Physics 101: Lecture 15
Rolling Objects

Today's lecture will cover Textbook Chapter 8.5-8.7

Overview

- Review
  - ➔ $K_{rotation} = ½ I \omega^2$
  - ➔ Torque = Force that causes rotation

    $\tau = F\, r \sin\theta$

  - ➔ Equilibrium

    $F_{Net} = 0$

    $\tau_{Net} = 0$
- Today
  - ➔ $\tau_{Net} = I\,\alpha$ (rotational F = ma)
  - ➔ Energy conservation revisited

Linear and Angular

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	½ m v2	$\frac{1}{2} I \omega^2$
N2L	F=ma	$\tau = I\alpha$
Momentum	p = mv	$L = I\omega$

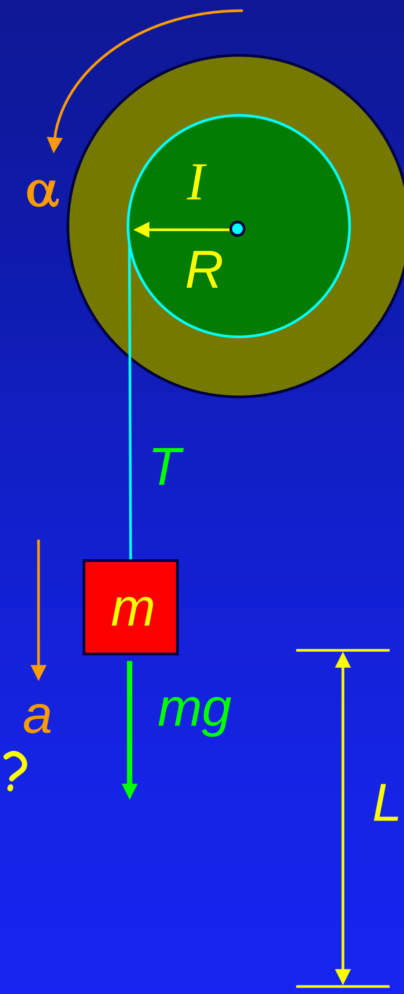
Today

Rotational Form Newton's 2nd Law

- $\tau_{Net} = I\,\alpha$
  - ➔Torque is amount of twist provide by a force
    - » Signs: positive = CCW 
  - ➔Moment of Inertia like mass. Large I means hard to start or stop from spinning.
- Problems Solved Like Newton's 2nd
  - ➔Draw FBD
  - ➔Write Newton's 2nd Law

Falling weight & pulley

- A mass m is hung by a string that is wrapped around a pulley of radius R attached to a heavy flywheel. The moment of inertia of the pulley + flywheel is I. The string does not slip on the pulley.
  Starting at rest, how long does it take for the mass to fall a distance L.



What method should we use to solve this problem?

A) Conservation of Energy (including rotational)

B) $\tau_{Net} = I\alpha$ and then use kinematics

Since it asks for time, we will use B.

Falling weight & pulley...

- For the hanging mass use $F_{Net} = ma$

$$\boxed{\rightarrow mg - T = ma}$$

- For the flywheel use $\tau_{Net} = I\alpha$

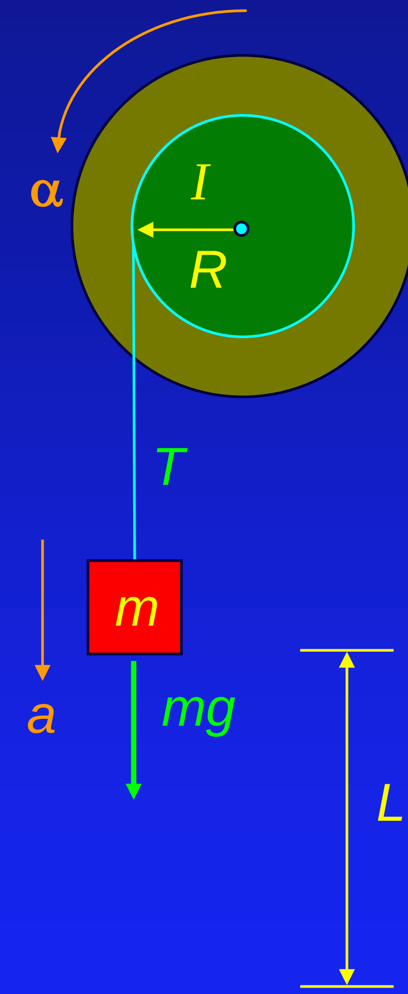
$\rightarrow TR \sin(90) = I\alpha$

- Realize that $a = \alpha R$

$$\boxed{\rightarrow \quad TR = I\frac{a}{R}}$$

- Now solve for a, eliminate T:

