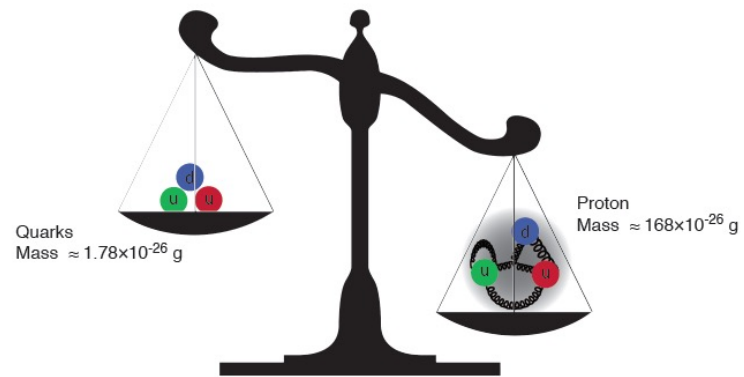


The glue that binds us all



Raju Venugopalan
Brookhaven National Laboratory

ICASU Inaugural Symposium, Urbana, May 19-21, 2022

Outline

QCD: the power and the glory

Emergent many-body phenomena in the QCD landscape

A classical lump and its quantum descendants in QCD at high energies

Universal features: examples in cold atoms and gravity

Quantum Chromodynamics (QCD)

- ◆ QCD - “nearly perfect” fundamental quantum theory of quark and gluon fields

F.Wilczek, hep-ph/9907340

- ◆ Theory is rich in symmetries: “Symmetries dictate interactions” – C.N Yang

$$\underbrace{SU(3)_c}_{\text{i}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\text{ii}} \times \underbrace{U(1)_A \times U(1)_B}_{\text{iii}}$$

- i) Gauge “color” symmetry: unbroken but confined
- ii) Global “chiral” symmetry: exact for massless quarks
- iii) Baryon number and axial charge (m=0) are conserved
- iv) Scale invariance of quark (m=0) and gluon fields
- v) Discrete C,P & T symmetries

- ◆ Chiral, Axial, Scale and (in principle) P & T broken by vacuum/quantum effects - “emergent” phenomena
- ◆ Inherent in QCD are the deepest aspects of relativistic Quantum Field Theories (confinement, asymptotic freedom, anomalies, spontaneous breaking of chiral symmetry)

Modern theory of the strong force: Quantum Chromodynamics

First QCD paper: Gell-Mann, Fritsch, Leutwyler (1973)



New Quantum Numbers
The Eightfold Way / Unitary Symmetry
Mesons ~ 8 Baryons $\sim 8 + 10$
Fundamental Representation Absent



The Quarks: Fractional Charge Triplets
Are They Real? (Constituents of Hadrons)
Are They Just a Mathematical Shorthand?
Relationship to Weak Currents?



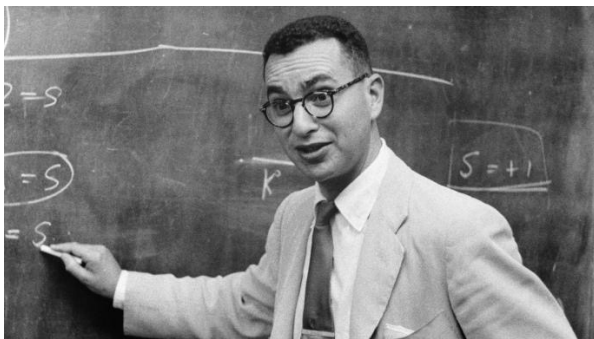
Thinking About Real Quarks –
Spin/Statistics Problem \rightarrow Parafermions
Color (New SU(3)!) – More Shorthand?
Still No Dynamics; Confinement a Mystery



Asymptotic Freedom \rightarrow Quarks = Partons
Promotion of Color to the Essence of
Strong Dynamics; Gluons a Color 8
QCD the Theory of Strong Interactions

From Gell-Mann's 8-fold way to QCD: A lepidopteral metaphor

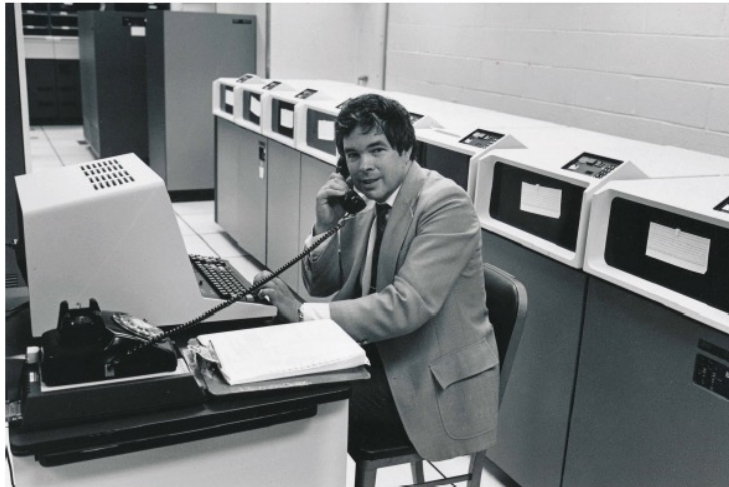
Jeffrey Mandula, Creutz-Fest 2014, BNL



M. Gell-Mann



Numerical realization: Lattice QCD



Kenneth G. Wilson

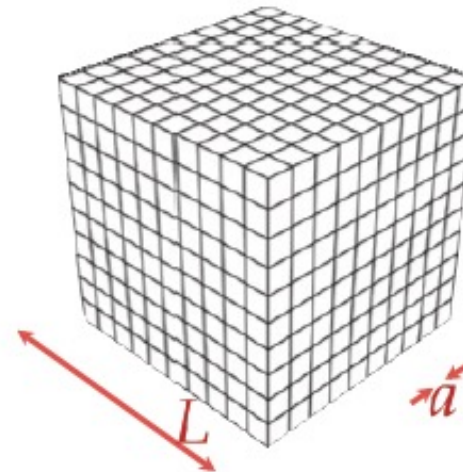
Lattice regularization (UV&IR) of QCD

First principles treatment of static properties of QCD: masses, moments, thermodynamics at finite T (& μ_B ?)

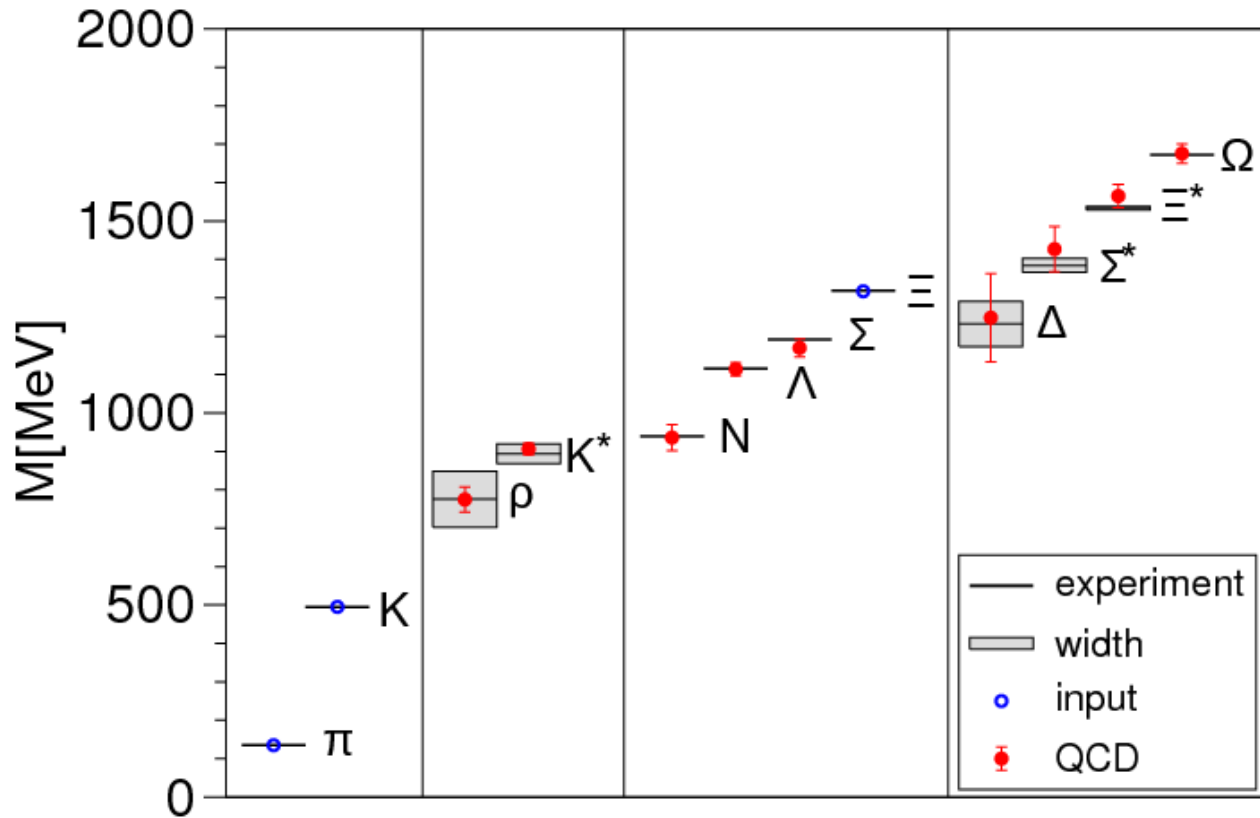


(1982)

CUBIC LATTICE



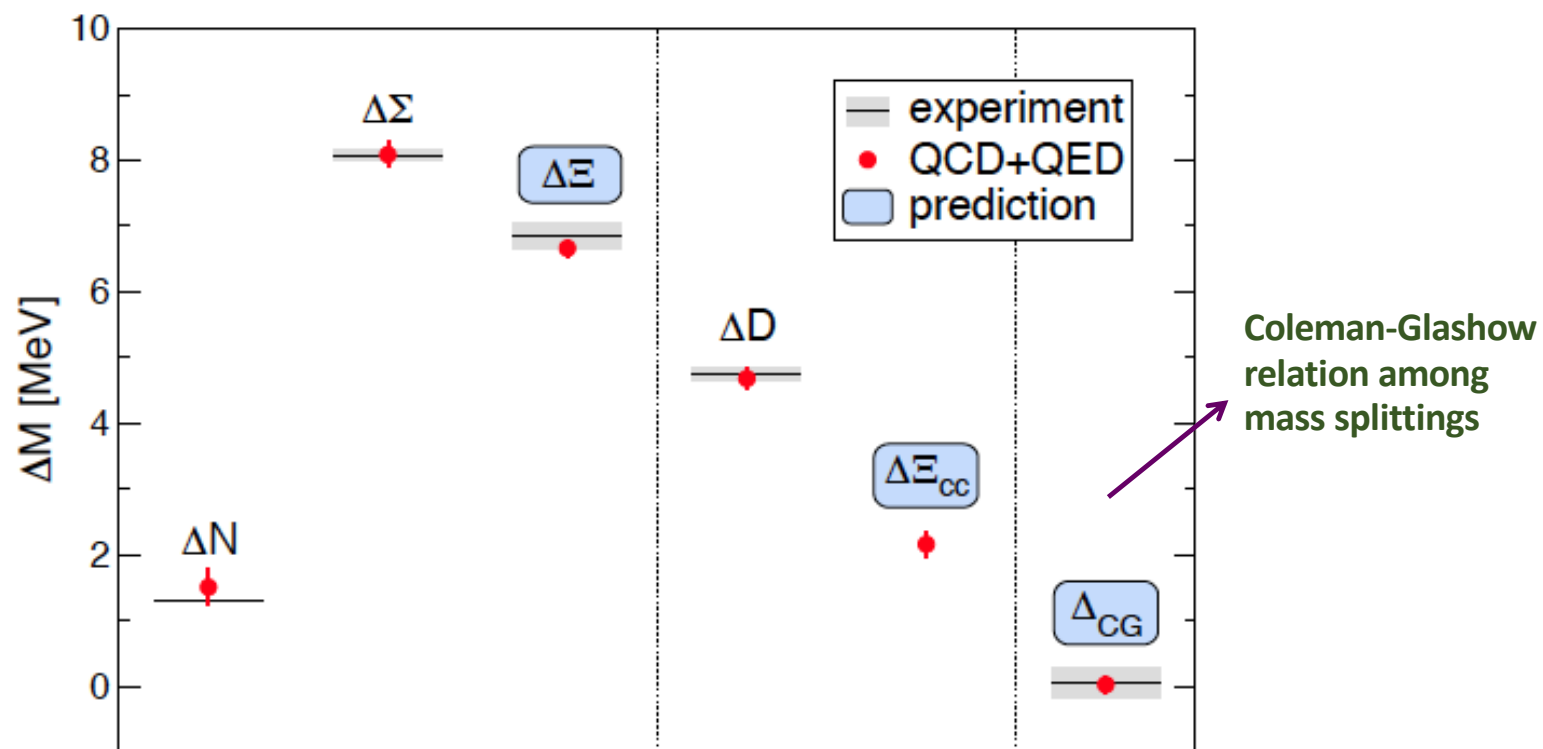
Precision QCD on the lattice



Durr et al., Science, 322 (2008)



