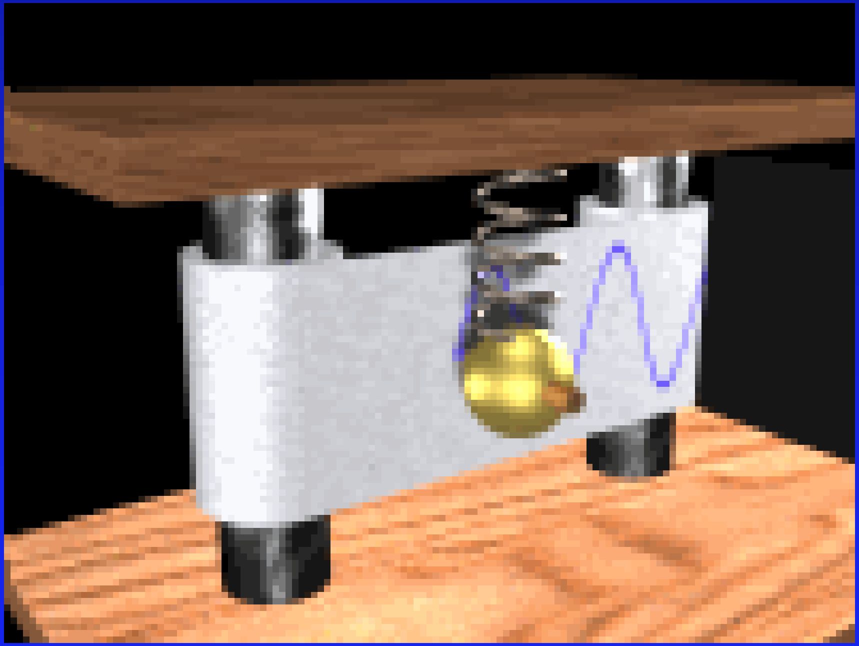
Physics 101: Lecture 19
Elasticity and Oscillations



Comments:

- Exam tonight – 7 PM (5:15 PM conflict)
- Know your section number (i.e. Dxx)
- Bring your iCard—you can be sent back home to get it if you forget
- Bring your own calculator
- Bring your own pencil

Overview

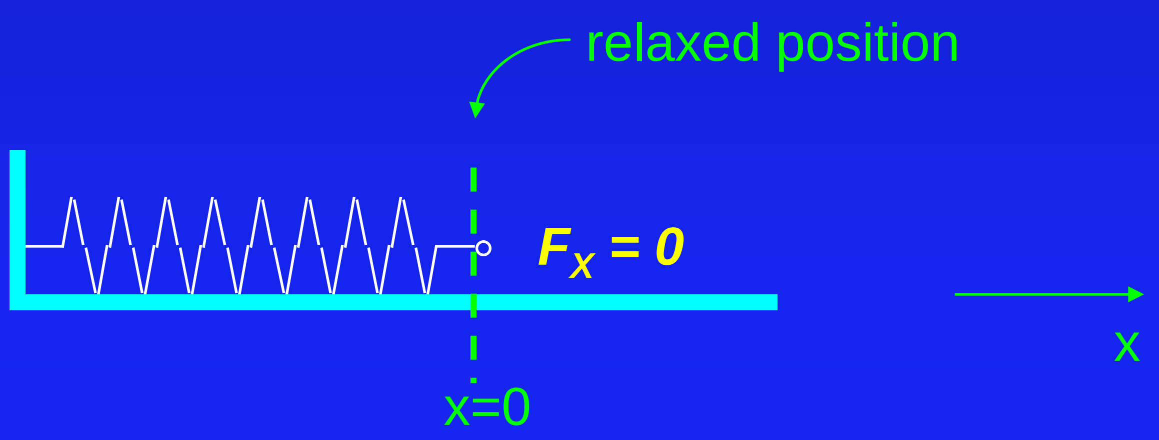
- Springs (review)
  - ➔ Restoring force proportional to displacement
  - ➔ $F = -k\ x$ (often a good approximation)
  - ➔ $U = \frac{1}{2} k\ x^2$
- Today
  - ➔ Simple Harmonic Motion
  - ➔ Springs Revisited
  - ➔ Young's Modulus (where does k come from?)

Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

➔ $F_X = -k\,x$

Where x is the displacement from the relaxed position and k is the constant of proportionality.

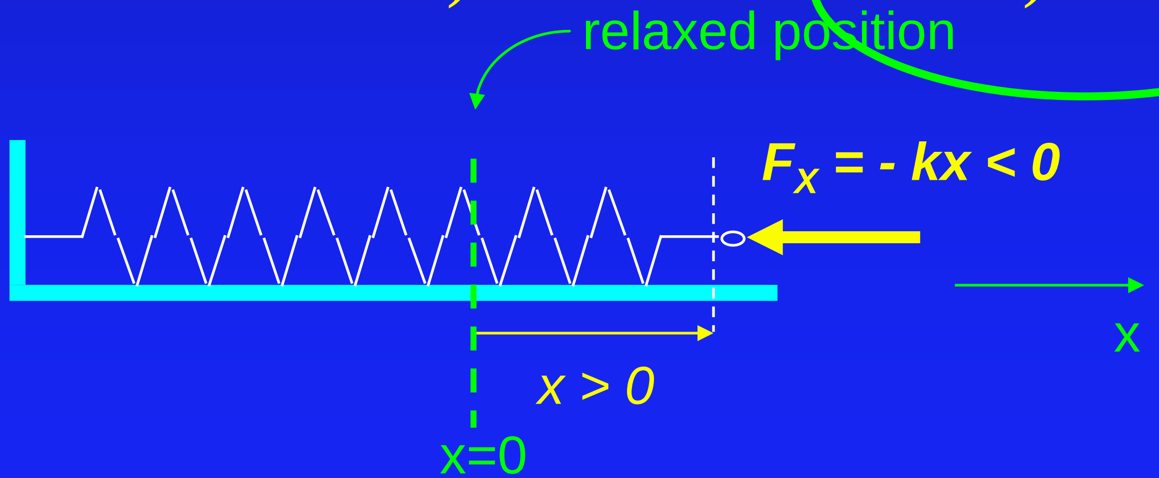


Springs ACT

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.
  - ➔ $F_X = -k\,x$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

What is force of spring when it is stretched as shown below.

A) F > 0 B) F = 0 C) F < 0



Springs

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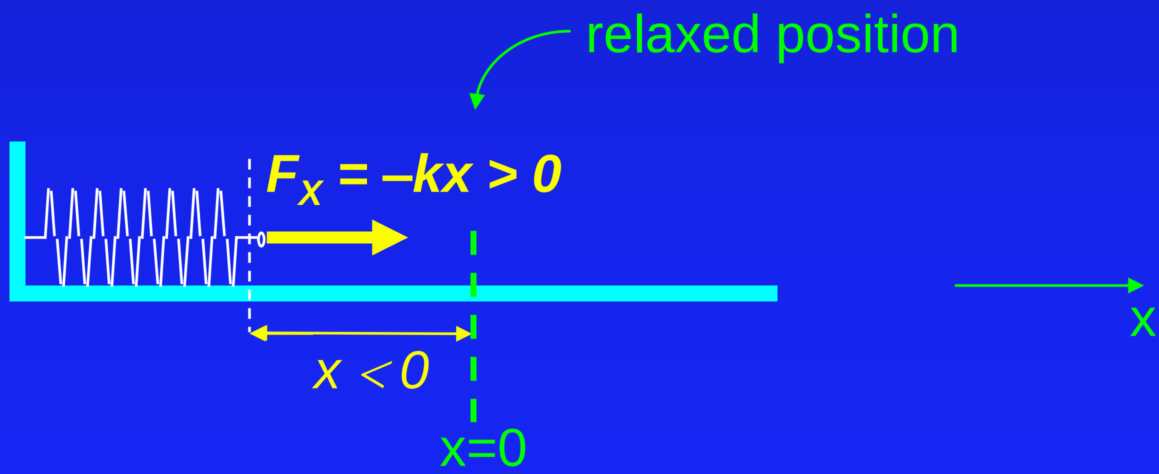
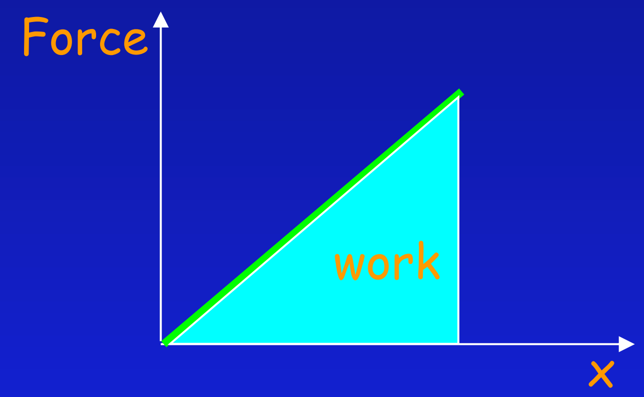


