Exam III

Physics 101: Lecture 20
Elasticity and Oscillations

Today's lecture will cover Textbook Chapter 10.5-10.10

Tuned mass damper (pendulum) in Taipei 101

Review Energy in SHM

- A mass is attached to a spring and set to motion. The maximum displacement is $x = A$
  - ➔ Energy = U + K = constant!

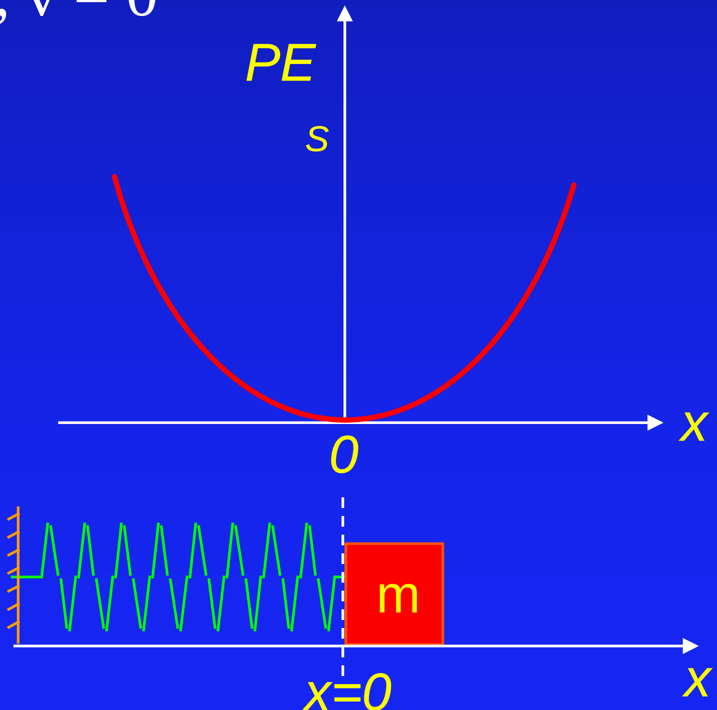
    $= \frac{1}{2} k x^2 + \frac{1}{2} m v^2$
  - ➔ At maximum displacement $x = A$, $v = 0$

    Energy $= \frac{1}{2} k A^2 + 0$
  - ➔ At zero displacement $x = 0$

    Energy $= 0 + \frac{1}{2} m v_m^2$

    $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$

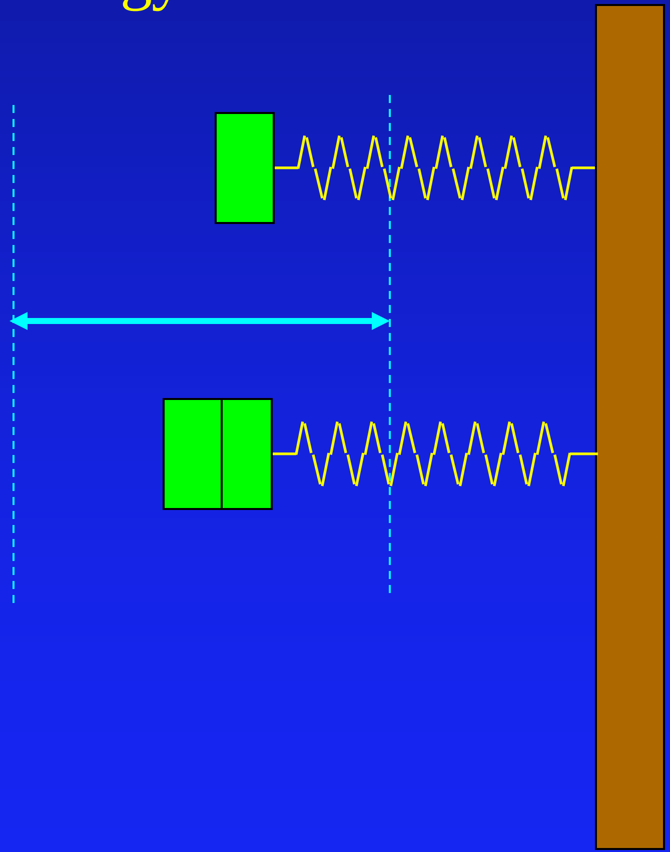
    $v_m = \mathrm{sqrt}(k/m)\, A$
  - ➔ Analogy with gravity/ball



Kinetic Energy ACT

In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1.