Precision QCD+QED on the lattice

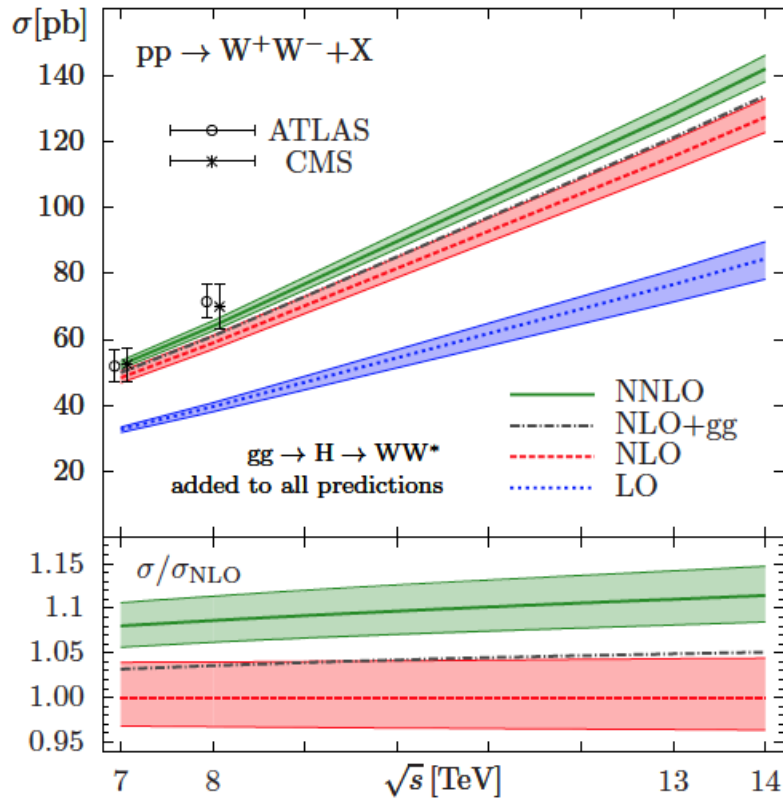


Budapest-Marseille-Wuppertal (BMW) Coll., arXiv:1406.4088

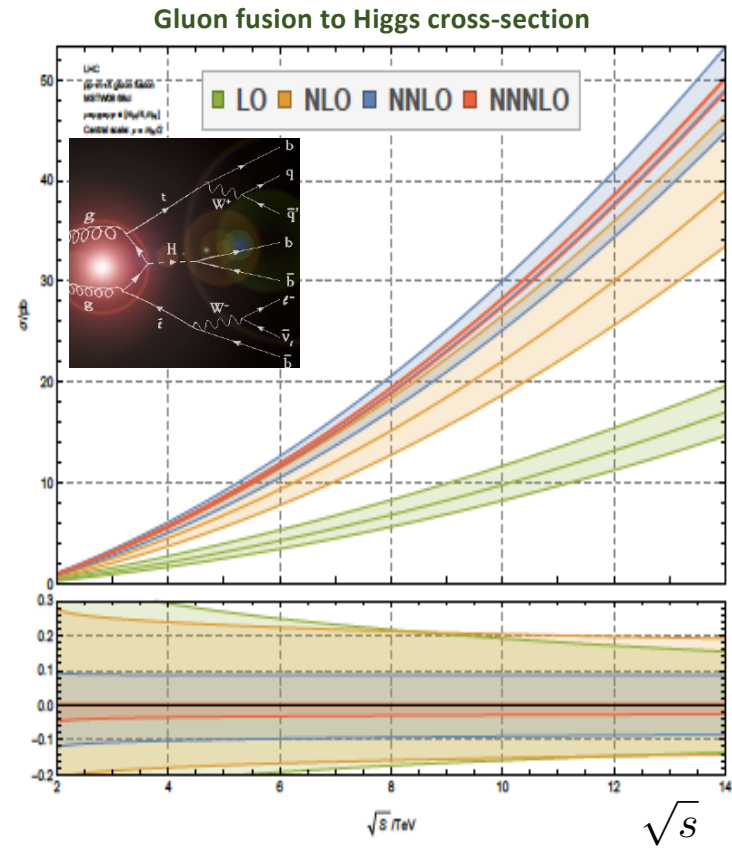
Noteworthy recent development: Constraining hadron matrix elements enabling discovery science with muon g-2

Muon g-2 Theory Initiative,
T. Aoyama et al., Phys. Repts. 887 (2020) 1

Perturbative QCD: benchmark for new physics



Gehrmann et al., arXiv:1408.5243



Anastasiou et al., arXiv:1503.06056

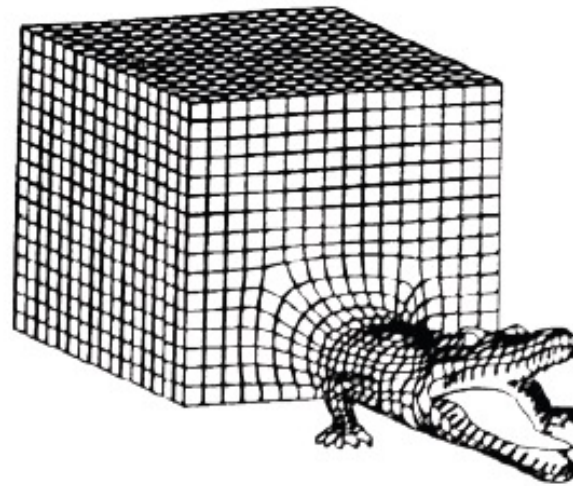
Are we done ?

The study of the strong interactions is now a mature subject - we have a theory of the fundamentals* (QCD) that is correct* and complete*.

In that sense, it is akin to atomic physics, condensed matter physics, or chemistry. The important questions involve emergent phenomena and “applications”.

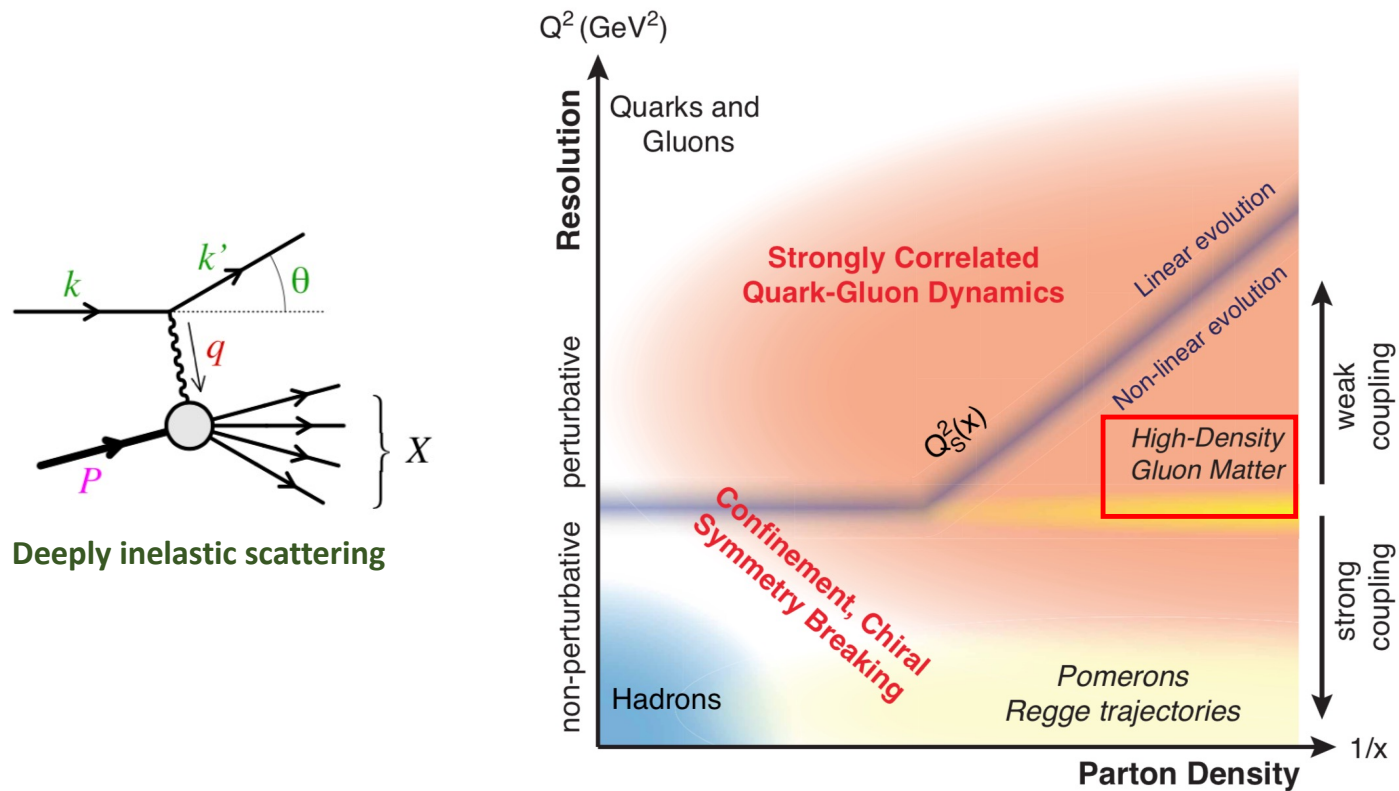
F. Wilczek , “Quarks (and Glue) at the Frontiers of Knowledge”, Talk at Quark Matter 2014

Emergent many-body phenomena in QCD at high energies



Don't try (yet) on the lattice...

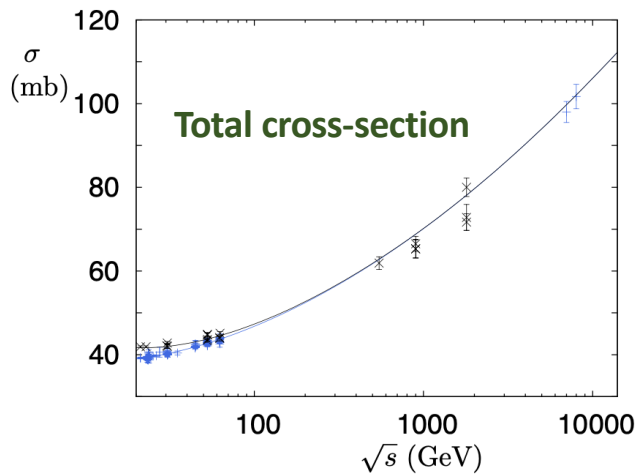
Landscape of QCD dynamics in resolution and energy



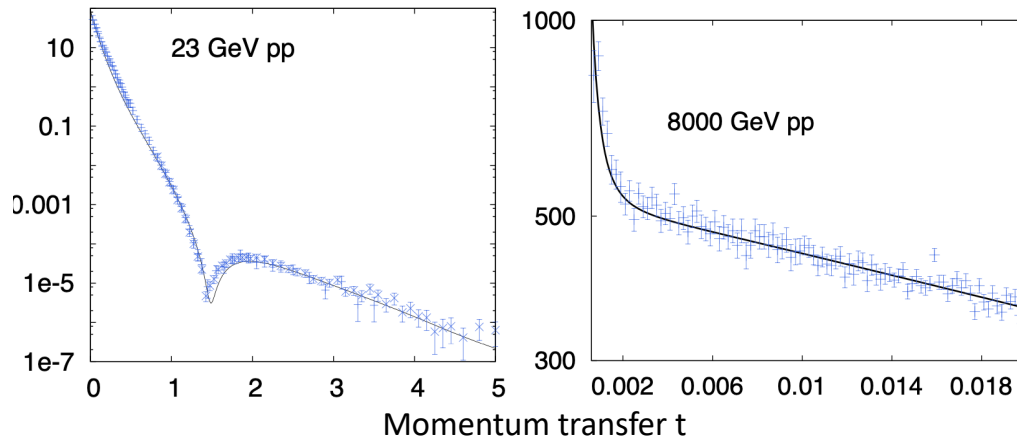
Aschenauer et al., arXiv:1708.01527
Rep.Prog. Phys. 82, 024301 (2019)

Many open questions: 3-D structure of quark-gluon structure of the proton, spin and orbital dynamics, many-body correlations, multi-particle production...

What is the Pomeron in QCD?



