Physics 101: Lecture 24
Ideal Gas Law and Kinetic Theory

- Today's lecture will cover Textbook Chapter 13.5-13.7

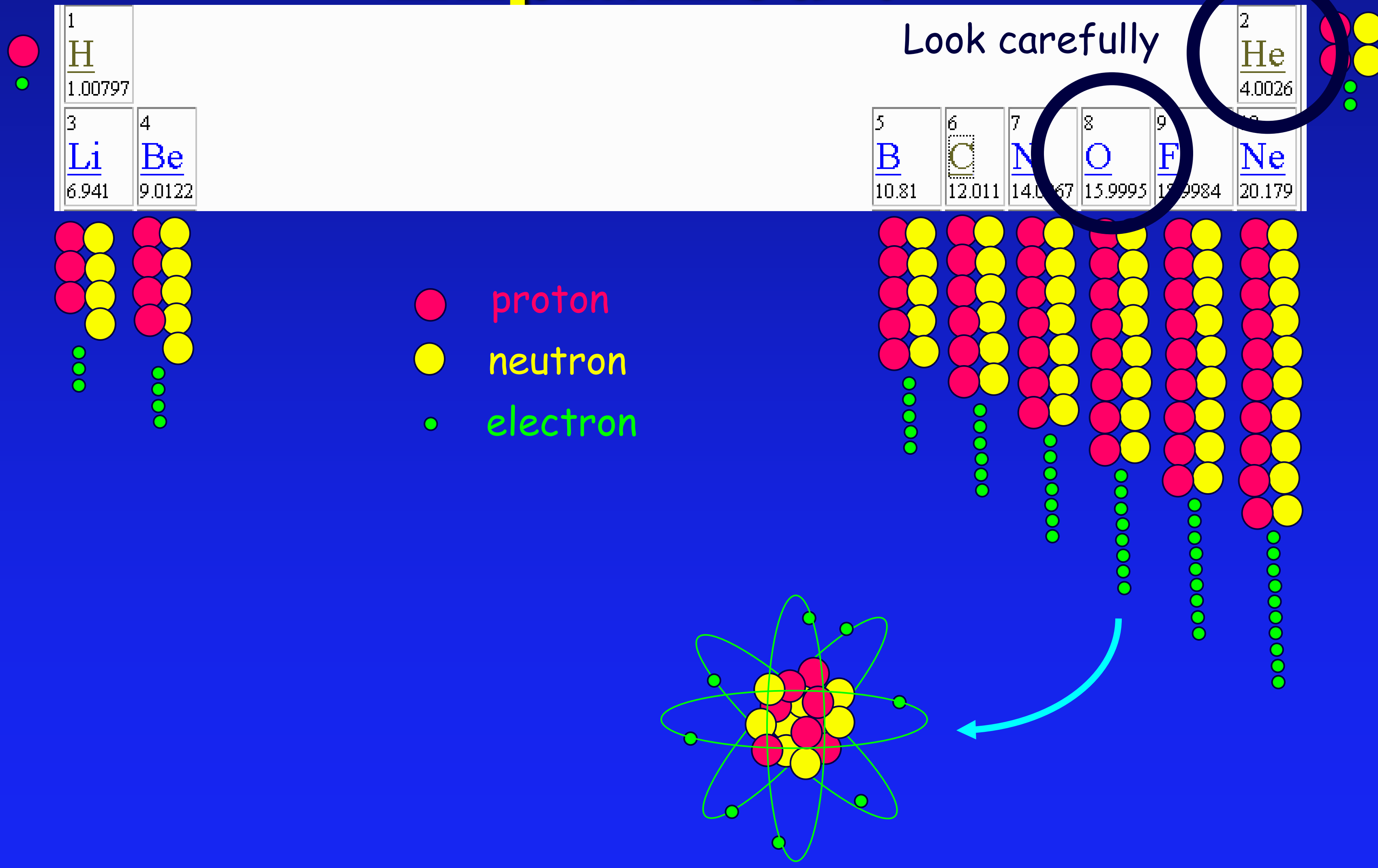
- Exam Monday, Dec. 1!

