

Physics 101: Lecture 24

Ideal Gas Law and Kinetic Theory

- Today's lecture will cover Textbook Chapter 13.5-13.7

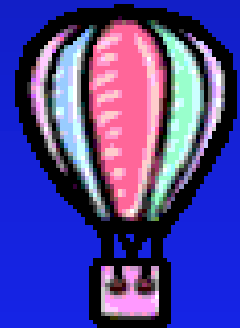


Molecular Picture of Gas

- Gas is made up of many individual molecules
- Number density is number of molecules/volume:
 - $N/V = \rho/m$
 - ρ is the mass density
 - m is the mass for one molecule

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 1/12 \text{ of a mass of } C^{12}$$

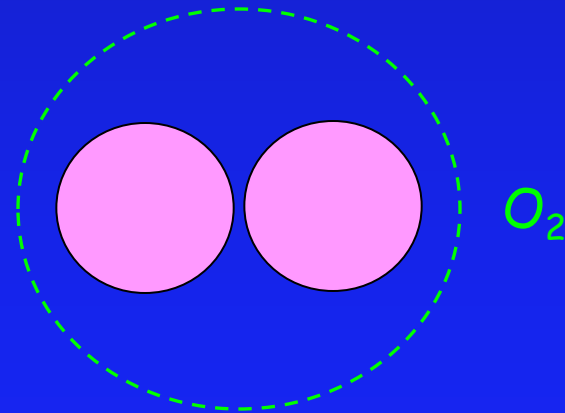
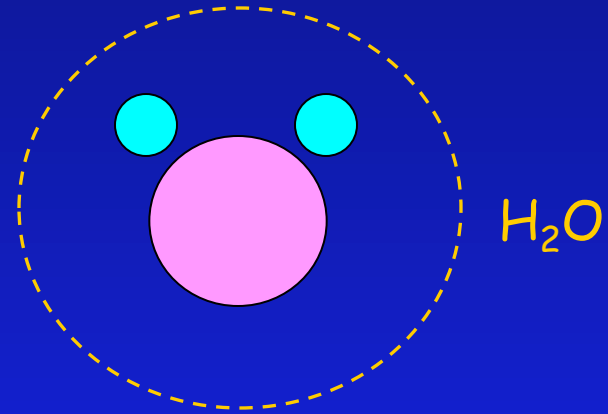
- Number of moles: $n = N / N_A$
 - N_A = Avogadro's Number = $6.022 \times 10^{23} \text{ mole}^{-1}$
- Mass of 1 mole of “stuff” in grams = molecular mass in u
 - e.g., 1 mole of N_2 has mass of $2 \times 14 = 28$ grams



Atomic Act I

Which contains the most molecules ?