Differential elastic cross-section



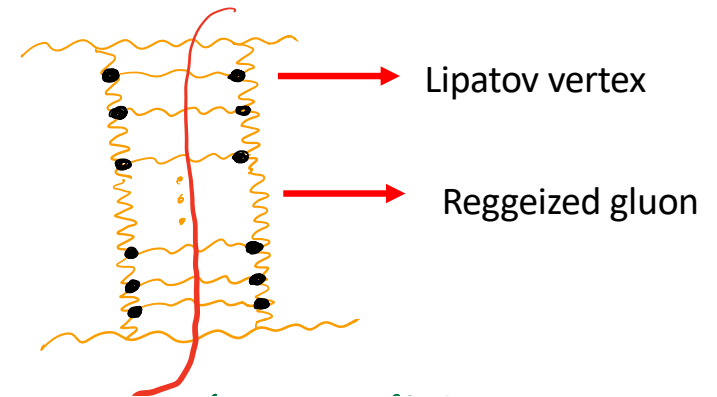
The behavior of total cross-sections over three orders of magnitude in energy (SPS \rightarrow LHC) is simply described in terms of effective Pomeron and Reggeon trajectories

Pomeron: t -channel exchange with vacuum quantum numbers dominates total hadron-hadron cross-sections:

Amplitude: $A(s, t) = s^{\alpha(t)}$ with $\alpha(t) = 1 + \varepsilon + \alpha' t$

Intercept $\varepsilon_P = 0.11$ String tension = 0.165 GeV^{-2}

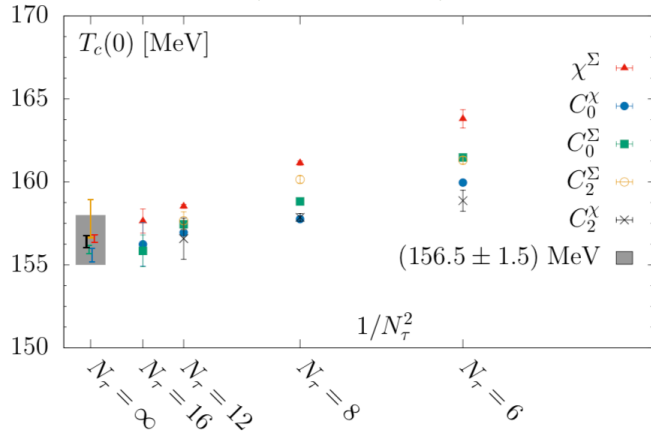
Donnachie, Landshoff, Phys. Lett. B750 (2015) 669



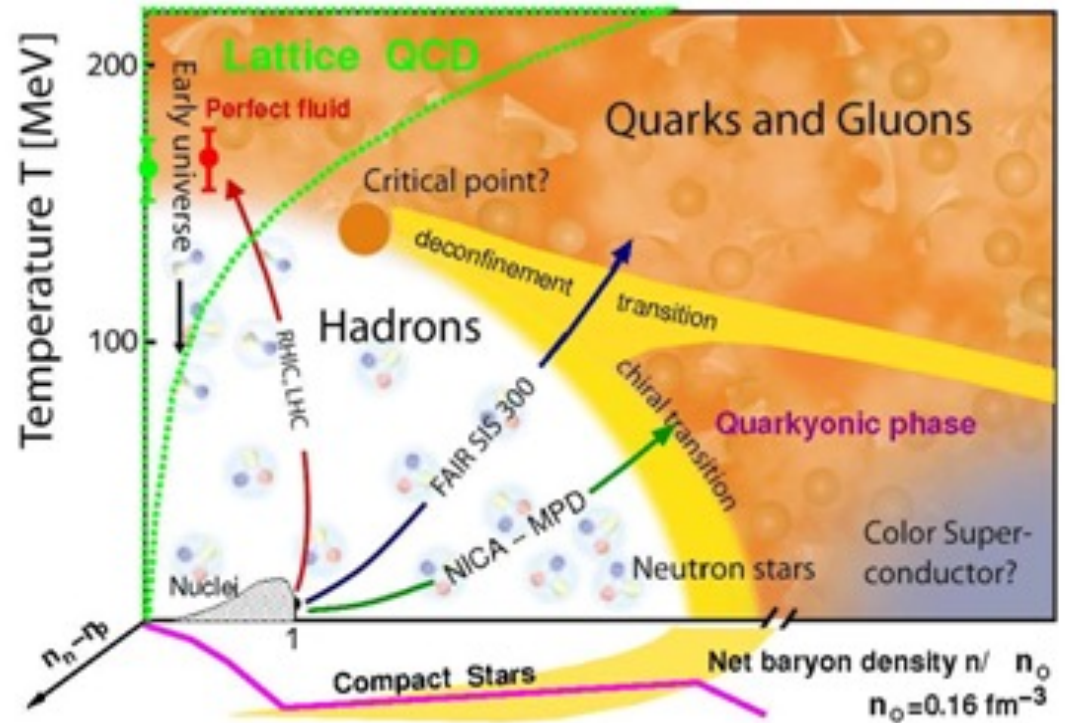
BFKL Pomeron- compound color singlet state of "reggeized gluons" with vacuum quantum numbers

The QCD phase diagram

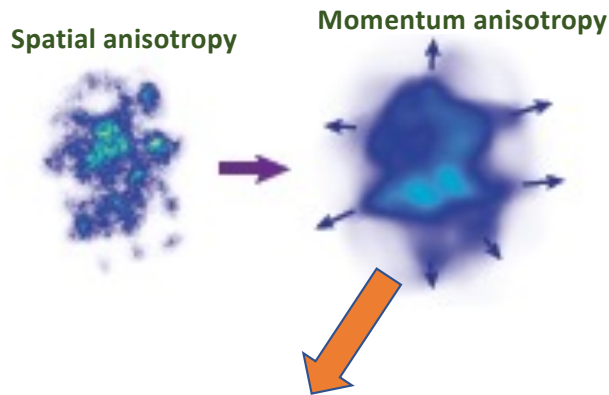
Hot QCD collaboration, Bazavov et al., arXiv:1812.08325



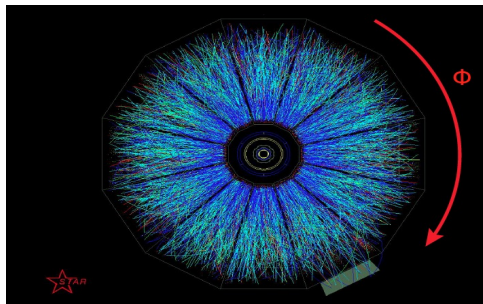
Cross-over temperature from hadron gas to quark-gluon plasma $T_c = 156.5 \pm 1.5$ MeV



The unreasonable effectiveness of hydrodynamics



$\sim 10^3$ subatomic particles produced in collisions of Gold nuclei at 200 GeV/nucleon



STAR detector @ BNL's Relativistic Heavy ion Collider (RHIC)

From the violence of a nuclear collision
...to the calm of a nearly perfect quark-gluon fluid, the **Quark-Gluon Plasma**



Initial state:
Far from equilibrium

*Non-equilibrium
dynamics*

Final state:
Thermal equilibrium

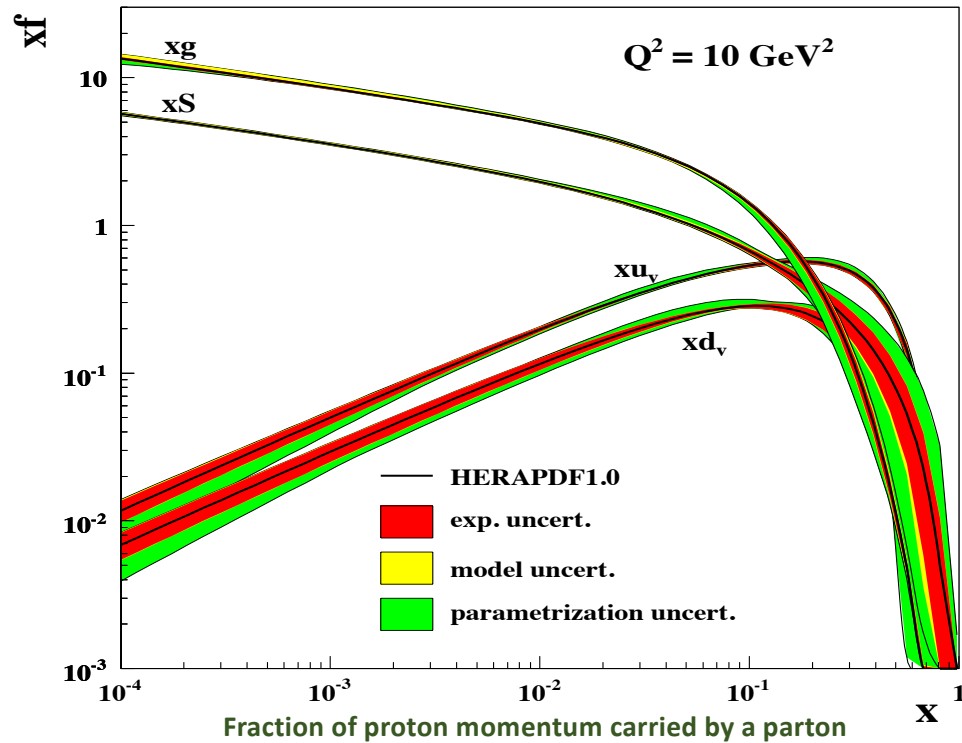
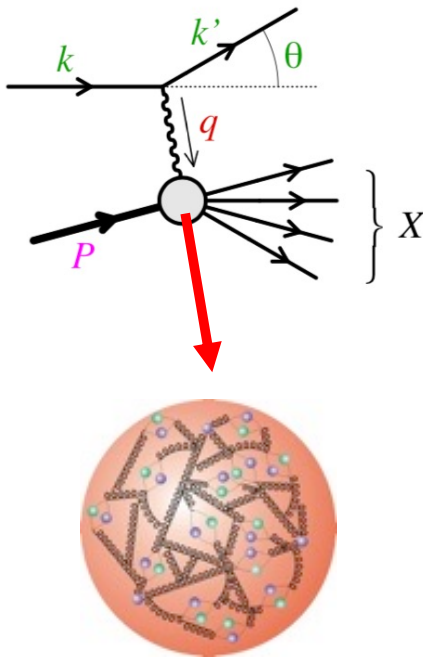


What is the thermalization process in heavy-ion collisions ?

Classical lumps and their quantum descendants in high energy QCD

The proton as a complex many-body system

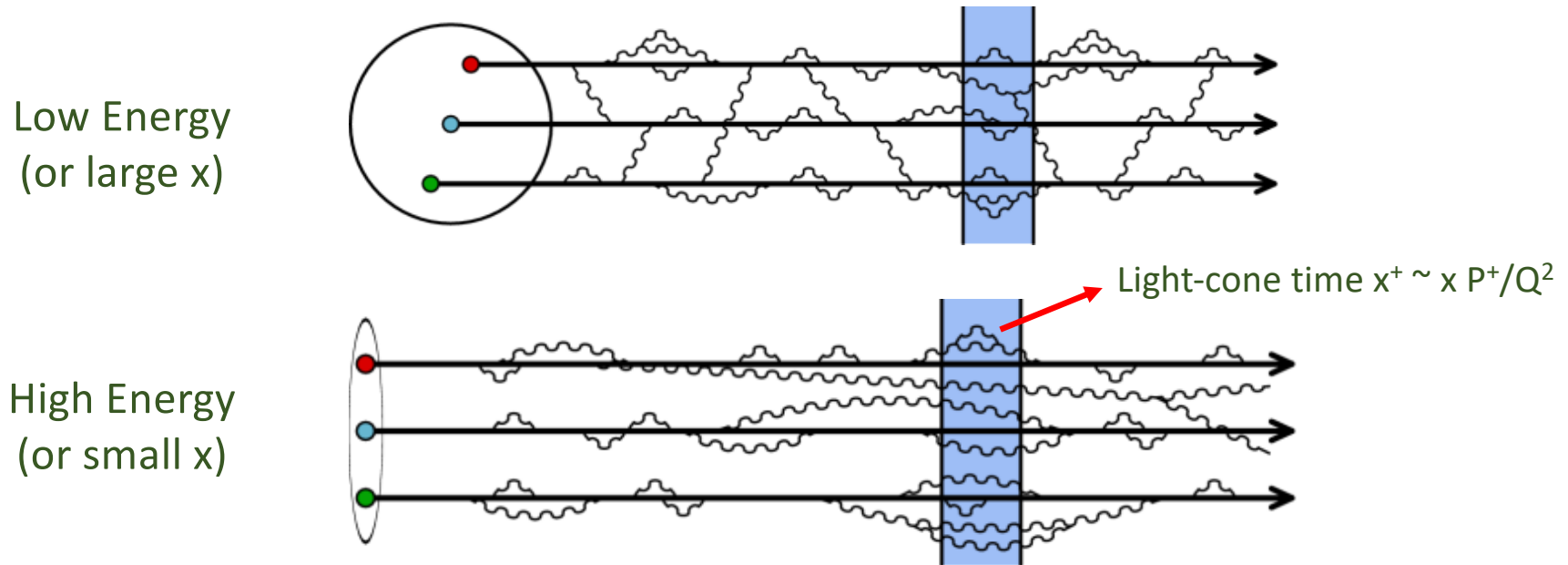
Deeply Inelastic Scattering



A key lesson from the HERA DIS collider:

Gluons and sea quarks dominate the proton wave-function at high energies

Lifting the veil: boosting the proton uncovers many-body structure

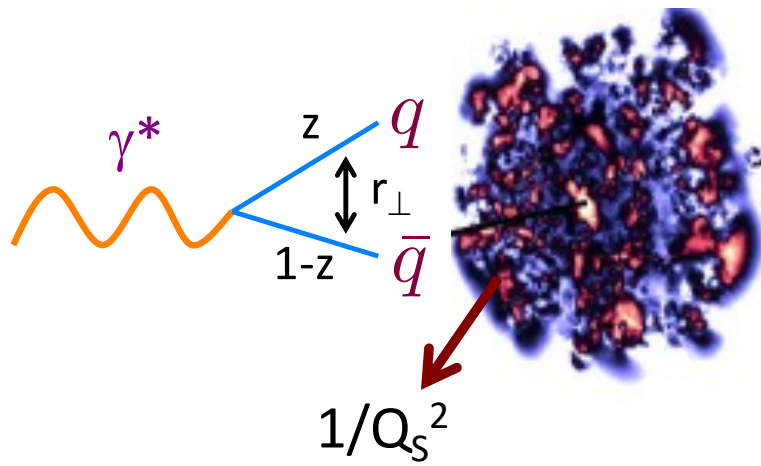


“Wee” parton fluctuations carrying a fraction $x \ll 1$ of proton’s momentum are time dilated on strong interaction time scales.