$$\boxed{a = \left(\frac{mR^2}{mR^2 + I}\right)g}$$



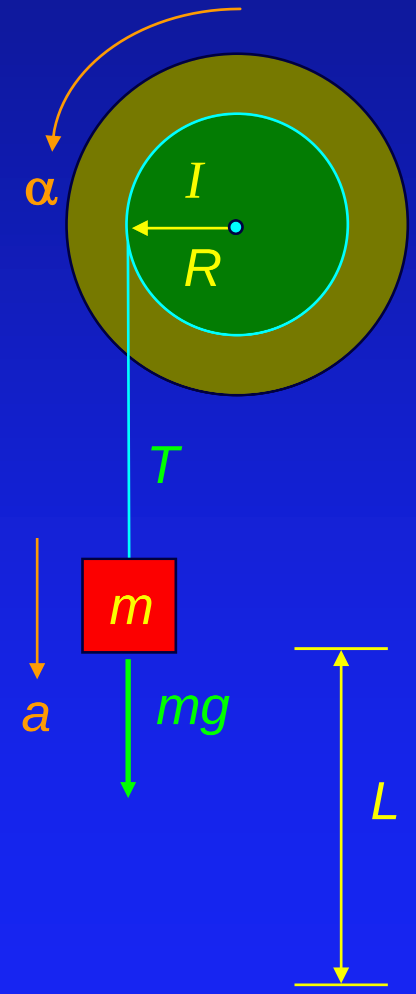
Falling weight & pulley...

- Using 1-D kinematics we can solve for the time required for the weight to fall a distance L:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$L = \frac{1}{2} a t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2L}{a}}$$

where $$a = \left(\frac{mR^2}{mR^2 + I} \right) g$$



Torque ACT

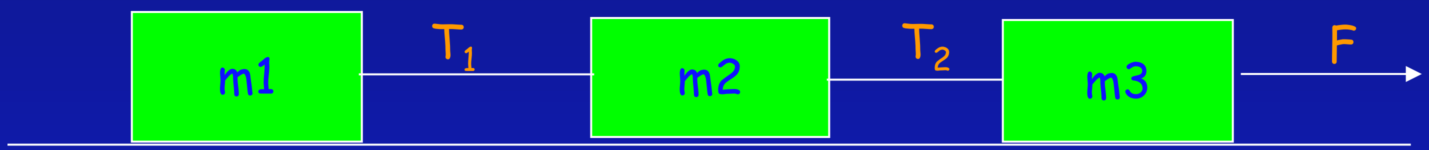
- Which pulley will make it drop fastest?

1) Small pulley
2) Large pulley
3) Same

$$a = \left(\frac{mR^2}{mR^2 + I} \right) g$$

Larger R, gives larger acceleration.

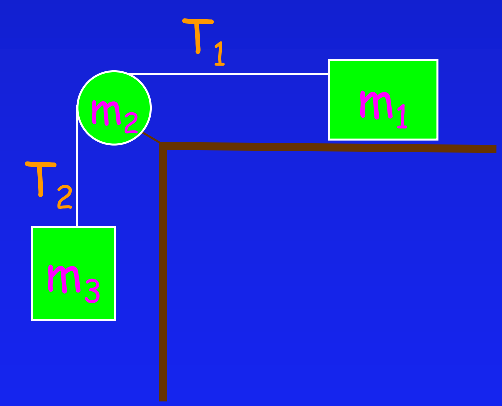
Tension...



Compare the tensions T_1 and T_2 as the blocks are accelerated to the right by the force F.

A) $T_1 < T_2$ B) $T_1 = T_2$ C) $T_1 > T_2$

$T_1 < T_2$ since $T_2 - T_1 = m_2 a$. It takes force to accelerate block 2.



