Beyond GR & standard objects: Issues and challenges

Luis Lehner

Perimeter Institute

[based on results with: *Ramiro Cayuso, Guillaume Dideron, Tiago Franca* & Pau Figueras;
Nicola Franchini, Miguel Bezares & Enrico Barausse*;*
Ulf Danielsson & Frans Pretorius]

NSERC CRSNG
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS
CIFAR CANADIAN INSTITUTE for ADVANCED RESEARCH

Ramiro

Tiago

Nicola

Miguel

Context…

- Many arguments point to considering extensions from GR (e.g. singularity resoln, DM, DE), and alternatives to black holes (e.g. information paradox, quantum arguments) $\rightarrow G_{ab} + \ldots = T_{ab} + \ldots$

- Exquisite data coming through: in *gravitational waves, EHT*, binary pulsars, large scale, CMB, etc.

- Especially in the context of compact binary mergers tests reach the highly dynamical (v/c ~ 0.5), strongly gravitating (M/R ~ 1), 'strong' curvature regimes ($M/R^2 \sim 10^{-9}$-1 M_O^{-1})
  - Search is suboptimal without guidance
  - Analysis is incomplete without guidance [phenomenological approach?]

- Many options, which can be *interesting and/or viable*

Beyond BHs: some examples…

- Boson stars, gravastars, fuzzballs, BHs (or NSs) with hair. Some of these as 'regarded' from a GR point of view or true solutions of extensions to GR

- Interesting features found in specific solutions… e.g. rotating black holes different from Kerr; rotating highly compact objects with $J/M^2>1$; echoes of gravitational waves… *but are these physically relevant?*

  - *Are they stable?*
  - *Can they be formed?*
  - *Can these questions even be asked?*

Beyond GR: some examples

- Pulled out from theorists imagination: “let this be a metric”
- Horndenski family (2nd order, 1 extra scalar field d.o.f; e.g. ESGB)
- ‘Breaking something else’: Dynamical Chern Simons; Horava-Lifschitz, Einstein-Aether, Coscouton …
- Effective field theory motivated theories (higher order theories with or without extra d.o.f at the ‘long-wavelength’ regime; e.g. dCS)

  - *Are these viable?*
  - *Can their non-linear dynamics be explored?*
  - *Can these questions be asked?*

why?....

Many of these questions, are difficult to answer or even impossible unless further issues are explored and understood

- Equations of motion are of unknown type

$$u_{,tt} = u_{,xx}$$
$$u_{,tt} = (1 + u)u_{,xx}$$
$$(1 + u_{,a})u_{,tt} = (1 + u_{,b})u_{,xx}$$
$$(1 + u_{,ab})u_{,tt} = (1 + u_{,cd})u_{,xx}$$

- Models aren't sufficiently complete
- New physical, non-linear in nature, mechanisms might arise. Relying on linear intuition might miss them

- *Not a goal here*: to advocate for/or analyze a particular theory or object

- *Instead:*
  - Illustrate such issues do arise & consequences
  - Discuss options to plow forward and examine some specific cases

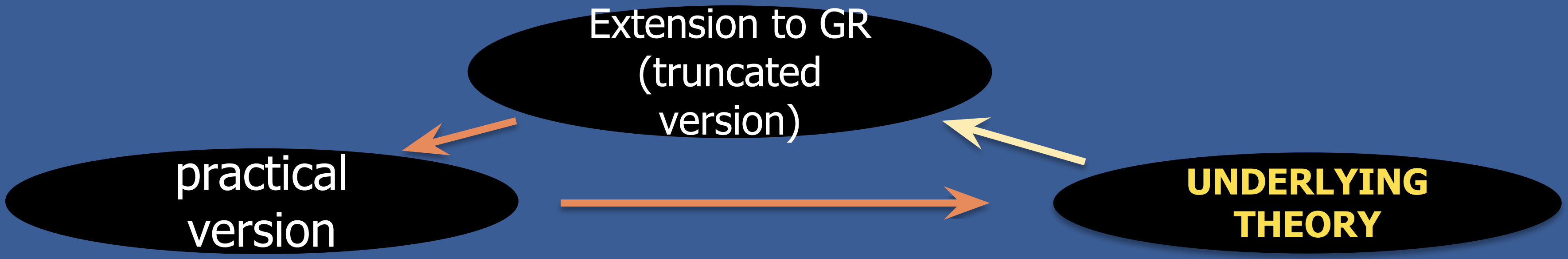
EFT path…

- Degrees of freedom relevant at higher energies, integrated out and their effects captured by higher order curvature corrections [and potentially others if other fields are considered]

$$L \sim R + a\, R^2 + b\, R^3 + c\, R^4 \ldots.$$

(e.g. Senatore+, deRham+, Hertog+, Trodden+....)

