EXAM II

Physics 101: Lecture 16
Angular Momentum

Today's lecture will cover Textbook Chapter 8.7-8.9

Overview

- Review
  - ➔ $K_{rotation} = \frac{1}{2} I \omega^2$
  - ➔ Torque = Force that causes rotation
  - ➔ Equilibrium
    - » $F_{Net} = 0$
    - » $\tau_{Net} = 0$

- Today
  - ➔ Angular Momentum $L = I\omega$
  - ➔ $\Delta L = 0$ if $\tau_{Net} = 0$

Linear and Angular

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	½ m v2	$\frac{1}{2} I \omega^2$
N2L	$F=ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$

Today

Define Angular Momentum

Momentum

$p = mV$

$F_{Net} = \Delta p/\Delta t$

conserved if $F_{ext} = 0$

Vector!

units: kg-m/s

Angular Momentum

$L = I\omega$

$\tau_{Net} = \Delta L/\Delta t$

conserved if $\tau_{ext} = 0$

Vector!

units: kg-m^2/s

Right Hand Rule

- Wrap fingers of right hand around direction of rotation, thumb gives direction of angular momentum.

- What is direction of angular momentum for wheel

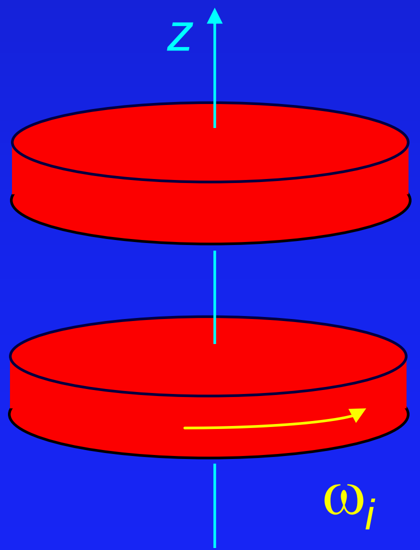
A) Up   B) Down   C) Left   D) Right

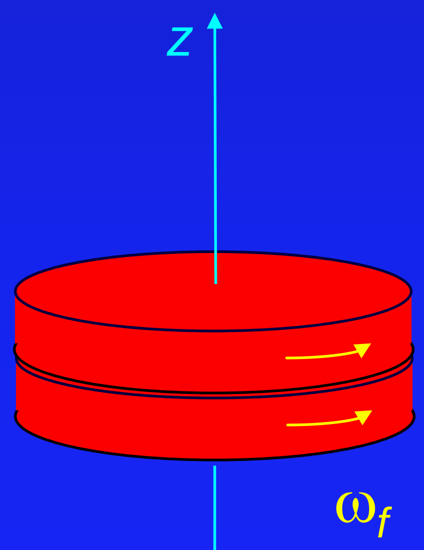
Wheel rim drop

Act: Two Disks

- A disk ($I_{disk} = \frac{1}{2} MR^2$) of mass M and radius R rotates around the z axis with angular velocity ω_i. A second identical disk, initially not rotating, is dropped on top of the first. There is friction between the disks, and eventually they rotate together with angular velocity ω_f.

A) $\omega_f = \omega_i$ B) $\omega_f = \frac{1}{2}\omega_i$ C) $\omega_f = \frac{1}{4}\omega_i$





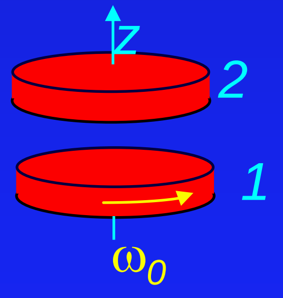
Act: Two Disks

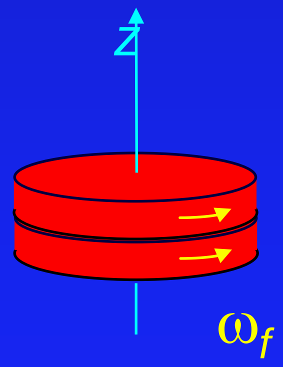
- First realize that there are no external torques acting on the two-disk system.
  - ➔ Angular momentum will be conserved!