Compare the tensions T_1 and T_2 as block 3 falls

A) $T_1 < T_2$ B) $T_1 = T_2$ C) $T_1 > T_2$

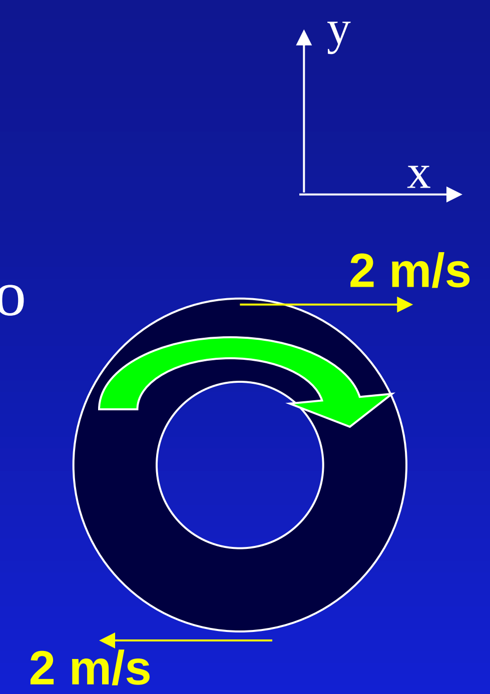
$T_2 > T_1$ since $RT_2 - RT_1 = I_2\,\alpha$. It takes force (torque) to accelerate the pulley.

Rolling

A wheel is spinning clockwise such that the speed of the outer rim is 2 m/s.

What is the velocity of the top of the wheel relative to the ground? + 2 m/s

What is the velocity of the bottom of the wheel relative to the ground? -2 m/s



You now carry the spinning wheel to the right at 2 m/s.

What is the velocity of the top of the wheel relative to the ground?

A) -4 m/s B) -2 m/s C) 0 m/s D) +2m/s E) +4 m/s

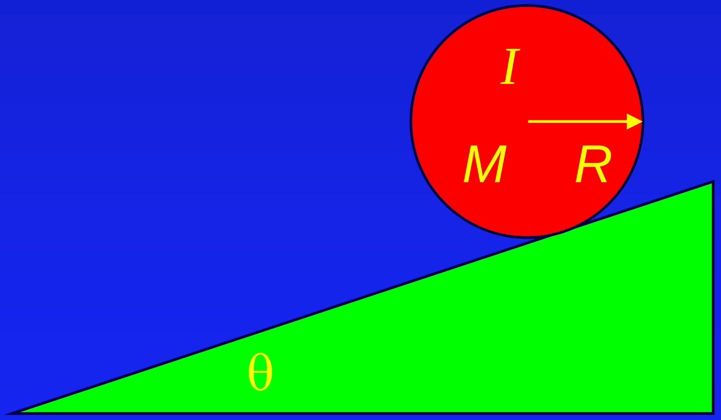
What is the velocity of the bottom of the wheel relative to the ground?

A) -4 m/s B) -2 m/s C) 0 m/s D) +2m/s E) +4 m/s

Rolling

- An object with mass M, radius R, and moment of inertia I rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

- Consider CM motion and rotation about the CM separately when solving this problem



Rolling...

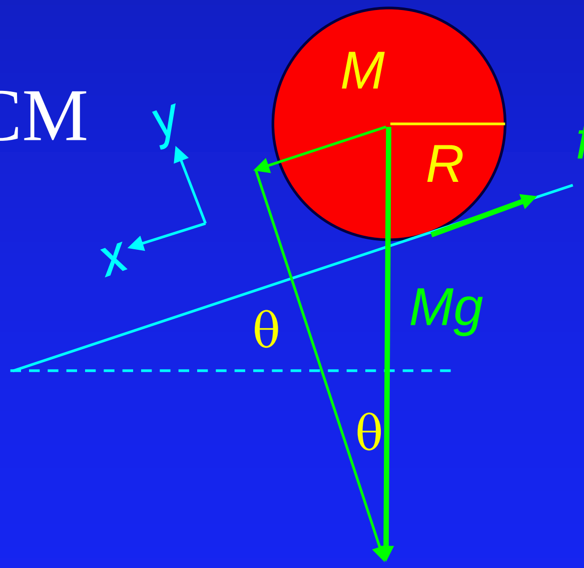
- Static friction *f* causes rolling. It is an unknown, so we must solve for it.
- First consider the free body diagram of the object and use $F_{NET} = Ma_{cm}$:

  In the *x* direction $\quad Mg \sin\theta - f = Ma_{cm}$

- Now consider rotation about the CM and use $\tau_{N\varepsilon\tau} = I\alpha$ realizing that

  $\tau = Rf$ and $a = \alpha R$

$$Rf = \mathrm{I}\frac{a}{R} \quad \Rightarrow \quad f = \mathrm{I}\frac{a}{R^2}$$



Rolling...