Potential Energy in Spring

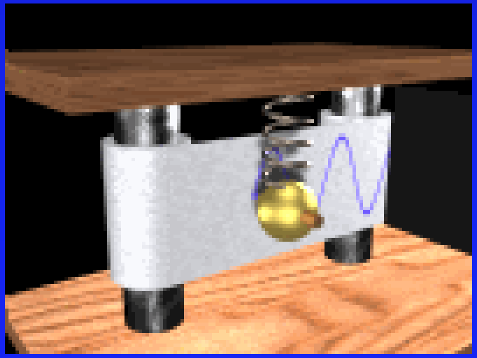
- Hooke's Law force is Conservative
  - ➔ $F = -k\ x$
  - ➔ $W = -1/2\ k\ x^2$



  - ➔ Work done only depends on initial and final position
  - ➔ Define Potential Energy $U_{spring} = ½\ k\ x^2$

Simple Harmonic Motion

- Vibrations
  - ➔ Vocal cords when singing/speaking
  - ➔ String/rubber band
- Simple Harmonic Motion
  - ➔ Restoring force proportional to displacement
  - ➔ Springs $F = -kx$

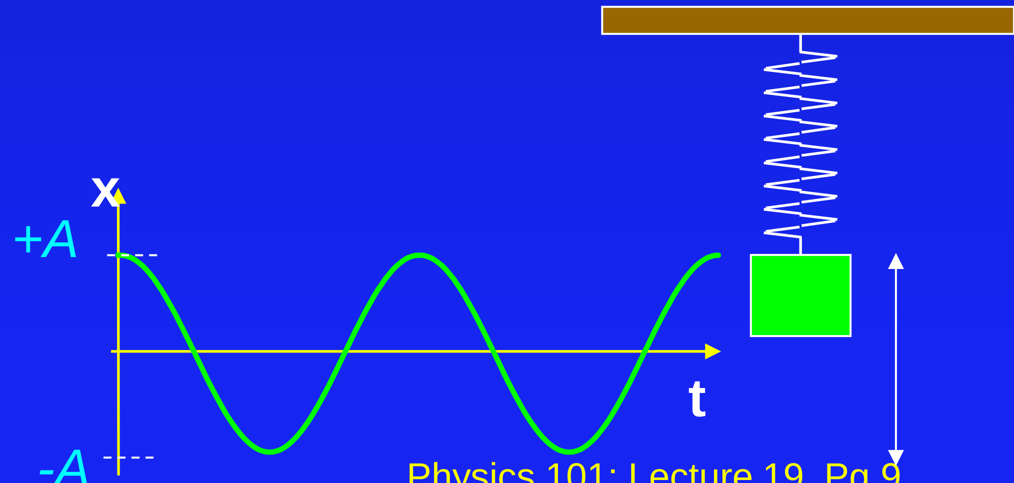


Spring ACT II

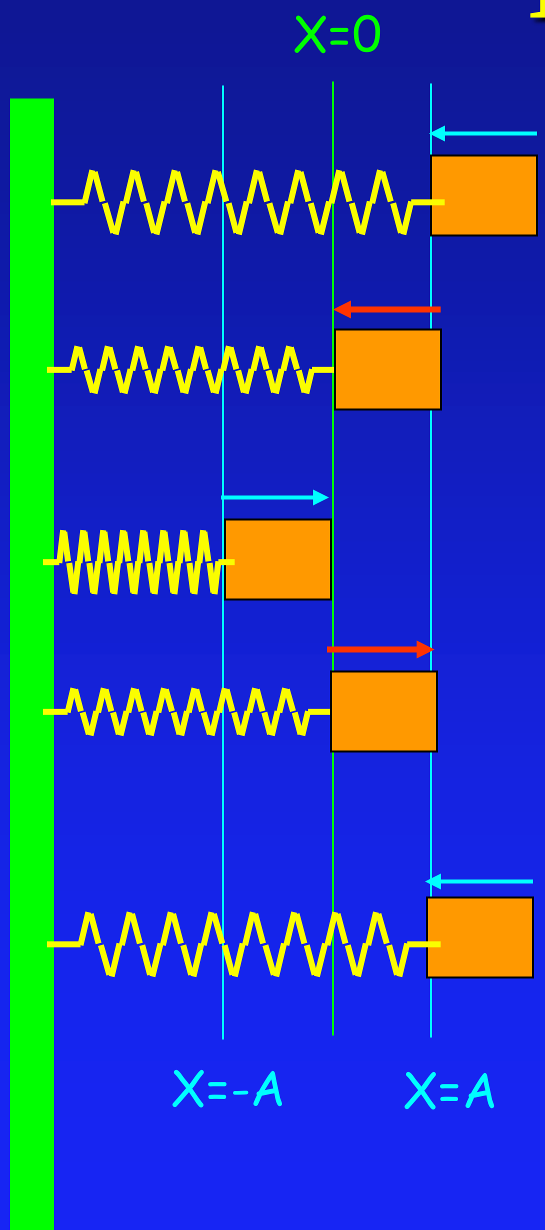
A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

1. When x = +A or -A (i.e. maximum displacement) ← CORRECT
2. When x = 0 (i.e. zero displacement)