  - used extensively in cosmology, and more recent efforts in compact binary systems… but is it sensible to “shut up and calculate”?



But what is in the equations of motion?

- Higher order curvature contributions, will give rise to equations with higher derivatives… e.g.

$$u_{,tt} = u_{,xx} + \lambda(u_{,xxx} + u_{,ttt})$$
$$u_{,tt} = u_{,xx} + \lambda(u_{,xxxx} + u_{,tttt})$$

- Now what?
  - Could use reduction of order (or field redefinitions) to have only 2nd order *time* derivatives… but spatial derivatives? →ill posedness becomes generic.
  - Regardless, one seeks to capture true physical solutions either within the EFT regime or through a suitable ‘completion’

What's the big deal? After all:

- One could introduce a 'cut-off' and bound frequencies:
- $\rightarrow$ Not so fast, equations are non-linear; low freqns -> high frequencies and also, high freqns-> low ones

- Option 1: we can solve, 'iteratively/hierarchically' (e.g. Okounkova+Stein)

$$u^0_{,tt} = u^0_{,xx}$$
$$u^1_{,tt} = u^1_{,xx} + \lambda(u^0_{,xxx})$$

- $\rightarrow$ Not so fast, no guarantees this would work, the problem at hand is very different from computing corrections to scattering. Secular effects can compound quite strongly and nonlinearities make things worse

- *Option 1:*

Take Navier Stokes, solution of:

$$\partial_t \hat{v} + \hat{v}\partial\hat{v} = 0 \ (\sim\eta\,\partial^2 v)$$

displays turbulence for all wavelengths. What recovers the laminar regime? [short wavelength!]

$$\partial_t v + \hat{v}\partial v + v\partial\hat{v} = \eta\partial^2 v$$

Introduces a secular dependence, and if resummation is possible, it would reveal the laminar/turbulent regimes.

Example, option (1) DCS

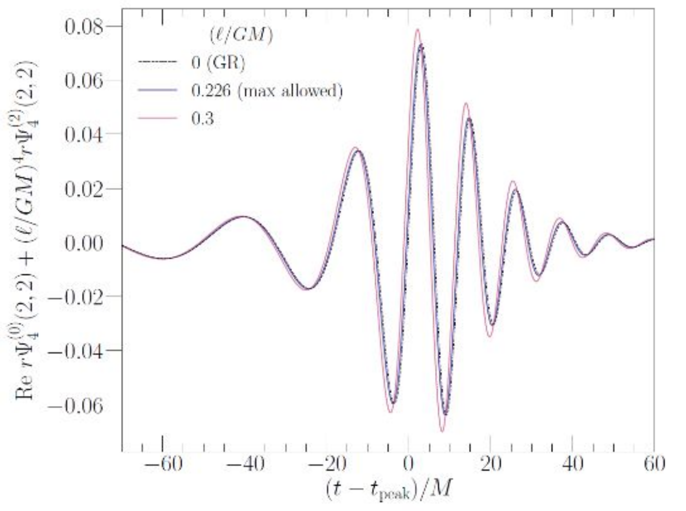


FIG. 3. Second-order accurate dCS gravitational waveforms, for three choices of dCS coupling constant, ℓ/GM. We add the leading-order dCS correction to the gravitational waveform (from Fig. 2) to the background GR gravitational waveform of the system, to give the total dCS waveform [cf. (10)]. The value $\ell/GM = 0$ corresponds to GR, with no dCS modifications. The value $\ell/GM = 0.226$ corresponds to the largest-allowed value for the perturbative scheme to be valid (cf. Sec. III F). The $\ell/GM = 0.3$ curve is included to visually emphasize the shape of alteration provided by the dCS correction.

