Exam 2 Overview

9/10) Simple Circuits and Kirchhoff's Laws

11) RC Circuits

12) Magnetic Force

13) Forces and Magnetic Dipoles

14) Biot-Savart Law

15) Ampere’s Law

16) Motional EMF
Current and KCL

Current (I) is the flow of charge per second

Units: Amperes (A) - Coulombs/second (C/s)

Kirchhoff's Current Law - KCL

- The amount of current going in is equal to the amount of current coming out

\[ I_{\text{in}} = I_{\text{out}} \]
Voltage and KVL

Voltage (V) is the amount of energy per unit charge

- Units: Volts (V) = Joules/Coulomb (J/C)

Kirchhoff's Voltage Law - KVL

- The total voltage in a loop is the sum of all the voltage drops and rises
  - Voltage drop - “+” to “-”
  - Voltage rise - “-” to “+”

You can solve all the circuit problems you will see in this course by applying KCL and KVL
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<tr>
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<th>Diagram</th>
<th>Formulas</th>
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<td>Series Resistors</td>
<td><img src="image" alt="Series Resistors Diagram" /></td>
<td>Equivalent resistance = $R_1 + R_2$</td>
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<tr>
<td>Voltage Divider</td>
<td><img src="image" alt="Voltage Divider Diagram" /></td>
<td>$V_1 = \frac{R_1}{R_1 + R_2} V_s$ [1] $V_2 = \frac{R_2}{R_1 + R_2} V_s$</td>
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<tr>
<td>Parallel Resistors</td>
<td><img src="image" alt="Parallel Resistors Diagram" /></td>
<td>Equivalent resistance = $R_1 \parallel R_2 = \frac{R_1R_2}{R_1 + R_2}$</td>
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<tr>
<td>Current Divider</td>
<td><img src="image" alt="Current Divider Diagram" /></td>
<td>$I_1 = \frac{R_2}{R_1 + R_2} I_s$ [2] $I_2 = \frac{R_1}{R_1 + R_2} I_s$</td>
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**Power**

Power is the amount of energy per second being delivered/absorbed

- Units: Watts \( (W) = \text{Joules/second} \ (J / s) \) = amount of energy per second
- \( P_{\text{resistor}} = IV = \frac{V^2}{R} = I^2R \) (These last 2 equations are for resistors ONLY)

The sign ("+" or "-") is very important when it comes to power (Not on your test)

- Negative power means that circuit element is delivering energy to the circuit (sources, capacitors, inductors)
- Positive power means that the circuit element is absorbing energy from the circuit (resistors, capacitors, inductors)
**RC Circuits**

- tau is the time constant which affects the rate of growth/decay

**Charging and Discharging Equations**

\[
Q(t) = Q(\infty) \left(1 - e^{-t/\tau}\right)
\]

\[
Q(t) = Q(0) e^{-t/\tau}
\]
RC Circuits cont.

Charging

\( t = 0 \rightarrow \text{capacitor acts like a wire (short circuit)} \)
- \( V = 0 \text{ V}, \text{ but there is a current} \)

\( t = \infty \rightarrow \text{no current thru capacitor (open circuit)} \)
- \( I = 0 \text{ A}, \text{ but there is a voltage} \)

Discharging

\( t = 0 \rightarrow \text{capacitor acts like a battery} \ (C = Q/V \text{ where } V \text{ is found when charging up}) \)

\( t = \infty \rightarrow \text{capacitor acts like a wire (all the charge is dissipated aka gone)} \)
Magnetic Force on Charges

- \( F_m = qv \times B \)
  - we know that \( F = ma \)
  - and for these problems \( a = a_c = \frac{v^2}{r} \)
  - If we substitute in for \( F \) we get \( mv^2/r = qv \times B \)
  - We use this to solve for any missing variable

Right-Hand Rule (1st RHR)

- Point fingers or hand along the direction of \( v \)
- Curl fingers in the direction of \( B \)
- Thumb points in the direction of the force*

*This works for positive charges, flip your thumb 180° for a negative charge
Forces on Current Wires and Loops

\[ F_{\text{wire}} = I L \times B \] (1st RHR)

- The force around an entire loop of current is always zero (assuming B is constant) but be careful because it may not be zero at a segment of the loop.

Currents traveling in the same direction - attract

Currents traveling in opposite directions - repel
Torques and Energy on Current Loops

Remember $\sin(\theta)$ goes with cross products and $\cos(\theta)$ goes with dot products.