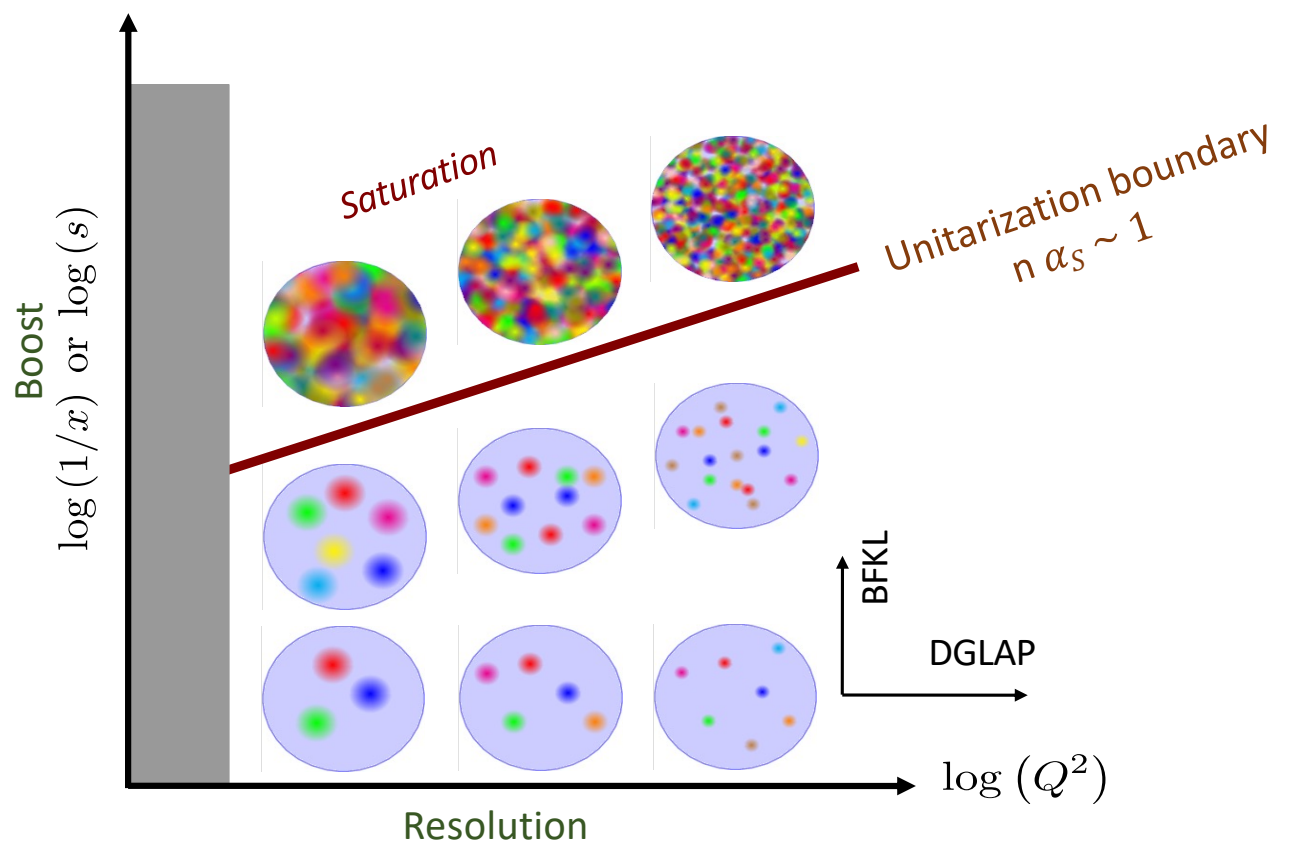
Long lived gluons radiate further small x gluons...Markovian process in $\log_e(x)$ generates power law growth of gluon distribution at small x

The boosted proton





Gluon saturation



At high occupancies, gluon proliferation tamed by many-body screening and recombination

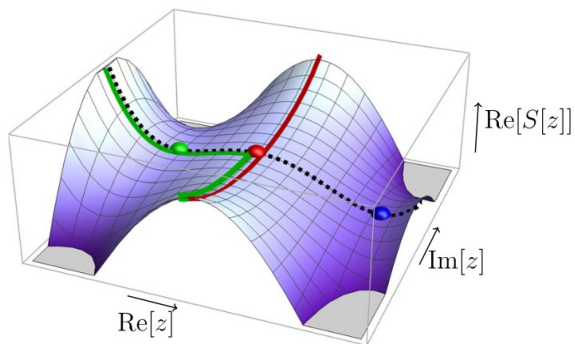
Gluon saturation characterized by emergent "close packing" scale

Formation of classical lumps of color charge that unitarize the cross-section for $r_\perp \approx 1/Q_S(x) \ll R_{proton}$

This scale is precocious in nuclei due to color coherence along path length of the interaction: $(Q_S^A)^2 \propto A^{1/3}$

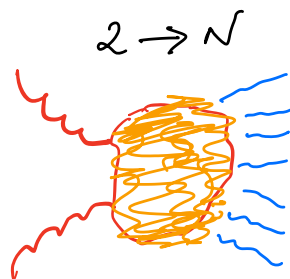
Classical lumps in $2 \rightarrow N$ scattering and unitarity

False vacuum decay of field configurations $\phi(z)$



Langer; Coleman; ...

Fig. from Andreassen, Frost, Schwartz, PRD97 (2018)



$$P_{2 \rightarrow N} \sim e^S \alpha_s^N N!$$

If $N \sim \frac{1}{\alpha_s}$

$$P_{2 \rightarrow N} \sim e^S \alpha_s^N \left(\frac{1}{\alpha_s}\right)^N e^{-1/\alpha_s}$$

Exponential suppression
of high occupancy states
(classical lumps) unless

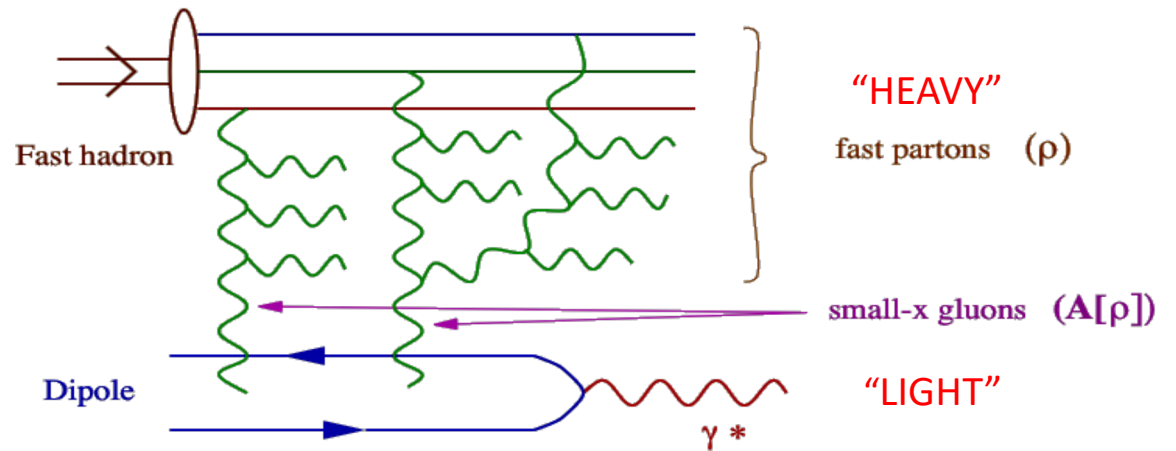
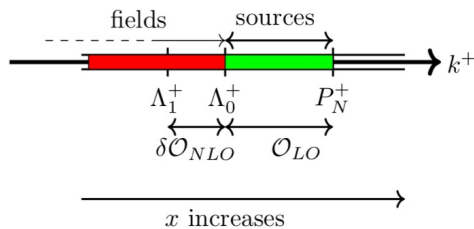
$$S \sim 1/\alpha_s \sim N$$

$$\Rightarrow P_{2 \rightarrow N} \sim O(1)$$

Classical EFT: the Color Glass Condensate

Born-Oppenheimer separation between fast and slow modes

Wilsonian RG describes their evolution with energy



CGC: classical Effective Field Theory of static parton sources and dynamical gluon fields

McLerran, RV (1994)
Iancu, Leonidov, McLerran (2001)

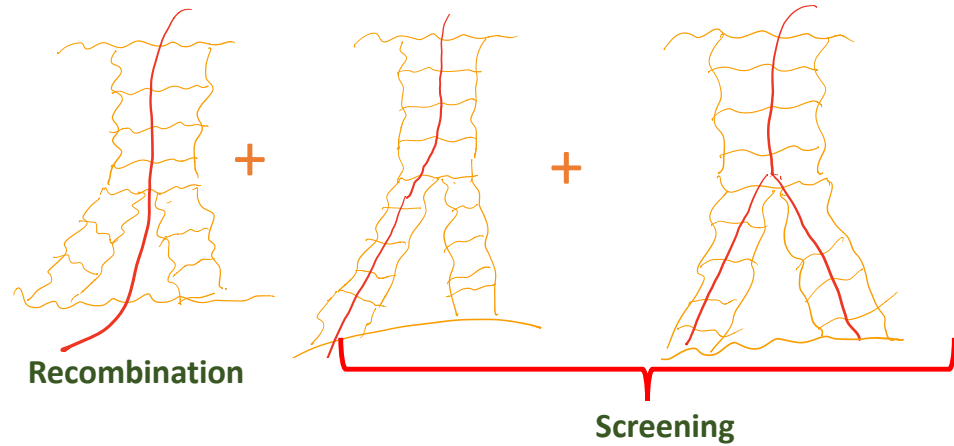
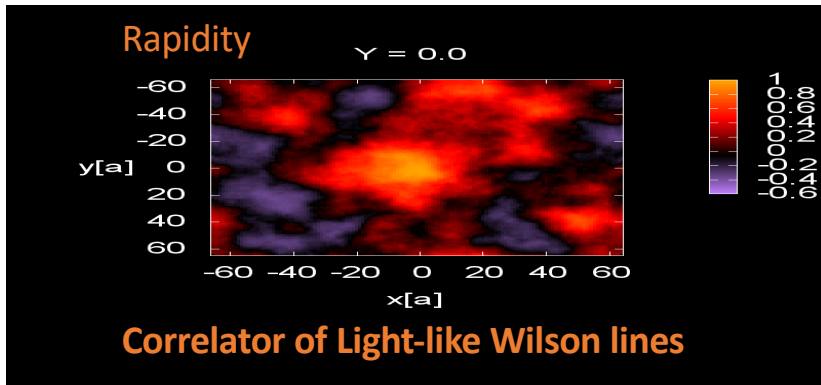
Color: self-evident

Glass: stochastic dynamics of static sources

Condensate: Highly occupied fields form a condensate in presence of color sources, on time scales much shorter than strong interaction time scales

Basic review: Jalilian-Marian, Gelis, Iancu, RV, arXiv:1002.0333
Textbook: Kovchegov, Levin, Cambridge Univ. Press (2012)