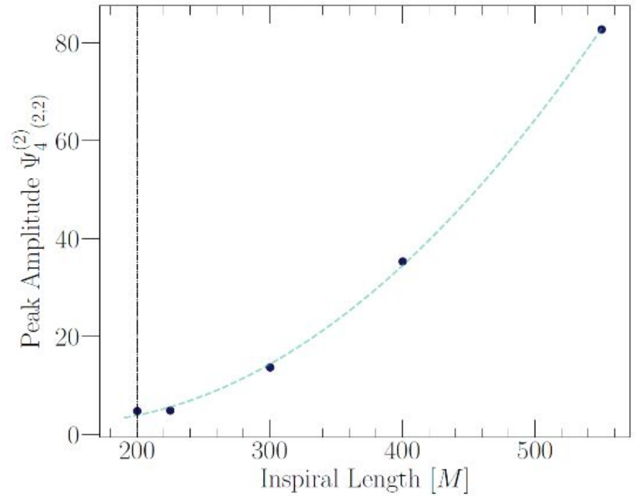


FIG. 7. Peak amplitude of the dCS correction to the gravitational waveform as a function of inspiral length. We show the length relative to the peak of the waveform (as in Fig. 6). The dashed black vertical line corresponds to the length of the dCS merger simulation we present in this paper. The peak amplitude serves as a measure of the amount of secular growth in the waveform (cf. Fig. 6). We see that the peak amplitude increases quadratically with inspiral length (as shown by the quadratic fit in dashed green).

Okounkova-Stein + (similar method used by Witek-Sperhake + for ESGB) [Aso, Galvez Ghersi-Stein]

- Option 2: Modify the system of eqns, in an ad-hoc manner to control higher gradients and prevent wild runaway to the UV

$$B(g) = F\,;\, F_t = -\lambda(F - S(g))$$

  - Modify system of equations to 'fix' problems [Cayuso+ '17]
  - Introduces a new timescale λ, can one guarantee the fidelity of the solution obtained?

- E.g. Israel-Stewart formultion of viscous relativistic hydrodynamics: T = T^{pf} + gradient terms
  - Define $\Pi =$ (shear/bulk)$_{ab}$ + Grad(shear/bulk..)$_{ab}$ as new and independent variable
  - Force an eqn on Π such that $\Pi \sim$ (shear/bulk)$_{ab}$ to leading order always

    $\tau\, \Pi_{,t}$ = - Π + (shear/bulk)$_{ab}$ …. [Geroch, details shouldn't matter]

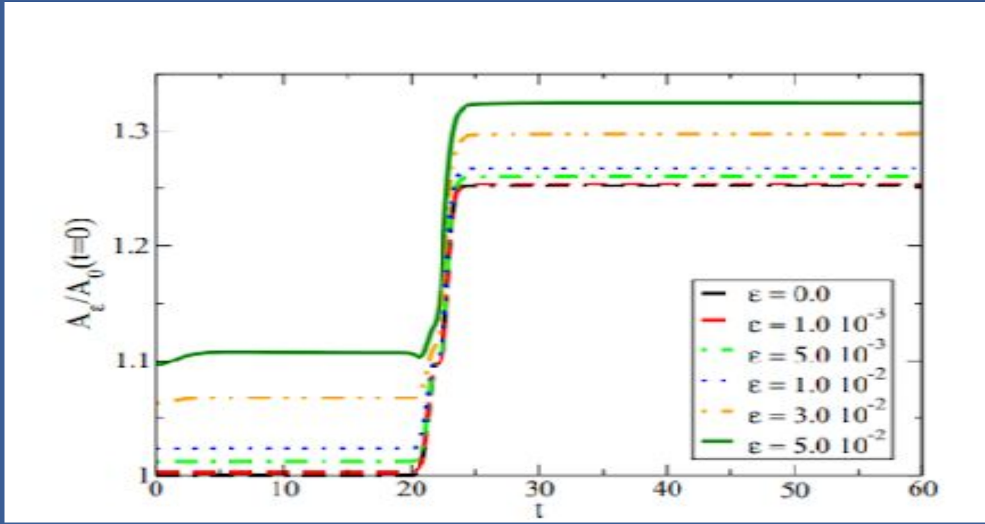
So, mathematically now 'in check'. How about physically?