In which case is the maximum kinetic energy of the mass the biggest?



A. Case 1

B. Case 2

C. Same

Potential Energy ACT

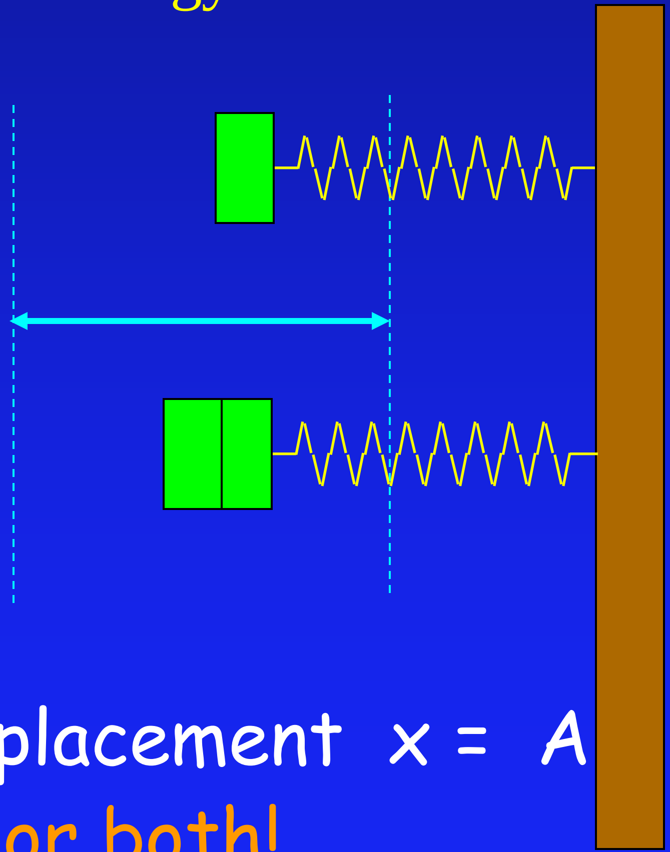
In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1.

In which case is the maximum potential energy of the mass and spring the biggest?

