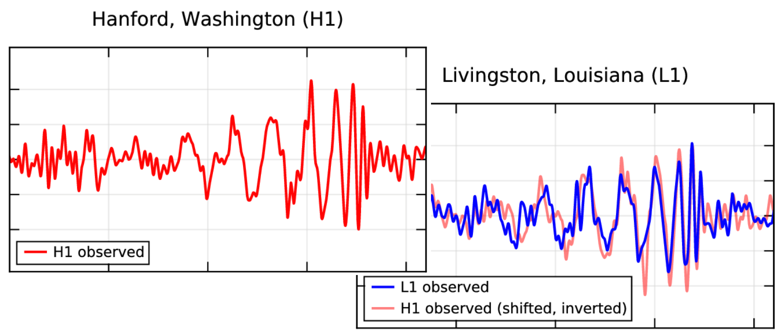
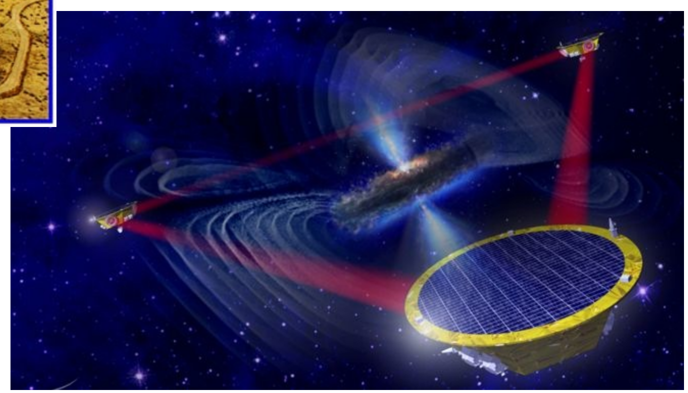
Probing strong gravity with gravitational waves





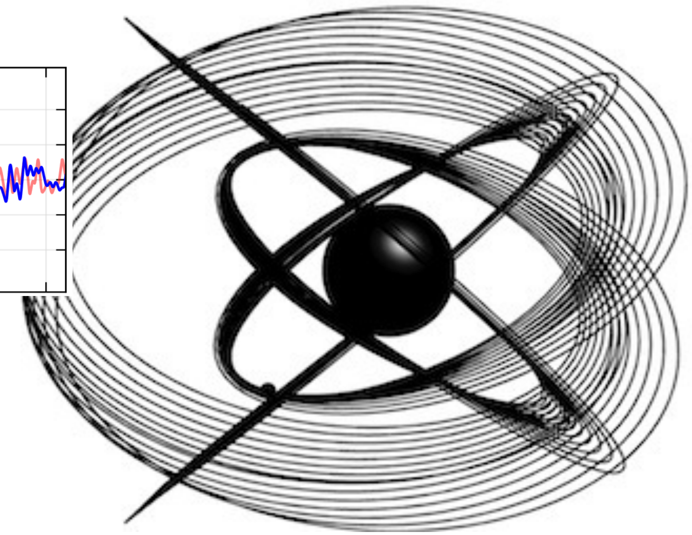


Image: Steve Drasco, CalPoly

Using gravitational wave measurements to learn about the nature of gravity far beyond what is accessible in the "laboratory"

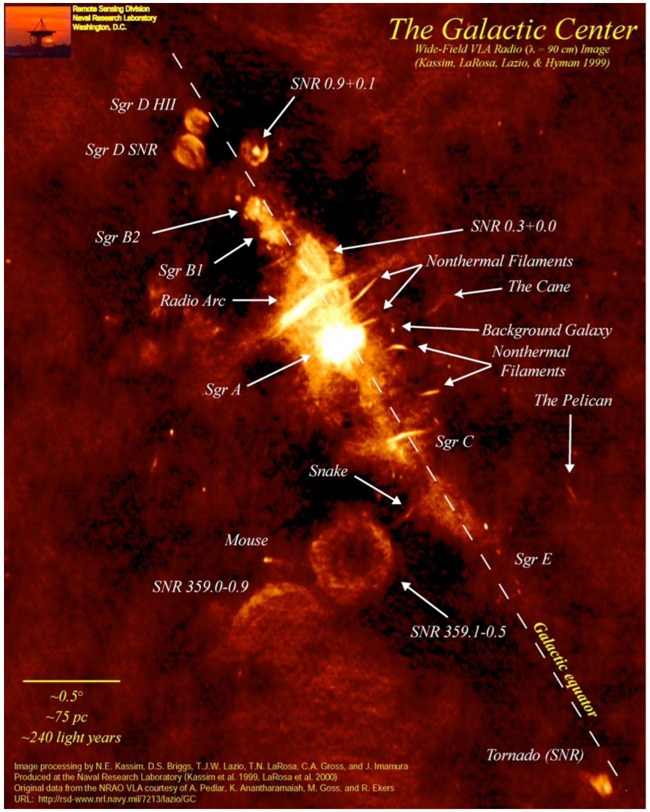
Black holes pre September 2015

If I were giving this talk 7 years ago, I would point to astronomical evidence for objects that are

Compact
Massive
Dark

Objects which all possess the ***necessary*** conditions to be black holes …

Example: Center of our galaxy. Long known to be a crowded, dense region …



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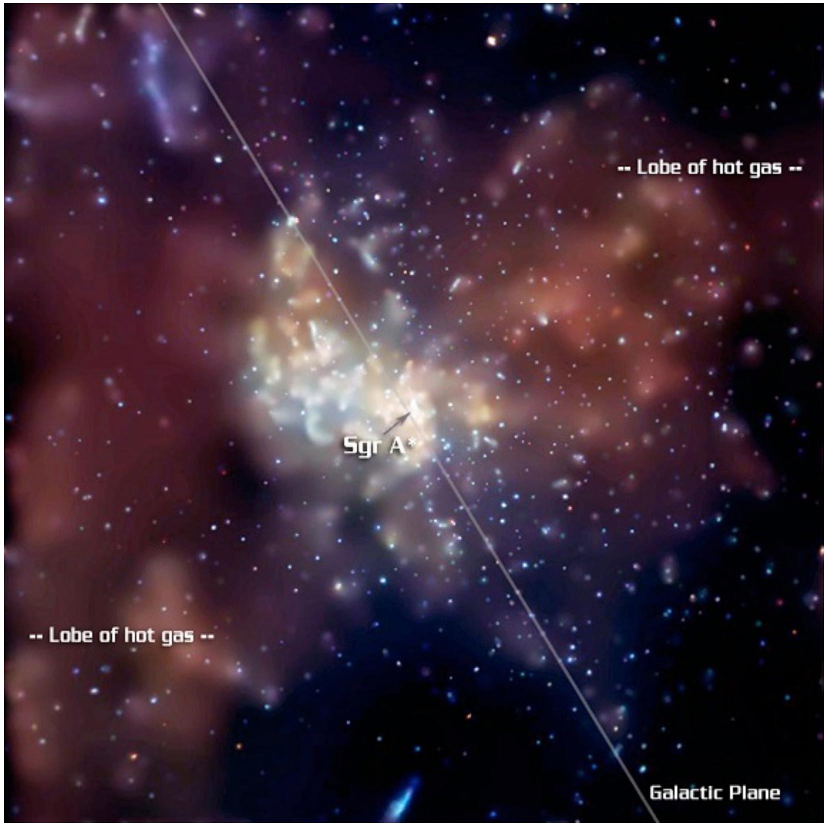
Point-like structure apparent as we zoom in to smaller and smaller length scales…



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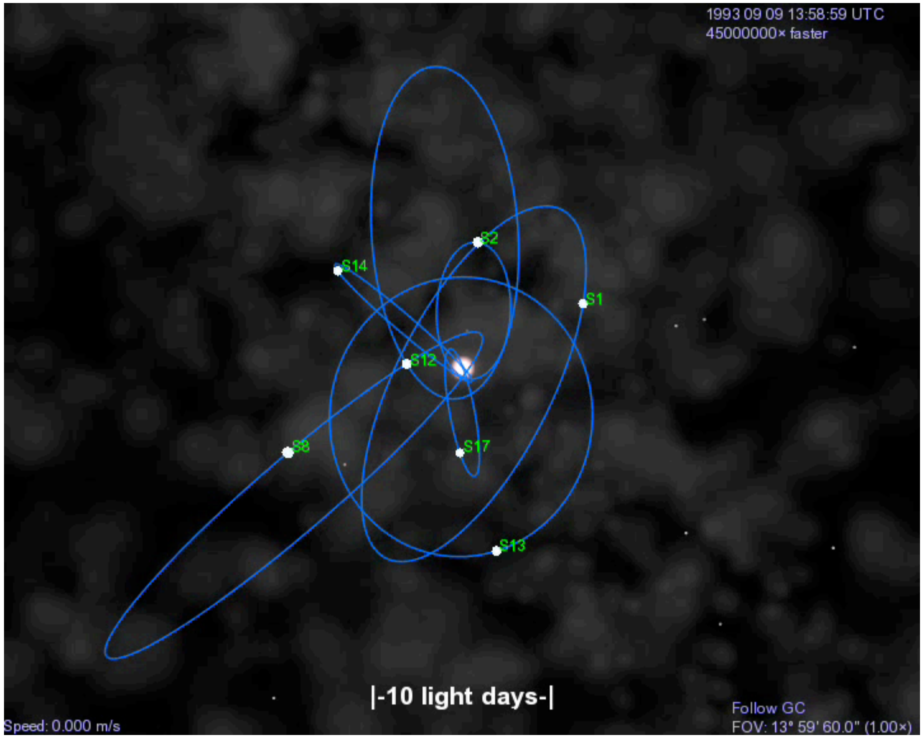
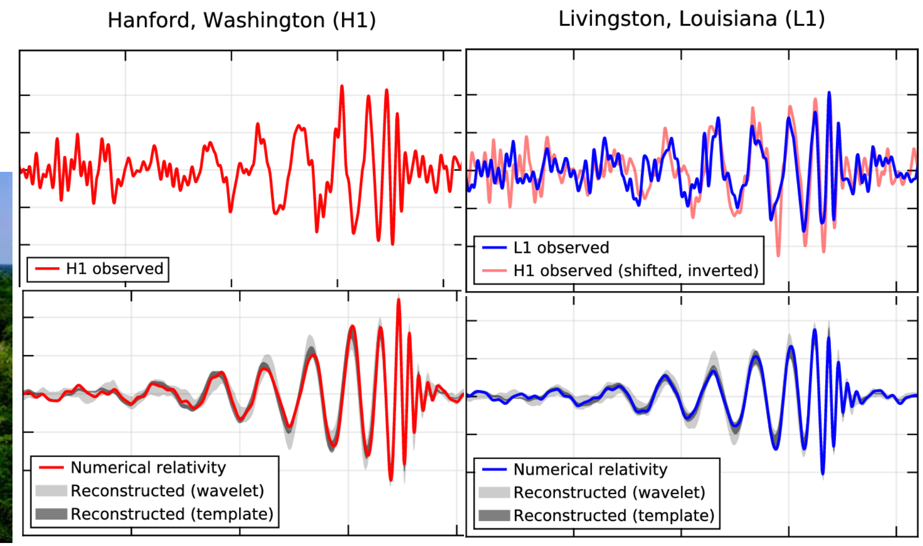
Objects which all possess the ***necessary*** conditions to be black holes …

Stars observed to orbit a dark object; orbits yield mass estimate of about 4 million Msun.



Black holes post 2015

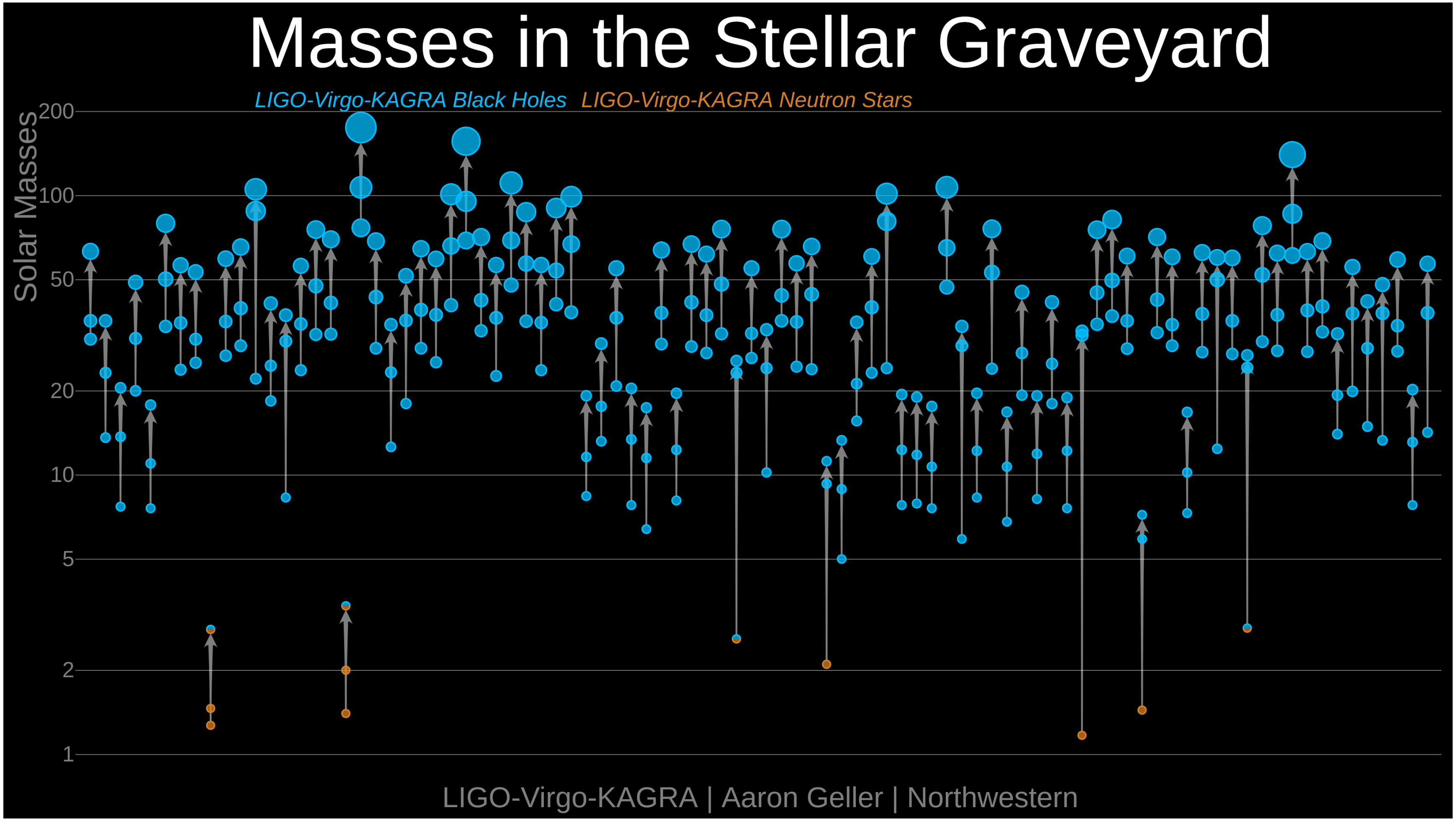
Situation changed in 0.2 seconds on the morning of 14 September 2015



LIGO measurements precisely match model of last several orbits of two black holes that merge and then settle down into a single black hole remnant…

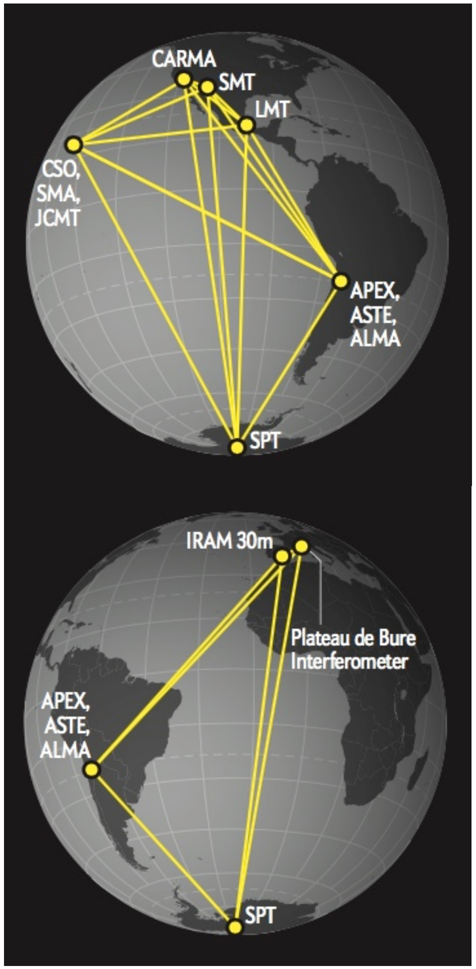
Consistency with GR predictions of global spacetime dynamics needed to explain data.