- Exam Review
  - ➔ No Problem Section this Sunday! (Thanksgiving Break)
  - ➔ Review is on Sunday, Nov. 30 7-9PM Loomis 141

Aside: The Periodic Table

Ia	IIa	IIIb	IVb	Vb	VIb	VIIb	VII			Ib	IIb	IIa	IVa	Va	VIa	VIIa	O
1 H 1.00797																	2 He 4.0026
3 Li 6.941	4 Be 9.0122											5 B 10.81	6 C 12.011	7 N 14.0067	8 O 15.9995	9 F 18.9984	10 Ne 20.179
11 Na 22.9898	12 Mg 24.305											13 Al 26.9815	14 Si 28.086	15 P 30.9738	16 S 32.06	17 Cl 35.453	18 Ar 39.948
19 K 39.098	20 Ca 40.08	21 Sc 44.956	22 Ti 47.90	23 V 50.9414	24 Cr 51.996	25 Mn 54.9830	26 Fe 55.847	27 Co 58.9332	28 Ni 58.70	29 Cu 63.546	30 Zn 65.38	31 Ga 69.72	32 Ge 72.59	33 As 74.9216	34 Se 78.96	35 Br 79.904	36 Kr 83.80
37 Rb 85.4678	38 Sr 87.62	39 Y 88.909	40 Zr 91.22	41 Nb 92.9064	42 Mo 95.94	43 Tc (97)	44 Ru 101.07	45 Rh 102.905	46 Pd 106.04	47 Ag 107.868	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.9046	54 Xe 131.30
55 Cs 132.905	56 Ba 137.34	57 * La 138.91	72 Hf 178.49	73 Ta 180.948	74 W 183.85	75 Re 186.207	76 Os 190.2	77 Ir 192.22	78 Pt 195.09	79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.2	83 Bi 208.980	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra 226.03	89 * Ac (227)															

The Periodic Table Explained ?



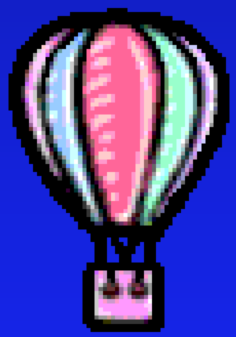
Molecular Picture of Gas

- Gas is made up of many individual molecules

- Number density is number of molecules/volume:
  - ➔ $N/V = \rho/m$
  - ➔ ρ is the mass density
  - ➔ m is the mass for one molecule

    $1\ u = 1.66 \times 10^{-27}\ kg$ = 1/12 of a mass of C^{12}

- Number of moles: $n = N / N_A$
  - ➔ N_A = Avogadro's Number = $6.022 \times 10^{23}\ mole^{-1}$

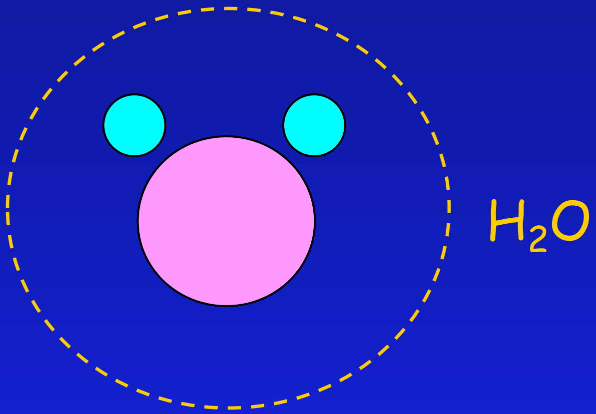


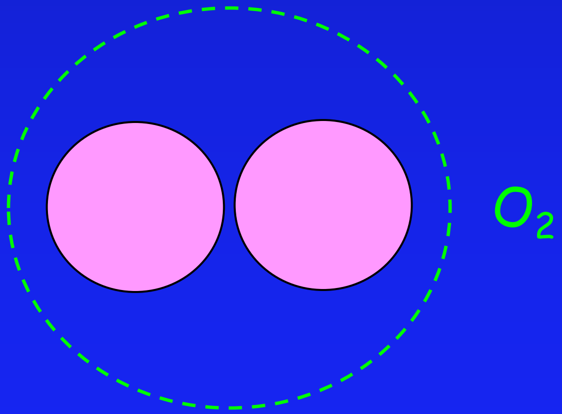
- Mass of 1 mole of "stuff" in grams = molecular mass in u
  - ➔ e.g., 1 mole of N_2 has mass of 2x14=28 grams

Atomic Act I

Which contains the most molecules ?

