

# EXAM I

## Physics 101: Lecture 08

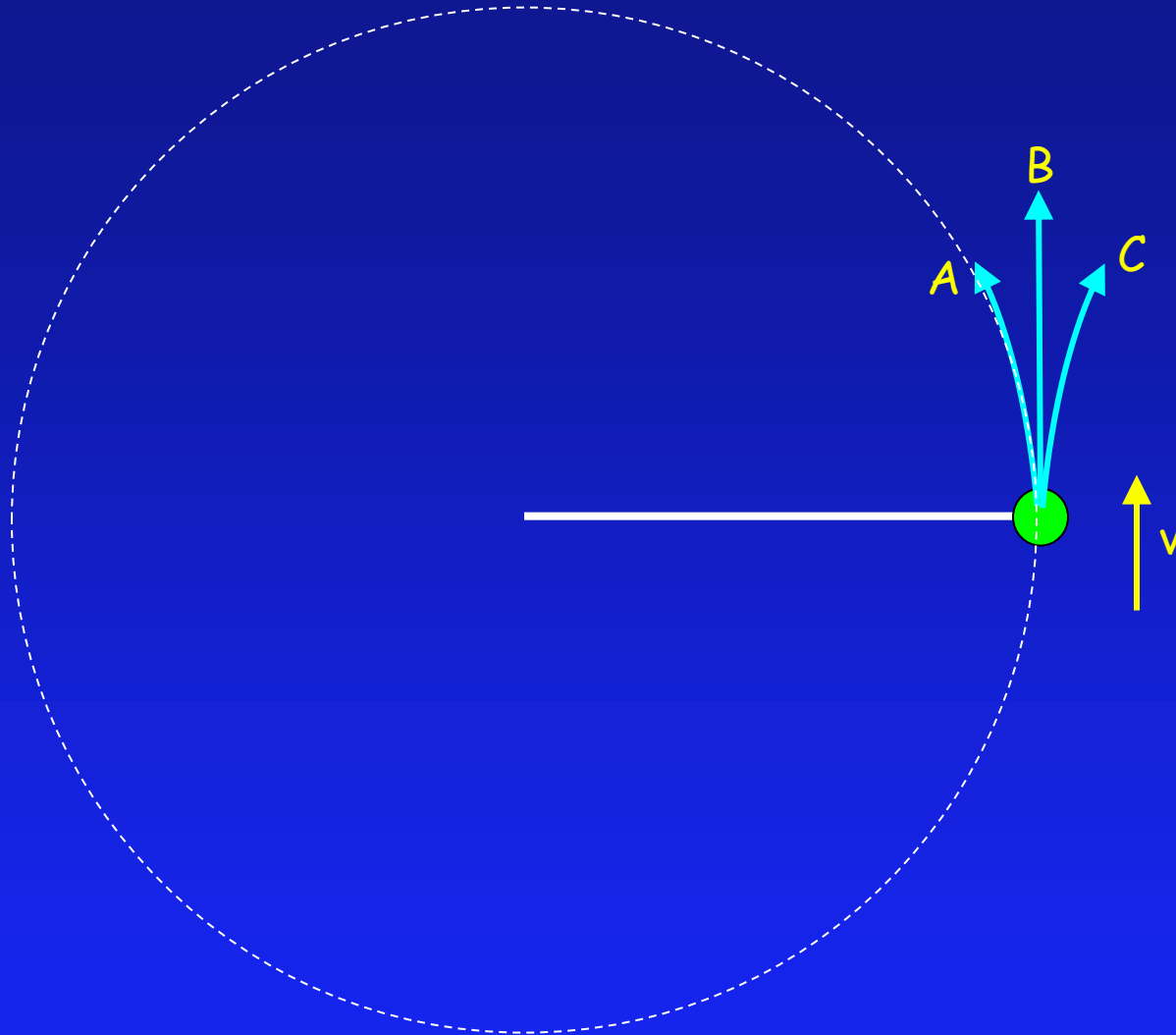
# Centripetal Acceleration and Circular Motion

<http://www.youtube.com/watch?v=ZyF5WemXRcI>

- Today's lecture will cover Chapter 5
- Exam I is Monday, Oct. 6 (2 weeks!)



