Simulations of matter under extreme conditions

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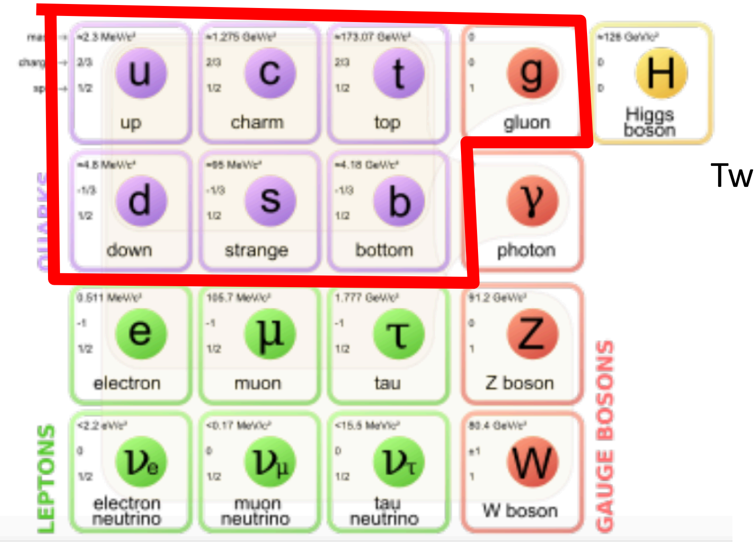
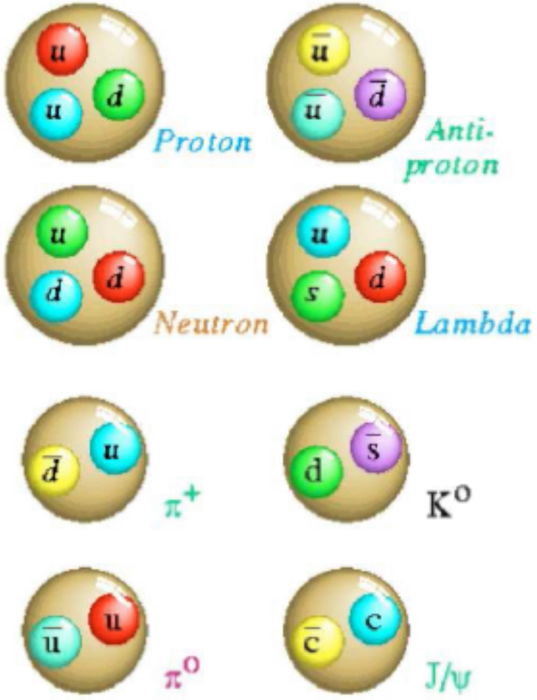
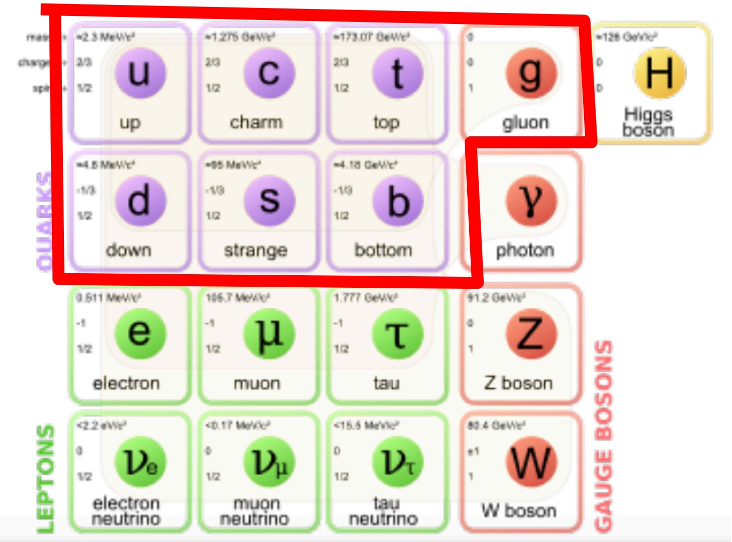
Matter in the Universe



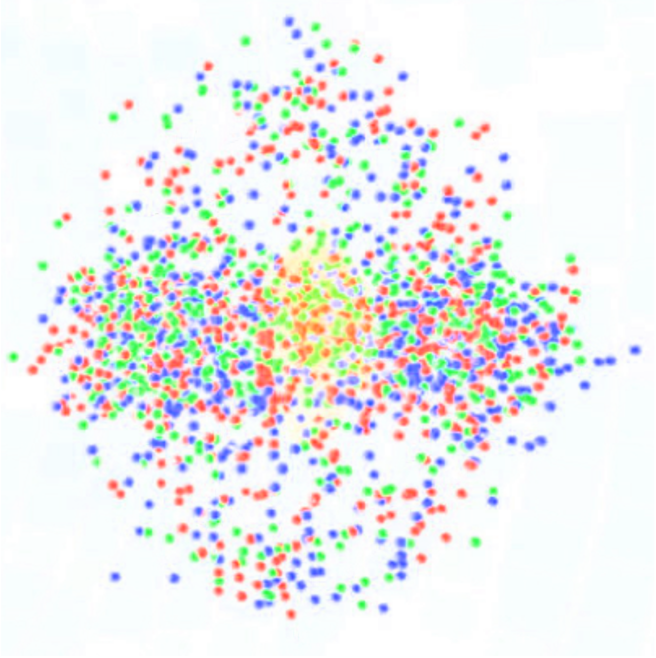
Two- and three-quark states only!



Matter in the Universe



Heat and compress matter



Quark-Gluon Plasma: new phase of matter at very high temperatures (or densities)

QCD matter under extreme conditions

Research Council of the National Academies: Eleven science questions for the new century

- How did the Universe begin?
- What are the new states of matter at exceedingly high density and temperature?
- What is Dark Matter?
- What is the nature of Dark Energy?
- What are the masses of the neutrinos, how have they shaped the evolution of the Universe?
- Did Enstein have the last word on Gravity?
- How do cosmic accelerators work and what are they accelerating?
- Are protons unstable?
- Are there additional space-time dimensions?
- How were the elements from Iron to Uranium made?
- Is a new theory of matter and light needed at the highest energies?

QCD matter under extreme conditions

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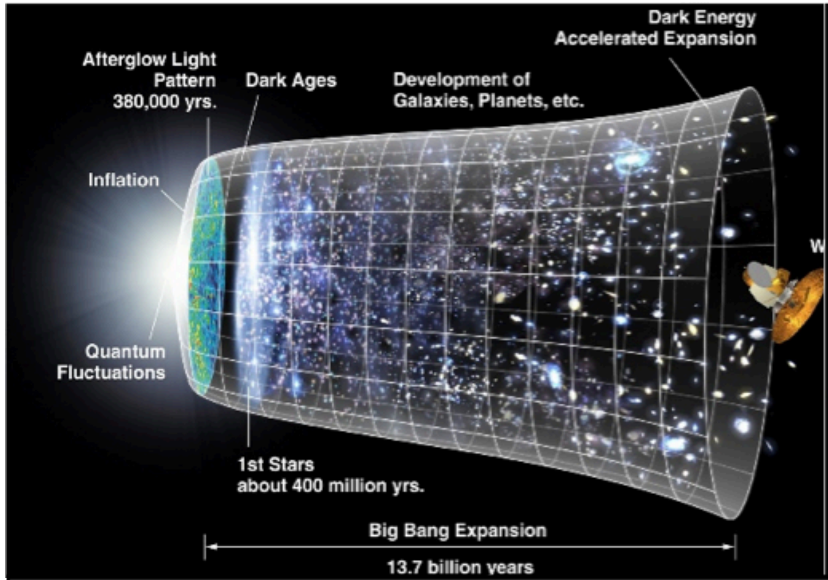
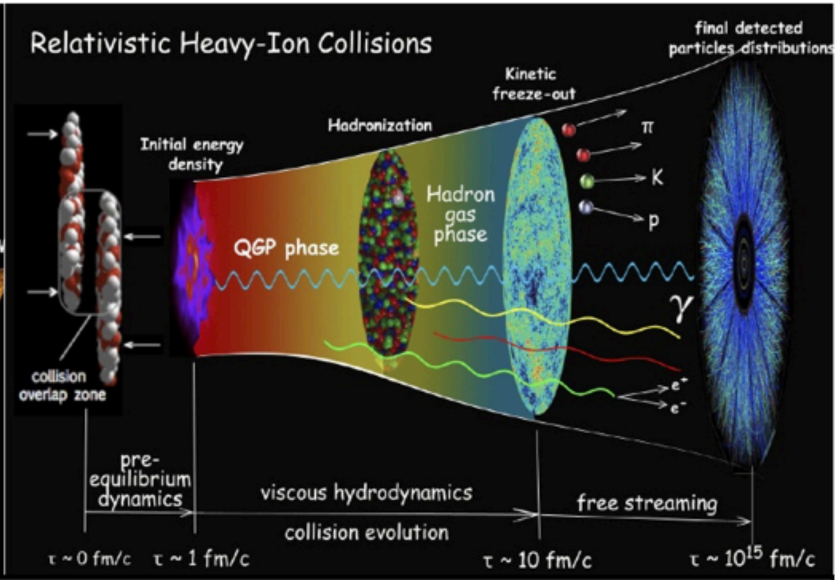




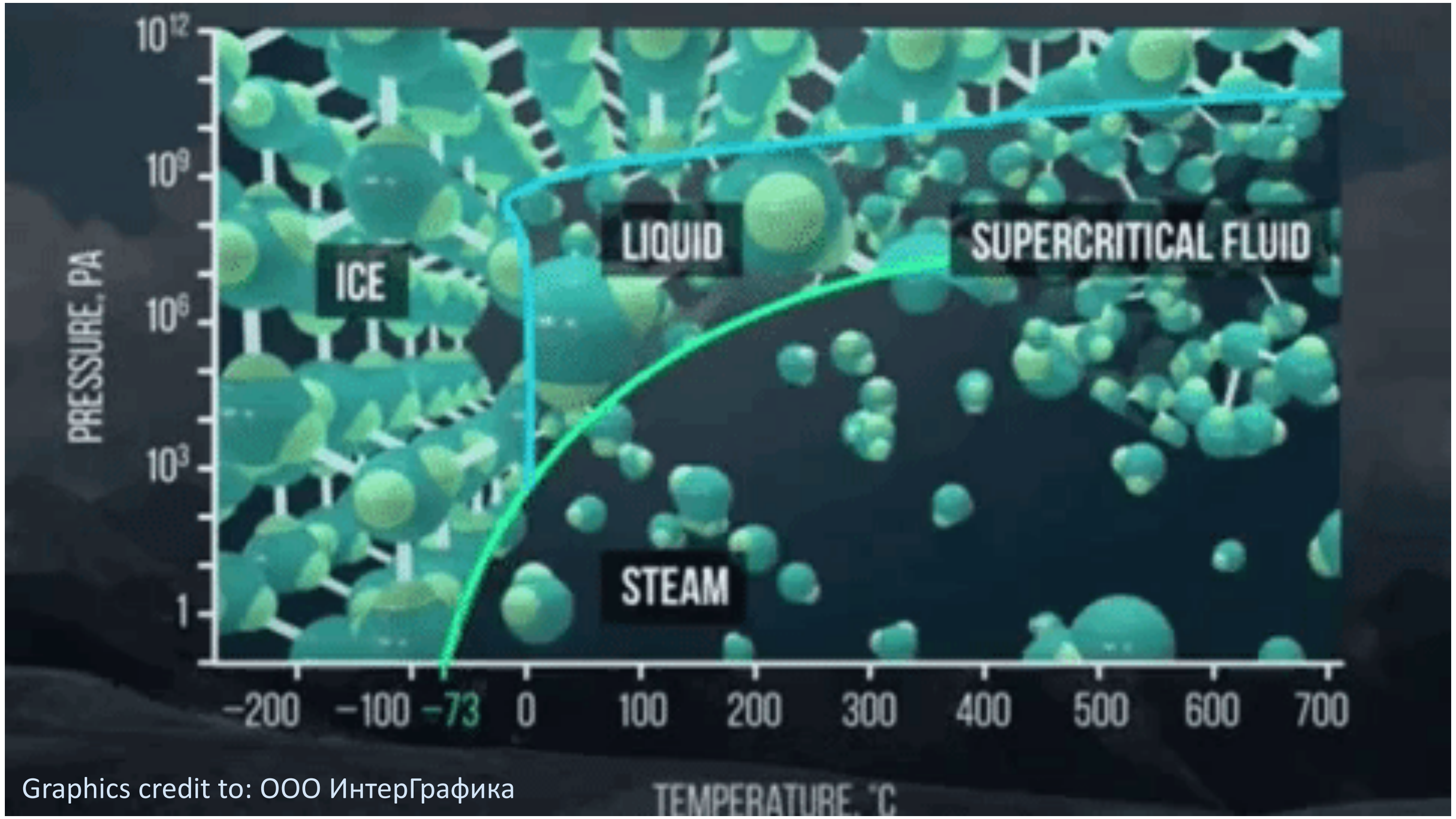
The two questions are related!