A. Case 1

B. Case 2

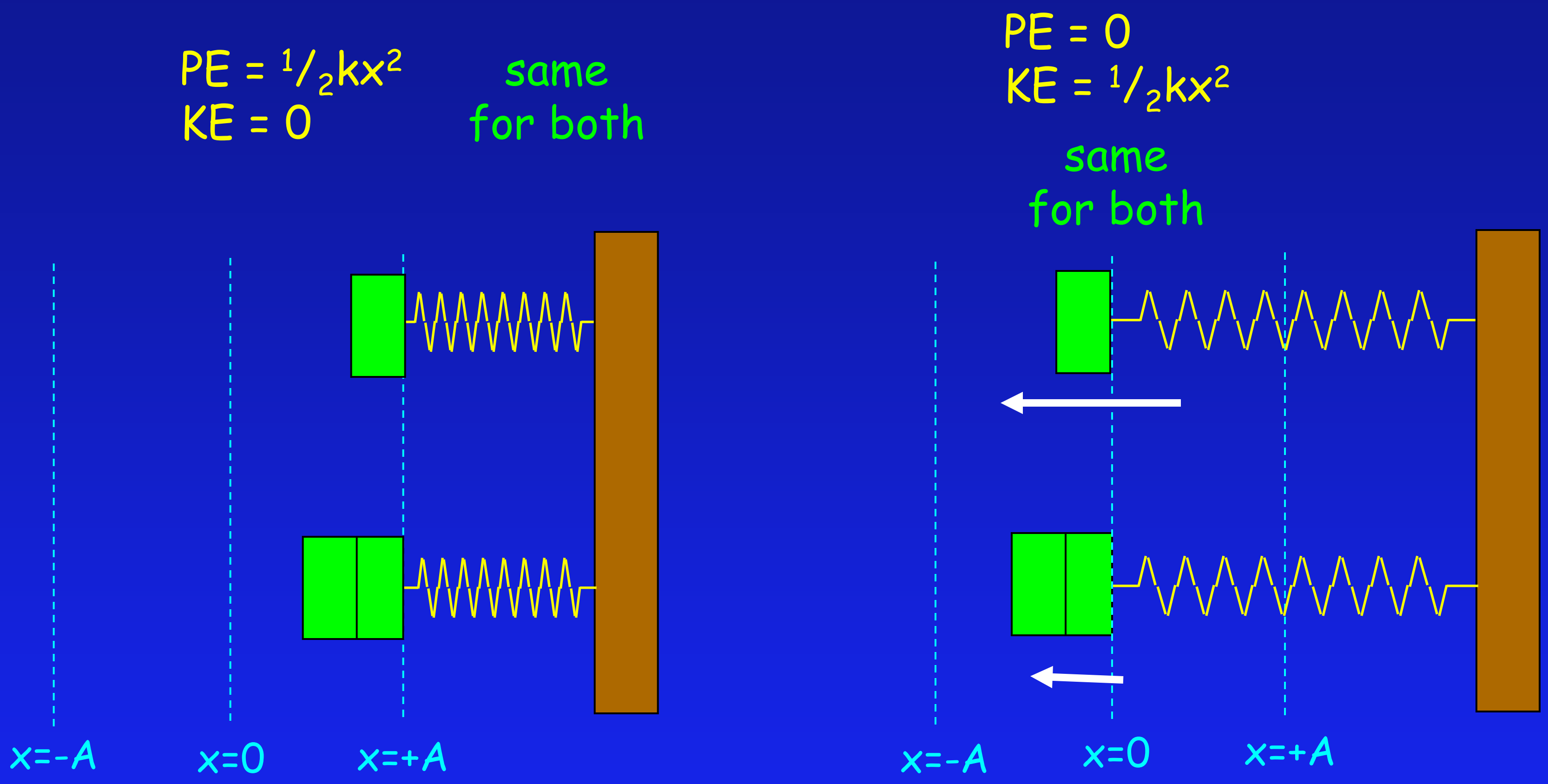
C. Same



Look at time of maximum displacement $x = A$

Energy $= \frac{1}{2} k A^2 + 0$ Same for both!

Kinetic Energy ACT



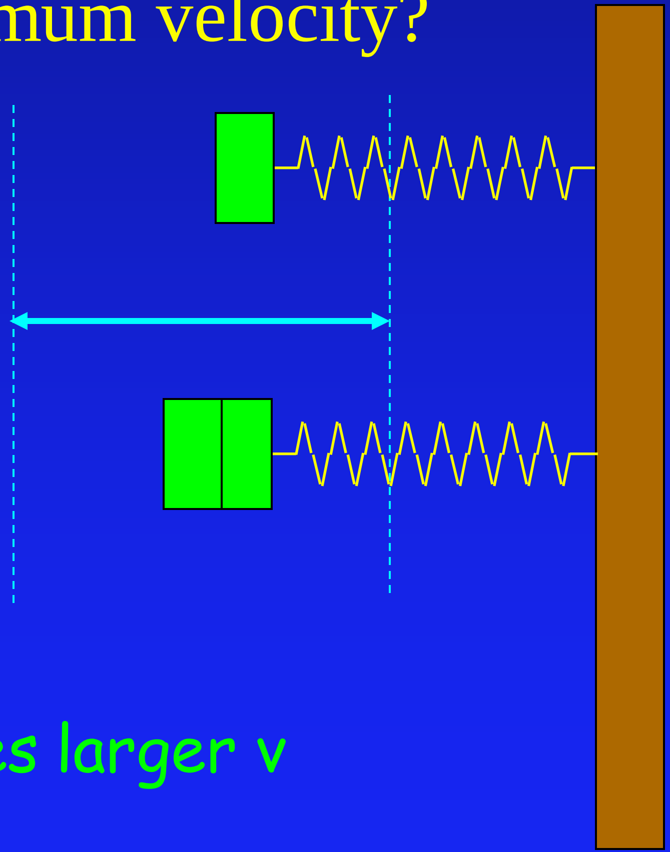
A) Case 1

B) Case 2

C) Same

Velocity ACT

In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1. Which case has the largest maximum velocity?

