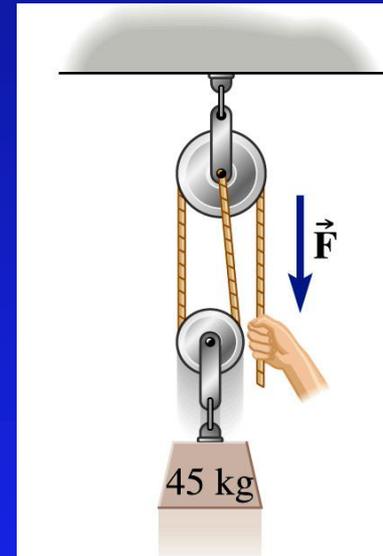
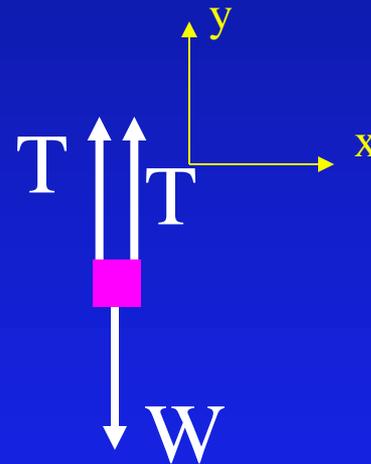
$$2T = W$$

$$T = m g / 2$$

$$= (45 \text{ kg} \times 9.8 \text{ m/s}^2) / 2$$

$$= 220 \text{ N}$$

- Remember the magnitude of the tension is the same everywhere along the rope!



Tension ACT II

- Determine the force exerted by the ceiling to suspend pulley holding the 45 kg mass as shown in the picture.

A) 220 N

B) 440 N

C) 660 N

D) 880 N

E) 1100 N

$$\Sigma F = m a$$

$$F_c - T - T - T = 0$$

$$F_c = 3 T$$

$$F_c = 3 \times 220 \text{ N} = 660 \text{ N}$$

- Remember the magnitude of the tension is the same everywhere along the rope!

