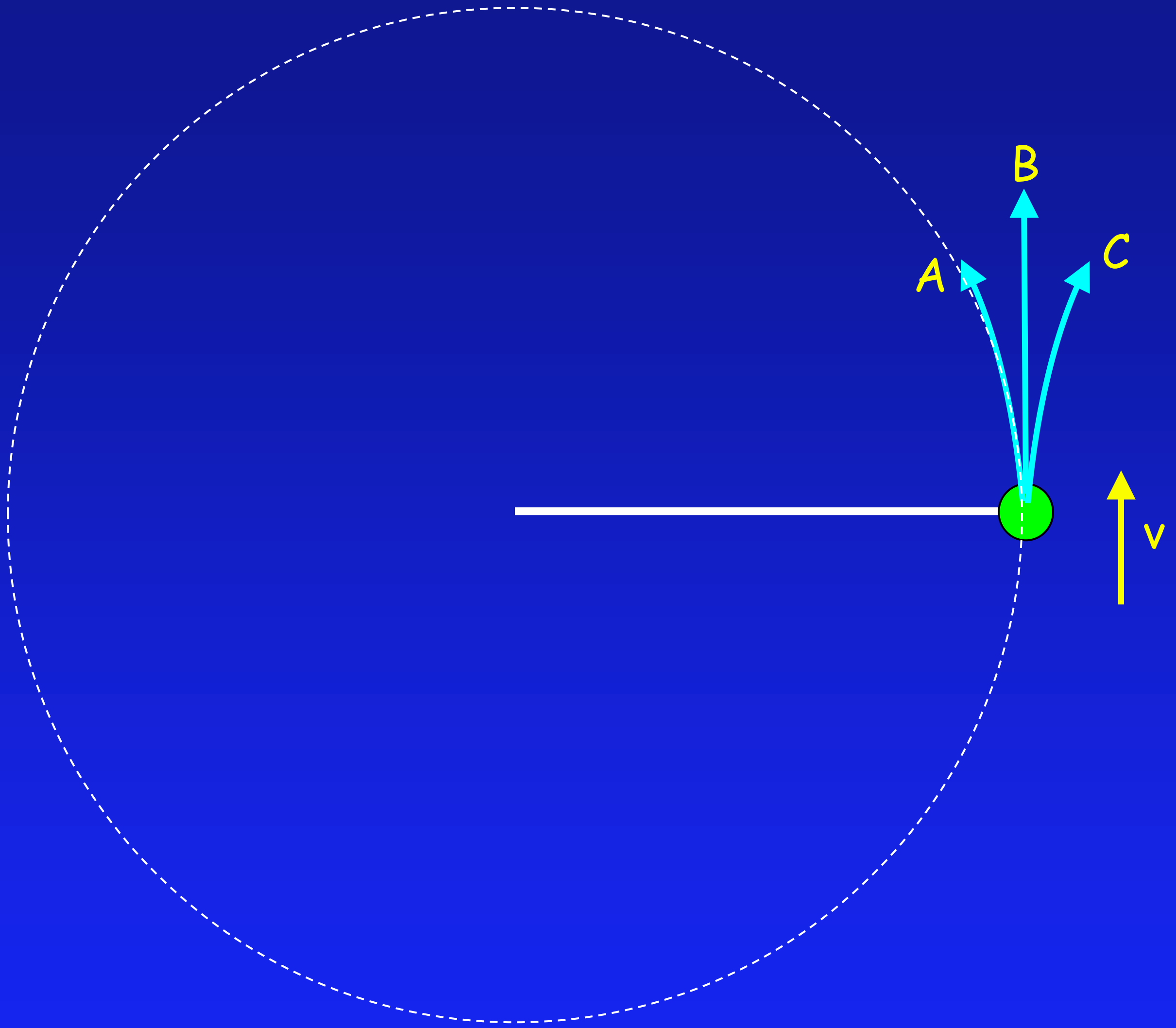
Physics 101: Lecture 08

Centripetal Acceleration and Circular Motion

http://www.youtube.com/watch?v=ZyF5WsmXRaI

- **Today's lecture will cover Chapter 5**
- **Exam I is Monday, March 3 (2 weeks!)**