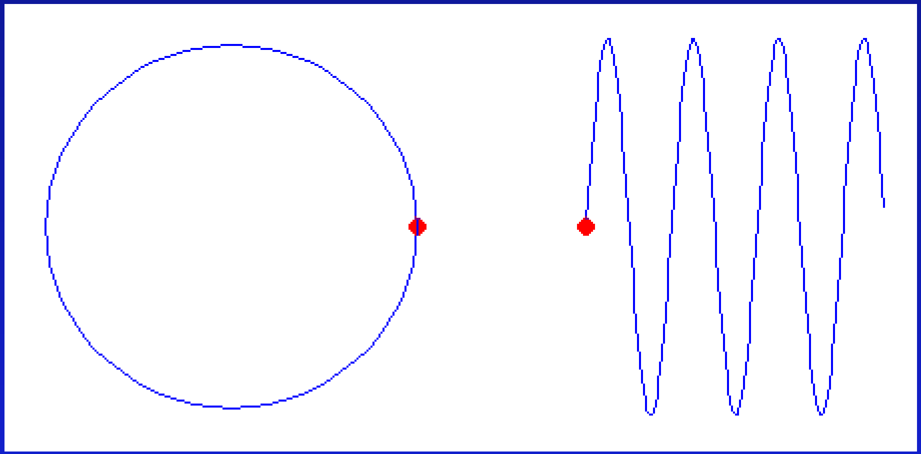


1. Case 1
2. Case 2
3. Same

Same maximum Kinetic Energy

$K = \frac{1}{2} m v^2$ smaller mass requires larger v

Review: Simple Harmonic Motion



Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = ω = $2\pi f$ = $2\pi/T$ (radians per second)

Period T of a Spring

- Simple Harmonic Oscillator
  - ➔ $\omega = 2\pi f = 2\pi / T$
  - ➔ $x(t) = [A] \cos(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$

Demos:

A,m,k dependence

- Draw FBD, write F=ma

$-k\,x = m\,a$

$-k\,A = m\,a_{max}$

$-k\,A = m\,(-A\,\omega^2)$

$A\omega^2 = (k/m)\,A$

$\omega = \text{sqrt}(k/m)$

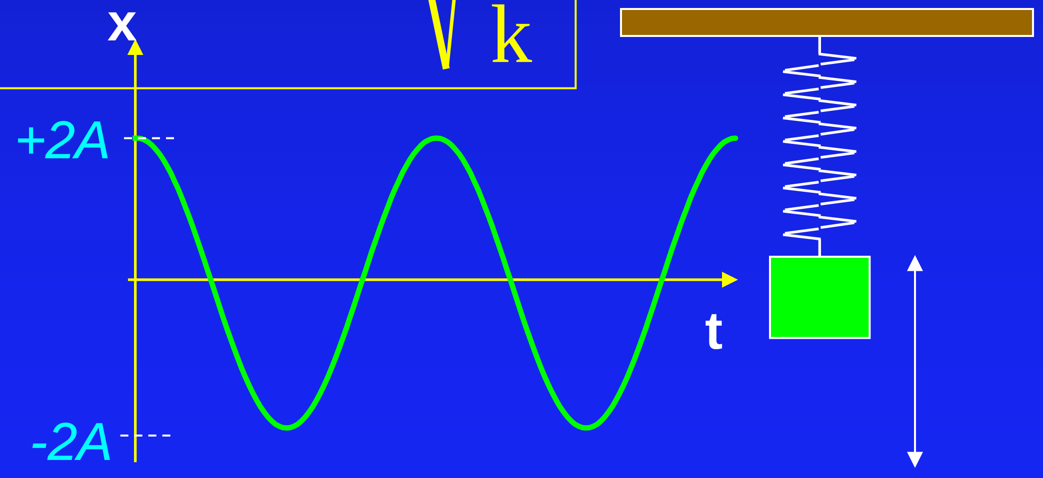
$$\boxed{\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}}$$

Period ACT

If the amplitude of the oscillation (same block and same spring) is doubled, how would the period of the oscillation change? (The period is the time it takes to make one complete oscillation)

A. The period of the oscillation would double.
B. The period of the oscillation would be halved
C. The period of the oscillation would stay the same ← CORRECT

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$



Vertical Mass and Spring

- If we include gravity, there are two forces acting on mass. With mass, new equilibrium position has spring stretched d
  - ➔ $F_{Net,\,y} = 0$