Gravity application

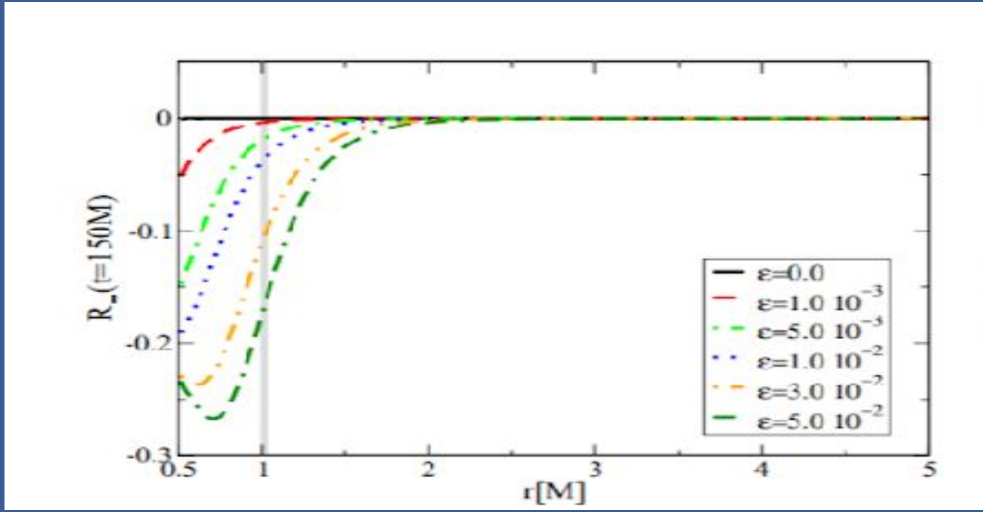
$$S_{eff} = 2M_{pl}^2 \int d^4x\sqrt{-g}\left(-R + c3\frac{R_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta\rho\sigma}}{\Lambda^4} \quad +\tilde{c3}\frac{\widetilde{R}_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta\rho\sigma}}{\Lambda^4} + \frac{\mathcal{C}^2}{\Lambda^6} + \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} + \frac{\mathcal{C}\tilde{\mathcal{C}}}{\Lambda_-^6} + \cdots\right)$$

$$\mathcal{C} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \quad \tilde{\mathcal{C}} \equiv \tilde{R}_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

- Focus on spherical case, and just one correction [R Cayuso, LL]
- Solve ID [going beyond BH solution, ie coupling it to ‘matter]
- Reduction of order/fixing and studying what’s ``new’’



- Horizon (non) growth



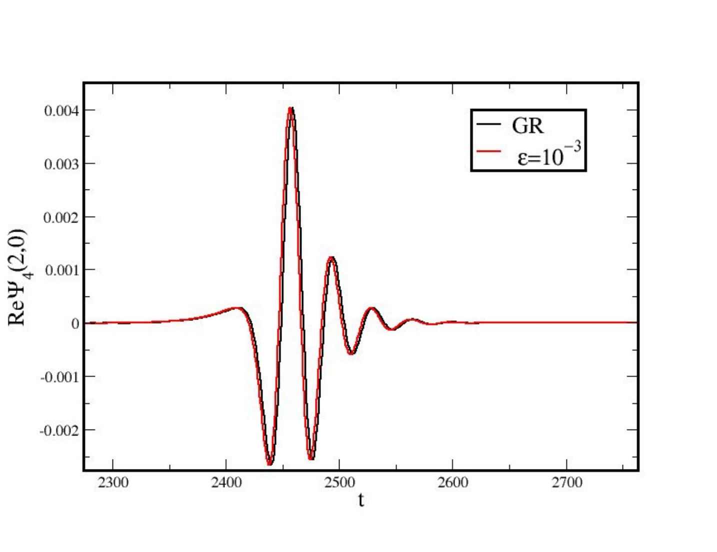
- Null convergence condition violation

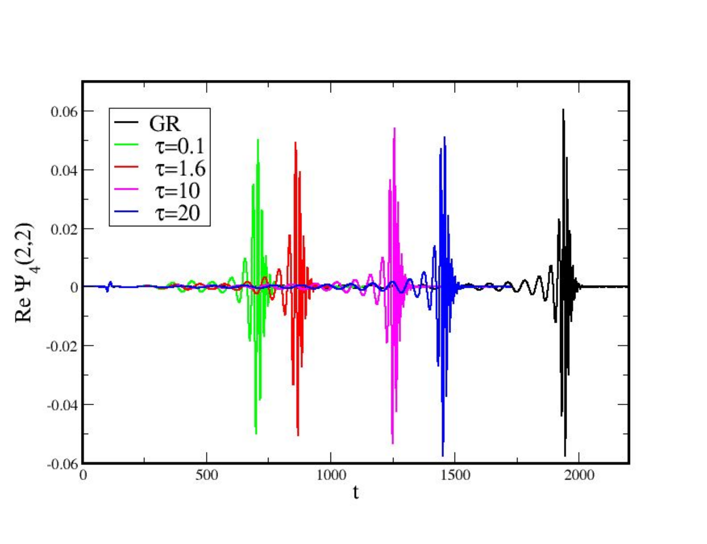
$$S_{eff} = 2M_{pl}^2 \int d^4x\sqrt{-g}\left(-R + c3\frac{R_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta\rho\sigma}}{\Lambda^4} + \tilde{c3}\frac{\widetilde{R}_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta\rho\sigma}}{\Lambda^4} + \frac{\mathcal{C}^2}{\Lambda^6} + \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} + \frac{\mathcal{C}\tilde{\mathcal{C}}}{\Lambda_-^6} + \cdots\right)$$