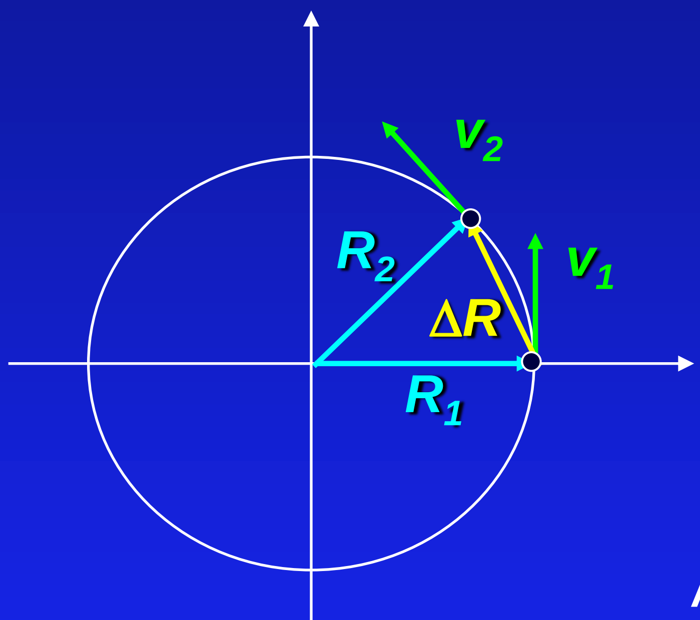
Circular Motion Act

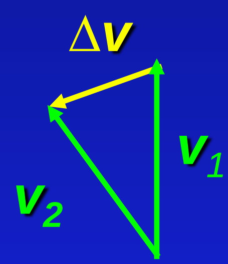


Answer: B

A ball is going around in a circle attached to a string. If the string breaks at the instant shown, which path will the ball follow (demo)?

Acceleration in Uniform Circular Motion





$\mathbf{a}_{ave} = \Delta\mathbf{v} / \Delta t$

Acceleration inward

$$a = \frac{v^2}{R}$$

Centripetal Acceleration

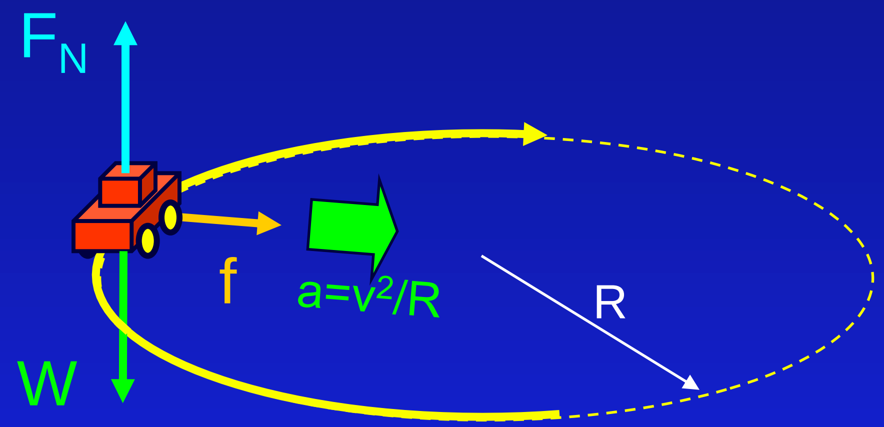
Directed radially inward!

Acceleration is due to change in direction, not speed. Since the object turns "toward" center, there must be a force toward center: "Centripetal Force"

Checkpoint

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. How many forces are acting on the car?

A) 1
B) 2
C) 3 ← correct
D) 4
E) 5



"Friction, Gravity, & Normal"

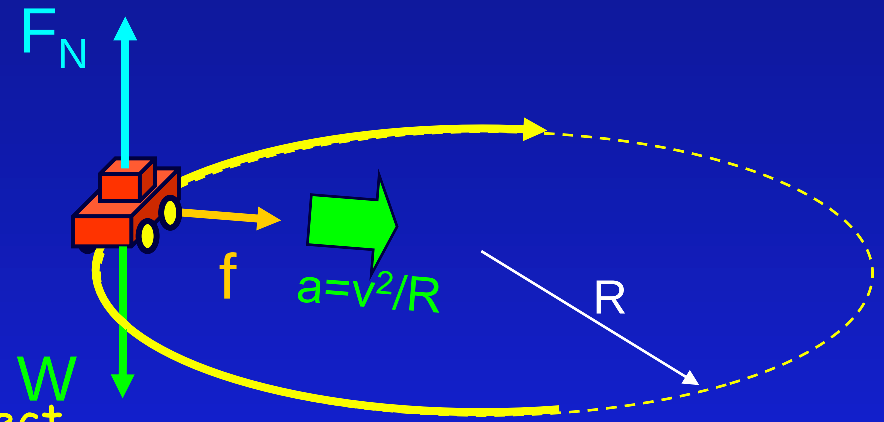
$\mathbf{F_{Net}} = m\mathbf{a} = mv^2/R$

"Centripetal Force" is NOT an additional force!