Quark-Gluon Plasma (QGP) is at $T>10^{12}$K and $\rho \sim 10^{40}$ cm^{-3}

The Universe was in the QGP phase a few μs after Big Bang

Ultimate goals

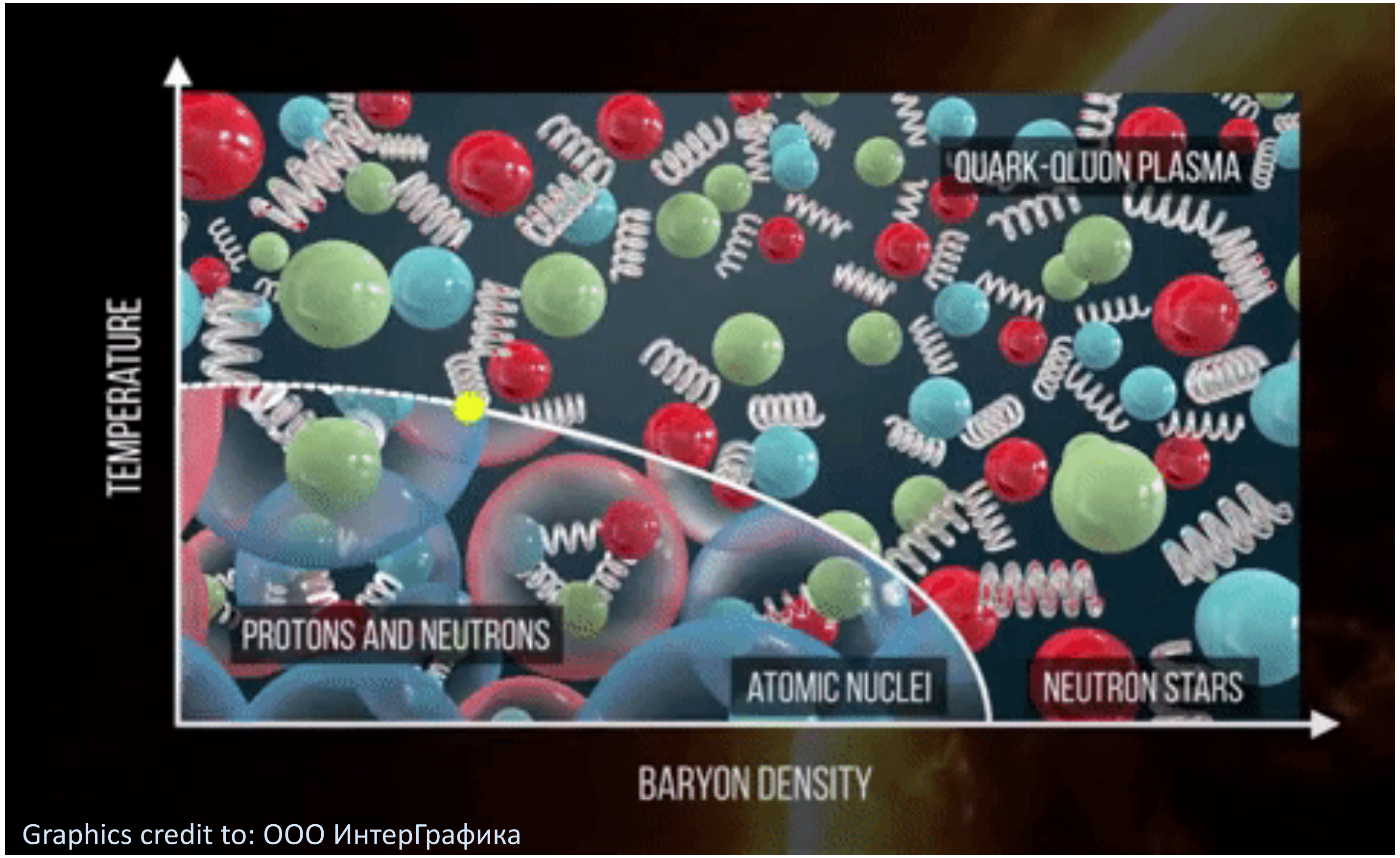
Phase diagram of water



Graphics credit to: ООО ИнтерГрафика

Ultimate goals

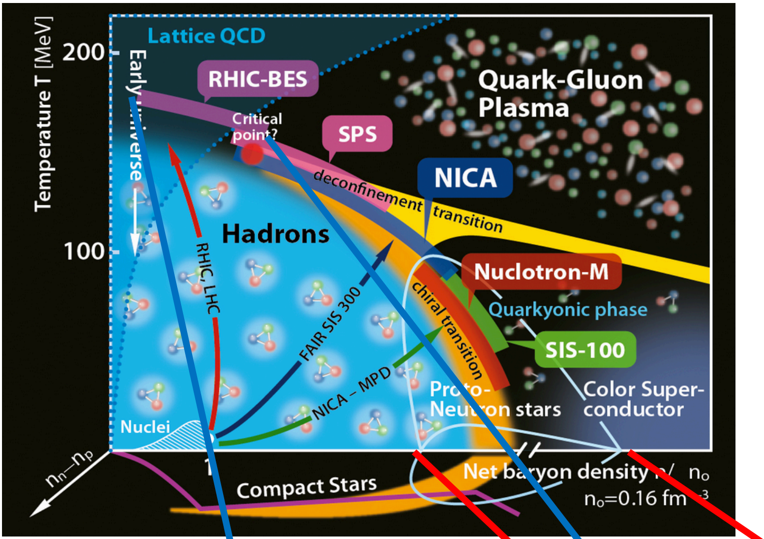
Phase diagram of strongly interacting matter



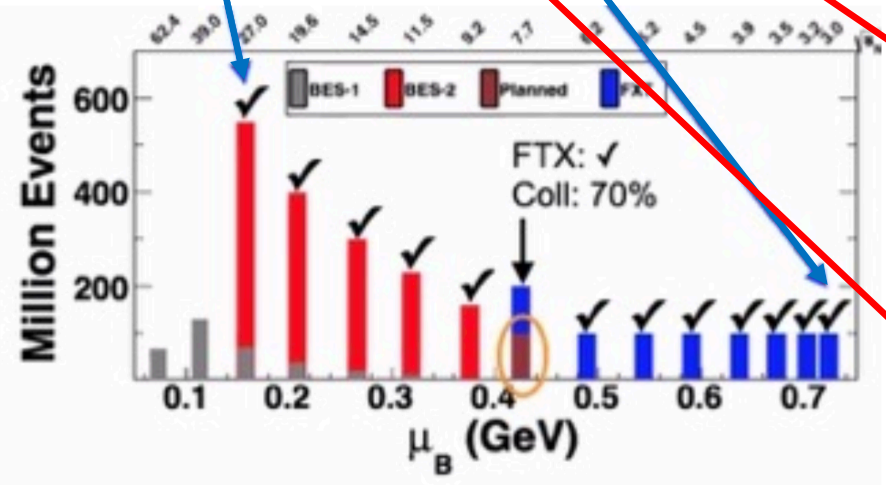
Graphics credit to: ООО ИнтерГрафика

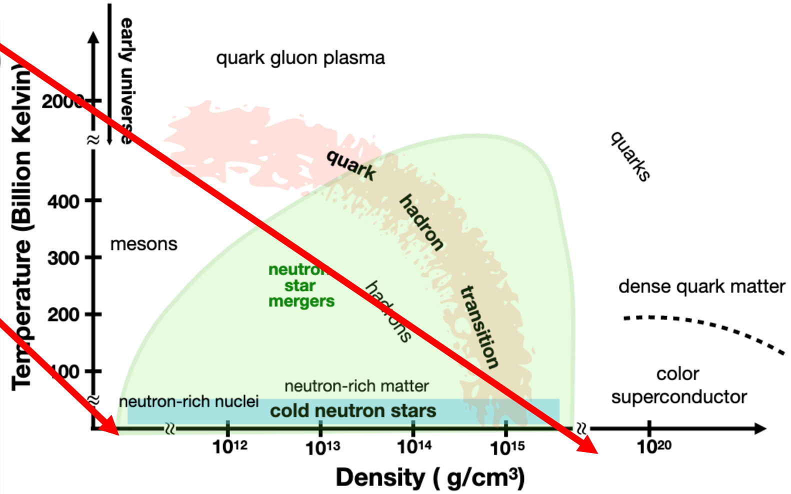
Motivating science goals

- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- Where is the transition line at high density?
- What are the phases of QCD at high density?
- Are we creating a thermal medium in experiments?



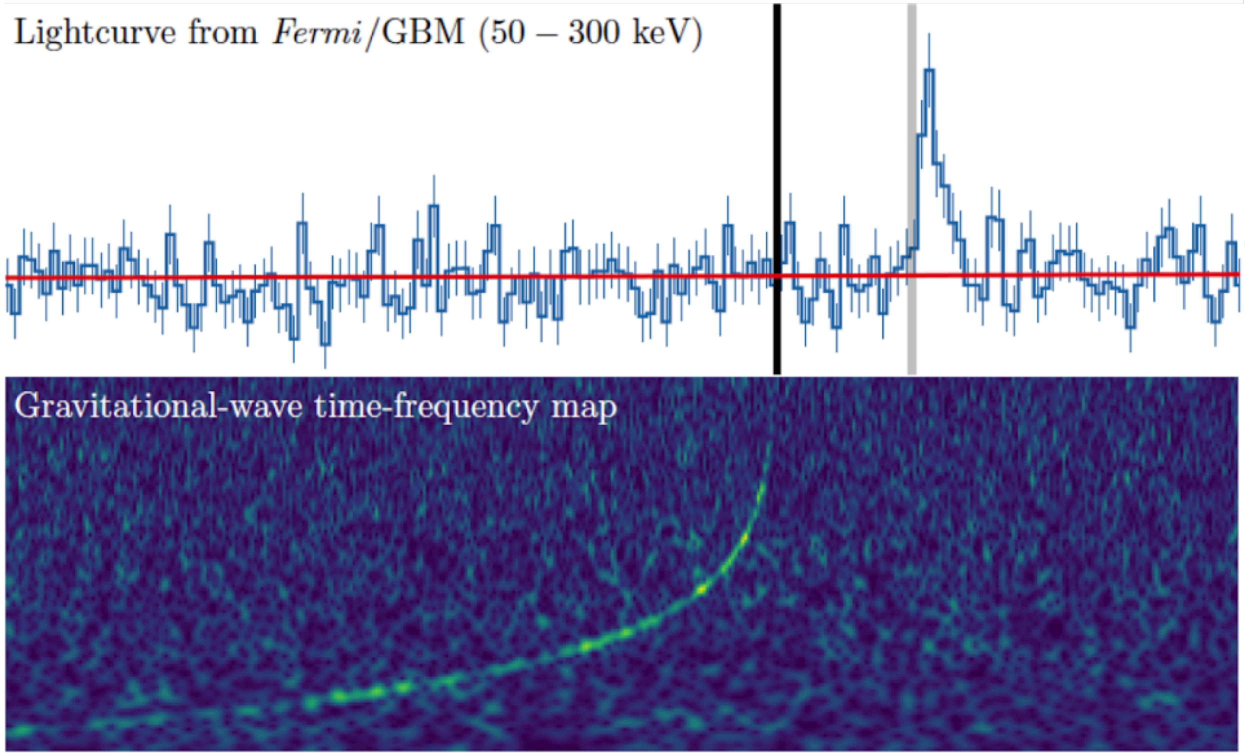
- Run 2019:
  - Collider: $\sqrt{s_{NN}}$=14.6, 19.6, 200 GeV
  - Fixed target: $\sqrt{s_{NN}}$=3.2 GeV
- Run 2020:
  - Collider: $\sqrt{s_{NN}}$=9.2, 11.5 GeV
  - Fixed target: $\sqrt{s_{NN}}$=3.5, 3.9, 4.5, 5.2, 6.2, 7.2, 7.7 GeV
- Run 2021:
  - Collider: $\sqrt{s_{NN}}$=7.7 GeV

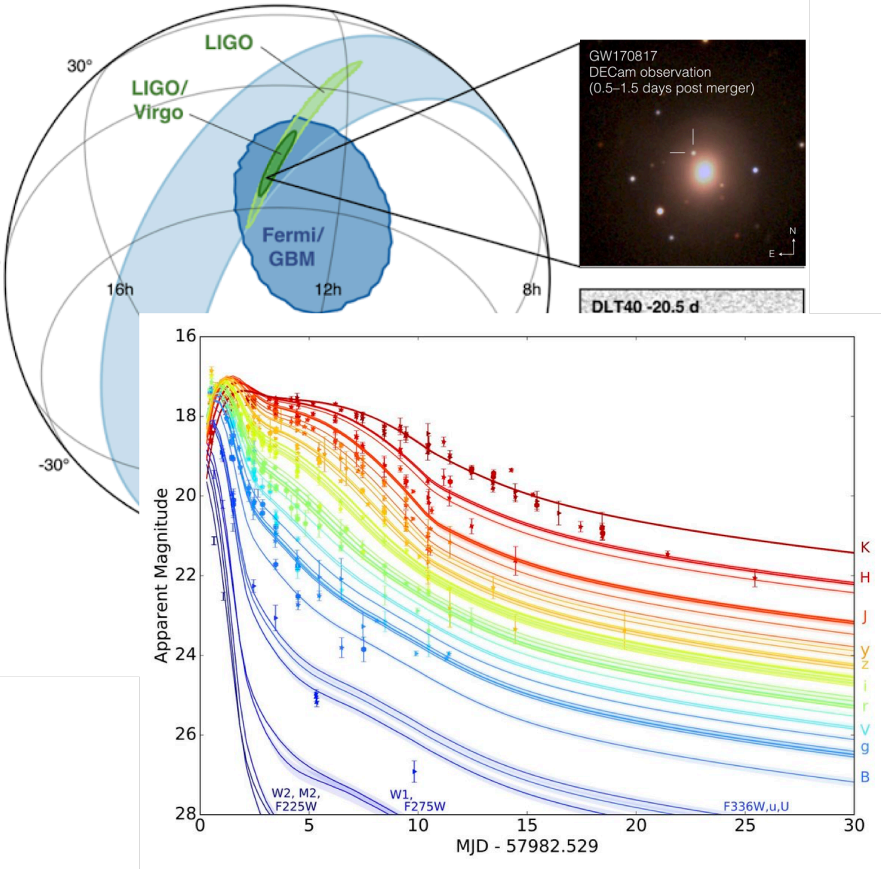




GW170817

Demonstrated the ability of mergers to advance Nuclear Physics





LIGO/Virgo PRL (2017)
P.S. Cowperthwaite et al., Astrophys. J. Lett. (2017)