$$\mathcal{C} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \quad \tilde{\mathcal{C}} \equiv \tilde{R}_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

$$R_{\mu\nu} = \epsilon\left(4\,\mathcal{C}\,C_\mu{}^{\alpha\beta\gamma}\,C_{\nu\alpha\beta\gamma} - \frac{3}{2}\mathcal{C}^2\,g_{\mu\nu} + 8\,C_\mu{}^\alpha{}_\nu{}^\beta\nabla_\alpha\nabla_\beta\mathcal{C}\right)$$

$$\epsilon \equiv \frac{1}{\Lambda^6}$$





- In general, do he wave the right to do whatever it takes?
  - Most definitively not in general (we are modifying the equations). But we can check a solution is sensible
  - Implicitly, any reduction of order assumes this is the case
  - Fluid-gravity correspondence suggests in 3+1 gravity this is sensible expectation
  - ‘Simplicity’ of waveforms in GR & in observations!

  - Potential re-interpretation of ‘fixing variables’ in ``physical terms’’ [discussions with Tolley]

Probing most general 2nd order theory

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(\Sigma_{i=1}^5 \mathcal{L}_i) \quad (1)$$

where,

$$\mathcal{L}_1 = R + X - V(\phi)\,, \quad (2)$$
$$\mathcal{L}_2 = \mathcal{G}_2(\phi, X)\,, \quad (3)$$
$$\mathcal{L}_3 = \mathcal{G}_3(\phi, X)\Box\phi\,, \quad (4)$$
$$,\mathcal{L}_4 = \mathcal{G}_4(\phi, X)R + \partial_X\mathcal{G}_4(\phi, X)\delta^{ac}_{bd}\nabla_a\nabla^b\phi\nabla_c\nabla^d\phi\,, \quad (5)$$
$$\mathcal{L}_5 = \mathcal{G}_5(\phi, X)G_{ab}\nabla^a\nabla^b\phi - \frac{1}{6}\partial_X\mathcal{G}_5(\phi, X)\delta^{ace}_{bdf}\nabla_a\nabla^b\phi\nabla_c\nabla^d\phi\nabla_e\nabla^g\phi\,. \quad (6)$$

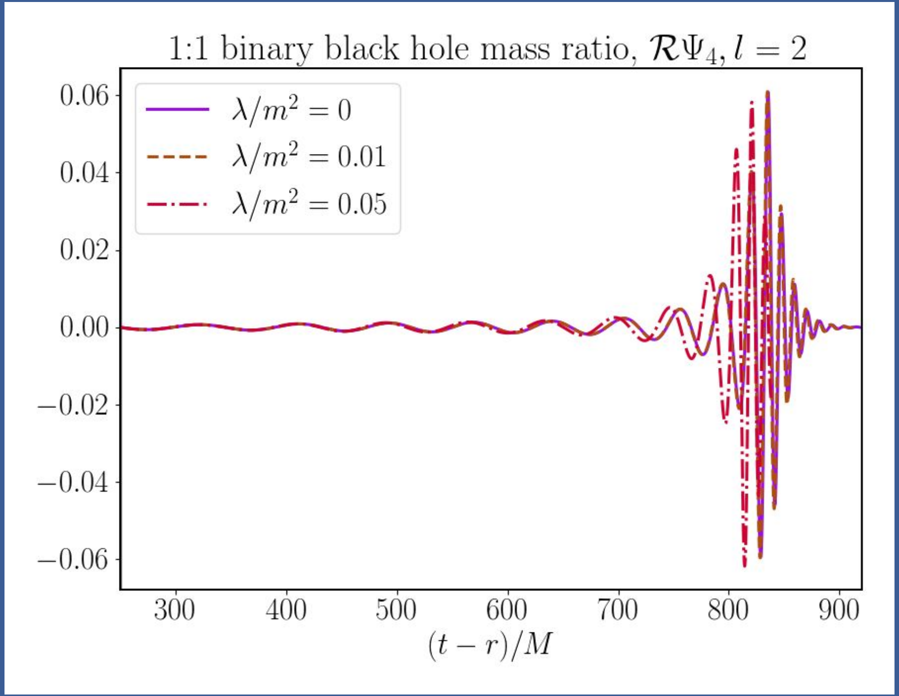
with $X = -1/2\nabla_a\phi\nabla^a\phi$, G_{ab} the Einstein tensor, $\mathcal{G}_i$ are functions of the scalars $\{\phi, X\}$, V is a potential and $\delta^{b_1..b_n}_{a_1..a_n}$ is the generalised Kronecker delta symbol.

