Keep in mind that this presentation was created by CARE tutors, and while it is thorough, it is not comprehensive.
QR Code to the Queue

The queue contains the worksheet and the solution to this review session
Fubini’s Theorem

- If \( f(x,y) \) is continuous on the rectangle

\[
R = \{(x, y) | a \leq x \leq b, c \leq y \leq d\}
\]

\[
\iint f(x, y) \, dA = \int_a^b \int_c^d f(x, y) \, dy \, dx = \int_c^d \int_a^b f(x, y) \, dx \, dy
\]
Double Integral Over a General Region

- Integrate $dy$ from $y=x$ to $y=1$
- Then integrate $dx$

- Integrate $dx$ from $x=0$ to $x=y$
- Then integrate $dy$
Center of Mass

- The x, y coordinates of the center of mass for an object that has a density function $\rho(x,y)$

$$\bar{x} = \frac{1}{m} \iint x \cdot \rho(x, y) dA \quad \bar{y} = \frac{1}{m} \iint y \cdot \rho(x, y) dA$$

, where mass is calculated as $m = \iint \rho(x, y) dA$
Let $E$ be the solid contained under the plane $2x + 3y + z = 6$ in the first octant. Compute the following:

$$\iiint_{E} 2x \, dV$$
Let $E$ be the solid contained under the plane $2x + 3y + z = 6$ in the first octant. Compute the following:

\[
\iiint_E 2x \, dV = \int_0^3 \int_0^{2-2x/3} \int_0^{6-2x-3y} 2x \, dz \, dy \, dx = \int_0^3 \int_0^{2-2x/3} 2x(6-2x-3y) \, dy \, dx
\]

\[
= \int_0^3 12x \left(2 - \frac{2x}{3}\right) - 4x^2 \left(2 - \frac{2x}{3}\right) - 3x \left(2 - \frac{2x}{3}\right)^2 \, dx = 9
\]
Example Question #1

- Match the integrals to their corresponding solid regions:

(A) \[ \int_0^1 \int_y^1 \int_0^{2-x^2-y^2} f(x, y, z) \, dz \, dx \, dy \]

(B) \[ \int_0^1 \int_0^{1-x} \int_0^{1-x^2-y^2} g(x, y, z) \, dz \, dy \, dx \]
Example Solution #1

\[(A) \int_0^1 \int_y^1 \int_0^{2-x^2-y^2} f(x, y, z) \, dz \, dx \, dy\]

\[(B) \int_0^1 \int_0^{1-x} \int_0^{1-x^2-y^2} g(x, y, z) \, dz \, dy \, dx\]
Polar Coordinates

\[ x = r \cos \theta \]
\[ y = r \sin \theta \]
\[ r^2 = x^2 + y^2 \]
\[ \theta = \arctan \left( \frac{y}{x} \right) \]
\[ \text{d}A = r \, \text{d}r \, \text{d}\theta \]

Cylindrical Coordinates

- Cylindrical coordinate is just an extension of polar coordinate to three dimension

\[ x = r \cos \theta \]
\[ y = r \sin \theta \]
\[ z = z \]
\[ r^2 = x^2 + y^2 \]
\[ \theta = \arctan \left( \frac{y}{x} \right) \]
\[ dV = rdzdrd\theta \]
Spherical Coordinates

\[ x = \rho \sin \varphi \cos \theta \]
\[ y = \rho \sin \varphi \sin \theta \]
\[ z = \rho \cos \varphi \]
\[ \rho^2 = x^2 + y^2 + z^2 \]
\[ dV = \rho^2 \sin \varphi \, d\rho \, d\theta \, d\varphi \]
Surface Area

- The area of the surface $A(S)$ with equation $z=f(x,y)$ can be calculated as:

$$A(S) = \iint_D \sqrt{1 + \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2} \, dA$$
Change of Variables Using Jacobian Matrix

- If there is a transformation such that \( x = g(u, v) \) and \( y = h(u, v) \), then:

\[
\iint_R f(x, y) \, dA = \iint_S f[g(u, v), h(u, v)] \cdot \left| \frac{\partial (x, y)}{\partial (u, v)} \right| \, dA
\]

, where the Jacobian Matrix is calculated as

\[
\frac{\partial (x, y)}{\partial (u, v)} = \begin{vmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{vmatrix} = \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u}
\]
Example Question #2

- Set up the integral to calculate the area of R when the transformation $T(u,v) = (u^2+v, v)$. 
Example Solution #2

- Set up the integral to calculate the area of $R$ when the transformation $T(u,v) = (u^2+v, v)$.

$$0 \leq v \leq 1 - \frac{u}{2} \quad 0 \leq u \leq 2$$

**Jacobian:**

$$\det \begin{bmatrix} 2u & 1 \\ 0 & 1 \end{bmatrix} = 2u$$

**Integral:**

$$\int_{0}^{2} \int_{0}^{1-u/2} 2u \, dv \, du$$