RG evolution of many-body correlators



Dumitru, Jalilian-Marian, Lappi, Schenke, RV
PLB706 (2011)219

$$\frac{\partial}{\partial Y} \langle \mathcal{O}[\rho] \rangle_Y = \frac{1}{2} \left\langle \int_{x,y} \frac{\partial}{\partial \rho^a(x)} \chi_{x,y}^{ab} \frac{\partial}{\partial \rho^b(y)} \mathcal{O}[\rho] \right\rangle_Y$$

→ "time"
← "diffusion coefficient"

RG in rapidity (boost) describes Langevin diffusion of the fuzz of "wee" (small x) partons in the functional space of colored fields

Blaziot, Iancu, Weigert (2003)

B-JIMWLK hierarchy:

Balitsky (1996)

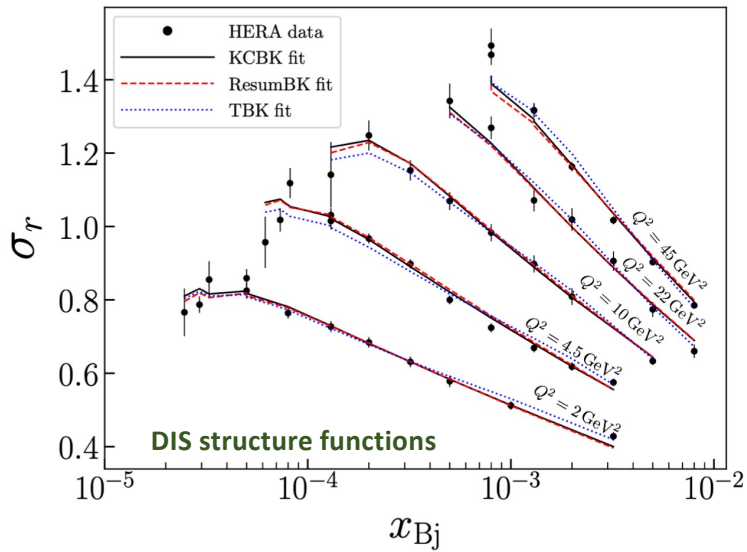
Jalilian-Marian, Kovner, Leonidov, Weigert (1997);

Iancu, Leonidov, McLerran (2001)

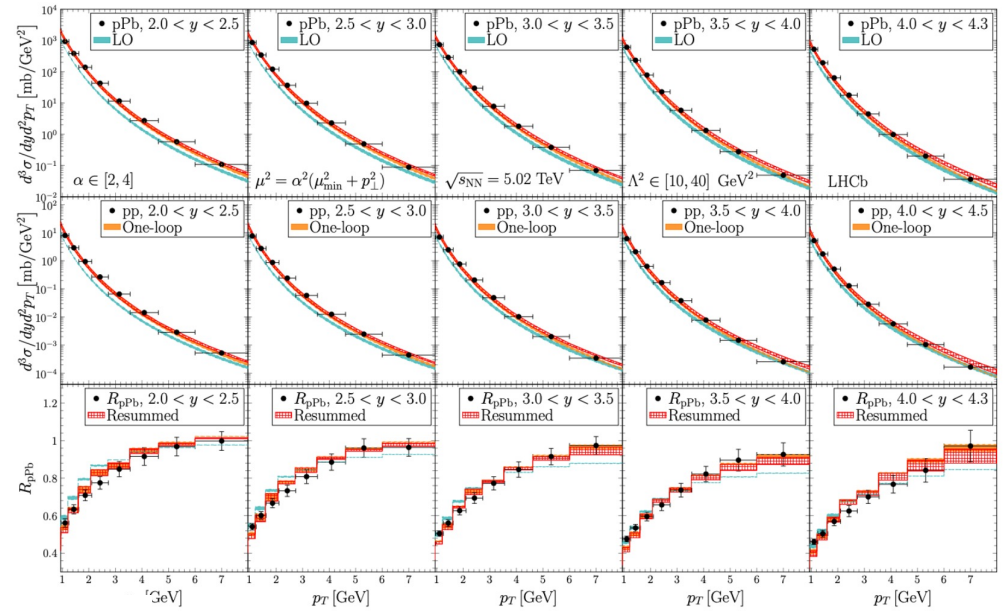
Captures multi-Pomeron dynamics going beyond the BFKL Pomeron

CGC state-of-the art

Beuf,Hanninen,Lappi,Mantysaari, arXiv:2007.01645

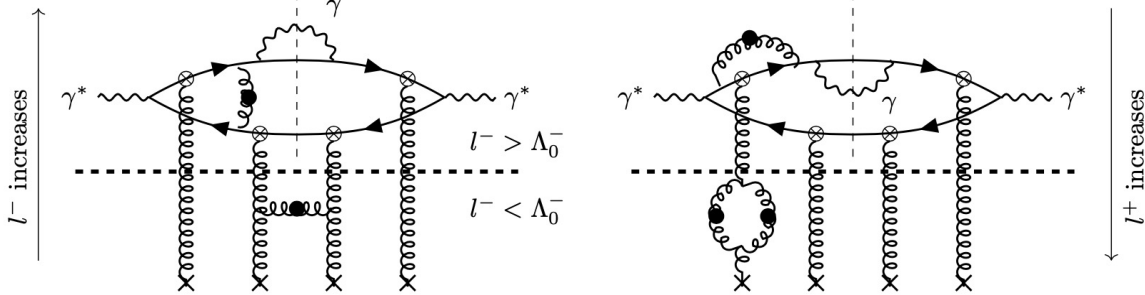


Single inclusive hadron distributions at the LHC



Shi,Wang,Wei,Xiao, arXiv:2112.06975

$e+A \rightarrow \text{photons} + \text{dijets} + X$

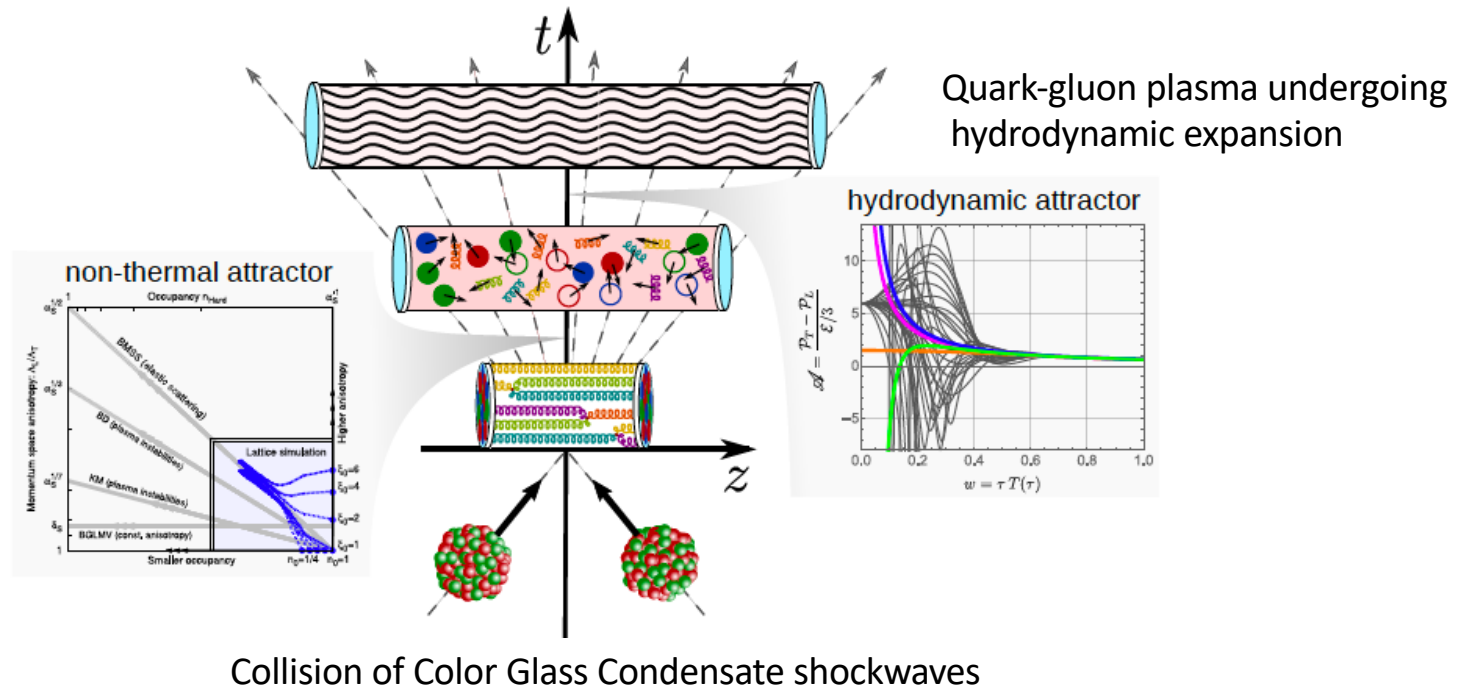


Systematic computations of differential final states sensitive to gluon saturation are becoming available

– precision tests at the Electron-Ion Collider

Roy, RV: 1911.04519, 1911.04530
Caucal,Salazar, RV: 2108.06347

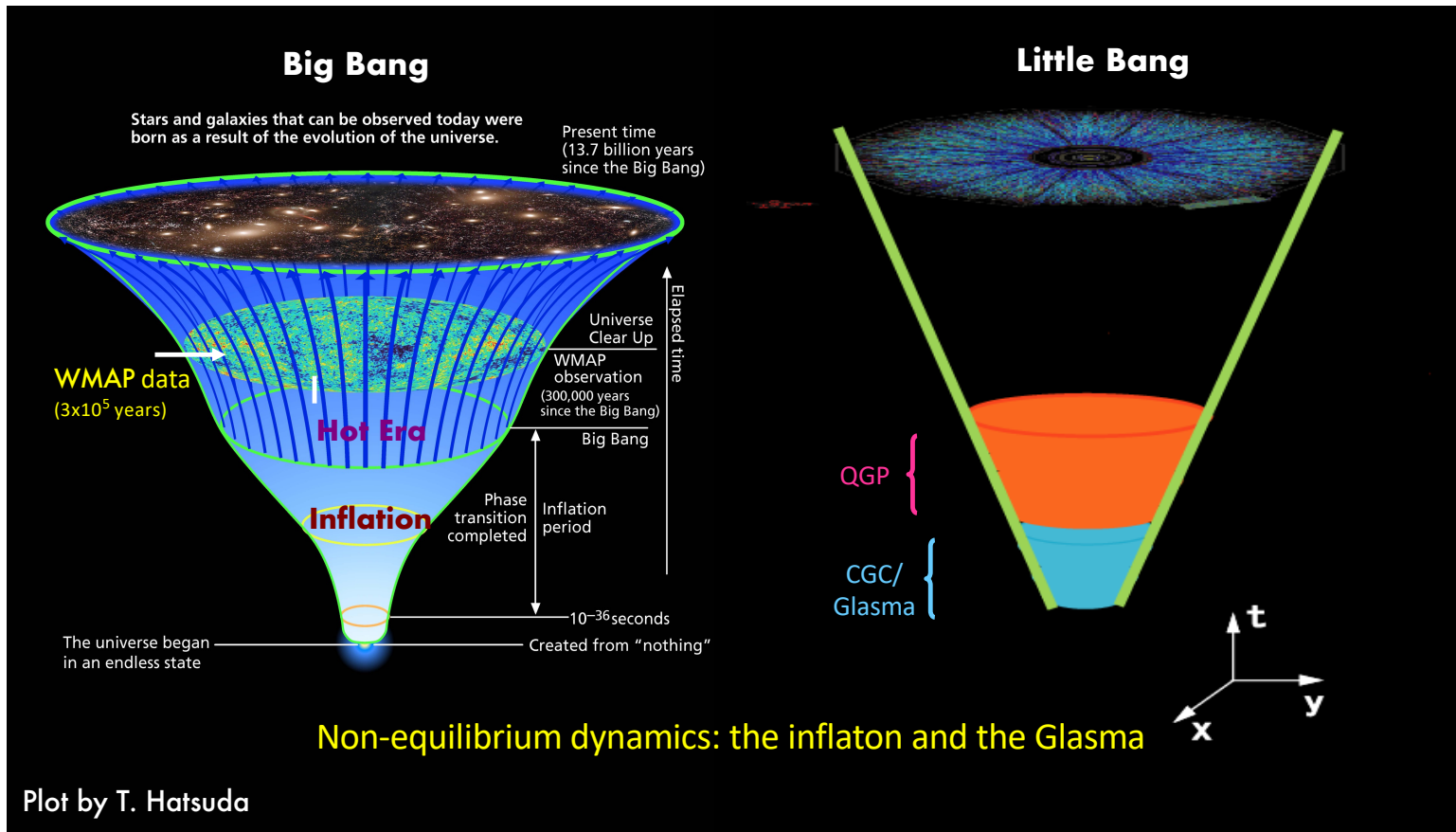
Spacetime evolution of a heavy-ion collision



QCD thermalization: Ab initio approaches and interdisciplinary connections

Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and Raju Venugopalan

Rev. Mod. Phys. **93**, 035003 (arXiv:2005.12299)



Big Bang vs. Little Bang

Decaying Inflaton
with occupation # $1/g^2$



Decaying Glasma
with occupation # $1/g^2$

Explosive amplification of low
momentum small fluctuations
(parametric resonance in preheating)



Explosive amplification of low
momentum small fluctuations
(Weibel instabilities)

Interaction of fluctuations/inflaton
-> thermalization?