$$L_i = \mathrm{I}_1\,\omega_1 + 0 = \frac{1}{2}MR^2\omega_i \qquad L_f = \mathrm{I}_1\,\omega_1 + \mathrm{I}_2\,\omega_2 = MR^2\omega_f$$

$$\frac{1}{2}MR^2\omega_i = MR^2\omega_f$$

$$\frac{1}{2}\omega_i = \omega_f$$

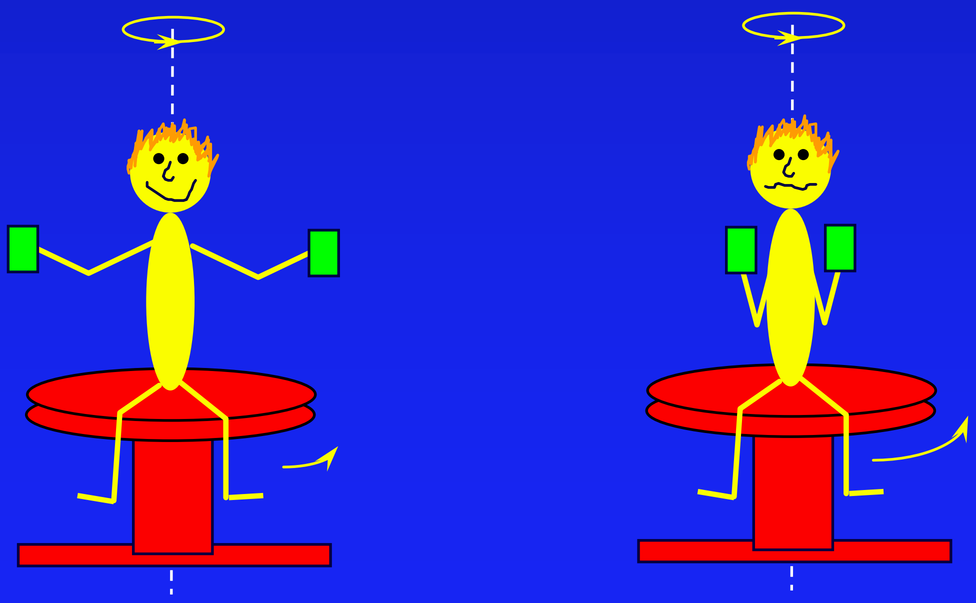




Lecture 16, Checkpoint

You are sitting on a freely rotating bar-stool with your arms stretched out and a heavy glass mug in each hand. Your friend gives you a twist and you start rotating around a vertical axis though the center of the stool. You can assume that the bearing the stool turns on is frictionless, and that there is no net external torque present once you have started spinning.

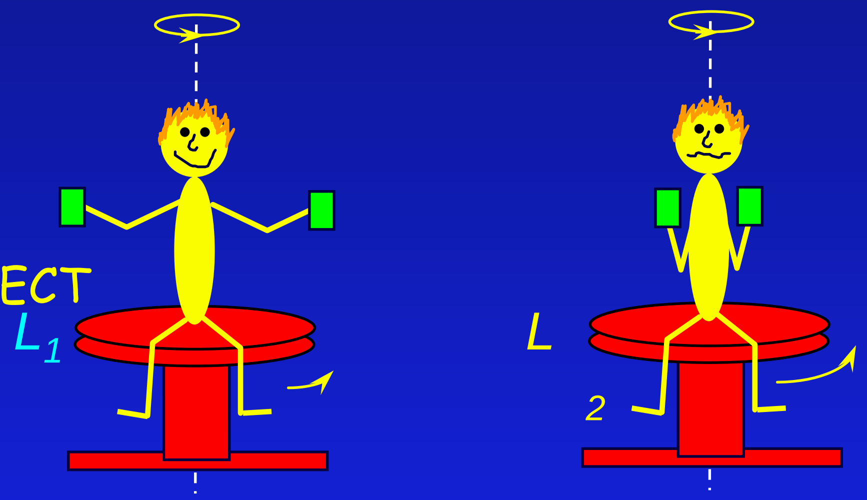
You now pull your arms and hands (and mugs) close to your body.