QCD matter under extreme conditions

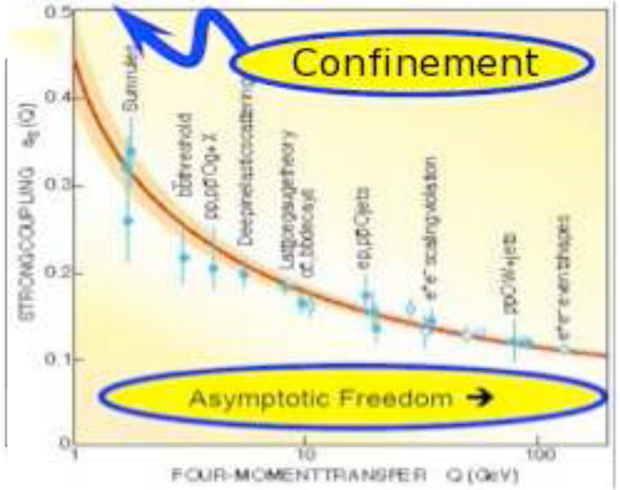
To address these questions, we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

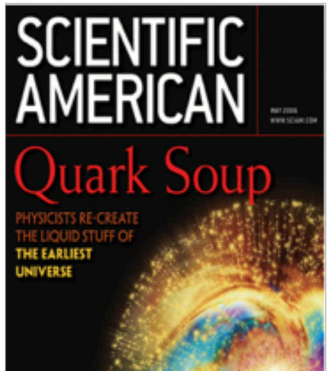
$$L_{QCD} = \sum_{i=1}^{n_f} \overline{\psi}_i \gamma_\mu \left(i\partial^\mu - g A_a^\mu \frac{\lambda_a}{2} \right) \psi_i - m_i \overline{\psi}_i \psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + i f_{abc} A_b^\mu A_c^\mu$$



Experiment: heavy-ion collisions

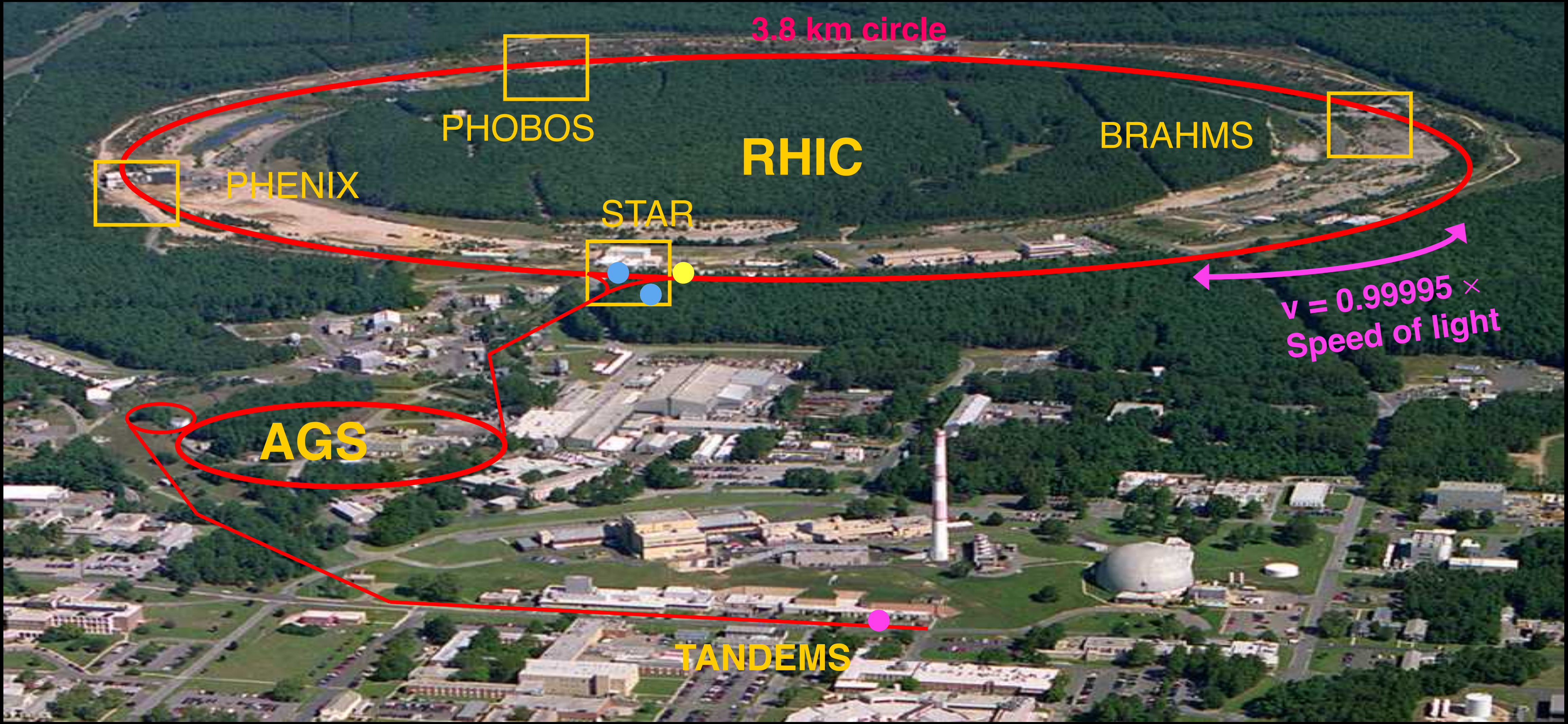
- Quark-Gluon Plasma (QGP) discovery at RHIC and LHC:
- SURPRISE!!! QGP is a PERFECT FLUID
- Changes our idea of QGP (no weak coupling)
- Microscopic origin still unknown



Geneva with the Large Hadron Collider

Speed: 0.999995 x speed of light
26.2 km circle

Relativistic Heavy Ion Collider



Gold nuclei, with 197 protons + neutrons each, are accelerated
The beams go through the experimental apparatus 100,000 times per second!

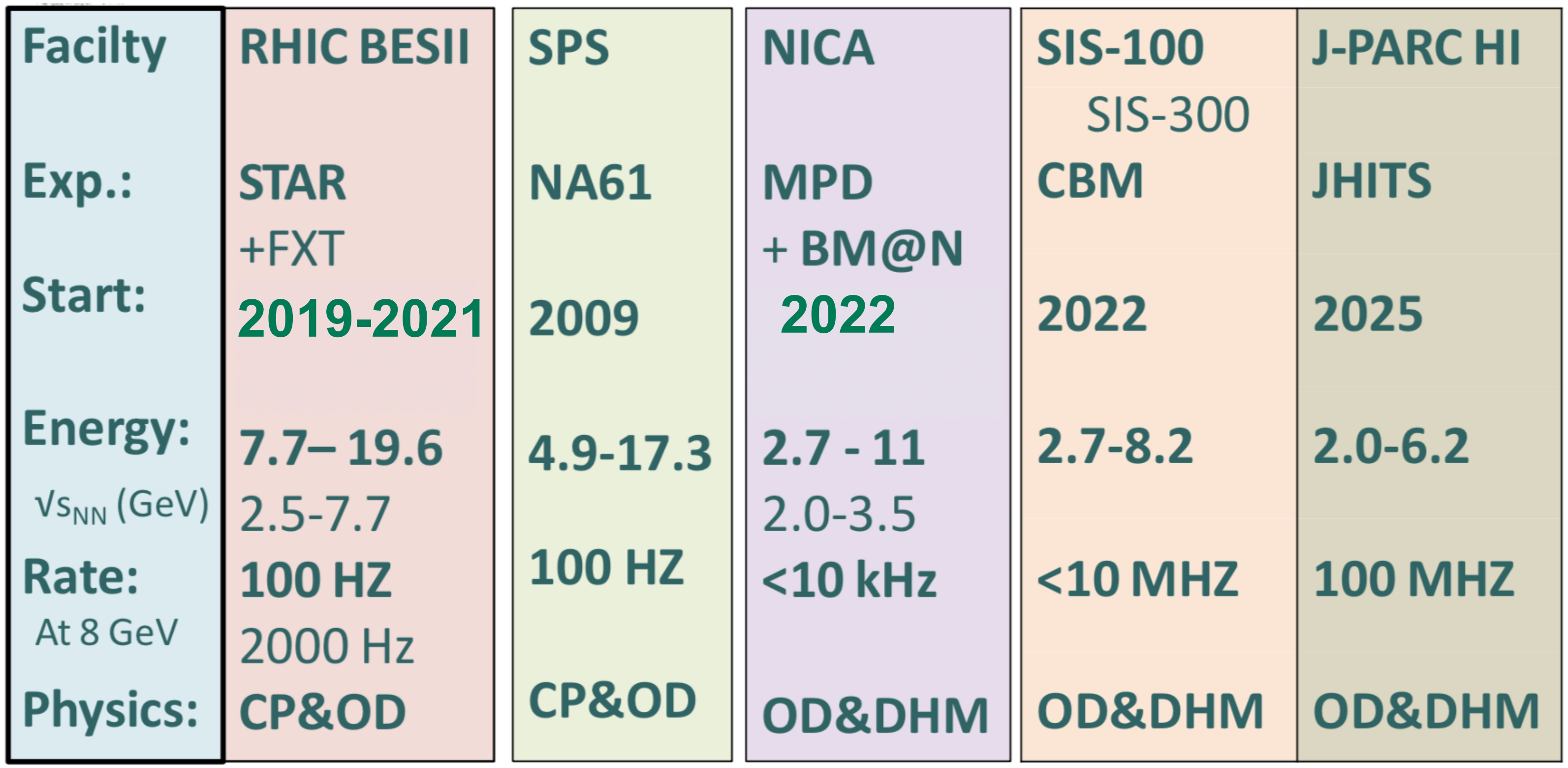
Comparison of the facilities

Compilation by D. Cebra

CP=Critical Point

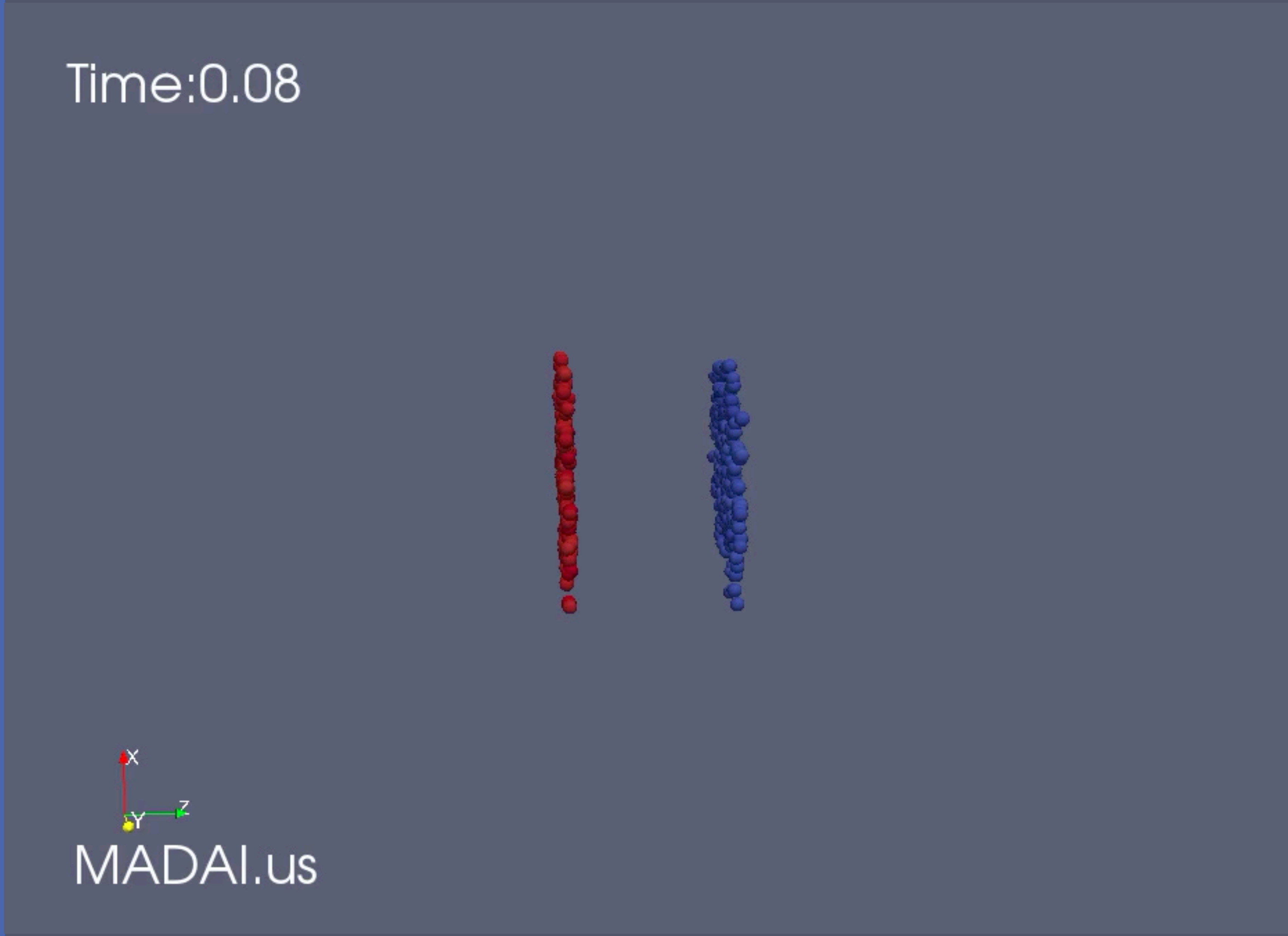
OD= Onset of Deconfinement

DHM=Dense Hadronic Matter



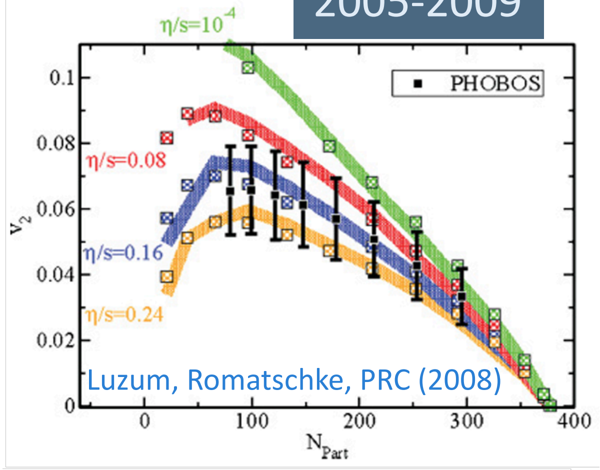
Facilty	RHIC BESII	SPS	NICA	SIS-100 SIS-300	J-PARC HI
Exp.:	STAR +FXT	NA61	MPD + BM@N	CBM	JHITS
Start:	2019-2021	2009	2022	2022	2025
Energy: $\sqrt{s_{NN}}$ (GeV)	7.7– 19.6 2.5-7.7	4.9-17.3	2.7 - 11 2.0-3.5	2.7-8.2	2.0-6.2
Rate: At 8 GeV	100 HZ 2000 Hz	100 HZ	<10 kHz	<10 MHZ	100 MHZ
Physics:	CP&OD	CP&OD	OD&DHM	OD&DHM	OD&DHM
	Collider Fixed target	Fixed target Lighter ion collisions	Collider Fixed target	Fixed target	Fixed target

What happens in a heavy-ion collision?