Interaction of fluctuations/Glasma
-> thermalization?

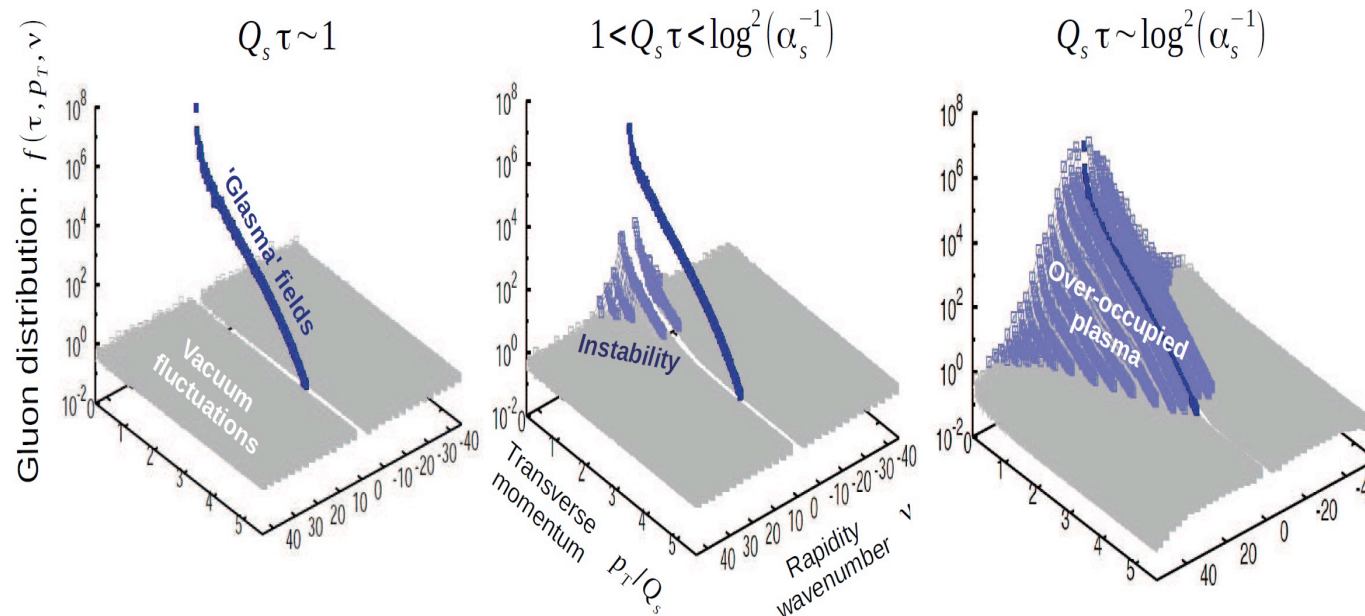
Other common features: turbulence, topological defects,...

Turbulent thermalization,
Micha and Tkachev, arXiv:hep-ph/0403101

From Glasma to Quark Gluon Plasma

Longitudinally expanding Glasma fields are unstable to quantum fluctuations... leading to an explosive “Weibel” instability.

Rapid decoherence and overpopulation of all momentum modes

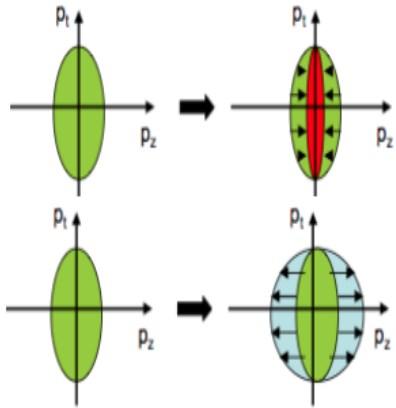


Classical-statistical lattice simulations of 3+1-D gluon fields exploding into the vacuum

Classical-statistical simulations of 3+1-D Yang-Mills

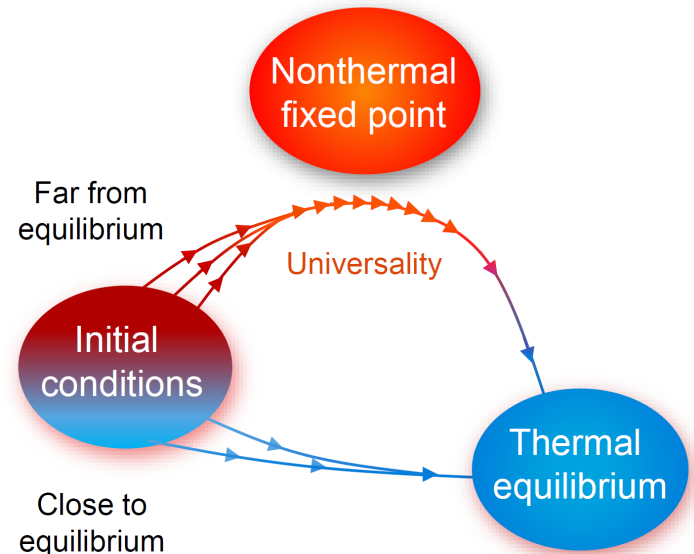
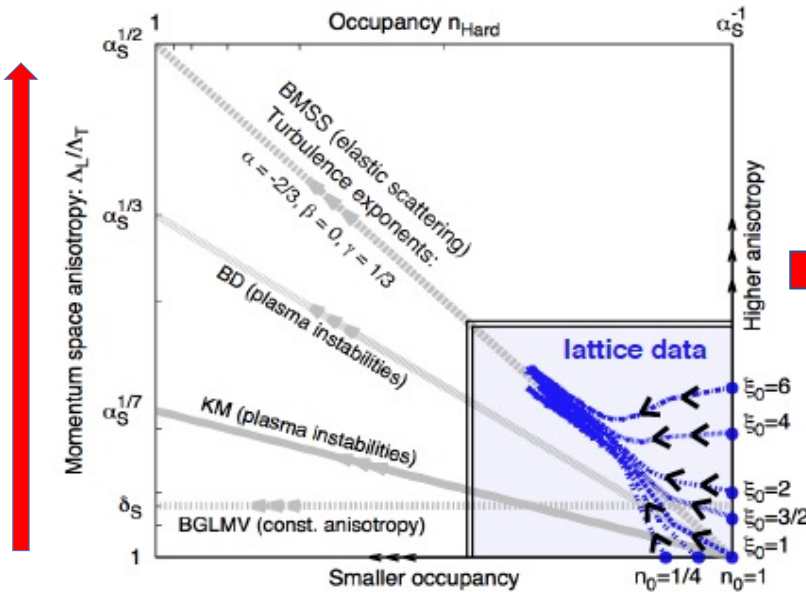
Rapid scrambling of information by quantum fluctuations,
 Competition between dilution due to expansion and isotropization due to scattering

Single particle distributions become self-similar in time characterized by universal exponents – helps identify “right” kinetic theory



$$f(p_{\perp}, p_z, t) = t^{\alpha} f_S(t^{\beta} p_T, t^{\gamma} p_z)$$

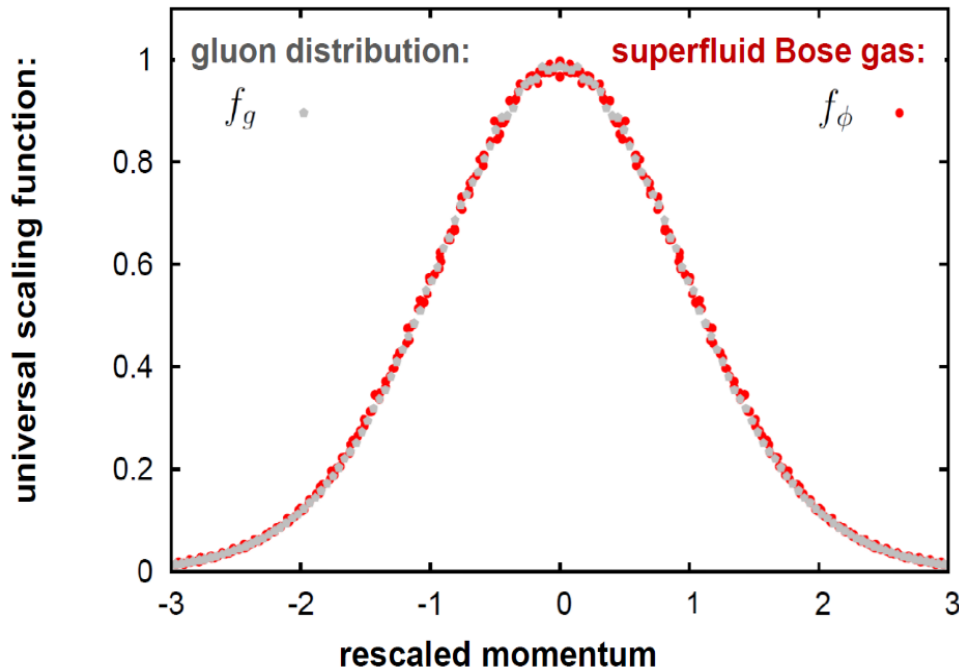
Increasing pressure anisotropy



Berges, Boguslavski, Schlichting, RV (2014)

The Glasma and overoccupied ultracold quantum gases

Simulations of self-interacting scalar fields with identical initial conditions demonstrates remarkable *universality of longitudinally expanding world's hottest and coolest fluids*



In a wide inertial range, scalar & gauge fields have *identical scaling exponents & functions*

$$f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)$$

$$\tau = \sqrt{t^2 - z^2}$$

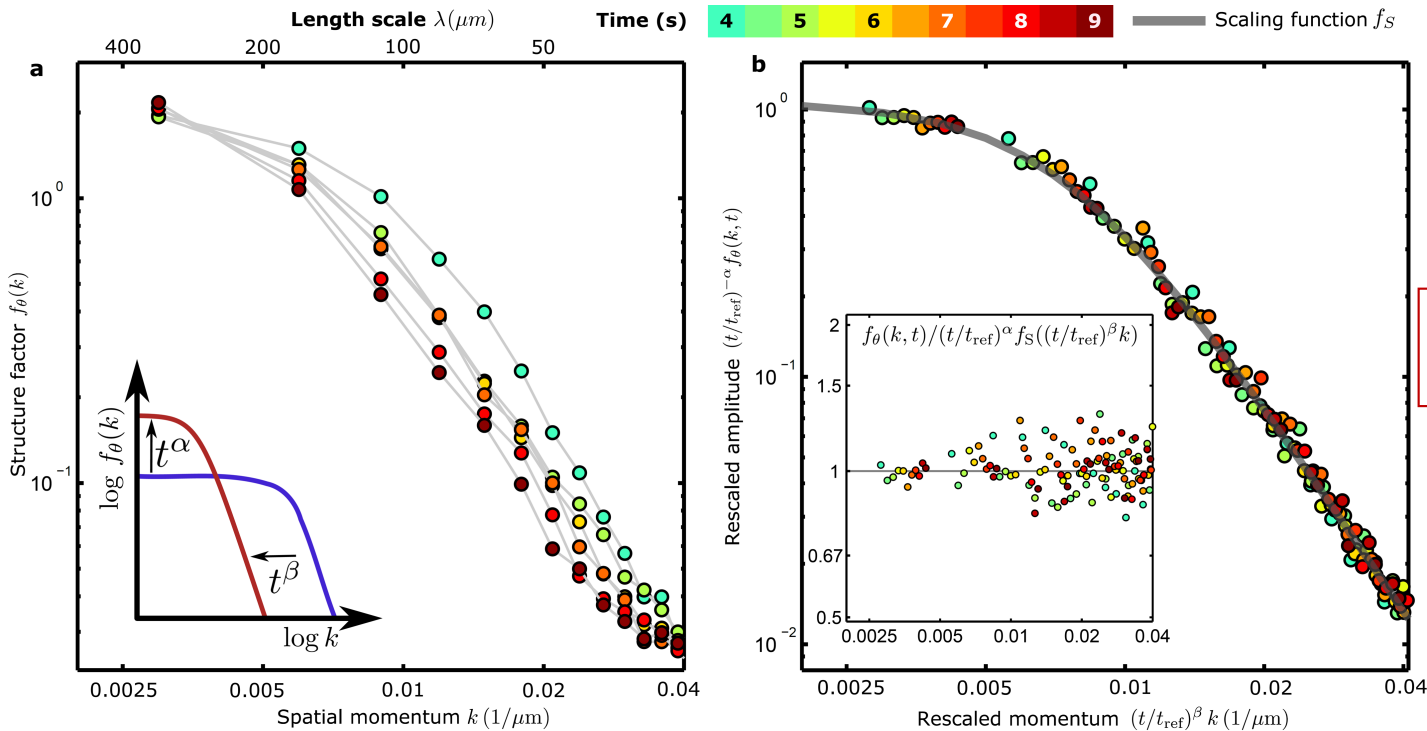
$$\alpha = -\frac{2}{3}, \beta = 0, \gamma = 1/3$$