Lecture 16, Prelecture 1

What happens to the angular momentum as you pull in your arms?

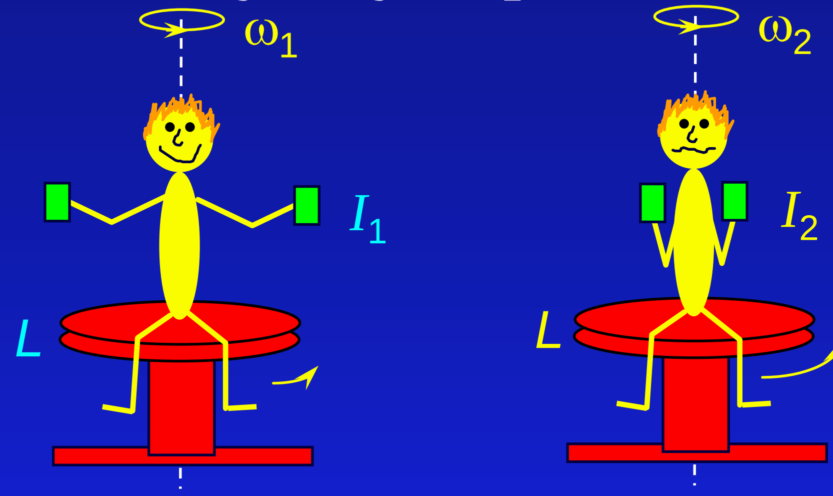
1. it increases
2. it decreases
3. it stays the same ← CORRECT



Lecture 16, Prelecture 2

What happens to your angular velocity as you pull in your arms?

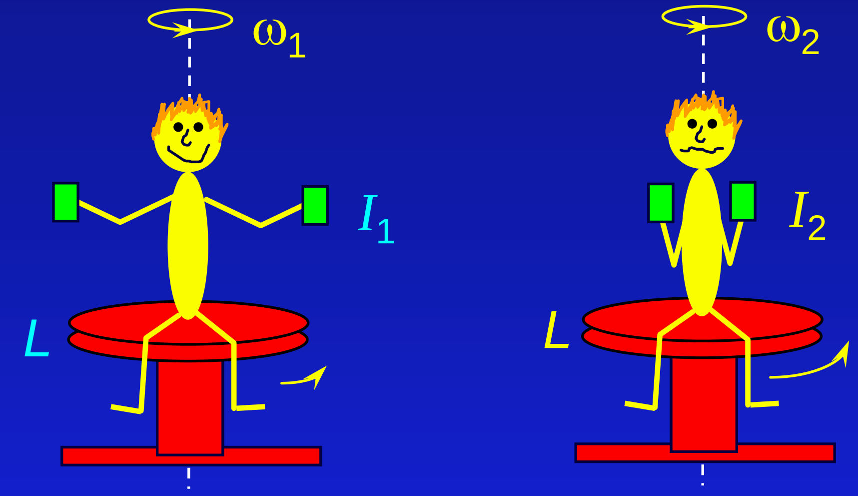
1. it increases ⟵ CORRECT
2. it decreases
3. it stays the same



Lecture 16, Prelecture 3

What happens to your kinetic energy as you pull in your arms?

1. it increases ⟵ CORRECT
2. it decreases
3. it stays the same



$$K = \frac{1}{2} I \omega^2 = \frac{1}{2I} I^2 \omega^2 = \frac{1}{2I} L^2$$ (using $L = I\omega$)

What about Energy Conservation?