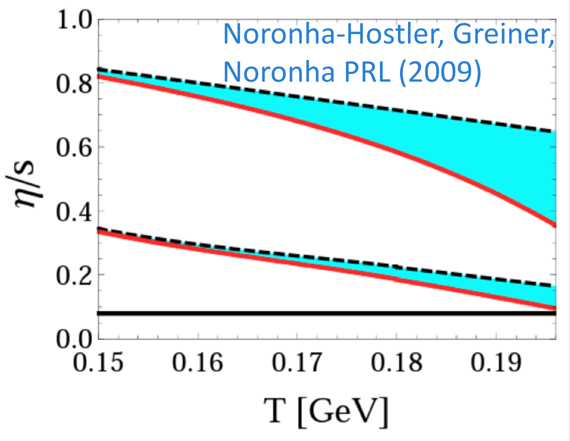


Nearly perfect fluidity

2005-2009



Luzum, Romatschke, PRC (2008)

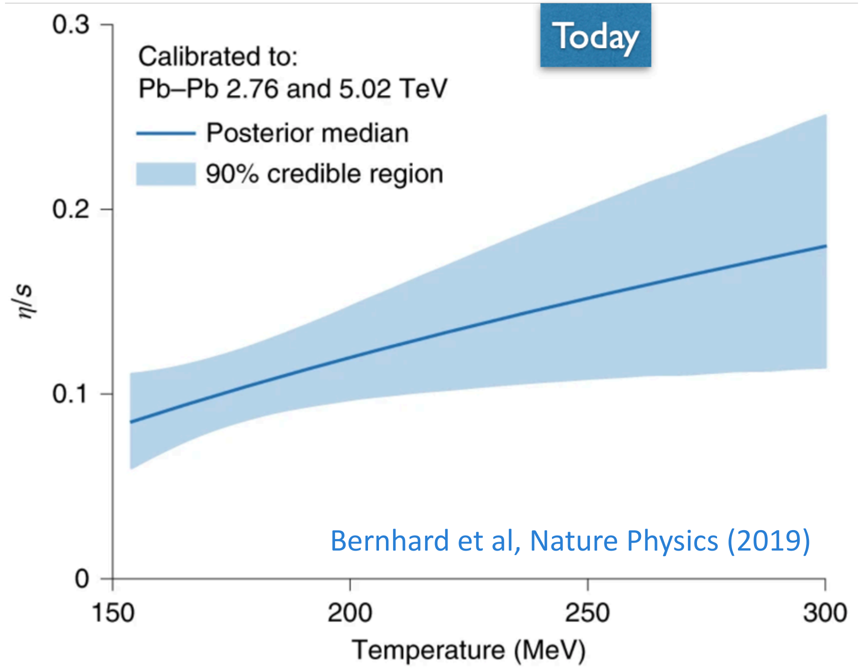


Noronha-Hostler, Greiner, Noronha PRL (2009)

Kovtun, Son, Starinets, PRL (2005)

$$\eta/s = 1/4\pi$$

Today

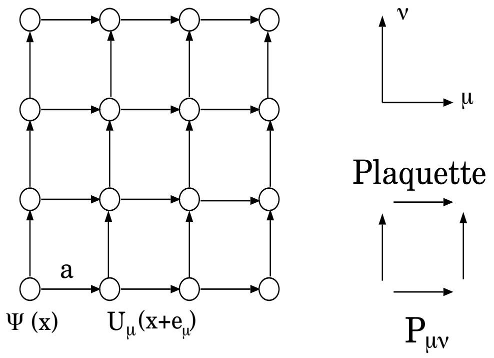


Bernhard et al, Nature Physics (2019)

- Hydrodynamic description of the system created in heavy-ion collisions works extremely well
- It needs an equation of state as input

Simulating strongly interacting matter

- Quantum ChromoDynamics (QCD) Nobel prize 2004
- Analytic solutions of QCD are not possible in the non-perturbative regime
- Numerical approach to solve QCD: discretization of 4D space-time
- Uncertainties:
  - Statistical: finite sample
  - Systematic: finite box size, unphysical quark masses
- Results from different groups, adopting different discretizations, must converge to consistent results
- Simulations are running on the most powerful supercomputers in the world



Fundamental fields

How can lattice QCD support the experiments?

Equation of state

- Needed for hydrodynamic description of the QGP

QCD phase diagram

- Transition line at finite density
- Constraints on the location of the critical point

Fluctuations of conserved charges

- Can be simulated on the lattice and measured in experiments
- Can give information on the evolution of heavy-ion collisions
- Can give information on the critical point

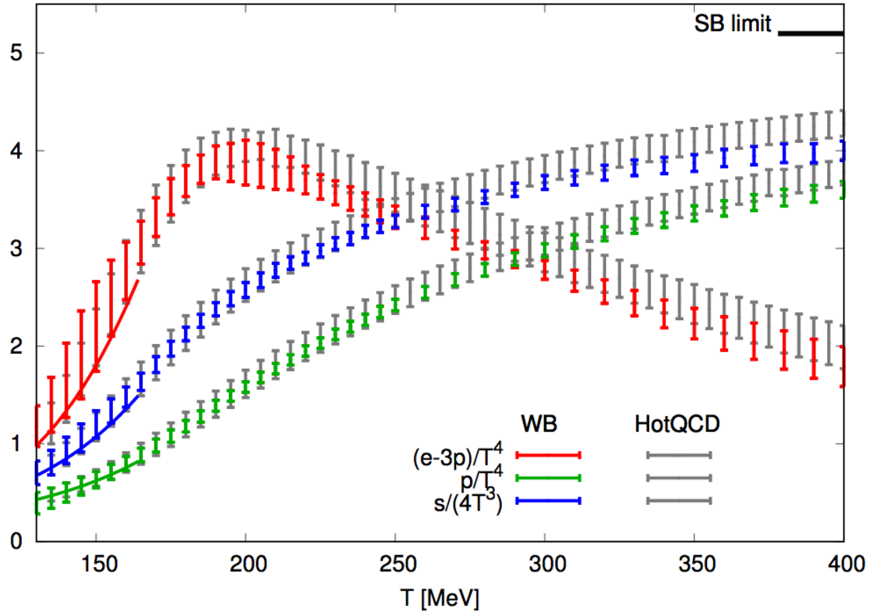
QCD Equation of State at finite density

TAYLOR EXPANSION

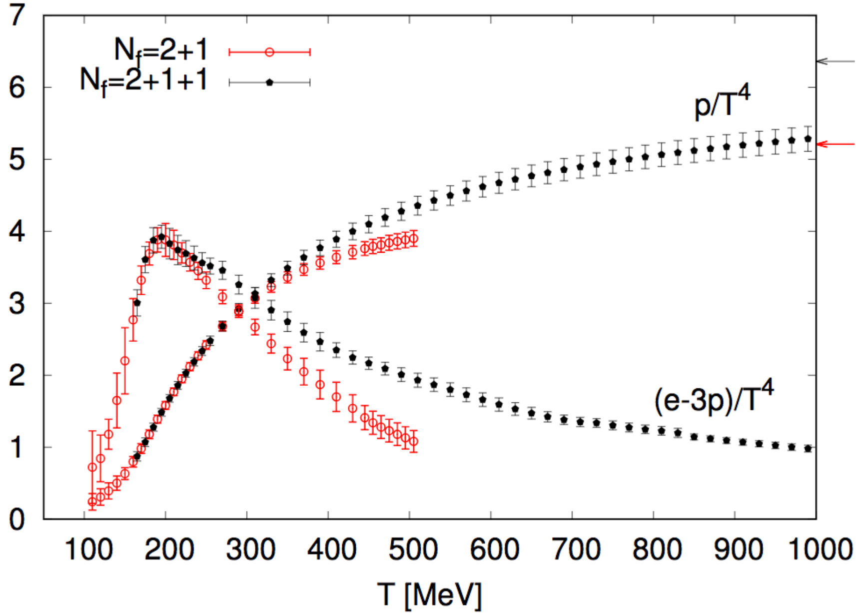
NEW EXPANSION SCHEME

QCD EoS at μ_B=0

WB: PLB (2014); HotQCD: PRD (2014)

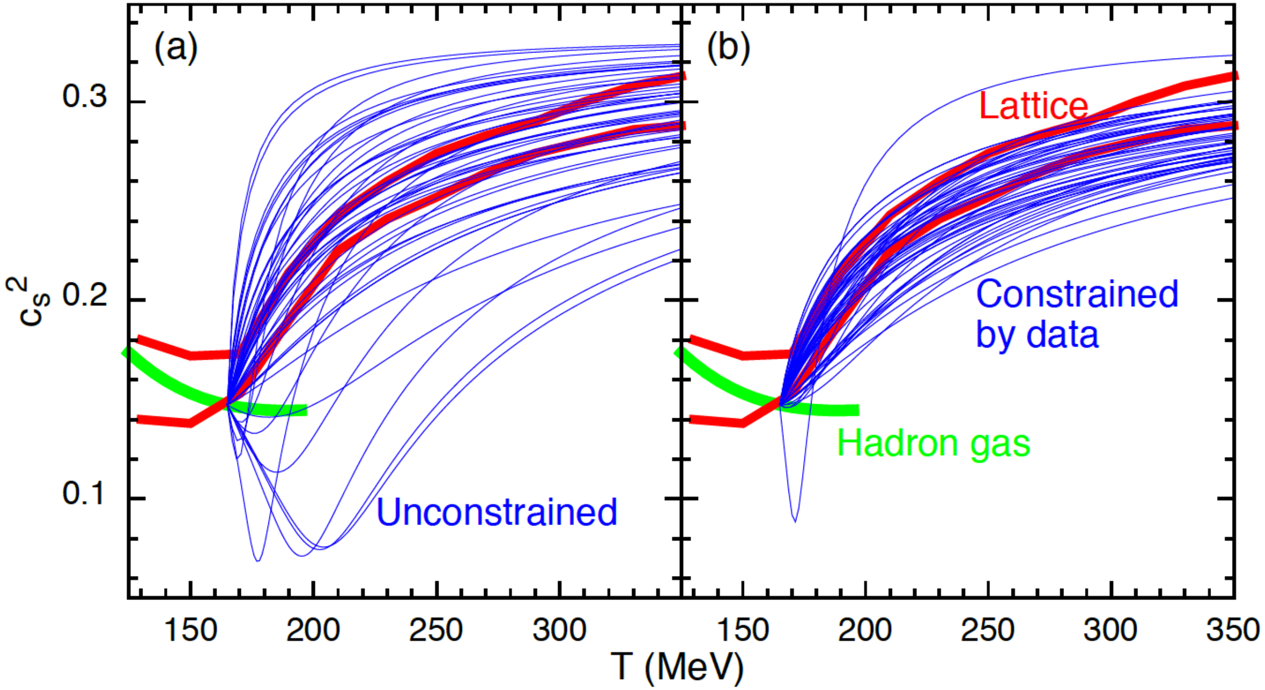


WB: Nature (2016)



- EoS for N_f=2+1 known in the continuum limit since 2013
- Good agreement with the HRG model at low temperature
- Charm quark relevant degree of freedom already at T~250 MeV

Constraints on the EoS from the experiments

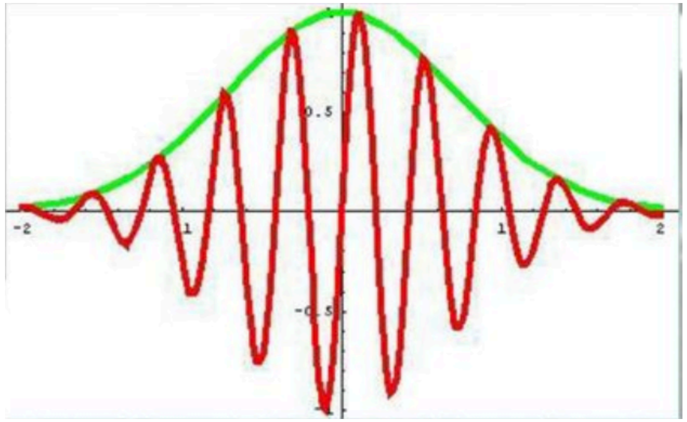


S. Pratt et al., PRL (2015)

- Comparison of data from RHIC and LHC to theoretical models through Bayesian analysis
- The posterior distribution of EoS is consistent with the lattice QCD one