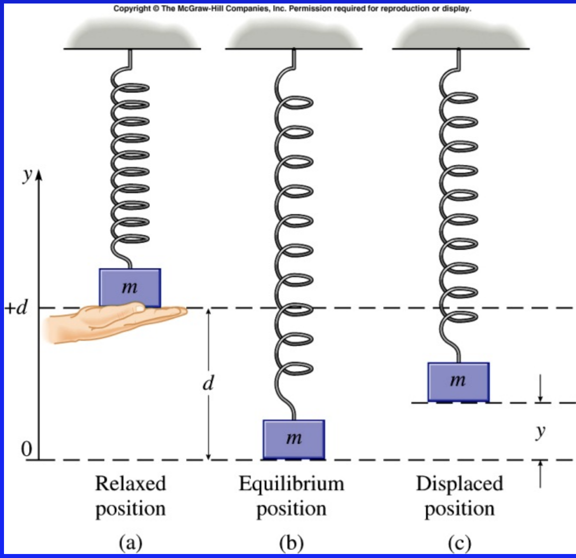
    $kd - mg = 0$

    $d = mg/k$

- Now displace a distance y:
  - ➔ $F_{Net} = ma$

    $k(d-y) - mg = ma$

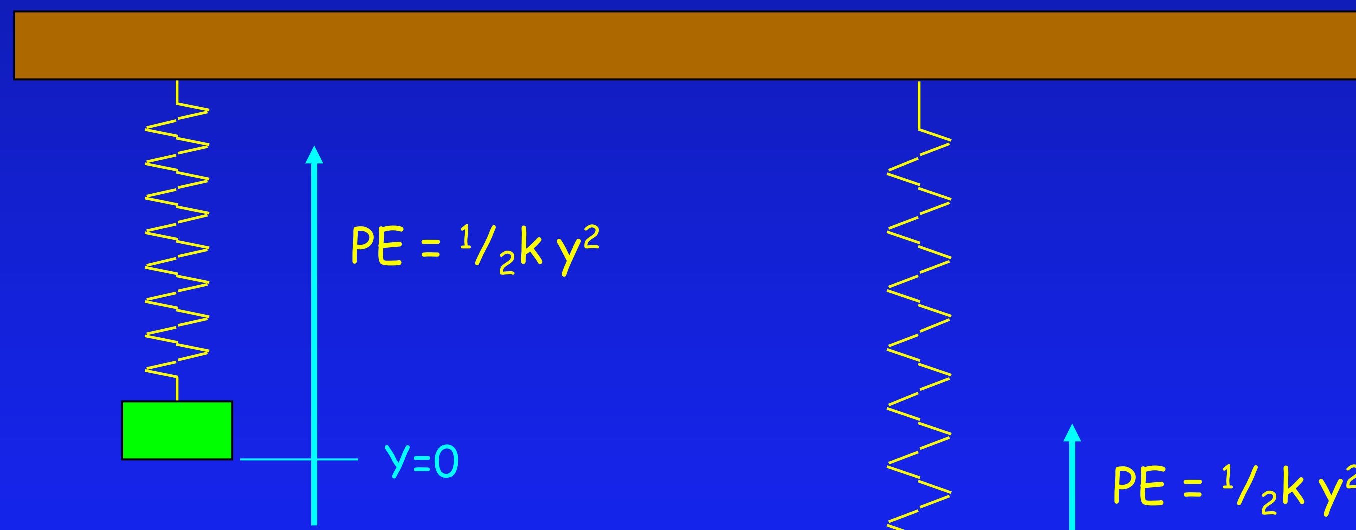
    $-k\,y = ma$
  - ➔ Same as horizontal! SHO
  - ➔ New equilibrium position

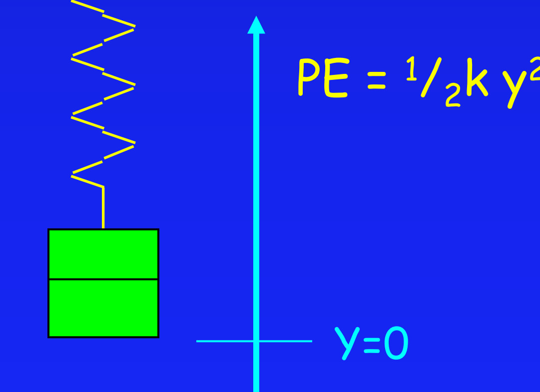


Vertical Spring ACT

If the springs were vertical, and stretched the same distance d from their equilibrium position and then released, which would have the largest maximum kinetic energy?