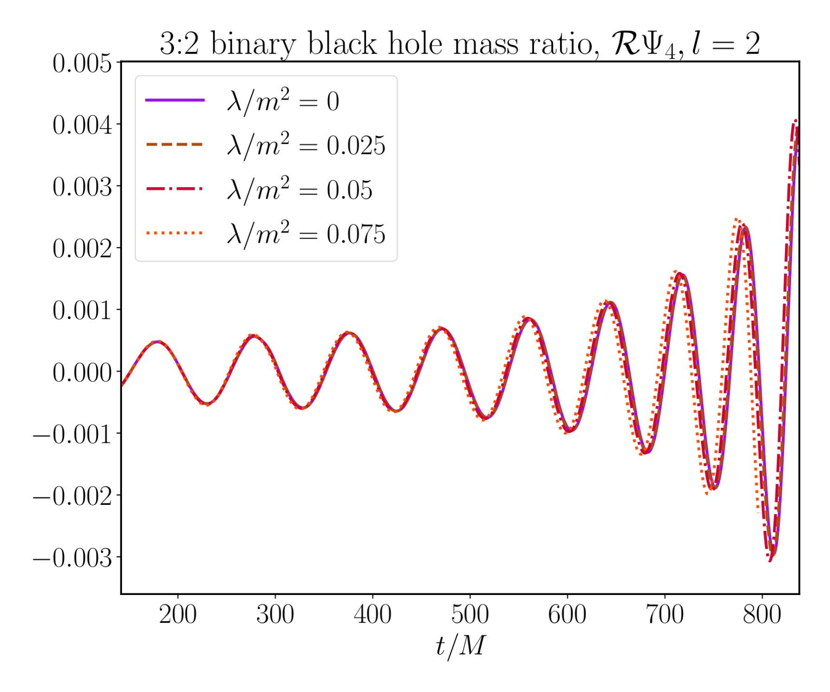
Linearized study: Papallo-Real's, in a special frame, better be that $G_4=G_5=0$ or *problem will be ill-posed*

- Counter argument: analysis relies in a particular gauge, could this be regarded as too restrictive?
- Kovacs-Reall: found a different gauge where local well posedness is attained (for suitable ID, and coupling values)

ESGB : [Corman, Rippley, East]

$$S = \int d^4x \sqrt{-g} \left(R - (\nabla\phi)^2 + \beta(\phi) \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta} \right) \right)$$





But… for larger couplings or not so smooth data, mathematical issues get in the way