3. The acceleration of the mass is constant

F=ma



Springs and Simple Harmonic Motion



X=A; v=0; $a=-a_{max}$

X=0; $v=-v_{max}$; a=0

X=-A; v=0; $a=a_{max}$

X=0; $v=v_{max}$; a=0

X=A; v=0; $a=-a_{max}$

Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

OR

$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

$\boxed{\text{For spring: } \omega^2 = k/m}$

***Energy ***

- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$
  - ➔ $\Sigma W_{nc} = \Delta K + \Delta U$
  - ➔ $0 = \Delta K + \Delta U$ or Energy U+K is constant!

    Energy $= \frac{1}{2} k x^2 + \frac{1}{2} m v^2$
  - ➔ At maximum displacement $x=A$, $v = 0$

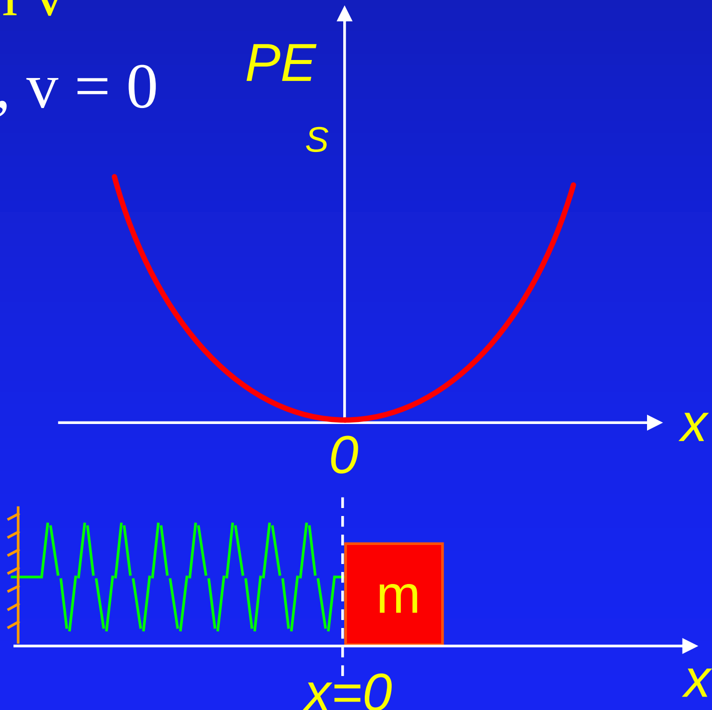
    Energy $= \frac{1}{2} k A^2 + 0$
  - ➔ At zero displacement $x = 0$