A) Energy isn't conserved here

B) Energy comes from weights

C) Gravitational energy is being converted to rotational kinetic energy

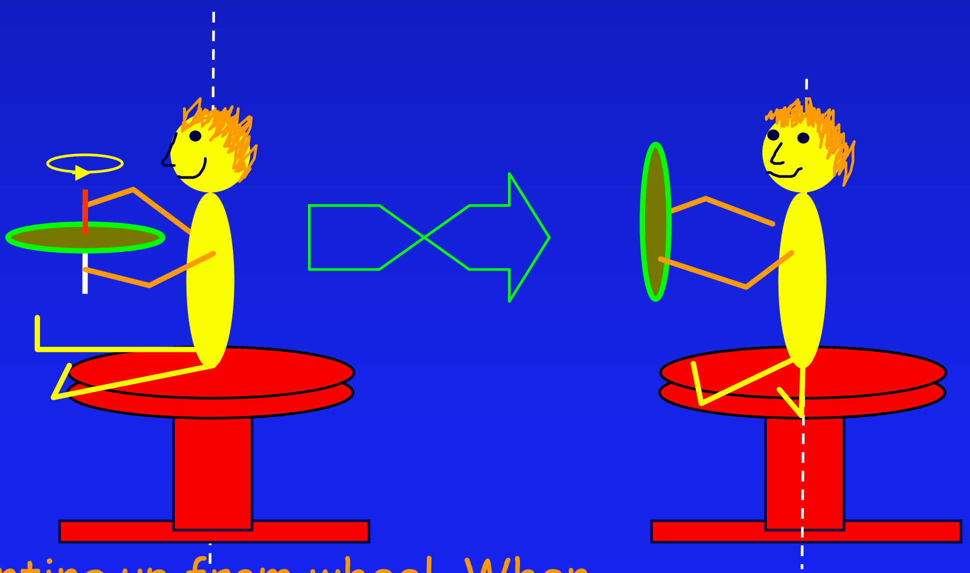
D) Energy comes from cookies.

E) I have no clue….

Turning the bike wheel

A student sits on a barstool holding a bike wheel. The wheel is initially spinning CCW in the horizontal plane (as viewed from above) $L = 25\ kg\ m^2/s$ She now turns the bike wheel over. What happens?

A. She starts to spin CCW. ⟵ CORRECT
B. She starts to spin CW.
C. Nothing

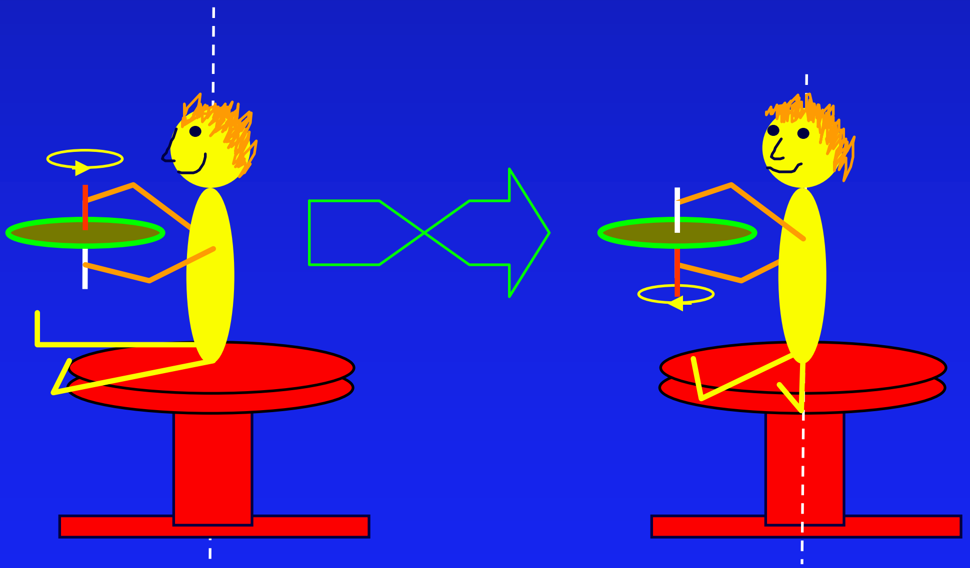


Start w/ angular momentum L pointing up from wheel. When wheel is flipped, no more angular momentum from it pointing up, so need to spin person/stool to conserve L!