1. A mole of water (H_2O)
2. A mole of oxygen gas (O_2)
3. Same ⟵ correct

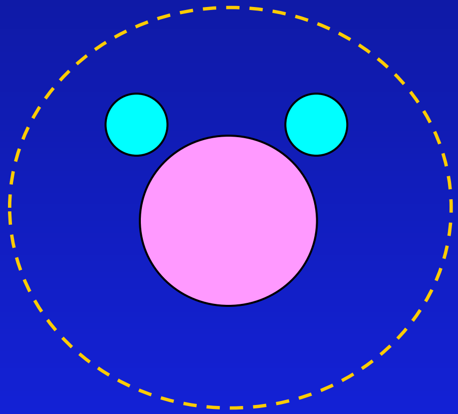




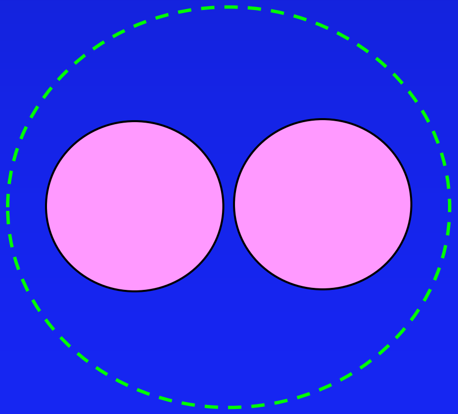
Atomic Act II

Which contains the most atoms ?

1. A mole of water (H_2O) ⟵ correct
2. A mole of oxygen gas (O_2)
3. Same



H_2O (3 atoms)



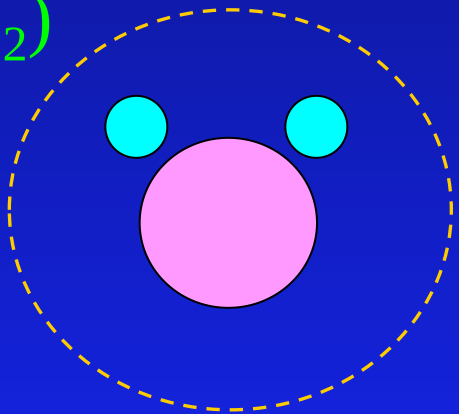
O_2 (2 atoms)

Atomic Act III

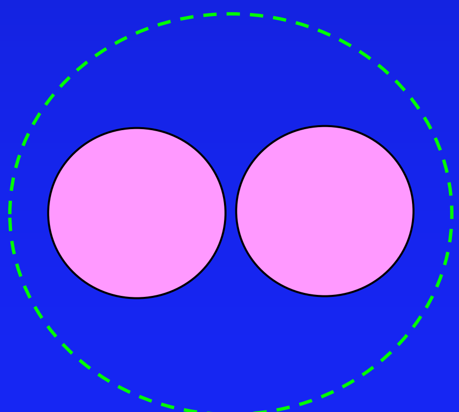
Which weighs the most ?

1. A mole of water (H_2O)
2. A mole of oxygen gas (O_2)
3. Same

correct



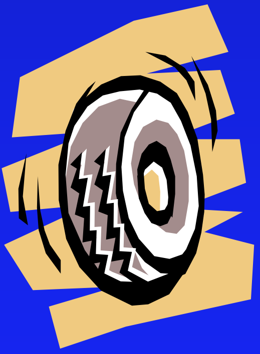
H_2O ($M = 16 + 1 + 1$)



O_2 ($M = 16 + 16$)