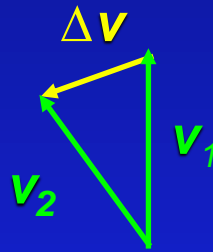
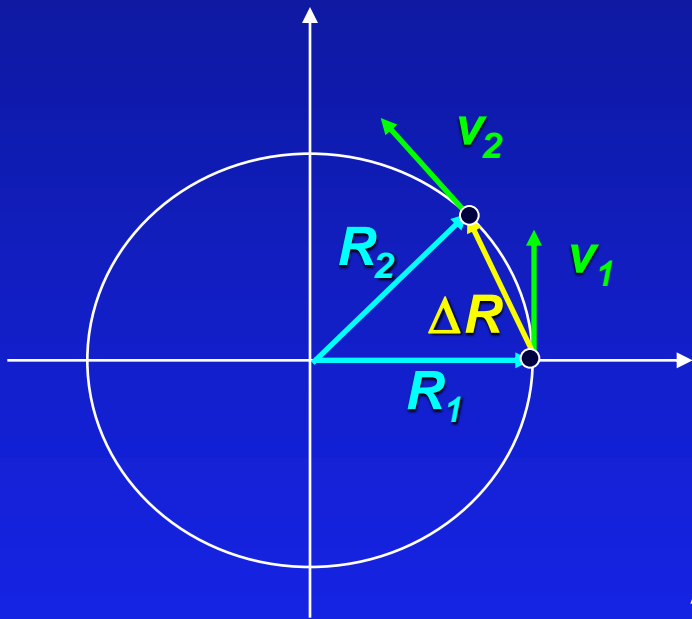
# Circular Motion Act



Answer: B

A ball is going around in a circle attached to a string. If the string breaks at the instant shown, which path will the ball follow (demo)?

# Acceleration in Uniform Circular Motion



$$\mathbf{a}_{\text{ave}} = \Delta \mathbf{v} / \Delta t$$

Acceleration inward

$$a = \frac{v^2}{R}$$

Centripetal Acceleration  
Directed radially inward!

Acceleration is due to change in direction, not speed. Since the object turns “toward” center, there must be a force toward center: “Centripetal Force”

# Prelectures

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. How many forces are acting on the car?

2% A) 1

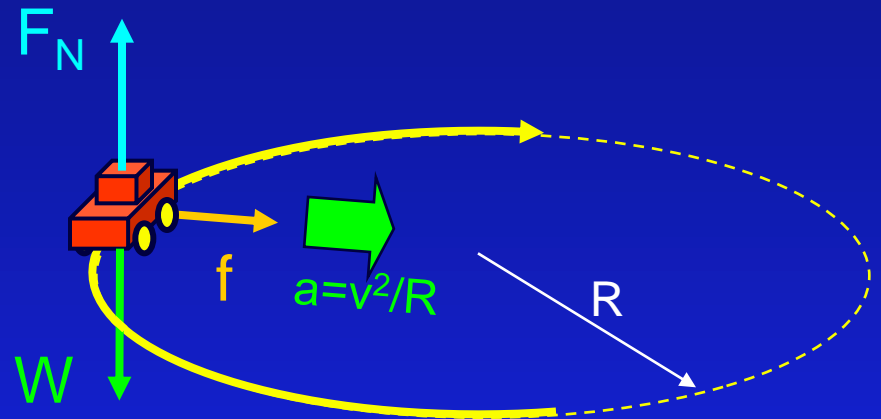
5% B) 2

46% C) 3

33% D) 4

14% E) 5

← correct



"Friction, Gravity, & Normal"