Fermionic sign problem

- The QCD path integral is computed by Monte Carlo algorithms which sample field configurations with a weight proportional to the exponential of the action



$$Z(\mu_B,T) = \mathrm{Tr}\left(e^{-\frac{H_{QCD}-\mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U,\mu_B]$$

- detM[μ_B] complex → Monte Carlo simulations are not feasible
- We can rely on a few approximate methods, viable for small μB/T:
  - Taylor expansion of physical quantities around μB=0

    Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003
  - Simulations at imaginary chemical potentials

    Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003

Taylor expansion of EoS

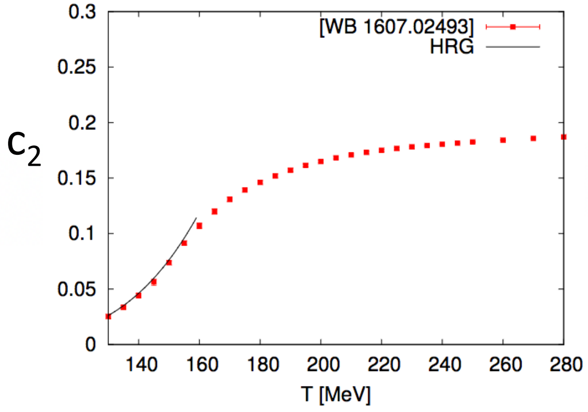
- Taylor expansion of the pressure:

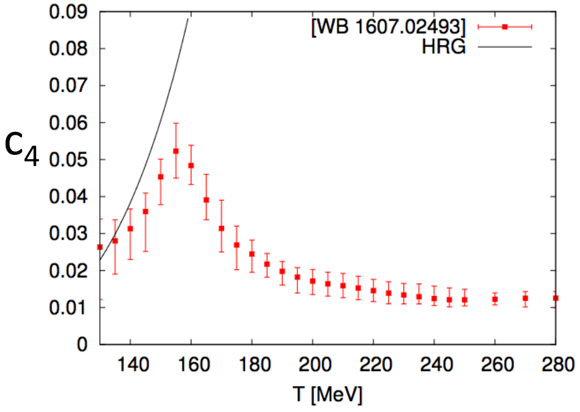
$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \overset{\chi_{2n}^{B}}{\boxed{\left.\frac{\mathrm{d}^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\right|_{\mu_B=0}}} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

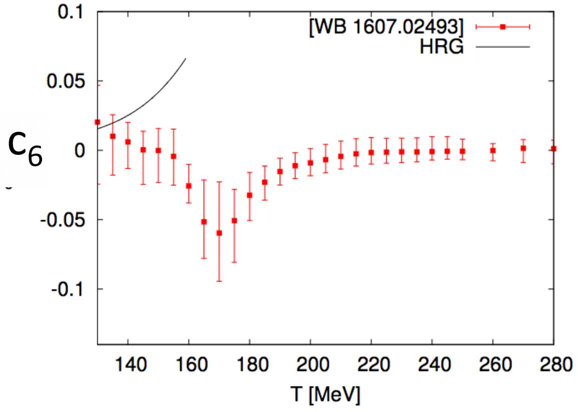
Simulations at imaginary μ_B:

Continuum, $O(10^4)$ configurations, errors include systematics

WB: NPA (2017)



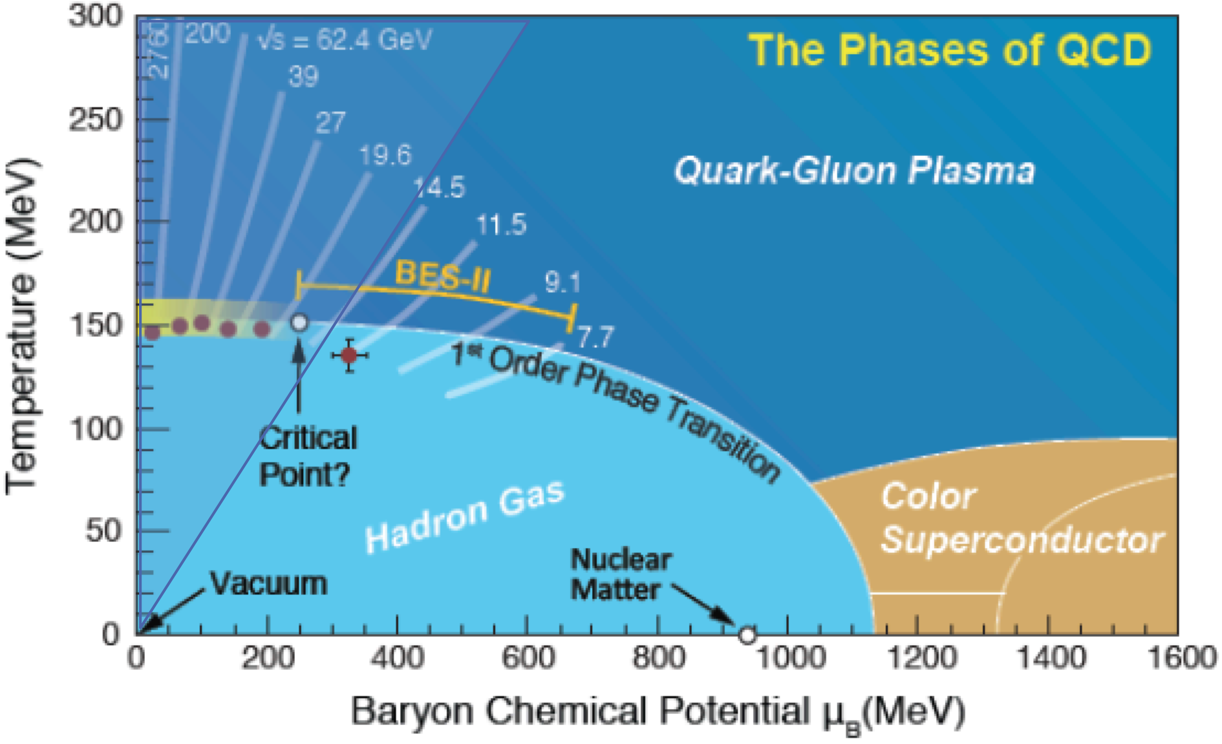


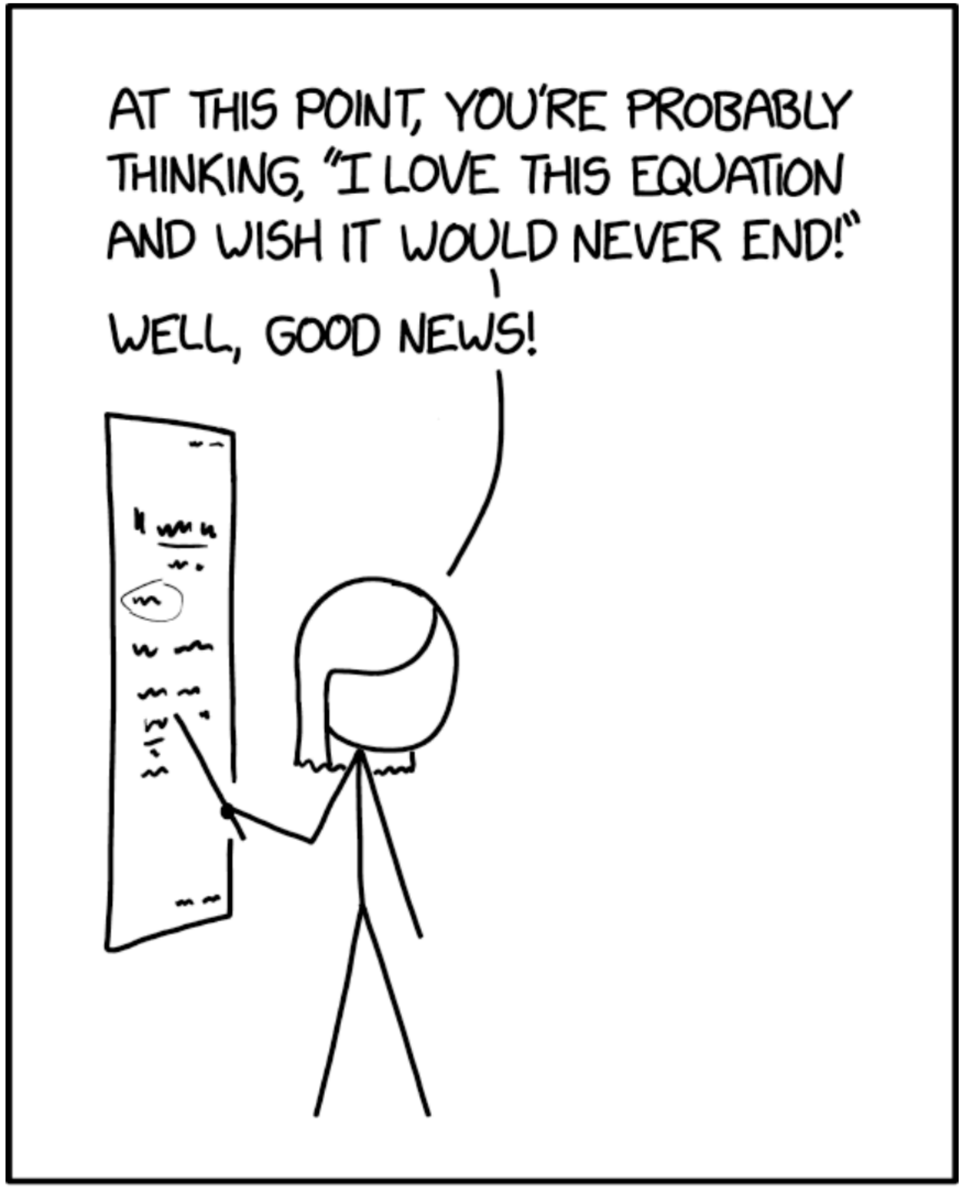


Range of validity of equation of state

- We now have the equation of state for $\mu_B/T \leq 2$ or in terms of the RHIC energy scan:

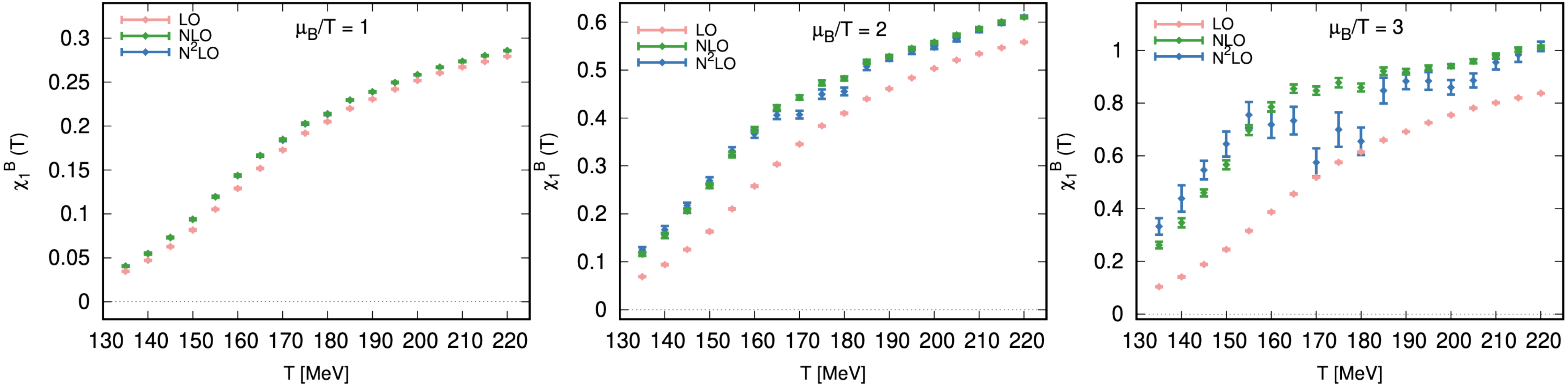
$$\sqrt{s} = 200,\ 62.4,\ 39,\ 27,\ 19.6,\ 14.5\text{GeV}$$





TAYLOR SERIES EXPANSION IS THE WORST.