1. A mole of water (H_2O)
2. A mole of oxygen gas (O_2)
3. Same ← correct



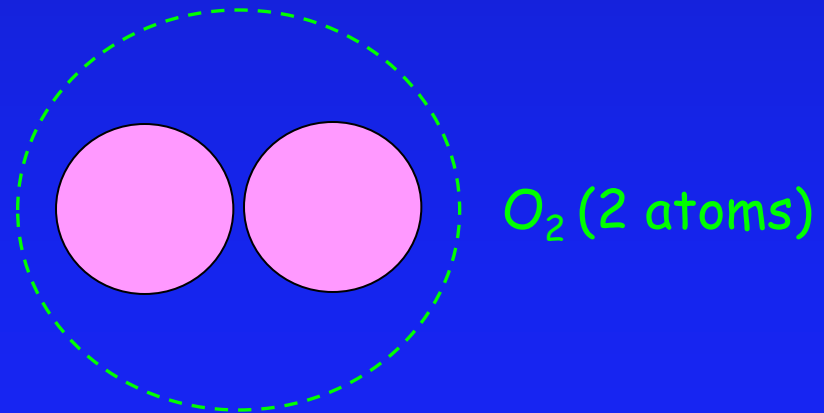
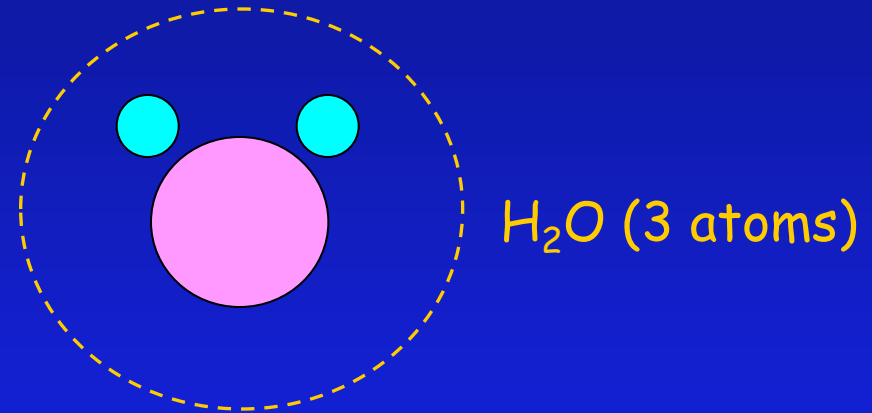
Atomic Act II

Which contains the most atoms ?

1. A mole of water (H_2O) ← correct

2. A mole of oxygen gas (O_2)

3. Same



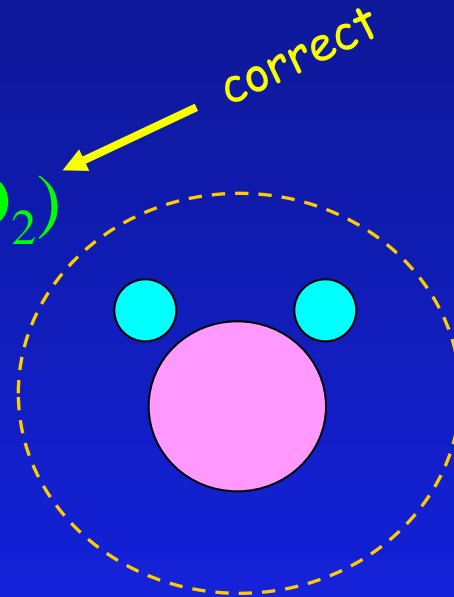
Atomic Act III

Which weighs the most ?

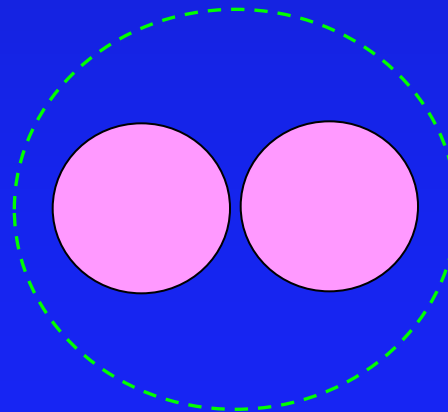
1. A mole of water (H_2O)

2. A mole of oxygen gas (O_2)

3. Same



H_2O ($M = 16 + 1 + 1$)



O_2 ($M = 16 + 16$)

The Ideal Gas Law

- $P V = N k_B T$

- P = pressure in N/m^2 (or Pascals)

- V = volume in m^3

- N = number of molecules

- T = absolute temperature in K

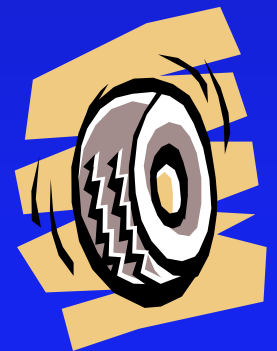
- k_B = Boltzmann's constant = $1.38 \times 10^{-23} \text{ J/K}$

- Note: $P V$ has units of N-m or J (energy!)