Black holes post 2015

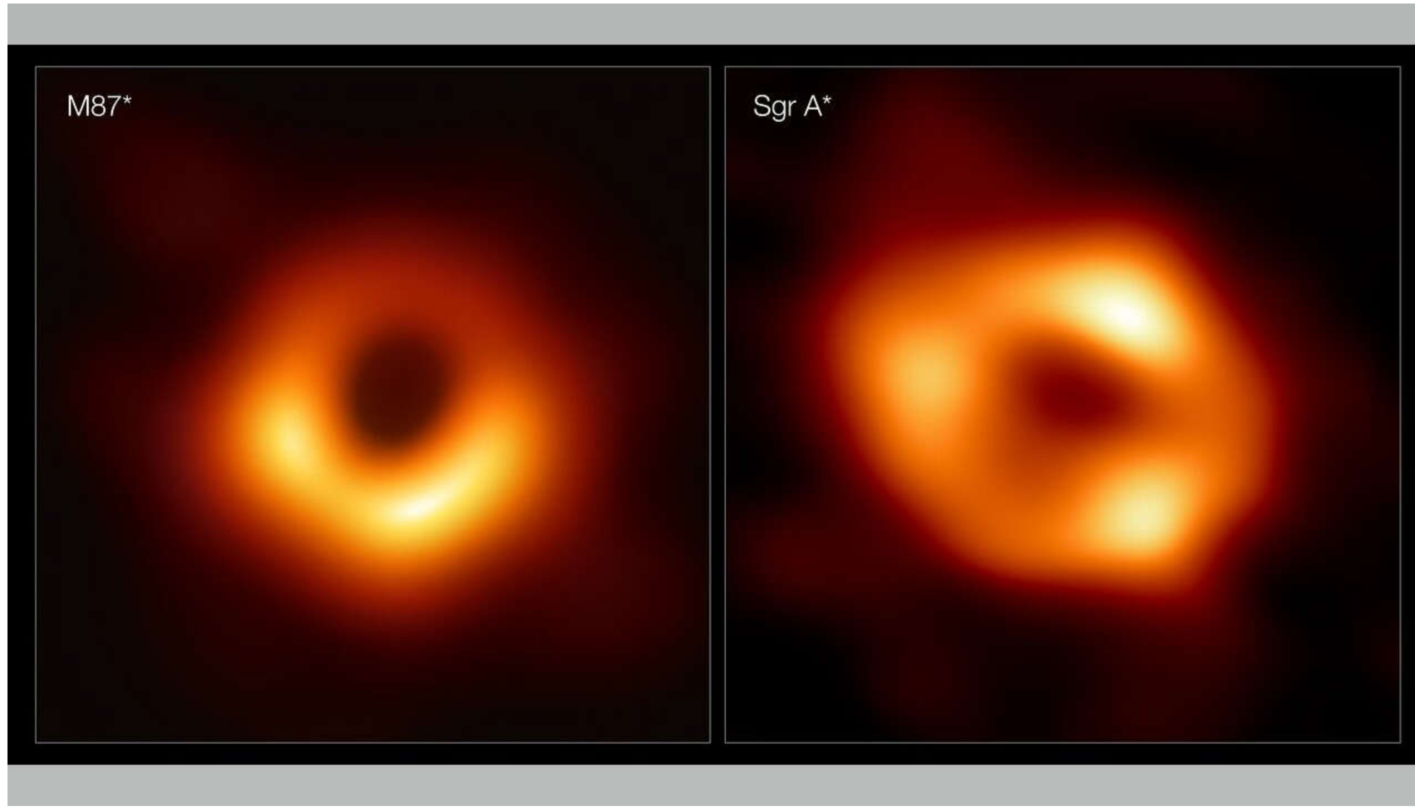


Many dozen events measured by gravitational-wave antennae since then continue to hammer this point home.

Event Horizon Telescope



Graphics provided by Dimitrios Psaltis, University of Arizona



Can now zoom in *much* farther on a few black holes, resolve features in the material accreting onto them.

GENERAL RELATIVITY WORKS

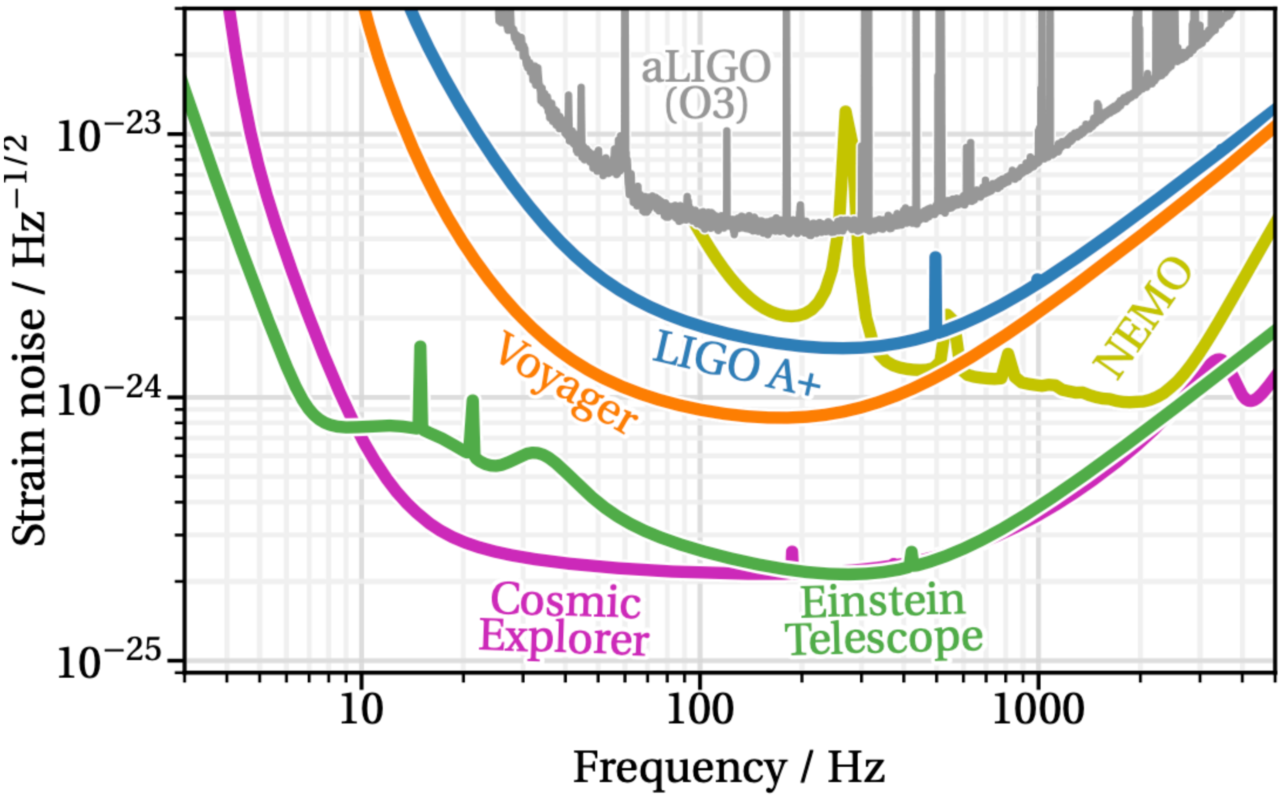
EINSTEIN WAS RIGHT

The predictions of general relativity suffice to explain the data so far.

Einstein has not yet been falsified.

Moving forward with gravitational waves: Frontier is in *precision*

As instruments are upgraded and sensitivity improves, three critical metrics to bear in mind: **Strain sensitivity, band** and **bandwidth.**



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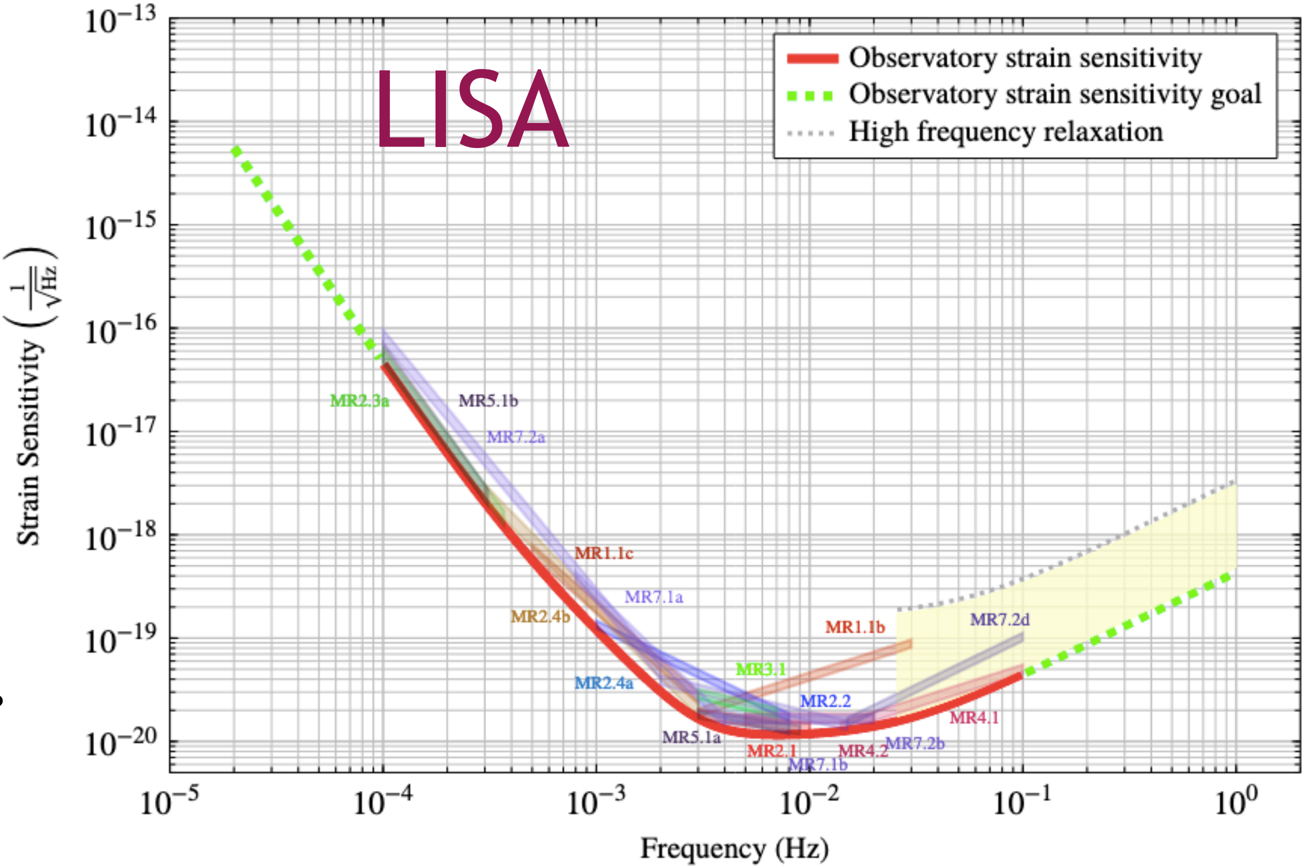

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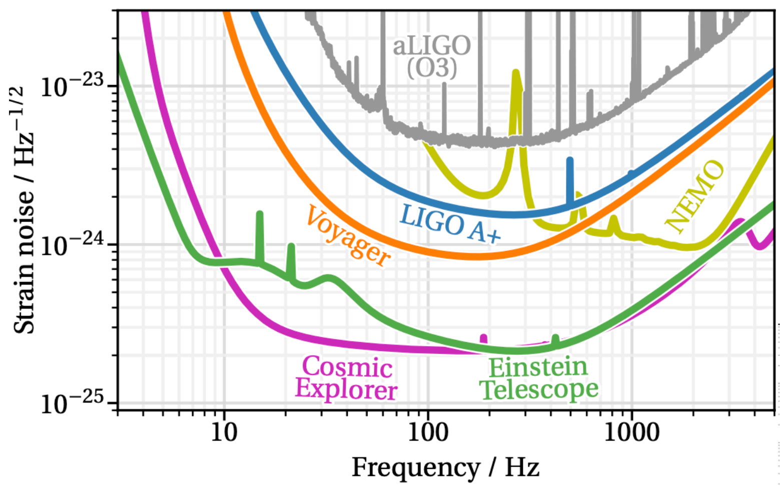

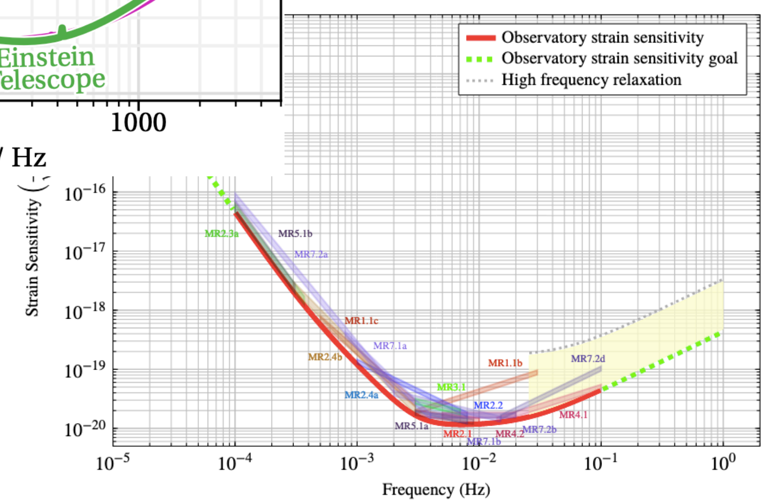

Sensitivity: More signal to noise for each cycle. Better able to resolve the nature of the signal.

