Exam II

Physics 101:
Lecture 13
Rotational Kinetic Energy and Rotational Inertia

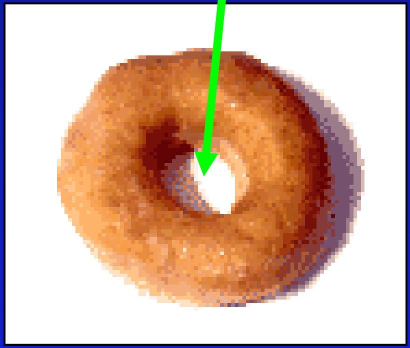
Center of Mass

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{\sum m_i}$$

Center of Mass = Balance point

Center of Mass!

- Shown is a yummy doughnut. Where would you expect the center of mass of this breakfast of champions to be located?



Typical wrong answer:

evenly distributed around the doughnut

in my stomach

doughnuts don't have a center of mass because they are removed and sold as doughnut hole

Center of Mass

$P_{tot} = M_{tot}V_{cm}$ $\qquad$ $F_{ext}\Delta t = \Delta P_{tot} = M_{tot}\Delta V_{cm}$

So if $F_{ext} = 0$ then V_{cm} is constant

Also: $F_{ext} = M_{tot}a_{cm}$

Center of Mass of a system behaves in a SIMPLE way

- moves like a point particle!
- velocity of CM is unaffected by collision if $F_{ext} = 0$

(pork chop demo)

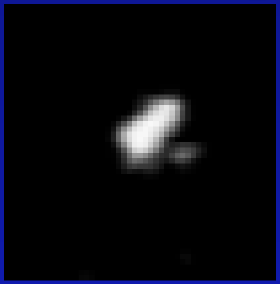
Overview of Semester

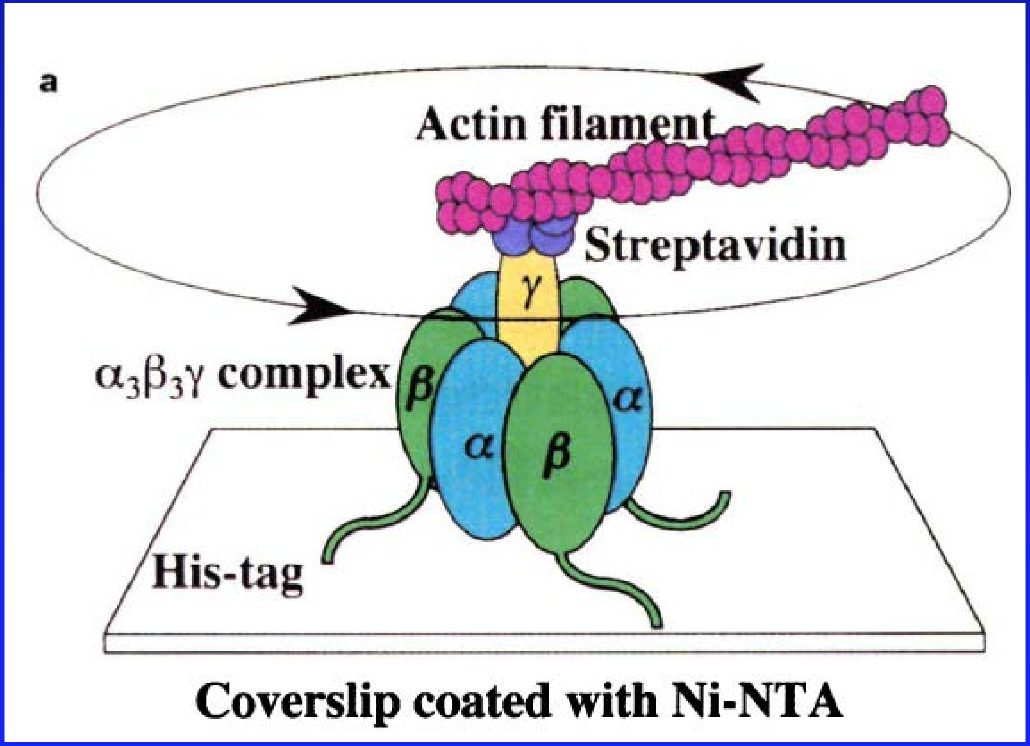
- Newton's Laws
  - ➔ $F_{Net} = m\,a$
- Work-Energy
  - ➔ $F_{Net} = m\,a$ multiply both sides by d
  - ➔ $W_{Net} = \Delta KE$ Energy is "conserved"
  - ➔ Useful when know Work done by forces
- Impulse-Momentum
  - ➔ $F_{Net} = m\,a = \Delta p/\Delta t$
  - ➔ Impulse = Δp
  - ➔ Momentum is conserved
  - ➔ Works in each direction independently