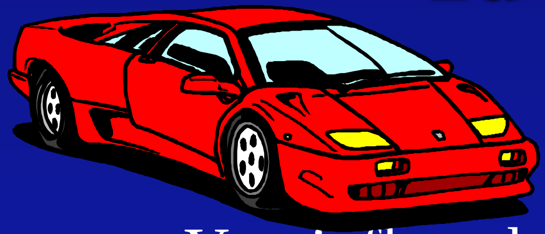
The Ideal Gas Law

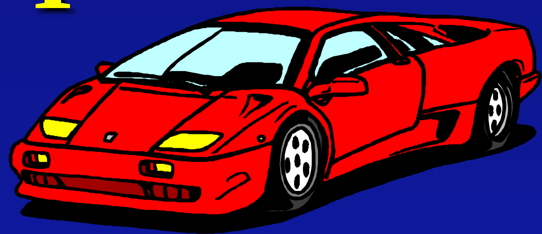
- $P\,V = N\,k_B\,T$
  - ➔ P = pressure in N/m^2 (or Pascals)
  - ➔ V = volume in m^3
  - ➔ N = number of molecules
  - ➔ T = absolute temperature in K
  - ➔ k_B = Boltzmann's constant = 1.38×10^{-23} J/K
  - ➔ Note: P V has units of N-m or J (energy!)



- $P\,V = n\,R\,T$
  - ➔ n = number of moles
  - ➔ R = ideal gas constant = $N_A k_B$ = 8.31 J/mol/K

Ideal Gas Law ACT I
PV = nRT





You inflate the tires of your car so the pressure is 30 psi, when the air inside the tires is at 20 degrees C. After driving on the highway for a while, the air inside the tires heats up to 38 C. Which number is closest to the new air pressure?

1) 16 psi

2) 32 psi

3) 57 psi

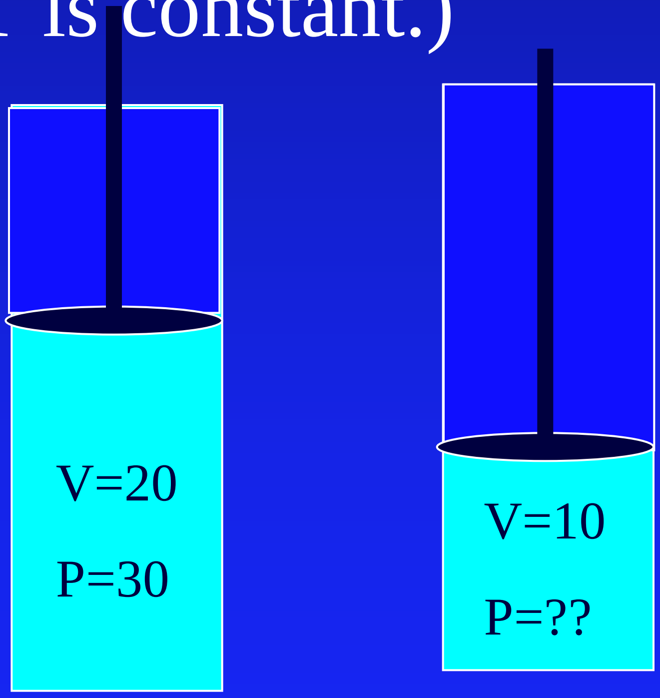
Careful, you need to use the temperature in K

$P = P_0\,(38+273)/(20+273)$

Ideal Gas Law: ACT II
$pV = nRT$

- A piston has volume 20 ml, and pressure of 30 psi. If the volume is decreased to 10 ml, what is the new pressure? (Assume T is constant.)

1) 60   2) 30   3) 15



- When n and T are constant, pV is constant (Boyle's Law)

Balloon ACT 1

- What happens to the pressure of the air inside a hot-air balloon when the air is heated? (Assume V is constant)

1) Increases 2) Same 3) Decreases

Balloon is still open to atmospheric pressure, so it stays at 1 atm

Balloon ACT 2

- What happens to the buoyant force on the balloon when the air is heated? (Assume V remains constant)

1) Increases   2) Same   3) Decreases

$F_B = \rho \, V \, g$

ρ is density of outside air!

Balloon ACT 3

- What happens to the number of air molecules inside the balloon when the air is heated? (Assume V remains constant)

1) Increases 2) Same 3) Decreases

$PV = NkT$

P and V are constant. If T increases N decreases.

Note! this is not a pressure effect, it is a density effect. As T increases, the density decreases, and the balloon floats due to Archimedes principle. The pressure remains constant!