Moving forward with gravitational waves: Frontier is in *precision*

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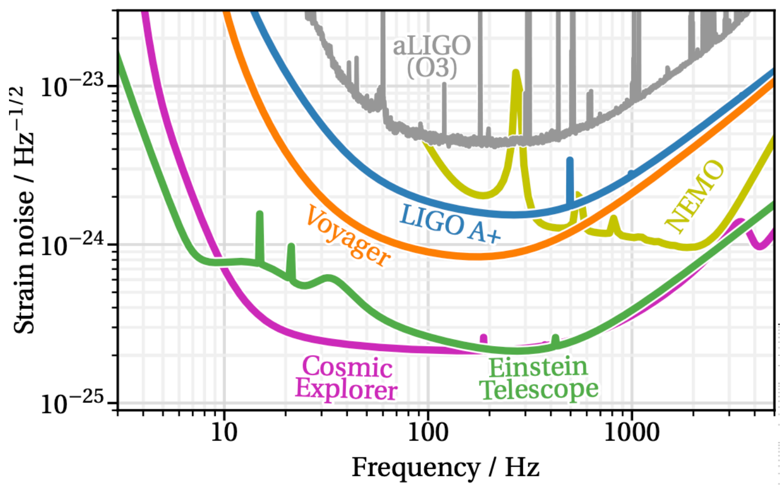
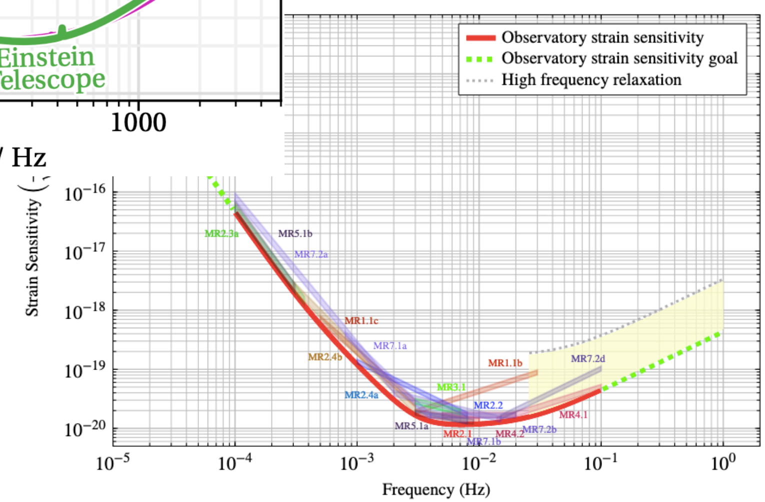
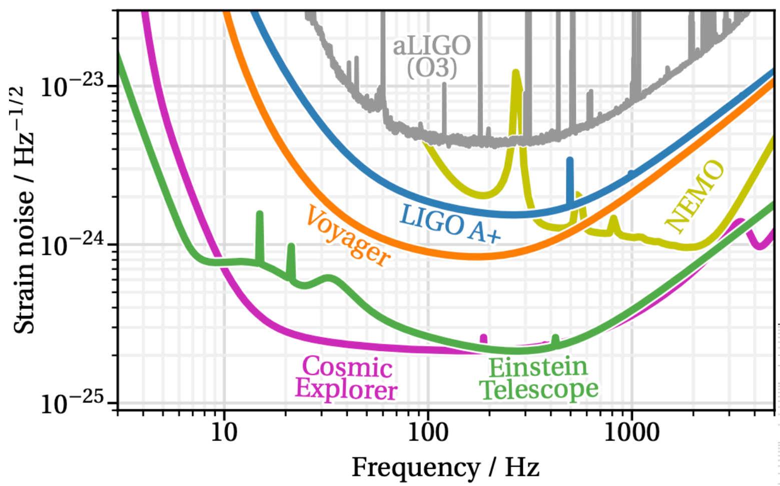


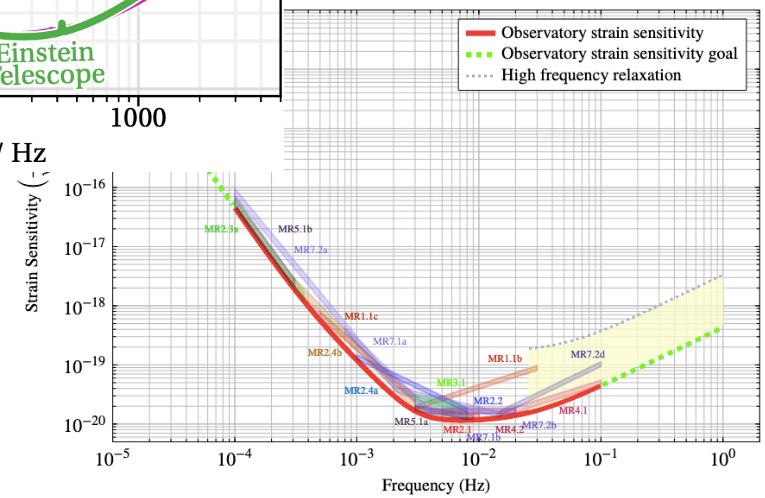


Band: Different future instruments sensitive to different parts of the GW spectrum, able to measure a wider astrophysical range of sources.

Moving forward with gravitational waves: Frontier is in *precision*

As instruments are upgraded and sensitivity improves, three critical metrics to bear in mind: **Strain sensitivity, band** and **bandwidth.**





Bandwidth: Many signals in band for longer, give us more details on its evolution and nature. More waveform cycles … high precision on GW properties.

Anatomy of measured waveform

What a detector like LIGO measures is a weighted sum of the two fundamental GW polarizations:

$$h = h_+(\cos\iota)F_+(\theta,\phi,\psi) + h_\times(\cos\iota)F_\times(\theta,\phi,\psi)$$

Two polarizations generated by source, depend on orientation with respect to line of sight

Antenna response functions, depend on source's position on sky, orientation at that position.

Anatomy of measured waveform

What a detector like LIGO measures is a weighted sum of the two fundamental GW polarizations:

$$h = h_+(\cos\iota)F_+(\theta,\phi,\psi) + h_\times(\cos\iota)F_\times(\theta,\phi,\psi)$$

Waveform example: Binary system with members widely separated. Leading order form is

$$h_+ = \frac{\mathcal{A}}{D}\mathcal{M}^{5/3}f^{2/3}(1+\cos^2\iota)\cos(2\Phi)$$

$$h_\times = \frac{\mathcal{A}}{D}\mathcal{M}^{5/3}f^{2/3}\cos\iota\sin(2\Phi)$$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

Important point: Waveform phase is measured **MUCH** more accurately than amplitude.

Anatomy of measured waveform

GW phase tracks the binary's orbital phase (modulo a factor of two for the loudest quadrupole mode).

Leading solution: how frequency changes due to backreaction of GW emission

$$\frac{df}{dt} = \frac{96\pi^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3} \qquad \mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

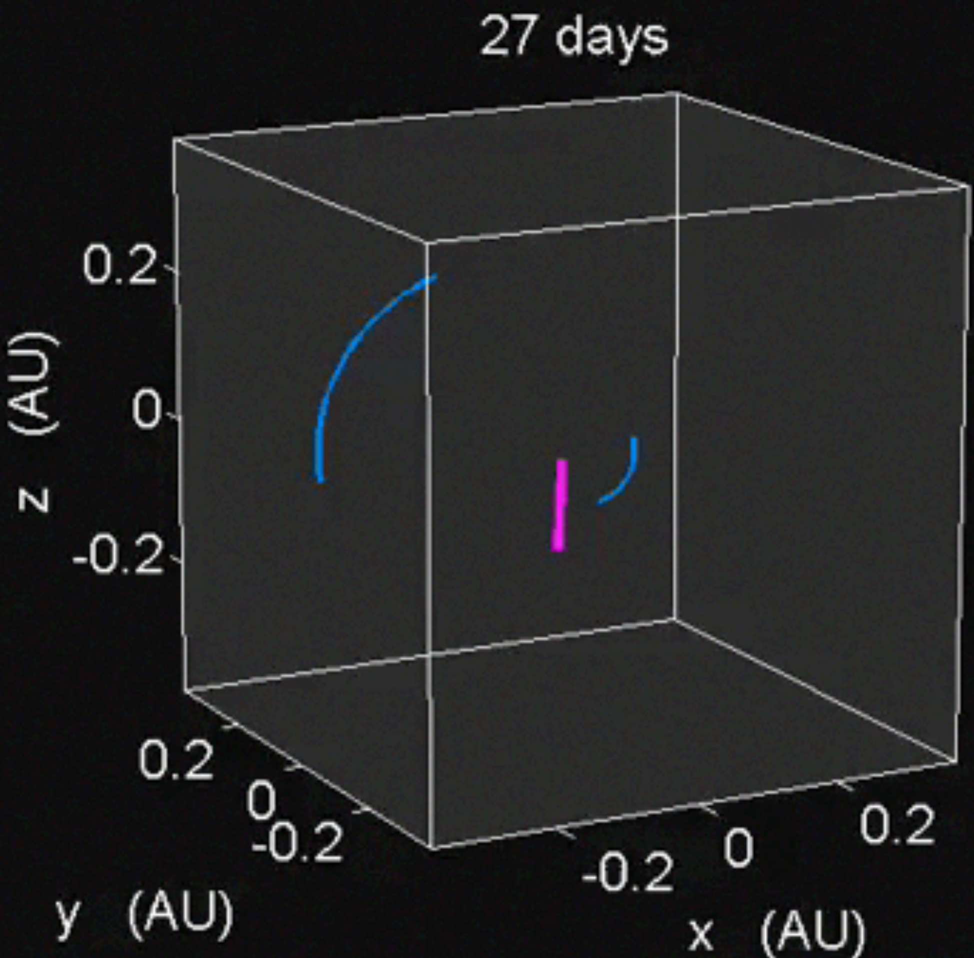
Anatomy of measured waveform

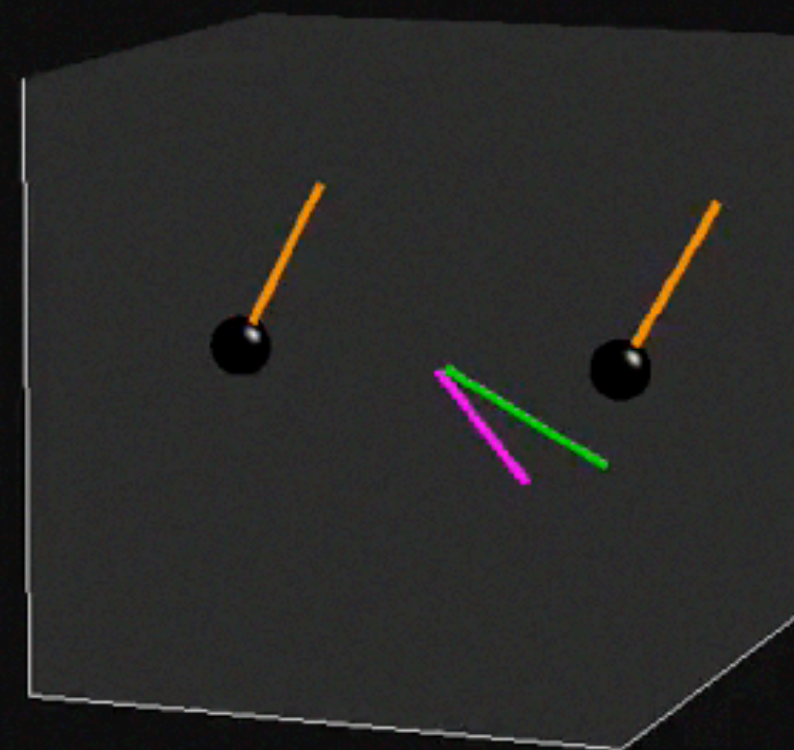
GW phase tracks the binary's orbital phase (modulo a factor of two for the loudest quadrupole mode).

Going beyond the leading solution incorporates additional physics about the nature of the binary's members into the inspiral law:

$$\frac{df}{dt} = \frac{96\pi^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}\left[1 - \left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)(\pi M f)^{2/3} + [4\pi - \beta(t)](\pi M f)\right.$$
$$\left. + \left(\frac{34103}{18144} + \frac{13661}{2016}\frac{\mu}{M} + \frac{59}{18}\frac{\mu^2}{M^2} + \sigma(t)\right)(\pi M f)^{4/3}\right]$$

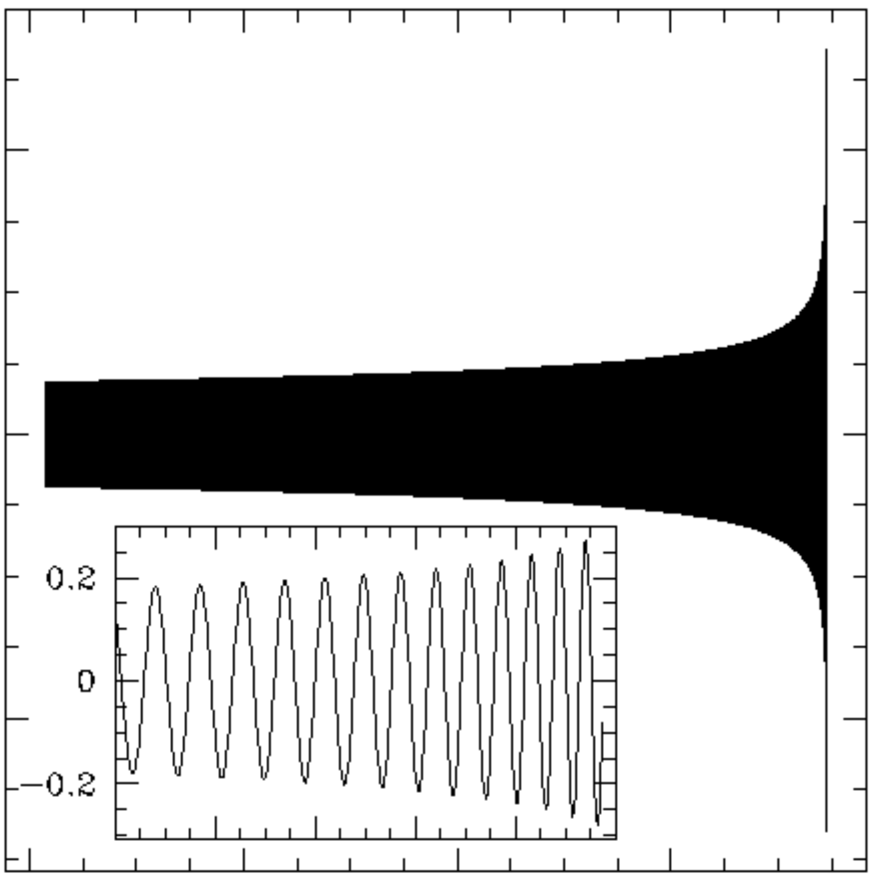
Higher order: introduces dependence on reduced mass μ; parameters β and σ depend on relative orientation of orbit $\mathbf{L}$ and members' spins $\mathbf{S}_1$ and $\mathbf{S}_2$


27 days
0.2
0
-0.2
z (AU)
0.2
0
-0.2
y (AU)
-0.2
0
0.2
x (AU)



GWs with spin vs GWs without

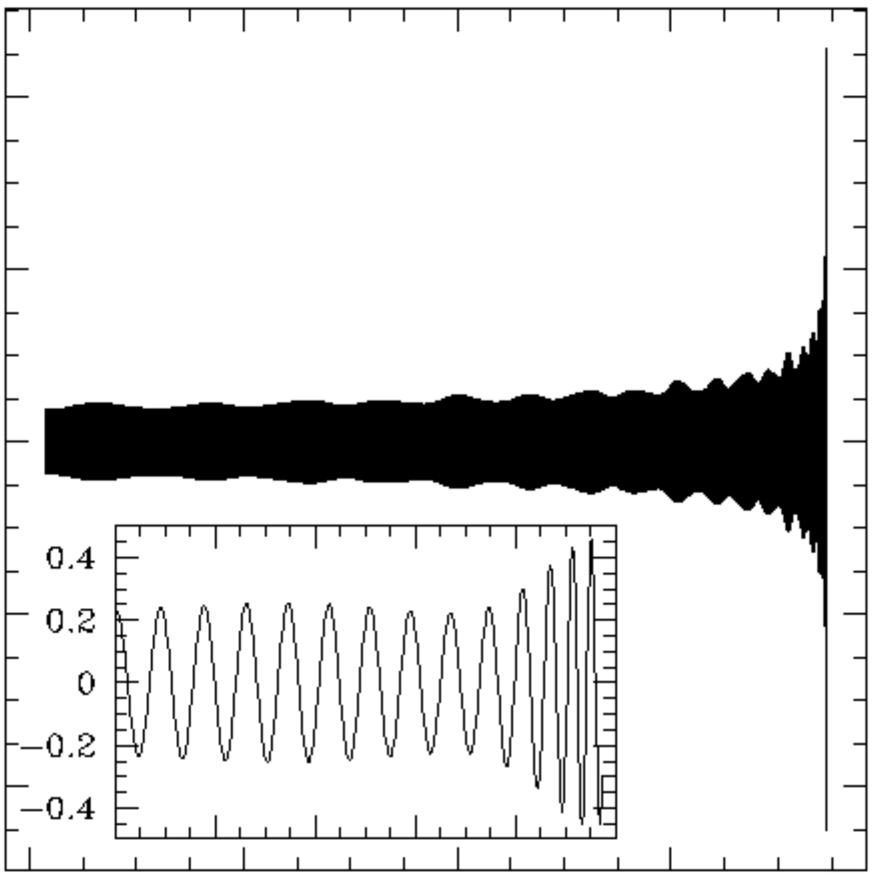
Influence of the spin terms in the phase leaves a strong imprint on the waveform.