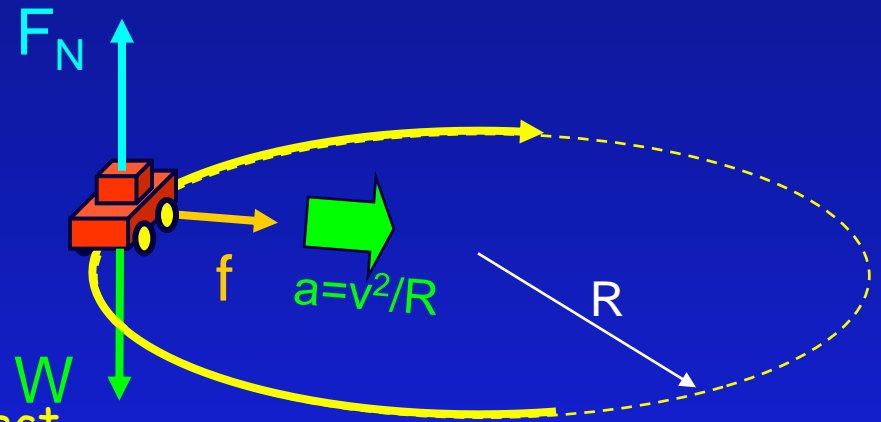
$$F_{\text{Net}} = ma = mv^2/R$$

"Centripetal Force" is NOT an additional force!

Draw your FBD as normal, and one of the forces will be the Centripetal Force!

# Prelecture

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. **The net force on the car is**



14% A. Zero

66% B. Pointing radially inward ← correct

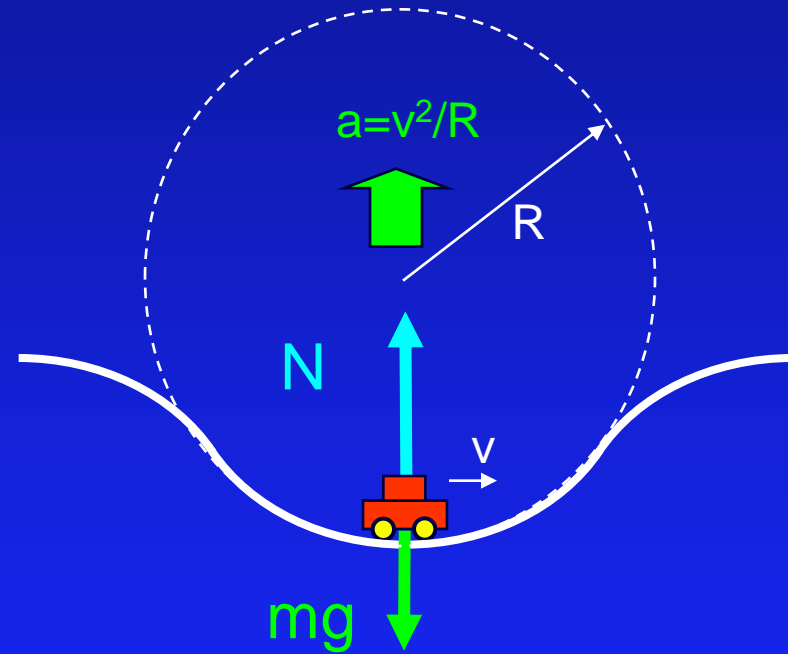
21% C. Pointing radially outward

$$F_{\text{Net}} = ma = mv^2/R$$

# ACT

Suppose you are driving through a valley whose bottom has a circular shape. If your mass is  $m$ , what is the magnitude of the normal force  $F_N$  exerted on you by the car seat as you drive past the bottom of the hill

- A.  $N < mg$
- B.  $N = mg$
- C.  $N > mg$  ← correct



$$F_{\text{Net}} = ma$$

$$N - mg = mv^2/R$$

$$N = mg + mv^2/R$$

# Roller Coaster Example

What is the minimum speed you must have at the top of a 20 meter roller coaster loop, to keep the wheels on the track?