Linear and Angular Motion

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	$\frac{1}{2} m v^2$	Today!
Newton's 2nd	$F=ma$	
Momentum	$p = mv$	

Rotary motor in biology #1





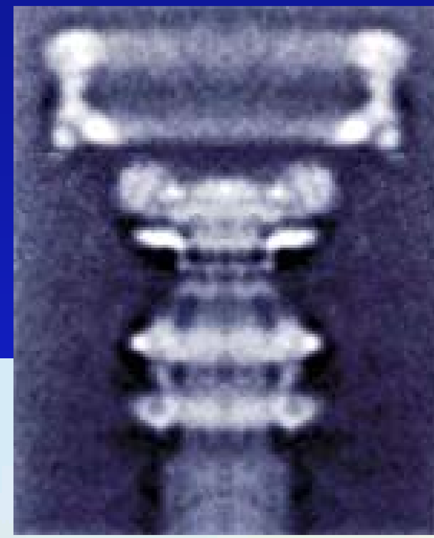
F1-ATPase

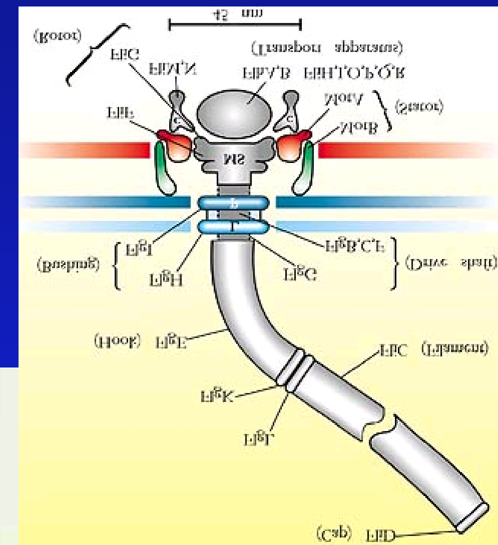
"Rotational catalysis"