Example 1:
Two non-spinning black holes.

GWs with spin vs GWs without

Influence of the spin terms in the phase leaves a strong imprint on the waveform.



Example 2:
Two *rapidly* spinning black holes.

Anatomy of measured waveform

When the final state is a black hole, we expect that the final waves will be quasi-normal modes of that hole: Damped sinusoids whose properties are determined by the hole's mass and spin.

$$h_{\text{ring}} = \sum_{lm} \mathcal{A}_{lm} e^{-t/\tau_{lm}} \sin(2\pi f_{lm} t + \varphi)$$

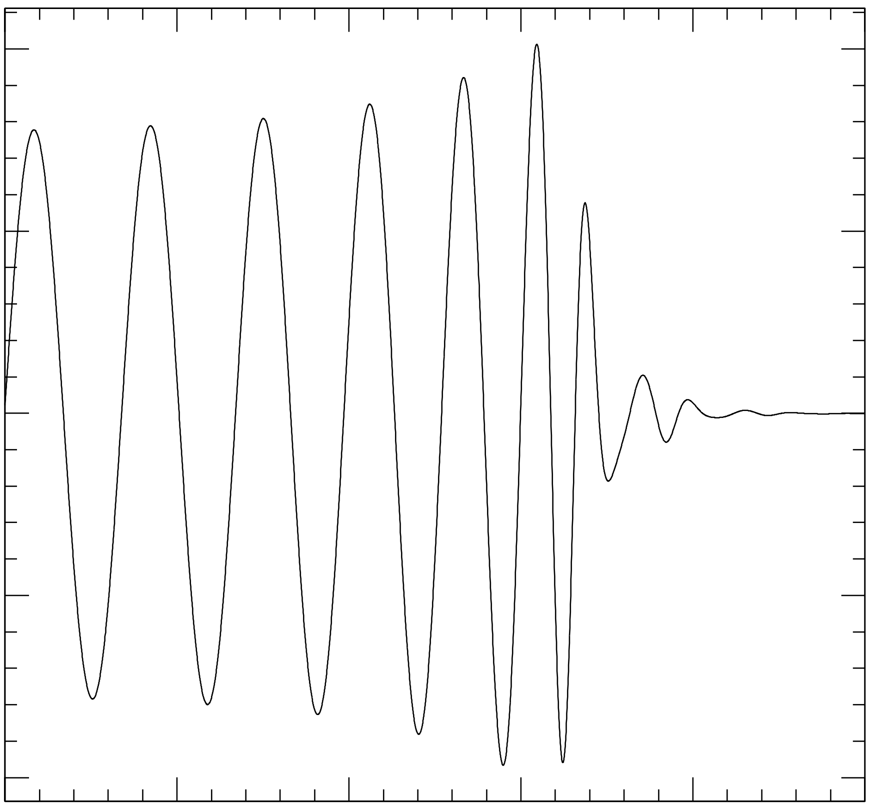
Longest lived modes have $l = m = 2$:

$$f_{22} \approx \frac{c^3}{2\pi G M}\left[1 - 0.63(1-a)^{3/10}\right]$$

$$Q_{22} \equiv \pi f_{22} \tau_{22} \approx 2(1-a)^{-9/20}$$

Listening to a black hole ring

Example of a signal dominated by the ringing of a black hole.



Final black hole has spin a = 0.6 … yields Q ≈ 4

“Probing the nature of gravity”

This catch-all phrase captures a number of important issues:

- Are we correctly describing objects with GR? E.g., if we assume general relativity, can we ascertain how well the Kerr metric describes the objects which we measure?
- Does general relativity describe all aspects of the sources that we measure?
- ***Can we distinguish these possibilities?***
- Do we understand systematic effects well enough to make strong statements about these possibilities?

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Major challenge: **GR works “too well.”**

Testing gravity requires comparing with alternatives … plausible alternatives are hard to make.

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Major challenge: **GR works “too well.”**

As we improve measurement precision, need much greater modeling precision!!

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Remainder of this talk: Look at examples that help to illustrate these issues.

Ringdown spectroscopy

If the spacetime is given by the Kerr solution, then late-time dynamics of merger should be dominated by black hole ***ringdown spectrum***:

$$h_{\rm ring} = \mathcal{A} e^{-t/\tau_{\rm ring}(M_{\rm fin}, a_{\rm fin})} \sin\left[2\pi f_{\rm ring}(M_{\rm fin}, a_{\rm fin}) + \varphi\right]$$

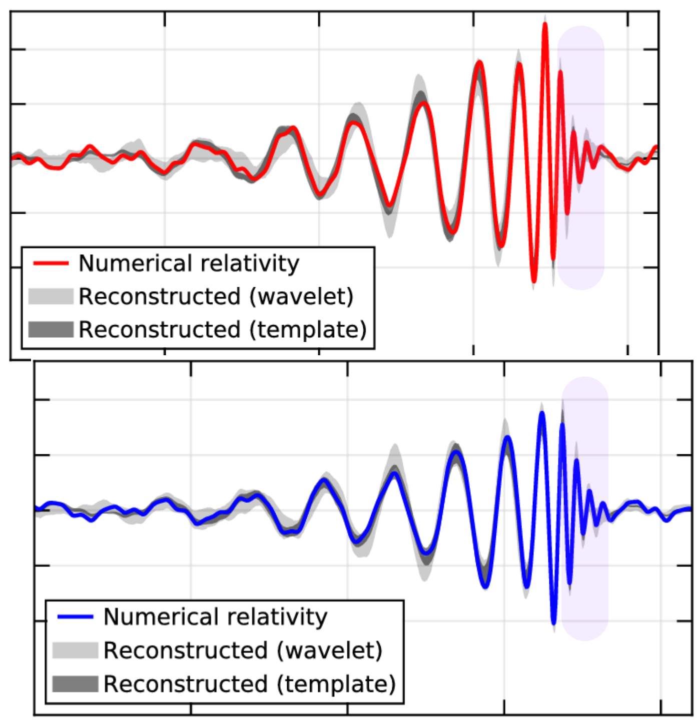
If end state of the system is a Kerr black hole, then the last waves we measure will be a superposition of modes with this form.

Ringdown spectroscopy

$$h_{\rm ring} = \mathcal{A}e^{-t/\tau_{\rm ring}(M_{\rm fin}, a_{\rm fin})}\sin\left[2\pi f_{\rm ring}(M_{\rm fin}, a_{\rm fin}) + \varphi\right]$$

Key point: Each mode has a frequency and damping time uniquely determined by the mass and the spin of the final state.

Longest lived: $l = m = 2$ mode. GW150914's last cycles consistent with this mode.



$l = m = 2$: Oscillations at about 270 Hz, amplitude falls by e after ~5 cycles.

Ringdown spectroscopy

$$h_{\mathrm{ring}} = \mathcal{A} e^{-t/\tau_{\mathrm{ring}}(M_{\mathrm{fin}}, a_{\mathrm{fin}})} \sin\left[2\pi f_{\mathrm{ring}}(M_{\mathrm{fin}}, a_{\mathrm{fin}}) + \varphi\right]$$

Need other modes … can be measured with louder events, more sensitive detectors:

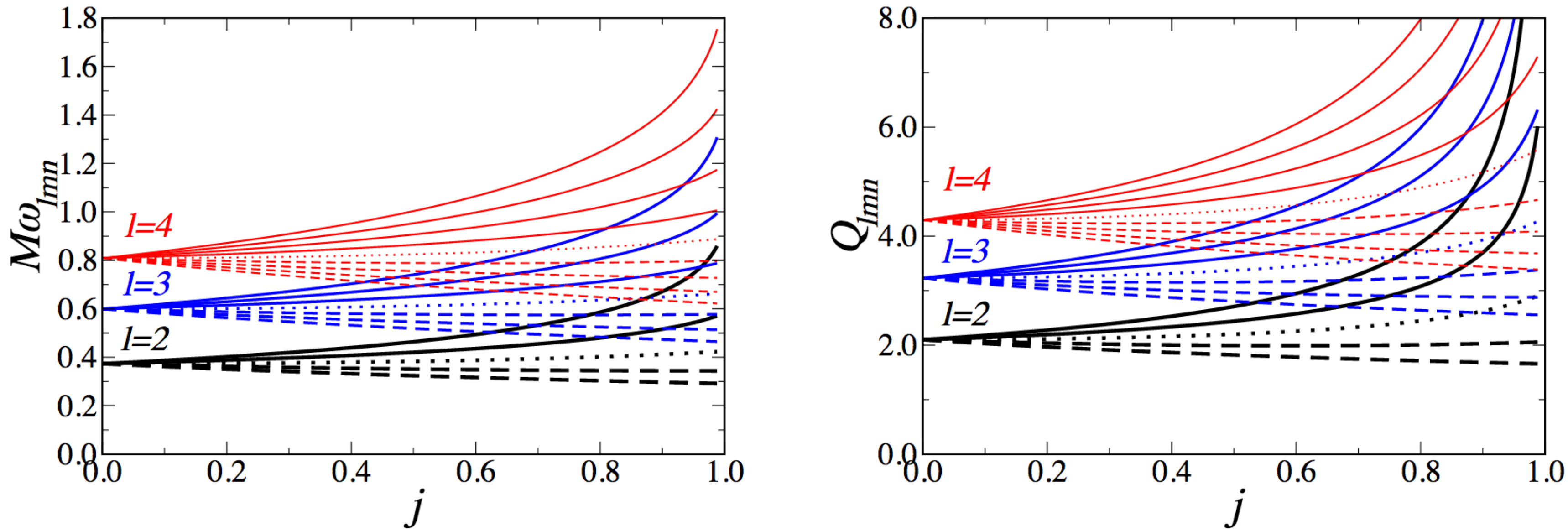
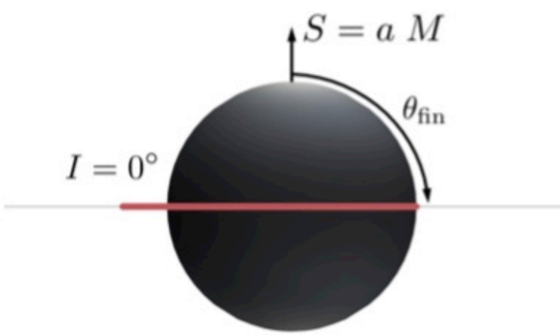
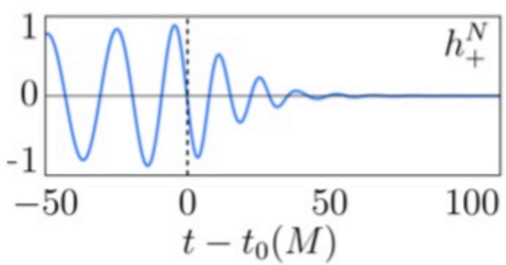


Figure from Berti, Cardoso, and Will, PRD **73**, 064030 (2005); discussion of multimode measurements in Berti, Sesana, Barausse, Cardoso, Belczynski, PRL **117**, 101102 (2016).

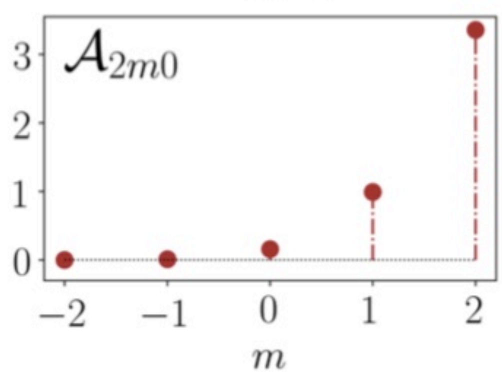
Amplitude of mode excitation depends on geometry



The nature of the final “whack” as the two black holes come together determines how loudly each of these modes rings out.