    Energy $= 0 + \frac{1}{2} m v_m^2$

    Since Total Energy is same

    $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$

    $v_m = \text{sqrt}(k/m)\, A$

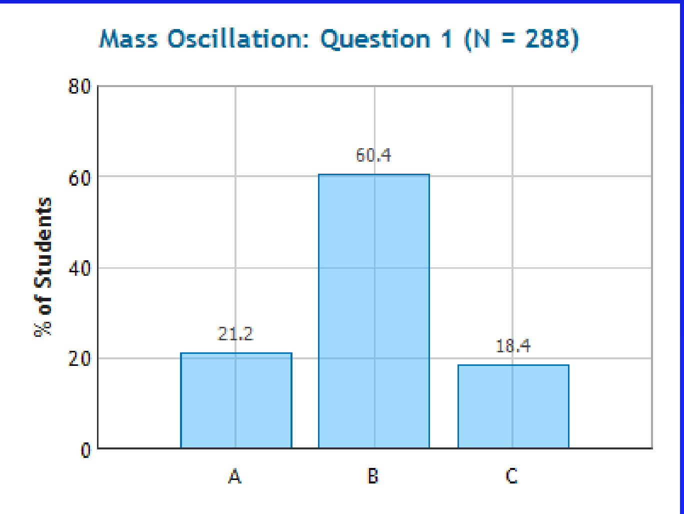


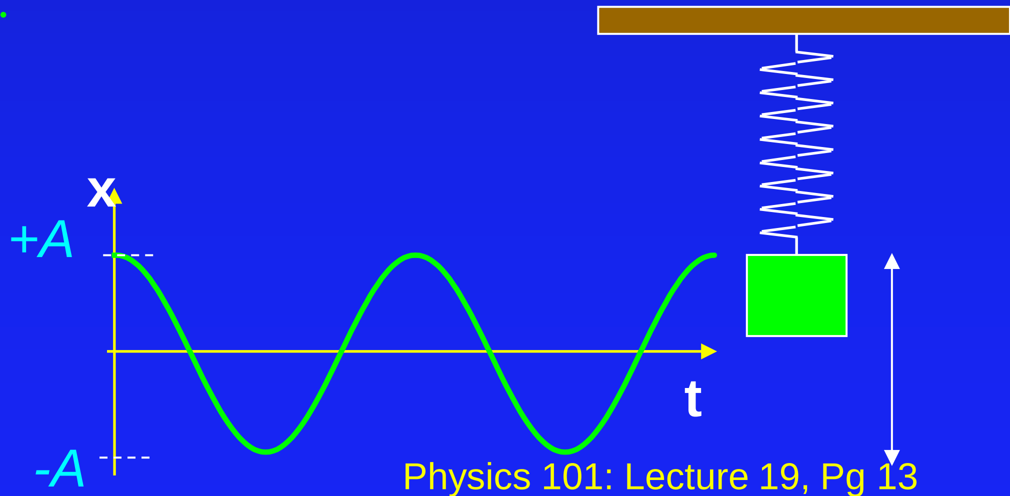
Prelecture 1+2

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement) ← CORRECT
3. The speed of the mass is constant

"At x=0 all spring potential energy is converted into kinetic energy and so the velocity will be greatest at this point."