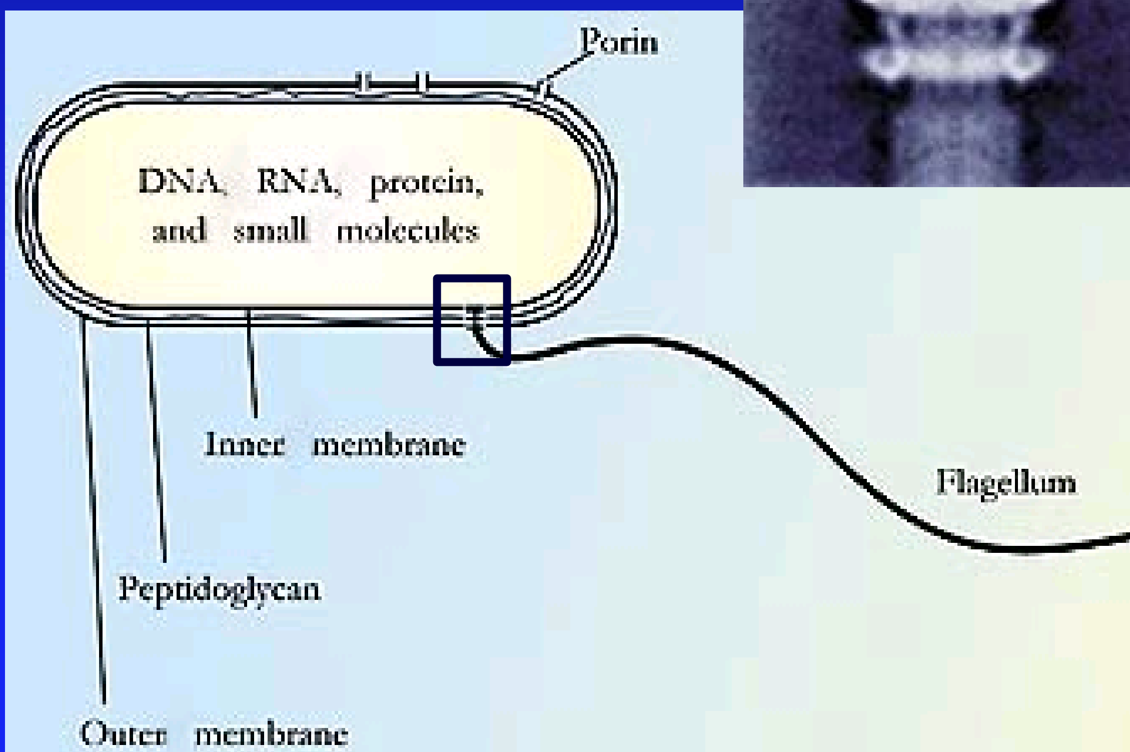
Rotational Energy ->
Chemical Energy (ATP)

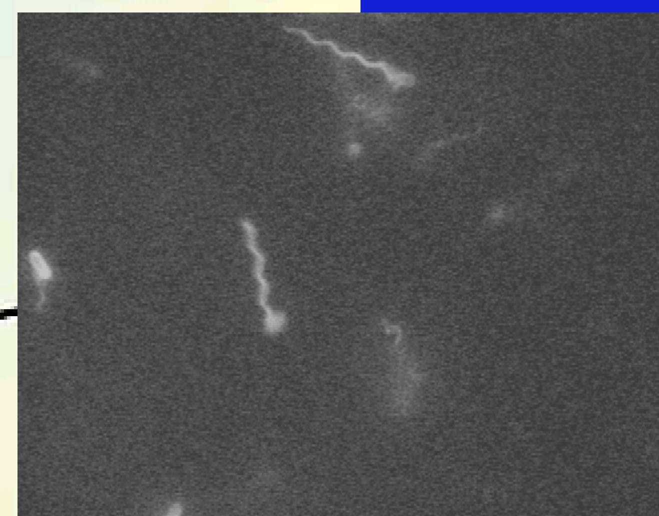
Rotary motor in biology #2

Bacterial flagellum







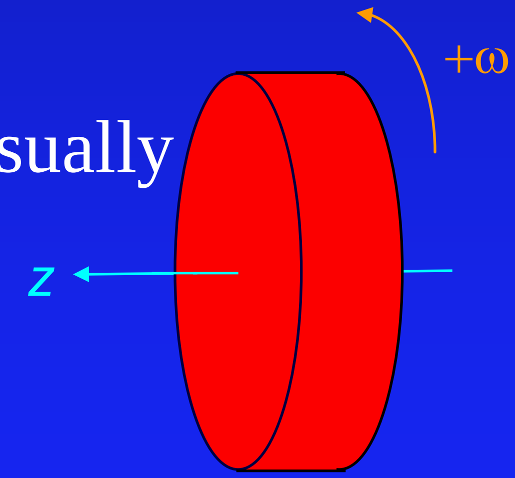


Comment on axes and sign
(i.e. what is positive and negative)

Whenever we talk about rotation, it is implied that there is a rotation "axis".