Berges, Boguslavski, Schlichting, RV, PRL (2015)
Editor's suggestion

The Glasma and over-occupied quantum gases

Similar non-thermal fixed points discovered in cold atom experiments
 - albeit only for static geometry so far

^{87}Rb BEC in a quasi 1D optical trap

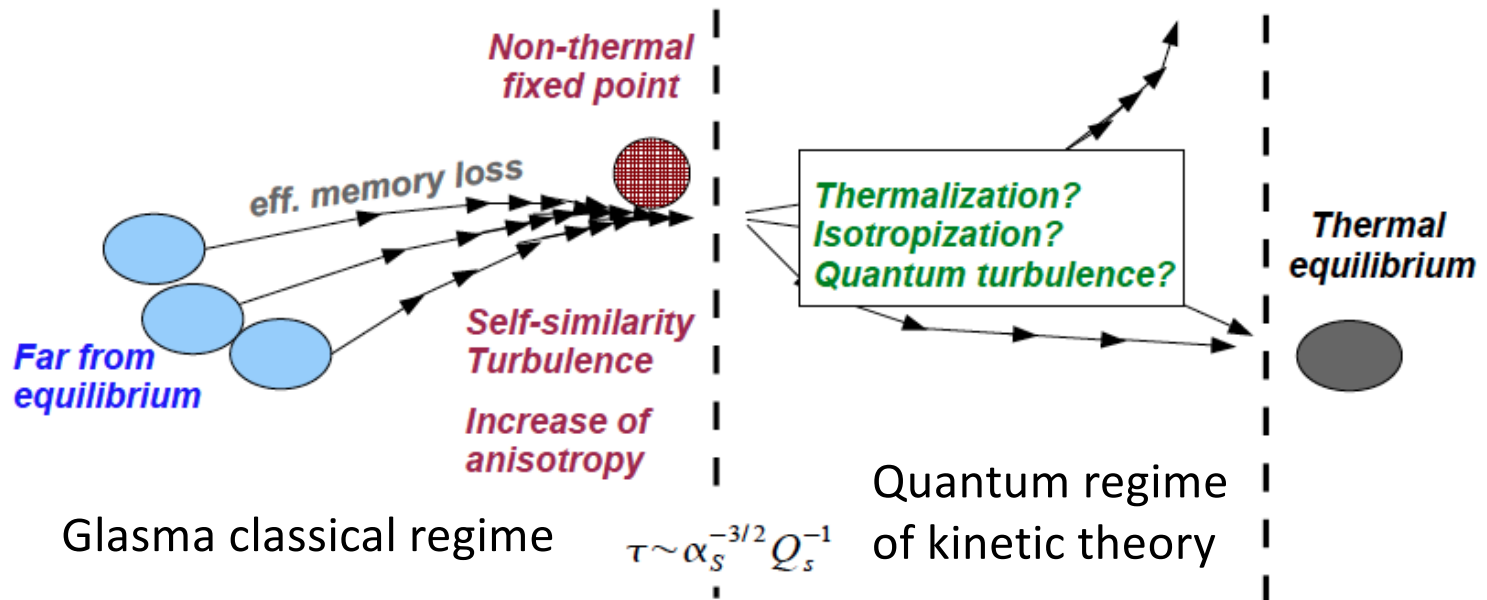


$$f_\theta(k, t) = t^\alpha f_S(t^\beta k)$$

$$\alpha = 0.33 \pm 0.08 \quad \beta = 0.54 \pm 0.06$$

Oberthaler BEC Labs
 Prüfer et al, arXiv:1805.11881, *Nature* (2018)

From nuts to soup: bottom-up thermalization



Thermalized soft bath of gluons for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

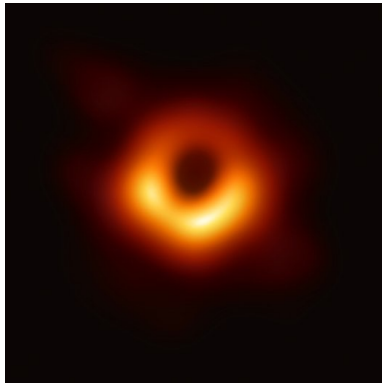
Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

Since $\alpha_S \propto \frac{1}{\log Q_S}$

$$\tau_{\text{therm}} \propto \frac{(\log Q_S)}{Q_S} \rightarrow 0 \text{ as } Q_S \rightarrow \infty$$

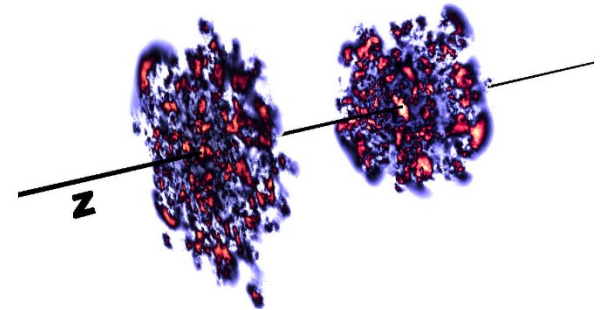
*High energy asymptotics:
Matter thermalizes almost instantaneously
even for the smallest systems...*

Classicalization and unitarization of wee partons in QCD and gravity: The CGC-Black Hole correspondence



$M_{\text{BH}} = (6.5 \pm 0.2_{\text{stat}} \pm 0.7_{\text{sys}}) \times 10^9 M_{\odot}$
 at center of Messier 87
 Event Horizon Telescope image of photon ring

$10^9 \text{ km} \longleftrightarrow 10^{-19} \text{ km}$



Collisions of Color Glass Condensate gluon states in nuclei, arXiv:1206.6805

A QFT picture of Black Holes: overoccupied ($N = M_{\text{BH}}^2/M_p^2 \gg 1, \sim 10^{66}$ for solar mass BH)
 “leaky” bound states of soft gravitons

Dvali, Gomez, arXiv:1203.6575, arXiv:1112.3359
 Dvali,Guidice,Gomez,Kehagis, arXiv:1010.1415

Conjecture: at a maximal occupancy ($\alpha_{\text{grav}} N \sim 1$) the physics of saturated gluons and gravitons is universal

Both systems characterized by a saturation scale ($Q_s = 1/R_s$, Schwarzschild radius) & saturate the Bekenstein entropy bound

Dvali, RV, arXiv:2106.11989

2 → N + 2 amplitudes Trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The S -matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

NPB 364 (1991) 614; 161 cites in INSPIRE

Effective action and all-order gravitational eikonal at planckian energies

AMATI, CIAFALONI, VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2 , where R , b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversely spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

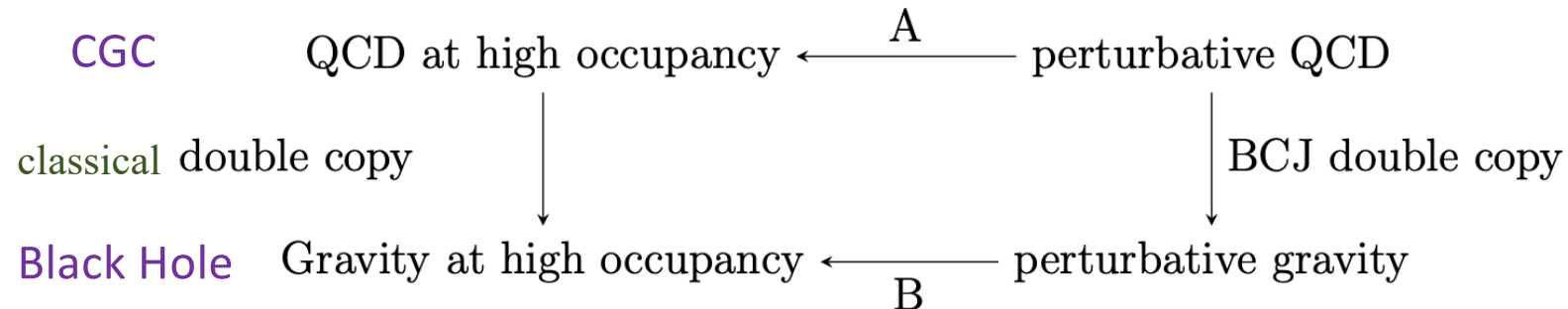
***J.Math.Phys.* 36 (1995) 6377; 3242 cites**

In Acknowledgements:

Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

These works do not explicitly discuss parton saturation which strongly informs our perspective

Gravity amplitudes as double copies of QCD amplitudes



Remarkable development: Gravity amplitudes as “squares” of QCD amplitudes (color-kinematic duality)
Inspiral Hamiltonian of binary gravitational wave sources computed to $O(G^4)$ and all orders in relative velocity

Bern et al., arXiv:2101.07254

BCJ double copy

Bern, Carrasco, Johansson, arXiv: arXiv:1004.0476

Classical double copy

Goldberger, Ridgeway, arXiv:1611.03493

Can one construct a quantitative double copy (from IR to UV) in gravity
in the strong field (weak coupling) regime

- analogous to the RG equations in the CGC (going from UV to IR)
- performing small fluctuations around the shockwave metric

H.Raj, RV, in progress

Thank you for your attention !

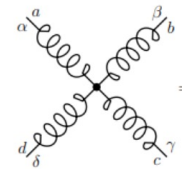
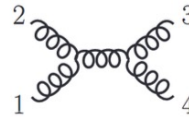
Gauge-Gravity correspondence

Double copy between QCD and Gravity amplitudes

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\text{tree}}(1, 2, 3, 4) = \left(\frac{\kappa}{2}\right)^2 s A_4^{\text{tree}}(1, 2, 3, 4) A_4^{\text{tree}}(1, 2, 4, 3)$$

$$\kappa = 32 \pi^2 G_N$$



Remarkable "BCJ" color-kinematics duality

Bern, Carrasco, Johansson, arXiv:0805.3993

Tree level $gg \rightarrow gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\text{tree}} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right) \quad \text{with the s channel color factor } c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$$

kinematic factor $n_s = -\frac{1}{2} \left\{ \left[(\epsilon_1 \cdot \epsilon_2) p_1^\mu + 2(\epsilon_1 \cdot p_2) \epsilon_2^\mu - (1 \leftrightarrow 2) \right] \left[(\epsilon_3 \cdot \epsilon_4) p_3^\mu + 2(\epsilon_3 \cdot p_4) \epsilon_4^\mu - (3 \leftrightarrow 4) \right] + s \left[(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3) \right] \right\}$

Tree level gravity amplitude obtained from replacing color factors by kinematic factors