So a coalescence in which the orbit is aligned with black hole spin rings with one tone …



From Hughes et al, arXiv:1901.05900

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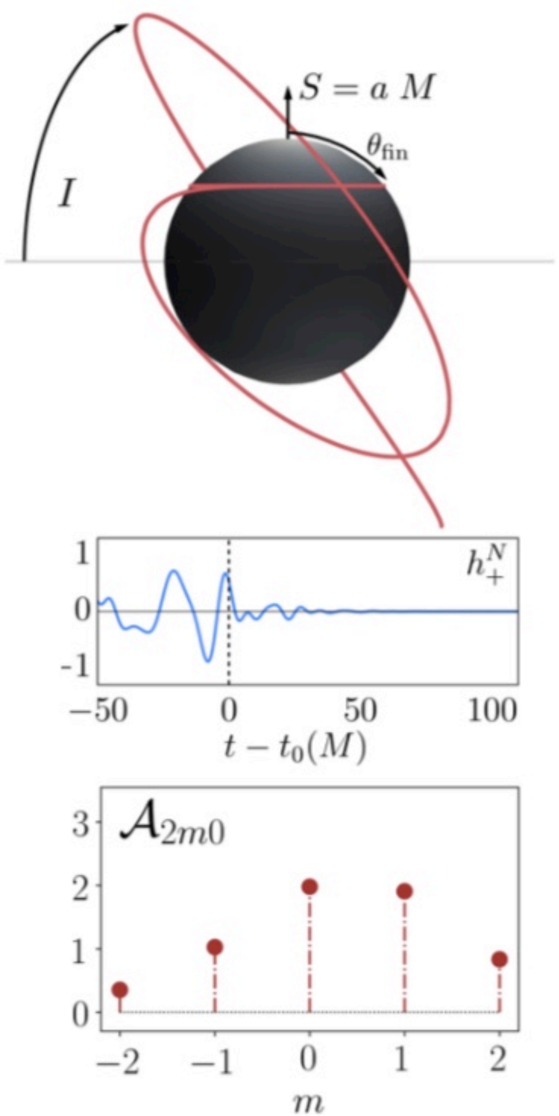
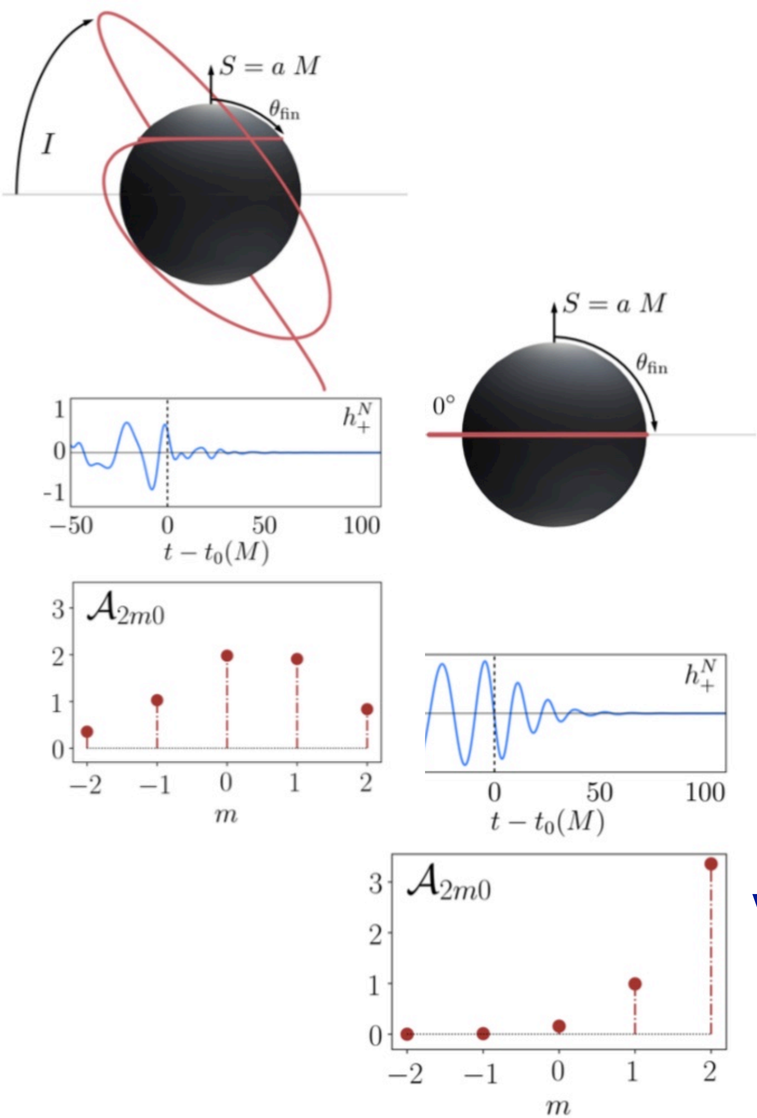


The nature of the final "whack" as the two black holes come together determines how loudly each of these modes rings out.

… which is quite different from one in which the orbit and the spin are highly misaligned.

From Hughes et al, arXiv:1901.05900

Amplitude of mode excitation depends on geometry

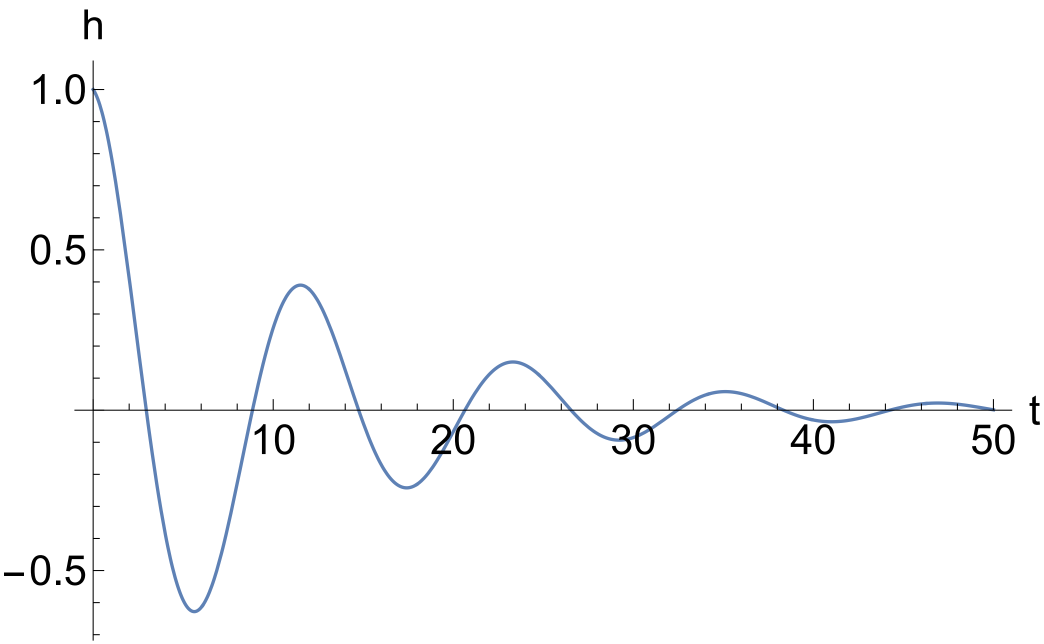


Opportunity: more details in the waveform that allow us to assess nature of system, provide more "handles" into source physics.

CAUTION: Easy to confuse modes with each other! An example of how systematic errors can skew what we infer from measurement.

Black holes ring badly

Although we describe black holes as "ringing like a bell," it must be emphasized that they are bells with *terrible* quality factors.



Example: The longest-lived mode for a coalescence similar to GW150914:

$Q \approx 6.6$

"Black holes don't ring; they thud."
(L.S. Finn, circa 1996)

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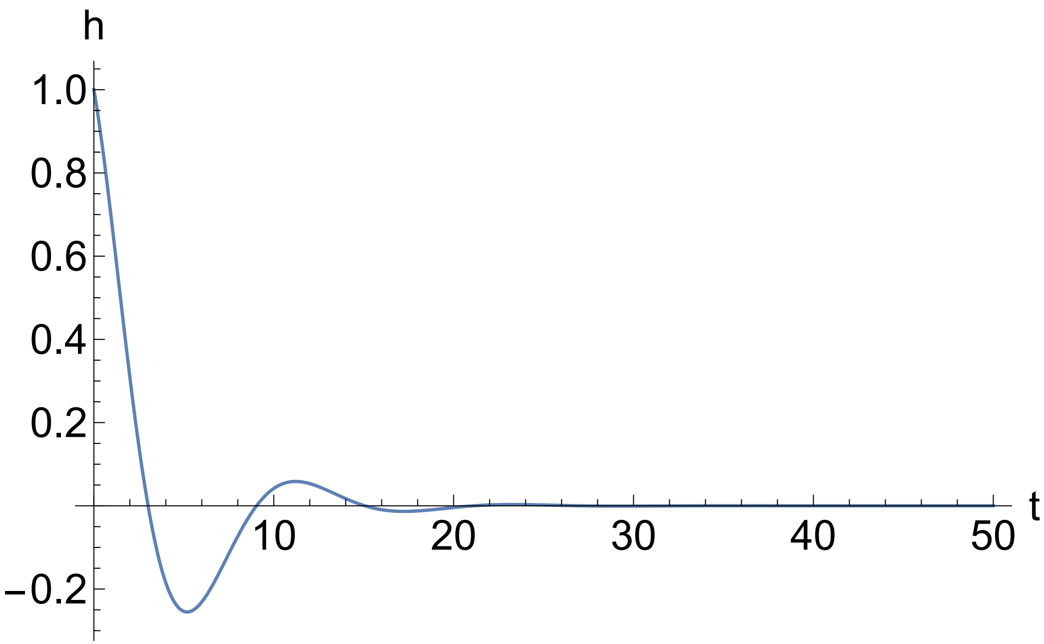

A very closely related mode (the first "overtone") that, in principle, should also be excited: Has very similar angular behavior, but different radial behavior:

$$Q \approx 2.1$$

Black holes ring badly

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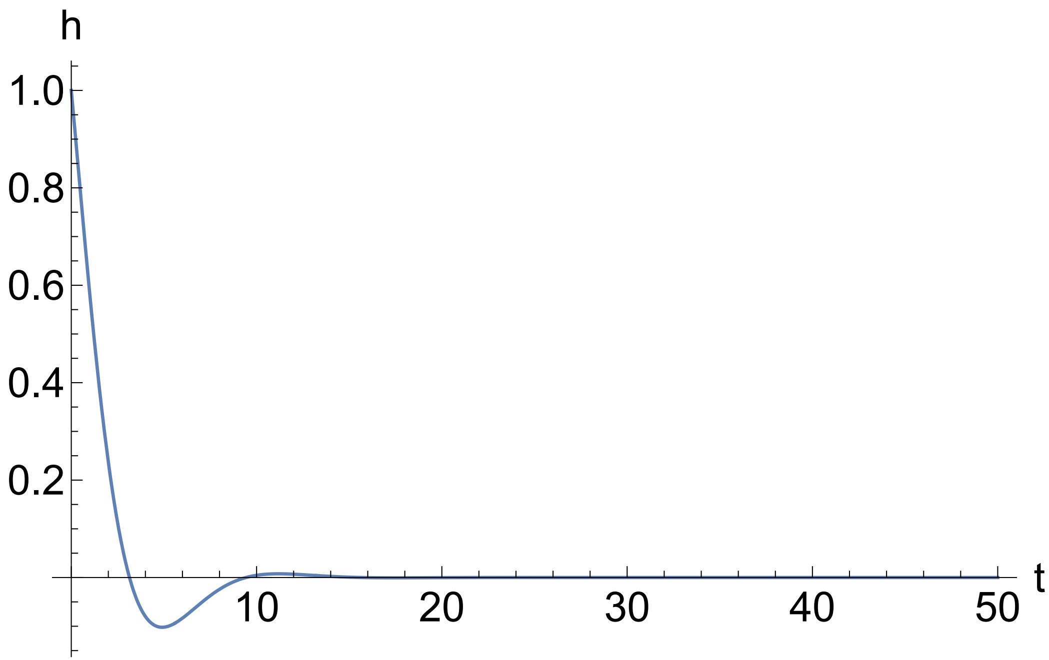


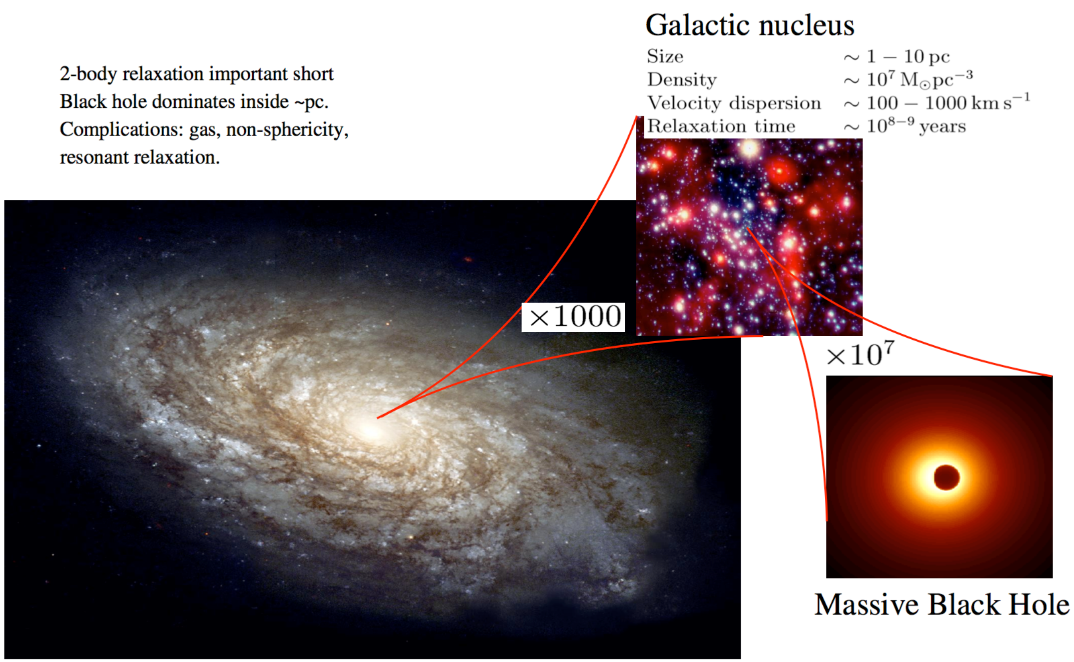
And the next such overtone — also present in principle, but is even more rapidly damped:

$$Q \approx 1.2$$

A lot of strong gravity physics in these modes, but difficult to identify them with certainty.

Extreme mass ratio inspirals

Astrophysical setting: Center of a “normal” galaxy. Typically hosts a black hole of 10^6—10^7 Msun; black hole in a nucleus with ~10^9 Msun of stars.



Graphic courtesy of Marc Freitag

The most massive of these stars tend to sink closest to the large black hole; these stars evolve through main sequence quickly, leave remnant stellar mass black holes behind.

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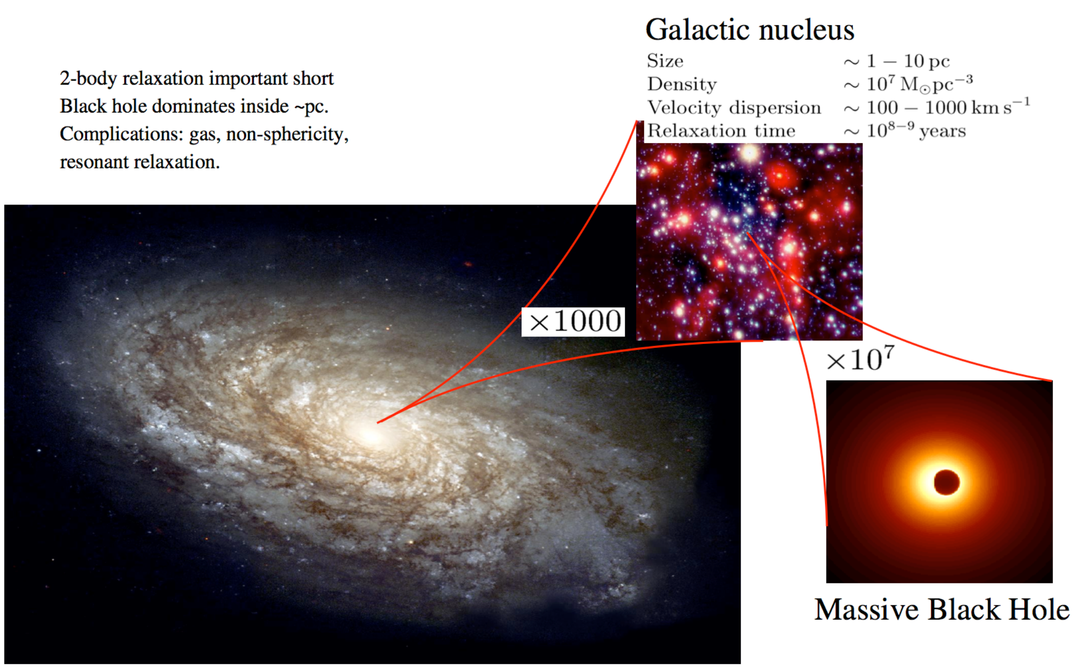