Problems with Taylor series



☐ Poor convergence of Taylor series: need to sum many terms to reach high μ_B

☐ Oscillatory/non-monotonic behavior in some observables at high μ_B

  ➢ Unphysical, due to truncation of Taylor series

Novel expansion method

WB, PRL (2021)

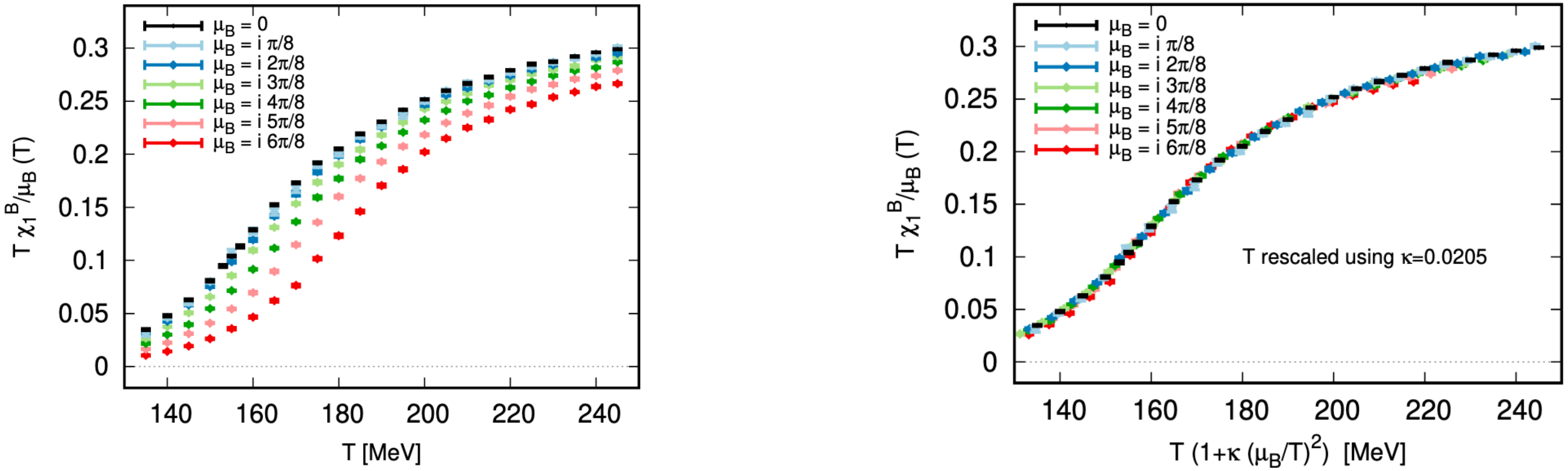
Observation: the temperature-dependence of baryonic density

$$n_B(T)/\hat{\mu}_B = \chi_1^B(T,\hat{\mu}_B)/\hat{\mu}_B$$

at finite imaginary chemical potential is just a shift in temperature from the $\mu_B = 0$ results for χ_2^B :

$$\frac{\chi_1^B(T,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)\ ,$$

$$T'(T,\hat{\mu}_B) = T\left(1+\kappa_2^{BB}(T)\hat{\mu}_B^2+\kappa_4^{BB}(T)\hat{\mu}_B^4+\mathcal{O}(\hat{\mu}_B^6)\right)$$

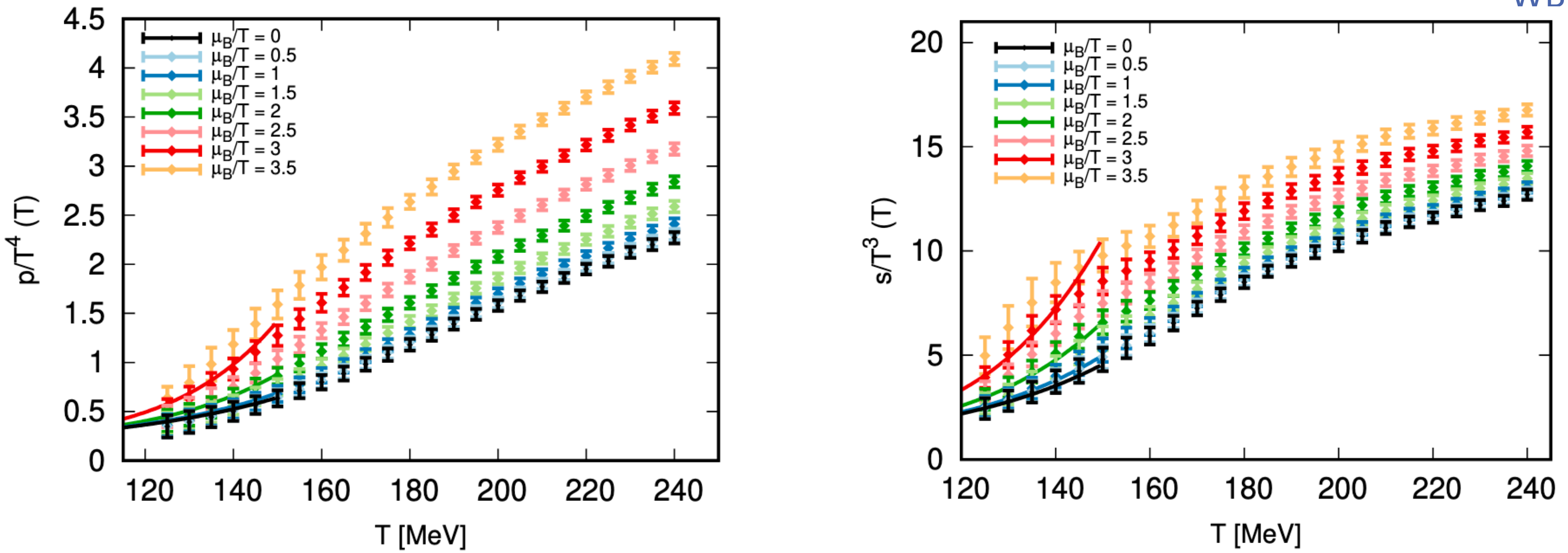


Novel expansion method

Once n_B is determined, we have everything we need to extract the other quantities

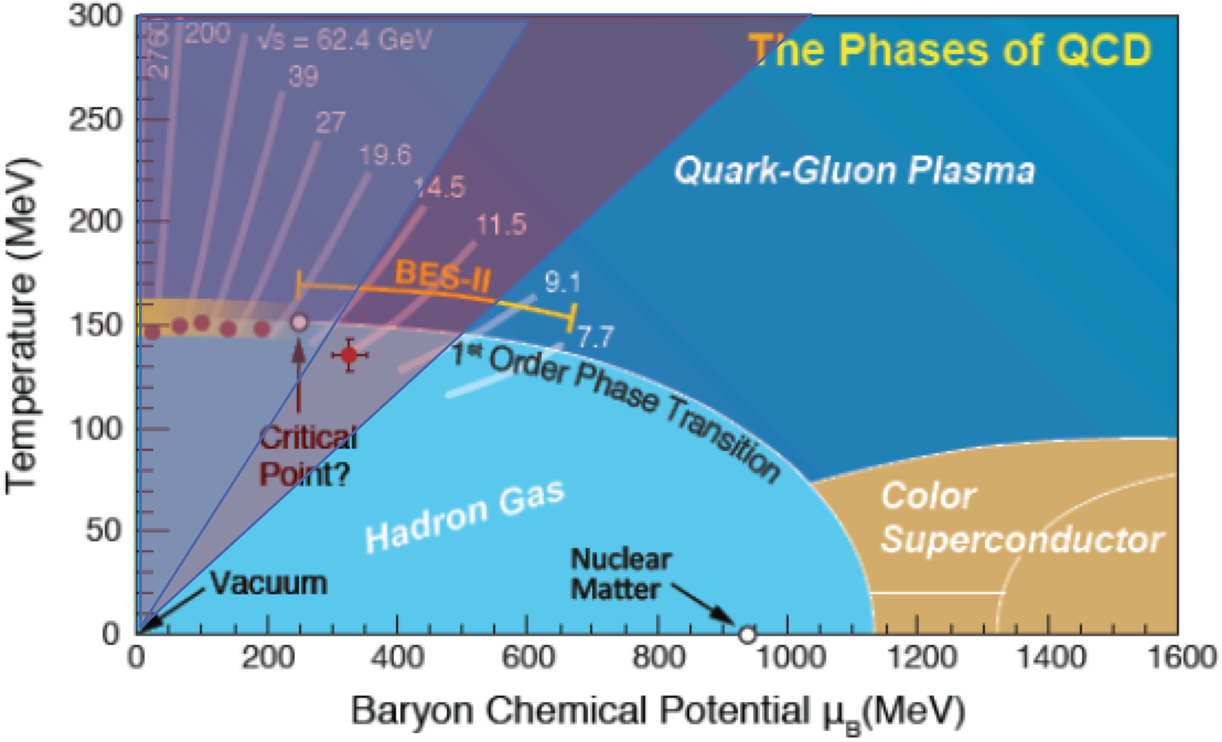
$$\frac{p(\mu_B,T)}{T^4} = \hat{p}(\hat{\mu}_B,T) = \hat{p}(0,T) + \int_0^{\hat{\mu}_B} \mathrm{d}\hat{\mu}'_B \, \hat{n}_B(\hat{\mu}'_B,T) \qquad s(\mu_B,T) = \left.\frac{\partial p(\mu_B,T)}{\partial T}\right|_{\mu}$$

WB, PRL (2021)



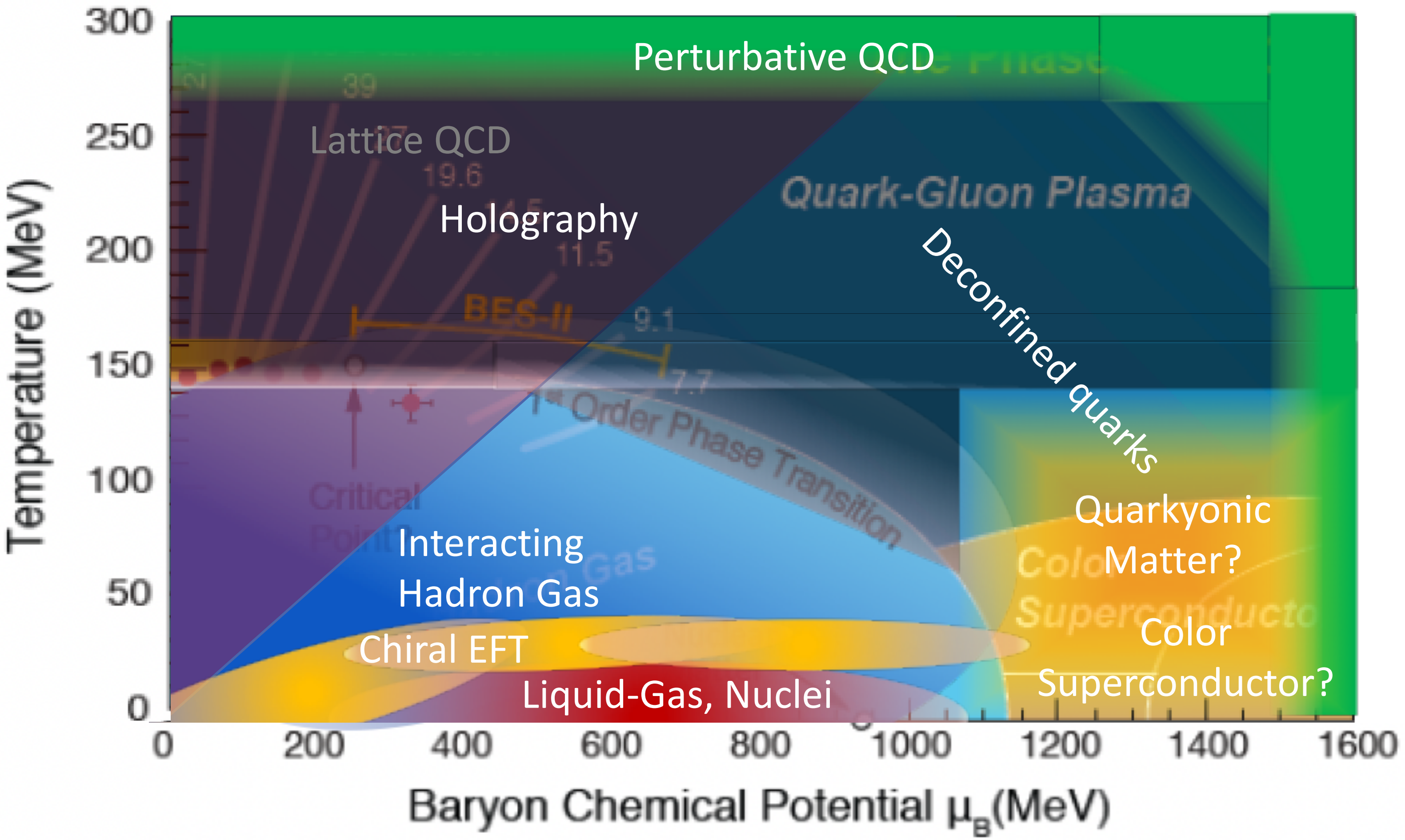
New range of validity of equation of state

- We now have the equation of state for $\mu_B/T \leq 3.5$



What happens at large densities?

- We need to merge the lattice QCD equation of state with other effective theories
- Careful study of their respective range of validity
- Constrain the parameters to reproduce known limits
- Test different possibilities and validate/exclude them



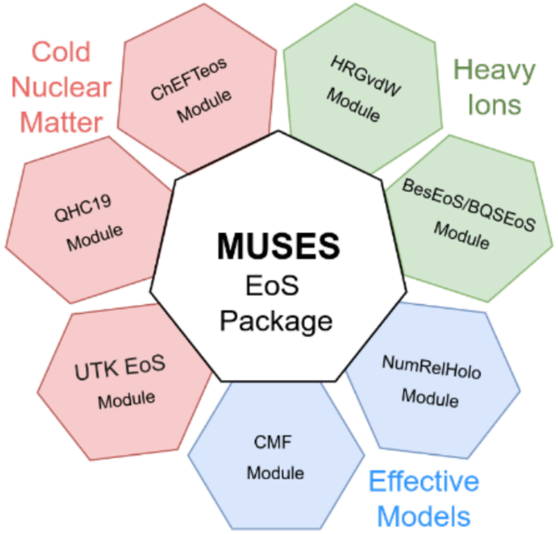
Lattice QCD: WB: PLB (2014)
Interacting HRG: V. Vovchenko et al., PRL (2017)
Liquid-gas, Nuclei: see e.g. Du et al. PRC (2019)
Chiral EFT: see e.g. Holt, Kaiser, PRD (2017)
Holography: see e.g. R. Critelli et al., PRD (2017)

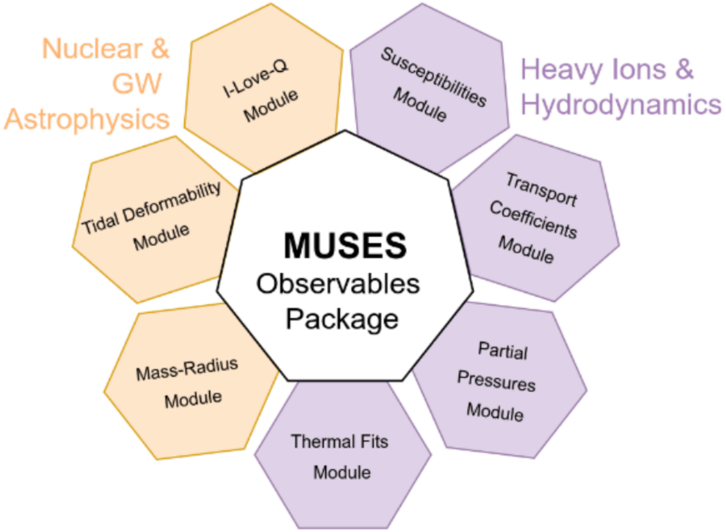
pQCD: Andersen et al., PRD (2002); Annala et al., Nat. Ph. (2020)
quarks: Dexheimer et al., PRC (2009); Baym et al., Astr. J. (2019)
quarkyonic: McLerran, Pisarski NPA (2007)
CSC: Alford et al., PLB (1998); Rapp et al., PRL (1998).