This is usually called the "z" axis (we usually omit the z subscript for simplicity).

Counter-clockwise (increasing θ) is usually called positive.



Clockwise (decreasing θ) is usually called negative.   [demo]

Energy ACT/demo

- When the bucket reaches the bottom, its potential energy has decreased by an amount mgh. Where has this energy gone?

A) Kinetic Energy of bucket

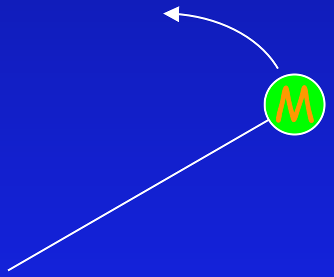
B) Kinetic Energy of flywheel

C) Both 1 and 2.

At bottom, bucket has zero velocity, energy must be in flywheel!

Rotational Kinetic Energy

- Consider a mass M on the end of a string being spun around in a circle with radius r and angular frequency ω [demo]



- ➔ Mass has speed $v = \omega r$
- ➔ Mass has kinetic energy
  - » $K = \frac{1}{2} M v^2$
  - » $\quad = \frac{1}{2} M \omega^2 r^2$
  - » $\quad = \frac{1}{2} (M r^2) \omega^2$
  - » $\quad \boxed{= \frac{1}{2} I \omega^2}$

- Rotational Kinetic Energy is energy due to circular motion of object.

Rotational Inertia I

- Tells how much "work" is required to get object spinning. Just like mass tells you how much "work" is required to get object moving.
  - ➔ $K_{tran} = \frac{1}{2} m v^2$ Linear Motion
  - ➔ $K_{rot} = \frac{1}{2} I \omega^2$ Rotational Motion

- $I = \Sigma m_i r_i^2$ (units kg m^2)

- Note! Rotational Inertia (or "Moment of Inertia") depends on what you are spinning about (basically the r_i in the equation).

Rotational Inertia Table

- For objects with finite number of masses, use $I = \Sigma\, m\, r^2$. For “continuous” objects, use table below (p. 263 of book).