Graphic courtesy of Marc Freitag

Multi-body scattering in nucleus puts stellar onto an orbit that evolves into a strong-gravity, GW-driven inspiral in band

$$\frac{c^3}{50GM} \lesssim f \lesssim \frac{c^3}{GM}$$

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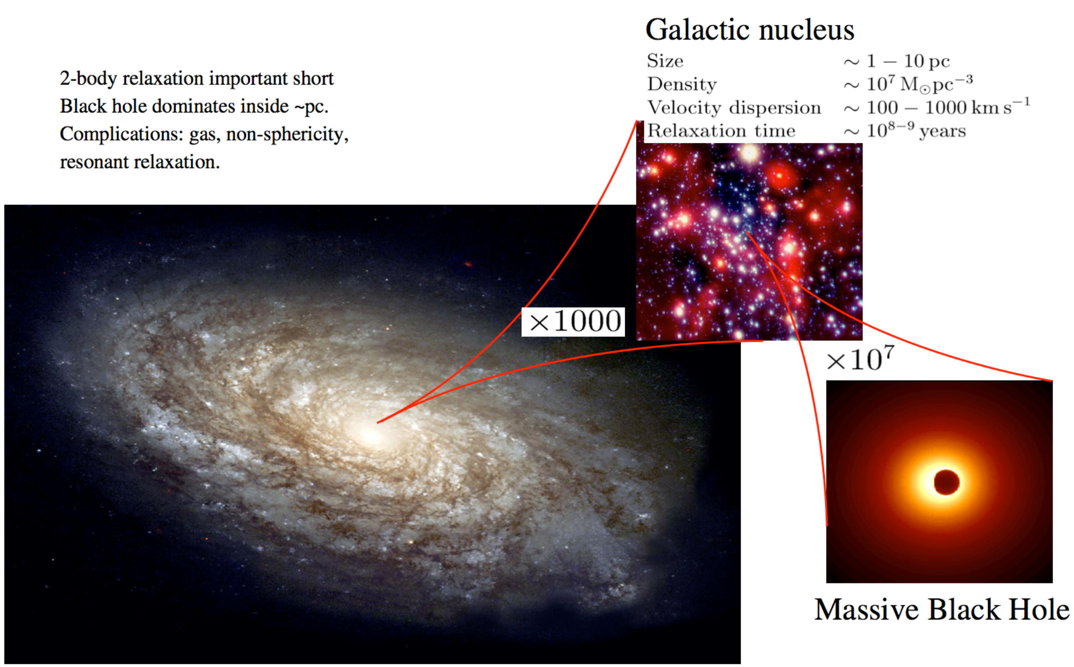


Graphic courtesy of Marc Freitag

Multi-body scattering in nucleus puts stellar onto an orbit that evolves into a strong-gravity, GW-driven inspiral in band

$$0.004\,\mathrm{Hz} \lesssim f \lesssim 0.2\,\mathrm{Hz}$$

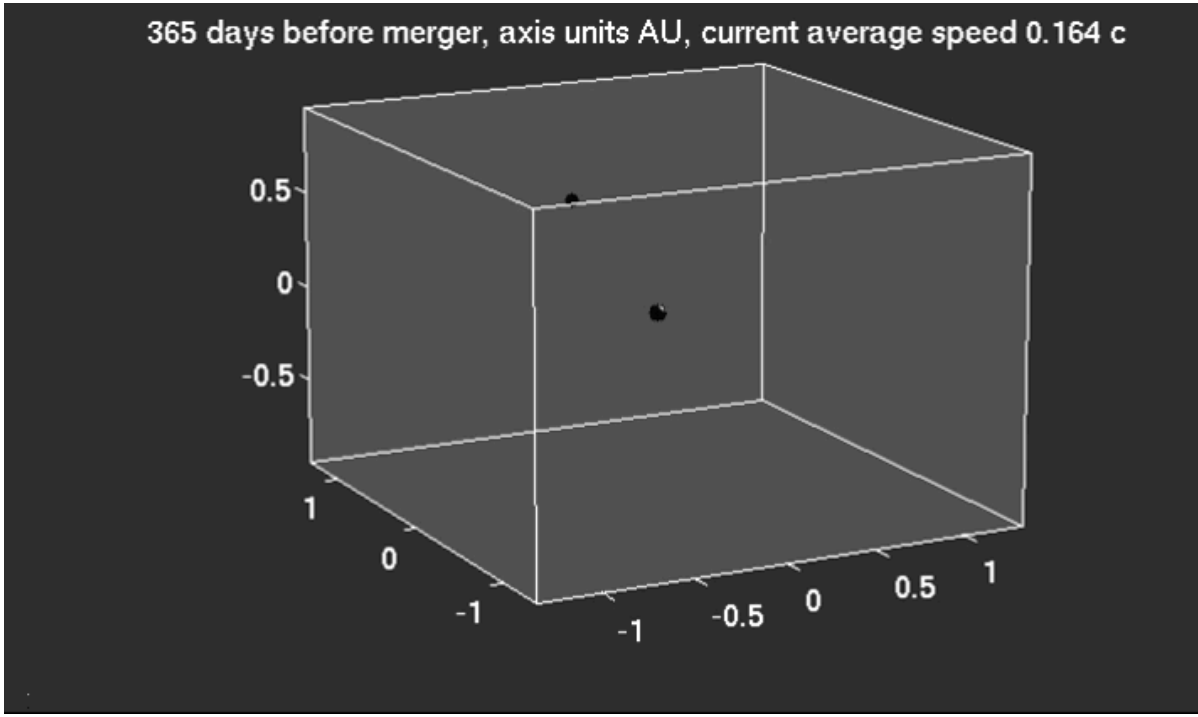
at 10^6 Msun

LISA band!

Character of an "EMRI"

Smaller body executes thousands of slowly evolving orbits in near-horizon region of large black hole … GWs they generate are sensitive to the nature of the near-horizon black hole spacetime.

If we can coherently track these GWs, can use them to measure spacetime properties; expect measurement errors to scale as $1/N_{orb}$ and 1/(signal to noise).



Modeling

Large mass ratio of these binaries makes it possible to model such binaries *perturbatively* … yielding equations that can be solved with high precision.

$$g_{\alpha\beta} = g^{\rm Kerr}_{\alpha\beta}(M,a) + h^{(1)}_{\alpha\beta} + h^{(2)}_{\alpha\beta} + \ldots$$

Compute corrections order by order in system's mass ratio. Motion of small body looks like a geodesic of $\boldsymbol{g}^{\rm Kerr}$ plus corrections that arise from perturbations $\boldsymbol{h}^{(n)}$:

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = f^\alpha$$

"Self force" — correction to geodesic black hole orbits due to ***h*** terms.

Also the limiting form of the *general* two-body problem in general relativity.


Moo.

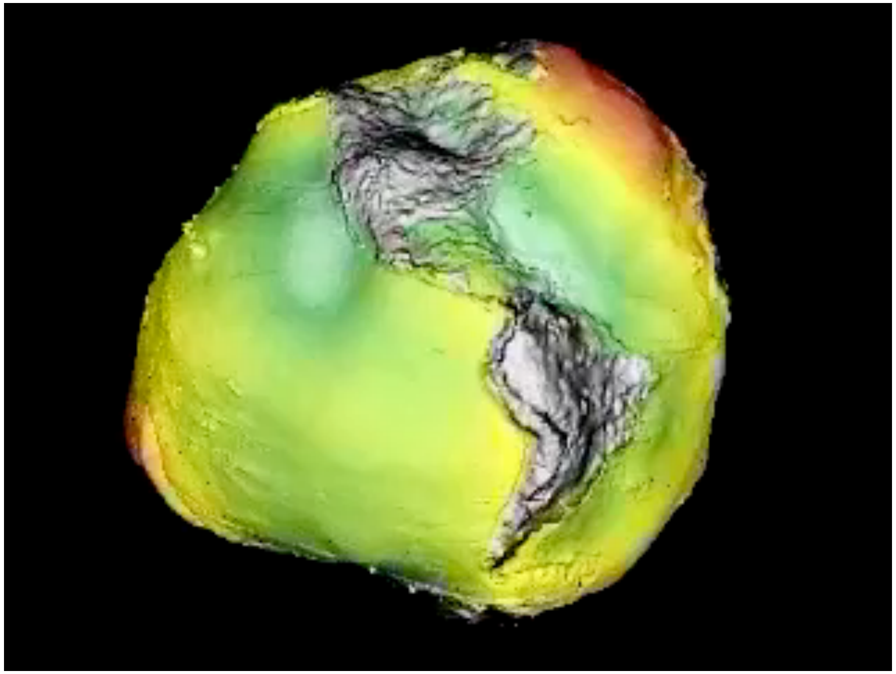
Probe of BH spacetime

Thanks to large mass ratio, EMRIs function as *nearly* a test particle probe of black hole spacetimes.

Measurements akin to *geodesy*: Inspiral is evolution through a sequence of orbits … orbits maps potential; enforce the gravitational field equation

$$\nabla^2 \Phi_g = 4\pi G \rho_m$$

and infer mass distribution.



GRACE gravity model

$$\Phi_g = -\frac{GM}{r} + \sum_{l=2}^{\infty} \sum_{m=-l}^{l} M_{lm} Y_{lm}(\theta, \phi)$$

Probe of BH spacetime

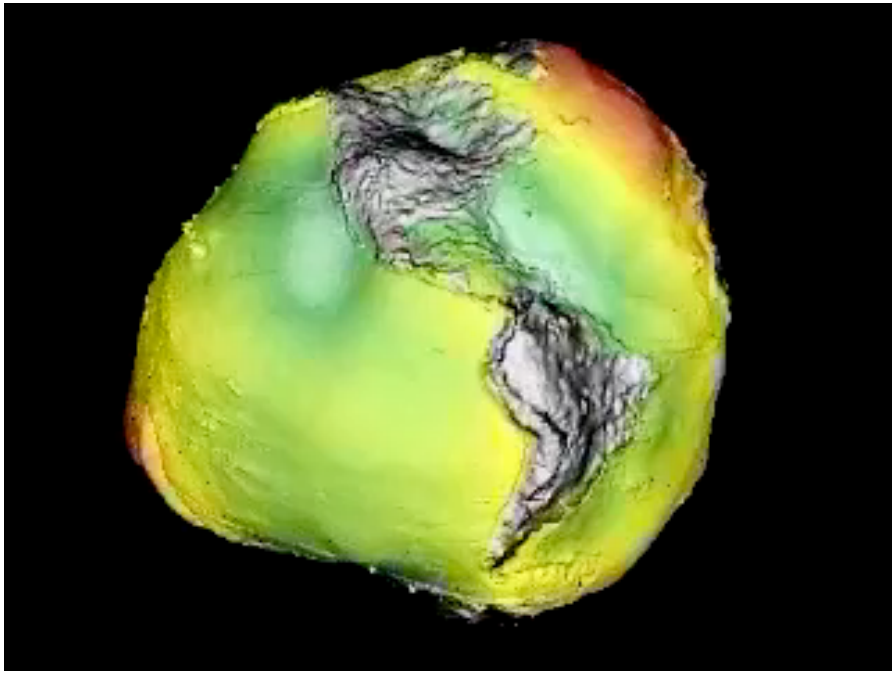
Thanks to large mass ratio, EMRIs function as *nearly* a test particle probe of black hole spacetimes.

"Bothrodesy": Mapping the multipoles that govern a black hole spacetime.

Kerr expectation: No non-zero *m* modes (axisymmetric)

Mass and current moments set by hole's mass and spin:

$$M_l + iS_l = M(ia)^l$$



GRACE gravity model

Probe of BH spacetime

Thanks to large mass ratio, EMRIs function as *nearly* a test particle probe of black hole spacetimes.

"Bothrodesy": Mapping the multipoles that govern a black hole spacetime.

Kerr expectation: No non-zero *m* modes (axisymmetric)

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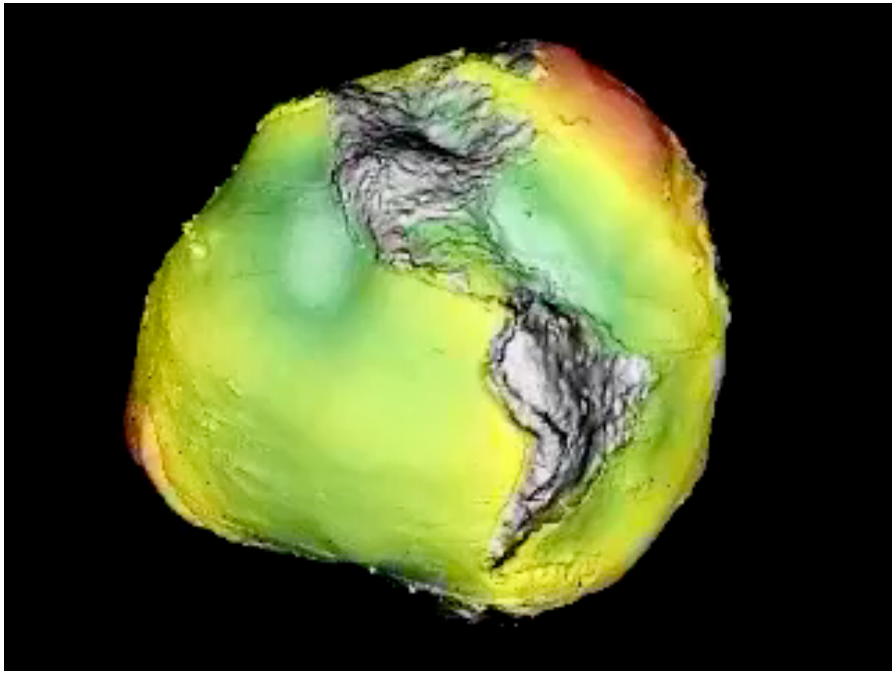
$$M_l + iS_l = M_0(iS_1/M_0)^l$$



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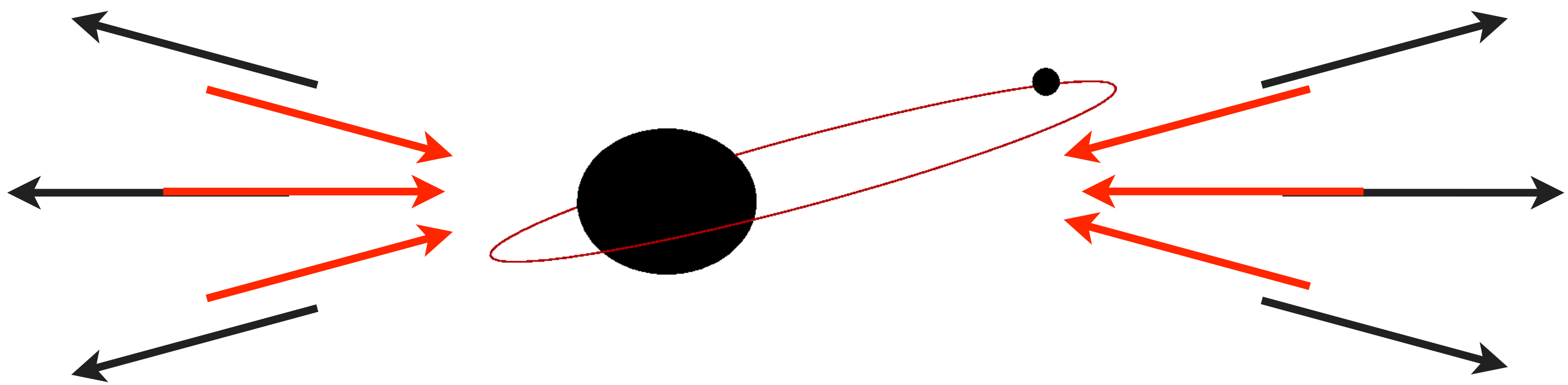
Can we "deform" black hole spacetimes in way that introduces testable deviations from Kerr without introducing pathologies that poison the analysis?