Draw your FBD as normal, and one of the forces will be the Centripetal Force!

Checkpoint

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. The net force on the car is



A. Zero
B. Pointing radially inward ← correct
C. Pointing radially outward

$$F_{Net} = m\mathbf{a} = mv^2/R$$

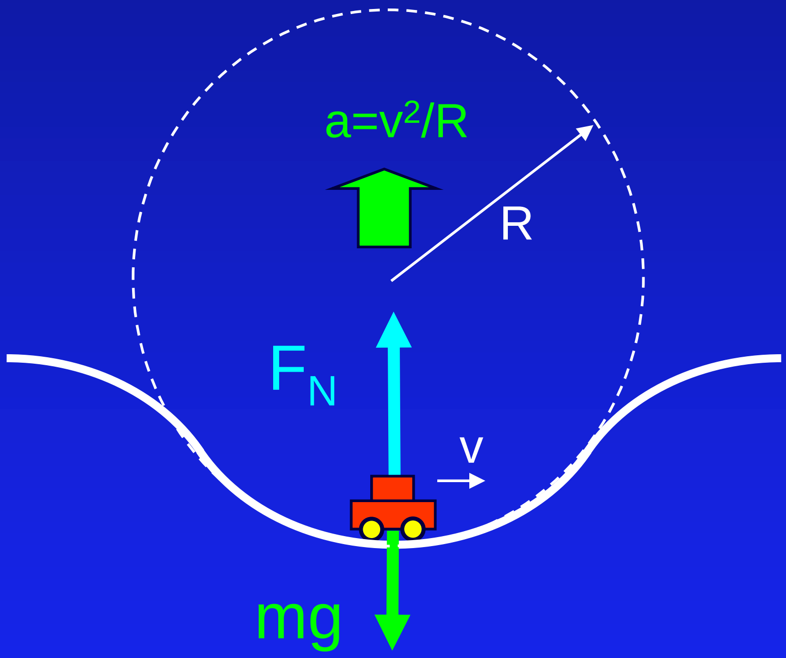
ACT

Suppose you are driving through a valley whose bottom has a circular shape. If your mass is m, what is the magnitude of the normal force F_N exerted on you by the car seat as you drive past the bottom of the hill

A. $F_N < mg$

B. $F_N = mg$

C. $F_N > mg$ ← correct



$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$F_N - mg = mv^2/R$$

$$F_N = mg + mv^2/R$$

Roller Coaster Example

What is the minimum speed you must have at the top of a 20 meter roller coaster loop, to keep the wheels on the track?

Y Direction: $F_{Net} = ma$

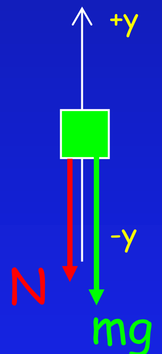
$$-N - mg = m\,a = m\,v^2/R$$

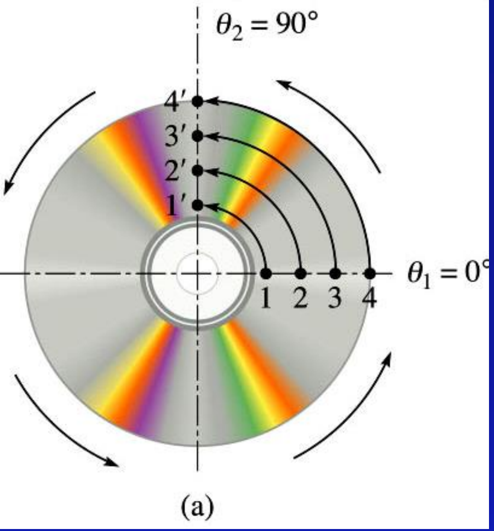
Let N = 0, just touching

$$-mg = -m\,v^2/R$$

$$g = v^2 / R$$

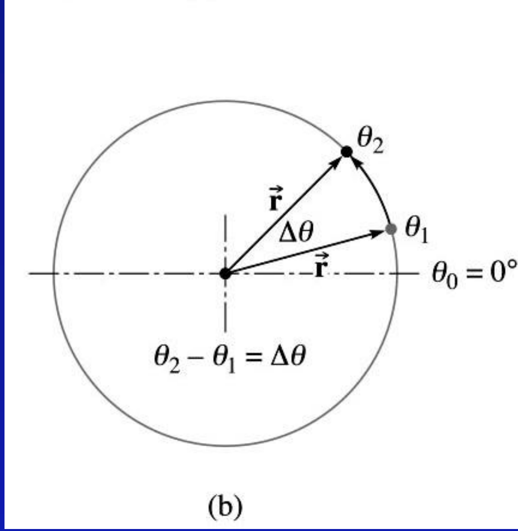
$$v = \mathrm{sqrt}(g*R) = 9.9 \text{ m/s}$$