Y Direction:  $F_{\text{Net}} = ma$

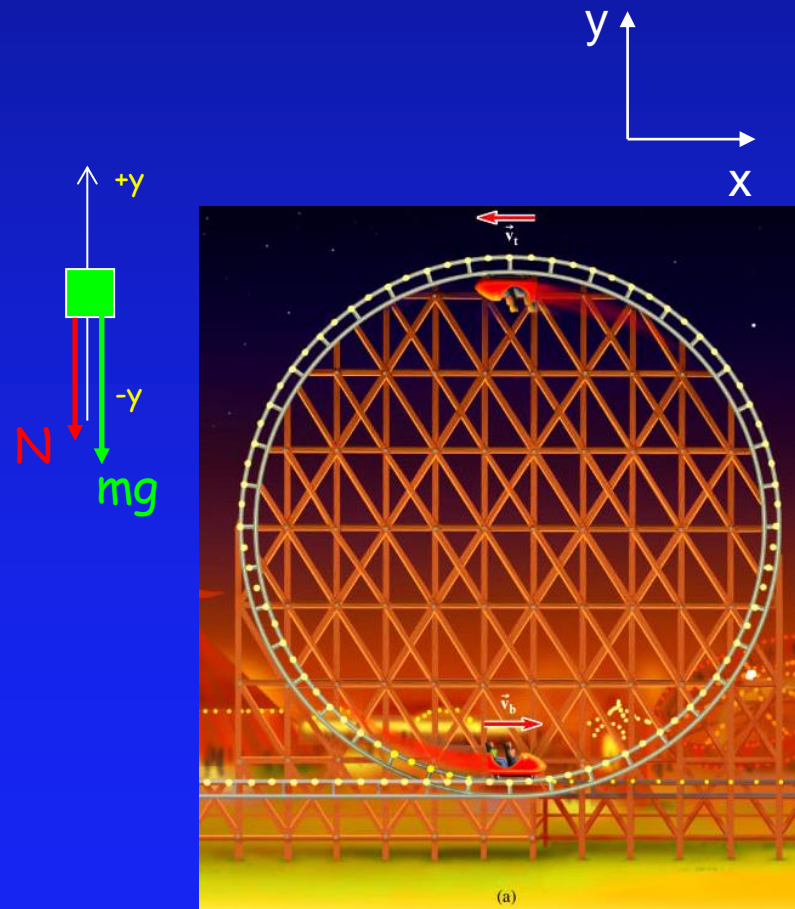
$$-N - mg = m a = -m v^2/R$$

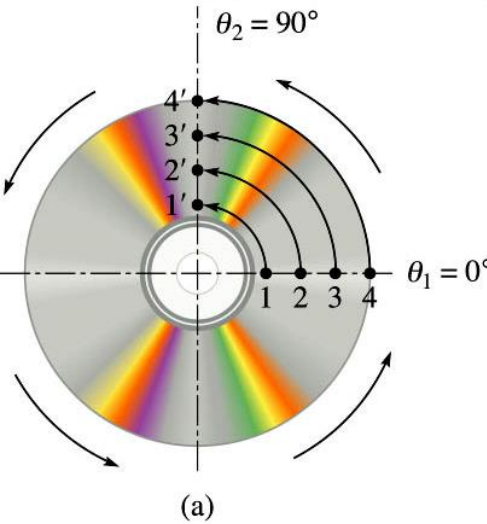
Let  $N = 0$ , just touching

$$-mg = -m v^2/R$$

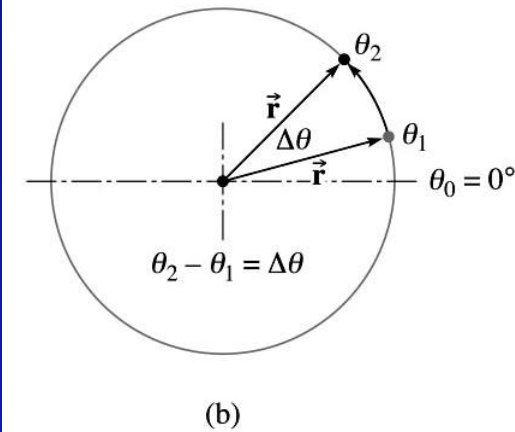
$$g = v^2 / R$$

$$v = \text{sqrt}(g * R) = 9.9 \text{ m/s}$$





# Circular Motion

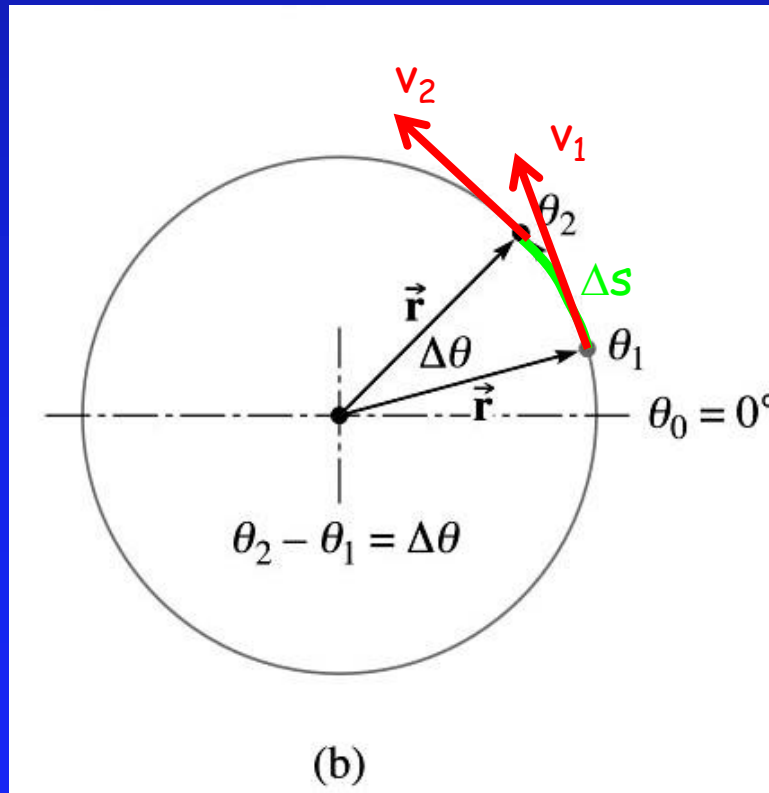


- Angular displacement  $\Delta\theta = \theta_2 - \theta_1$ 
  - How far it has rotated
  - Units radians ( $2\pi = 1$  revolution)
- Angular velocity  $\omega = \Delta\theta / \Delta t$ 