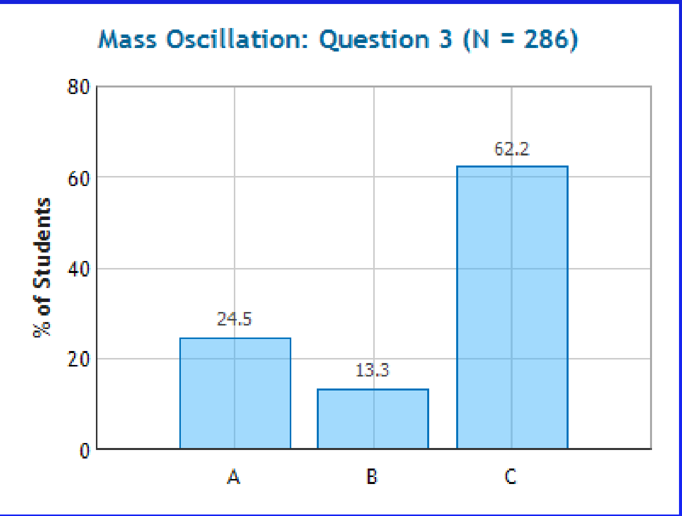




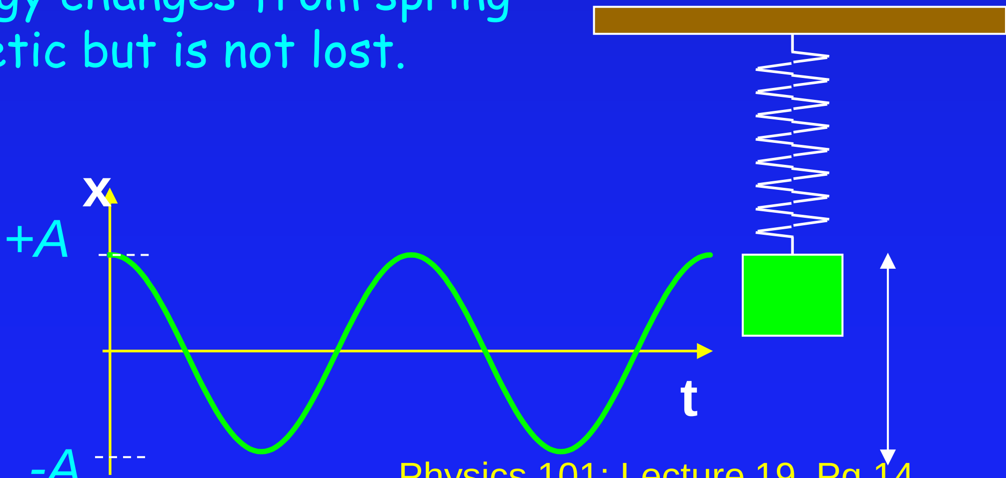
Prelecture 3+4

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total energy (K+U) of the mass and spring a maximum? (Ignore gravity).

1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The energy of the system is constant ← CORRECT



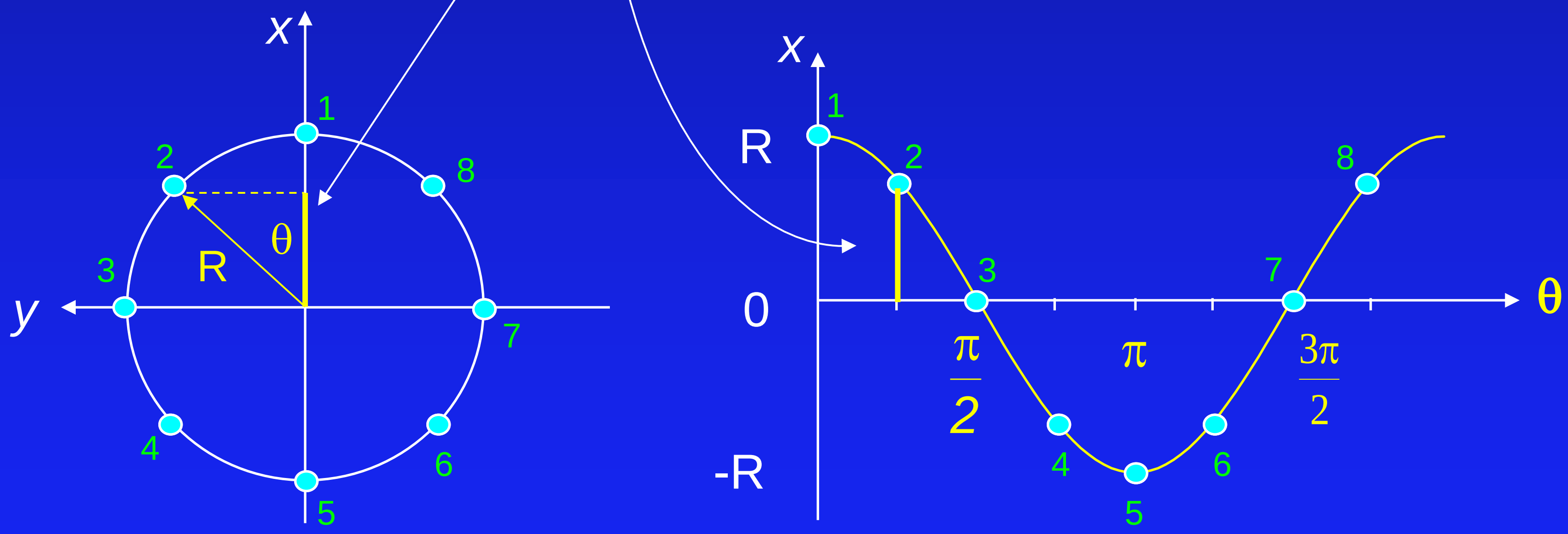
The energy changes from spring to kinetic but is not lost.