Turning the bike wheel (more)

She is holding the bike wheel and spinning counter clockwise. What happens if she turns it the other ½ rotation (so it is basically upside down from how it started).

A) Spins Faster B) Stays same C) Stops



Turning the bike wheel...

- Since there is no net external torque acting on the student-stool system, angular momentum is conserved.

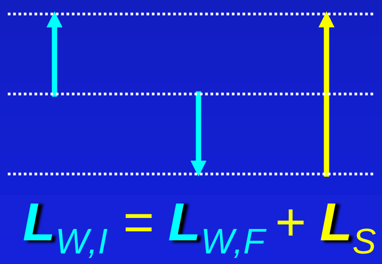
  ➔Remenber, $\boldsymbol{L}$ has a <u>direction</u> as well as a <u>magnitude</u>!

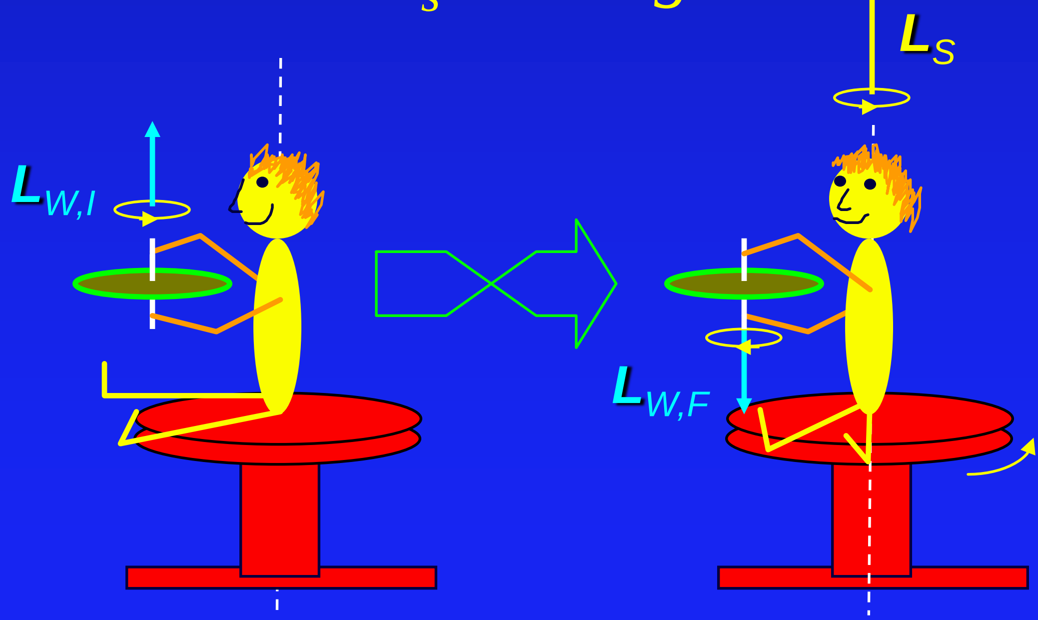
Initially: $\boldsymbol{L}_{INI} = \boldsymbol{L}_{W,I} = +\,25\ kg\ m^2/s$