  - How fast it is rotating
  - Units radians/second
- Period = 1/frequency  $T = 1/f = 2\pi / \omega$ 
  - Time to complete 1 revolution



# Circular to Linear

- Displacement  $\Delta s = r \Delta\theta$  ( $\theta$  in radians)
- Speed  $|v| = \Delta s / \Delta t = r \Delta\theta / \Delta t = r\omega$
- Direction of  $v$  is tangent to circle

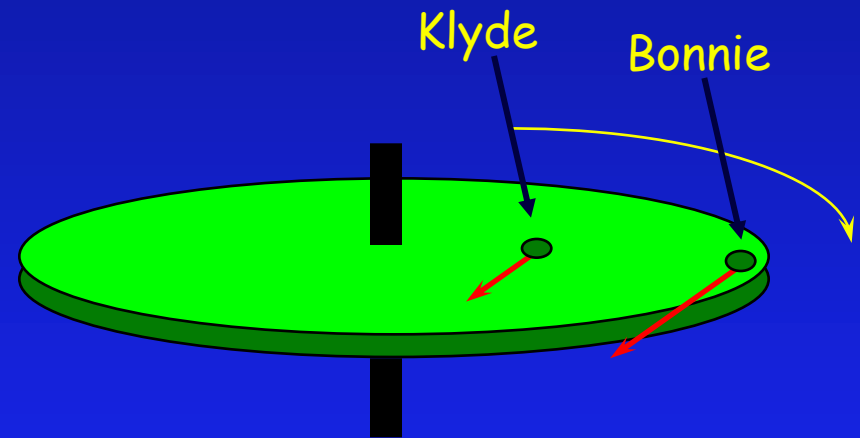


# Merry-Go-Round ACT

- Bonnie sits on the outer rim of a merry-go-round with radius 3 meters, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds (demo).