1) M 2) 2M 3) Same





Just before being released, v=0 y=d

$E_{tot} = 0 + \frac{1}{2}\, k\, d^2$ Same total energy for both

When pass through equilibrium all of this energy will be kinetic energy again same for both!

Pendulum Motion

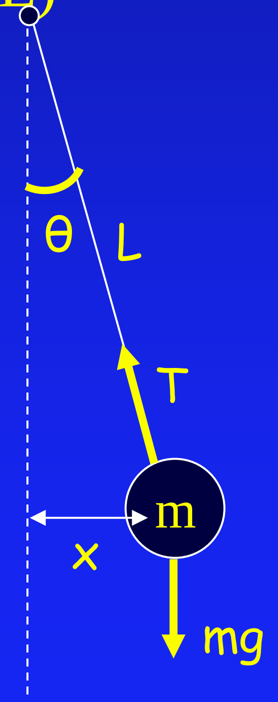
- For *small angles*
  - ➔ $T = mg$
  - ➔ $T_x = -T \sin\theta$
  - ➔ $T_x = -mg\,(x/L)$ Note: $F = -kx$! ($k = mg/L$)
  - ➔ $F_{Net,\,x} = m\,a_x$

    $-mg\,(x/L) = m\,a_x$

    $a_x = -(g/L)\,x$
  - ➔ Recall for SHO $a = -\omega^2\,x$

    $\omega = \mathrm{sqrt}(g/L)$

    $T = 2\,\pi\,\mathrm{sqrt}(L/g)$

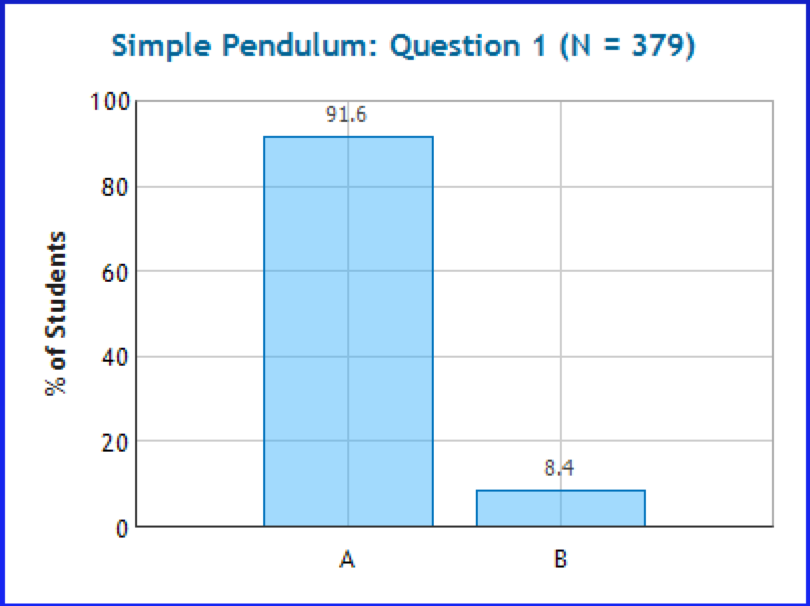


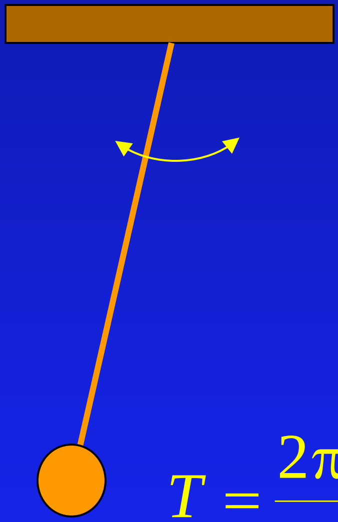
Period does not depend on A, or m!