Modular Unified Solver of the Equation of State collaboration

- Funded by NSF through CSSI program
- Developers and Users are working together to create a sustainable software to generate equations of state in the whole phase space
- Modular: Different models (``modules”) to describe the EoS in different regimes of phase space
- Unified: Modules smoothly integrated to (i) ensure maximal coverage of phase space, and (ii) respects constraints

muses





QCD phase diagram

TRANSITION TEMPERATURE

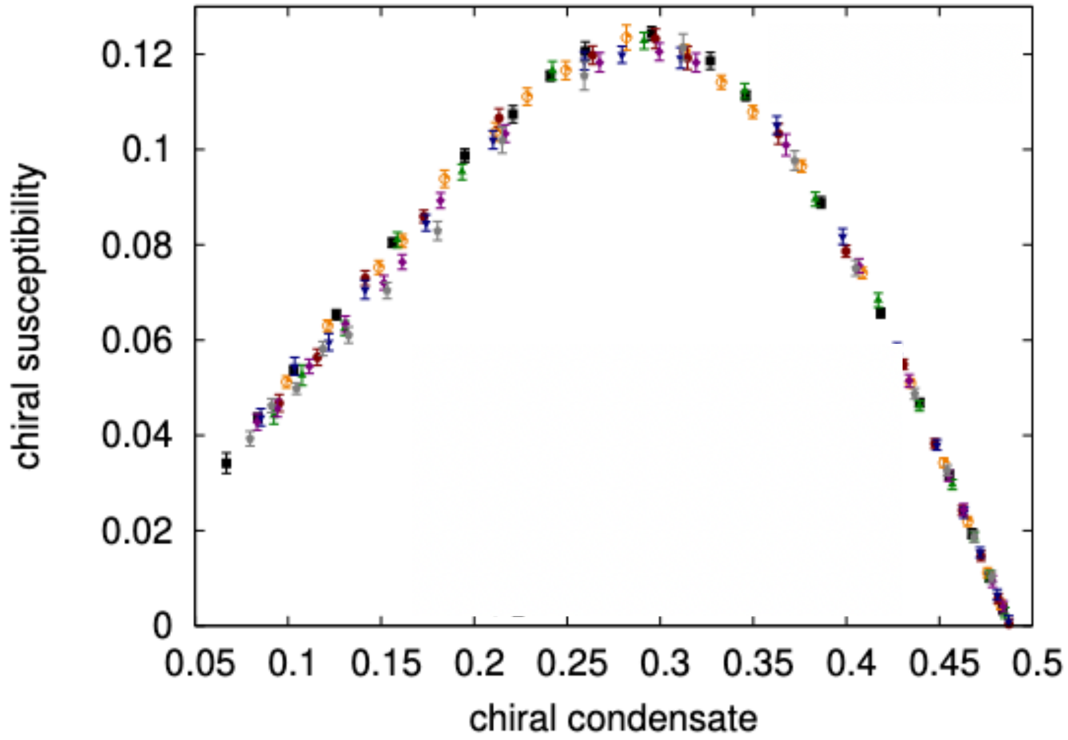
TRANSITION LINE

TRANSITION WIDTH

Phase Diagram from Lattice QCD

The transition at μ_B=0 is a smooth crossover

Aoki et al., Nature (2006)
Borsanyi et al., JHEP (2010)
Bazavov et al., PRD (2012)



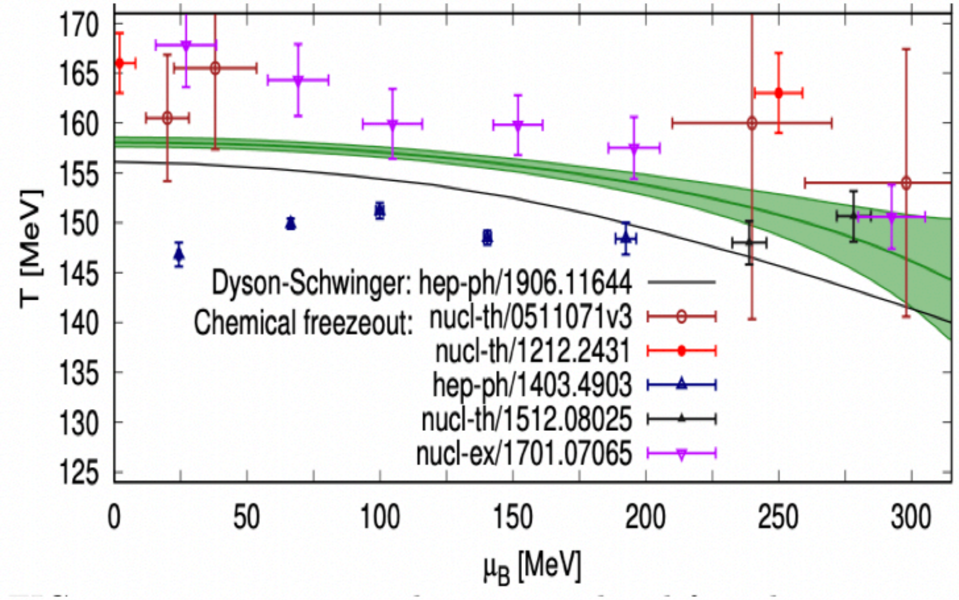
QCD transition temperature and curvature

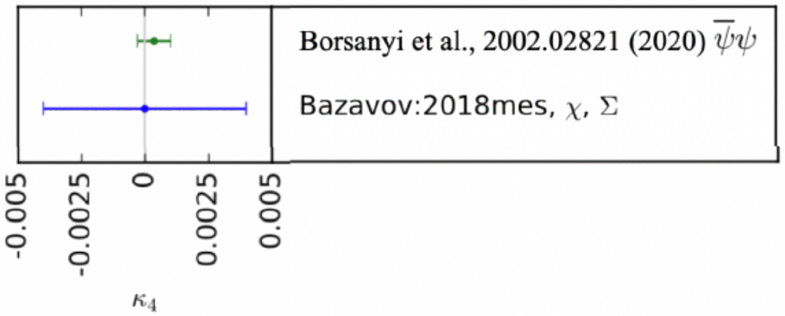
$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

Borsanyi, C. R. et al. PRL (2020)

- Latest results on T_O from WB collaboration based on subtracted chiral condensate and chiral susceptibility

T_O=158.0±0.6 MeV



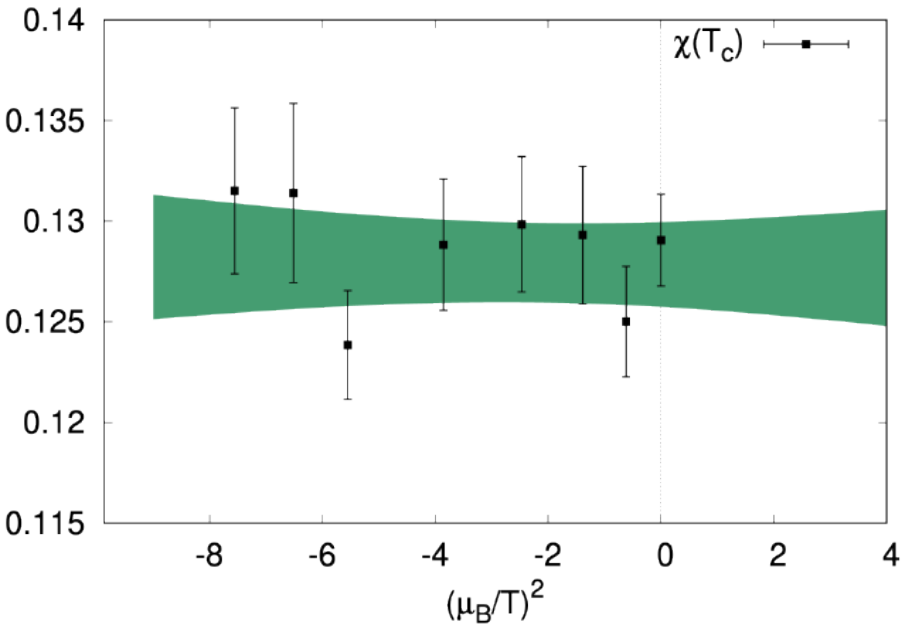


$$\kappa_2 = 0.0153 \pm 0.0018 \,,$$
$$\kappa_4 = 0.00032 \pm 0.00067$$

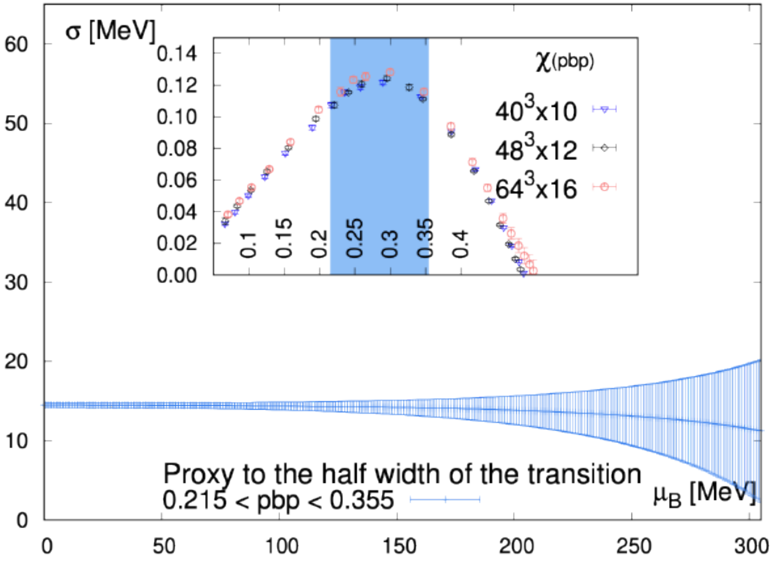
Limit on the location of the critical point

For a genuine phase transition, the height of the peak of the chiral susceptibility diverges and the width shrinks to zero

Height of chiral susceptibility peak



Width of chiral susceptibility peak



No sign of criticality for μ_B<300 MeV

Borsanyi, C. R. et al. PRL (2020)

Fluctuations of conserved charges

COMPARISON TO EXPERIMENT

CHEMICAL FREEZE-OUT PARAMETERS

Fluctuations of conserved charges

Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{\,l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

Relationship between chemical potentials:

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S. \end{aligned}$$

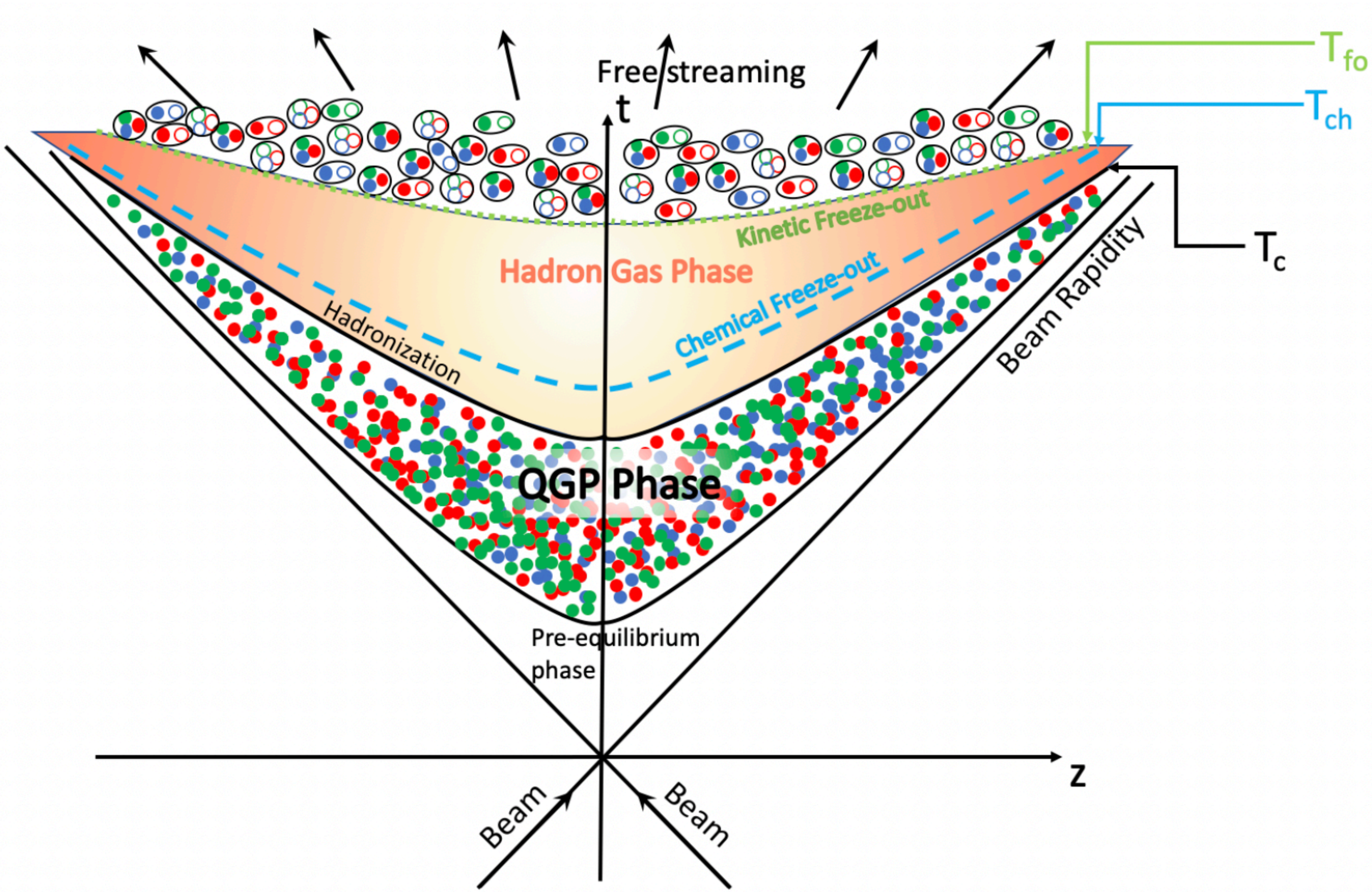
They can be calculated on the lattice and compared to experiment