Scalar Gauss Bonnet

$$S_{\text{GB}} = \int \frac{d^4x\sqrt{-g}}{16\pi}\Big[R$$

where the Gauss-Bonnet
constructed with the R
tensor R_{ab} and the Ricci

$$\mathscr{G} = R_{abcd}R^{a}$$

A variation of the action

$$R_{ab} - \frac{1}{2}g_{a}$$

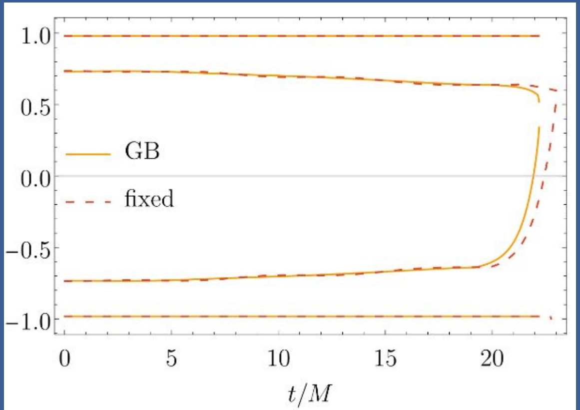
$$\Box\phi = S^{(\phi)}$$

- Black holes with 'scalar hair'
t from Kerr

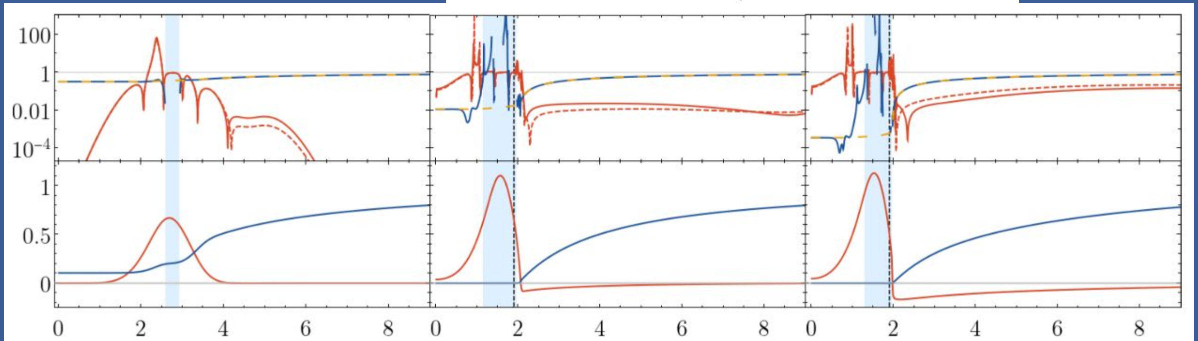
Ms)
lied [Witek+]
ear behavior
anks to new
n
l

Does the

Full and Fixed ⟶

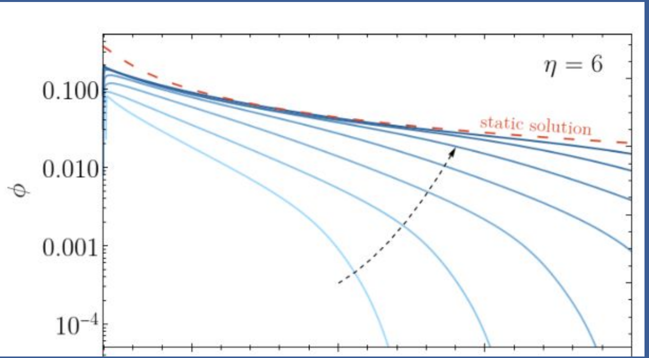


Original theory, take reasonable ID to a regime where hyperbolicity is broken



‘Fided’ version, controls this breaking locally and evolution proeeds

Solution that develops agrees with the expected ‘special’ solution of a static scalarized black hole!



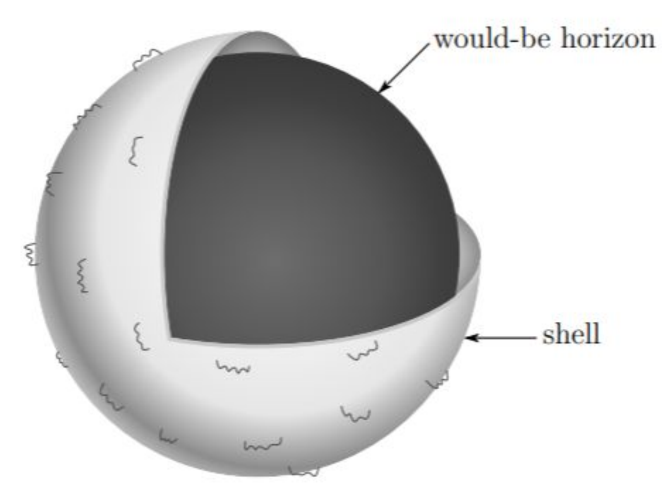
Exotic compact objects

- Collapse $\longrightarrow$ leads to a singularity, unless QG considerations replace the interior somehow. Resulting object does not have a '1-way membrane'. Radiation/matter can bounce off its surface, and/or fall in and come out. How will it?
  - With arguably 1 exception (boson stars), models are incomplete $\rightarrow$stable? non-linearly stable? And other qns can't be answered unless further development is given (tough!)