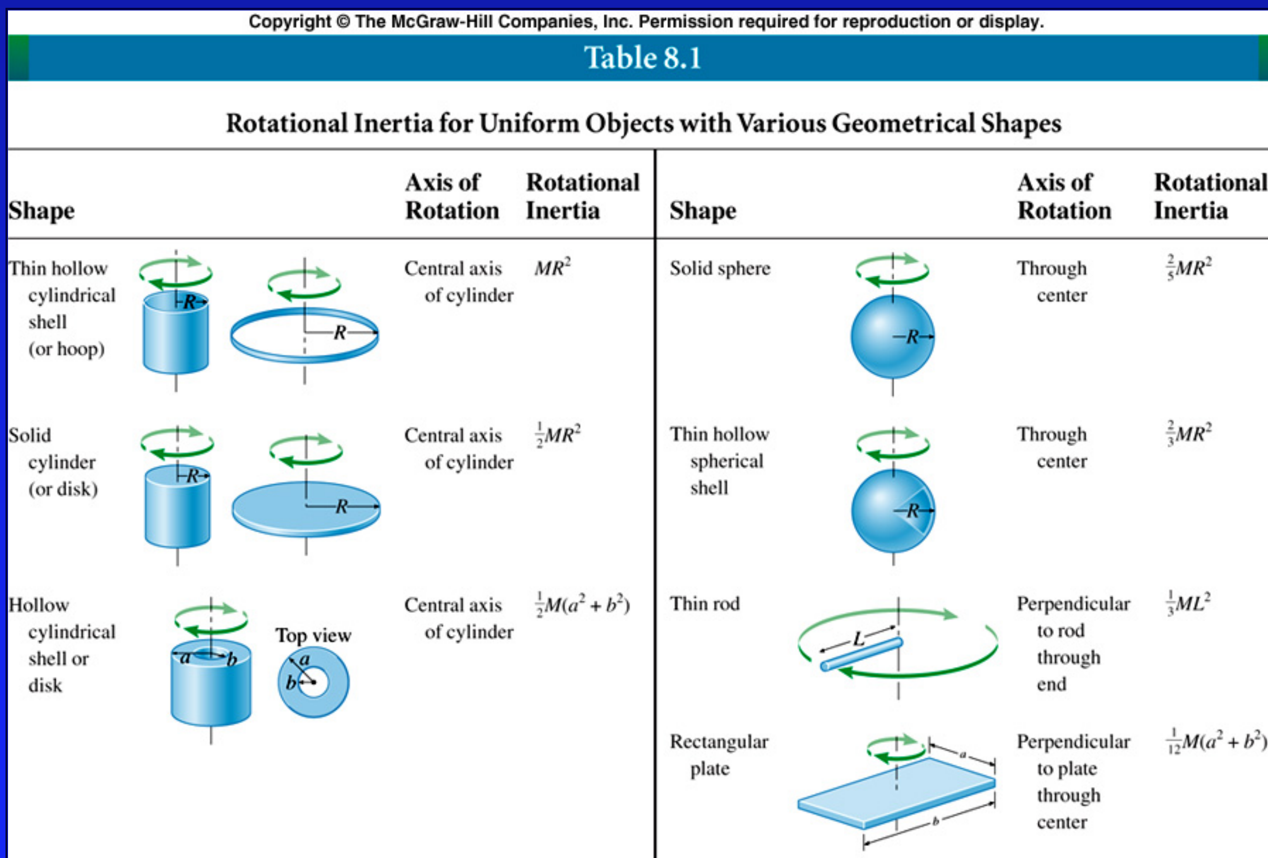


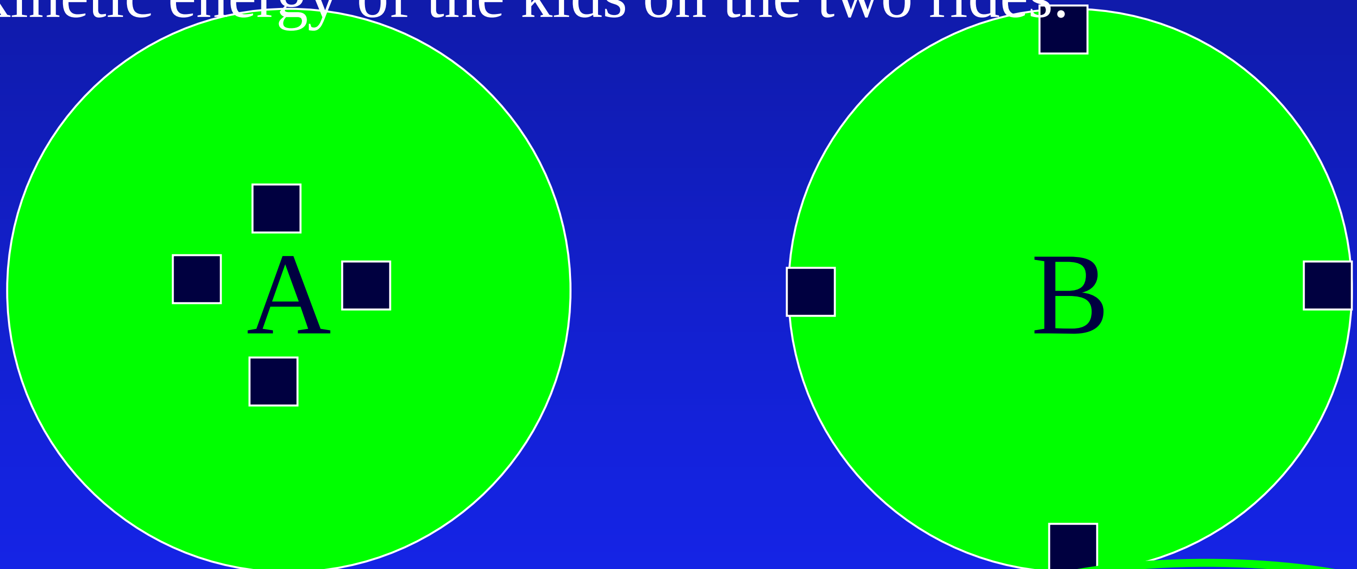
Table 8.1

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

Shape	Axis of Rotation	Rotational Inertia	Shape	Axis of Rotation	Rotational Inertia
Thin hollow cylindrical shell (or hoop)	Central axis of cylinder	MR^2	Solid sphere	Through center	$\frac{2}{5}MR^2$
Solid cylinder (or disk)	Central axis of cylinder	$\frac{1}{2}MR^2$	Thin hollow spherical shell	Through center	$\frac{2}{3}MR^2$
Hollow cylindrical shell or disk (Top view)	Central axis of cylinder	$\frac{1}{2}M(a^2+b^2)$	Thin rod	Perpendicular to rod through end	$\frac{1}{3}ML^2$
			Rectangular plate	Perpendicular to plate through center	$\frac{1}{12}M(a^2+b^2)$

Merry Go Round