Ideal Gas Law: Demos
$pV = nRT$

- When T is constant, PV is constant (Boyle's Law)
  - ➔ Boyle's law demo
- When P is constant, V is proportional to T
  - ➔ Helium and oxygen in LN_2
- When V is constant, P is proportional to T
  - ➔ Explosion!

Kinetic Theory:

The relationship between energy and temperature (for monatomic ideal gas)

$$\Delta p_x = 2mv_x$$

$$\Delta t = \frac{2L}{v_x}$$

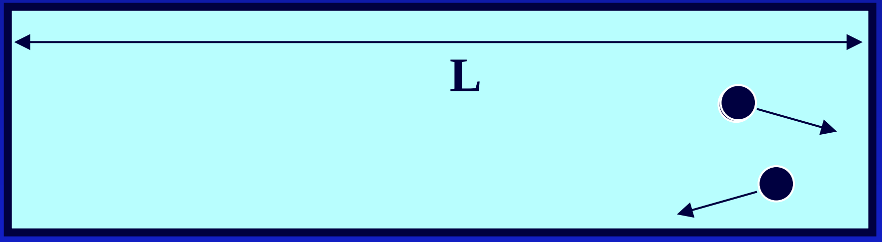
$$F_{avg} = \frac{\Delta p_x}{\Delta t} = \frac{mv_x^2}{L}$$

For N molecules, multiply by N

$$P = \frac{F}{A} = \frac{1}{A}\frac{Nmv_x^2}{L} = \frac{N}{V}mv_x^2$$

Note $K = \frac{1}{2} m v^2$ => $m v_x^2 = 2/3\ K$

$(v^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2)$



$$\boxed{P = \frac{2}{3}\frac{N}{V}\langle K\rangle}$$

⟨⟩ means *average.*

Using PV = NkT

$$\boxed{\langle K\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT}$$

Root Mean Square

$$\boxed{v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{\frac{3kT}{m}}}$$

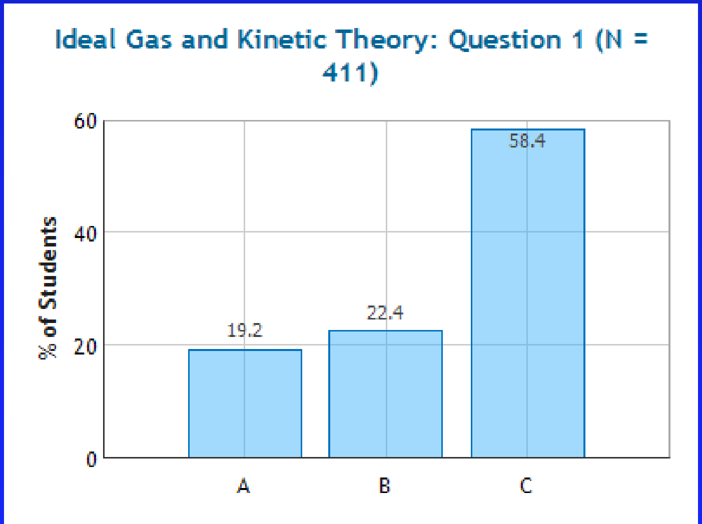
Prelecture 1

root-mean-square?

Suppose you want the rms (*root-mean-square*) speed of molecules in a sample of gas to double. By what factor should you increase the temperature of the gas?

1. 2
2. $\sqrt{2}$
3. 4 ← correct

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$



Example

- What is the rms speed of a nitrogen (N_2) molecule in this classroom?

$$\langle K \rangle = \frac{3}{2} k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3 k_B T}{m}$$

$$\langle v^2 \rangle = \frac{3(1.38\times 10^{-23}\ \text{J/K})(273+20)\text{K}}{(28\ \text{u})\times(1.66\times 10^{-27}\ \text{kg/u})}$$

$$v_{rms} = 510\ \text{m/s}$$

$$= 1150\ \text{mph!}$$

Summary

- Ideal Gas Law $PV = n R T$
  - ➔ P = pressure in N/m^2 (or Pascals)
  - ➔ V = volume in m^3
  - ➔ n = # moles
  - ➔ $R = 8.31$ J/ (K mole)
  - ➔ T = Temperature (K)
- Kinetic Theory of Monatomic Ideal Gas
  - ➔ $\langle K \rangle = 3/2\ k_B\ T$