**Magnetic Dipole:** $\mu = n \ast I \ast A$ (2nd RHR)

- $n = \#$ of turns
- $I = \text{current through loop}$
- $A = \text{area of the loop}$

**Torque:** $\tau = \mu \times B = |\mu||B|\sin(\theta)$ (1st RHR)

**Potential Energy:** $U = \mu \cdot B = |\mu||B|\cos(\theta)$

**Work:** $W = -U$
Torques and Energy Cont.

Remember $\sin(\theta)$ goes with cross products and $\cos(\theta)$ goes with dot products

Torque: $\tau = \mu \times B = |\mu||B|\sin(\theta)$

Max when $\sin(\theta) = 1 \rightarrow \theta = 90^\circ \rightarrow$ when $\mu$ and $B$ are perpendicular

Potential Energy: $U = \mu \cdot B = |\mu||B|\cos(\theta)$

Max when $\cos(\theta) = 1 \rightarrow \theta = 0^\circ \rightarrow$ when $\mu$ and $B$ are parallel in the same direction

Min when $\cos(\theta) = -1 \rightarrow \theta = 180^\circ \rightarrow$ $\mu$ and $B$ are parallel in opposite directions

Work: $W = -U$
Biot-Savart Law

By using the Biot-Savart Law, we were able to derive the equation for the magnetic field produced by a current carrying wire (in orange)

Direction of B is always tangent to the circle (3rd RHR)

(Not used often, painful to integrate)

\[ B_z = \frac{\mu_0 IR^2}{2(z^2 + R^2)^{3/2}} \]
Right-Hand Rules (3 Total)

1st RHR - Cross Products

- Place your fingers along the first vector, curl your fingers in the direction of the second vector, your thumb gives you the direction of the force, torque, etc.

2nd RHR - Magnetic Dipole

- Curl your fingers along the direction in which the current is flowing, your thumb gives you the direction of the magnetic dipole

3rd RHR - Magnetic Fields

- Place your thumb along the direction of current, curl your fingers to give you the direction of the “circular path”, B is tangent to the “circular path”
**Ampere’s Law**

Think of it as the 2D version of Gauss’s Law, but for magnetic fields now.

By convention for line integrals, **traversing a closed loop counter-clockwise (CCW)** is positive and traversing it **clockwise (CW)** is negative.

Current density: \( J = \frac{I}{A} \)

Units: \((\text{A/m}^2)\)

- \(I\) - Current
- \(A\) - Area
Ampere’s Law Cont.

Magnetic field equations inside and outside a current-carrying wire

Memorize inside equation (#1), it will save you time from deriving it on the exam

\[ B_{\text{max}} = \frac{\mu_0 I}{2\pi a} \]

\[ B = \frac{\mu_0 I}{2\pi a^2 r} \quad \text{(for } r < a) \]

\[ B = \frac{\mu_0 I}{2\pi r} \quad \text{(for } r > a) \]
Ampere’s Law Cont.

Magnetic field equation for an infinite sheet of current

\[ B = \frac{1}{2} \mu_0 n I \]
Motional EMF

Potential difference = Voltage = Electromagnetic Force (EMF)

\[ \mathcal{E} = \nu BL \]

- \( \mathcal{E} \) - voltage
- \( \nu \) - velocity
- \( B \) - magnetic field
- \( L \) - length of the loop

To find direction of current: 1st RHR

- RHR wrt the magnet: \( \mathbf{F} = q\mathbf{v} \times \mathbf{B} \)
- Your thumb gives you the direction of the current
Faraday’s Law

Main Idea: A changing magnetic flux creates an electric field

The induced EMF (voltage) always opposes the change in magnetic flux

The induced EMF gets multiplied by N turns if the loop has N turns in it

3 ways to change the magnetic flux

- Making the area of the loop smaller or larger
- Moving the loop around in a constant magnetic field
- Having a time-varying magnetic field (i.e. B is not constant with time)
Faraday’s Law cont.

Steps for solving Faraday’s Law problems (2 types)

**Type 1:** (Usually given B as a function of time or on a graph)

1) Find the magnetic flux \((B \cdot A)\)
2) Solve for the induced EMF by take the negative derivative of the magnetic flux with respect to time \((-\frac{d}{dt} \text{ of the magnetic flux})\)

**Type 2:** (Usually a picture with one or “N” conducting loops)

1) **Determine the change in magnetic flux, \(B_{\text{induced}}\) will always point in the opposite direction** to the change in magnetic flux
2) **Use the 3rd RHR:** Point your fingers in the direction of \(B_{\text{induced}}\) and curl your fingers to give you the direction of the induced current
Kahoot :)  
Exam 2 Kahoot