Physics of inspiral

Bothrodesy essentially uses each orbit in the sequence … what do we get from the whole inspiral?

Waves from binary have two pieces: Flux to infinity, and flux down the black hole's event horizon.



Flux to infinity has simple behavior:

$$\left(\frac{dE}{dt}\right)^{\infty} > 0$$

Always takes energy away from the binary.

Physics of inspiral

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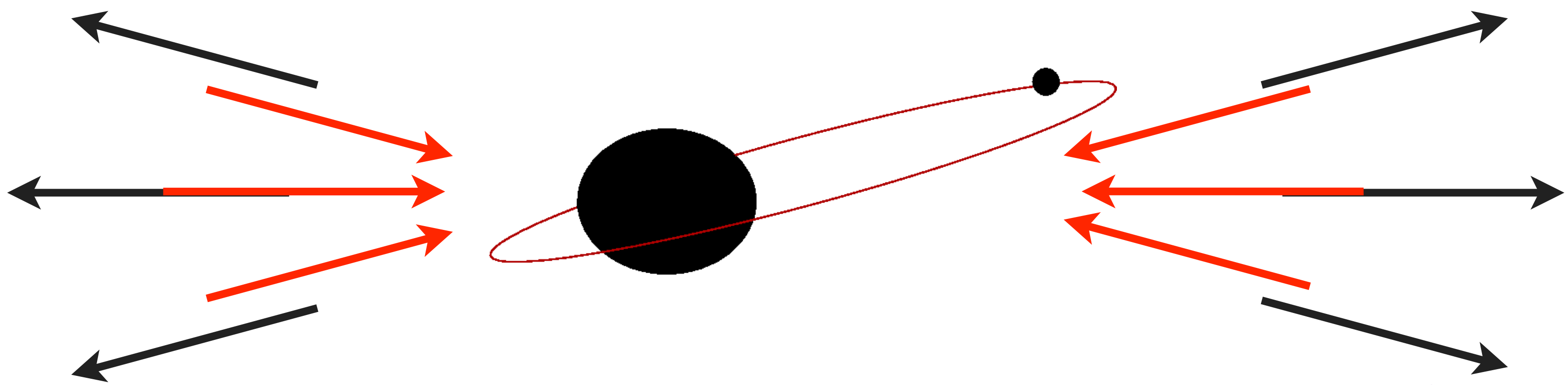

Sign of horizon flux depends on ***relative*** frequency of orbit and BH spin.

$$\left(\frac{dE}{dt}\right)^{\rm H} > 0$$

If

$\Omega_{\rm orb} > \Omega_{\rm H}$

Takes energy away if orbit is faster than hole's spin …

Physics of inspiral

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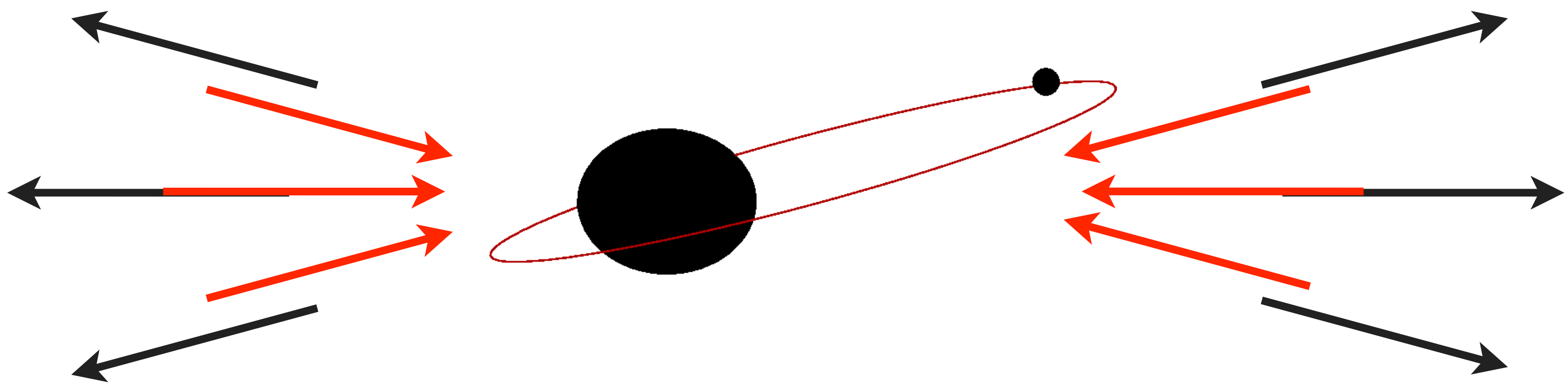


Sign of horizon flux depends on ***relative*** frequency of orbit and BH spin.

$$\left(\frac{dE}{dt}\right)^{\mathrm{H}} < 0$$

If

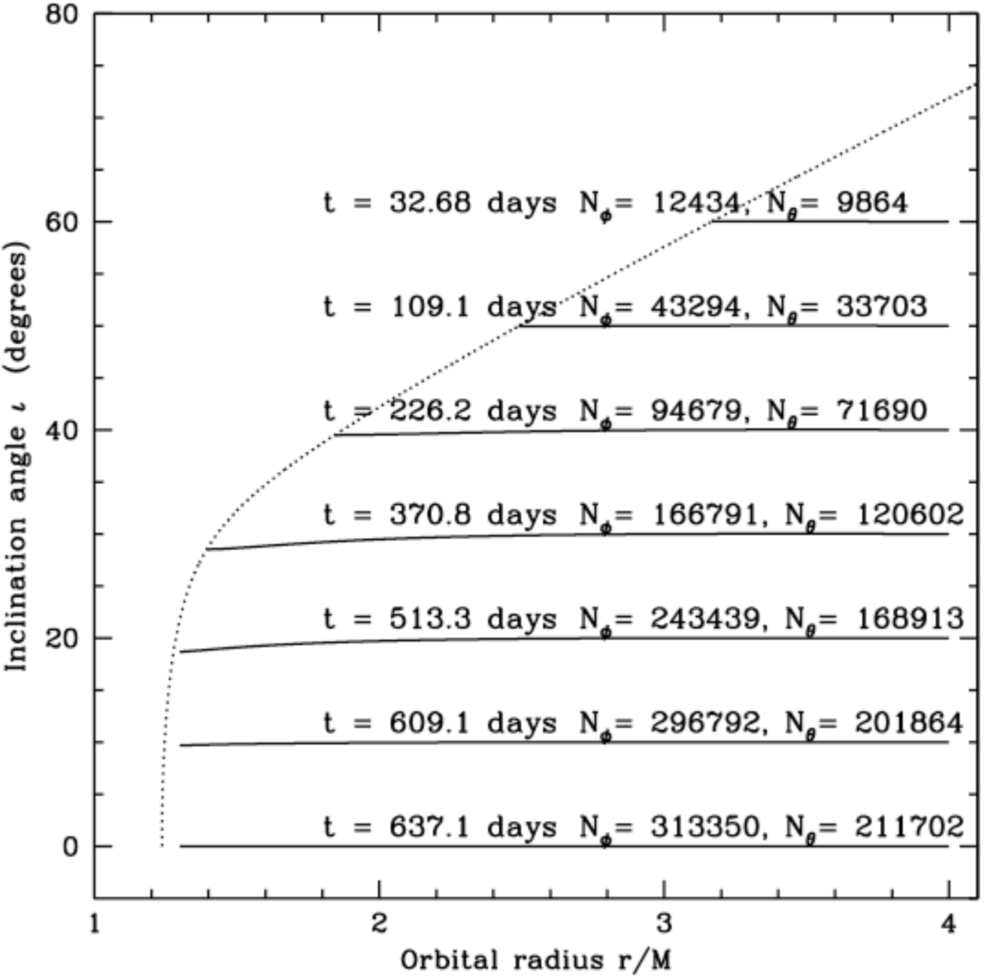
$\Omega_{\mathrm{orb}} < \Omega_{\mathrm{H}}$

… but ***adds energy to orbit*** if hole spins faster than orbit.

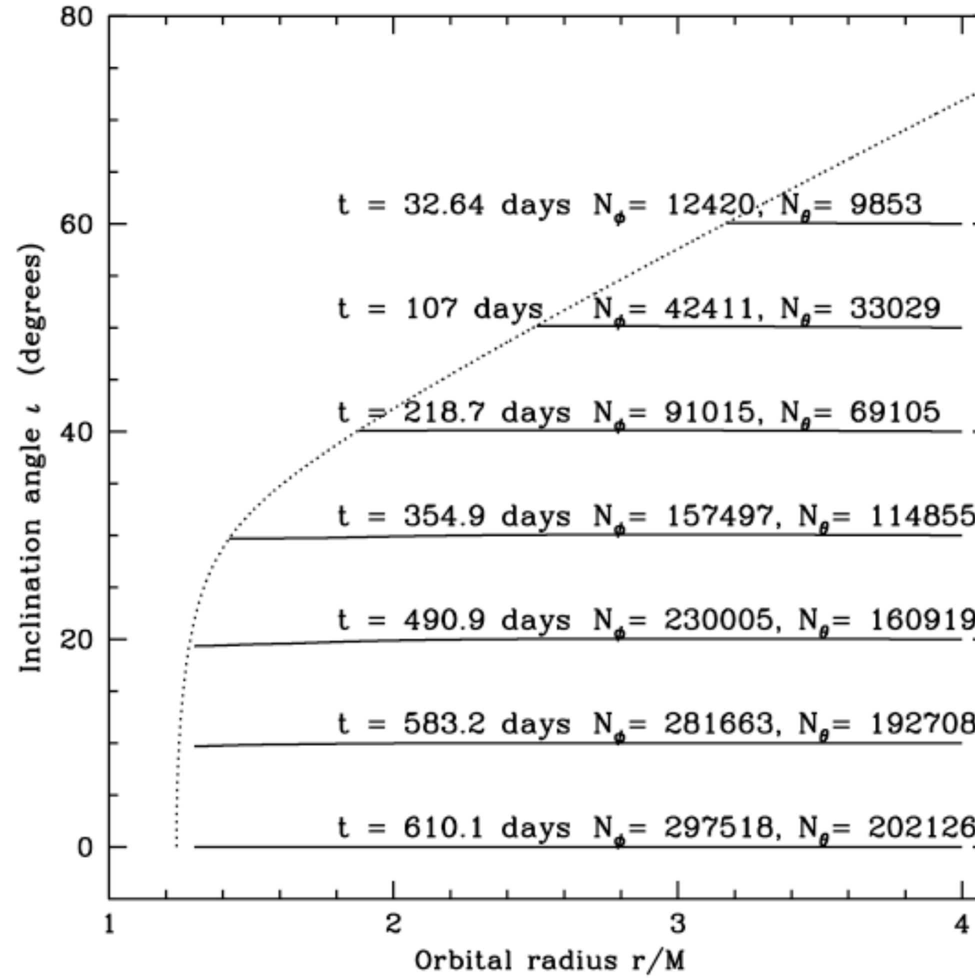
Not a small effect!

Example of inspiral into a very rapidly spinning BH:

With horizon flux



Without



Horizon flux *slows* inspiral by up to 4 weeks

Now digging into how to take these simple, clean ideas from "principle" to "practice" … and learning details that make things *much* messier.

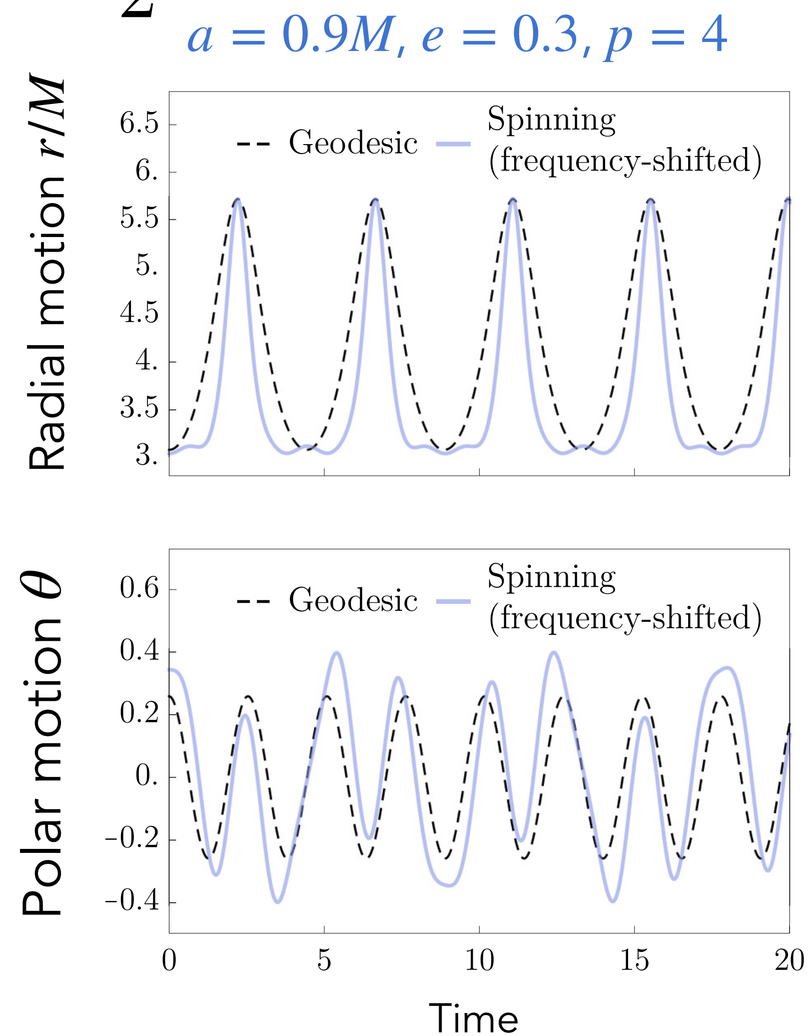
Example: Small body spin. Binary's secondary is not a featureless point mass! Spin couples to spacetime; The body precesses, and feels additional "forces":

$$\frac{DS^{\alpha\beta}}{d\tau} = 0 \qquad\qquad F^{\alpha}_{\mathrm{S}} = -\frac{1}{2} R^{\alpha}{}_{\nu\lambda\sigma} u^{\nu} S^{\lambda\sigma}$$

Forces change the orbits

$$\frac{DS^{\alpha\beta}}{d\tau} = 0 \qquad F^{\alpha}_{\mathrm{S}} = -\frac{1}{2} R^{\alpha}{}_{\nu\lambda\sigma} u^{\nu} S^{\lambda\sigma}$$

When we include this effect, it adds significant new harmonic structure to the orbital motion, and thus to waveforms.



[From Drummond and Hughes, arXiv:2201.13335; PRD, in press.]