→ Klyde's speed is:

- (a) the same as Bonnie's
- (b) twice Bonnie's
- (c) half Bonnie's



$$V_{Klyde} = \frac{1}{2} V_{Bonnie}$$

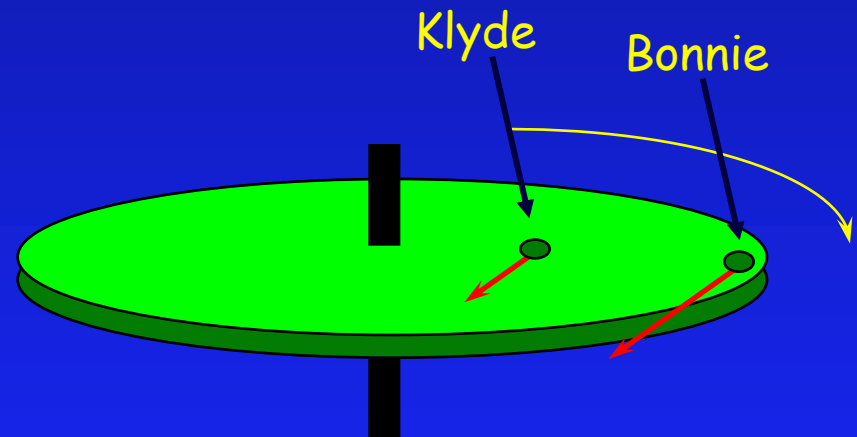
Bonnie travels  $2\pi R$  in 2 seconds  $v_B = 2\pi R / 2 = 9.42 \text{ m/s}$

Klyde travels  $2\pi (R/2)$  in 2 seconds  $v_K = 2\pi (R/2) / 2 = 4.71 \text{ m/s}$

# Merry-Go-Round ACT II

- Bonnie sits on the outer rim of a merry-go-round, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds.  
→ Klyde's angular velocity is:

- (a) **the same as Bonnie's**
- (b) **twice Bonnie's**
- (c) **half Bonnie's**



- The angular velocity  $\omega$  of any point on a solid object rotating about a fixed axis is the same.  
→ Both Bonnie & Klyde go around once ( $2\pi$  radians) every two seconds.

# Angular Acceleration

- Angular acceleration is the change in angular velocity  $\omega$  divided by the change in time.

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_0}{\Delta t}$$

- If the speed of a roller coaster car is 15 m/s at the top of a 20 m loop, and 25 m/s at the bottom. What is the cars average angular acceleration if it takes 1.6 seconds to go from the top to the bottom?

$$\omega = \frac{V}{R}$$

$$\omega_f = \frac{25}{10} = 2.5$$

$$\omega_0 = \frac{15}{10} = 1.5$$

$$\bar{\alpha} \equiv \frac{2.5 - 1.5}{1.6} = 0.64 \text{ rad/s}^2$$

# Summary

(with comparison to 1-D kinematics)

Angular	Linear
$\alpha = \text{constant}$	$a = \text{constant}$
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$
<p>And for a point at a distance <math>R</math> from the rotation axis:</p> $x = R\theta \qquad v = \omega R \qquad a = \alpha R$	

# CD Player Example

- The CD in a disk player spins at about 20 radians/second. If it accelerates uniformly from rest with angular acceleration of 15 rad/s<sup>2</sup>, how many revolutions does the disk make before it is at the proper speed?

$$\omega_0 = 0$$

$$\omega_f = 20 \text{ rad/s}$$

$$\alpha = 15 \text{ rad/s}^2$$

$$\Delta\theta = ?$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\frac{\omega_f^2 - \omega_0^2}{2\alpha} = \Delta\theta$$

$$\frac{20^2 - 0^2}{2 \times 15} = \Delta\theta$$

$$\Delta\theta = 13.3 \text{ radians}$$

$$1 \text{ Revolution} = 2\pi \text{ radians}$$

$$\Delta\theta = 13.3 \text{ radians}$$

$$= 2.12 \text{ revolutions}$$

# Summary of Concepts

- Uniform Circular Motion
  - Speed is constant
  - Direction is changing
  - Acceleration toward center  $a = v^2 / r$
  - Newton's Second Law  $F = ma$
- Circular Motion
  - $\theta$  = angular position radians
  - $\omega$  = angular velocity radians/second
  - $\alpha$  = angular acceleration radians/second<sup>2</sup>
  - Linear to Circular conversions  $s = r \theta$
- Uniform Circular Acceleration Kinematics
  - Similar to linear!