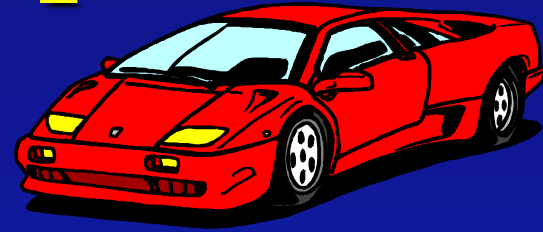
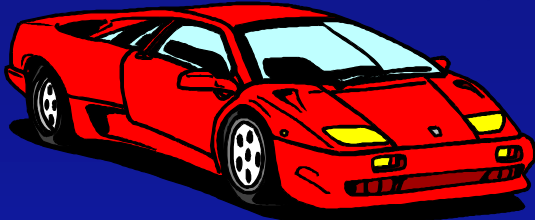
- $P V = n R T$

- n = number of moles

- R = ideal gas constant = $N_A k_B = 8.31 \text{ J/mol/K}$



Ideal Gas Law ACT I

$$PV = nRT$$


You inflate the tires of your car so the pressure is 30 psi, when the air inside the tires is at 20 degrees C. After driving on the highway for a while, the air inside the tires heats up to 38 C. Which number is closest to the new air pressure?

1) 16 psi

2) 32 psi

3) 57 psi

Careful, you need to use the temperature in K

$$P = P_0 (38+273)/(20+273)$$

Ideal Gas Law: ACT II

$$pV = nRT$$

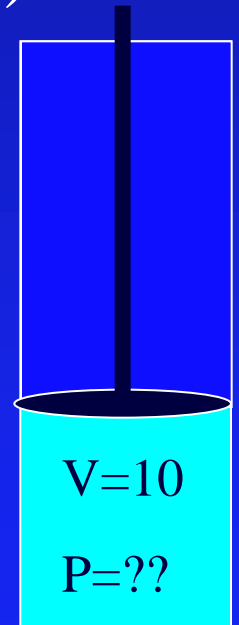
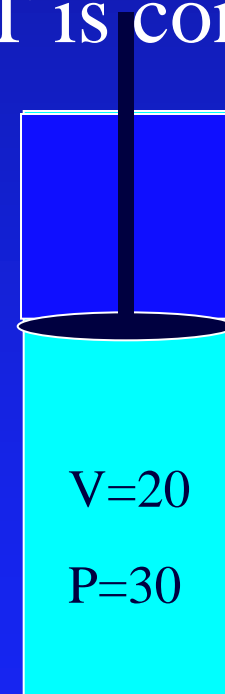
- A piston has volume 20 ml, and pressure of 30 psi. If the volume is decreased to 10 ml, what is the new pressure? (Assume T is constant.)

1) 60

2) 30

3) 15

- When n and T are constant, pV is constant (Boyle's Law)



Balloon ACT 1



- What happens to the pressure of the air inside a hot-air balloon when the air is heated? (Assume V is constant)

1) Increases 2) Same 3) Decreases

Balloon is still open to atmospheric pressure,
so it stays at 1 atm

Balloon ACT 2



- What happens to the buoyant force on the balloon when the air is heated?
(Assume V remains constant)

1) Increases 2) Same 3) Decreases

$$F_B = \rho V g$$

ρ is density of outside air!



Balloon ACT 3



- What happens to the number of air molecules inside the balloon when the air is heated? (Assume V remains constant)

1) Increases 2) Same 3) Decreases

$$PV = NkT$$

P and V are constant. If T increases N decreases.

Note! this is not a pressure effect, it is a density effect. As T increases, the density decreases the balloon then floats due to Archimedes principle. The pressure remains constant!

Ideal Gas Law: Demos

$$pV = nRT$$

- When T is constant, PV is constant (Boyle's Law)
 - Boyle's law demo
- When P is constant, V is proportional to T
 - Hot air balloon, helium and oxygen in LN_2
- When V is constant, P is proportional to T
 - Explosion!