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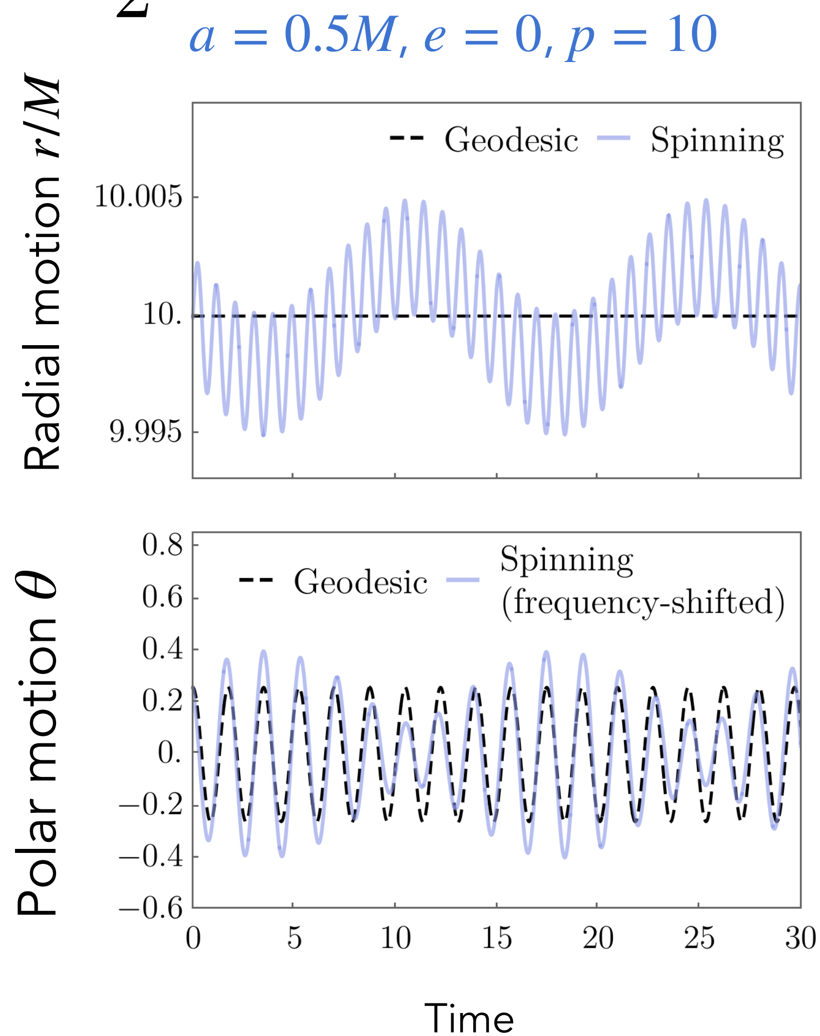
These changes are competitive with "effects of "strong gravity" that we hope to probe … must be carefully included in our models.



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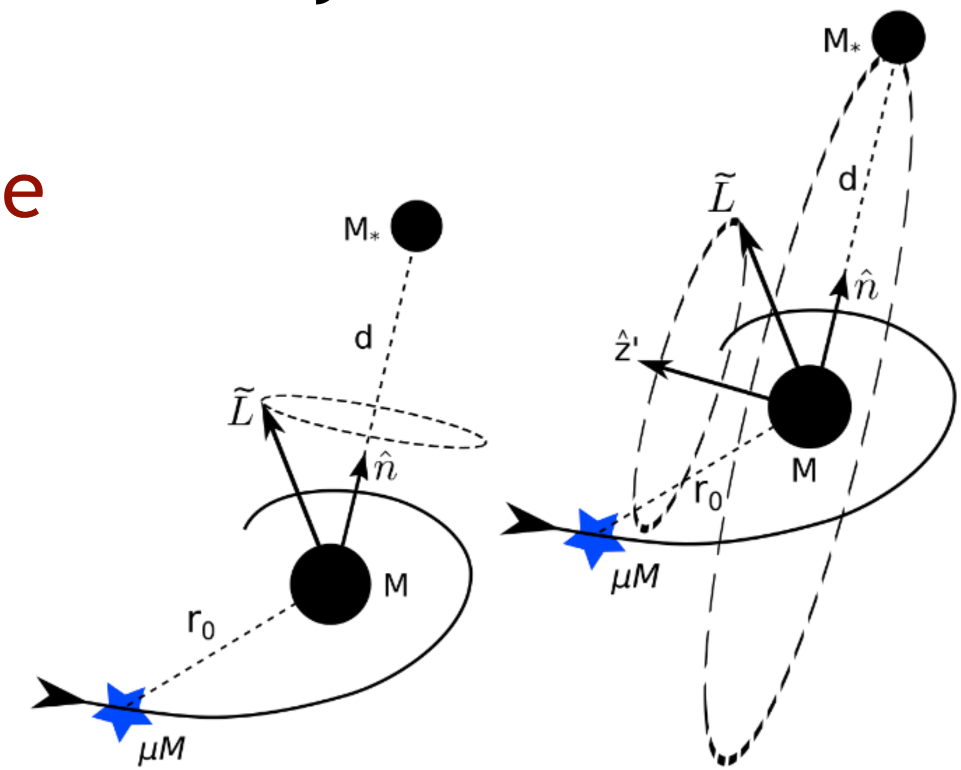
Another example

Real binaries don't exist in an empty universe: There will be other bodies in their vicinity, and those bodies affect the binary.

Yang & Casals (2017): What happens if a 10^7 Msun black hole is 0.1 parsecs from an EMRI?



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Another example

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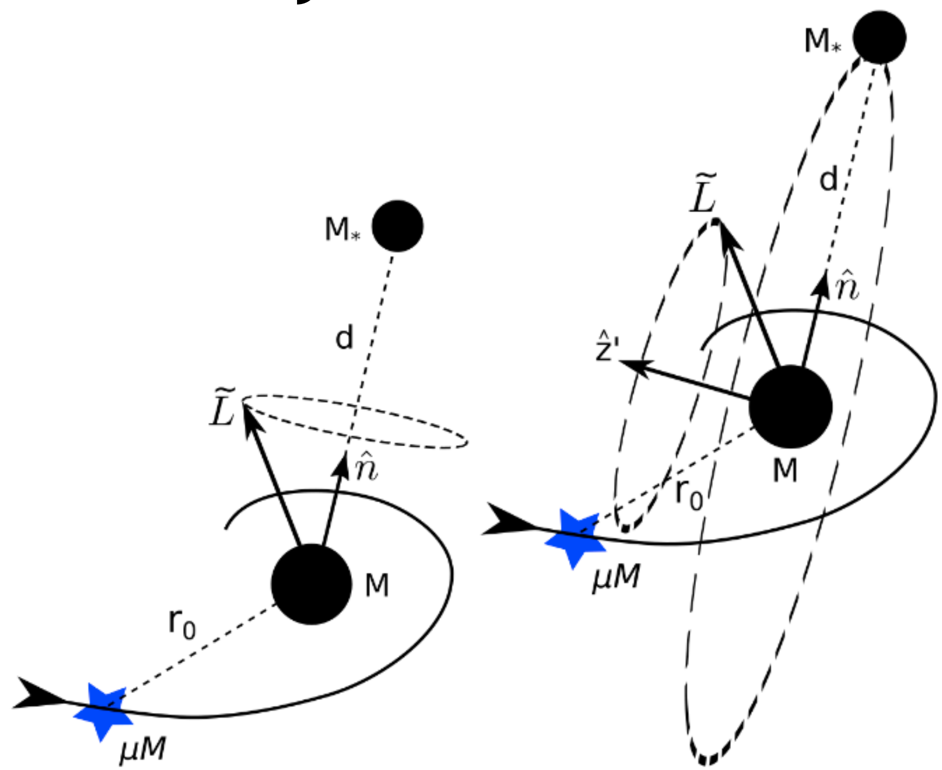
Answer: 3rd body breaks spacetime axisymmetry, tidally squeezes binary.

Tidal effects "*almost always*" average out … except near orbits where r and φ motions are in a low order resonance. Kicks the binary, driving precession of EMRI's orbit.



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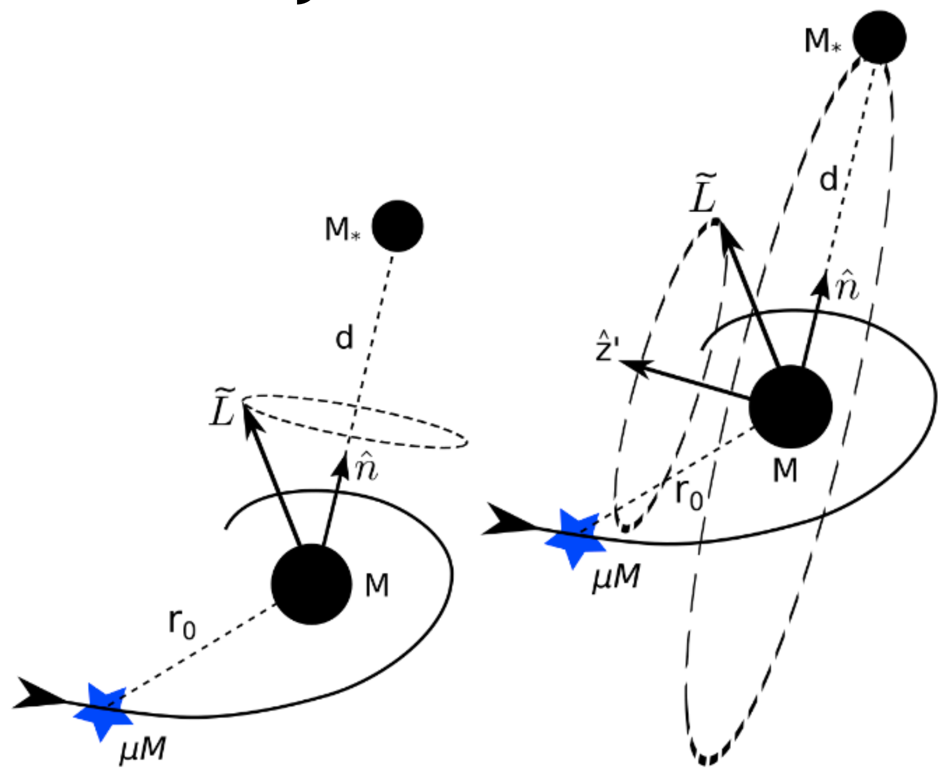
Another example

Real binaries don't exist in an empty universe: There will be other bodies in their vicinity, and those bodies affect the binary.

Tide from a 10^7 Msun body at 0.1 parsecs is identical to tide from a 10 Msun body at 0.001 parsecs … 0.001 parsecs = 1 light day.



Our galactic center has several stars of this mass which come within a light day of Sgr A*.

Another example

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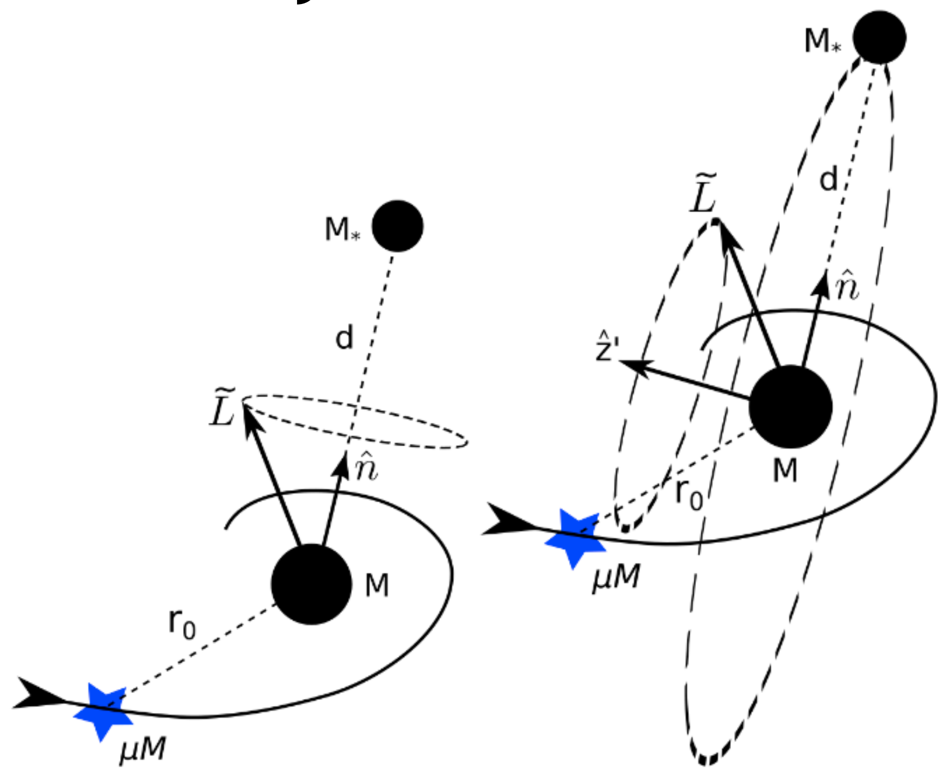
Punchline: We need to account for tides from various stellar-mass bodies in galactic nucleus near the EMRI!



B. Bonga, H. Yang, and S. A. Hughes, PRL **123**, 101103 (2019); arXiv:1905.00030
P. Gupta, B. Bonga, A. J. K. Chua, and T. Tanaka, PRD **104**, 044056 (2021); arXiv:2104.03422
P. Gupta, L. Speri, B. Bonga, A. J. K. Chua, and T. Tanaka, arXiv:2205.04808


Moo.

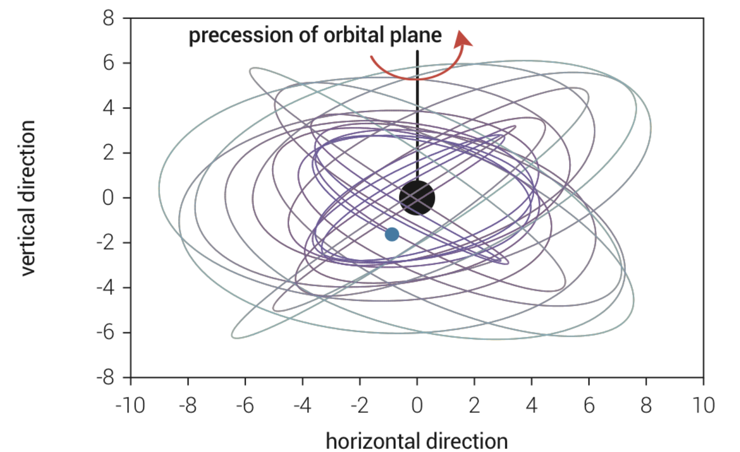

Moo.

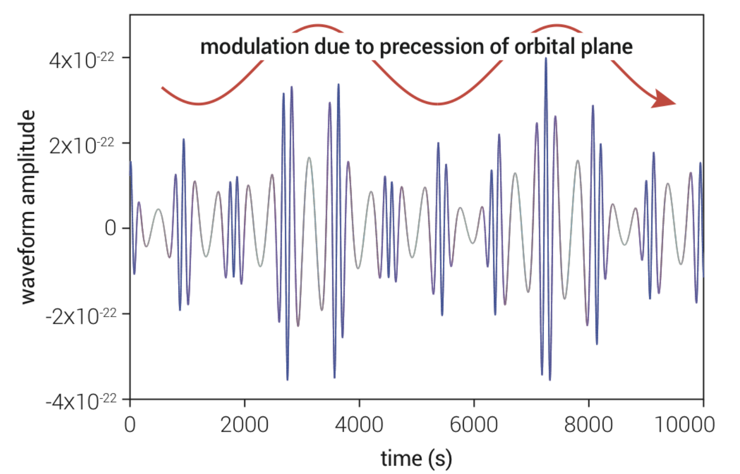
Conclusion

In the past 6 years, we have observationally probed strong gravity more effectively and thoroughly than ever in human history.

The next generation of facilities promises to continue pushing these probes, giving us ***high precision*** data on the strongest gravitational domains.

Theoretical modeling will be ***key*** to success in this endeavor.





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Get your students started on these problems early!!