Four kids (mass m) are riding on a (light) merry-go-round rotating with angular velocity ω=3 rad/s. In case A the kids are near the center (r=1.5 m), in case B they are near the edge (r=3 m). Compare the kinetic energy of the kids on the two rides.



A) $K_A > K_B$

B) $K_A = K_B$

C) $K_A < K_B$

KE $= 4 \times \frac{1}{2}\, m\, v^2$

$= 4 \times \frac{1}{2}\, m\, \omega\, r^2 = \frac{1}{2}\, I\, \omega^2$ Where $I = 4\, m\, r^2$

Further mass is from axis of rotation, greater KE it has.
[strength contest]

Inertia Rods

Two batons have equal mass and length.

Which will be "easier" to spin

A) Mass on ends

B) Same

C) Mass in center

$I = \Sigma\, m\, r^2$ Further mass is from axis of rotation, greater moment of inertia (harder to spin)

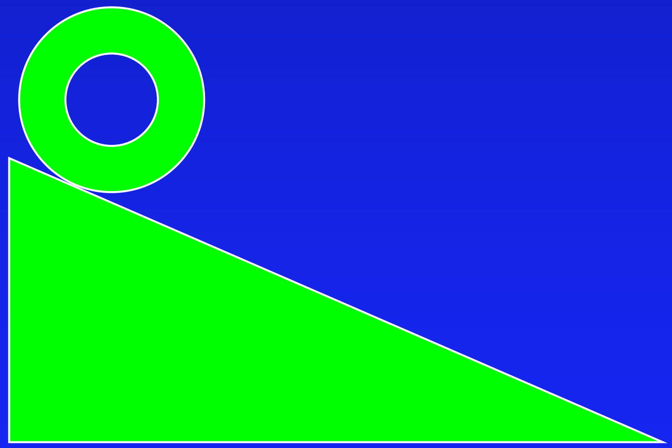
Prelecture: Rolling Race (Hoop vs Cylinder)