(a)

Circular Motion

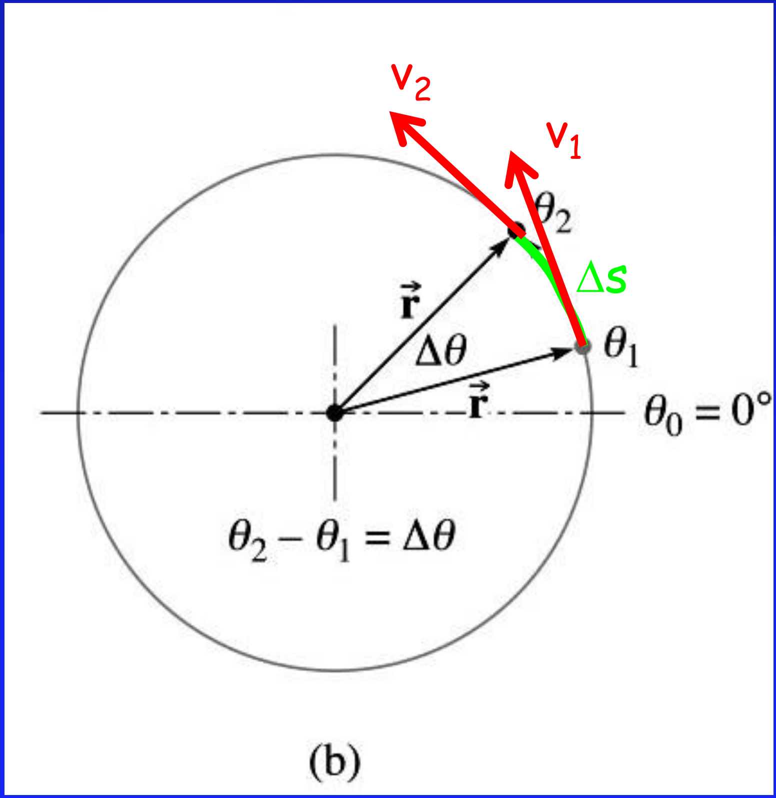


(b)

- Angular displacement $\Delta\theta = \theta_2 - \theta_1$
  - ➔How far it has rotated
  - ➔Units radians (2π = 1 revolution)
- Angular velocity $\omega = \Delta\theta/\Delta t$
  - ➔How fast it is rotating
  - ➔Units radians/second
- Period =1/frequency $T = 1/f = 2\pi / \omega$
  - ➔Time to complete 1 revolution

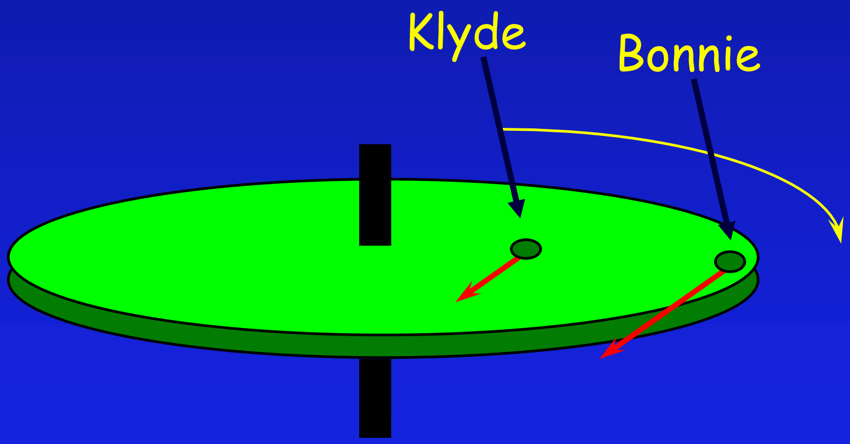
Circular to Linear

- Displacement $\Delta s = r\,\Delta\theta$ (θ in radians)
- Speed $|v| = \Delta s/\Delta t = r\,\Delta\theta/\Delta t = r\omega$
- Direction of v is tangent to circle