Kinetic Theory:

The relationship between energy and temperature
(for monatomic ideal gas)

$$\Delta p_x = 2mv_x$$

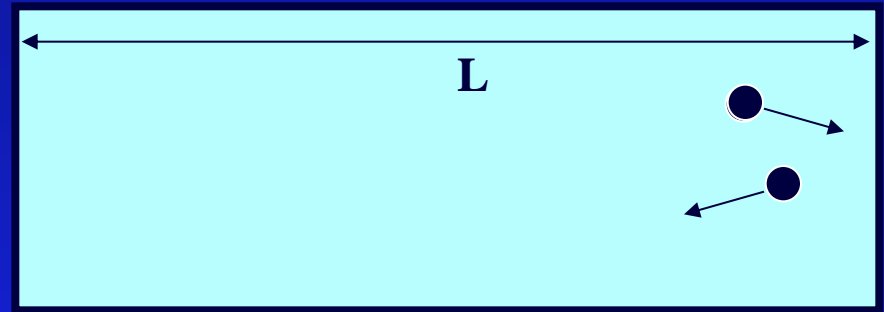
$$\Delta t = 2 \frac{L}{v_x}$$

$$F_{avg} = \frac{\Delta p_x}{\Delta t} = \frac{mv_x^2}{L}$$

For N molecules, multiply by N

$$P = \frac{F}{A} = \frac{Nmv_x^2}{V}$$

Note $KE = \frac{1}{2} m v^2 = \frac{3}{2} m v_x^2$



$$P = \frac{2}{3} \frac{N}{V} \langle K_{tr} \rangle$$

Using $PV = NkT$

$$\langle K_{tr} \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$\langle \rangle$ means *average*.

$kT/2$ energy per
degree of
freedom =
equipartition
theorem

Prelecture 1

root-mean-square?

Suppose you want the rms (*root-mean-square*) speed of molecules in a sample of gas to double. By what factor should you increase the temperature of the gas?

1. 2

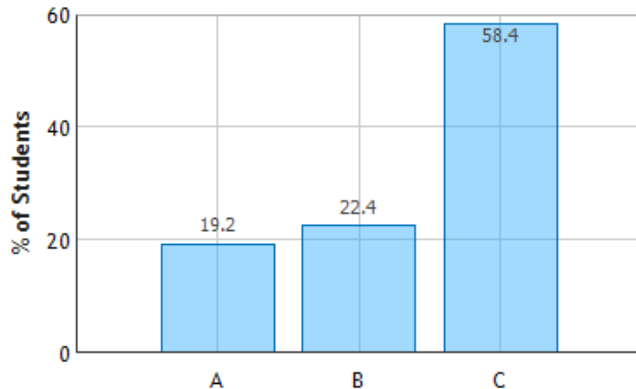
2. $\sqrt{2}$

3. 4 ← correct

$$\langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

- If v doubles, v^2 quadruples
- Therefore, T quadruples

Ideal Gas and Kinetic Theory: Question 1 (N = 411)



$$x_{\text{rms}} = \sqrt{\frac{1}{n} (x_1^2 + x_2^2 + \cdots + x_n^2)}$$

Example

- What is the rms speed of a nitrogen (N_2) molecule in this classroom?

$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3 k_B T}{m}$$

$$v = 510 \text{ m/s}$$

$$= 1150 \text{ mph!}$$

$$\langle v^2 \rangle = \frac{3(1.38 \times 10^{-23} \text{ J/K})(273 + 20) \text{ K}}{(28 \text{ u}) \times (1.66 \times 10^{-27} \text{ kg/u})}$$

Summary

- Ideal Gas Law $PV = n R T$
 - P = pressure in N/m^2 (or Pascals)
 - V = volume in m^3
 - n = # moles
 - $R = 8.31 \text{ J/ (K mole)}$
 - T = Temperature (K)
- Kinetic Theory of Monatomic Ideal Gas
 - $\langle K_{\text{tr}} \rangle = \frac{3}{2} k_B T$