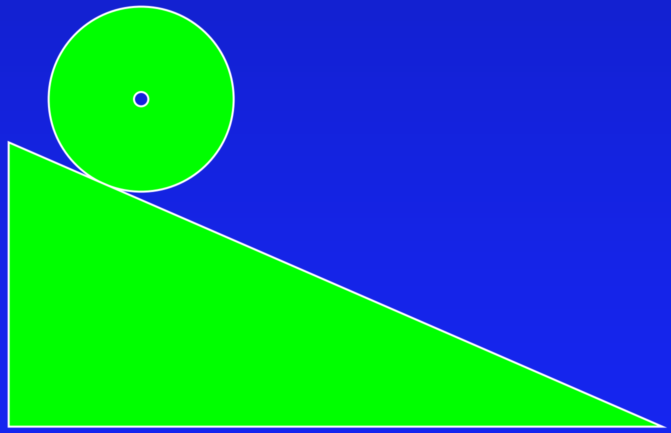
A solid and hollow cylinder of equal mass roll down a ramp with height h. Which has greatest KE at bottom?

A) Solid 19%

B) Hollow 5%

C) Same 76%



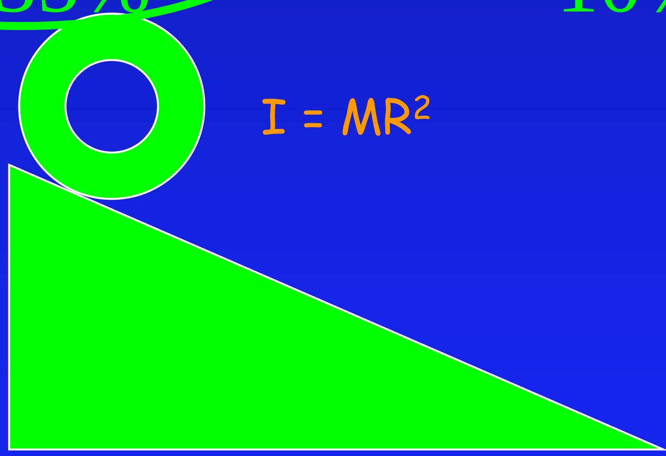


"Both start with same PE so they both end with same KE."

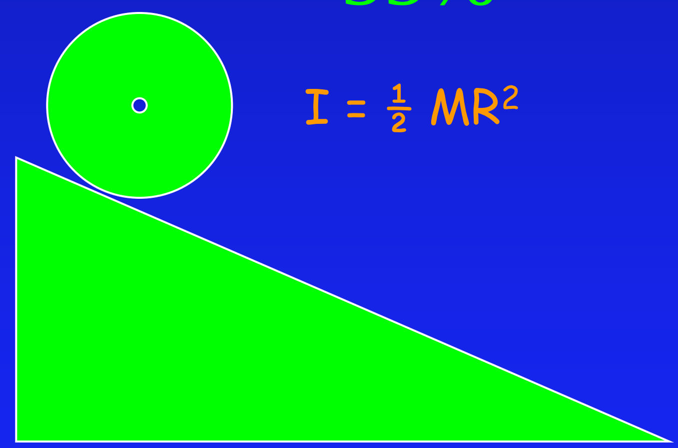
Prelecture: Rolling Race (Hoop vs Cylinder)

A solid and hollow cylinder of equal mass roll down a ramp with height h. Which has greatest speed at the bottom of the ramp?

A) Solid 35%   B) Hollow 10%   C) Same 55%



$I = MR^2$



$I = \frac{1}{2} MR^2$

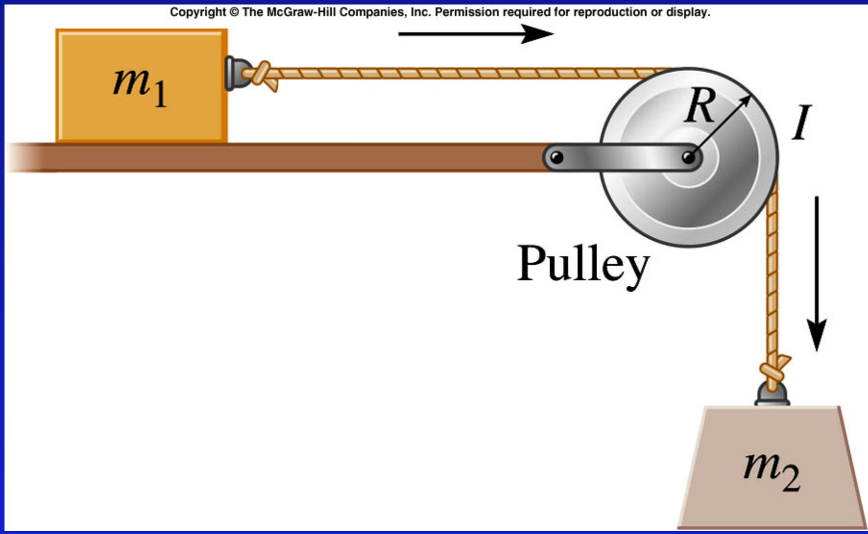
$$K = K_{trans} + K_{rot} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