Evolution of a heavy-ion collision

- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)

- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)

- Hadrons reach the detector



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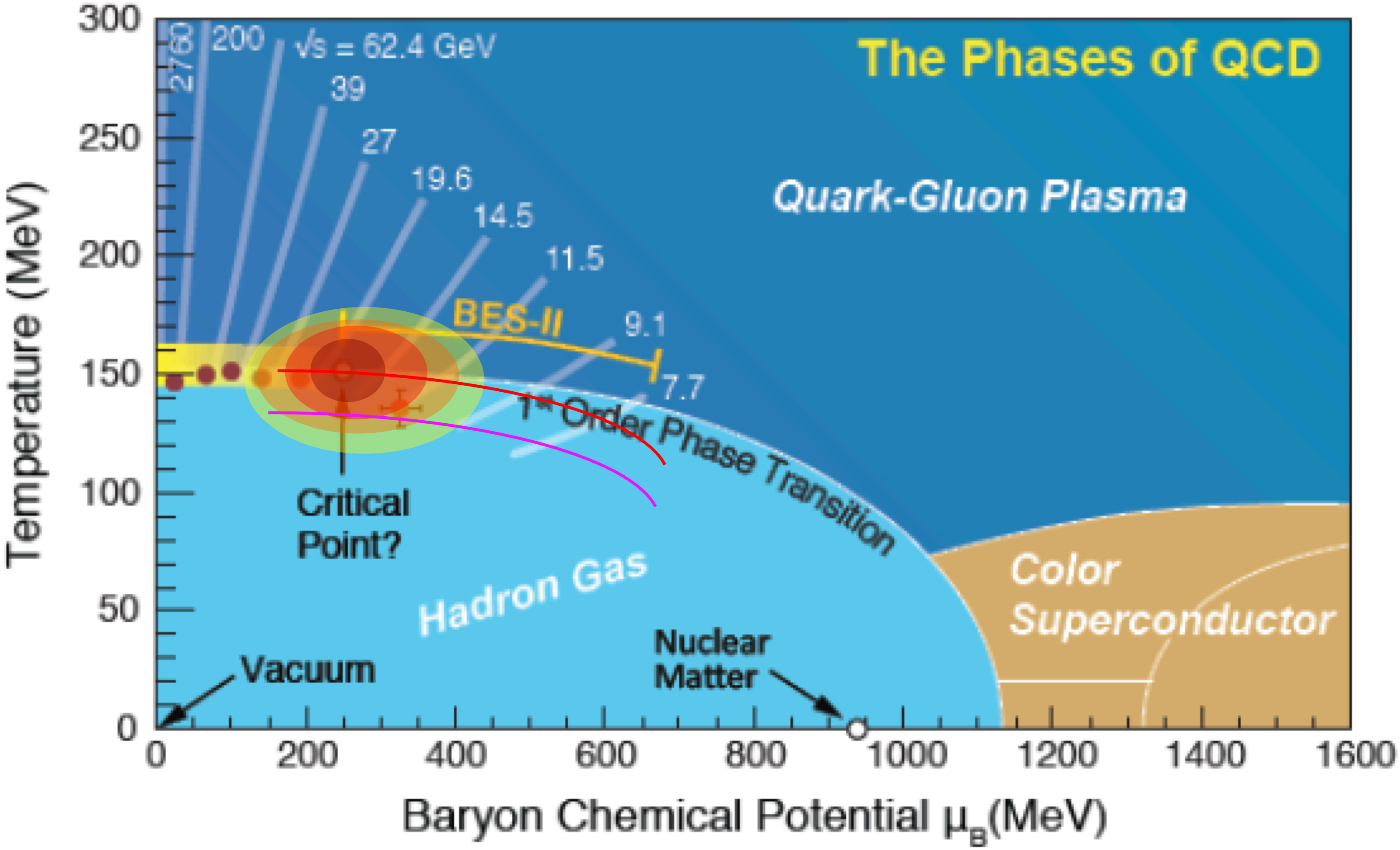
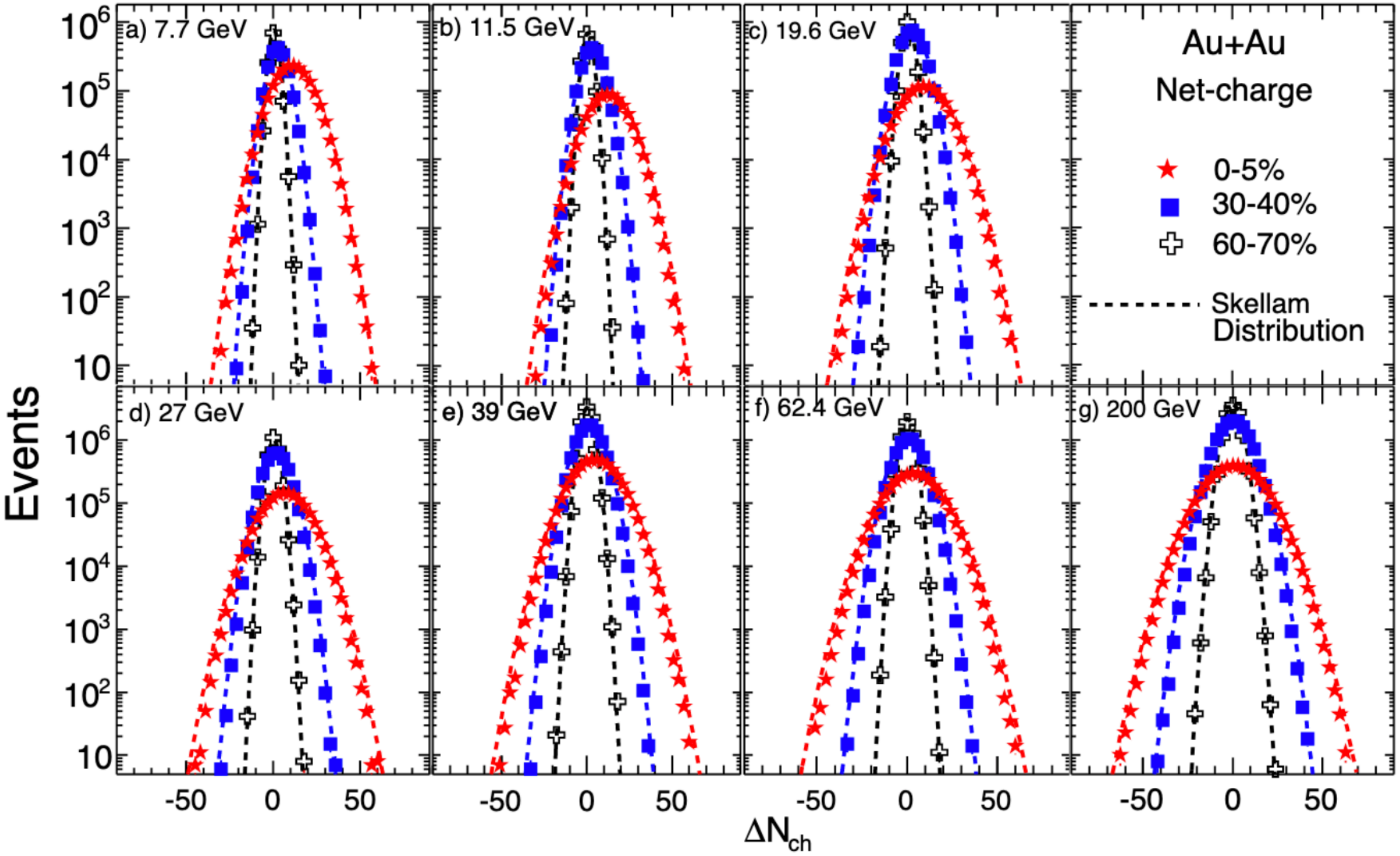


Connection to experiment

- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to measure its event-by-event distribution



STAR Collab., PRL (2014)

Connection to experiment

Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

The chemical potentials are not independent: fixed to match the experimental conditions:

$\langle n_S \rangle = 0$ $\qquad$ $\langle n_Q \rangle = 0.4 \langle n_B \rangle$

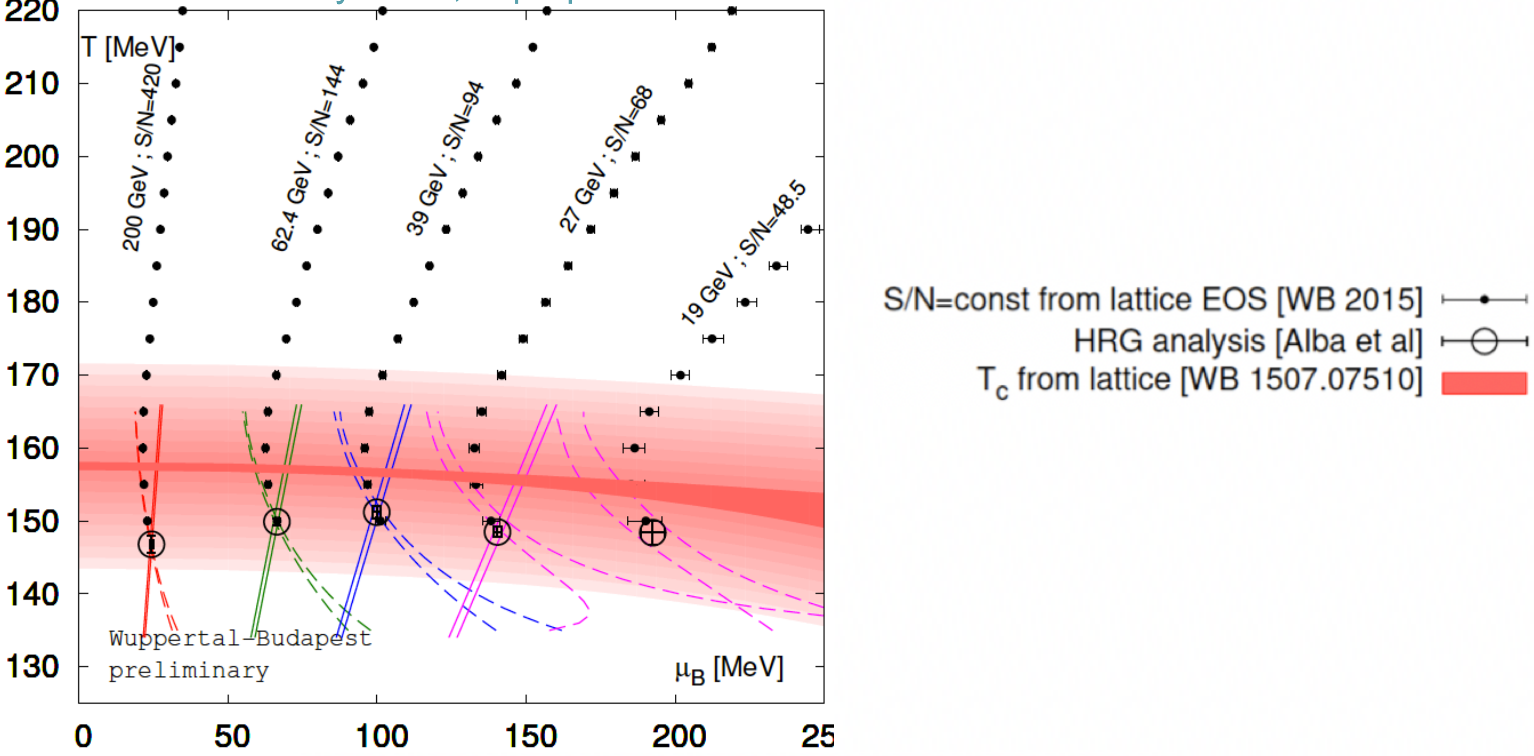
Freeze-out line from first principles

Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation



Conclusions

- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 3.5$
- Extensions to higher densities by means of lattice-based models
- No indication of Critical Point from lattice QCD in the explored μ_B range