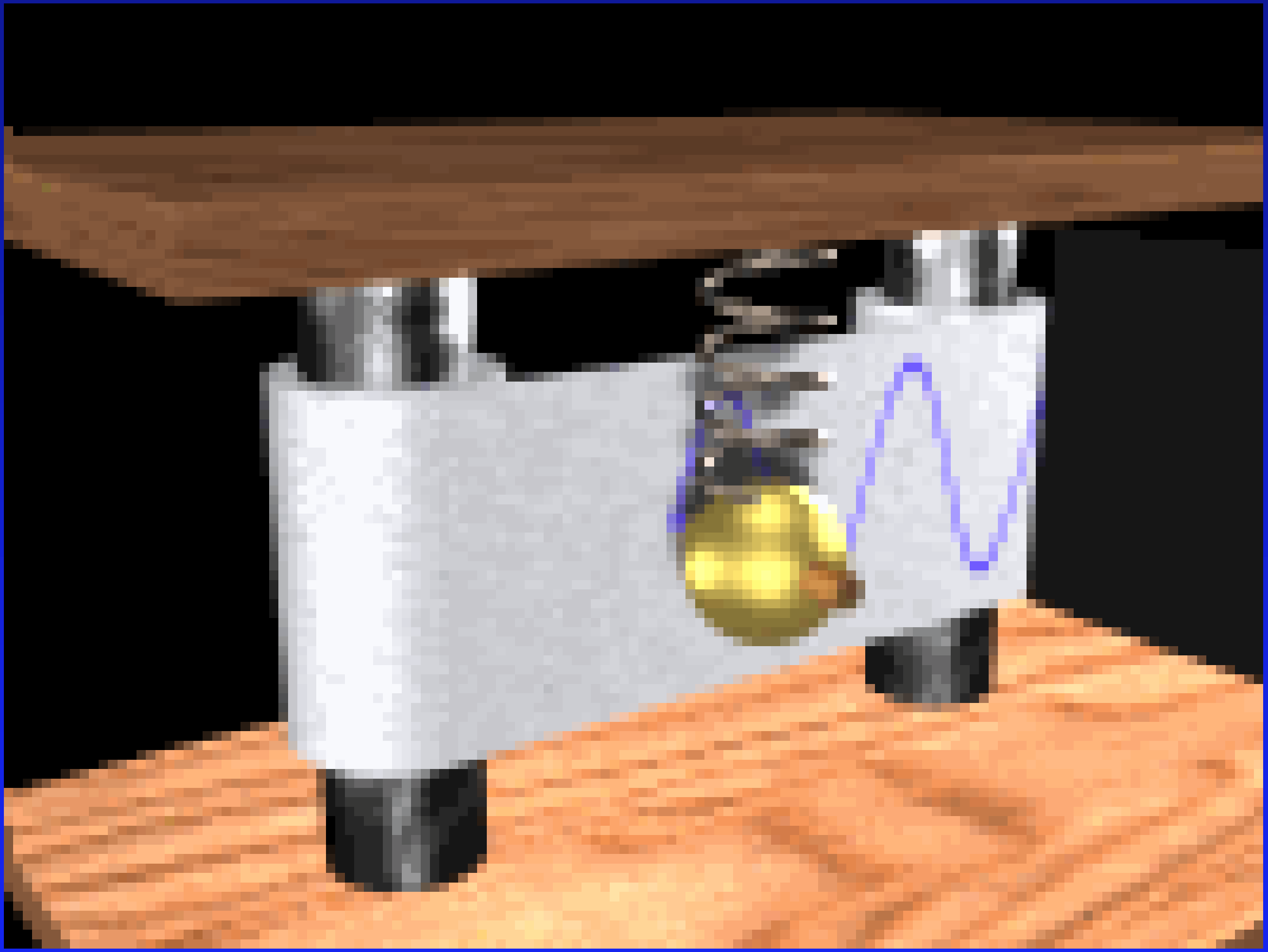
What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$$x = R\cos\theta = R\cos(\omega t)$$