Main Ideas

- Rotating objects have kinetic energy
  - ➔ $KE = \frac{1}{2} I \omega^2$
- Moment of Inertia $I = \Sigma mr^2$
  - ➔ Depends on Mass
  - ➔ Depends on axis of rotation
- Energy is conserved but need to include rotational energy too: $K_{rot} = \frac{1}{2} I \omega^2$

Massless Pulley Example

Consider the two masses connected by a pulley as shown. Use conservation of energy to calculate the speed of the blocks after m_2 has dropped a distance h. Assume the pulley is massless.



$$E = K + U$$

$$E_0 = E_f$$

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$0 + 0 = -m_2 gh + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$2m_2 gh = m_1 v^2 + m_2 v^2$$

$$v = \sqrt{\frac{2m_2 gh}{m_1 + m_2}}$$

Massive Pulley Act

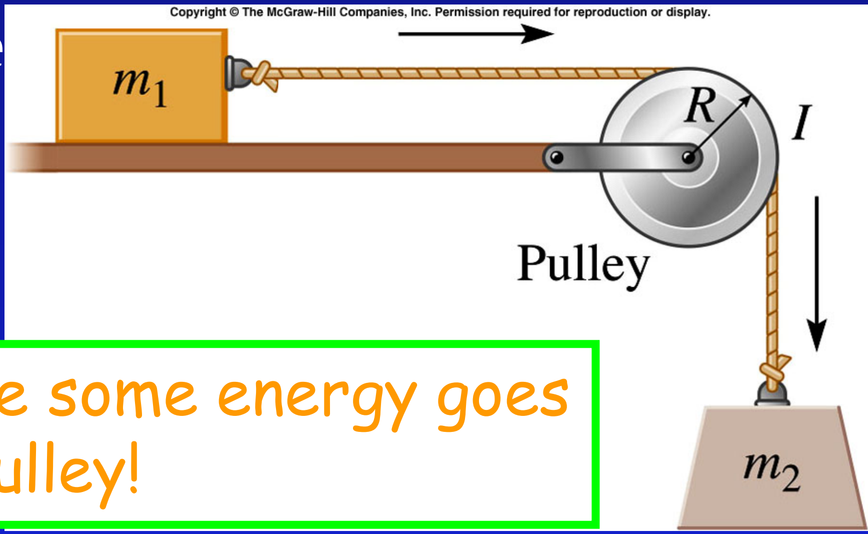
Consider the two masses connected by a pulley as shown. If the pulley is massive after m2 drops a distance h, the blocks will be moving

A) faster than

B) the same speed as

C) slower than

if it was a massless pulley



Slower because some energy goes into spinning pulley!

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$0 = -m_2gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

$$m_2gh = +\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$m_2gh = +\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{4}Mv^2$$

$$\boxed{v = \sqrt{\frac{2m_2gh}{m_1 + m_2 + M/2}}}$$

Summary

- Rotational Kinetic Energy $K_{rot} = \frac{1}{2} I \omega^2$

- Rotational Inertia $I = \Sigma m_i r_i^2$

- Energy Still Conserved!