- One example. *AdS black bubbles* [Danielsson,Dibitetto,Girii '17]

AdS Bubbles - ECO

- Main idea: spacetime is unstable to decay to an AdS spacetime. Heavily suppressed, but when matters ‘threatens’ to collapse and form a BH such nucleation is entropically enhanced.
- If an AdS bubble forms, matter can turn into *massless* open strings and a *shell* divides the regions
- How will such object affect radiation?



interior/shell/exterior

- Interior: Anti de Sitter ($\Lambda<0$)
- Shell has 3 components, described by perfect fluids with EOS:
  - Brane (shell) with $p_\tau = -\rho_\tau$
  - Massless particles $p_g = \rho_g$ *[infalling matter -> massless open strings]*
  - Stiff fluid $p_s = \rho_s$ *[endpoints supported by lower dimensional branes dissolved in the brane]*
- Exterior: ($\Lambda=0$)

- positive energy of shell ~ negative energy of interior
- mass of system -> matter on the shell
- Israel-Darmois conditions massless particles
- For gas to have entropy ~ BH entropy large number of d.o.g lower dimensional branes dissolved on the brane

stability

- Can construct a static bubble that sits at R=9/4 m (with a given {ρ, p})
- Stability?
  - Non-trivial energy exchange among components needed.
  - 1.- Surface at constant radii → accelerated →shell heated to Unruh temperature. If the shell is at $T < T_H$ must absorb energy → coming from shell's tension (and viceversa ➔ flux term connecting them ~ $T_{,t}/T$ ~ $a_{,t}/a$)
  - 2.- Area change → number of dissolved branes needs to change ➔ flux term ~ $R_{,t}/R$)

- Thus: $D^a T^g_{ab} = -j_b$; $D^a T^\tau_{ab} = j_b$; $D^a T^s_{ab} = 0$
- With $j = 3\rho_g(\frac{\dot{a}}{a} + \frac{V}{2R})$

Stability 2: new ingredients

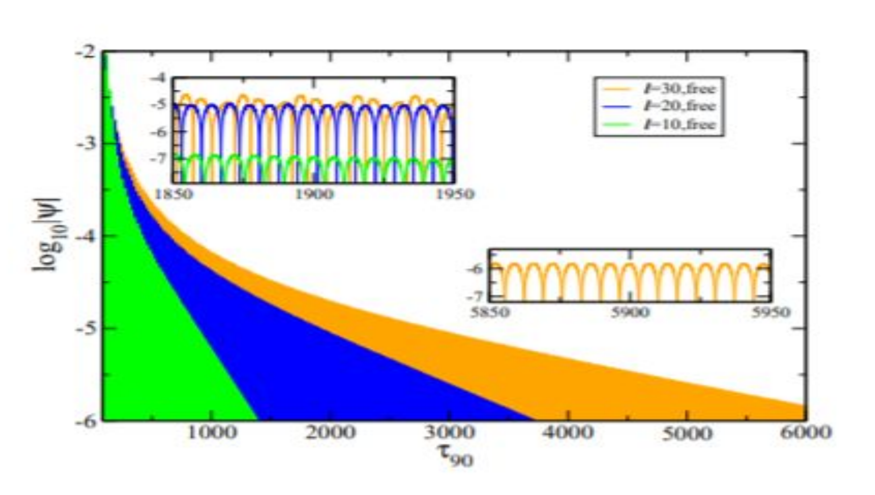
- But, as is, there is a dynamical component in the current that has a destabilizing contribution. Consider instead:
- $j = 3\rho_g(\alpha \frac{\dot{a}}{a} + \beta \frac{\dot{R}}{2R})$
- Also: include a viscous contribution to reflect components' entropy growth [introducing a new par: ζ]

$$
\begin{aligned}
{}^{(\rho)}T_{ab} &\equiv {}^{(\rho_g)}T_{ab} + {}^{(\rho_s)}T_{ab} + {}^{(\rho_\tau)}T_{ab},\\
{}^{(\rho_g)}T_{ab} &= (\rho_g + \mathcal{A})u_a u_b + (p_g + \Pi)\Delta_{ab},\\
{}^{(\rho_s)}T_{ab} &= \rho_s u_a u_b + p_s \Delta_{ab},\\
{}^{(\rho_\tau)}T_{ab} &= \rho_\tau u_a u_b + p_\tau \Delta_{ab},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A} &= \tau_e \left[u^a D_a \rho_g + (\rho_g + p_g) D_a u^a\right],\\
\Pi &= -\zeta D_a u^a + \tau_p \left[u^a D_a \rho_g + (\rho_g + p_g) D_a u^a\right],
\end{aligned}
$$

One consequence

- if scalar field falls into interior and leaks out, sustained signal afterwards. Though characteristic frequencies tied to its size and interior modes **not just characteristics at infall**



Final words

- Interesting questions & motivations taking us to uncharted regions, where strengths of different corners and their intuition/standard practices might not quite work. ‘Bold steps’ together with careful reassessments called for making progress

- New ideas and new methods required for new/deeper/more complete applications

- Multidisciplinary approach is key to take raise new questions and bring forward new ideas