- We have two equations: $Mg \sin\theta - f = Ma$

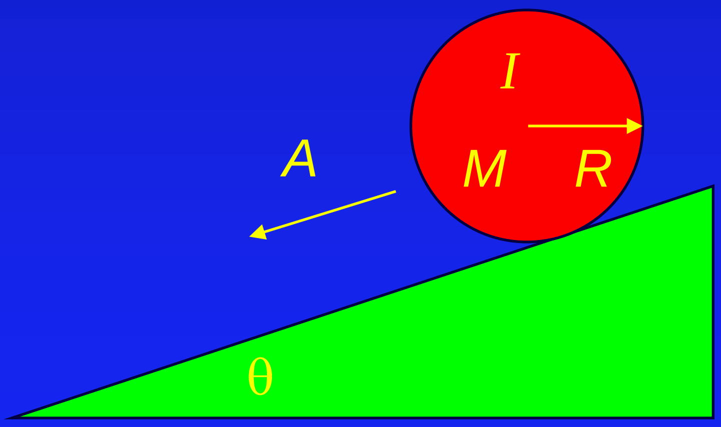
$$f = I\frac{a}{R^2}$$

- We can combine these to eliminate f:

$$a = g\left(\frac{MR^2 \sin\theta}{MR^2 + I}\right)$$

For a sphere:

$$a = g\left(\frac{MR^2 \sin\theta}{MR^2 + \frac{2}{5}MR^2}\right) = \frac{5}{7} g \sin\theta$$



Energy Conservation!

- Friction causes object to roll, but if it rolls w/o slipping friction does NO work!
  - ➔ $W = F\, d \cos\theta$ d is zero for point in contact
- No dissipated work, energy is conserved
- Need to include both translational and rotational kinetic energy.
  - ➔ $K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

Translational + Rotational KE

- Consider a cylinder with radius R and mass M, rolling w/o slipping down a ramp. Determine the ratio of the translational to rotational KE.

Translational: $K_T = \frac{1}{2} M v^2$

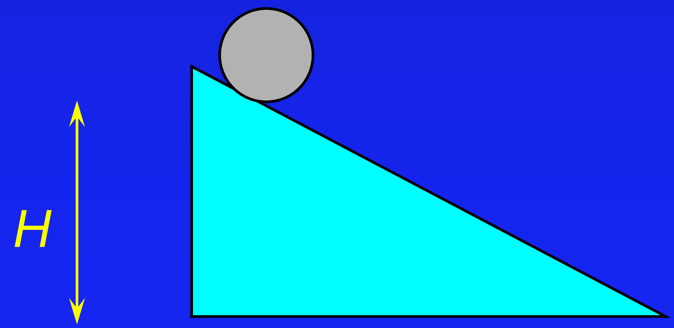
Rotational: $K_R = \frac{1}{2} I \omega^2$

use $I = \frac{1}{2} MR^2$ and $\omega = \frac{V}{R}$

Rotational: $K_R = \frac{1}{2} (\frac{1}{2} M R^2) (V/R)^2$

$= \frac{1}{4} M v^2$

$= \frac{1}{2} K_T$



Rolling Act

- Two uniform cylinders are machined out of solid aluminum. One has twice the radius of the other.
  - ➔ If both are placed at the top of the same ramp and released, which is moving faster at the bottom?

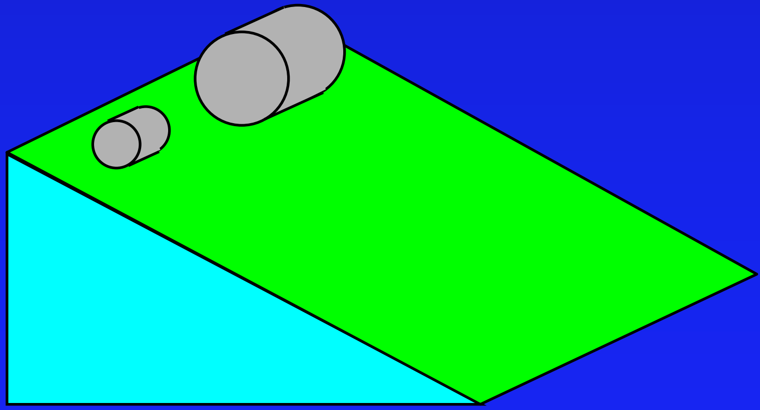
(a) bigger one (b) smaller one (c) same

$K_i + U_i = K_f + U_f$

$$MgH = \frac{1}{2} I \omega^2 + \frac{1}{2} MV^2$$

$$MgH = \frac{1}{2}\left(\frac{1}{2} MR^2\right)\frac{V^2}{R^2} + \frac{1}{2} MV^2$$

$$V = \sqrt{\frac{4}{3} gH}$$



Summary

- $\tau = I\,\alpha$

- Energy is Conserved
  - ➔ Need to include translational and rotational