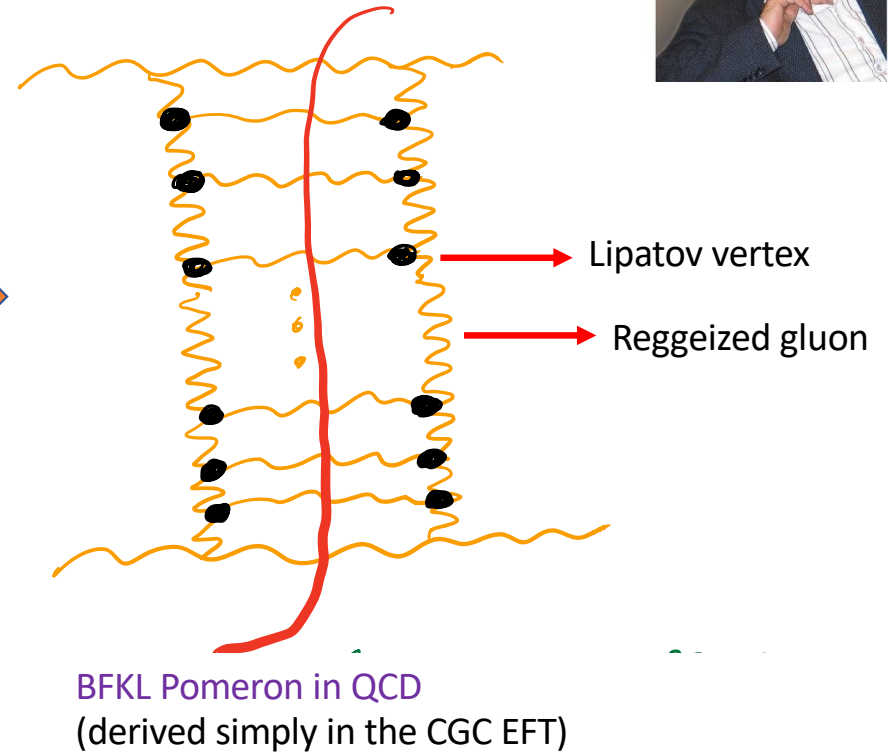
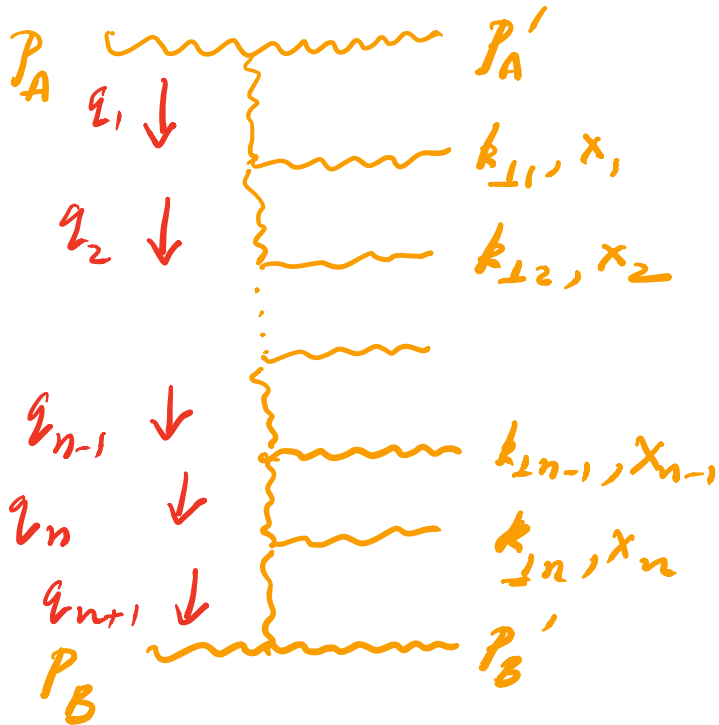
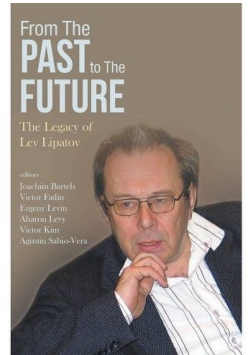
$$i\mathcal{A}_4^{\text{tree}}|_{c_i \rightarrow n_i, g \rightarrow \kappa/2} = i\mathcal{M}_4^{\text{tree}} = \left(\frac{\kappa}{2}\right)^2 \left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u} \right) \quad \text{Significant on-going work on extension to loop amplitudes}$$

Review: Bern et al., arXiv: 1909.01358

Lipatov's EFT for wee partons in QCD and gravity

$2 \rightarrow N + 2$ amplitude in the Regge (high energy) limit of QCD and gravity

Powerful double copy was discovered by Lipatov in high energy (Regge) asymptotics 40 years ago



2 → N + 2 amplitudes in the Regge limit of gravity

Double copy between QCD and Gravity amplitudes in Regge asymptotics

Lipatov, PLB 116B (1982) 411

$$\mathcal{M}_n \simeq -s^2 \mathcal{C}(2; 3) \frac{-1}{|q_4^\perp|^2} \mathcal{V}(q_4; 4; q_5) \cdots \frac{-1}{|q_{n-1}^\perp|^2} \mathcal{V}(q_{n-1}; n-1; q_n) \frac{-1}{|q_n^\perp|^2} \mathcal{C}(1; n)$$

Gravitational effective vertex: $\mathcal{V}(q_i, i, q_{i+1}) = \Gamma_{i, \mu\nu}(q_i, q_{i+1}) \epsilon_i^{\mu\nu}(k_i)$

is the double copy $\Gamma_i^{\mu\nu}(q_i, q_{i+1}) \equiv 2(C_i^\mu C_i^\nu - N_i^\mu N_i^\nu)$

Lipatov Vertex

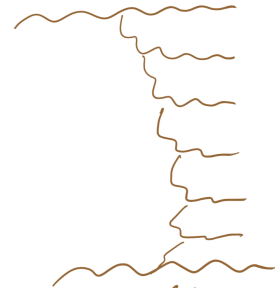
QED Bremsstrahlung vertex

The $\mathcal{C}(2;3)$ and $\mathcal{C}(1;n)$ are gravitational “impact factors” that can also be expressed as a double copy of Lipatov’s gluon-gluon-reggeized gluon vertex

Graviton reggeization was also shown in Lipatov’s 1981 paper with Regge trajectory: $j(q^2) \approx 2 + q^2 \text{Ln}(q^2)$

Following Lipatov, large body of work by Amati, Ciafaloni, Veneziano, and collaborators; also interesting work in AdS/CFT by Strassler and Polchinski

High energy scattering of Gravitons



Same multi-Regge kinematics as previously

Black Holes “demystified”: The Black Hole N Portrait

Dvali, Gomez, arXiv:1203.6575

Dvali, Gomez, arXiv:1112.3359

Dvali,Guidice,Gomez,Kehagis, arXiv:1010.1415

Classical description: Macroscopic objects in GR with geometric, thermodynamic properties

QFT understanding in the BHNP: Black Holes are highly occupied

($N = M_{BH}^2/M_P^2 \gg 1$, $\sim 10^{66}$ for solar mass BH) “leaky” bound states of soft gravitons

$$L_P^2 = \hbar G$$

$$M_P^2 = \frac{\hbar}{G}$$

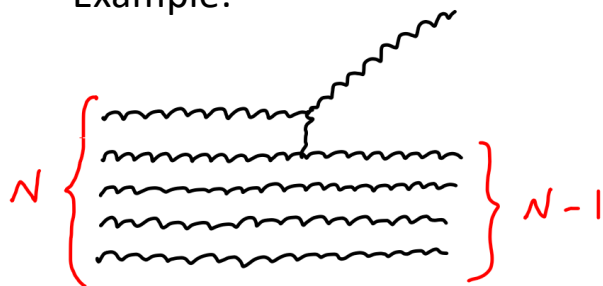
Semi-classical limit: $N \rightarrow \infty$, $L_P \rightarrow 0$, Schwarzschild radius $R_S = 2 G M_{BH} = L_P \sqrt{N}$ = finite, \hbar = finite

Event horizon, BH thermodynamics, no-hair theorem, understood simply in this limit

Hawking Temperature

$$T_H = \frac{\hbar}{R_S} = \frac{\hbar}{L_P \sqrt{N}}$$

Example:



Black Hole evaporation Rate

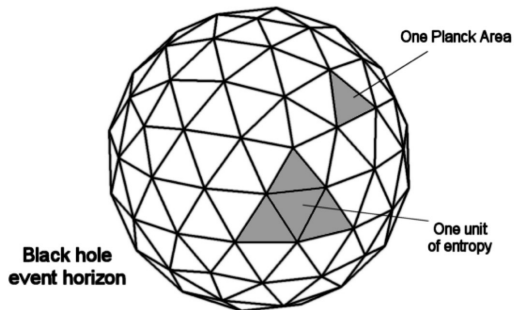
$$\Gamma = 6 n_h (1+N) \sim \frac{1}{8\pi R_S^2} \frac{N^2}{R_S^3} \equiv \frac{1}{R_S}$$

Rate can be equivalently be written as $\frac{dM}{dt} = - \frac{T_H^2}{\hbar^2}$

With Black Hole half life $t_{BH} = \frac{\hbar^2}{T_H^3 G} = N^{3/2} L_P$

Black Hole N Portrait and entropy bound

Bekenstein-Hawking bound



(for a nice discussion, see Bousso, arxiv:1810.01880)

The Bekenstein entropy bound is given by $S \leq 2\pi ER/\hbar$

In the Black Hole N portrait, this is given by $S \leq 2\pi N Q_S R_S$ which is saturated when $N = \frac{1}{\alpha_{gr}} \rightarrow S_{Bek} = \frac{1}{\alpha_{gr}}$ consistent with unitarity

Here $E = N Q_S$ is the energy in a critically packed volume $= R_S^3$ of quanta saturating unitarity (maximal information) and $Q_S = 1/R_S$

$$S_{Bek} = \frac{1}{\alpha_{gr}} = \frac{R_S^2}{L_P^2} = \frac{Area}{4G} = S_{BH}$$

The entropy can also be expressed in terms of a Goldstone decay constant corresponding to spontaneous breaking of Poincare invariance by the graviton condensate:

Decay constant of Goldstone field ϕ is $f = R \partial_x \phi = \sqrt{N}/R$; In gravity, this is nothing but M_P

Hence one can equivalently express the entropy as $S_{BH} = Area \times f^2$

Bekenstein entropy of the CGC

Since both BHNP and the CGC are macrostate “lumps” that saturate unitarity at $\alpha N = 1$, we conjecture that

$$S_{\text{CGC}} = \frac{1}{\alpha_S} = N = f^2 * \text{Area}$$

If $f = \sqrt{N} Q_S$

Physical interpretation of f in this picture is the scale denoting the onset of shadowing
Above this scale one has unscreened partons...

Since at small x , $\alpha_S Y \sim 1$, $\frac{dS_{\text{CGC}}}{dY} = \text{constant}$

Consistent with the estimate of
Kharzeev and Levin

[arXiv:1702.03489](#)

The Goldstone scale that we have postulated is identical to that found by Duan, Skokov and Kovner
by explicit computation of the CGC reduced density matrix...!

[arXiv:2111.06475](#)

Their massless quasi-particles (with creation (annihilation) operators c^\dagger (c)) are our Goldstones and are related to those of the perturbative vacuum by a simple Bogoliubov transformation

Conjecture: CGC/BHNP correspondence

Dvali, RV, arXiv:2106.11989, PRD (2022)

We begin to glimpse the elements of the CGC/BHNP correspondence for a region of space $R_S = Q_S^{-1}$ of the CGC mapped on to a Black Hole of analogous radius determined by the saturation of the graviton condensate:

The scale at which this occurs is a weak coupling scale in both theories: $\begin{array}{l} \text{for BHNP, } Q_S^{-1} \gg M_P^{-1} \\ \text{for CGC, } Q_S \gg \Lambda_{QCD} \end{array}$

This scale is determined by a maximal occupancy of $N = \frac{1}{\alpha}$ where (perturbative) unitarization occurs

The direct correspondence between the two theories only occurs at $Q=Q_S$ where the physics is universal- It is independent of the details of the dynamics of the wee partons

The existence of a double copy between the two theories is however suggestive that it can be extended to a “classical” double copy between gluon and graviton classical shockwaves

Review on classical double copy: C. White, arXiv:1708.07056

Open questions II. S-matrix symmetries

Can one formulate the Goldstone description of the CGC in the language of asymptotic (“BMS”) spacetime symmetries ?

Recent developments in understanding soft theorems in terms of broken global symmetries on a 2-D celestial sphere at null infinity

Strominger, arXiv:1703.05448

Soft photons, gravitons, gluons (?) can be interpreted as the corresponding Goldstone modes

In Yang-Mills, as in gravity, there is a color memory effect analogous to gravitational memory

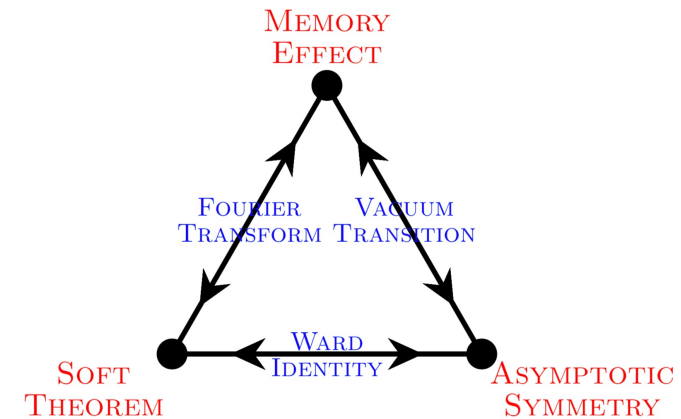
Pate, Raclariu, Strominger, PRL (2017)

This is the Q_5 kick experienced by a probe across the shock wave

Ball, Pate, Raclariu, Strominger, RV, Annals of Physics (407 2019) 15

This suggests that the soft gluons in the shockwave are Goldstones of the broken global BMS symmetry with associated BMS charges that satisfy a 2-D Kac-Moody algebra

He, Mitra, Strominger (2015)



$$A_i = 0 \quad \Big|_{x^- = 0} \quad A_i = -\frac{1}{ig} U \partial_i U^\dagger$$