since $\theta = \omega t$



SHM and Circles



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

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OR

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$x_{max} = A$

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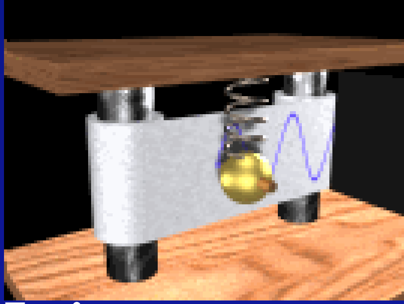
Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

$\boxed{\text{For spring: } \omega^2 = k/m}$

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

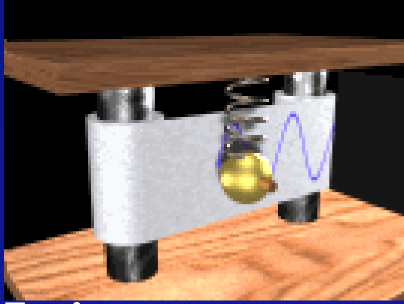
Which equation describes the position as a function of time x(t) =

A) 5 sin(ωt) B) 5 cos(ωt) C) 24 sin(ωt)

D) 24 cos(ωt) E) -24 cos(ωt)

We are told at t=0, x = +5 cm. x(t) = 5 cos(ωt) only one that works.

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the total energy of the block spring system?

A) 0.03 J　　B) .05 J　　C) .08 J

E = U + K

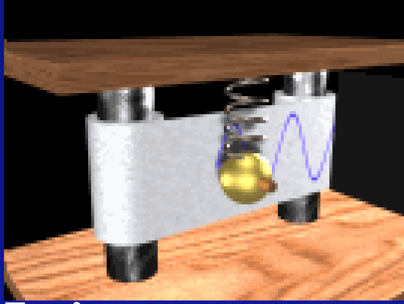
At t=0, x = 5 cm and v=0:

$E = \frac{1}{2} k x^2 + 0$

$= \frac{1}{2}$ (24 N/m) (5 cm)^2

= 0.03 J

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the maximum speed of the block?

A) .45 m/s B) .23 m/s C) .14 m/s

$E = U + K$

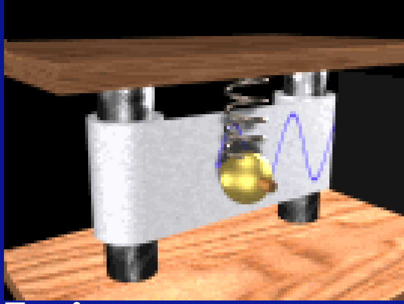
When $x = 0$, maximum speed:

$E = \frac{1}{2} m v^2 + 0$

$.03 = \frac{1}{2}\ 3\ \text{kg}\ v^2$

$v = .14$ m/s

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

How long does it take for the block to return to x=+5cm?

A) 1.4 s B) 2.2 s C) 3.5 s

ω = sqrt(k/m)

= sqrt(24/3)

= 2.83 radians/sec

Returns to original position after 2 π radians

T = 2 π / ω = 6.28 / 2.83 = 2.2 seconds

Summary

- Springs
  - ➔ $F = -kx$
  - ➔ $U = \frac{1}{2} k x^2$
  - ➔ $\omega = \mathrm{sqrt}(k/m)$

- Simple Harmonic Motion
  - ➔ Occurs when have linear restoring force $F = -kx$
  - ➔ $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$

Young's Modulus

- Spring $F = -k\,x$ [demo]
  - ➔ What happens to "k" if cut spring in half?
  - ➔ A) decreases B) same C) increases
- k is inversely proportional to length!
- Define
  - ➔ Strain = $\Delta L / L$
  - ➔ Stress = F/A
- Now
  - ➔ Stress = Y Strain
  - ➔ $F/A = Y \; \Delta L/L$
  - ➔ $k = Y\,A/L$ from $F = k\,x$
- Y (Young's Modules) independent of L