Checkpoint 1

Suppose a grandfather clock (a simple pendulum) runs slow. In order to make it run on time you should:

1. Make the pendulum shorter ⟵ CORRECT
2. Make the pendulum longer





$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

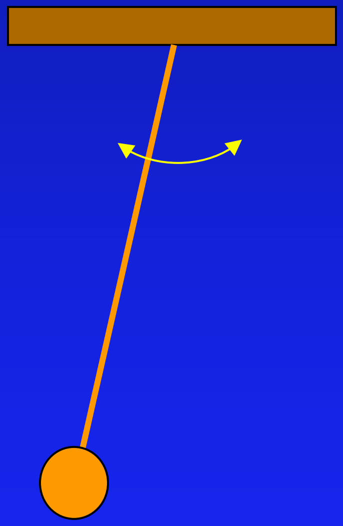
Elevator ACT

A pendulum is hanging vertically from the ceiling of an elevator. Initially the elevator is at rest and the period of the pendulum is T. Now the pendulum accelerates upward. The period of the pendulum will now be

A. greater than T

B. equal to T

C. less than T

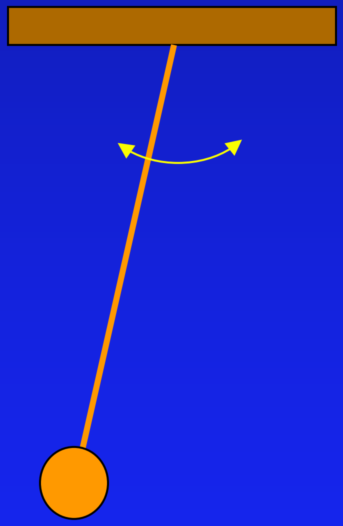


ACT

A pendulum is hanging vertically from the ceiling of an elevator. Initially the elevator is at rest and the period of the pendulum is T. Now the pendulum accelerates upward.

If you are accelerating upward your weight is the same as if g had

1. increased
2. same
3. decreased



"Effective g" is larger when accelerating upward

(you feel heavier)

Elevator ACT

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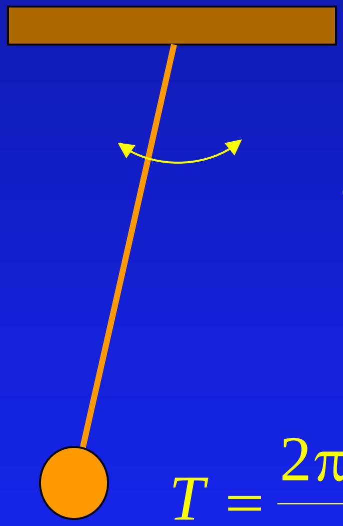
1. greater than T
2. equal to T
3. less than T ⟵ CORRECT



$$\omega = \sqrt{\frac{g}{L}}$$

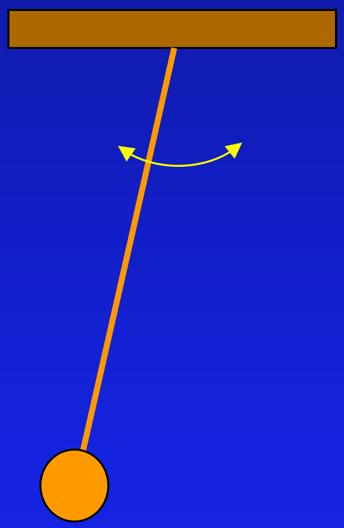
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

"Effective g" is larger when accelerating upward

(you feel heavier)

Checkpoint 2

Imagine you have been kidnapped by space invaders and are being held prisoner in a room with no windows. All you have is a cheap digital wristwatch and a pair of shoes (including shoelaces of known length). Explain how you might figure out whether this room is on the earth or on the moon



$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$g = (2\pi)^2 \frac{L}{T^2}$$

make a pendulum with the shoelace and shoes and use the wristwatch to determine the length of each period.

Summary

- Simple Harmonic Motion
  - ➔ Occurs when have linear restoring force $F = -kx$
  - ➔ $x(t) = [A] \cos(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$
- Springs
  - ➔ $F = -kx$
  - ➔ $U = \frac{1}{2} k x^2$
  - ➔ $\omega = \mathrm{sqrt}(k/m)$
- Pendulum (Small oscillations)
  - ➔ $\omega = \mathrm{sqrt}(L/g)$