Merry-Go-Round ACT

- Bonnie sits on the outer rim of a merry-go-round with radius 3 meters, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds (demo).
  - ➔Klyde's speed is:



(a) **the same as Bonnie's**

(b) **twice Bonnie's**

(c) **half Bonnie's**

$$V_{Klyde} = \frac{1}{2} V_{Bonnie}$$

Bonnie travels $2\pi R$ in 2 seconds $v_B = 2\pi R / 2 = 9.42$ m/s

Klyde travels $2\pi (R/2)$ in 2 seconds $v_K = 2\pi (R/2) / 2 = 4.71$ m/s

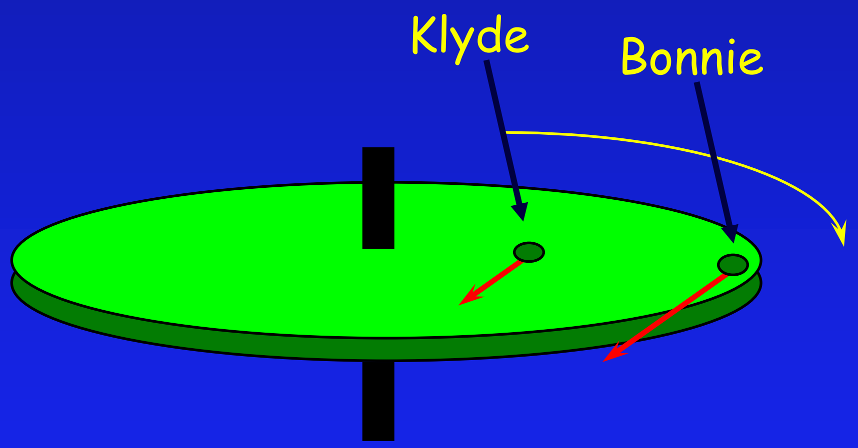
Merry-Go-Round ACT II

- Bonnie sits on the outer rim of a merry-go-round, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds.
  - ➔Klyde's angular velocity is:

(a) **the same as Bonnie's**

(b) **twice Bonnie's**

(c) **half Bonnie's**



- The angular velocity ω of any point on a solid object rotating about a fixed axis is the same.
  - ➔Both Bonnie & Klyde go around once (2π radians) every two seconds.

Angular Acceleration

- Angular acceleration is the change in angular velocity ω divided by the change in time.

$$\overline{\alpha} \equiv \frac{\omega_f - \omega_0}{\Delta t}$$

- If the speed of a roller coaster car is 15 m/s at the top of a 20 m loop, and 25 m/s at the bottom. What is the cars average angular acceleration if it takes 1.6 seconds to go from the top to the bottom?

$$\omega = \frac{V}{R}$$

$$\omega_f = \frac{25}{10} = 2.5$$

$$\omega_0 = \frac{15}{10} = 1.5$$

$$\overline{\alpha} \equiv \frac{2.5 - 1.5}{1.6} = 0.64 \text{ rad/s}^2$$

Summary
(with comparison to 1-D kinematics)

Angular	Linear
$\alpha = constant$	$a = constant$
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$

And for a point at a distance R from the rotation axis:

$x = R\theta$ $v = \omega R$ $a = \alpha R$

CD Player Example

- The CD in a disk player spins at about 20 radians/second. If it accelerates uniformly from rest with angular acceleration of 15 rad/s^2, how many revolutions does the disk make before it is at the proper speed?

$$\omega_0 = 0$$

$$\omega_f = 20 \text{ rad/s}$$

$$\alpha = 15 \text{ rad/s}^2$$

$$\Delta\theta = ?$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\frac{\omega_f^2 - \omega_0^2}{2\alpha} = \Delta\theta$$

$$\frac{20^2 - 0^2}{2\times 15} = \Delta\theta$$

$\Delta\theta$ = 13.3 radians

1 Revolutions = 2 π radians

$\Delta\theta$ = 13.3 radians

= 2.12 revolutions

Summary of Concepts

- Uniform Circular Motion
  - ➔ Speed is constant
  - ➔ Direction is changing
  - ➔ Acceleration toward center $a = v^2 / r$
  - ➔ Newton's Second Law $F = ma$

- Circular Motion
  - ➔ θ = angular position radians
  - ➔ ω = angular velocity radians/second
  - ➔ α = angular acceleration radians/second^2
  - ➔ Linear to Circular conversions $s = r\,\theta$

- Uniform Circular Acceleration Kinematics
  - ➔ Similar to linear!