Finally: $\boldsymbol{L}_{FIN} = \boldsymbol{L}_{W,F} + \boldsymbol{L}_S$

$= -25\ kg\ m^2/s + L_s$

$L_s = 50\ kg\ m^2/s$



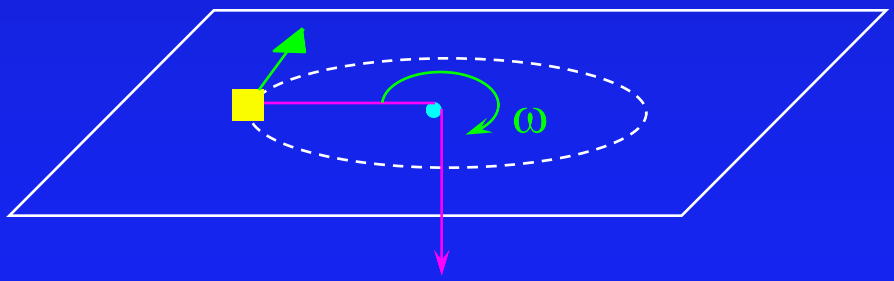


Act 2 Rotations

Puck on ice

- A puck slides in a circular path on a horizontal frictionless table. It is held at a constant radius by a string threaded through a frictionless hole at the center of the table. If you pull on the string such that the radius decreases by a factor of *2*, by what factor does the angular velocity of the puck increase?

(a) 2 (b) 4 (c) 8



Act 2 Solution

- Since the string is pulled through a hole at the center of rotation, there is no torque: Angular momentum is conserved.

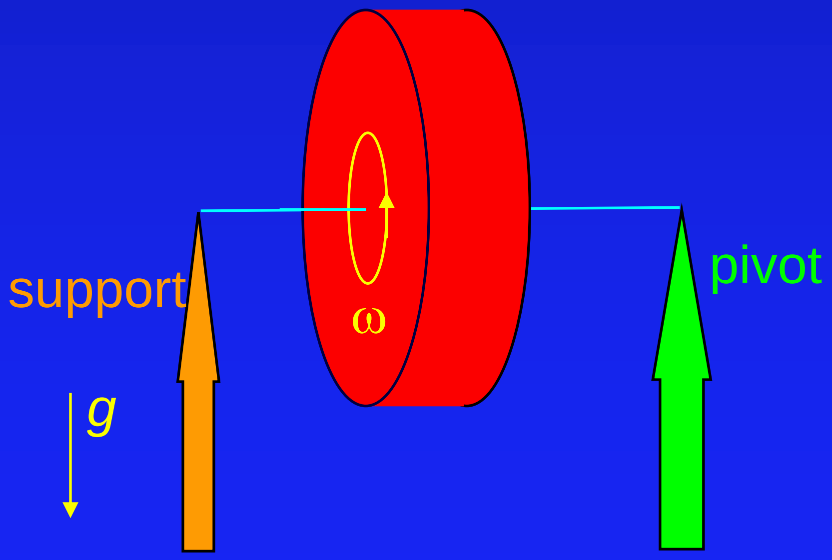
$$L_1 = I_1\omega_1 = mR^2\omega_1 \qquad = \qquad L_2 = I_2\omega_2 = m\left(\frac{R}{2}\right)^2\omega_2$$

$$mR^2\omega_1 = m\frac{1}{4}R^2\omega_2$$

$$\omega_1 = \frac{1}{4}\omega_2 \quad \Rightarrow \quad \boxed{\omega_2 = 4\omega_1}$$

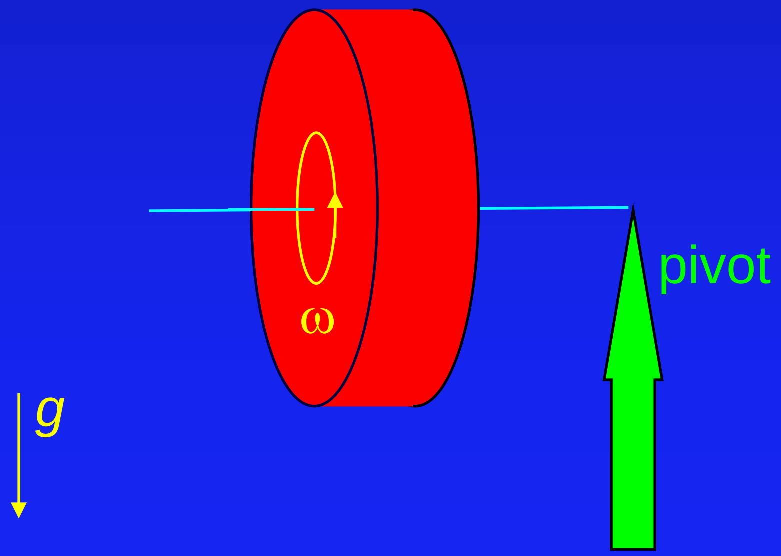
Gyroscopic Motion:

- Suppose you have a spinning gyroscope in the configuration shown below:
- If the left support is removed, what will happen??



Gyroscopic Motion...

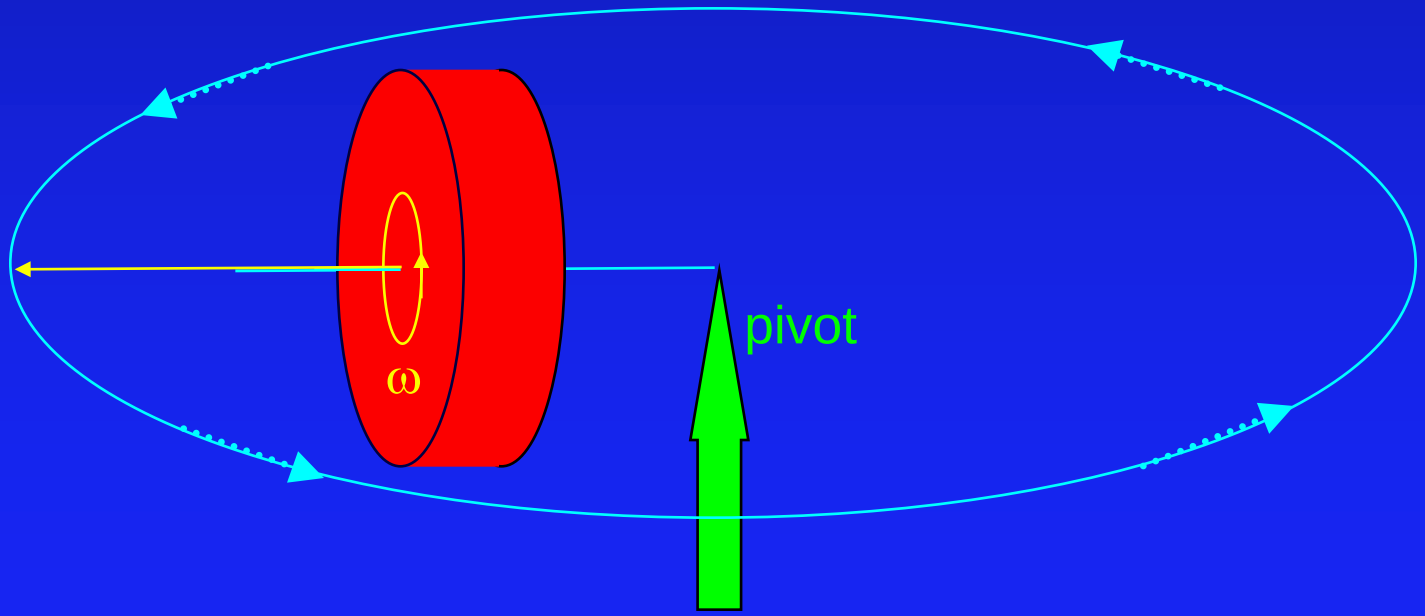
- Suppose you have a spinning gyroscope in the configuration shown below:
- If the left support is removed, what will happen?
  - ➔ The gyroscope does not fall down!



Gyroscopic Motion...

Bicycle wheel

- ... instead it *precesses* around its pivot axis !



Summary

- $\tau_{Net} = I\,\alpha$
- $L = I\,\omega$
  - ➔ Right Hand Rule gives direction
  - ➔ If $\tau_{Net} = 0$, L is conserved